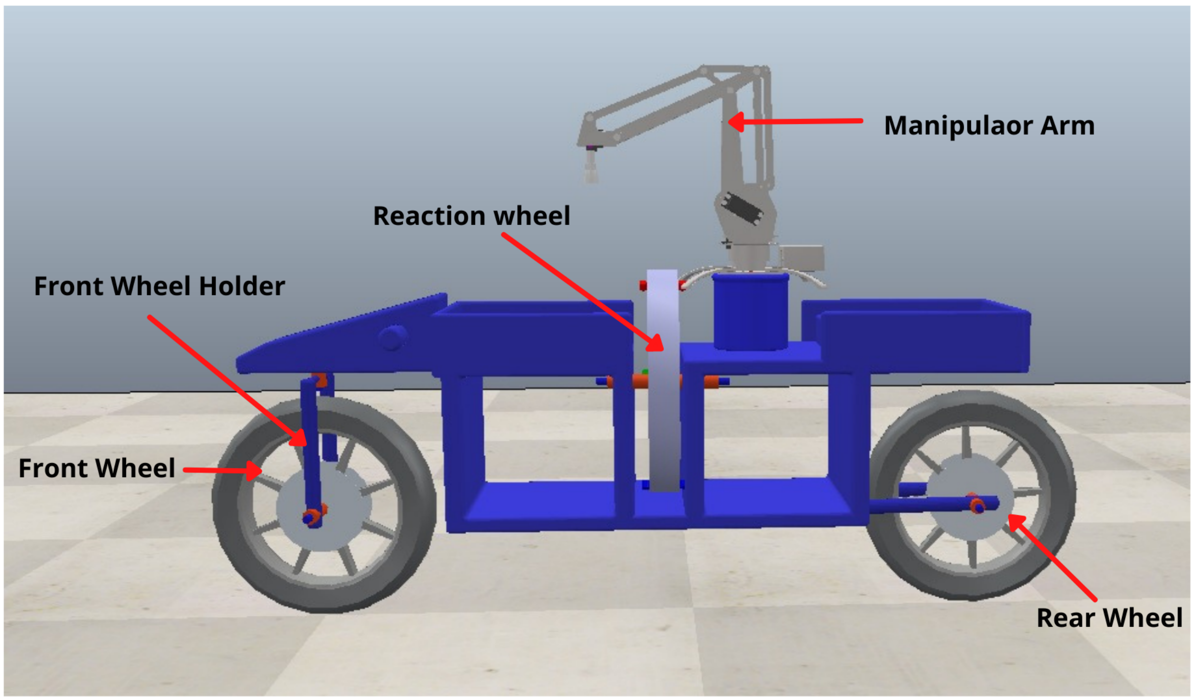
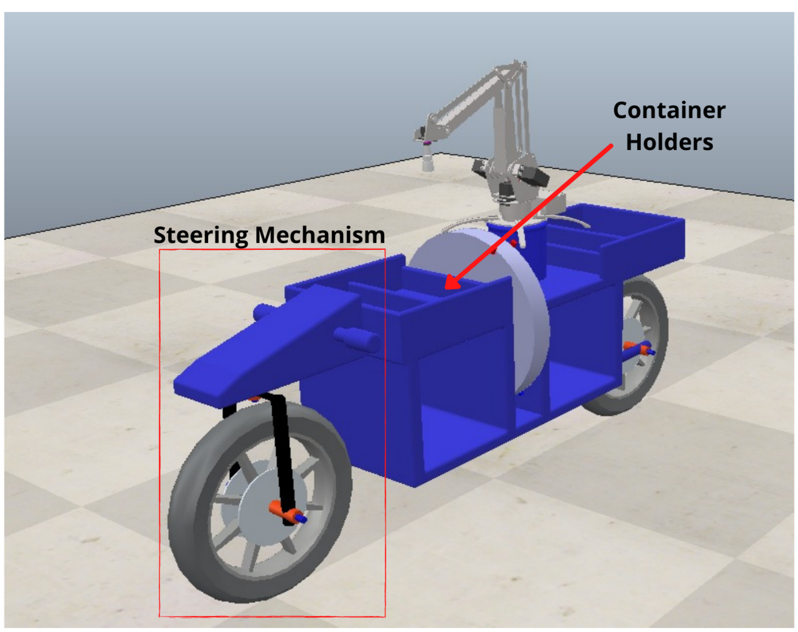
<u>Dairy-Bike Task 2.2</u>

Team ID : 2338

Q1. Components





The bike has the following components:

1. Base:

We selected such a base so that we have enough space on it to fit the arm in between the two sides where we store the dairy product.

Mass of the base = 10.44 kg

Moment of inertia of the base = 0.531 kg-m^2

2. Front and rear wheels:

Mass of each wheel = 1.238 kg

Moment of inertia of each wheel = 0.006389 kg-m^2

3. Front wheel holder:

It is used to hold the front wheel while allowing it to move in the direction the bike wants to move.

Mass of the front wheel holder = 0.1335 kg

Moment of inertia of the front wheel holder = 0.0001357 kg-m^2

4. Reaction wheel:

It is placed just above the center of mass of the system to ensure that the bike will remain balanced perfectly.

Mass of the reaction wheel = 4.952 kg

Moment of inertia of the reaction wheel = 0.27855 kg-m^2

5. Arm:

We have selected the 'uarm' model because it has a good reach and is just the right size for our dairy bike.

Mass of the arm = 2.0 kg

Moment of inertia of the arm = 0.009738 kg-m^2

Q2. Kinetic and Potential Energies

Parameter List:

1] <u>For Bike :</u>

m_1 = Bike Mass(without Reaction Wheel) = 15.048 kgs
O_1 = COM of Bike(without Reaction Wheel) = (0,-0.0381,0.231) m
I_1 = MOI of Bike about O1 = 0.8045 kg.m^2
L_1 = Distance from Origin to COM = 0.231 m
θ = Angle between Bike and Vertical direction

2] <u>For Reaction Wheel :</u>

m_2 = Mass of Reaction Wheel = 4.952 kgs
O_2 = COM of Reaction Wheel = (0,-0.0381,0.317) m
I_2 = MOI of Reaction Wheel = 0.2785 kg.m^2
L_2 = Distance from Origin to COM = 0.317 m
Φ = Angle of Rotation of Reaction Wheel

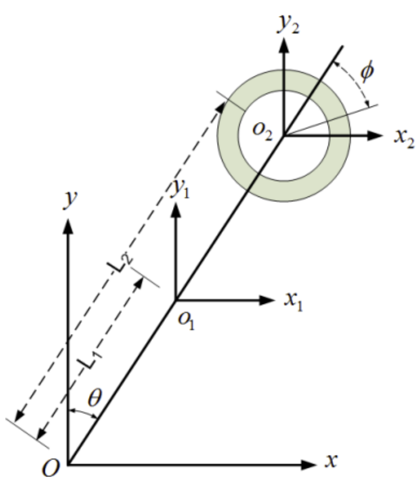
The equations of kinetic and potential energy are as follows:

$$KE = \frac{1}{2}(m_1 L_1^2 + m_2 L_2^2 + I_1 + I_2)\dot{\theta}^2 + I_2\dot{\theta}\dot{\phi} + \frac{1}{2}I_2\dot{\phi}^2$$

$$PE = (m_1 L_1 + m_2 L_2) g \cos\theta$$

The chosen states of the system are as follows:

- Angle between bike and vertical direction - **θ**
- Angular velocity of bike - **θ'**
- Angle of rotation of reaction wheel - ***ϕ***
- Angular velocity of reaction wheel - ***ϕ'***



Q3. State-Space equations

The Lagrangian function is as follows:

$$
\begin{aligned}
L =& KE - PE \\
=& \frac{1}{2}(m_1 L_1^2 + m_2 L_2^2 + I_1 + I_2)\dot{\theta}^2 + I_2\dot{\theta}\dot{\phi} + \frac{1}{2} I_2 \dot{\phi}^2 - (m_1 L_1 + m_2 L_2) g \cos\theta .
\end{aligned}
$$

The dynamical equations using Euler-Lagrange mechanics are:

$$\theta' = y(2)$$

$$\theta'' = (b \sin(\theta) - u)/a$$

$$\Phi' = y(4)$$

$$\Phi'' = (u / I2) - dy(2)$$

Q4. Equilibrium Points

To find the equilibrium points we have to make the above equations 0. Doing so gives the following equilibrium points:

1. **$\theta = \pi$:**
   Vertically Downward Position.
   Stable Equilibrium.
   Technically not possible in Bike, so it corresponds to bike fallen flat on the ground.
2. **$\theta = 0$:**
   Vertically Upright Position.
   Unstable Equilibrium.
   Has to be maintained using External motor torque.

Q5. A and B Matrices

Let,

$$a = m_1*(L_1)^2 + m_2*(L_2)^2 + I_1$$
$$b = (m_1*L_1 + m_2*L_2)*g$$

The state matrix A and input matrix B for the robot designed are as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ b/a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -b/a & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -1/a \\ 0 \\ (a+I2)/(a*I2) \end{bmatrix}$$

Substituting the values of ‘a’ and ‘b’ in these matrices, we get:

$$A = \begin{bmatrix} 0.00000 & 1.00000 & 0.00000 & 0.00000 \\ -23.49038 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 1.00000 \\ 23.49038 & 0.00000 & 0.00000 & 0.00000 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.00000 \\ -0.47504 \\ 0.00000 \\ 4.06570 \end{bmatrix}$$

(Jacobian is same as A since sinθ ≅ θ as θ is small)

Q6. Controllability

The controllability matrix [C] is:

$$C = \begin{bmatrix} 0.00000 & -0.47504 & 0.00000 & 11.15881 \\ -0.47504 & 0.00000 & 11.15881 & 0.00000 \\ 0.00000 & 4.06570 & 0.00000 & -11.15881 \\ 4.06570 & 0.00000 & -11.15881 & 0.00000 \end{bmatrix}$$

Here the rank of the matrix is 4. Also, the number of state variables is 4.
Since, rank = number of state variables, therefore the system is fully controllable.

Q7. Discrete Time System

A Continuous Time System the sampling of system states is done continuously w.r.t time. In a Discrete Time System, the sampling is done at periodic intervals of time. This makes Discrete systems more flexible and reliable.
The *c2d* function in Octave allows us to convert our LTI continuous system to discrete time.

```
#Converts to discrete time system with sampling rate of 0.1s
sys = c2d(ss(A,B),0.1)
```

Q8. Gain K

```
Q = 12*eye(4);
R = 16.6*[1];
K = lqr(A,B,Q,R)
```

The value of matrix ‘K’ is:

$$K = \begin{bmatrix} -1.225643 & -0.051416 & 0.850230 & 1.099185 \end{bmatrix}$$

The values of the matrix ‘y’ come out to be:

$$y = \begin{bmatrix} 3.13861 & 0.00517 & 6.28463 & -0.01398 \end{bmatrix}$$

The ‘ysetpoint’ matrix that we had set was:

$$ysetpoint = \begin{bmatrix} \Pi & 0.00000 & 2\Pi & 0.00000 \end{bmatrix}$$

By comparing the values in the matrix ‘K’ and the matrix ‘ysetpoint’, we can conclude that the system is relatively quite stable.

Therefore the matrix $\overline{A}$ = A - BK has a value of:

$$A - BK = \begin{bmatrix} 0.00000 & 1.00000 & 0.00000 & 0.00000 \\ -24.07260 & -0.02442 & 0.40389 & 0.52215 \\ 0.00000 & 0.00000 & 0.00000 & 1.00000 \\ 28.47348 & 0.20904 & -3.45678 & -4.46896 \end{bmatrix}$$