

In quantum mechanics, the **uncertainty principle** (also known as **Heisenberg's uncertainty principle**) is any of a variety of mathematical inequalities^[1] asserting a fundamental limit to the precision with which certain pairs of physical properties of a particle, known as complementary variables, such as position x and momentum p , can be known.

Introduced first in 1927, by the German physicist Werner Heisenberg, it states that the more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa.^[2] The formal inequality relating the standard deviation of position σ_x and the standard deviation of momentum σ_p

Forms of Heisenberg's uncertainty principle

Heisenberg Uncertainty Relationships

$$\Delta x \cdot \Delta p \sim \hbar$$

$$\Delta E \cdot \Delta t \sim \hbar$$

Applications of Heisenberg's uncertainty principle

The most commonly known one can be used to quantify broadening of spectral lines, predict quantum fluctuations and of course, set a fundamental limit to various simultaneous observations.

Whenever the energy interacts with the matter the atoms get excited and the excited atom gives up its excess energy by emitting a photon of certain frequency which leads to the spectrum. Whatever may be the resolving power of the spectrometer is used to record these spectral lines no spectral line is observed perfectly sharp. The broadening in the spectral lines is observed due to the indeterminateness in the atomic energies. To measure the energies accurately the time (Δt) required is more that is Δt tends to infinity or otherwise. According to Heisenberg's uncertainty relation

$$\Delta E = h / 2\pi\Delta t$$

Where ΔE is the uncertainty in the measurement of energies and Δt is the mean life time of the level is finite and of the order of 10^{-8} s which is a finite value. Therefore ΔE must have a finite energy spread that means the energy levels are not sharp and hence the broadening of the spectral lines. Thus broadening of spectral line which cannot be reduced further for any reason is known as natural broadening.

$$\Delta E \Delta t \approx \frac{h}{2\pi} \quad \Delta t \approx \frac{h}{2\pi \cdot h \Delta \nu} \approx \frac{1}{2\pi \Delta \nu}$$

$$\Delta \nu \geq 1/2\pi \Delta t$$

Broadening due to this effect is called the natural linewidth.

2) It has its origin in string theory, black hole physics and double special relativity. It can be solved as a quadratic inequality in momentum (p) to show that existence of minimal length. Also, the formalism predicts many effects in basic quantum mechanical models.

3) measure of uncertainty was no more taken to be standard deviation owing to its fallibility for different wavefunctions. It predicted a different onset of classicality in different quantum mechanical wavefunctions.

Consider a single hydrogen atom:

an electron of *charge* = $-e$ free to move around
in the electric field of a fixed proton of *charge* = $+e$
(proton is ~2000 times heavier than electron, so we consider it fixed).

The electron has a potential energy due to the attraction to proton of:

$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r} \quad \text{where } r \text{ is the electron-proton separation}$$

The electron has a kinetic energy of $K.E. = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$$\text{The total energy is then } E(r) = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r}$$

Classically, the minimum energy of the hydrogen atom is $-\infty$
the state in which the electron is on top of the proton $\rightarrow p = 0, r = 0$.

Quantum mechanically, the uncertainty principle forces the electron to
have non-zero momentum and non-zero expectation value of position.

If a is an average distance electron-proton distance,
the uncertainty principle informs us that the minimum electron
momentum is on the order of \hbar/a .

The energy as a function of a is then:

$$E(a) = \frac{\hbar^2}{2ma^2} - \frac{e^2}{4\pi\epsilon_0 a}$$