Experiment: 06

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1. AIM

To solve a system of linear equations using the Gauss–Jordan Reduction Method by converting the augmented matrix into reduced row echelon form (RREF) using Python.

2. S/W REQUIREMENT

- Python 3.11
- NumPy Library
- IDE / Platform: Jupyter Notebook / Google Colab / VS Code

3. THEORY

A system of linear equations can be written in matrix form as:

$$AX = B$$

Where:
A = Coefficient matrix
X = Column vector of unknowns
B = Constant column vector

Given,

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 8 \\ 13 \\ 14 \end{bmatrix}$$

Hence,

$$AX = B$$

The Gauss–Jordan Method is a direct technique used to solve such systems.
It converts the augmented matrix directly into RREF form using elementary row operations.
If determinant of matrix $A \neq 0$, the system has a unique solution.

4. PROBLEM STATEMENT

Solve the following system of equations:

$$x + 2y - z = 8$$
$$2x - y + 3z = 13$$
$$3x + y + z = 14$$

FORMATION OF AUGMENTED MATRIX

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 14 \end{bmatrix}$$

GAUSS–JORDAN REDUCTION

Step 1: Eliminate elements below first pivot (1)

Row operations:

$$R_2 \to R_2 - 2R_1$$
$$R_3 \to R_3 - 3R_1$$

After performing operations:

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & -5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ 10 \end{bmatrix}$$

Step 2: Make second pivot = 1

$$R_2 \to \frac{R_2}{-5}$$
$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & -5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0.6 \\ 10 \end{bmatrix}$$

Eliminate above and below:

$$R_1 \rightarrow R_1 - 2R_2$$
$$R_3 \rightarrow R_3 + 5R_2$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6.8 \\ 0.6 \\ -7 \end{bmatrix}$$

Step 3: Make third pivot = 1

$$R_3 \rightarrow -R_3$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6.8 \\ 0.6 \\ 7 \end{bmatrix}$$

Eliminate above third pivot:

$$R_1 \rightarrow R_1 - R_3$$
$$R_2 \rightarrow R_2 + R_3$$

Final RREF:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.2 \\ 7.6 \\ 7 \end{bmatrix}$$

FINAL SOLUTION

$$x = -0.2$$
$$y = 7.6$$
$$z = 7$$

5. IMPLEMENTATION USING PYTHON

```
import numpy as np

# Augmented Matrix
A = np.array([[1.0, 2.0, -1.0, 8.0],
              [2.0, -1.0, 3.0, 13.0],
              [3.0, 1.0, 1.0, 14.0]])

n = 3

for i in range(n):
```

```
        # Make pivot = 1
        A[i] = A[i] / A[i][i]

        # Make other elements in column = 0
        for j in range(n):
            if i != j:
                A[j] = A[j] - A[j][i] * A[i]

print("Reduced Row Echelon Form:")
print(A)

print("\nSolution:")
print("x =", A[0][3])
print("y =", A[1][3])
print("z =", A[2][3])
```

6. OUTPUT

```
Output

Reduced Row Echelon Form:
[[ 1.   0.   0.  -0.2]
 [-0.   1.   0.   7.6]
 [-0.  -0.   1.   7. ]]

Solution:
x = -0.20000000000000018
y = 7.6
z = 7.0

=== Code Execution Successful ===
```

7. VERIFICATION TABLE

Substitute values into original equations:

Equation	LHS	RHS	Error
$x + 2y - z$	8	8	0
$2x - y + 3z$	13	13	0
$3x + y + z$	14	14	0

Hence the computed solution satisfies the system exactly.

8. GEOMETRICAL REPRESENTATION OF THE LINEAR SYSTEM

In 3D Cartesian space:

- Every equation corresponds to one plane.
- The solution represents the point where all three planes intersect.
- If the planes intersect at a single point, the system has a unique solution.

For the given system, the three planes intersect at:

$$(-0.2, 7.6, 7)$$

3D Intersection Representation

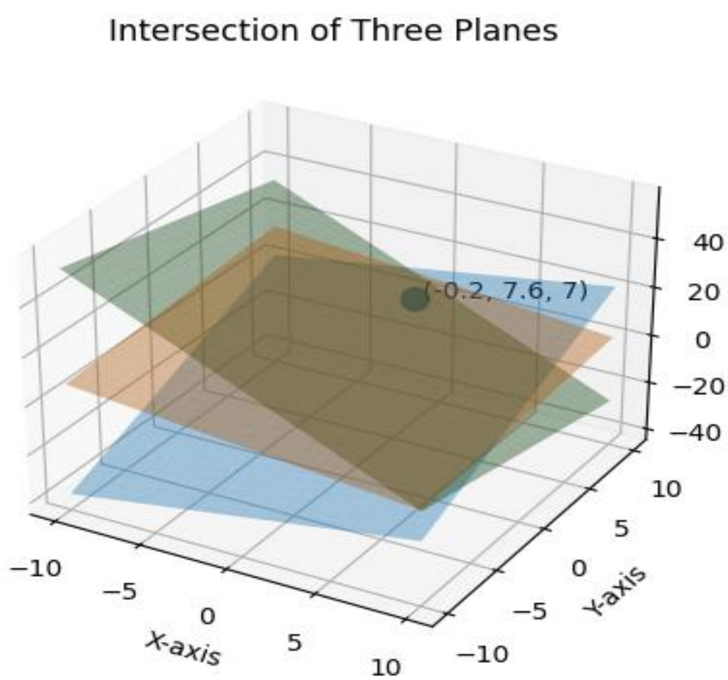

Fig: Intersection of three planes at solution point (-0.2, 7.6, 7)

9. LEARNING OUTCOME

- Converted augmented matrix into RREF form
- Applied Gauss–Jordan elimination correctly
- Implemented solution using Python
- Verified solution using substitution
- Understood geometric meaning of linear systems