Method of Mitigating Polarization Aberrations in Reflective Telescopes: Homogeneous Thin Films

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ABSTRACT

Next-generation ground and space telescopes aim to characterize earthlike exoplanets at closer angular separations and deeper contrasts that have been demonstrated before. One of the key factors limiting the performance of these instruments is the aberration of the wavefront due to polarization. Polarization aberrations present a unique problem in high-contrast imaging, because the orthogonally polarized parts of the incoming field are incoherent. This means that standard wavefront control techniques will be incapable of removing the aberration from both polarization states. A new method of controlling the polarization aberrations in astronomical telescopes is necessary to meet the contrast goals set by the Habitable Worlds Observatory and the Giant Segmented Mirror Telescopes. We outline a method of polarization aberration control using optimized thin-film multilayer stacks to minimize the contrast degradation in a 6 meter off-axis space telescope, and a 30 meter on-axis ground telescope. By optimizing the spatially-varying thickness of the multilayer thin film stacks, we are able to reduce the polarization leakage of a vector vortex coronagraph by an order of magnitude. We also conduct a Monte Carlo tolerance analysis to analyze the sensitivity of our optimized thin films to manufacturing errors. Polarization aberration minimized to X pico meter across Y-Z wavelengths.

Keywords: Manuscript format, template, SPIE Proceedings, LaTeX

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1. INTRODUCTION

1.1 Astrophysical Goals for 2020

Polarimetry, high-contrast imaging

1.2 Telescope Architectures

Next-generation telescopes for astronomy will largely take two forms. The ground-based telescopes are the Giant Segmented Mirror Telescopes (GSMTs): The European Extremely Large Telescope (ELT), The Thirty Meter Telescope (TMT), and the Giant Magellan Telescope (GMT). The GSMT's employ on-axis designs, which results in a central obscuration from the secondary mirror, and rotational symmetry of the optical system. Architechtures being considered for the future Habitable Worlds Observatory (HWO) are off-axis, meaning that the primary mirror is unobscured. This is more ideal for wavefront control, but can introduce more polarization aberration due to the higher angle of incidence. To analyze the polarization aberrations of these systems, we design two "prototype" designs to approximate the polarization aberrations of a ground-based 30 meter on-axis telescope, and a space-based 6 meter off-axis telescope.

Figure 1. Ray traces of the 30 meter prototype (a) and 6 meter prototype (b).

Surface	RoC	Conic Constant	Tilt	Diameter
1*				
2				
3				

Table 1. Caption

Surface	RoC	Conic Constant	Tilt	Diameter
1*				
2				
3				

Table 2. Caption

1.3 Polarization Aberrations

Polarization aberrations are a physical optics phenomena that has recently gained attention in the high-contrast imaging community. The goal of directly imaging exoplanets in reflected light at small angular separations is extremely sensitive to aberrations of low spatial order. Polarization aberrations arise when a beam has variable angle of incidence across against a given optical surface. The fundamental theory of polarization aberration was derived by McGuire and Chipman,[?,?] and was discussed in the context of high-contrast imaging by Breckenridge.[?] The aberrations described are typically of low amplitude, but largely manifest as phase and amplitude aberrations with the same geometry as second-order aberrations (tilt, defocus) and astigmatism. The phase aberrations are described by retardance (δ, shown in Equation 1), which describes the phase delay between orthogonal polarization states of a given beam.

$$\delta = \phi_s - \phi_p \tag{1}$$

Where ϕ_s, ϕ_p are the phases of orthogonal $s-$ and $p-$polarizations, respectively. The amplitude aberrations are described by diattenuation (D, shown in Equation 2)

$$D = \frac{A_s - A_p}{A_s + A_p} \tag{2}$$

Where A_s, A_p are the amplitudes of orthogonal $s-$ and $p-$polarizations, respectively. Polarization aberrations can be completely decomposed into diattenuation and retardance, which we must compute for every point in the exit pupil of our prototype reflective telescopes.

1.4 Polarization Ray Tracing and The Jones Pupil

The Jones Pupil is an integral tool in the analysis of an optical system's polarization aberrations. To construct the Jones Pupil, we must construct a full three-dimensional polarization ray trace (PRT) of the optical system. We refer readers interested in the exact mechanics of this technique to the extensive literature that covers its implementation.[?,?,?] The basic principle of PRT is to represent each surface in an optical system by a 3×3 PRT matrix $\mathbf{P}_q$, which is composed of a diagonal matrix $\mathbf{J}_q$ and two orthogonal transformation matrices $\mathbf{O}_{in}, \mathbf{O}_{out}$ (shown in Equation 3).

$$\mathbf{P}_q = \mathbf{O}_{out}\mathbf{J}_q\mathbf{O}_{in}^{-1} = \mathbf{O}_{out} \begin{pmatrix} r_{s,q} & 0 & 0 \\ 0 & r_{p,q} & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{O}_{in}^{-1} \tag{3}$$

Where $\mathbf{J}_q$ contains our complex reflection coefficients for the light-matter interaction. The optical system can then be represented by a matrix product of these PRT matrices, shown in Equation 4,

$$\mathbf{P}_{total} = \prod_{q=1}^{Q} \mathbf{P}_q. \tag{4}$$

The resulting $\mathbf{P}_{total}$ matrix represents the three-dimensional transformation of any polarization state in the entrance pupil to the exit pupil in global coordinates. To analyze the influence on the point-spread function (PSF) of optical systems, we must transform the PRT matrix into the coordinates of the exit pupil of the instrument we want to analyze. There are several methods in the litterature detailing how to perform this transformation.[?] For this investigation, we elect to conduct this transformation using the double-pole basis due to its insensitivity to polarization singularities in the resultant pupil.

Deriving the double-pole basis vectors begins with the selection of a direction for the *anti-pole* a_{loc}. This is the direction opposite the two coincident poles in the vector field. We generally chose this to be the point where the optical axis intersects the exit pupil because we are interested in generating Jones pupils for diffraction models. When a suitable a_{loc} is chosen, the first basis vector must be chosen for this point. In practice this choice is constrained to be orthogonal to the propagation direction because this is the plane light is confined to, but can otherwise be specified arbitrarily. We chose the first basis vector to be the horizontal dimension of our Jones pupil x_o. The next basis vector is orthogonal to both a_{loc} and x_o, so it is defined by their cross product.

$$y_o = a_{loc} \times x_o \tag{5}$$

The multiplication order is chosen to maintain a right-handed coordinate system. To construct a set of double-pole basis vectors the basis vectors x_o and y_o are rotated by an angle θ_{dbl} about an axis $\vec{r}_{dbl}$.

$$\theta_{dbl} = -arccos(\vec{k}_Q \cdot \vec{a_{loc}}) \tag{6}$$

$$\vec{r}_{dbl} = \vec{k}_Q \times \vec{a_{loc}} \tag{7}$$

This is accomplished using a rotation matrix expressed in terms of equations 6 and 7 where $s = sin(\theta_{dbl})$ and $c = cos(\theta_{dbl})$ and $r_{x,y,z}$ are the scalar components of the vector r_{dbl}.

$$\mathbf{R} = \begin{pmatrix} (1-c)r_x^2 + c & (1-c)r_x r_y - s r_z & (1-c)r_x r_z + s r_y \\ (1-c)r_y r_x + s r_z & (1-c)r_x^2 + c & (1-c)r_y r_z - s r_x \\ (1-c)r_z r_x - s r_y & (1-c)r_y r_z + s r_x & (1-c)r_z^2 + c \end{pmatrix} \tag{8}$$

The basis vectors are computed by multiplying x_o and y_o by R.

$$< x_{loc}, y_{loc} >=< Rx_o, Ry_o > \tag{9}$$

This operation rotates the basis vectors about the wavefront at the plane of interest based on the direction of the wave vector to be tangent to the wavefront. This operation is done at both the entrance and exit pupil spheres to create the orthogonal transformation matrices O_{XP} and O_{EP} using the format of equation **??** and **??**. Equation 10 will yield the final Jones pupil.

$$J_{tot} = O_{XP}^{-1} P_{tot} O_{EP} \tag{10}$$

2. METHODS

To conduct the thin film optimization we first must derive the equations that describe the equations for the complex reflection coefficients for $s-$ and $p-$polarization.

2.1 Thin film characteristic matrix

The Fresnel equations are sufficient for describing uncoated optical surfaces, but rarely is this the case. Most optical surfaces have thin dielectric coatings to enhance the transmitted or reflected signal. These coatings can be inhomogeneous, which can cause further polarization aberration. To accurately capture these effects, we need a method of computing an effective Fresnel reflection coefficient. This is commonly done (assuming the coatings are isotropic) by computing the characteristic matrix of the dielectric stack.[?,?] The derivation for these is quite involved so it will not be recreated here, but the full derivation and set of equations is shown in the free online textbook by Peatross and Ware.[?] For p-polarized light, the characteristic matrix $A^{(p)}$ is given by the following relation.

$$A^{(p)} = \frac{1}{2n_o cos(\theta_o)} \begin{pmatrix} n_o & cos(\theta_o) \\ n_o & -cos(\theta_o) \end{pmatrix} \prod_{j=1}^{N} M_j^{(p)} \begin{pmatrix} cos(\theta_{N+1}) & 0 \\ n_{N+1} & 0 \end{pmatrix} \tag{11}$$

Where $M_j^{(p)}$ is the j-th matrix of a layer in the thin film coating described by

$$M_j^{(p)} = \begin{pmatrix} cos(\beta_j) & -isin(\beta_j)cos(\theta_j)/n_j \\ -in_j sin(\beta_j)/cos(\theta_j) & cos(\beta_j) \end{pmatrix} \tag{12}$$

And B_j is the optical path length along the propagation direction in the dielectric film defined by the path length from the first interface.

$$\beta_j = \begin{cases} 0 & j = 0 \\ k_j d_j cos(\theta_j) & 1 \leq j \leq N \end{cases} \tag{13}$$

Similarly for the s-polarized characteristic matrix, we have

$$A^{(s)} = \frac{1}{2n_o cos(\theta_o)} \begin{pmatrix} n_o cos(\theta_o) & 1 \\ n_o cos(\theta_o) & -1 \end{pmatrix} \prod_{j=1}^{N} M_j^{(s)} \begin{pmatrix} 1 & 0 \\ n_{N+1} cos(\theta_{N+1}) & 0 \end{pmatrix} \tag{14}$$

Where $M_j^{(s)}$ is

$$M_j^{(s)} = \begin{pmatrix} cos(\beta_j) & -isin(\beta_j)/n_j cos(\theta_j) \\ -in_j sin(\beta_j)cos(\theta_j) & cos(\beta_j) \end{pmatrix} \tag{15}$$

The effective Fresnel transmission and reflection coefficients are then derived simply in terms of the characteristic matrices.

$$r_p^{tot} = \frac{a_{10}^{(p)}}{a_{00}^{(p)}} \tag{16}$$

$$r_s^{tot} = \frac{a_{10}^{(s)}}{a_{00}^{(s)}} \tag{17}$$

To optimize these reflection coefficients we must express one of the parameters in the above expression as a variable. To do so we look to the phase thickness β_j. This parameter is a function of refractive index, wavelength, angle of incidence, and film thickness. Of these variables:

Figure 2. Runtime to compute Jones pupil v.s. number of rays traced

- The refractive index can be chosen, but is not a continuous function that we can vary
- The wavelength is a set quantity
- The angle of incidence is set by the instrument. Changing it would result in scalar wavefront aberrations, which would further impede performance.
- The film thickness is not explicitly constrained

Therefore, we choose thickness as our primary variable of optimization. To perform an expeditious optimization of the thin film thickness, it is convenient to lower the dimensionality of the problem. We expand the thickness of the j-th film as a sum over N low-order polynomials Z_i, as shown in Equation 18

$$d_j = \sum_{i=0}^{N} a_i Z_i, \tag{18}$$

These physics were implemented in the open-source Python package Poke, which supports polarization ray tracing so that we can perform optimizations on the thin film stacks.

2.2 Poke: Integrating ray and diffraction models

Poke is an open-source Python package that was originally developed to simulate the polarization aberrations of astronomical telescopes.[?] It allows the user to perform PRT calculations independent of the ray tracer used, effectively open-sourcing physical optics calculations. For this study, we vectorized the Jones pupil computation so that optimizations can be performed in a reasonable time.

2.3 Constant raypaths, variable films

Keep the orthogonal transformations the same, change the fresnel coefficients

2.4 Telescope Architectures for Design Study

6 meter space telescope, 30m ground telescope. These don't have to be official, but can be simmilar enough in geometry to be relevant.

2.5 Optimization methodology

Polarization ray tracing is conducted via a matrix product of the constituent PRT matrices to produce the system PRT matrix ($\mathbf{P}_{tot}$ in equation 19)

$$\mathbf{P}_{tot} = \prod_{i=0}^{Q} \mathbf{P}_i \tag{19}$$

The i-th PRT matrix is simply a product of two orthogonal transformation matrices ($\mathbf{O}$) with a diagonal jones matrix.

$$\mathbf{P}_i = \mathbf{O}_{out,i} \begin{pmatrix} \Lambda_s & 0 & 0 \\ 0 & \Lambda_p & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{O}_{in,i}^{-1} \tag{20}$$

Where Λ_s, Λ_p are the fresnel coefficients for s- and p- polarizations, respectively. The matrices for orthogonal transformation are determined by the wave vector $\mathbf{k}$ and corresponding eigenpolarizations of the local surface interaction $\mathbf{s}$ and $\mathbf{p}$.

$$\mathbf{O} = \begin{pmatrix} s_x & s_y & s_z \\ p_x & p_y & p_z \\ k_x & k_y & k_z \end{pmatrix} \tag{21}$$

The orthogonal transformation matrices are uniquely determined by the ray paths through the optical system. In the limit of interactions with thin film optical filters on entirely reflective optical systems, the raypaths through these optical systems do not change as a function of the film thickness or wavelength. This means that the orthogonal transformation matrices need only be computed once per optimization procedure, and the only operation that needs to be updated per optimization iteration is the computation of the Jones pupil.

The cost function we define is a simple summation over the ray index (i) and wavelengths (λ) of the polarization aberrations normalized by the total number of pixels (N).

$$\mathcal{C} = \sum_{\lambda=600nm}^{800nm} \sum_{i=0}^{N} \frac{\delta_i^2}{N} + \frac{\mathcal{D}_i^2}{N} + \frac{(1-\mathcal{R}_i)^2}{N} \tag{22}$$

3. RESULTS

3.1 Jones Pupils before optimization

We begin by comparing the Jones pupils of a bare aluminum coating v.s. a protected silver coating for the two prototypes.

3.2 Diattenuation, Retardance pupils v.s. Spectrum

ACKNOWLEDGMENTS

This unnumbered section is used to identify those who have aided the authors in understanding or accomplishing the work presented and to acknowledge sources of funding.