

NCERT - 9.4.2

EE224BTECH11031 - Jashwanth.M

I. DIFFERENTIAL EQUATIONS

Question: Solve $\frac{dy}{dx} = \frac{x+y}{x}$ **Solution: (Theoretical Solution)** The differential equation is

$$\frac{dy}{dx} = 1 + \frac{y}{x} \quad (1)$$

To solve the equation, substitute y as xt

$$y = xt \quad (2)$$

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \quad (3)$$

Substitute in the equation

$$t + x \frac{dt}{dx} = 1 + t \quad (4)$$

$$x \frac{dt}{dx} = 1 \quad (5)$$

$$dt = \frac{dx}{x} \quad (6)$$

$$(7)$$

Integrate on both sides

$$\int dt = \int \frac{dx}{x} \quad (8)$$

$$t = \ln x \text{ (Let's assume integration constant as zero)} \quad (9)$$

$$(10)$$

Substituting $\frac{y}{x}$ in place of t

$$\frac{y}{x} = \ln x \quad (11)$$

$$y = x \ln x \quad (12)$$

Solution by the method of finite differences:

$$\frac{dy}{dx} = \frac{x+y}{x} \quad (13)$$

In this method, we approximate derivative as

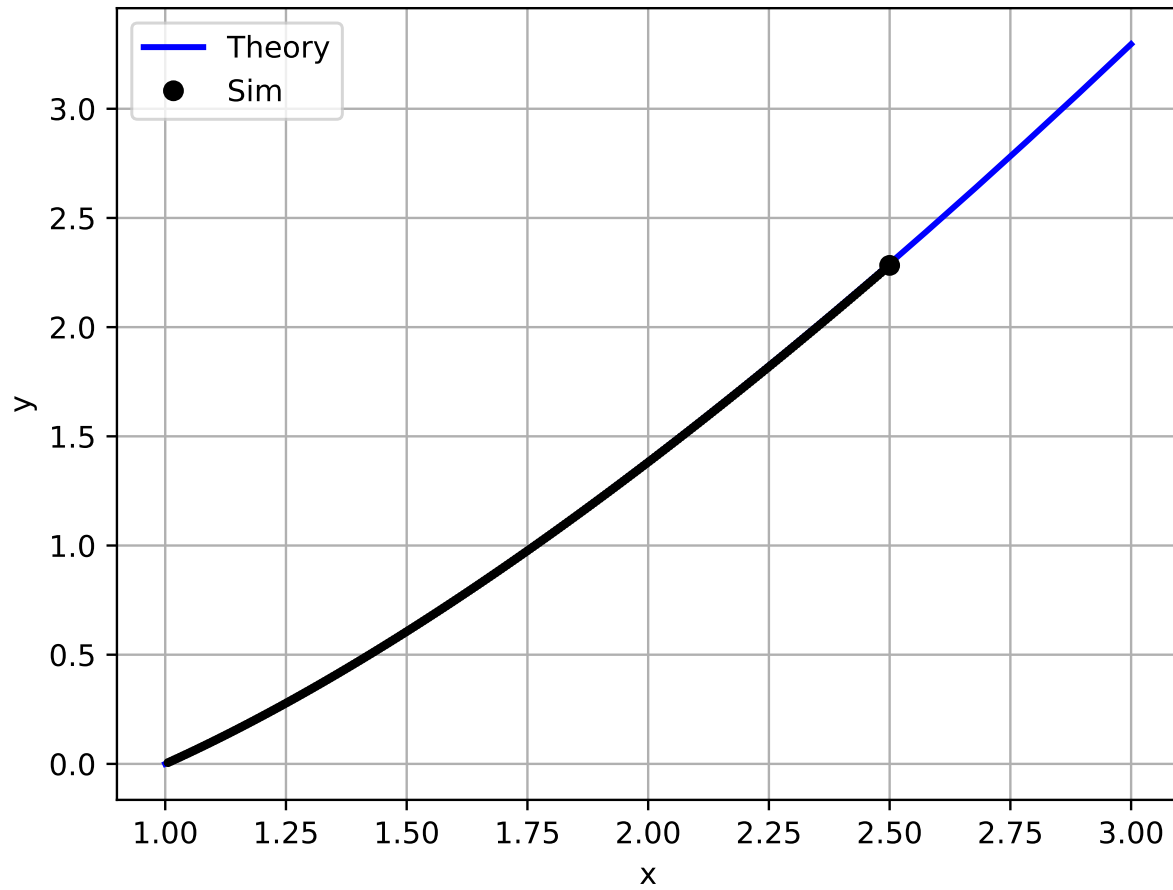
$$\frac{dy}{dx} \approx \frac{y_{n+1} - y_n}{h} \quad (14)$$

Substitute equation(14) in equation(13)

$$\frac{y_{n+1} - y_n}{h} = \frac{x_n + y_n}{x_n} \quad (15)$$

$$y_{n+1} - y_n = h \left(\frac{x_n + y_n}{x_n} \right) \quad (16)$$

$$y_{n+1} = y_n + h \left(\frac{x_n + y_n}{x_n} \right) \quad (17)$$



I'm taking the initial conditions as $x = 1$, $y = 0$ and $h = 0.006$. We compute the values of x_n and y_n using above recurrence relation. These values can be used to approximate the solution numerically for a given range of x .