NCERT - 9.4.2

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I. DIFFERENTIAL EQUATIONS

Question: Solve $\frac{dy}{dx} = \frac{x+y}{x}$ **Solution: (Theoretical Solution)** The differential equation is

$$\frac{dy}{dx} = 1 + \frac{y}{x} \tag{1}$$

To solve the equation, substitute y as xt

$$y = xt \tag{2}$$

$$\frac{dy}{dx} = t + x\frac{dt}{dx} \tag{3}$$

Substitute in the equation

$$t + x\frac{dt}{dx} = 1 + t \tag{4}$$

$$x\frac{dt}{dx} = 1 \tag{5}$$

$$dt = \frac{dx}{x} \tag{6}$$

$$\tag{7}$$

Integrate on both sides

$$\int dt = \int \frac{dx}{x} \tag{8}$$

$$t = \ln x (Let's assume integration constant as zero) \tag{9}$$

$$\tag{10}$$

Substituting $\frac{y}{x}$ in place of t

$$\frac{y}{x} = \ln x \tag{11}$$

$$y = x \ln x \tag{12}$$

Solution by the method of finite differences:

$$\frac{dy}{dx} = \frac{x+y}{x} \tag{13}$$

In this method, we approximate derivative as

$$\frac{dy}{dx} \approx \frac{y_{n+1} - y_n}{h} \tag{14}$$

Substitute equation(14) in equation(13)

$$\frac{y_{n+1} - y_n}{h} = \frac{x + y}{x} \tag{15}$$

$$y_{n+1} - y_n = h(\frac{x + y}{x}) \tag{16}$$

$$y_{n+1} = y_n + h(\frac{x + y}{x}) \tag{17}$$

I'm taking the initial conditions as $x = 1$, $y = 0$ and $h = 0.006$. We compute the values of x_n and y_n using above recurrence relation. These values can be used to approximate the solution numerically for a given range of x.