ASSIGNMENT 1

EE24BTECH11031 - Jashwanth

1) The normal at a point **P** on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at **Q**. If **M** is the mid point of the line segment **PQ**, then the locus of **M** interests the latusrectums of the given ellipse at the points (2009)

   a) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$
   b) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\sqrt{\frac{19}{4}}\right)$
   c) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$
   d) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

2) The locus of the orthocentre of the traingle formed by the lines

$$(1+p)x - py + p(1+p) = 0,$$
$$(1+q)x - qy + q(1+q) = 0,$$

   and $y = 0$, where $p \neq q$, is (2009)

   a) a hyperbola
   b) a parabola
   c) an ellipse
   d) a straight line

3) Let $\mathbf{P}(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point **P** intersects the x-axis at $(9,0)$, then the eccentricity of the hyperbola is (2011)

   a) $\sqrt{\frac{5}{2}}$
   b) $\sqrt{\frac{3}{2}}$
   c) $\sqrt{2}$
   d) $\sqrt{3}$

4) Let (x, y) be any point on the parabola $y^2 = 4x$. Let **P** be the point that divides the line segment from $(0,0)$ to (x, y) in the ratio $1:3$. Then the locus of **P** is (2011)

   a) $x^2 = y$
   b) $y^2 = 2x$
   c) $y^2 = x$
   d) $x^2 = 2y$

5) The ellipse E_1:$\frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle **R** whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0,4)$ circumscribes the rectangle **R**.The eccentricity of the ellipse E_2 is (2012)

   a) $\frac{\sqrt{2}}{2}$
   b) $\frac{\sqrt{3}}{2}$
   c) $\frac{1}{2}$
   d) $\frac{3}{4}$

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6) The common tangents to the circie $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points **P**, **Q** and the parabola at the points **R**, **S**.Then the area of the quadrilateral **PQRS** is (JEE Adv. 2014)

a) 3
b) 6
c) 9
d) 15

D.MCQs with One or More than One Correct

1) The number of the values of c such that the straight line $y = 4x + c$ touches the curves $(x^2/4) + y^2 = 1$ is (1998 - 2 Marks)

a) 0
b) 1
c) 2
d) infinite

2) If $\mathbf{P} = (x, y)$, $\mathbf{F}_1 = (3, 0)$, $\mathbf{F}_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $\mathbf{PF}_1 + \mathbf{PF}_2$ equals (1998-2 Marks)

a) 8
b) 6
c) 10
d) 12

3) On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are (1999-3 Marks)

a) $\left(\frac{2}{5}, \frac{1}{5}\right)$
b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$
c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$
d) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

4) The equations of the common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are (2006-5M,-1)

a) $y = 4(x-1)$
b) $y = 0$
c) $y = -4(x-1)$
d) $y = -30x - 50$

5) Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then (2006-5M,-1)

a) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$
b) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$
c) focus of hyperbola is $(5, 0)$
d) vertex of hyperbola is $\left(5\sqrt{3}, 0\right)$

6) Let $\mathbf{P}(x_1, y_1)$ and $\mathbf{Q}(x_2, y_2)$, $y_1 < 0, y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum $\mathbf{PQ}$ are (2008)

a) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$
b) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$
c) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$
d) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

7) In a traingle **ABC** with fixed base **BC**, the vertex **A** moves such that

$$\cos B + \cos C = 4\sin^2\frac{A}{2}$$

. If a, b and c denote the lengths of the sides of the traingle opposite to the angles A, B and C, respectively, then (2009)

a) $b + c = 4a$
b) $b + c = 2a$
c) locus of the point **A** is an ellipse
d) locus of the point **A** is a pair of straight lines

8) The tangent **PT** and the normal **PN** to the parabola $y^2 = 4ax$ at a point **T** and **N**, respectively. The locus of the centroid of the traingle **PTN** is a parabola whose (2009)

a) vertex is $\left(\frac{2a}{3}, 0\right)$
b) directrix is $x = 0$
c) latus rectum is $\frac{2a}{3}$
d) focus is $(a, 0)$

9) An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then (2009)

a) equation of the ellipse is $x^2 + 2y^2 = 2$
b) the foci of ellipse are $(\pm 1, 0)$
c) equation of the ellipse is $x^2 + 2y^2 = 4$
d) the foci of ellipse are $\left(\pm\sqrt{2}, 0\right)$