Question 3

b) i) 1100111_2 (using Horner's Rule)

$= (1\times2)+1\times2)+0\times2)+0\times2)+1\times2)+1\times2)+1\times2)$

$= \underline{103_{10}}$

$\therefore 1100111_2$ (using Horner's Rule) $= 103_{10}$

ii) 0.0011101_2

$= (0\times2^0)+(0\times2^{-1})+(0\times2^{-2})+(1\times2^{-3})+(1\times2^{-4})+(1\times2^{-5})+(0\times2^{-6})+(1\times2^{-7})$

$= 0+0+0+0.125+0.0625+0.03125+0+0.0078125$

$= \underline{0.2265625_{10}}$

iii) $2BF4_{16}$

$= (2\times16^3)+(B\times16^2)+(F\times16^1)+(4\times16^0)$

$= (2\times16^3)+(11\times16^2)+(15\times16^1)+(4\times1)$

$= 8192+2816+240+4$

$= \underline{11252_{10}}$

iv) 3733_8

$= (3\times8^3)+(7\times8^2)+(3\times8^1)+(3\times8^0)$

$= 1536+448+24+3$

$= \underline{2011_{10}}$

c)

i) 11011.10011_2 to Base ten (Decimal)

$= (1\times2^4)+(1\times2^3)+(0\times2^2)+(1\times2^1)+(1\times2^0)+(1\times2^{-1})+(0\times2^{-2})+(0\times2^{-3})+$
$(1\times2^{-4})+(1\times2^{-5})$

$= 16+8+0+2+1+0.5+0+0+0.0625+$
0.03125

$= \underline{27.59375}$

ii) 11011.10011_2 to Base 8 (octal

$011_2 = 3_8$

$011_2 = 3_8$

100

$11011_2 \simeq 33_8$

0.10011_2

$100_2 = 4_8$

$110_2 = 6_8$

$0.10011_2 \simeq 46_8$

∴ We add them to form base 8 $= 33.46_8$

iii) 11011.10011_2 to base 16 (Hexadecimal)

$0001_2 - 1$

$1011_2 - B$

$11011_2 \simeq 1B_{16}$

0.10011

$1001_2 - 9$

$1000_2 - 8$

$0.10011_2 \simeq 98_{16}$

∴ We add them to form base 16

$= 1B.98_{16}$

Question 4

(a) i) Binary

23.875_{10}

2	23	1
2	11	1
2	5	1
2	2	0
	1	

$23_{10} \simeq 10111_2$

Fraction

$$\begin{array}{r} 0.875_{10} \\ \times \quad 2 \\ \hline 1.750 \end{array} \quad \begin{array}{r} 0.75 \\ \times \quad 2 \\ \hline 1.50 \end{array} \quad \begin{array}{r} 0.5 \\ \times \quad 2 \\ \hline 1.0 \end{array}$$

$= 0.111_2$

∴ We combine $= 10111.111_2$

∴ $23.875_{10} = 10111.111_2$

ii) Hexadecimal

23.875_{10}

16	23	7
	1	

$23_{10} \simeq 17_{16}$

Fraction

$$\begin{array}{r} 0.875 \\ \times \quad 16 \\ \hline 14 \end{array}$$

$14 \sim E_{16}$

∴ We combine $= 17.E_{16}$

∴ $23.875_{10} = 17.E_{16}$

iii) The typical 32-bit floating point format

$23.875_{10} = 10111.111_2$

Normalise $= 0.10111111 \times 2^5$

Exponent $= 128 + 5 = 133$

$= 10000101_2$

Exponent	Sign	Fraction
1000 0101	0	1011 1111 1000 0000 0000 0000
8 5	5	F 8 0 0 0

$\therefore 23.875_{10} \simeq 855F8000_{16}$

(iv) Single precission IEEE / INTEL floating point

$23.875_{10} = 10111.111_2$

Normalise $= 1.0111111 \times 2^4$

Exponent $= 127 + 4 = 131$

$= 10000011_2$

Sign	Exponent	Fraction
0	1000 0011	011 1111 0000 0000 0000 0000
4	1	B F 0 0 0 0

$23.875_{10} \simeq 41BF0000_{16}$

v) Double precission

$23.875_{10} = 10111.111_2$

Normalise $= 1.0111111 \times 2^4$

Exponent $= 1023 + 4 = 1027_{10}$

$= 10000000011_2$

Sign	Exponent	Fraction
0	100 0000 0011	0111 1110 0000 0000 0000 0000 0000 0000 ...

$403\,7E00000000_{16}$

b) i)

$$
\begin{array}{r}
11000111_2 \\
10011100_2 \\
+\quad 10011110_2 \\
\hline
1\,1\,1\,100101
\end{array}
$$

$= 11100101$

ii)

$$
\begin{array}{r}
10011_2 \\
\times \quad 101_2 \\
\hline
10011 \\
00000 \\
10011 \\
\hline
1011111
\end{array}
$$

$= 1011111_2$

iii) $BEAF_{16} - FED_{16}$

$BEAF_{16} = 48815_{10}$

$FED_{16} = 4077_{10}$

$48815 - 4077_{10}$

$= 44,738$

$= AE42_{16}$

c)

i) 11001100_2

$= (1\times2^7)+(1\times2^6)+(1\times2^5)+(1\times2^4)+ \ldots$

$= (1\times2^7)+(1\times2^6)+(0\times2^5)+(0\times2^4)+(1\times2^3)+(1\times2^2)+(0\times2^1)+(0\times2^0)$

$= 128+64+0+0+8+4+0+0$

$= 204_{10}$

ii) 11001100_2

1 = negative

1001100_2

$= (1\times2^6)+(0\times2^5)+(0\times2^4)+(1\times2^3)+(1\times2^2)+(0\times2^1)+(0\times2^0)$

$= 64+0+0+8+4+0$

$= 76_{10}$

11001100_2 sign magnitude number $= -76$

$11001100_2 \simeq -76_{10}$

iii) 1's compliment Number

11001100_2

1's compliment $= 00110011_2$

$= (0\times2^7)+(0\times2^6)+(1\times2^5)+(1\times2^4)+(0\times2^3)+(0\times2^2)+(1\times2^1)+(1\times2^0)$

$= 0+0+32+16+0+0+2+1$

$= 51_{10}$

$\therefore$ 1's compliment number of 11001100 is -51_{10}

$11001100_2 \simeq -51_{10}$

iv) 2's compliment

11001100_2

1's compliment = 00110011_2

2's compliment Add 1

$$
\begin{array}{r}
00110011 \\
+ \quad 1 \\
\hline
00110100
\end{array}
$$

$\therefore$ 2's compliment is 00110100_2

$(0\times2^7)+(0\times2^6)+(1\times2^5)+(1\times2^4)+(0\times2^3)+(1\times2^2)+(0\times2^1)+(0\times2^0)$

$= 0+0+32+16+0+4+0+0$

$= 52_{10}$

$\therefore$ 2's compliment number represents -52_{10}

v) Signed BCD number

11001100_2 to base 10

$=(1\times2^7)+(1\times2^6)+(0\times2^5)+(0\times2^4)+(1\times2^3)+(1\times2^2)+(0\times2^1)+(0\times2^0)$

$= 128+64+0+0+8+4+0+0$

$= 204_{10}$

BCD representation = 0010 0000 0100

Sign (+ve) = 1

$\therefore$ 1001000000100_2

d)

i) Sign Magnitude numbers

A = ~~to~~ 01101101_2 0 - positive

B = 11011100_2 1 - negative

A: 01101101_2

$= (1\times2^6)+(1\times2^5)+(0\times2^4)+(1\times2^3)+(1\times2^2)+(0\times2^1)+(1\times2^0)$

$= 64+32+8+4+1$

$= 109_{10}$ (+ve)

B = 1011100_2

$= (1\times2^6)+(0\times2^5)+(1\times2^4)+(1\times2^3)+(1\times2^2)+(0\times2^1)+(0\times2^0)$

$= 64+16+8+4$

$= 92_{10}$ (-ve)

$= A + B$

$= 109$

$\quad - 92$

$\quad \overline{\quad 17 \quad}$

17_{10} +ve result

17_{10} to binary

$= 00010001_2$

$\therefore A + B = 00010001_2$

(ii) 2's compliment numbers

$A + B$

$$
\begin{array}{r}
1\,1\,1\,1\,1\,1 \quad \\
0\,1\,1\,0\,1\,1\,0\,1 \\
+\ 1\,1\,0\,1\,1\,1\,0\,0 \\
\hline
1\,0\,1\,0\,0\,1\,0\,0\,1
\end{array}
\qquad\qquad
\begin{array}{r}
1\,1\,1\,1\,1 \quad \\
0\,1\,1\,0\,1\,1\,0\,1 \\
+\ 1\,1\,0\,1\,1\,1\,0\,0 \\
\hline
1\,0\,1\,0\,0\,1\,0\,0\,1 \\
+\ 1 \\
\hline
1\,0\,1\,0\,0\,1\,0\,1\,0 \\
\hline
\end{array}
$$

$\therefore$ 2's compliment of $A+B = 101001010_2$