



SAPIENZA  
UNIVERSITÀ DI ROMA

# On the Sensitivity and Uncertainty of Convolution Neural Networks to Adversarial Perturbations

Senad Beadini

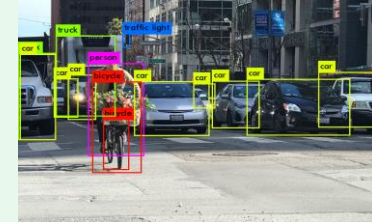
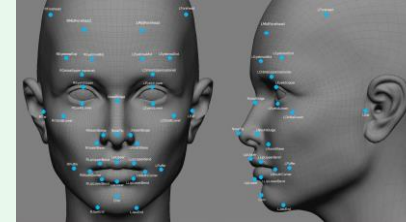
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# Deep Convolutional Networks for Classification



State-of-the-art models get **remarkable results in complex cognitive tasks.**

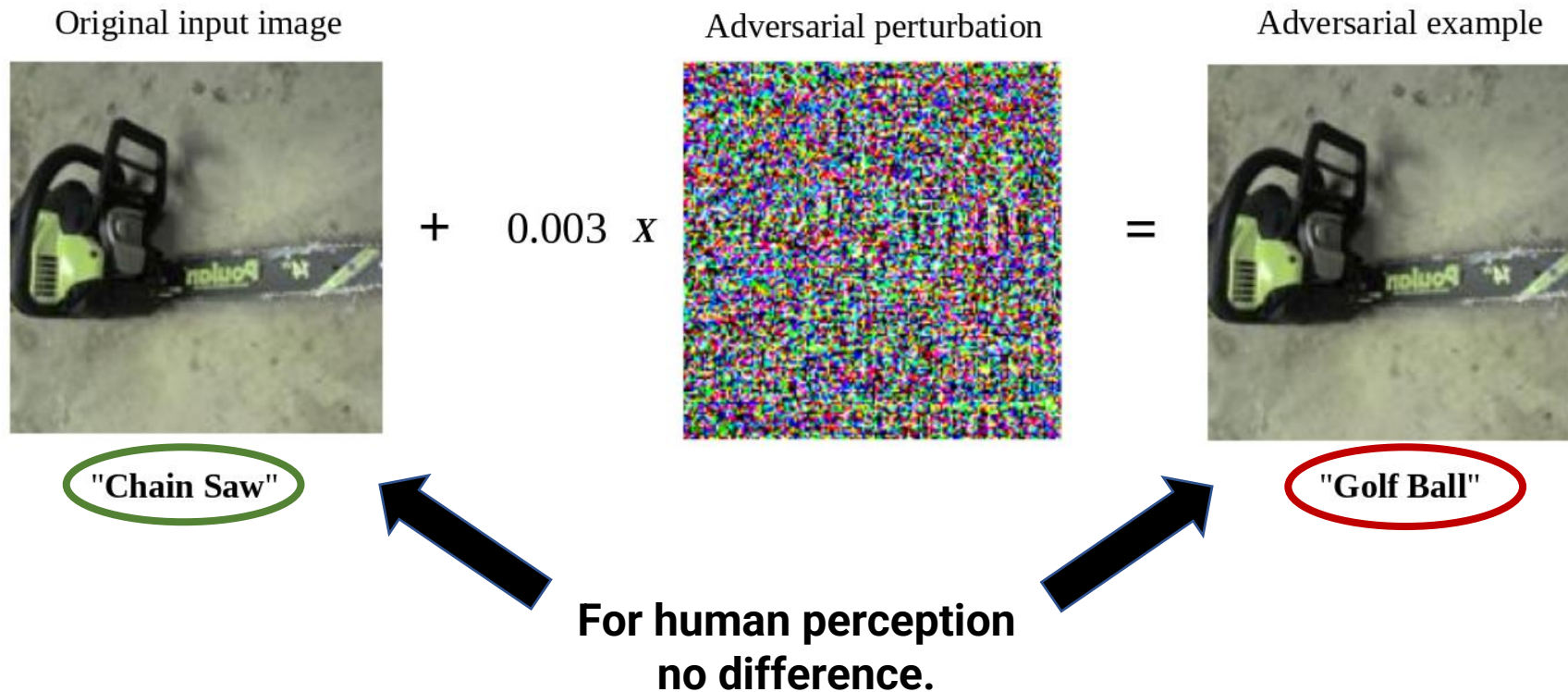


On ImageNet, convolutional neural networks (CNNs) **achieve accuracy on par with humans.**

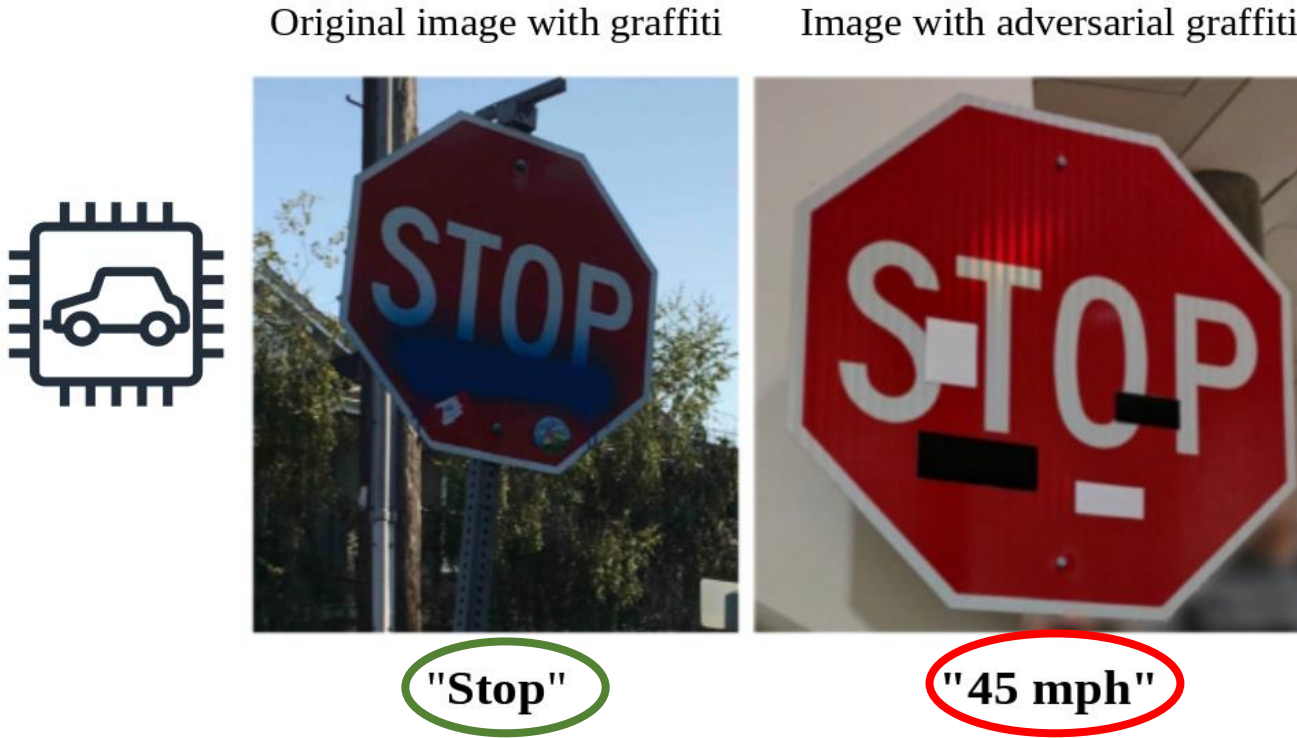


# Adversarial Perturbations Make CNNs non-Robust

An imperceptible perturbation could break the performance of any model.



# Why Is Important to Study Adversarial Perturbations?

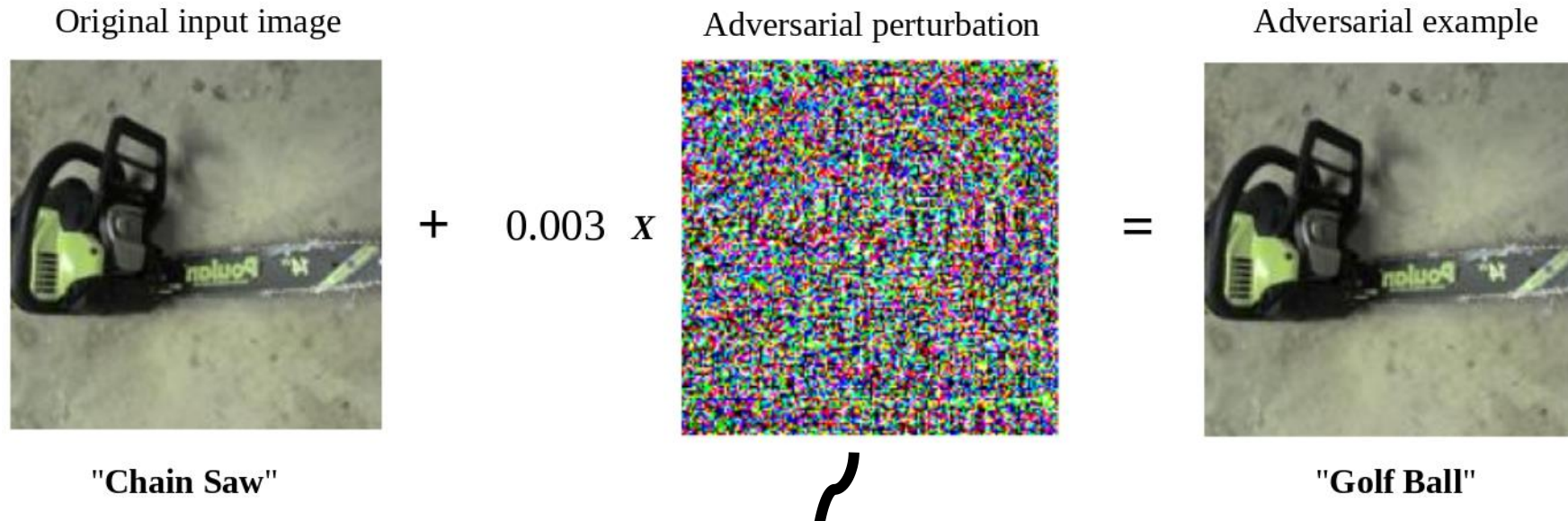


[ Eykholt et al. CVPR 2018 ]

**What kinds of perturbations exist?**

# Pixel-wise Perturbations :

The perturbation in the pixel space is limited by a threshold  $\epsilon$  under a norm  $p$ .



**The CNN achieves 85% accuracy without any perturbation.**  
**The accuracy of the CNN under attack is 0%.**



# Transformation-based Adversarial Example :

We apply a transformation to the input image to get a new image.

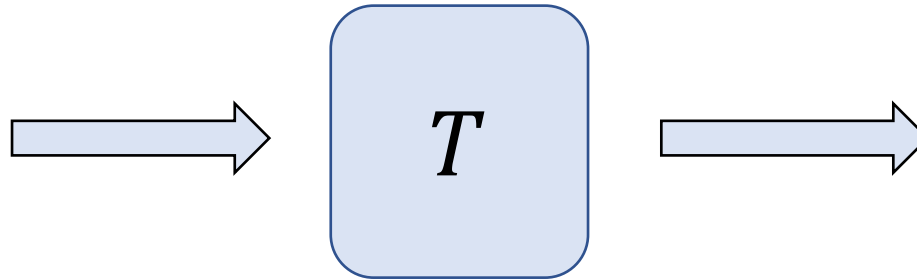
[ Engstrom et al. ICML 2018 ]

Original input image



$x$

$$x' = T(x, \gamma)$$



$T$  parametrized by  $\gamma$



The perturbation is created in the parameter space of  $T$

Transformed image



$x'$

# Contribution and Related Work

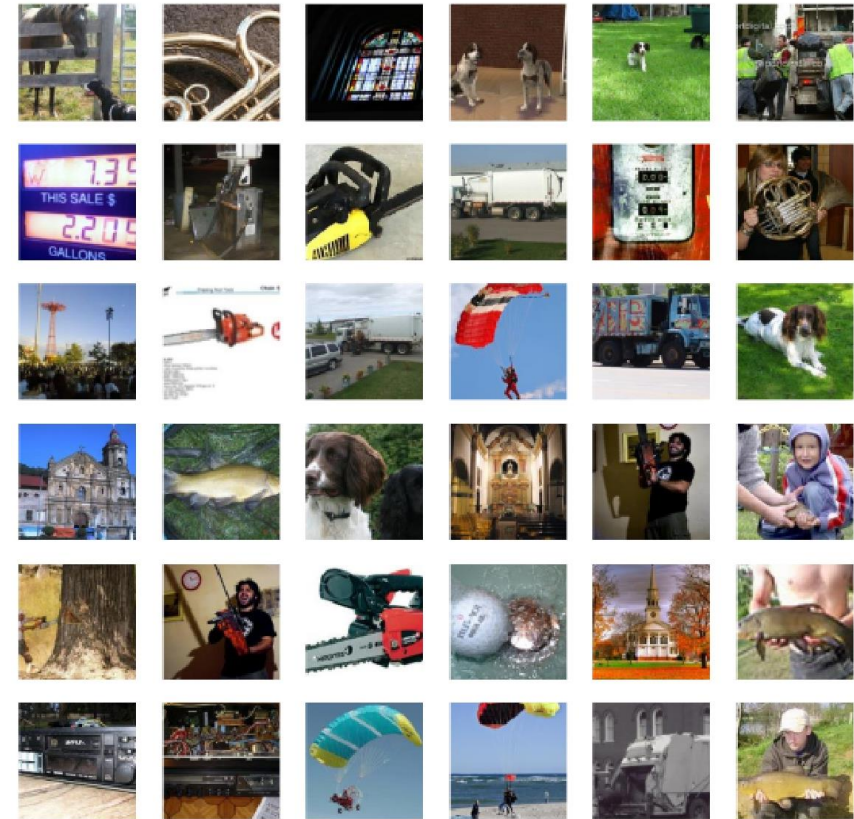
- 1) **Kamath et al. NeurIPS 2021** proved that exists a trade-off between pixel-wise robustness and Rotation/Translation robustness.
  - **We replicate the results of this work by showing the trade-off for rotation and extend the analysis to the perspective and other transformations.**
- 2) **Grathwohl et al. ICLR 2020** and **Zhu et al. ICCV 2021** find that CNNs can be reinterpreted as an energy-based model.
  - **Exploiting these results, we show that there's a correlation between the energy of a discriminative model and the strength of a pixel-wise adversarial attack.**
  - **We provide a detector based on the energy function to detect adversarial attacks.**
  - **We propose an algorithm for generating low-energy adversarial data.**



# Dataset and Models

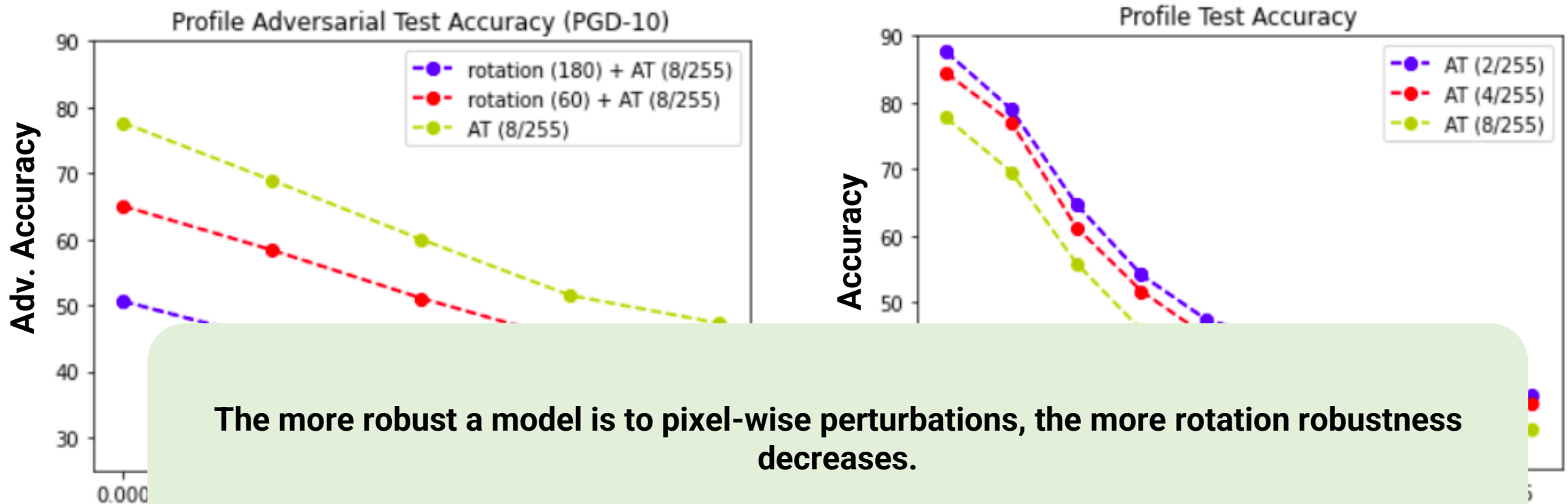
The datasets used are CIFAR-10 and Imagenette, a subset of ImageNet.

All the following experiments have been performed using a CNN with residual connections (i.e., ResNet10) .

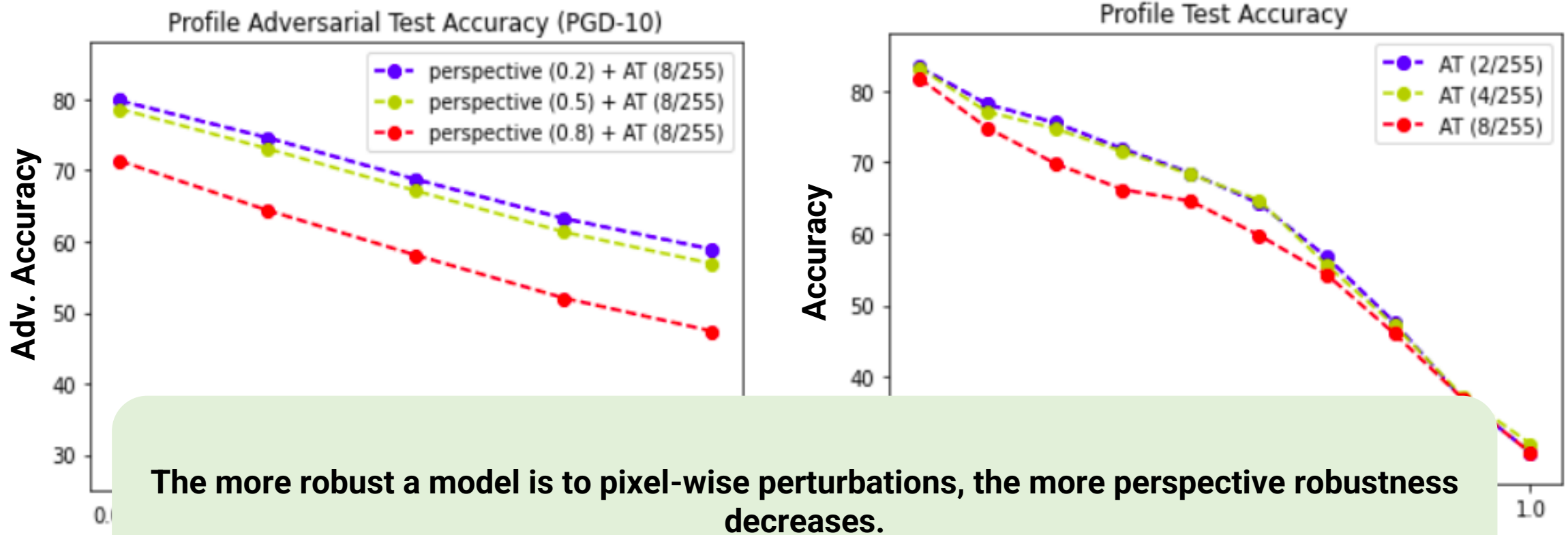


# Trade-off Analysis for Different Transformations

The basic defense for robustness consists of generating at training time perturbations and perform training on perturbed data.

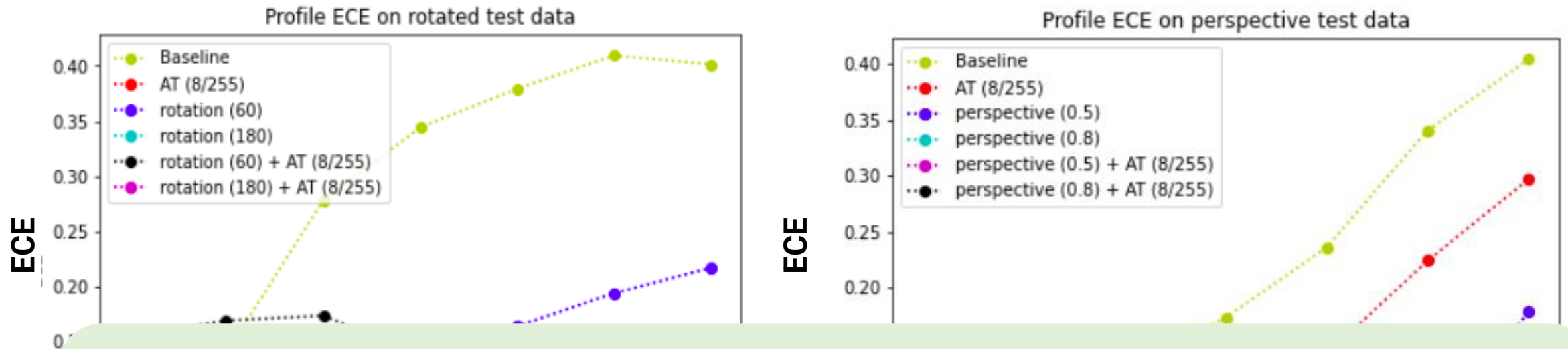


# Trade-off Analysis for Perspective Transformation and Others



# Calibration Analysis for Different Transformation

We measure calibration using the Expected Calibration Error (ECE).



**Pixel-wise robustness provides more calibrated CNNs.**

**Combining both robustnesses still provides more calibrated CNNs.  
This holds for other transformations like Gaussian Noise and Motion Blur.**

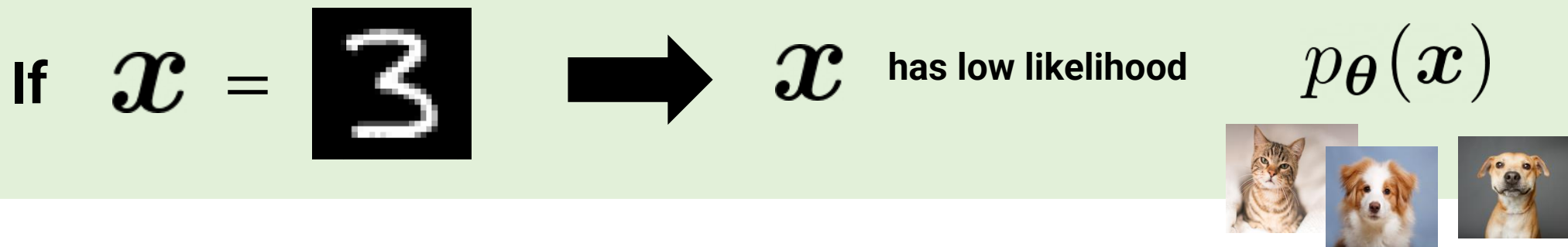
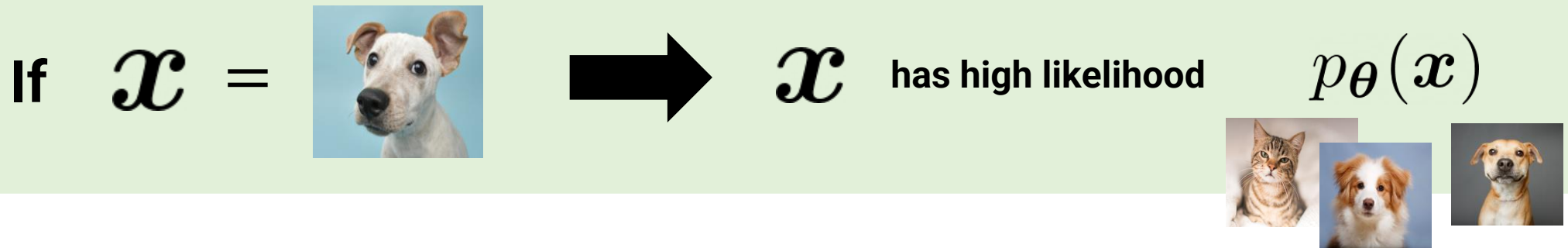
# Hypothesis :

**Pixel-wise adversarial examples are out-of-distribution (OOD) data.**

# Energy-based Model (EBM)

While classifiers are discriminative, EBMs are motivated from a different perspective: density estimation.

Density estimation : given a set of images that represents dogs and cats we want to learn a probability density function  $p_{\theta}(x)$  over all possible images such that :



# Energy-based Model (EBM)

While classifiers are discriminative, EBMs are motivated from a different perspective: density estimation.

The fundamental idea of EBM is the energy function such that we approximate  $p(x)$  via :

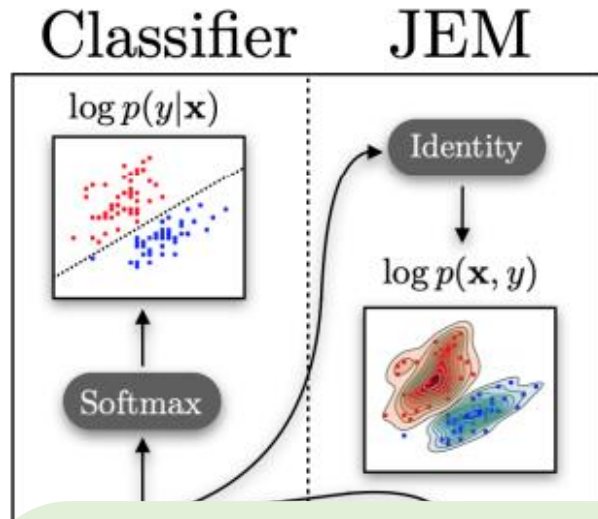
$$p_{\theta}(x) = \frac{\exp(-E_{\theta}(x))}{Z_{\theta}}$$

Since  $Z_{\theta}$  is intractable for high dimensional data, EBM focuses directly on energy  $E_{\theta}(x)$  and not  $p_{\theta}(x)$ .

In this framework, data points with high likelihood have a low energy, while data points with low likelihood have a high energy.



# A CNN is Actually an EBM



A CNN  $F$  with softmax is a discriminative model :

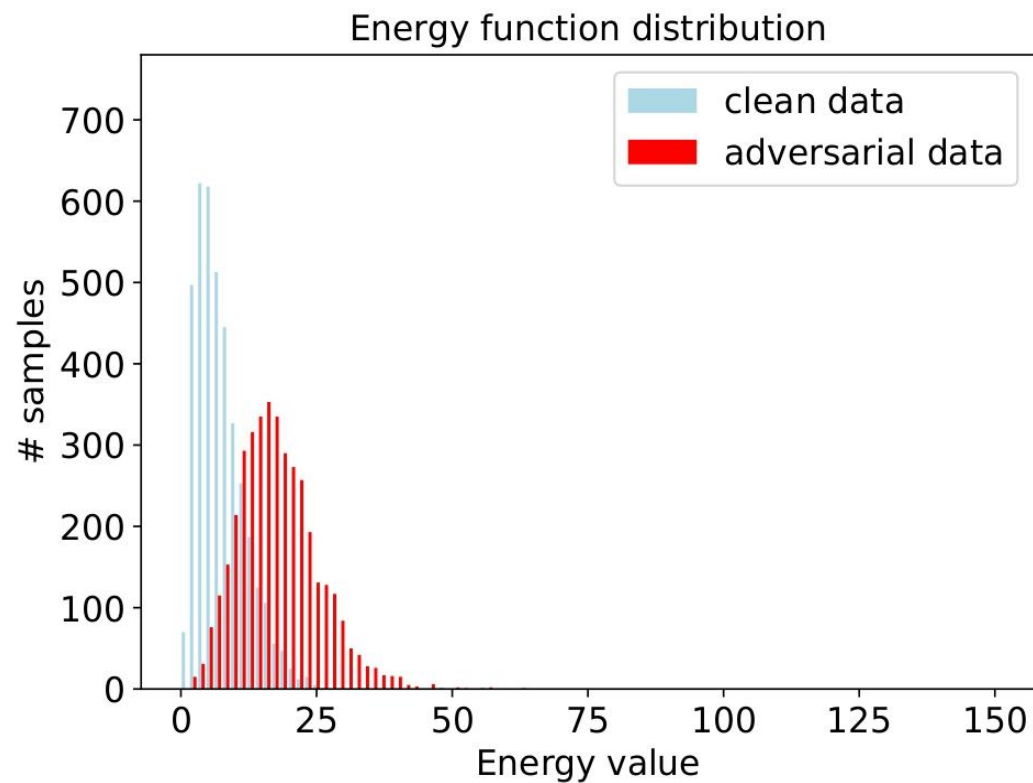
$$p(y = i | \mathbf{x}) = \frac{\exp F_{\theta}(\mathbf{x})[i]}{\sum_{j=1}^K \exp F_{\theta}(\mathbf{x})[j]}$$

Grathwohl et al. ICLR 2020 observed that we can reinterpret without any change  $F$  as an EBM :

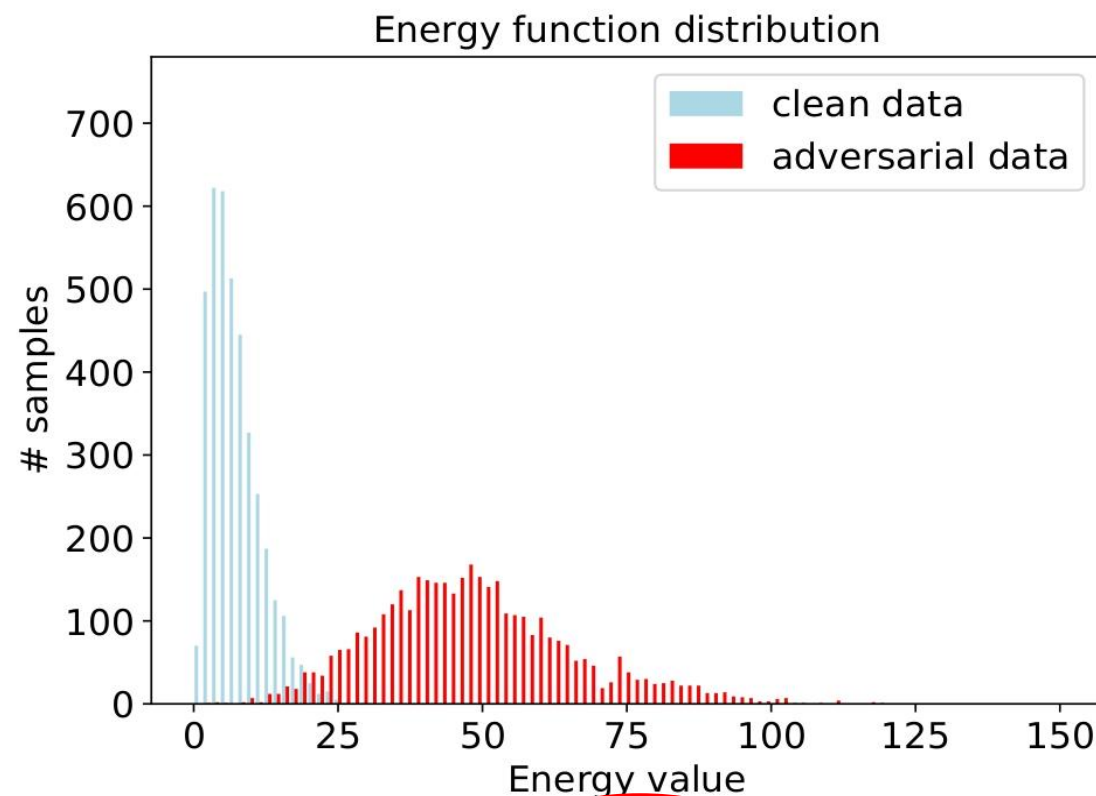
**The energy function is the denominator of the softmax operation.**

[taken from ICLR 2020]

# Energy Function vs Attack Strength



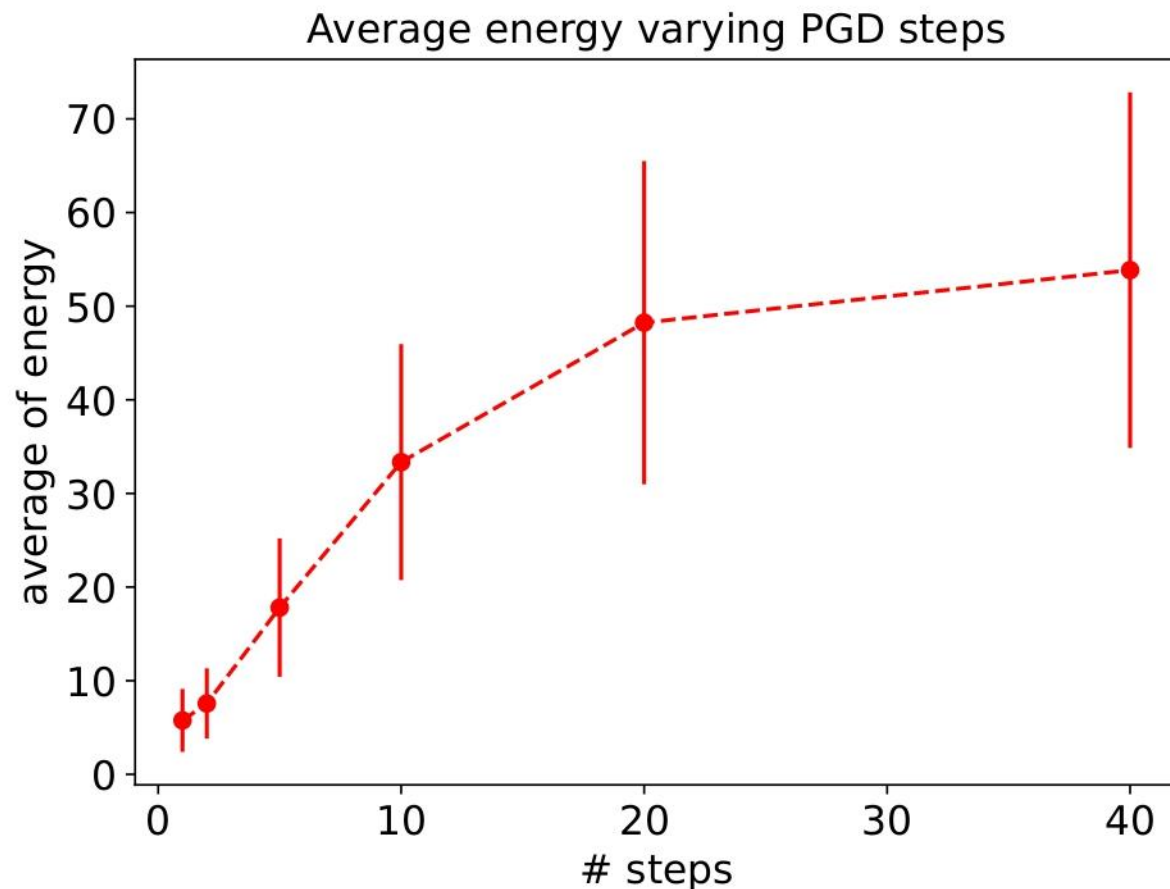
**PGD-5**



**PGD-20**

Projected Gradient Descent (PGD) Madry et al. ICLR2018

# Energy Function vs Attack Strength



**The attack strength is correlated with the energy value.**

**Notice that the norm of the perturbation is bounded, we only increase the iterations of the attack.**

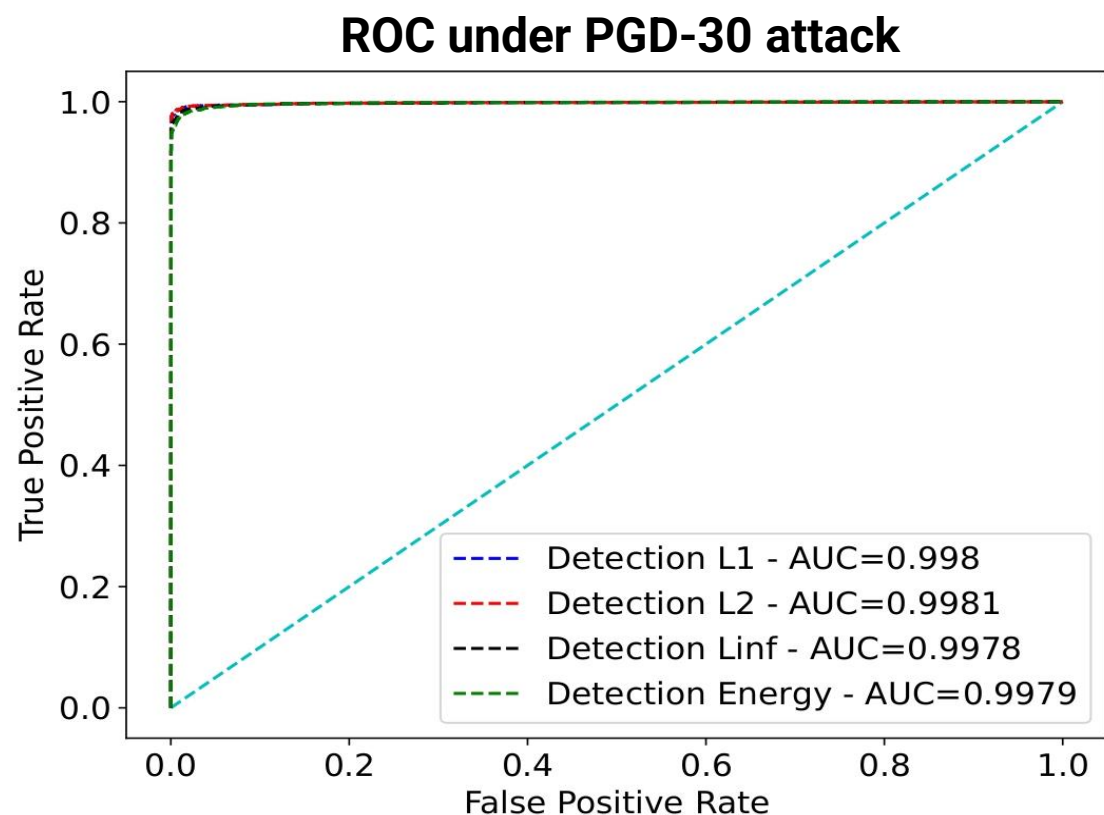
**The increase in energy is a "clue" that we are generating OOD data.**

Projected Gradient Descent (PGD) Madry et al. ICLR2018

# Energy-based Adversarial Examples Detection

Exploiting the previous result is possible to build a detector for adversarial data on the top of energy.

The threshold can be optimized on a given metric in the validation set.



# Energy-based Adversarial Examples Detection

True Label	Clean	Adversarial
	Predicted Label	Adversarial
Clean	Detection TN = 98.78% (3877/3925) Class. Accuracy on TN = 83.83% (3250/3877)	Detection FP = 1.22% (48/3925) Class. Accuracy on FP = 100.00% (48/48)
Adversarial	Detection FN = 2.14% (84/3925) Class. Accuracy on FN = 0.00% (0/84)	Detection TP = 97.86% (3841/3925) Class. Accuracy on TP = 0.00% (0/3841)

**Under PGD-30**

**TPR = 98%**

**FPR = 1%**

**Can we bypass the energy-based detection algorithm?**

**That is, can we craft an adversarial attack that fools the classifier yet has low energy?**

# Low Energy Projected Gradient Descent (LE-PGD)

PGD is a constrained optimization procedure.

PGD builds a perturbation through an iterative procedure in which at each step the direction of the gradient is followed to maximize the loss. The result is clipped to bound the Lp norm.

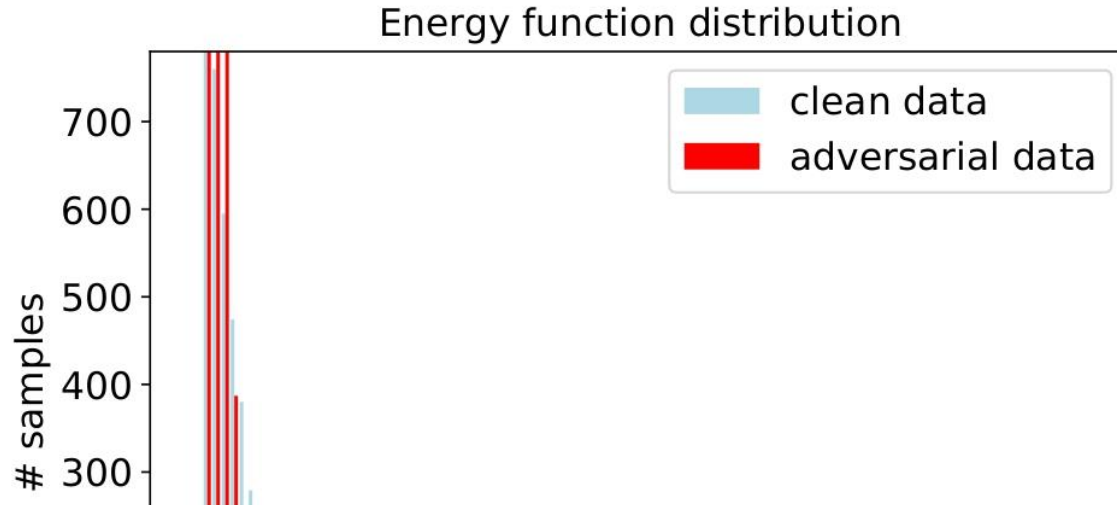
$$\text{PGD} \quad \mathbf{x}^* = \text{clip}_\epsilon \left( \mathbf{x}^* + \alpha \nabla_{\mathbf{x}^*} \mathcal{L}_\theta(\mathbf{x}^*, y) \right)$$

$$\text{LE-PGD} \quad \mathbf{x}^* = \text{clip}_\epsilon \left[ \mathbf{x}^* + \alpha \nabla_{\mathbf{x}^*} \left( \mathcal{L}_\theta(\mathbf{x}^*, y) - \lambda E_\theta(\mathbf{x}^*) \right) \right]$$

We add an energy regularizer in the optimization procedure.



# Low Energy Projected Gradient Descent (LE-PGD)



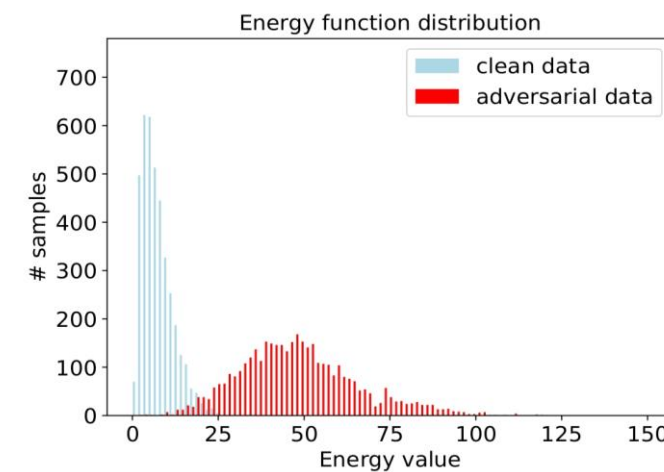
**The two distributions completely overlap.**

**Under LE-PGD the CNN accuracy still drops to 0%.**

**LE-PGD produces adversarial data that are «in-distribution» according to the model but the accuracy is still 0%.**

# Conclusion and Take Home Message

- We illustrated that it is not always possible to have both pixel-wise and transformation robustness for CNNs.
- We have shown that there is a correlation between energy and attack strength. Under this validation, we provide a detector for pixel-wise adversarial examples.
- We proposed a modification of the PGD attack to produce low energy adversarial examples to break the energy-based detector.



$$\mathbf{x}^* = \text{clip}_\epsilon \left[ \mathbf{x}^* + \alpha \nabla_{\mathbf{x}^*} \left( \mathcal{L}_\theta(\mathbf{x}^*, y) - \lambda E_\theta(\mathbf{x}^*) \right) \right]$$

Thank you!

# Detector Performances for Different Attacks

Attack	$\epsilon$	DR	FPR	FNR
PGD-5	8/255	83%	19%	17%
PGD-10	8/255	93%	4%	7%
PGD-30	8/255	98%	1%	2%
PGD-50	8/255	98%	0.7%	2%
PGD-50	16/255	99%	0.02%	0.1%

# Energy-based Model (EBM)

While most of the previous models had the goal of classification, EBMs are motivated from a different perspective: density estimation.

The fundamental idea of EBM is the energy function such that we approximate  $p(x)$  via :

$$p_{\theta}(x) = \frac{\exp(-E_{\theta}(x))}{\int_x \exp(-E_{\theta}(x))dx}$$

Notice that :  $\log p_{\theta}(x) = -E_{\theta}(x) - \underbrace{Z_{\theta}}_{\text{Constant for all } x}$

**Data points with high likelihood have a low energy, while data points with low likelihood have a high energy.**

## **Beyond Accuracy Metric :**

**We would like to have probabilistic models that are highly confident in their correct predictions and low in their incorrect ones.**

**Given 100 prediction, each with confidence 0.8, we expect that 80 should be correctly classified.**

**Despite their high accuracy modern CNNs are not well-calibrated.**

**A CNN overestimates/underestimates its predictions.**

[ Guo et al. ICML 2017 ]

## ECE Metric :

**Divide the interval  $[0,1]$  in  $m$  bins. Then, sort the predictions according the winning confidence class.**

$$\text{ECE} = \sum_{m=1}^M \frac{|B_m|}{n} \left| \text{Acc}(B_m) - \text{Conf}(B_m) \right|$$



# What is Classification?

