Estimating investment decisions: A comparison of two approaches*

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1 Antecedent estimation attempts

There is a long literature in economics that attempts to estimate the responsiveness of investment to changes in the costs of capital.[1] Early techniques were built around an accelerator model of capital investment. This permitted a simple relation of investment as a function of output. Unfortunately, these specifications used endogenous variables to explain investment decisions. They also ignored the costs and benefits of installing new capacity, a point emphasized by neoclassical investment theory. These critiques together asserted that the models were too reduced-form—improved models would need to incorporate some structural features to test neoclassical theory and to eliminate exogenous variables.

Early attempts were unsuccessful. Models with structural features often lagged in prediction horse races against accelerator models. They also suggested that coefficients on structural features, including user costs and taxes, were insignificant, an implausible result. The breakthrough required adding an additional structural feature: adjustment costs of capital.[2] Previous models theoretically permitted firms to change their capitals stocks by great amounts from one period to the next. Adding adjustment costs not only improved the realism of the models, but also added an explanation of the consistent correlation between output and investment; this relationship would arise if changes in output can be used by firms to predict future fundamentals.

From here, two approaches could be used to estimate investment models. The first is to use an Euler equation describing the optimal path of investment—investment today will be a function

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[1]This section is drawn from Hassett and Hubbard (2002).

[2]Original neoclassical models assumed that the supply of capital was perfectly elastic. Relaxation of this assumption also motivates the inclusion of adjustment costs.

of prices, taxes, interest rates, and the expectation of investment in the next period. An alternative approach is to model investment as a function of the user cost of capital today and the expectation of its future values. Additional assumptions can express these user costs as average q, the ratio of the firm's market value to the replacement cost of its capital.[3] If the previous generation of models implied implausibly-low effects of fundamentals, these reformulations suggested implausibly-high marginal adjustment costs, between \$1 and \$5 per dollar of investment, and still fell behind in horse races with accelerator models.

These models continued to perform poorly because they failed to address the issue of endogeneity. If the changes in the explanatory variables were not exogenous, then the coefficients on the structural parameters will be biased toward zero. Both of the papers compared here aim for exogeneity. Cummins, Hassett and Hubbard (1994) sought exogeneity by using major tax changes as natural experiments shocking the cost of capital. Hubbard, Kashyap and Whited (1995) used tax payments as an instrument. The papers also differ in their modeling strategies; Cummins, Hassett and Hubbard followed the q-theory approach, while Hubbard, Kashyap and Whited estimated the Euler equation.

2 Comparing the models

2.1 Cummins, Hassett, and Hubbard: q theory

This paper begins with a simple model:

$$\frac{I_{i,t}}{K_{i,t-1}} = \mathbb{E}_{t-1}[S_{i,t}\gamma] + \epsilon_{i,t}$$

where $I_{i,t}$ is investment by firm i at time t, $K_{i,t-1}$ is the capital stock at time $t-1$, $S_{i,t}$ is an identifying structural variable, including tax-adjusted user cost of capital or q, and γ receives a structural interpretation based upon adjustment cost assumptions. The model assumed that S was known immediately following the tax reforms that the authors examined, eliminating the need to

[3] Implementations based upon user costs represent demand for investment, while q theory models the supply of investment.

model the firm's expectation-setting process. The expectations operator, then, can be removed:

$$\frac{I_{i,t}}{K_{i,t-1}} = S_{i,t}\gamma + \epsilon_{i,t}$$

The use of taxes as a natural experiment assumed that taxes were not predictable by the firms. Hence, the model can be reformulated as the *unexpected* change in the investment-to-capital ratio as a function of the unexpected, tax-induced change in the structural parameter. The result is that the observed investment ratio is differenced from its conditional expectation, as opposed to its mean, to control for heterogeneity among firms. If the firms use linear projections to form expectations, the model becomes

$$\frac{I_{i,t}}{K_{i,t-1}} - P_{i,t-1}\left[\frac{I_{i,t}}{K_{i,t-1}}\right] = \left(S_{i,t} - P_{i,t-1}[S_{i,t}]\right)\gamma + \epsilon_{i,t} \tag{1a}$$

where $P_{i,t-1}$ is the linear projection operator based upon information from period $t-1$. More conveniently,

$$\omega_{i,t} = \gamma\psi_{i,t} + \epsilon_{i,t} \tag{1b}$$

where $\omega_{i,t}$ measures the deviation of investment due to the shock and $\psi_{i,t}$ measures the shock. This expression clearly shows how the authors imparted an exogenous interpretation to the model, as will be discussed in Section 3.1. The empirical approach will use first-stage regressions to estimate the projections and subsequent deviations as characterized in equation (1a) and a pooled cross-section will estimate γ via equation (1b).

To ensure exogeneity, the authors assumed that the expectation of all non-tax components of S in the next period will equal their value in the previous period. For example, the value of the interest rate expected in 1987 would be the end-of-year value from 1985. If another formulation was used, it might introduce endogeneity into the second-stage regression because tax changes may be caused by or cause changes in market fundamentals.

The authors estimated the model using both user costs of capital and q under quadratic adjustment costs. The authors also assumed that firms perfectly faced statutory tax parameters, ignoring gain-loss asymmetries and other quirks of the tax code. For Hubbard, Kashyap and Whited, however, the imperfect link between taxes and profits drove identification in their empirical strategy (see Section 3.2). Another contrast is that this model assumed perfect capital markets, while Hubbard, Kashyap and Whited attempted to disprove this theory. A final assumption is that firms perceive tax changes to be permanent. The authors argued that the typical length of a tax change, which is three years, is a first-order approximation to permanence under reasonable discount factors. This ignored the mean-reverting nature of investment tax credits, for example.

These assumptions led the authors to estimate the following model:

$$\frac{I_{i,t}}{K_{i,t-1}} - \widehat{\left(\frac{I_{i,t}}{K_{i,t-1}}\right)} = \mu_i + \Omega\left(Q_{i,t} - \hat{Q}_{i,t}\right) + \epsilon_{i,t} \tag{2}$$

where variables with hats are projections based on information available in period $t-2$, μ_i is a firm-specific constant, Ω is inversely related to adjustment costs, and $Q_{i,t}$, the tax-adjusted value of q, is defined as

$$Q_{i,t} = \frac{q_{i,t-2} - p_{t-2}(1-\Gamma_{i,t})}{1-\tau_t}$$

with $q_{i,t-2}$ being marginal q, $p_{i,t-2}$ the price of capital relative to output, $\Gamma_{i,t}$ the tax wedge, and τ_t the corporate tax rate. In practice, average q is calculated rather than marginal q. These two values are the same under perfect competition and constant returns to scale.

2.2 Hubbard, Kashyap, and Whited: An Euler equation approach

Hubbard, Kashyap and Whited questioned the reliability of estimates for marginal q. Reflecting this apprehension, this paper followed an Euler equation approach. To derive this expression, the authors equated the return generated by shares of a firm to the holders' required return. Solving this forward and assuming a no-bubble condition yielded an expression for the firm's value. This value is maximized by the firm subject to five constraints: the capital accounting identity, a cash flow equation that defines dividend payments and includes adjustment costs, non-negativity of dividends, a limitation on the level of share repurchases, and a transversality condition. Additionally, it is assumed that adjustment costs $\Psi(I_{it}, K_{i,t-1})$ take the form

$$\Psi(I_{it}, K_{i,t-1}) = \frac{\alpha}{2}\left(\frac{I_{it}}{K_{i,t-1}} - \nu\right) I_{it}$$

where ν is the firm's investment "bliss point." This specification conforms with the standard assumption of linear homogeneity in investment and capital, which yields a marginal q that is equal to the more easily observable average q. The result is the following model:

$$\left(\frac{1-\hat{\omega}_{it}}{1+(1-\tau)i_t-\pi_t^e}\right)\left[F_K(K_{it},N_{i,t+1})+\frac{\alpha}{2}\left(\frac{I_{i,t+1}}{K_{it}}\right)^2+(1-\delta)\left(\alpha\,\frac{I_{i,t+1}}{K_{it}}+\frac{p_{i,t+1}}{1-\tau}-\nu\right)\right] \\ -\alpha\left(\frac{I_{it}}{K_{i,t-1}}\right)-\frac{p_{it}}{1-\tau}+\nu=e_{i,t+1} \tag{3}$$

i_t is the nominal interest rate
π_t^e is the expected inflation rate at time t
p_{it} is the price of capital goods at time t relative to output, including taxes
τ is the corporate tax rate
K_{it} is the capital stock of firm i at time t
I_{it} is the firm's investment at time t
δ is the constant rate of economic depreciation
N_{it} is a vector of variable factors of production for firm i at time t
$F(K_{i,t-1},N_{it})$ is the firm's real revenue function, with $F_K>0$ and $F_{KK}<0$
$e_{i,t+1}$ is the expectational error, with mean 0 and is uncorrelated with anything known at time t

The parameter $\hat{\omega}_{it}$ captures deviations from perfect capital markets, manifested by differing constraints faced under debt and equity financing scenarios. The authors motivated this interpretation via three different theoretical derivations in the paper. They tested its significance from zero, the null hypothesis of perfect markets. Specifically, the test generated a statistic of orthogonality between the error term at $t+1$ and the variables observed at time t. These variables should be orthogonal under perfect capital markets; prediction errors should not be correlated with information that could have been used in making the prediction.

To estimate equation (3), the marginal cost of capital was parameterized under a constant mark-up (equal to μ) framework

$$F_K=\frac{Y_{i,t+1}-\mu C_{i,t+1}}{K_{it}} \tag{4}$$

where $Y_{i,t+1}$ is output and $C_{i,t+1}$ is real variable cost at time $t+1$. The parameter $\hat{\omega}_{it}$ was parameterized as

$$\hat{\omega}_{it}=\gamma_0+\gamma_1\frac{CF_{it}}{K_{i,t-1}} \tag{5}$$

where CF_{it} is cash flow. This is the simplest formulation that permits financing constraints to vary with internal funds. The authors hypothesized that γ_1 is less than 0; higher internal funds weaken the importance of the constraint.

2.3 Summary

While the two models appear to be fashioned in different ways, under certain assumptions, they are actually quite similar. Indeed, q, user cost, and Euler equation methods all arise from the same maximization problem if they utilized the same assumptions regarding technology, competition, and adjustment costs (Hassett and Hubbard 2002). While both models employed quadratic adjustment costs, the differences in assumptions include:

> Cummins, Hassett and Hubbard required perfect competition in output and capital markets and constant marginal costs to calculate marginal q;
>
> Hubbard, Kashyap and Whited permitted imperfect competition and returns to scale by using a mark-up formulation to derive marginal product and allowed for imperfect capital markets.

Hence, we would expect to achieve different results unless the mark-up parameter happened to be near unity and $\hat{\omega}$, the measure of imperfection in capital markets, was 0.

3 Comparing the sources of exogeneity

3.1 Cummins, Hassett, and Hubbard: Tax changes as natural experiments

This paper identifies four major changes in corporate tax policy to use as natural experiments. The criteria used for selection were that the tax wedge changed by more than 10%, no other tax changes occurred in the previous or subsequent year, and that the change was unanticipated during the previous year. These requirements made the events significant, isolated, and unexpected in order to acheive clear results. Cross-sectional variation in asset types, costs of investing, and tax wedges also aided identification.

The authors claimed that the changes are exogenous by assuming that they are unpredictable. They did admit, however, that aggregate variables are certainly correlated with tax changes, since tax policy is often aimed to be countercyclical. They claimed that this endogeneity does not carry down to the micro level, but they did not provide compelling evidence to assure the reader of this point. They explained that the wide variation of capital stock composition among firms would prevent endogeneity concerns from arising for any particular firm. An empirical verification of this results would have been interesting.

3.2 Hubbard, Kashyap, and Whited: Instrumental variables

One problem encountered in identifying equation (3) is that simultaneity arises from the presence of expected marginal product of capital. The authors attempted to overcome this issue by using instrumental variables. The instruments included twice-lagged values of the variables in the model, but replaced cash flow with tax payments.

The use of tax payments as an instrument was a novel approach by the paper. This variable was included to reduce the effect of marginal product of capital mismeasurement. If equation (4) was misspecified, then cash flow may carry information about future values of the marginal product. If serial correlation existed alongside measurement error, then *lagged* values of cash flow may also contain information, rendering those values useless as instruments in specifying the value of $\hat{\omega}_{it}$ in equation (5). While tax payments are well correlated with cash flow, they are only imperfectly correlated with profitability because of the asymmetric treatments of gains and losses, for example. The use of tax payments as an instrument for cash flow, the authors argued, reduced the import of measurement error in the marginal product.

Like Cummins, Hassett and Hubbard, the authors made claims of exogeneity, but did not provide empirical results to verify their assertions. This is especially worrisome here because the authors are relying upon tests of endogeneity to make claims about the presence of financial constraints. If tax payments did not actually combat marginal product mismeasurement, then incorrect inferences regarding financial constraints could have been made.

4 Conclusion

The estimate of the coefficient on Q found by Cummins, Hassett and Hubbard was 0.66. A similar estimate for Hubbard, Kashyap and Whited would be 1.28. While these numbers are quite different, both are more reasonable than previous estimates of this parameter. A bigger contrast is that the former paper found no evidence of financial constraints, while the latter concluded that the constraints were quite important to low-dividend-payout firms. Additional research incorporating the insights of these papers will lead to improved estimation attempts.

A natural experiment is a more compelling strategy than an instrumental variables approach because exogeneity is more readily apparent. Unfortunately, Cummins, Hassett and Hubbard did not entirely prove that the tax changes were unpredictable based upon firm characteristics. Still, I found their attempt at exogeneity more effective than that of Hubbard, Kashyap and Whited. And while similar neoclassical theory underlies both models, Cummins, Hassett and Hubbard were able to create a more transparent model as a result of their identification strategy.

Both models were static and neither accounted for general equilibrium effects. Prior to these modeling innovations, however, few models incorporating structural parameters were successful. Hence, it is unreasonable to fault these authors for not adding advanced considerations before beginning the bridge from theory to empirical validation.

References

Cummins, Jason G., Kevin A. Hassett and R. Glenn Hubbard. 1994. "A Reconsideration of Investment Behavior Using Tax Reforms as Natural Experiments." *Brookings Papers on Economic Activity* 1994(2):1–74.

Hassett, Kevin A. and R. Glenn Hubbard. 2002. Tax Policy and Business Investment. In *Handbook of Public Economics*, ed. Alan Auerbach and Martin Feldstein. Vol. 3 Elsevier Science.

Hubbard, R. Glenn, Anil K. Kashyap and Toni M. Whited. 1995. "Internal Finance and Firm Investment." *Journal of Money, Credit, and Banking* 27(3):683–701.