1) We first show that

$$r_0 \perp\!\!\!\perp Z | e(X_{Z=1}) \tag{1}$$

where $e(X_{Z=1})$ is the distribution of the propensity score for the treated units.

Note that $P(Z = 1|X) = E(Z|X)$. Following Rosenbaum and Rubin, we have:

$$\begin{aligned} \mathbb{E}(Z|r_0, e(X_{Z=1})) &= \mathbb{E}[\mathbb{E}[Z|r_0, X_{Z=1}]|r_0, e(X_{Z=1})] = \mathbb{E}[\mathbb{E}[Z|X_{Z=1}]|r_0, e(X_{Z=1})] \\ &= \mathbb{E}[e(X_{Z=1})|r_0, e(X_{Z=1})] = e(X_{Z=1}) \end{aligned}$$

and

$$\mathbb{E}(Z|e(X_{Z=1})) = \mathbb{E}[\mathbb{E}(Z|X_{Z=1})|e(X_{Z=1})] = \mathbb{E}(e(X_{Z=1})|e(X_{Z=1})) = e(X_{Z=1})$$

Thus, $\mathbb{E}(Z|r_0, e(X_{Z=1})) = \mathbb{E}(Z|e(X_{Z=1}))$, and so, (1) must hold.

Now, by (1) and the law of iterated expectations,

$$\begin{aligned} ATT &= \mathbb{E}(r_1 - r_0|Z = 1) = \mathbb{E}[\mathbb{E}(r_1|Z = 1, e(X_{Z=1})] - \mathbb{E}[\mathbb{E}(r_0|Z = 1, e(X_{Z=1})] \\ &= \mathbb{E}[\mathbb{E}(r_1|Z = 1, e(X_{Z=1})] - \mathbb{E}[\mathbb{E}(r_0|Z = 0, e(X_{Z=1})] \end{aligned}$$

We can compute this last expression using actual data. By the assumption that $e(X_{Z=1}) < 1$, the expectation $\mathbb{E}[\mathbb{E}(r_0|Z = 0, e(X_{Z=1})]$ is well defined.

To estimate the ATE unbiasedly, we need to strengthen the conditions to

$$\begin{gathered} r_0, r_1 \perp\!\!\!\perp Z|X \\ 0 < e(X) < 1 \end{gathered}$$

These are the conditions outlined in Rosenbaum and Rubin (1983).