Evidence of Excessive Buchanan Vote Share in Palm Beach County, Florida*

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Abstract

To assess the claim that the butterfly ballot used in Palm Beach County, Florida, affected the outcome of the 2000 presidential election, we provide a statistical analysis of the vote share received by Reform Party Presidential candidate Patrick Buchanan. Our analysis demonstrates that Buchanan's vote share in the county was much greater than would be expected given plausible models of the popular vote. In particular, using robust estimation of an overdispersed binomial regression model, we show that in a collection of 3,015 counties from across the United States, Palm Beach County had the second most anomalous Buchanan vote share. Palm Beach County also had the most anomalous Buchanan vote share in Florida. We also find that Buchanan's vote share among Palm Beach County's election day voters, who used the butterfly ballot, was four times as large as his vote share among the county's absentee voters, who did not use the butterfly ballot. Finally, we analyze precinct-level data from Palm Beach County and find patterns that are consistent with the hypothesis that the butterfly ballot caused Democrats to vote mistakenly for Buchanan.

Beginning on election day, November 7, 2000, Palm Beach County, Florida, attracted national and ultimately international attention because thousands of voters in the county complained that they had a difficult time understanding the now infamous *butterfly ballot* and that they consequently cast invalid or erroneous presidential votes.[1] Eleven groups of Palm Beach County voters filed independent lawsuits seeking relief because, they claimed, they and others had made mistakes in their votes for President because of the misleading and confusing format of the ballot.[2] Within four days of the election, more than a dozen scholars had posted to the Internet analyses of what appeared to be an exceptional electoral outcome in Palm Beach County. [3] Early analytical work drew attention to an apparent excess in the number of votes received in the county by 2000 Reform Party presidential candidate Patrick Buchanan who, in the initial and uncertified count of Palm Beach County ballots, received 3,407 votes. The technical approaches used in the early analytical work varied greatly, but most analysts agreed that 3,407 was many times more than the number of votes to be expected for Buchanan, with 2,058 being a conservative minimum for the number of votes that were counted for Buchanan but that were irregular.[4]

The number of votes—at least 2,000—that appear to have been given erroneously to Buchanan was larger than the final, certified vote difference that separated Gore from his Republican challenger, Texas Governor George Bush. Final election results gave Gore 2,912,253 and Bush 2,912,790 votes in Florida, a difference of only 537 votes.[5] Because Florida was a pivotal state in the overall

[1]Lawyers working for the Democratic party reportedly collected 10,000 affidavits sworn by voters with complaints about some aspect of their election day experiences in Palm Beach County. "Voting Scrutinized All Over Florida," *Associated Press*, Thursday, November 9, 2000. Don Van Natta Jr. and Michael Moss, "Democratic 'War Room' Tries to Oversee the Battle for Florida, to Mixed Results," *New York Times*, Saturday, November 11, 2000. David Firestone, "Party Mobilized on Ballot Even as Votes Were Cast," *New York Times*, Saturday, November 11, 2000. David Firestone, "Democrats Still Prefer to Keep Counting," *New York Times*, Saturday, November 11, 2000. Don Van Natta Jr., "Gore to Contest Recount Result in Palm Beach," *New York Times*, Saturday, November 25, 2000.

[2]The suits filed by such voters in the Fifteenth Judicial Circuit of Florida, West Palm Beach, were cases CL 00-10965, CL 00-10970, CL 00-10988AE, CL 00-109922AF, CL 00-11000AH, CL 00-11084AH, CL 00-11098AO, CL 00-1146AB, CL 00-1240AB, CL 00-129OAB and CL 00-113O2AO. The suits were consolidated by Administrative Order No. 2.061-11/00. Texts of the filings and of the Circuit Court rulings in the cases are available from `http://www.pbcountyclerk.com/`.

[3]Brady posted analysis on November 9, and Wand, Shotts, Sekhon, Mebane, and Herron posted on November 11. Brady, Herron, Mebane, Sekhon, Shotts, and Wand (2001) list others who posted such early results. Lists of empirical work posted on the Internet through the end of November, 2000, appear at `http://www.bestbookmarks.com/election/` (created by Jonathan O'Keeffe), `http://economics.about.com/money/economics/library/weekly/aa111000.htm` (created by John S. Irons), `http://www.sbgo.com/election.htm` (created by Sebago Associates) and `http://madison.hss.cmu.edu` (created by Greg Adams and Chris Fastnow).

[4]The estimated bound of 2,058 is reported in "Who Won Florida? Are the Palm Beach Votes Irregular?" by Bruce E. Hansen, posted on November 11, 2000. Paper posted at `http://www.ssc.wisc.edu/~bhansen/vote/vote.html`.

[5]In the days after the election, the Associated Press reported a 327 vote margin based on the initial, automatic

presidential election, the Palm Beach County butterfly ballot may have itself been pivotal.

What characterizes a butterfly ballot is the presence of two columns of candidate names that together sandwich a single column of punch holes. Figure 1 contains a picture of the Palm Beach County presidential ballot. The alleged source of confusion which some Gore supporters claim led them to vote for Buchanan concerns the second valid punch hole (#4). This hole corresponds to Buchanan, the first candidate on the right hand side of the ballot. But, from the perspective of the left hand side of the ballot, the second candidate listed is Gore. An individual who scanned down the left hand column of his or her presidential ballot and only later considered the right hand column could mistakenly assume that the punch holes correspond, respectively, to Bush, Gore, Browne, and so forth.

The question of whether the Palm Beach County butterfly ballot caused systematic voting errors is ultimately a question about ballot design.[6] As of this paper's writing, published research on ballot design is not extensive. Sinclair, Mark, Moore, Lavis, and Soldat (2000) report experimental evidence showing that a double-column ballot format like the one used in Palm Beach County can be more confusing and cause more voter errors than a single-column ballot. Hamilton and Ladd (1996) use a collection of North Carolina counties to show that the extent to which ballot format made straight-party voting easy had measurable though small effects on voting outcomes. Finally, Darcy (1986) argues that position effects on ballots—namely, which candidate is listed first—can have small consequences for candidate vote shares but that these effects do not matter much for races in which party labels are very important. In general, very little is known about optimal ballot design, the relationship between ballot format and error rates, and so forth.

Our paper's analysis of the Buchanan vote share in Palm Beach County combines a strategy of triangulation with the elimination of plausible alternative explanations. We show that different models using different types of data all point to the same conclusion regarding the Buchanan vote in Palm Beach County. We begin, in Section 1, by examining a collection of 3,015 counties from across the United States to show that the Buchanan vote share in the Palm Beach County was

recount across Florida. Daniel J. Wakin, "Bush Campaign Pushes for a Gore Concession," *New York Times*, Friday, November 10, 2000. In many counties the first recount did not include late-arriving overseas absentee ballots, which were expected to and ultimately did tilt heavily for Bush.

[6]The butterfly ballot format was an untested innovation of Theresa LePore, Supervisor of Elections for Palm Beach County, Florida.

highly unusual and excessively large. We also show that Palm Beach County, which did not use the butterfly ballot for the previous presidential election, did not have an unusual level of Reform support in 1996.

Section 2 describes the results of a natural experiment: Buchanan's vote share among Palm Beach County absentee voters, who did not use a butterfly ballot, was much lower than his vote share among election day voters, who did use the butterfly ballot. In Section 3 we examine precinct-level election day returns within Palm Beach County and find that Buchanan votes tended to come from the most liberal parts of the county. This pattern, which is absent in voting returns for other Reform candidates, is consistent with the claim that Buchanan's votes came from mistaken Gore supporters. Finally, Section 4 concludes.

1 Palm Beach County and Counties across the United States

We show that in Palm Beach County in the 2000 election the discrepancy between the share of votes expected for Buchanan on the basis of previous election results and the share of votes Buchanan actually received was exceptionally large. The discrepancy in Palm Beach County was the second largest among the 3,015 counties in the U.S. for which discrepancies could be estimated.[7] Most of the other counties for which we observe extremely large discrepancies either trace to documented balloting problems or occur in patterns that suggest that systematic and distinctive patterns of support for Buchanan developed in those places.

To measure discrepancy we use residuals from robust estimation of an overdispersed binomial regression model of the number of votes cast for Buchanan compared to the votes cast for all other presidential candidates.[8] We estimate a separate model for each state. The binomial model respects the fact that the basic data are counts of votes. Following McCullagh and Nelder (1989), we allow for overdispersion because we believe that the county-level data (and, further below, precinct-level data) are subject to unobserved internal clustering effects.

We use robust estimators for several reasons. The primary reason is obvious: the voter com-

[7] Our collection of counties (or county equivalents) excludes those of Alaska, Michigan, Delaware, Hawaii and Rhode Island. In Michigan Buchanan appeared only as a write-in candidate. Delaware, Hawaii and Rhode Island contain too few counties for us to analyze. Alaska does not report election returns by county.

[8] Western (1995) introduces a class of robust estimators for generalized linear models.

plaints, legal cases and media reports strongly suggest that the processes that produced the electoral results in Palm Beach County were substantially different from the processes that produced the results elsewhere in Florida. The robust estimators we use have a high *breakdown point* (Hampel, 1971; Donoho and Huber, 1983) so that they are consistent and produce reliable measures of discrepancy even if unusual voting processes occurred in several counties in a state.[9] A large anomaly in one county will not mask (Atkinson, 1986) comparable or perhaps somewhat smaller anomalies that occur in other counties (Hampel, Ronchetti, Rousseeuw, and Stahel, 1986, 67). An estimator that lacks a high breakdown point will underestimate the frequency of highly anomalous election results. Another reason to use robust estimators is the fact that the regression models we use are at best rough approximations for the processes that produced the vote counts (Hampel et al., 1986, 82). The estimators we use produce reliable measures of discrepancy even under such conditions, as long as the model gives a pretty good approximation for Buchanan's expected vote share in most counties in each state. Data weakness is another reason for robust estimation. Because our regressors include functions of results from the previous election, there is a generic kind of problem. If anomalies occurred in the earlier election at roughly the same rate as in the current one, then the data include observations that have distorted regressor values. The robust estimators we use protect against the influence such distorted regressors might otherwise have on the results. An observation that has a substantially distorted regressor will not affect the results for the other observations and will itself appear as an observation that has a large discrepancy.

A county that has a discrepancy larger in magnitude than a certain threshold is an *outlier*. The threshold is defined in terms of quantiles of the standard normal distribution.[10] If the threshold is large, it is highly likely that the relationship between current and previous election results in an outlier county differs from the relationship that approximates the data elsewhere in the state. The outlier county may have a different disturbance (e.g., a nonzero mean, a fat-tailed distribution), different regression coefficients or contaminated regressors. To explain why a county is an outlier requires investigation using additional information. There are many possible reasons for Palm

[9]Intuitively, the finite sample breakdown point of an estimator is the smallest proportion of the observations that must be replaced by arbitrary values in order to force the estimator to produce values arbitrarily far from the parameter values that generated the original data (Donoho and Huber, 1983).

[10]The threshold value is the value of tuning parameter c of the tanh estimator defined in the Appendix. We use $c = 4$.

Beach County to be an outlier. In later sections we provide further support for our claim that the cause of Palm Beach County's exceptional status is the butterfly ballot.

1.1 Robust Estimation of an Overdispersed Binomial Model

We use an overdispersed binomial model for the count y_i of votes for Buchanan out of m_i ballots cast in county i, $i = 1, \ldots, n$. For each county there are k observed regressors (including a constant) collected in a vector x_i. Following McCullagh and Nelder (1989, 125, eqn. 4.20), the mean and variance of y_i are

$$E(y_i \mid x_i, m_i) = m_i \pi_i, \tag{1}$$

$$E[(y_i - m_i\pi_i)^2 \mid x_i, m_i] = \sigma^2 m_i \pi_i (1 - \pi_i), \tag{2}$$

with $\sigma^2 > 0$ and, for an unknown constant vector of coefficient parameters β,

$$\pi_i = \frac{1}{1 + \exp(-x_i'\beta)}. \tag{3}$$

If $\sigma^2 > 1$ then there is overdispersion relative to a purely binomial model.

To estimate σ^2 we use a *least quartile difference* (LQD) estimator (Croux, Rousseeuw, and Hossjer, 1994; Rousseeuw and Croux, 1993) for the scale $\sigma = \sqrt{\sigma^2}$. Let $\hat{\sigma}$ denote the estimated scale value. Given $\hat{\sigma}$, we use a *hyperbolic tangent* (tanh) estimator (Hampel, Rousseeuw, and Ronchetti, 1981) for β. Let $\hat{\beta}$ denote the estimated coefficient vector. The estimators are described in more detail in the Appendix. An important product of the estimation is a weight $w_i \in [0, 1]$ for each county. If $w_i = 0$ then data from county i had no effect on $\hat{\beta}$, given $\hat{\sigma}$: the tanh estimator completely rejected the county as an outlier.

Given expected proportions $\hat{\pi}_i = [1 + \exp(-x_i'\hat{\beta})]^{-1}$, we use studentized residuals (Carroll and Ruppert, 1988, 31–34) to measure the discrepancy between actual and expected votes for Buchanan. The studentized residuals may be compared across counties, both within and across states.[11] A

[11] Strictly speaking, direct comparability holds asymptotically in the number of counties in each state, subject to various assumptions. The distribution of the studentized residuals for the non-outlying counties in each state probably approximates closely to a t-distribution for degrees of freedom roughly proportional to the number of counties in the state minus the number of regressors. The variation across states due to the varying degrees of freedom should be negligible compared to the variation due to the seriously anomalous processes that occurred in some counties.

standardized residual is

$$r_i = \frac{y_i - m_i\hat{\pi}_i}{\hat{\sigma}\sqrt{m_i\hat{\pi}_i(1-\hat{\pi}_i)}}. \tag{4}$$

To obtain studentized residuals we need to make a weighting adjustment for leverage (applying to the counties that have $w_i > 0$) or for forecasting error (applying to the counties that have $w_i = 0$). Let W denote the matrix that has diagonal entries $W_{ii} = w_i$ and off-diagonal entries equal to zero ($W_{ij} = 0$ for $i \neq j$). Let V denote the matrix that has $V_{ii} = [m_i\hat{\pi}_i(1-\hat{\pi}_i)]^{-1/2}$ and $V_{ij} = 0$ for $i \neq j$. Let X be the $n \times k$ matrix of the regressors (row i of X is x_i'). The diagonal values of

$$H = VX(X'VWVX)^{-1}X'V$$

provide robust estimates of the additional weights[12]. Let $h_i = H_{ii}$ if $w_i > 0$ and $h_i = -H_{ii}$ if $w_i = 0$. The studentized residual is

$$\tilde{r}_i = r_i/\sqrt{1-h_i}. \tag{5}$$

1.2 The Buchanan Vote Across the Country in 2000

To analyze the results of the 2000 election in counties across the country, we define y_i to be the number of votes counted for Patrick Buchanan in the 2000 presidential election in county i. The linear predictor is

$$x_i'\beta = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}, \tag{6}$$

where x_{1i} is the proportion of votes officially received by the Republican candidate in the 1996 presidential election in county i and x_{2i} is the proportion of votes officially received by the Reform Party candidate in the 1996 presidential election. It should be clear that our independent variables may be contaminated. The same kinds of irregularities that led to outliers in the 2000 election may have led to outliers in 1996. It is important that the estimator we have chosen is robust to contamination from x_i values as well as robust to contamination from the disturbance.

[12] McCullagh and Nelder (1989, 397) motivate a nonrobust version of H.

We estimate the model *separately* for each state in the United States. This is done because results not presented here show that we cannot pool the coefficient or dispersion parameters across states. Therefore we obtain separate estimates of β and σ in each state in the analysis. But the residuals of interest are comparable across states because of the studentization described above.

Figure 2 contains a histogram of studentized residuals for the 3,015 counties from across the United States in our data set. Palm Beach County, Florida, is the second largest outlier of all counties. Detailed results for all counties having a studentized residual greater than 4 or less than -4 are presented in Tables 1, 2 and 3.

With respect to Buchanan vote share, the most unusual county in the United States is Jasper County, South Carolina: the vote for Buchanan vastly exceeds expectations there. Jasper County did not receive much national media attention, because the county's total vote was a fraction of Bush's margin of victory and because the county's vote was immaterial to the outcome of the 2000 presidential contest.[13] Nonetheless, "the State Board of Canvassers unanimously said [...] that problems in the county council election were so numerous that a new election should be held."[14] In the Tillman precinct Gore and Bush each received one vote while Buchanan received 239 and Nader received 111.[15] A re-vote was ordered in Jasper County although not for the presidential race.

Palm Beach County is the second most unusual county. Additional perspective on Palm Beach County can be gained through Figure 4, which shows the studentized residuals for all counties in Florida. The figure plots the expected or predicted Buchanan vote share, based on 1996 presidential returns, versus corresponding studentized residuals. With the exception of Palm Beach County—which is pointed out explicitly—there appears to be no systematic relationship between expected vote share and studentized residual. Similarly, a normal quantile-quantile plot, seen in the Figure 5, describes the extent to which Palm Beach County was an outlier in 2000. The quantile-quantile plot is relatively smooth, excluding Palm Beach County, and the jump in the plot indicates that the studentized residual for this county lies far in the tail of the distribution of studentized residuals

[13] Bush easily won South Carolina—beating Gore by over 200,000 votes—and the total Jasper County presidential vote amounted to 6,406 cast ballots.

[14] "One Election Result Confirmed; Second Election Ruled a Do-over." *Associated Press State and Local Wire.* December 13, 2000.

[15] See the December 28, 2000 issue of *The Beaufort Gazette* for allegations regarding the Tillman precinct.

from Florida.

After Palm Beach County the next most anomalous county with respect to 2000 Buchanan vote share is Hancock County, West Virginia. A remarkable feature of Hancock County is that it is part of a geographically contiguous cluster of seven counties that had unusually high levels of votes for Buchanan. Five of the counties are in West Virginia and two are in the state of Ohio. Table 4 lists the counties in the cluster. Because it spans two states, it is highly unlikely that this exceptional burst of support for Buchanan reflects problems of ballot format or electoral administration. Most likely the reason is special success in mobilizing voters for Buchanan in those areas, perhaps on a basis of economic interests special to those areas. Another cluster of outlier counties that spans state boundaries includes three counties in Nebraska and two in Iowa.[16]

In contrast, and as shown in Figure 3, Jasper County is geographically somewhat isolated from other pro-Buchanan counties. The isolation bolsters our tracing of the anomaly in that county to problems in a single precinct.

Palm Beach County also does not belong to a cluster of exceptionally pro-Buchanan counties. In Figure 3 Palm Beach County clearly sticks out compared to its neighbors: Martin County to the north, Hendry County to the west, and Broward County to the south. Therefore, the explanation for Palm Beach County's large positive residual is most likely at the geographic resolution of county or lower. It is possible that Palm Beach County is an outlier because of only a few anomalous precincts. We engage this issue in Section 3.

Among counties with negative studentized residuals, the one with the largest residual is Cook County, Illinois, which is traditionally a heavily Democratic county. Cook County is surrounded by other Illinois counties that contributed very few votes to Buchanan. In other words, while Cook County was unusual insofar as its extreme lack of Buchanan votes, it is not unusual in the larger sense of being located in an area that is generally pro-Buchanan. Post-election news stories documented serious balloting problems in Cook County, although not focusing on the aberration in the vote recorded for Buchanan.[17]

Tables 3 and 4 show that there is a large cluster of negative outliers in Colorado. The Colorado

[16] A point of technical interest: The existence of such clusters of anomalous counties reinforces the importance of using a robust estimator that has a high breakdown point.

[17] John Mintz and Dan Keating, "A Racial Gap in Voided Votes," *Washington Post*, Wednesday, December 27, 2000.

cluster reflects a huge rift in the Reform Party there, which in addition abetted an extreme kind of ballot problem. John Hagelin—not Buchanan—was on the Colorado ballot as the Reform Party candidate. Buchanan appeared as the Colorado Freedom Party nominee.[18]

There are several outliers that look suspicious—such as Los Angeles, California, which has a studentized residual of 10.7. In this paper, we do not examine those counties in detail. The detection of an outlier is grounds for further investigation and not proof of voting irregularities as was definitively determined in the case of Jasper County, South Carolina. For our purposes, it is sufficient to note that Palm Beach County is one of the most unusual counties in the country. This finding supports—but of course on its own does not prove—that Buchanan received a larger number of votes than Palm Beach County voters intended to give him.

1.3 Outlier Detection in the 1996 Florida Election

The 1996 presidential ballot in Palm Beach County was not of the butterfly type. If the county also had excessive Reform Party vote share in 1996, compared to other Florida counties, this could provide a plausible alternative explanation for the apparently excessive Buchanan votes in 2000. The idea would be that support for the Reform Party has often been unusually high there. On the other hand, if the Reform Party vote share was exceptionally low in Palm Beach County in 1996, then the anomality of the 2000 vote share could be exaggerated. The 1996 data support neither of these alternatives.

The presidential election of 1996 featured three main contestants, one of whom was Reform Party leader Ross Perot. The other candidates on the ballot included Democratic incumbent Bill Clinton and Republican challenger Senator Bob Dole. In the overdispersed binomial model of equations (1), (2) and (3) for an analysis of Florida county data in 1996, y_i is the count of votes received by the Reform Party candidate in the 1996 presidential election in county i. Our regressors are the proportion of votes officially received by the Republican candidate in the 1992 presidential election in county i (x_{1i}), and the proportion of votes officially received by the Reform Party candidate in the 1992 presidential election (x_{2i}).

Figures 6 and 7 show the 1996 Florida county residuals. These contrast sharply with those

[18] "Judge Won't Rule on Reform Party." *Associated Press.* August 30, 2000.

shown for the 2000 residuals. In the 1996 data there are three outliers, all with negative residuals: Pinellas County ($\tilde{r}_i = -4.4$), St. Lucie County ($\tilde{r}_i = -4.0$) and Okaloosa County ($\tilde{r}_i = -3.9$). Palm Beach County is in the tail of the distribution, with a residual of $\tilde{r}_i = -2.2$, but it is not an outlier. Certainly it is not a *positive* outlier. This result undermines a potential critique based on historically high support for the Reform Party.

Palm Beach County was not a negative outlier in 1996. In terms of raw rank position, the Reform Party vote in 1996 was not exceptionally low compared to 1992. In the 1992 presidential election Palm Beach County had the 11th lowest Reform vote share among counties in Florida and in the 1996 presidential election it was the 7th lowest. As a counterfactual thought experiment we consider how large a vote share in 1996 would have been required for the 2000 vote share not to be an outlier. A simulation exercise shows that the 1996 vote share of 8 percent would have to have been 18 percent in order to bring the 2000 studentized residual below 4.0. A value of 18 percent in 1996 would have put Palm Beach County among the five *highest* counties in terms of support for the Reform Party's presidential candidate in the state.

2 A Natural Experiment: Election day and Absentee Voters in 2000

The previous section demonstrates that Buchanan's Palm Beach County vote share in the 2000 presidential election was anomalous and that this support was not a reflection of historical support for the Reform Party. Although it is reasonable to conclude that the butterfly ballot caused this anomaly, it is also conceivable that some other factor was responsible. For example, it could be that the county experienced an exceptional surge in voter support for either the Reform Party in general or for Patrick Buchanan in particular. We now address this possibility.

Since the 2000 absentee presidential ballot in Palm Beach County was not a butterfly ballot, we can make use of a natural experiment: one group of Palm Beach County voters (election day) used a butterfly ballot but a second group (absentee) did not. If Buchanan's vote total in Palm Beach county reflects true support among the voters, then this support should be present in both pools of ballots. But if the butterfly ballot is responsible for Buchanan's vote share then his support should

come disproportionately from election day ballots.

A limitation of this natural experiment is that the selection mechanism which allocates voters to either the election day pool or the absentee pool is not truly random. Voters self-select to be in the absentee pool; for instance, certain demographic or ideological groups may be prone to taking November vacations and hence casting absentee ballots. Furthermore, some Florida voters—e.g., military officers—must cast absentee ballots. Thus this group may not be representative of voters in general.

Absentee voters in Florida are generally thought to be more politically conservative than election day voters, and this means that the natural experiment which fosters a comparison of election day and absentee ballots may be biased. Specifically, the absentee ballots should disproportionately go to conservative candidates such as Bush or Buchanan.

Figure 8 contains a chart displaying the distribution of Buchanan vote share across election day and absentee ballots in ten Florida counties. Ignoring Palm Beach County for the moment, it is apparent from Figure 8 that, in general, support for Buchanan in absentee ballots was similar or slightly greater than the corresponding support in election day ballots.[19] This is intuitive in light of the conventional wisdom that absentee voters are more conservative than their election day counterparts.

What is striking about Figure 8 is the gross discrepancy between election day and absentee support for Buchanan in Palm Beach County. In particular, the former dwarfs the latter, in sharp contrast to the patterns observed in the other nine counties.

It is possible, of course, that Buchanan supporters in Palm Beach County are peculiarly averse to casting absentee ballots. However, it is difficult to see why this would be the case, particularly considering the fact that no such aversion is observable in the other nine Florida counties we examined. Overall, the results of the election day versus absentee ballot natural experiment support the conclusion that Buchanan's anomalous support was a product of the butterfly ballot.

[19] Although it is difficult to see, in Brevard County Buchanan vote share on absentee ballots was just slightly greater than Buchanan vote share on election day.

3 Precinct-Level Analysis of Palm Beach County Returns

The results of the previous sections strongly suggest that Buchanan's vote total in Palm Beach County was not a true reflection of voter intentions. A crucial question remains: Who made the mistakes? Was it voters who wanted to vote for Bush or those who wanted to vote for Gore?

At an intuitive level, mistakes seem less likely for Bush voters, who had to match the first candidate with the first punch hole, than for Gore voters, who had to match the second candidate with the third punch hole. Of course, a large number of Democratic voters complained that the butterfly ballot caused them to vote for Buchanan by mistake. Nonetheless, it is important to examine the possibility that Buchanan received votes intended for both candidates.[20] To do that, we estimate an overdispersed binomial model for precinct-level election returns across Palm Beach County. The linear predictor of (6) now has for regressors the vote share in each election-day precinct for U.S. Senate candidates Democrat Bill Nelson (x_{1i}) and Reform Party candidate Joel Deckard (x_{2i}). We use the robust tanh estimator. If the butterfly ballot did not cause any systematic voting errors, we would expect that support for Nelson (D) in a precinct would be *negatively* associated with support for Buchanan and that support for Deckard (Ref.) would be *positively* associated with support for Buchanan. If the butterfly ballot did cause asymmetric voting errors, we would expect support for Nelson (D) to be *positively*—instead of *negatively*—associated with support for Buchanan.

The first line in Table 5 presents the results for election-day precincts across the whole of Palm Beach County. Support for Nelson (D) is *positively* associated with support for Buchanan. Such a pattern is what we would expect to observe if the butterfly ballot caused many Democrats to vote for Buchanan. The pattern supports the claim that Buchanan's votes tended to come from mistaken Gore supporters.

The pattern does not occur for votes for other offices that included a Reform candidate but did not use the butterfly ballot. In 2000 in Palm Beach County, Reform Party candidates contested the legislative races in both Florida State Senate District 35 and Florida U.S. Congressional District 16. Only the presidential portion of the 2000 Palm Beach County ballot used the butterfly format, so

[20] It is worth noting that even if Republicans and Democrats were equally likely to mistakenly vote for Buchanan, the net effect of the butterfly ballot would strongly favor Bush since Palm Beach County is heavily Democratic.

it is probably safe to assume that the outcomes of the non-presidential races in Palm Beach County were not contaminated by the butterfly ballot. Using this assumption, we are able to examine the relationship between Reform Party support and Democratic Party support both in the presence and in the absence of the butterfly ballot.

Restricting attention to Palm Beach County election-day precincts in State Senate District 35, we robustly estimate two separate models. In the first model y_i is the count of votes received by Buchanan, and in the second y_i is defined as the count of votes received by Reform Party State Senate candidate Sherree Lowe. We also estimate two parallel models restricting attention to Palm Beach County election-day precincts in Congressional District 16. In the first model y_i is the count of votes received by Buchanan, and in the second model y_i is the count of votes received by Reform Party candidate John McGuire. Because the butterfly ballot was relevant only for the presidential vote, we expect support for Nelson (D) to be *negatively* associated with support both for Lowe and for McGuire.

Turning first to the results in Table 5 for District 35, we see that support for Nelson (D) remains *positively* associated with support for Buchanan across precincts in the district. But support for Nelson (D) is *negatively* associated with support for Lowe (Ref.). As expected, the coefficient associated with the Nelson (D) vote share switches sign. The results for Congressional District 16 parallel those of State Senate District 35. Across the precincts of Congressional District 16, support for Nelson (D) is *positively* associated with support for Buchanan, but support for Nelson (D) is *negatively* associated with support for McGuire (Ref.). We also find that in both districts support for Deckard (Ref.) is not significantly associated with support for Buchanan, but it is positively associated with support for McGuire (Ref.).

In the absence of claims about the butterfly ballot, the precinct-level findings would be quite counter-intuitive. One would expect precincts that are *more* Democratically-inclined with respect to the U.S. Senate race to be *less* Reform Party-inclined when it comes to other races. The results presented in Table 5 support the claim that the butterfly ballot caused systematic, biased voter errors that cost Gore more lost votes than Bush. Democratic-inclined precincts (as measured by the Nelson (D) vote share) are less likely to vote for Reform party candidates in general (e.g., Lowe and McGuire) but are more likely to vote for Buchanan. The difference is the butterfly ballot.

The key features of Table 5 are the contrasts between Buchanan and Lowe and between Buchanan and McGuire. It is possible to explain the positive association between the Nelson (D) vote share and the Buchanan vote share by asserting that Reform Party members in Palm Beach County chose to live among Democrats. The unintuitive nature of such an assertion notwithstanding, the assertion contradicts the finding that the Nelson (D) vote is negatively associated with both the Lowe (Ref.) and Mcguire (Ref.) vote shares. We have the additional oddity that there is no significant relationship between votes for Deckard, the Reform candidate for the U.S.. Senate, and votes for Buchanan. But there is, as one would expect, a strong positive relationship between votes for Deckard and votes for at least one of the other two Reform party candidates.

The results in this section also enable us to examine, and refute, one final alternative explanation for Buchanan's anomalous vote total in Palm Beach County. It is conceivable that Buchanan's Palm Beach County vote total was caused by a group of anomalous precincts within the county. Anomalous results concentrated within a few precincts would suggest that excess votes were the result of localized phenomena, rather than the butterfly ballot, which was used uniformly throughout the county. For example malfunctioning vote machines such as the one in Tillman precinct in Jasper County, South Carolina, could have recorded extra votes for Buchanan in a few precincts. Alternatively, intentional fraud in a few precincts could be the source of Buchanan's success. Finally, pockets of intense election-day Reform Party mobilization could have delivered the extra votes. Such explanations are, however, quite difficult to reconcile with the precinct-level patterns documented in this section. Given our use of a robust estimator, the coefficients would not be affected by a few precinct-level anomalies produced by irregular voting processes. Moreover, a localized mobilization effort should affect outcomes in multiple races whereas the peculiar relationship between Nelson (D) vote share and Reform vote share is present only in the presidential race, which used the butterfly ballot.

4 Discussion

Palm Beach County is traditionally considered a politically liberal county, one that, according to conventional wisdom, should provide relatively few Buchanan votes. Despite this, in the 2000 presidential election the county supplied 19.6% of Buchanan's votes in the state of Florida. In

contrast, in the 1996 Republican presidential primary—which did not use the controversial butterfly ballot—only 5.4% of Buchanan's Florida votes came from Palm Beach County. Similarly, in the 1996 presidential election the county was less pro-Reform Party than Florida as a whole, but this pattern was reversed in 2000.[21]

We have examined the source of this puzzle—the seemingly anomalous Buchanan support in Palm Beach County—by focusing on allegations that the county's use of a butterfly ballot caused systematic voting errors that boosted the vote share of Patrick Buchanan.[22] In particular, we presented a robust outlier detection method and showed that, with respect to the Reform Party vote in 2000, Palm Beach county is the second largest outlier among all counties in the United States. We also showed that Palm Beach county was not a Reform Party vote outlier in 1996, the year of a presidential election in which the county did not use a butterfly ballot. As part of our outlier detection study we have identified counties other than Palm Beach County in the U.S. which may be anomalous in 2000. But, for the purposes of this paper, we focus on Palm Beach County because of the allegations voters made on election day and, more importantly, because Palm Beach County may have been pivotal in the 2000 presidential election.

Having identified Palm Beach County as an outlier, we have sought to determine whether the butterfly ballot was indeed the mechanism that caused this anomaly. A comparison of election day versus absentee results in Palm Beach County shows that Buchanan's success in Palm Beach County did not extend to absentee voters, who did not use the butterfly ballot. Finally, we have examined the claim that Democratic presidential candidate Al Gore in particular was harmed by the butterfly ballot. We found that Buchanan's support in Palm Beach County tended to come from more Democratic precincts, a pattern that supports the claim that Buchanan's votes tended to come from mistaken Gore supporters.

[21] In 2000, Buchanan received 0.787% of the presidential vote in Palm Beach County while garnering only 0.3% of the overall Florida presidential vote. In contrast, Ross Perot, the Reform Party candidate for president in 1996, received 9.1% of the Florida vote share while garnering only 7.7% of the Palm Beach County vote. These data are from the Florida Department of State and are available at `http://election.dos.state.fl.us/online/elecresu.shtml`.

[22] Another butterfly ballot related allegation is that the Palm Beach County ballot led to excessive overvoting in the presidential race. Ballots which contain presidential overvotes are those in which there are votes for more than one presidential candidate. The subject of overvoting is beyond the scope of this paper, and the authors are currently engaged in research on overvoting and undervoting in the 2000 presidential election.

5 Appendix: Robust Estimation Methodology

5.1 Robust Overdispersed Binomial Model Implementation

The point of departure for our estimation method is the fact that if the overdispersed binomial model of (1) and (2) is correctly specified then given a consistent estimate $\hat{\beta}$ for β and hence $\hat{\pi}_i = [1 + \exp(-x_i'\hat{\beta})]^{-1}$, residuals of the form

$$r_i^* = \frac{y_i - m_i\hat{\pi}_i}{\sqrt{m_i\hat{\pi}_i(1-\hat{\pi}_i)}} \tag{7}$$

are approximately normal.[23] Indeed, a good moment estimator for σ^2 may be defined in terms of r_i^* (McCullagh and Nelder, 1989, 126–127, eqn 4.23).

The LQD estimator focuses on the $\binom{h_k}{2}$ order statistic of the set $\{|r_i^* - r_j^*| : i < j\}$ of absolute differences, where $\{|r_i^* - r_j^*| : i < j\}$ has $\binom{n}{2}$ elements and $h_k = \lceil (n+k)/2 \rceil$. Following Croux et al. (1994) we use the notation

$$Q_n^* = \{|r_i^* - r_j^*| : i < j\}_{\binom{h_k}{2}:\binom{n}{2}} \tag{8}$$

to denote that order statistic. To implement LQD we choose estimates $\hat{\beta}$ to minimize Q_n^*. Let $\hat{\beta}_{\text{LQD}^*}$ designate the estimated coefficient vector and let $\hat{Q}_n^*$ designate the corresponding minimized value of Q_n^*. The LQD scale estimate is

$$\hat{\sigma} = \hat{Q}_n^* \frac{1}{\sqrt{2}\Phi^{-1}(5/8)}, \tag{9}$$

where Φ^{-1} is the quantile function for the standard normal distribution (Rousseeuw and Croux, 1993, 1277).

[23] The approximate normality depends on conditions such as independence across i and large values for m_i and $m_i\pi_i(1-\pi_i)$.

The tanh estimator is based on the function

$$\psi(x) = \begin{cases} x, & \text{for } 0 \le |x| \le p \\ (A(d-1))^{1/2} \tanh[\frac{1}{2}((d-1)B^2/A)^{1/2}(c-|x|)]\,\text{sign}(x), & \text{for } p \le |x| \le c \\ 0, & \text{for } c \le |x| \end{cases}$$

where p, c, d, A and B are constants satisfying $0 < p < c$ and other conditions.[24] Given a scale estimate $\hat{\sigma}$ and a vector of trial estimates $\hat{\beta}$, we compute the standardized residuals r_i of (4) and then weights

$$w_i = \begin{cases} \psi(|r_i|)/|r_i|, & \text{for } r_i \neq 0 \\ 1, & \text{for } r_i = 0. \end{cases}$$

Observation i is weighted by w_i in what is otherwise the usual iteratively reweighted least squares algorithm to estimate β. The given scale value remains unchanged but the weights are updated to match the current coefficient estimates. Numerical convergence is required for both the $\hat{\beta}$ values and the weights. Because redescending M-estimators such as the tanh estimator have multiple solutions, starting values affect the results. We use $\hat{\beta}_{\text{LQD}^*}$ to start the coefficients and use the LQD values $(r_i^* - \text{med}_i\, r_i^*)/\hat{\sigma}$ for an initial set of residuals ($\text{med}_i\, r_i^*$ denotes the median of the r_i^* values, $i = 1, \ldots, n$). To estimate the asymptotic variance of the coefficient estimates we use the sandwich estimator (White, 1994, 92) $\text{avar}(\hat{\beta}) = \hat{J}^{-1}\hat{I}\hat{J}^{-1}$ where $\hat{I}$ denotes the outer product of the score and $\hat{J}$ denotes the Hessian matrix.

The particular tanh estimator we use has $c = 4.0$, $d = 5.0$ and values for p, A and B as given in the indicated row of Table 2 in Hampel et al. (1981, 645).[25] The value of c fixes the threshold for the magnitude of r_i beyond which an observation is completely rejected by assigning it a weight $w_i = 0$.

[24] For details see Hampel et al. (1981, 645) or Hampel et al. (1986, 160).

[25] Hampel et al. (1981) use k for the tuning parameter of ψ that we have denoted by d. The same information about the tuning parameters appears in Table 2 of (Hampel et al., 1986, 163) with notation r and k used for the parameters we have denoted by c and d.

5.2 Background

The classical linear regression model for n observations and k regressors has the form $y_i = x_i'\beta + \epsilon_i$, $i = 1, \ldots, n$, where β is a vector of unknown coefficients, the data y_i and x_i are observed, and the unobserved disturbance ϵ_i has conditional mean $E(\epsilon_i \mid x_i) = 0$ and variance σ^2. We seek parameter estimates $\hat{\beta}$ that converge in probability to β as n gets large: we want *consistent* estimates.[26] Least squares (LS) chooses $\hat{\beta}$ to minimize the sum of squared residuals $r_i = y_i - x_i\hat{\beta}$ over all $i = 1, \ldots, n$:

$$\hat{\beta}_{\text{LS}} = \underset{\hat{\beta}}{\text{argmin}} \sum_{i=1}^{n} r_i^2. \tag{10}$$

Because even one contaminated data point can cause $\hat{\beta}_{\text{LS}}$ to take values arbitrarily different from β, LS has a breakdown point of $1/n$ (asymptotically, 0). LS is not robust.

For the regression model the maximum possible breakdown point is, asymptotically, 0.5. One popular estimator that achieves that maximum is *least median of squares* (LMS):

$$\hat{\beta}_{\text{LMS}} = \underset{\hat{\beta}}{\text{argmin}} \underset{i}{\text{med}}\, r_i^2 \tag{11}$$

(Rousseeuw, 1984; Rousseeuw and Leroy, 1987).[27] LMS has two important defects. $\hat{\beta}_{\text{LMS}}$ is consistent for β, but the estimator converges at the slow rate of $n^{-1/3}$. LMS also is inefficient when the disturbance e_i is an identically and independently distributed Gaussian random variable (i.e., no ϵ_i outliers) and the model is otherwise correctly specified. One way to achieve greater efficiency is to use LMS estimates as starting values for a redescending M-estimator (Hampel et al., 1981).

Other estimators exist that achieve the maximum breakdown point while having a $n^{-1/2}$ convergence rate and better Gaussian efficiency than LMS does. The LQD estimator (Croux et al., 1994) is defined by choosing $\hat{\beta}$ to minimize the (approximately) first quartile of the absolute differences between pairs of residuals. Let

$$Q_n = \{|r_i - r_j| : i < j\}_{\binom{h_k}{2}:\binom{n}{2}} \tag{12}$$

[26] To be technically precise, the statistical theory that supports the development of the tanh and LQD estimators assumes the existence of a *Fisher-consistent* estimator (Hampel et al., 1986, 83).

[27] Western (1995) presents LMS. Christmann (1994) discusses application of LMS to a grouped binomial model (albeit without overdispersion).

denote the $\binom{h_k}{2}$ order statistic of the set $\{|r_i - r_j| : i < j\}$ of absolute differences. The LQD estimator is

$$\hat{\beta}_{\mathrm{LQD}} = \underset{\hat{\beta}}{\operatorname{argmin}}\, Q_n. \tag{13}$$

For the linear regression model, LQD does not estimate the intercept term, because the differences $r_i - r_j$ do not depend on the overall mean. If necessary an intercept must be estimated separately.[28] For the remaining elements of β, LQD is consistent with $n^{-1/2}$ convergence rate and Gaussian efficiency of 67.1% (Croux et al., 1994). In addition to the efficiency gain over LMS, LQD provides a superior estimate of the scale (i.e., σ) when ϵ_i has an asymmetric distribution, because LQD does not estimate the scale by measuring a symmetric spread of the residuals around a central location value (Rousseeuw and Croux, 1993).

The LQD objective function is difficult to optimize. Because high breakdown point estimators are not smooth functions of the data, optimization techniques that are based solely on derivative information, such as Newton-Raphson, are highly unreliable (Stromberg, 1993). In general, high breakdown point objective functions have multiple minima. Therefore, the use of local optimization techniques is not reliable. But in our application, there does appear to be local hill-climbing information contained in the derivatives. Therefore, we use a global optimizer which makes use of derivative information: GENetic Optimization Using Derivatives (GENOUD) (Sekhon and Mebane, 1998). GENOUD combines evolutionary algorithm methods with a derivative-based, quasi-Newton method to solve difficult unconstrained optimization problems.[29]

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[28] In the overdispersed binomial model there is information about the constant from the factor $[\hat{\pi}_i(1-\hat{\pi}_i)]^{-1/2}$ in r_i^*.

[29] Please see `http://jsekhon.fas.harvard.edu/genoud/` for software.

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Table 1: Counties with the Largest Positive Studentized Residuals, National Analysis

State	County Name	Studentized	Predicted VP†	Actual VP	Order
SC	Jasper	26.95	0.0012	0.0379	1
FL	Palm Beach	20.08	0.0016	0.0079	2
WV	Hancock	19.15	0.0052	0.0289	3
WV	Brooke	16.77	0.0048	0.0286	4
KS	Pottawatomie	16.09	0.0112	0.0528	5
OH	Jefferson	12.70	0.0079	0.0303	6
OH	Athens	12.32	0.0061	0.0292	7
IA	Woodbury	11.20	0.0039	0.0107	8
WV	Ohio	11.12	0.0033	0.0126	9
WV	Marshall	11.07	0.0055	0.0196	10
CA	Los Angeles	10.67	0.0027	0.0043	11
MT	Silver Bow	8.70	0.0101	0.0265	12
AR	Union	8.32	0.0060	0.0186	13
IA	Plymouth	8.28	0.0042	0.0143	14
AR	Faulkner	8.26	0.0057	0.0148	15
NC	Durham	8.20	0.0015	0.0047	16
NE	Thurston	8.09	0.0053	0.0245	17
NE	Dakota	7.74	0.0046	0.0144	18
NE	Dixon	6.65	0.0077	0.0233	19
KS	Shawnee	6.49	0.0058	0.0097	20
VA	Warren	6.37	0.0026	0.0088	21
OH	Belmont	6.37	0.0089	0.0218	22
OH	Ashland	6.24	0.0057	0.0176	23
NY	Erie	6.19	0.0051	0.0079	24
PA	Allegheny	6.14	0.0021	0.0036	25

Notes: Results based on 3,015 counties. This table presents all counties with studentized residuals greater than 4. Please see Section 1 for details.

† VP is the vote proportion.

Table 2: Counties with the Largest Positive Studentized Residuals, National Analysis (cont'd)

State	County Name	Studentized	Predicted VP†	Actual VP	Order
IA	Dubuque	5.95	0.0039	0.0074	26
MS	Leflore	5.52	0.0011	0.0039	27
WI	Marathon	5.37	0.0050	0.0116	28
IA	Sioux	5.32	0.0016	0.0056	29
GA	Whitfield	5.31	0.0045	0.0081	30
MS	Washington	5.26	0.0012	0.0033	31
KY	Boone	5.19	0.0021	0.0043	32
GA	Richmond	5.14	0.0027	0.0044	33
WI	Lincoln	5.04	0.0051	0.0178	34
KY	Kenton	4.97	0.0021	0.0036	35
MN	Clay	4.94	0.0045	0.0183	36
MS	Wilkinson	4.81	0.0022	0.0078	37
IL	Sangamon	4.41	0.0028	0.0050	38
WI	Wood	4.35	0.0053	0.0123	39
IA	Benton	4.28	0.0036	0.0081	40
IN	Allen	4.26	0.0058	0.0083	41
IL	Effingham	4.18	0.0037	0.0099	42
WV	Wetzel	4.16	0.0055	0.0129	43
KS	Wyandotte	4.10	0.0032	0.0057	44
AL	Lauderdale	4.04	0.0052	0.0092	45
VA	Manassas City	4.03	0.0017	0.0047	46
MD	Somerset	4.03	0.0024	0.0064	47
SC	Abbeville	4.01	0.0021	0.0084	48
MS	Jones	4.00	0.0019	0.0037	49

Notes: Results based on 3,015 counties. This table presents all counties with studentized residuals greater than 4. Please see Section 1 for details.

† VP is the vote proportion.

Table 3: Counties with the Largest Negative Studentized Residuals, National Analysis

State	County Name	Studentized	Predicted VP†	Actual VP	Order
IL	Cook	−13.42	0.0038	0.0020	1
LA	Orleans	−12.66	0.0198	0.0035	2
CO	Arapahoe	−7.66	0.0105	0.0048	3
CO	Denver	−7.58	0.0094	0.0042	4
CO	Douglas	−6.93	0.0114	0.0034	5
KS	Johnson	−6.92	0.0051	0.0028	6
CO	Boulder	−6.92	0.0090	0.0034	7
CO	Jefferson	−5.87	0.0096	0.0058	8
LA	Caddo	−5.37	0.0105	0.0035	9
TX	Dallas	−5.03	0.0019	0.0015	10
LA	East Baton Rouge	−4.75	0.0082	0.0041	11
MO	St. Charles	−4.74	0.0051	0.0026	12
CO	Larimer	−4.68	0.0090	0.0049	13
TN	Davidson	−4.58	0.0020	0.0010	14
NV	Washoe	−4.30	0.0077	0.0056	15
MO	St. Louis	−4.25	0.0033	0.0024	16
TN	Shelby	−4.21	0.0015	0.0009	17
WY	Albany	−4.17	0.0126	0.0055	18
CO	El Paso	−4.03	0.0102	0.0073	19

Notes: Results based on 3,015 counties. This table presents all counties with studentized residuals less than −4. Please see Section 1 for details.

† VP is the vote proportion.

Table 4: Contiguous Counties Among Those that Have the Largest Studentized Residuals

State	County Name	Studentized
WV	Hancock	19.15
WV	Brooke	16.77
OH	Jefferson	12.70
WV	Ohio	11.12
WV	Marshall	11.07
OH	Belmont	6.37
WV	Wetzel	4.16
KS	Pottawatomie	16.49
KS	Shawnee	6.50
IA	Woodbury	11.20
IA	Plymouth	8.28
NE	Thurston	7.98
NE	Dakota	7.73
NE	Dixon	6.63
WI	Marathon	5.84
WI	Lincoln	5.37
WI	Wood	4.80
KY	Boone	5.19
KY	Kenton	4.97
CO	Arapahoe	−7.66
CO	Denver	−7.58
CO	Douglas	−6.93
CO	Boulder	−6.92
CO	Jefferson	−5.87
CO	Larimer	−4.68
CO	El Paso	−4.03
MO	St. Charles	−4.74
MO	St. Louis	−4.25

Notes: Results based on 3,015 counties. This table presents all counties with studentized residuals of magnitude greater than 4.0 that are contiguous. Please see Section 1 for details.

Table 5: Reform Party Vote Shares across Palm Beach County

Scope	Candidate	Intercept	Nelson	Deckard
All Precincts	Buchanan	−6.17 (0.14)	2.06 (0.20)	−12.74 (13.56)
District 35	Buchanan	−7.48 (0.47)	3.85 (0.66)	13.13 (23.16)
District 35	Lowe	−1.98 (0.32)	−1.86 (0.48)	18.06 (14.19)
District 16	Buchanan	−7.00 (0.27)	3.32 (0.43)	3.95 (14.62)
District 16	McGuire	−3.37 (0.29)	−1.00 (0.49)	25.96 (9.09)

Notes: tanh estimator with precinct-level data from the 2000 election. Sandwich standard errors in parentheses.

Figure 1: The Palm Beach County Butterfly Ballot

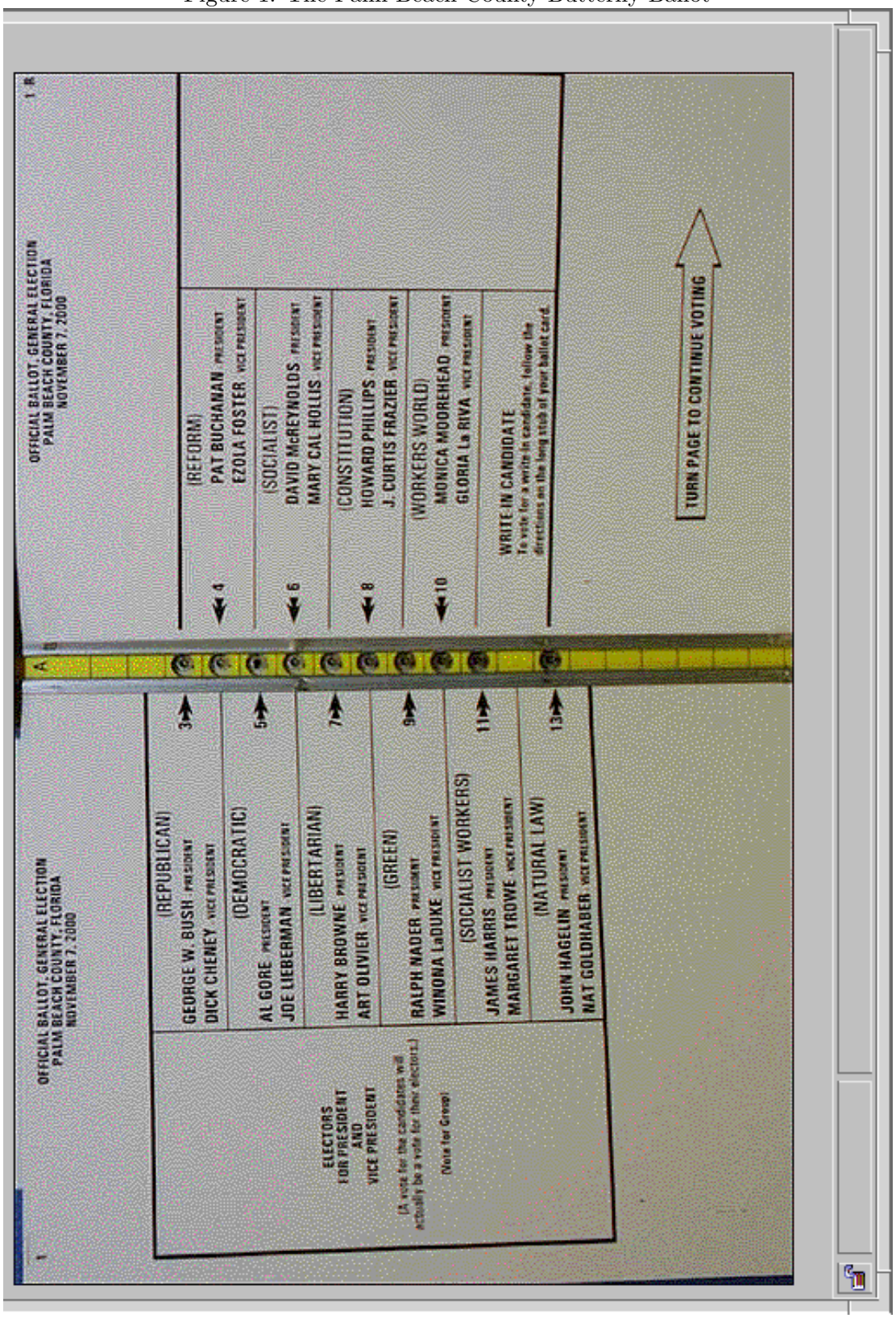

Figure 2: Histogram of Studentized Residuals from United States Counties

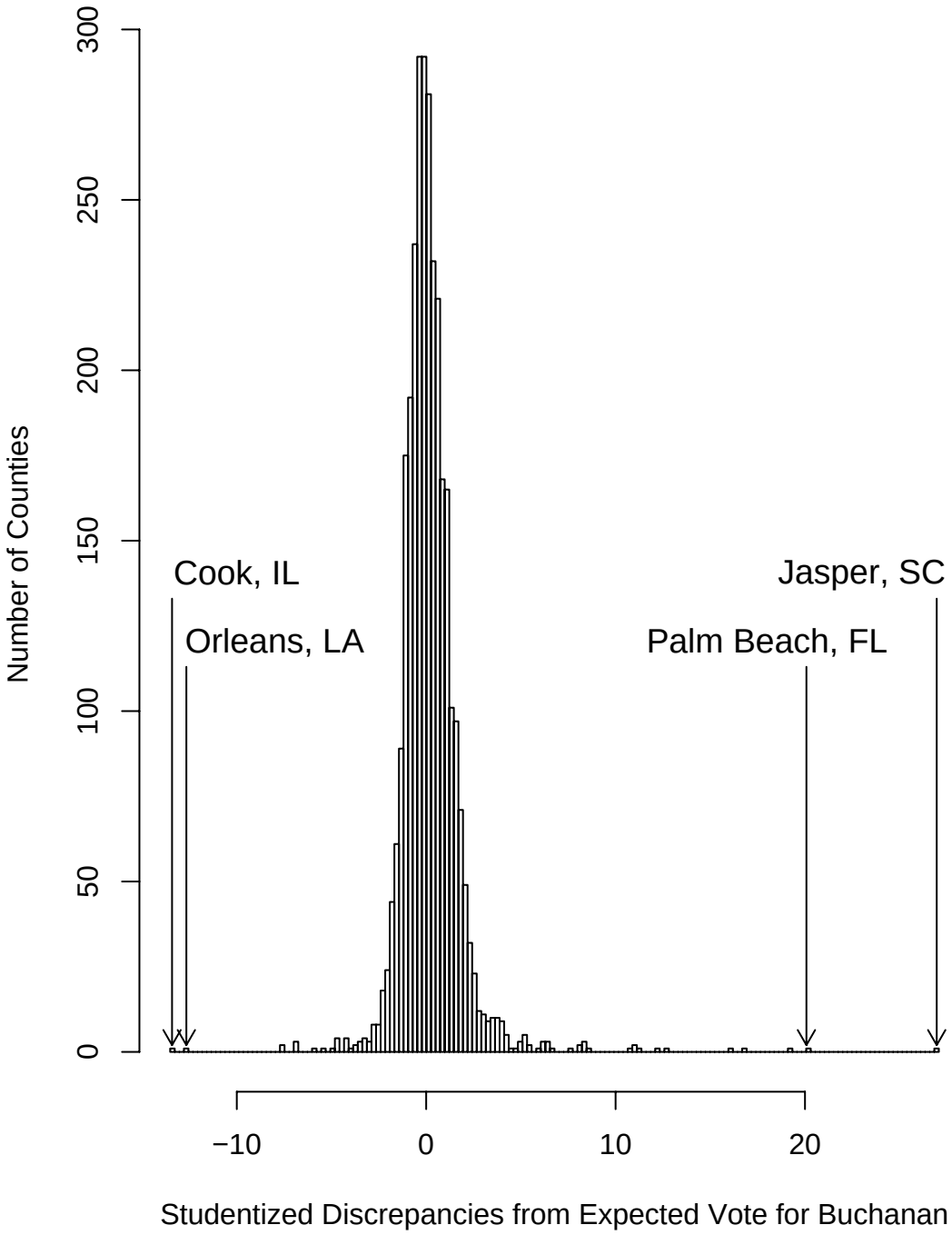

Figure 3: Map of Buchanan Vote Share across the United States

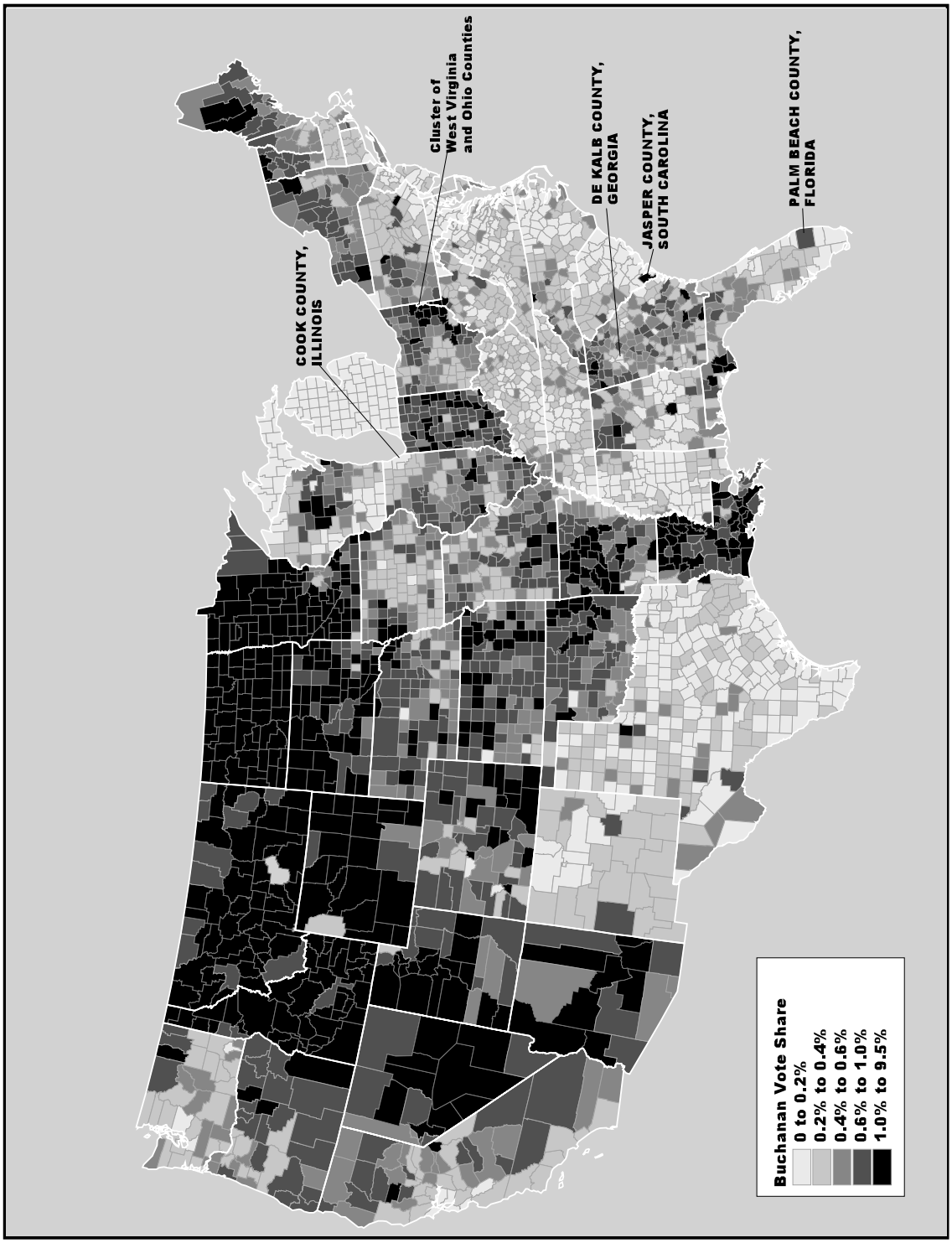

Figure 4: Florida Residuals for the 2000 Election

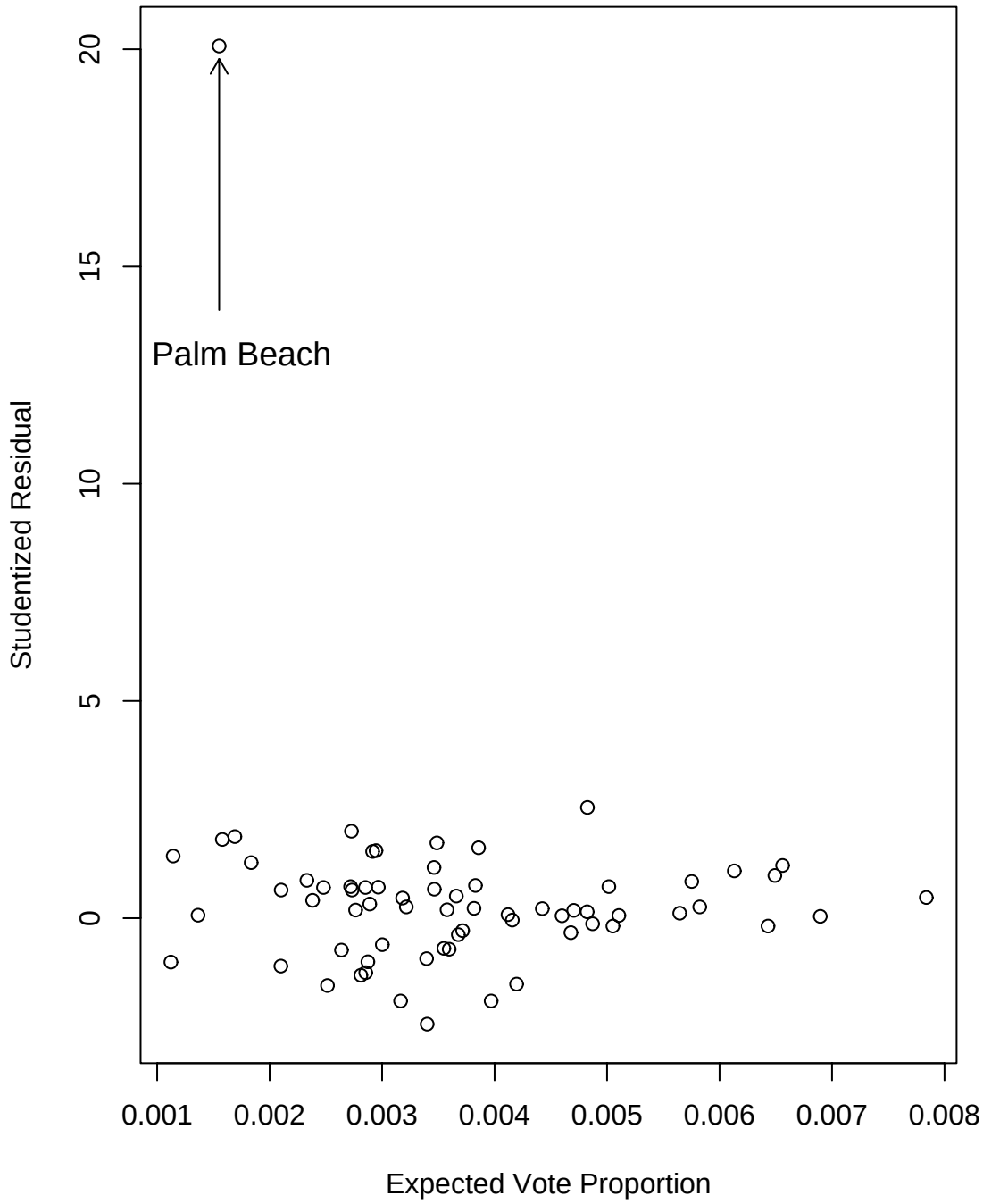

Figure 5: Normal Quantile-Quantile Plot of 2000 Florida Residuals

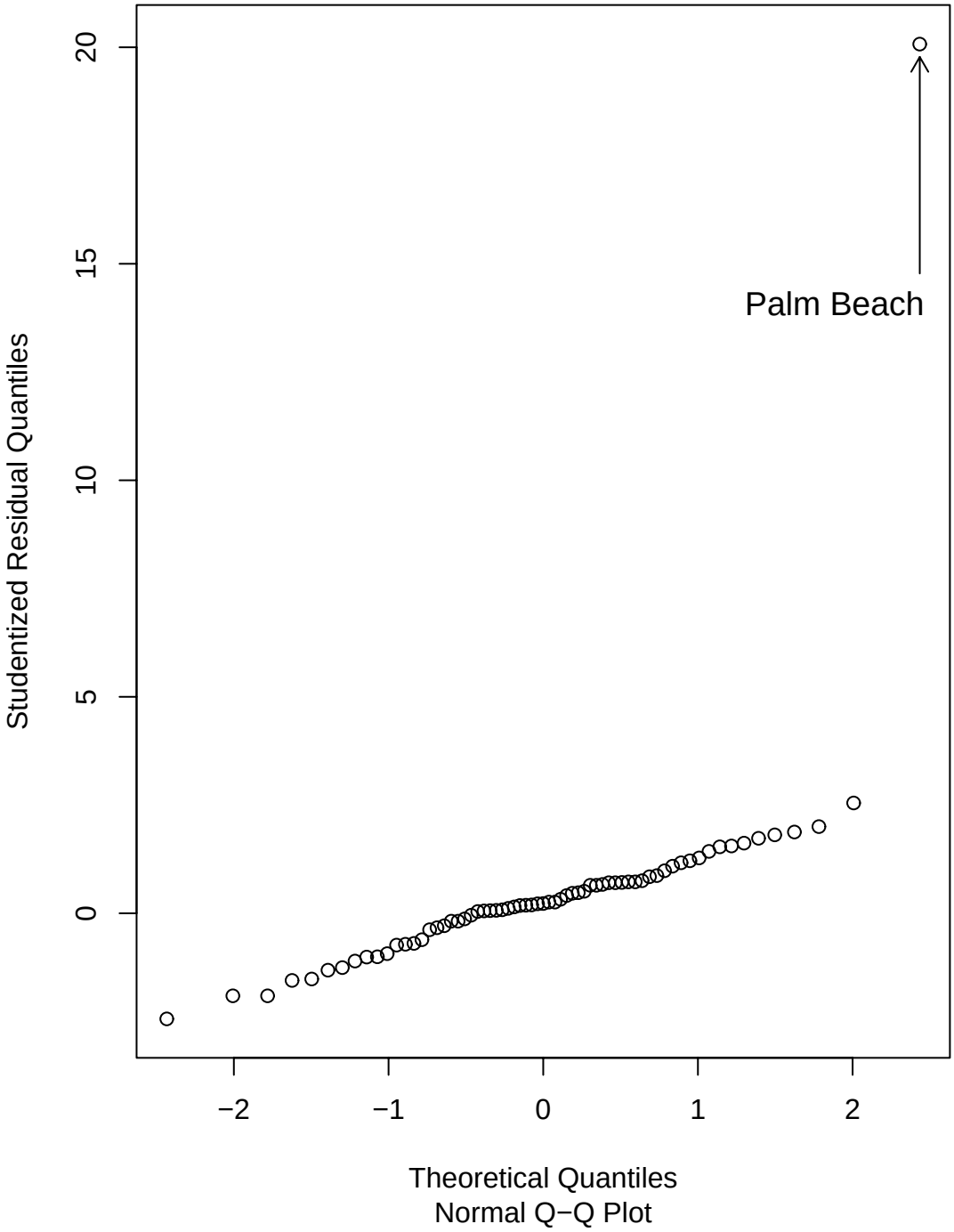

Figure 6: Florida Residuals for the 1996 Election

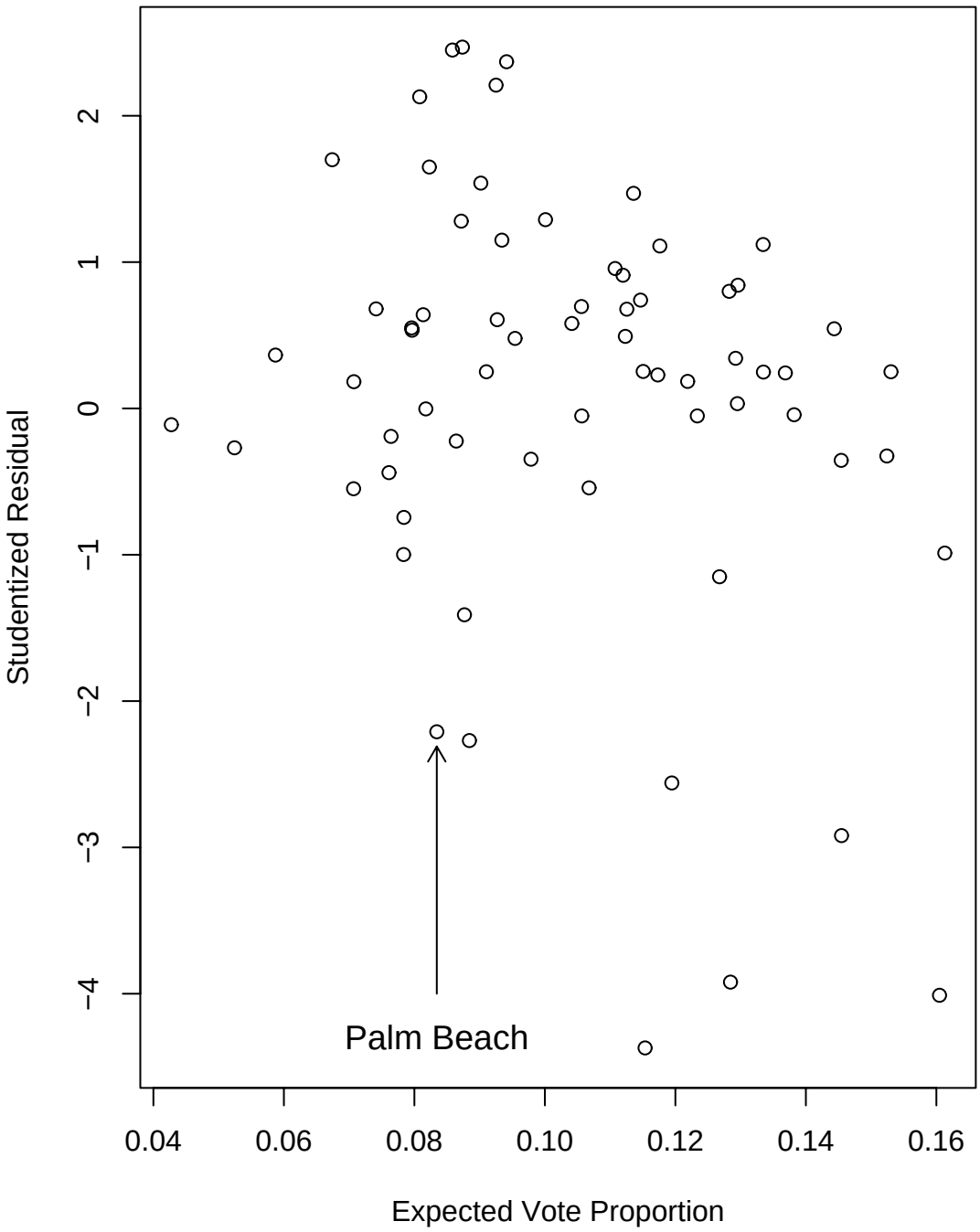

Figure 7: Normal Quantile-Quantile Plot of 1996 Florida Residuals

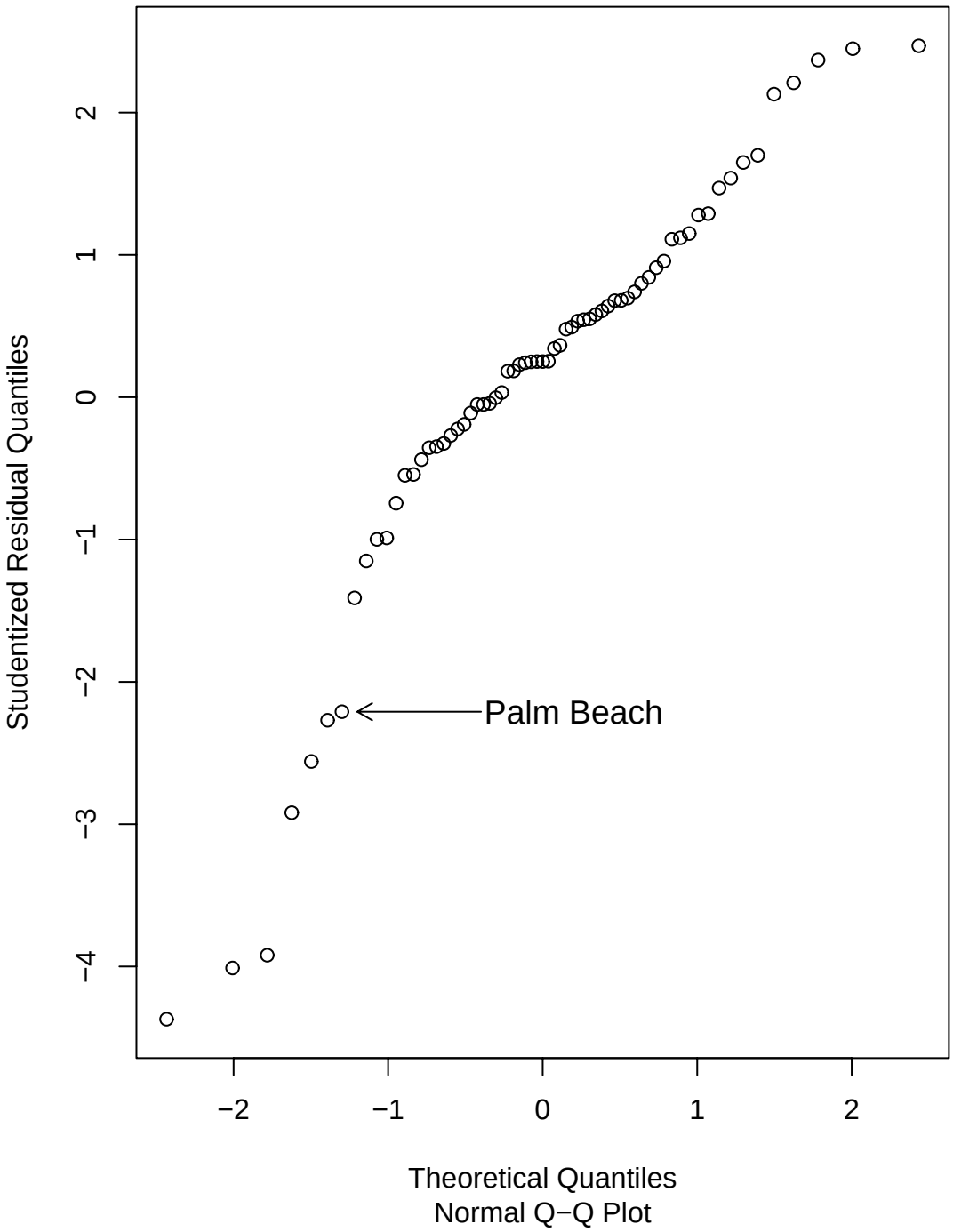

Figure 8: Comparison of Election day and Absentee Voters