The Butterfly Did It: The Aberrant Vote for Buchanan in Palm Beach County, Florida*

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Abstract

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To assess the claim that the butterfly ballot used in Palm Beach County, Florida, affected the outcome of the 2000 presidential election, we provide a statistical analysis of the vote received by Reform Party Presidential candidate Patrick Buchanan. Our analysis demonstrates that Buchanan's vote in the county is much greater than would be expected given plausible models of the popular vote. Using new methods we have developed for robust estimation of overdispersed binomial regression models, we show that Palm Beach County has the most anomalous Buchanan vote among all of the 2,998 counties in the United States we are able to examine. We also find that Buchanan's vote share among Palm Beach County's election day voters, who used the butterfly ballot, is four times as large as his vote share among the county's absentee voters, who did not use the butterfly ballot. We analyze precinct-level data and ballot-level data from Palm Beach County and find patterns that are consistent with the hypothesis that the butterfly ballot caused Democrats to vote mistakenly for Buchanan. Using a variety of models and methods, we show that Buchanan mistakenly received more than 2,000 additional election day votes, and that the vast majority of these votes were from Democrats. Although this article focuses on the butterfly ballot in Palm Beach county, our methods could be used on a regular basis as part of an ongoing effort to identify election anomalies and to develop improvements in the administration of elections.

Beginning on election day November 7, 2000, Palm Beach County, Florida, attracted national and eventually international attention because thousands of voters in the county complained that they had difficulty understanding the now infamous *butterfly ballot* and consequently cast invalid or erroneous presidential votes.[1] Shortly after election day, eleven groups of Palm Beach County voters filed independent lawsuits seeking relief because, they claimed, they and others had made mistakes in their votes for president because of the misleading and confusing format of the ballot.[2] Many of the complaining voters stated that they intended to vote for Democratic candidate Al Gore but instead had voted by mistake for Reform candidate Patrick Buchanan. The number of votes involved was more than enough to have tipped the presidential vote in Florida to Gore, thus giving him Florida's 25 electoral votes and the presidency.[3]

Palm Beach County is a heavily Democratic, politically liberal county that conventional wisdom says should provide relatively few Buchanan votes.[4] Yet in the 2000 presidential election the county supplied 19.6% of Buchanan's votes in Florida. In contrast, only 5.4% of Buchanan's Florida votes came from Palm Beach County in the 1996 Republican presidential primary, which did not use a butterfly ballot. In the 1996 presidential election the county gave less support to Reform than did

[1]Lawyers working for the Democratic party reportedly collected 10,000 affidavits sworn by voters with complaints about some aspect of their election day experiences in Palm Beach County. "Voting Scrutinized All Over Florida," *Associated Press*, Thursday, November 9, 2000. Don Van Natta Jr. and Michael Moss, "Democratic 'War Room' Tries to Oversee the Battle for Florida, to Mixed Results," *New York Times*, Saturday, November 11, 2000. David Firestone, "Party Mobilized on Ballot Even as Votes Were Cast," *New York Times*, Saturday, November 11, 2000. David Firestone, "Democrats Still Prefer to Keep Counting," *New York Times*, Saturday, November 11, 2000. Don Van Natta Jr., "Gore to Contest Recount Result in Palm Beach," *New York Times*, Saturday, November 25, 2000.

[2]The suits filed by such voters in the Fifteenth Judicial Circuit of Florida, West Palm Beach, were cases CL 00-10965, CL 00-10970, CL 00-10988AE, CL 00-109922AF, CL 00-11000AH, CL 00-11084AH, CL 00-11098AO, CL 00-1146AB, CL 00-1240AB, CL 00-129OAB and CL 00-113O2AO. The suits were consolidated by Administrative Order No. 2.061-11/00. Texts of the filings and of the Circuit Court rulings in the cases are available from `http://www.pbcountyclerk.com/`.

[3]Bush received 271 electoral votes, one more than the 270 needed to win, while Gore received 266 electoral votes. One Elector who was pledged to Gore from Washington, D.C., left her ballot blank, hence reducing Gore's count from 267. David Stout, "As Electors Cast Their Votes, Bush Confers in Washington," *New York Times*, Monday, December 18, 2000. Alison Mitchell, "Results Official, at Last: Bush Defeats Gore," *New York Times*, Sunday, January 7, 2001.

[4]In general Pat Buchanan would not be expected to attract much voter support according to standard models of third-party candidates (Rosenstone, Behr, and Lazarus, 1984). Regarding Palm Beach County in particular, Bay Buchanan (Pat Buchanan's sister and campaign manager) said "she was startled to hear Bush strategist Karl Rove argue Thursday that Buchanan has strong support in a county where his campaign never bought an ad and never paid a visit." "Bay Buchanan Sees Something Peculiar in Palm Beach Voting," Megan Garvey, *Los Angeles Times*, Friday, November 10, 2000. The story continues, "In fact, said David Goldman, who until October was the state Reform Party's vice chair, Florida, and the membership in Palm Beach County in particular, were 'hotbeds' of anti-Buchanan sentiment. It was such sentiment that led to the national party's implosion at their convention in Long Beach in August, when the party split into two warring factions, one supporting and the other backing a rival." "Goldman and other longtime Reform members in the state described a party in 'disarray' with little organization, much less a groundswell of support for Buchanan in a place even he concedes is not his base."

Florida as a whole, but in 2000 this pattern was dramatically reversed.[5]

A consensus quickly formed among scholars that the vote for Buchanan in Palm Beach County was anomalously large. According to the initial, uncertified count of Palm Beach County ballots, Buchanan received 3,407 votes. Within four days of the election, more than a dozen scholars had posted to the Internet analyses of the electoral outcome in Palm Beach County.[6] The technical approaches used in the early analytical work varied greatly, but most analysts agreed that 3,407 was many times greater than the number of votes to be expected for Buchanan under normal circumstances. Given various measurements of county political configuration and demographic composition, a typical estimate was that Buchanan received about 2,800 more votes than were to be expected based on voting patterns across the other 66 counties of Florida.[7] In an especially careful study, Bruce Hansen reported 2,058 as a conservative minimum for the number of votes that were counted for Buchanan but that were irregular.[8]

The possibility that so many votes were given erroneously to Buchanan made the butterfly ballot of immediate practical interest. In the final, certified vote in Florida, Republican candidate George W. Bush had a margin over Gore of only 537 votes.[9] The number of apparently accidental votes for Buchanan—almost certainly more than 2,000—was greater than Bush's official margin of victory in the state. If those votes had gone to Gore, Gore would have won Florida and become

[5]In 2000, Buchanan received 0.787% of the presidential vote in Palm Beach County while garnering only 0.3% of the overall Florida presidential vote. In contrast, Ross Perot, the Reform Party candidate for president in 1996, received 9.1% of the Florida vote while garnering only 7.7% of the Palm Beach County vote. These data are from the Florida Department of State and are available at `http://election.dos.state.fl.us/online/elecresu.shtml`.

[6]In Brady, Herron, Mebane, Sekhon, Shotts, and Wand (2001) we list the early posters, including ourselves: Brady posted analysis on November 9, and Wand, Shotts, Sekhon, Mebane, and Herron posted on November 11. Lists of empirical work posted on the Internet through the end of November, 2000, appear at `http://www.bestbookmarks.com/election/` (created by Jonathan O'Keeffe), `http://economics.about.com/money/economics/library/weekly/aa111000.htm` (created by John S. Irons), `http://www.sbgo.com/election.htm` (created by Sebago Associates) and `http://madison.hss.cmu.edu` (created by Greg Adams and Chris Fastnow).

[7]Our own county-level model for Buchanan's vote in Florida (see Section 1.3) gives a point estimate of 438 for the number of votes to be expected for Buchanan based on previous election results and demographic factors. Our estimate is based on the final, certified election returns.

[8]The number 2,058 is the lower bound of a 99% confidence interval for the number of irregular Buchanan votes. "Who Won Florida? Are the Palm Beach Votes Irregular?" by Bruce E. Hansen, posted on November 11, 2000. Paper posted at `http://www.ssc.wisc.edu/~bhansen/vote/vote.html`.

[9]The final, certified results gave Bush 2,912,790 votes and Gore 2,912,253 votes in Florida. A few days after the election, the Associated Press reported a 327 vote margin based on the initial, automatic recount across Florida. Daniel J. Wakin, "Bush Campaign Pushes for a Gore Concession," *New York Times*, Friday, November 10, 2000. In many counties the first recount did not include late-arriving overseas absentee ballots, which were expected to and ultimately did tilt heavily for Bush. In the final, certified results Buchanan received 3,411 votes in Palm Beach County.

President. If the butterfly ballot caused those voters mistakenly to choose Buchanan, then in a sense the butterfly ballot itself was pivotal in the election.[10]

Did the butterfly ballot cost Al Gore the presidency? The lawsuits filed by citizens of Palm Beach County were thrown out because the Supreme Court of Florida ruled that the butterfly ballot was not illegal.[11] But the Court's ruling did not depend upon and implied nothing about the impact the ballot actually had on the behavior of voters in Palm Beach County. Our goal in this paper is to assess the extent to which the butterfly ballot caused Gore supporters to vote mistakenly for Buchanan.

The butterfly format was an innovation of Theresa LePore, Supervisor of Elections for Palm Beach County, Florida.[12] The distinctive format was used only in Palm Beach County and only for the election-day ballots for the presidential race. Figure 1 displays a picture of the ballot. It is a "butterfly" because there are two columns of names of candidates (the "wings of the butterfly"), all for the same office, that sandwich a single column of punch holes between the names. These punch holes are alternately for the left-hand and right-hand side candidates. Thus, the first valid punch hole (#3) is for Bush, the first candidate on the left-hand side of the ballot. The second valid punch hole (#4) is for Buchanan, the first candidate on the right-hand side of the ballot. But from the perspective of the left-hand side of the ballot, the second candidate listed is Gore. An individual who scanned down the left-hand column of the ballot and only later considered the right-hand column could mistakenly conclude that the first two punch holes corresponded, respectively, to Bush and Gore. Even having made such an incorrect reading of the ballot, a Bush voter would be likely to punch the hole for Bush correctly, but a Gore voter might mistakenly punch the hole for Buchanan. Sinclair, Mark, Moore, Lavis, and Soldat (2000) report experimental evidence showing that a double-column ballot format like the one used in Palm Beach County can be more confusing and cause more voter errors than a single-column ballot.

[10] Of course in a close election many events are pivotal. From a normative point of view, a biased misallocation of votes which is decisive in determining the outcome may be considered particularly troubling and worthy of note, even if the misallocation was unintentional.

[11] The Court's ruling states, "even accepting appellants' allegations, we conclude as a matter of law that the Palm Beach County ballot does not constitute substantial noncompliance with the statutory requirements mandating the voiding of the election" (Supreme Court of Florida, Fladell, et al. Vs. Palm Beach County Canvassing Board, etc. et al. Case Nos. SC 00-2373 and SC 00-2376). The cases did not progress to hearings regarding the facts.

[12] Reportedly LePore "split the names over two pages to make the type larger." Two days after the election LePore was quoted as saying, "Hindsight is 20-20, but I'll never do it again." Joel Engelhardt, "Elections Chief on Firing Line," *Palm Beach Post*, Thursday, November 9, 2000.

*** Figure 1 about here ***

The question of whether the Palm Beach County butterfly ballot caused systematic voting errors is ultimately a question about ballot design. Beyond the paper by Sinclair et al. (2000), published research on the effects of ballot design is not extensive. Hamilton and Ladd (1996) use a collection of North Carolina counties to show that the extent to which ballot format made straight-party voting easy had measurable although small effects on voting outcomes. Darcy (1986) argues that position effects on ballots—namely, which candidate is listed first—can have small consequences for candidate vote shares but that these effects do not matter much for races in which party labels are very important. Because so little is known about optimal ballot design, about the relationship between ballot format and error rates, or about the types of voting errors that particular ballots tend to elicit, the existing literature does not provide much guidance regarding the particular pattern of errors the Palm Beach County butterfly ballot may have induced voters to make.

In order to detect irregular voting outcomes we use new methods we have developed for robust estimation of overdispersed binomial regression models. These methods allow us accurately to detect irregular vote outcomes when they occur. But determining *why* a given irregular outcome occurred requires investigation using additional information. There may be many plausible explanations for the observed anomaly.[13] Our analysis of the impact of the butterfly ballot uses a strategy of triangulation to eliminate plausible alternative explanations. We show that different models using different types of data all point to the same conclusion regarding the Buchanan vote in Palm Beach County: the butterfly ballot did cause at least 2,000 voters to vote mistakenly for Buchanan, and the vast majority of these votes were from Democrats.

The research strategy we follow here could be used on a regular basis as part of an ongoing project to identify election anomalies. Such a project would seek to identify ballot design and other administrative practices that are likely to distort election outcomes. The goal would be to eliminate distortionary practices and hence improve the administration of elections.

[13] In a previous version of this paper, which was posted in "real-time" at `http://elections.fas.harvard.edu/`, we found outlier counties which turned out to be anomalous because of innocuous data reporting problems. The counties in question did not report their election returns properly on election night to the relevant news gathering organizations from which (at that time) we were obtaining vote return data. The most notable example of this is Mississippi County, AR. We found it to have the most irregular Buchanan vote return in the country in our original work (posted on the web November 11, 2000). This finding was the a result of a data reporting error propagated by both the CNN and ABC news websites. This example shows both the power of our outlier detection technology and the need to closely examine an outlier before coming to a conclusion as to why a given vote outcome is anomalous.

In Section 1 we show that the vote for Buchanan in Palm Beach County truly was unusual. Out of 2,998 counties across the United States—all the counties for which estimation is possible by our methods—Palm Beach County has the largest discrepancy between the actual 2000 vote for Buchanan and the vote one would expect based on each county's electoral history and demographic profile. To ensure against various threats to the statistical validity of that finding, we use our new methods for robust estimation to estimate the discrepancy between actual and expected votes. The point of this analysis is to see whether anomalies as large as the one observed in Palm Beach County occur in many other places. If comparable anomalies are relatively frequent, then maybe it is unreasonable to credit the butterfly ballot for the unexpected outcome in Palm Beach County. We find that anomalies comparable to the one in Palm Beach County do not occur elsewhere for the 2000 election.

But perhaps the Reform Party vote in Palm Beach County has been anomalous in the past. In Section 1 we also show that the result for the Reform candidate in Palm Beach County in the previous presidential election—where a butterfly ballot format was not used—was not anomalous when compared to the vote for Reform in the other 66 counties of Florida.

Nevertheless it is possible that the large Buchanan vote in Palm Beach County in 2000 is a one-time anomaly because of some idiosyncratic factor other than the butterfly ballot. If so, we would expect that the anomaly would appear in both the election-day results that used the butterfly ballot and the absentee results that did not use the butterfly ballot. In fact, because we expect absentee voters to be politically more conservative than election-day voters, we should find that the absentee results have an even larger percentage vote for Buchanan. In Section 2 we show that in Palm Beach County the reverse is true. There is a much larger percentage Buchanan vote among election-day voters than among absentee voters, and the difference between the support for Buchanan among election-day voters and the support for Buchanan among absentee voters is extraordinarily large compared to the same difference in all other Florida counties. We conclude that the anomalous results in Palm Beach County occurred only among election-day voters who used the butterfly ballot. In effect, the use of two different ballot forms in Palm Beach County presents us with a natural experiment that allows us to conclude that the unexpectedly large Buchanan vote in Palm Beach County is almost certainly the result of the butterfly ballot.

But were these Buchanan voters people who meant to vote for Al Gore? In general we expect that party loyalty and ideological positions make it likely that those voting Democratic for one office in an election will vote for Democrats in another office. When Democrats do defect, we expect that they will choose someone who is ideologically close to them, and not someone who is ideologically distant. These commonplace observations make it unlikely that those voting for the Democratic Senatorial candidate in the 2000 election would vote for Reform candidates for other offices. Therefore, we would not expect to find much Buchanan support in those precincts that voted more heavily for the Democratic U.S. Senate candidate. We would expect the share of votes for Buchanan to decrease as the share of votes for the Democratic U.S. Senate candidate increases. Such expectations are wrong for Palm Beach County. In Section 3 we use precinct-level data from Palm Beach County to show that precincts that voted more heavily for the Democratic U.S. Senate candidate also voted more heavily for Buchanan for president.

The abnormality of this pattern is further underscored by the fact that precincts that voted more heavily for the Democratic U.S. Senate candidate did not tend to vote heavily for other Reform party candidates. In Section 3 we also analyze the precinct-level votes for two Reform candidates who were running, respectively, for the U.S. House and for the Florida State Senate. Unlike the case of Buchanan, the votes for these two Reform candidates decreased as votes for the Democratic U.S. Senate candidate increased. It does not appear that Democratic voters in Palm Beach County were usually attracted to Reform Party candidates. Consequently, the association between Democratic votes for Senate and the vote for Buchanan is surprising. The most reasonable explanation is that the butterfly ballot caused many Democratic voters to choose Buchanan by mistake.

The vote for Buchanan in Palm Beach County is so small, however, that a few pockets of concentrated Buchanan support might explain these results. Perhaps a few highly Democratic precincts had concentrations of Buchanan supporters leading to the results described above. In such a situation, a few anomalous precincts may bias our inferences about the county as a whole. And since the butterfly ballot hypothesis concerns a county level administrative phenomena (i.e., ballot design), a result supported only in a few anomalous precincts would not be consistent with it. Fortunately, our new methods for the robust estimation of overdispersed binomial models allow us to avoid such erroneous inferences. These methods, which we use in our Section 3 analysis,

produce estimates that require much more than isolated instances of a strong relationship between Democratic precincts and Buchanan supporters. They require that the pattern must prevail across most of the precincts in the county.

In Section 4 we present an analysis of individual ballots that reconfirms the patterns found in Sections 2 and 3. Democratic voters (as measured by votes in the U.S. Senate election) who voted on election day were much more likely to vote for Buchanan than were absentee Democratic voters. These findings, as well as other patterns in the ballot-level data, support the conclusions we reached through the analysis of aggregate data: in every case, it is the butterfly ballot that best explains the unusual outcomes for Buchanan in Palm Beach County.

1 Palm Beach County and Counties across the United States

For counties across the U.S. we estimate the discrepancy between the votes to be expected for Buchanan in the 2000 election and the votes Buchanan actually received. Our purpose is to identify counties where there are large discrepancies to see if the apparently large vote for Patrick Buchanan in Palm Beach County is exceptional or ordinary. We estimate expected votes from a model that uses information about the previous election results and the demographic characteristics of each county. Because the counties differ dramatically in size (some have only a few hundred residents while others have millions), a direct comparison of the discrepancies would be statistically misleading. A few Buchanan votes in a county with a few hundred voters can lead to large percentage discrepancies that are merely the result of statistical fluctuations that should not be taken to be significant. We guard against these and other problems by using studentized residuals from overdispersed binomial regression models that we estimate using robust methods.

1.1 The Rationale for Robust Estimation

We use robust estimators for several reasons.[14] The primary reason is that voter complaints, legal cases and media reports strongly suggest that the processes that produced the electoral results in

[14]Western (1995) provides a practically minded review of robust estimation. Following Pregibon (1981, 1982), he suggests an approach to robust estimation of generalized linear models. He also discusses high-breakdown estimation of linear models using least median of squares (Rousseeuw, 1984). Christmann (1994) discusses application of LMS to a grouped binomial model (albeit without overdispersion). Our direct application of high-breakdown estimation to an overdispersed binomial model, with separate robust estimation of the dispersion and mean parameters, is new.

Palm Beach County were substantially different from the standard political forces (party identification, liberalism-conservatism, and policy positions) that produced the results elsewhere in Florida. We do not want our models to be distorted by idiosyncratic factors (such as poor ballot design) that cause deviations from the standard political factors that cause voters to act predictably from one election to another. By developing models that pick-up only standard political forces, we can be reasonably sure that deviations from our predictions are the result of idiosyncratic factors.

Because the robust estimators we use have a high *breakdown point* (Hampel, 1971; Donoho and Huber, 1983), they are consistent and produce reliable measures of discrepancy even if unusual voting processes occurred in several counties in a state.[15] A large anomaly in one county will not mask (Atkinson, 1986) comparable or perhaps somewhat smaller anomalies that occur in other counties (Hampel, Ronchetti, Rousseeuw, and Stahel, 1986, 67). An estimator that lacks a high breakdown point will underestimate the frequency of highly anomalous election results. The robust methods we use also perform well in the absence of anomalies.[16]

Another reason to use robust estimators is the fact that the regression models we use are at best rough approximations for the processes that produced the vote counts (Hampel et al., 1986, 82). The estimators we use produce reliable measures of discrepancy even under such conditions, as long as the model gives a pretty good approximation for Buchanan's expected vote in most counties in each state.

Data weakness is another reason for robust estimation. Because our models are based on the results from the previous election, anomalies in that election will give a distorted picture of the standard political forces that predict the vote in the current one. The robust estimators we use protect against the influence such distortions might otherwise have on the results. A county whose previous election results are highly distorted will not affect the results for the other counties and will itself appear as an observation that has a large discrepancy.

A county that has a discrepancy larger in magnitude than a certain threshold is an *outlier*. The threshold is defined in terms of quantiles of the standard normal distribution.[17] If a county's

[15]Intuitively, the finite sample breakdown point of an estimator is the smallest proportion of the observations that must be replaced by arbitrary values in order to force the estimator to produce values arbitrarily far from the parameter values that generated the original data (Donoho and Huber, 1983).

[16]If there are no anomalies, the robust estimator remains consistent and is almost as efficient as an estimator such as simple iteratively reweighted least squares that ignores the possibility of anomalous observations.

[17]The threshold value we mainly use is the value of tuning parameter c of the tanh estimator defined in the

discrepancy is large, it is highly likely that the relationship between the current election results and the previous results and demographic profile of the county differs from the relationship that approximates the data elsewhere in the state. The outlier county may have a different disturbance (e.g., a nonzero mean, a fat-tailed distribution), different regression coefficients or contaminated regressors. To explain *why* a county is an outlier requires investigation using additional information. Subsequent sections provide such information to show that the butterfly ballot explains why Palm Beach County is an outlier.

1.2 Robust Estimation of an Overdispersed Binomial Model

The goal of our model is to provide the best possible prediction of votes for Buchanan in each county in 2000. Two kinds of information, the results of previous elections and the demographic characteristics of the county, are available and highly relevant for making these predictions. The previous election outcome is a proxy not only for the array of interests and party sentiments in each county but also for the strength of local party mobilization. We use two variables to represent the preceding election result in each county: the proportion of votes officially received by the Republican candidate in the 1996 presidential election; and the proportion of votes officially received by the Reform Party candidate in the 1996 presidential election. Because those two variables may not be adequate to capture the electoral environment in each county in 2000, we supplement them with a set of nine demographic variables.[18]

We cannot use all eleven variables (plus a constant) in our models at once, because with that many variables a state would need to have about 40 to 60 counties to produce reliable estimates.

Appendix. We use $c = 4$. If the absolute magnitude of an observation's studentized residual is greater than c, then the observation has zero weight in the estimate of the model's parameters (details about the estimation appear in the Appendix). In effect, such an observation is omitted from the data used to estimate the parameters. But observations that have studentized residuals of magnitude greater than the tuning value p, but less than c, are downweighted in estimating the parameters. The value we use for p is $p = 1.8$. One might reasonably interpret observations that have studentized residuals of magnitude greater than some value less than c, but greater than p, to be to some extent outliers.

[18]The demographic variables include the 2000 Census of Population and Housing proportions of county population in each of four Census Bureau race categories (White, Black, Asian and Pacific Islander, and American Indian or Alaska Native), 2000 proportion Hispanic, 2000 population density (i.e., 2000 population/1990 square miles) and 2000 population. The 2000 Census data were built from Census 2000 Redistricting Data (Public Law 94-171) Summary File, Matrices PL1, PL2, PL3, and PL4 (U.S. Census Bureau. 2001. "FactFinder Tables." Accessed April 7, 2001, at `http://factfinder.census.gov`). We also include the 1990 proportion of population with college degree, and 1989 median household money income. We do not use the number of voters registered as Reform Party members because voter registration is nonpartisan in many states.

But 21 states (including D.C.) have fewer than 50 counties. We take the approach of including the vote proportions variables in all of the models while using principal components of the demographic data. To maximize the efficiency of the information gained from the demographic variables, we compute principal components of the set of residuals obtained by regressing each demographic variable on the previous election proportion variables and the constant.[19] The regressions on the election variables and the computation of principal components are done separately for each state.[20] For the detailed results we report here we use only the first principal component. As we will explain below, using more principal components does not substantively change the results that bear on Palm Beach County. Hence the expected vote for Buchanan in county i in state s is based on a linear predictor defined by:

$$x_i'\beta_s = \beta_{s0} + \beta_{s1}x_{1i} + \beta_{s2}x_{2i} + \beta_{s3}x_{3i} \tag{1}$$

where x_{1i} is the 1996 proportion of votes received by the Republican candidate, x_{2i} is the 1996 proportion of votes received by the Reform candidate, x_{3i} is the principal component and β_{s0}, β_{s1}, β_{s2} and β_{s3} are constant coefficients to be estimated. We complete the definition of the model for the expected vote for Buchanan in equations (2) and (3) below.

We model the number of votes cast for Buchanan out of all votes cast for presidential candidates. The other candidates whose votes we include are Gore, Bush, Ralph Nader (Green Party), Harry Browne (Libertarian), Howard Phillips (Constitution Party), John Hagelin (Natural Law Party) and any write-in candidates.[21] Because the dependent variable records counts we use binomial models. Following McCullagh and Nelder (1989), we allow for overdispersion because we believe that the county-level data (and, further below, precinct-level data) are subject to unobserved internal clustering effects. To estimate the parameters we use robust methods.

The overdispersed binomial model is defined as follows. Let i indicate one of the n_s counties in

[19] We standardize the residuals of the demographic variables to have variance equal to 1.0 before computing the principal components.

[20] Tests not presented here show that the parameters cannot be pooled across states.

[21] We ignore votes for other candidates, undervotes (no apparent vote recorded on the ballot), overvotes (votes for more than one presidential candidate on a single ballot) and other spoiled ballots. For discussions of undervotes and overvotes in Palm Beach County see Joel Engelhardt and Scott McCabe, "Election 2000: Under-votes Could Have Meant Victory for Gore," *Palm Beach Post*, Sunday, March 11, 2001, and Joel Engelhardt and Scott McCabe, "Election 2000: Over-votes Cost Gore the Election in FL," *Palm Beach Post*, Sunday, March 11, 2001.

state s, $i = 1, \ldots, n_s$. Given a vector of k regressors x_i for that county, the count y_i of votes for Buchanan out of the m_i total ballots cast for presidential candidates in county i has a mean equal to the number of total ballots m_i times the probability π_i of someone voting for Buchanan in that county:

$$E(y_i \mid x_i, m_i) = m_i \pi_i, \tag{2}$$

with π_i being a logistic function of the linear predictor $x_i' \beta_s$ specified in (1):

$$\pi_i = \frac{1}{1 + \exp(-x_i' \beta_s)}, \tag{3}$$

where β_s is an unknown constant vector of coefficient parameters. The model departs from a standard binomial because the variance is:

$$E[(y_i - m_i \pi_i)^2 \mid x_i, m_i] = \sigma_s^2 m_i \pi_i (1 - \pi_i), \tag{4}$$

with $\sigma_s^2 > 0$ (McCullagh and Nelder, 1989, 125, eqn. 4.20). If $\sigma_s^2 > 1$ then there is overdispersion relative to a purely binomial model.

We use robust estimators to estimate σ_s^2 and β_s. Instead of estimating σ_s^2 directly, we use a *least quartile difference* (LQD) estimator (Croux, Rousseeuw, and Hossjer, 1994; Rousseeuw and Croux, 1993) to obtain an estimate of the scale $\sigma_s = \sqrt{\sigma_s^2}$. Let $\hat{\sigma}_s$ denote the estimated scale value. Given $\hat{\sigma}_s$, we use a *hyperbolic tangent* (tanh) estimator (Hampel, Rousseeuw, and Ronchetti, 1981) for β_s. Let $\hat{\beta}_s$ denote the estimated coefficient vector. The estimators are described in more detail in the Appendix. As far as we know they have not previously been used to estimate overdispersed binomial models, so that our application represents a practical innovation. An important product of the estimation is a weight $w_i \in [0, 1]$ for each county. If $w_i = 0$ then data from county i had no effect on $\hat{\beta}_s$, given $\hat{\sigma}_s$. That is, the tanh estimator did not use information from county i because the estimator determined that the county was an outlier.

Given expected proportions $\hat{\pi}_i = [1 + \exp(-x_i' \hat{\beta}_s)]^{-1}$ of votes for Buchanan, we use studentized residuals (Carroll and Ruppert, 1988, 31–34) to measure the discrepancy between actual and

expected votes for Buchanan. The studentized residuals may be compared across counties, both within and across states.[22] The calculation of the studentized residual starts with a standardized residual. The standardized residual takes into account the different number of ballots in each county and the degree of overdispersion in the state:

$$r_i = \frac{y_i - m_i\hat{\pi}_i}{\hat{\sigma}_s\sqrt{m_i\hat{\pi}_i(1-\hat{\pi}_i)}}. \tag{5}$$

To obtain studentized residuals we need to make a weighting adjustment for leverage (applying to the counties that have $w_i > 0$) or for forecasting error (applying to the counties that have $w_i = 0$). Let W denote the matrix that has diagonal entries $W_{ii} = w_i$ and off-diagonal entries equal to zero ($W_{ij} = 0$ for $i \neq j$). Let V denote the matrix that has $V_{ii} = [m_i\hat{\pi}_i(1-\hat{\pi}_i)]^{-1/2}$ and $V_{ij} = 0$ for $i \neq j$. Let X be the $n \times k$ matrix of the regressors (row i of X is x_i'). The diagonal values of

$$H = VX(X'VWVX)^{-1}X'V$$

provide robust estimates of the additional weights[23]. Let $h_i = H_{ii}$ if $w_i > 0$ and $h_i = -H_{ii}$ if $w_i = 0$. The studentized residual is:

$$\tilde{r}_i = r_i/\sqrt{1-h_i}. \tag{6}$$

Dividing by $(1-h_i)^{1/2}$ corrects for differences in the variability of the residuals that arise because of variations across counties in the magnitude and relationships among the regressors.

1.3 The Buchanan Vote Across the Country in 2000

We compare Palm Beach County to all of the 2,998 counties in the U.S. for which we could robustly estimate the overdispersed binomial models described by equations (1) through (3) and hence

[22] Strictly speaking, direct comparability holds asymptotically in the number of counties in each state, subject to various assumptions. The distribution of the studentized residuals for the counties in each state that are not outliers should approximate closely to a t-distribution for degrees of freedom roughly proportional to the number of counties in the state minus the number of regressors. The variation across states due to the varying degrees of freedom should be negligible compared to the variation due to the seriously anomalous processes that occurred in some counties.

[23] McCullagh and Nelder (1989, 397) motivate a nonrobust version of H.

compute studentized residuals.[24] The dependent variable is the count y_i of votes for Buchanan in each county i in the 2000 election. We estimate the model *separately* for each state in the United States. Tests not presented here show that the coefficient and dispersion parameters cannot be pooled across states. But the residuals of interest are comparable across states because of the studentization of equation (6).

Palm Beach County has the largest studentized residual among the 2,998 counties included in our analysis. This result can be seen in Figure 2, which presents a histogram that displays the distribution of all 2,998 studentized residuals. Not only does the residual[25] for Palm Beach County have the largest positive value but it is the largest in absolute magnitude. In Palm Beach County Buchanan received vastly more votes than were to be expected based on the county's electoral history and demographic profile.

*** Figure 2 about here ***

Tables 1, 2 and 3 list the studentized residuals, expected Buchanan vote proportion and actual Buchanan vote proportion for all counties for which the studentized residual is greater than or equal to 4.0 (Tables 1 and 2) or less than or equal to -4.0 (Table 3). In all, the tables list 67 positive outliers and eight negative outliers. The difference between the number of positive and the number of negative outliers reflects the overall positive skew of the residuals that is visible in Figure 2.[26] Using the threshold of magnitude 4.0 to classify a county as an outlier, outliers occur in 30 of the 43 states covered by our analysis.

*** Tables 1, 2 and 3 about here ***

The outlier status of some counties can readily be explained. The most easily explained outlier is Jasper County, South Carolina, which has the second largest residual in our analysis (see Table 1). Jasper County did not receive much national media attention, because the county's total vote was a fraction of Bush's margin of victory and hence immaterial to the outcome of the 2000 presidential contest. Bush easily won South Carolina—beating Gore by over 200,000 votes—while only 6,406

[24] Our collection of counties (or county equivalents) includes the counties of all states except Michigan, Connecticut, Delaware, Hawaii, Rhode Island and Alaska. In Michigan Buchanan appeared only as a write-in candidate. Connecticut, Delaware, Hawaii, New Hampshire and Rhode Island contain too few counties for us to analyze. Alaska does not report election returns by county.

[25] Throughout the rest of this paper the word "residual" always refers to the studentized residual of equation (6), unless otherwise indicated.

[26] In most states the distribution of the residuals that are not outliers is well described by a t-distribution with degrees of freedom equal to the number of counties minus the number of parameters. Recall footnote 22.

presidential ballots were cast in Jasper county. Nonetheless there were serious problems with a voting machine in Jasper County's Tillman precinct. In that precinct Gore and Bush each received one vote while Buchanan received 239 and Nader received 111.[27] The problems in the precinct affected vote totals for other offices. Indeed, "the State Board of Canvassers unanimously said [...] that problems in the county council election were so numerous that a new election should be held."[28]

Palm Beach County is not geographically contiguous to any other outlier county, but many of the counties listed in Tables 1, 2 and 3 are contiguous to another outlier.[29] Table 4 displays the sets of counties from Tables 1, 2 and 3 that are geographically contiguous.[30] Sixteen of the twenty-five largest positive outliers belong to such a geographic cluster. Two clusters includes counties from two states (West Virginia and Ohio, and Kansas and Missouri) and another includes counties from three states (South Dakota, Iowa and Nebraska). Because they span more than one state, it is highly unlikely that those exceptional bursts of support for Buchanan reflect problems of ballot format or electoral administration. Most likely the reason is special success in mobilizing voters for Buchanan in those areas.

*** Table 4 about here ***

There are also a few other outliers for which we have some information that may be relevant.[31] We do not emphasize such counties here, in part because much of the information about them seems only indirectly connected to the vote for Buchanan and partly because the residuals for some of these counties are not stable over variations in the model specifications. Such instability is in

[27] See the December 28, 2000 issue of *The Beaufort Gazette* for allegations regarding the Tillman precinct.

[28] "One Election Result Confirmed; Second Election Ruled a Do-over." *Associated Press State and Local Wire*. December 13, 2000.

[29] The county maps used to assess geographic contiguity appear in U.S. Bureau of the Census (1983).

[30] The existence of such clusters of outlier counties reinforces the importance of using a robust estimator that has a high breakdown point.

[31] For example, regarding St. Louis City and St. Louis County, Missouri (see Table 1), there is a history of balloting problems in St. Louis and on election day, 2000, events occurred there that involved allegations of racial discrimination ("Voters Complain Around the Country," *Associated Press*, Thursday, November 9, 2000. B. Drummond Ayres, Jr., "St. Louis Sees Specter of Vote Fraud," *New York Times*, Sunday, March 4, 2001.). Possibly relevant to the positive outliers in Colorado (Arapahoe, El Paso, Jefferson and Adams, see Tables 1 and 2) may be the profound rift in the Reform Party in that state—to the point that John Hagelin, not Buchanan, was on the Colorado ballot as the Reform Party candidate. Buchanan appeared as the Colorado Freedom Party nominee ("Judge Won't Rule on Reform Party." *Associated Press*. August 30, 2000.). The negative residual for Fulton County, Georgia (see Table 3) may reflect differences in the political mobilization of blacks in Atlanta, which is located partly in Fulton, compared to black mobilization in the rest of the state (Pam Belluck, "Georgia Could Be Big Test for Bush," *New York Times*, Sunday, September 17, 2000.). De Kalb County, which also contains part of Atlanta, has a residual of $\tilde{r}_i = -3.95$

contrast to the stability we see in the size and relative positions of the residuals for counties such as Palm Beach FL or Jasper SC, for which voting irregularities were reported, or Hancock WV and Pottawatomie KS which are in contiguous clusters of unusual Buchanan votes. Palm Beach County has the largest residual and Jasper County the second largest residual whether we use one or two principal components. In particular, the studentized residual for Palm Beach County is $\tilde{r}_i = 36.1$ for the one component model and $\tilde{r}_i = 29.4$ for the two component model, and for Jasper County the studentized residual is $\tilde{r}_i = 28.3$ for the one component model and $\tilde{r}_i = 25.7$ for the two component model. With three principal components, Palm Beach County has the second largest studentized residual ($\tilde{r}_i = 35.5$) and Jasper County has the third largest ($\tilde{r}_i = 20.9$). With no principal components (i.e., only past vote proportions) Jasper County has the largest studentized residual ($\tilde{r}_i = 25.6$) and Palm Beach County has the second largest ($\tilde{r}_i = 21.5$).

In Florida, besides Palm Beach County the only other outlier is Pinellas County—according to the measure of having a residual of magnitide 4.0 or greater (see Table 1). Broward County has a residual of ($\tilde{r}_i = 3.9$), only trivially below the threshold. The residual for Broward County is not an outlier in any of our other models, however. Unlike our result for Palm Beach County, the Broward result is unstable across model specification.[32] The next largest positive residuals in Florida counties occur for Hillsborough County ($\tilde{r}_i = 3.1$) and Calhoun County ($\tilde{r}_i = 2.3$). The two negative residuals of largest magnitude in Florida occur for Lee County ($\tilde{r}_i = -2.4$) and Collier County ($\tilde{r}_i = -2.2$). No county in Florida comes even remotely close to Palm Beach County in terms of having an excess of votes for Buchanan in the 2000 election. The parameter estimates for Florida give a point estimate of 438 for the number of votes expected for Buchanan in Palm Beach County—implying an excess of 2,973 accidental votes in the official tally of 3,411 votes for Buchanan.

As we have tried to exemplify in our discussion of some of the outlier counties, an explanation of why a county is an outlier requires further investigation using additional information. In some counties we examined we found clear auxiliary evidence of problems with ballots, voting machines or election administration. In still others there are strong indications that the reason Buchanan received an exceptional number of votes is that he truly enjoyed exceptional political support in

[32] With no principal components, the residual is $\tilde{r}_i = 0.07$, with two principal components the residual is $\tilde{r}_i = 1.87$, and with three principal components the residual is $\tilde{r}_i = 2.94$.

those places. But there is no reason to believe that Buchanan enjoyed such mobilized support in Palm Beach County.[33]

The 2000 vote for Buchanan in Palm Beach County was extremely unusual—clearly among the most unusual in the entire country. The vast difference between the residual for Palm Beach County and the residuals for the other counties of Florida shows that something anomalous occurred in Palm Beach County in terms of the vote we would expected Buchanan to have received based on the voting history and demographic profile of the county. This county level analysis does not provide direct evidence that the butterfly ballot is to blame. But the analysis does show that the Buchanan vote in Palm Beach County diverged from expectations in an extreme way. Starting with Section 2, we provide direct evidence for the butterfly ballot hypothesis. But first we shall show that the Reform vote in Palm Beach County was not anomalous in 1996.

1.4 The Vote for the Reform Party in Florida in 1996

The 1996 presidential ballot in Palm Beach County did not have a butterfly format. An excessively high number of Reform votes in 1996, compared to other Florida counties, could provide an alternative explanation for the apparently excessive Buchanan votes in 2000. Maybe support for Reform is just unusually high in Palm Beach County. On the other hand, if the Reform vote was exceptionally low in Palm Beach County in 1996, then the anomality of the 2000 vote could be exaggerated. The 2000 vote would appear to be excessive even if the county were simply returning to normalcy. The 1996 data support neither of these alternatives.

We analyze the count of votes received by the Reform candidate Ross Perot in the 1996 presidential election across the counties of Florida. We model the number of votes cast for Perot out of all votes cast either for Perot, Democrat Bill Clinton or Republican Bob Dole. We use the overdispersed binomial model of equations (2), (3) and (4). The regressors are defined the same way as in equation 1, except that x_{1i} is the proportion of votes officially received by the Republican candidate in the 1992 presidential election in county i, x_{2i} is the proportion of votes officially received by the Reform Party candidate in the 1992 presidential election and we use earlier demographic data.[34]

[33] Recall footnote 4.

[34] The race, Hispanic ethnicity and population variables (see footnote 18) are taken from the 1990 Census instead of the 2000 Census.

We use our robust combination of the LQD and tanh estimators.

The 1996 results contrast sharply with the 2000 findings. In 1996 no county in Florida has a residual even remotely as large as the one for Palm Beach County in 2000. In 1996 only St. Lucie County has a residual of absolute magnitude greater than 4.0 ($\tilde{r}_i = -4.92$). The largest positive residual is for Holmes County ($\tilde{r}_i = 2.30$). Palm Beach County has the seventh most negative residual ($\tilde{r}_i = -1.86$). Palm Beach County was not an outlier in 1996.

2 A Natural Experiment: Election day and Absentee Voters in 2000

The previous section demonstrates that Buchanan's vote total in Palm Beach County in the 2000 presidential election was anomalously large and that this support was neither a reflection of historical support for the Reform Party nor consistent with the demographic composition of the county. It may be reasonable to conclude that the butterfly ballot caused this anomaly, but it is also conceivable that some other factor was responsible. For example, it could be that the county experienced an exceptional surge in voter support for either the Reform Party in general or for Patrick Buchanan in particular. We now address this possibility.

Because the 2000 absentee presidential ballot in Palm Beach County was not a butterfly ballot, the election gives us a natural experiment: one group of Palm Beach County voters (election day) used a butterfly ballot but a second group (absentee) did not. If Buchanan's vote total in Palm Beach county reflects true support among the voters, then this support should be present in both pools of ballots. But if the butterfly ballot is responsible for Buchanan's vote then his support should have come disproportionately from election-day ballots.

A limitation of this natural experiment is that the selection mechanism that allocates voters to either the election-day pool or the absentee pool is not random assignment (Achen, 1986). Voters self-select to be in the absentee pool. For instance, certain demographic or ideological groups may be prone to take November vacations and hence cast absentee ballots. Furthermore, some Florida voters, e.g., military officers stationed overseas, must cast absentee ballots. Thus absentee voters may not be representative of voters in general. Absentee voters in Florida are generally thought to

be more politically conservative than election-day voters. This means that the natural experiment which fosters a comparison of election-day and absentee ballots may be biased. Absentee voters may be more likely to vote for conservative candidates such as Bush or Buchanan than election-day voters are. If such a conservative bias does exist in Florida, it would make it more difficult to find that Buchanan received significantly greater support among election-day voters than absentee voters. The bias would work against finding evidence for the butterfly ballot hypothesis. The potential for such a bias may be neutralized if absentee voters are more likely than election-day voters are to choose Bush over Buchanan.

The differences between the election-day and absentee proportions of 2000 presidential votes that went to Buchanan show that there was disproportionate support for Buchanan among election-day voters in Palm Beach County. Figure 3 plots the differences—election-day proportion minus absentee proportion—for all 67 counties of Florida by the number of presidential ballots cast in each county. One can see that Palm Beach County (which is labeled) has one of the largest differences in the state, albeit not the largest. Four counties have larger differences, and a few other counties have differences close in value to that of Palm Beach County. But all of those counties have voting populations much smaller than Palm Beach County's. In Palm Beach County 433,186 ballots were cast for president. In the largest of the counties that have differences larger than Palm Beach County there were 5,174 ballots cast for president.[35]

*** Figure 3 about here ***

The significance of the disparity in population sizes is that even if identical voting processes exist for each vote cast in all the counties, there will be greater variability in the differences of proportions in the counties that have smaller numbers of ballots cast. With identical processes in all counties, the standard deviation of the differences of proportions will vary in proportion to the reciprocal of the square root of the total number of ballots. If in all counties the proportion of election-day ballots cast for Buchanan is not systematically different from the proportion of absentee ballots cast for Buchanan, then the observed differences between the proportions should

[35]The election-day minus absentee difference in Palm Beach County is 0.00634. Liberty, Calhoun, Hamilton and Dixie counties have differences respectively of 0.0177, 0.00959, 0.00731 and 0.00706, based on total numbers of ballots cast for president respectively of 2,410, 5,174, 3,964 and 4,666. Union and Baker counties have the next largest differences after Palm Beach County. For both counties the proportion difference is 0.00620. The numbers of ballots cast in those counties are, respectively, 3,826 and 8,154.

be contained within bounds defined by the reciprocal of the square root of the number of ballots.[36] In Figure 3 we have drawn dashed lines to indicate the location of the bounds. One can see that only Palm Beach County falls outside the bounds. The difference between the election-day and absentee proportions voting for Buchanan in Palm Beach County appears to be much greater than one would expect by chance. The differences for the other counties appear to fall within the range one would expect given only random deviations from equality in the processes that generate votes by election-day and absentee voters.

The distorting effects of vastly different population sizes for counties may be fully corrected by explicitly setting up the comparison between election-day and absentee ballots as a test of the hypothesis that the proportion of votes for Buchanan is equal for the two ballot forms. Under that hypothesis, the best measure of the proportion of votes for Buchanan is the proportion voting for him among all the votes cast for president. Let A_i denote the total number of absentee votes cast for president in county i and let a_i denote the number of absentee votes cast for Buchanan. Let B_i denote the total number of election-day votes cast for president and let b_i denote the number cast for Buchanan. Using p_i to denote the proportion for Buchanan among all the votes for president in county i, we have $p_i = (a_i + b_i)/(A_i + B_i)$. Consistent with Section 1, we use an overdispersed binomial model for the totals a_i and b_i, with statewide dispersion parameter σ^2. Under the equality hypothesis the expected number of absentee votes for Buchanan is $A_i p_i$ and the expected number of election-day votes for Buchanan is $B_i p_i$. The variance of a_i is $\sigma^2 A_i p_i (1 - p_i)$ and the variance of b_i is $\sigma^2 B_i p_i (1 - p_i)$. Under the equality hypothesis and using the overdispersed binomial model, each of the following z-scores is approximately normal with mean zero and unit variance:

$$z_{ai} = \frac{a_i - A_i p_i}{\sigma \sqrt{A_i p_i (1 - p_i)}} \tag{7a}$$

$$z_{bi} = \frac{b_i - B_i p_i}{\sigma \sqrt{B_i p_i (1 - p_i)}}. \tag{7b}$$

The difference between the z-scores is a good measure of whether the proportion of votes for Buchanan is equal across the two ballot forms. Under the equality hypothesis the difference $z_{bi} - z_{ai}$ is approximately normal with mean zero and variance equal to 2.0. If $z_{bi} - z_{ai}$ is significantly greater

[36] The containment within the inverse square root bounds is an implication of the binomial model we use more explicitly in the next paragraph, as long as the overdispersion measured by σ^2 is not too large.

than zero, then in proportional terms election-day voters cast ballots for Buchanan much more often than absentee voters did. If $z_{bi} - z_{ai}$ is significantly less than zero, then support for Buchanan was disproportionately great among absentee voters. To compute the z-scores we need an estimate of σ. We use the LQD estimate of the scale parameter that we obtained for the 2000 Florida data when we estimated the the overdispersed binomial model defined by equations (1) through (4). That estimate is $\hat{\sigma}_s = 3.81$.

Table 5 reports the value of the z-score difference $z_{bi} - z_{ai}$ in the 2000 election for each of the 67 counties of Florida.[37] Far and away the largest difference is the positive value of 4.8 for Palm Beach County. The z-score difference for Palm Beach County is more than three times larger than the next largest positive difference. The z-score difference for Palm Beach County is more than three standard deviations away from the value of zero that is expected under the hypothesis of equality. Only one other value, the z-score difference of -1.74 for Duval County, is more than one standard deviation away from zero.

*** Table 5 about here ***

In Palm Beach County, election-day votes went to Buchanan vastly more often, in proportional terms, than absentee votes did. The results of the election-day versus absentee ballot natural experiment strongly support the conclusion that Buchanan's anomalous support was caused by the butterfly ballot.

Underlying the dramatic test statistic value for Palm Beach County is the fact that Palm Beach County election-day voters supported Buchanan at a rate approximately four times that of absentee voters: for Palm Beach County, $b_i/B_i = 0.0085$ but $a_i/A_i = 0.0022$. If election-day voters had cast ballots for Buchanan at the rate that absentee voters did, then Buchanan would have received $B_i(a_i/A_i) = 387356(0.0022) \approx 854$ election-day votes. In fact he received 3,310 election-day votes. By this method one might gauge the number of accidental votes Buchanan received because of the butterfly ballot to be approximately $3310 - 854 = 2,456$ votes.

[37] The data used to compute the statistics reported in Table 5 are based on certified numbers from the Florida Department of State and precinct-level returns provided by the 67 Florida counties. We used the precinct data to calculate the absentee returns.

3 Precinct-Level Analysis of Palm Beach County Returns

The results so far strongly suggest that Buchanan's vote total in Palm Beach County was not a true reflection of voter intentions. A crucial question remains: Who made the mistakes? Was it voters who wanted to vote for Bush or those who wanted to vote for Gore?

At an intuitive level, mistakes seem less likely for Bush voters, who had to match the first candidate with the first punch hole, than for Gore voters, who had to match the second candidate with the third punch hole. Furthermore, it was a large number of Democratic voters who complained that the butterfly ballot caused them to vote for Buchanan by mistake. Nonetheless, it is important to examine the possibility that Buchanan received votes intended for both candidates.[38] The analysis in this section is intended to assess the asymmetry of voters' errors: whether Democratic voters mistakenly chose Buchanan at a substantially greater rate than Republican voters did. To do that, we estimate an overdispersed binomial model for precinct-level election returns across Palm Beach County. The linear predictor of equation (1) now includes the proportions of the vote in each election-day precinct i for U.S. Senate candidates Democrat Bill Nelson (x_{1i}) and Reform candidate Joel Deckard (x_{2i}): $x_i'\beta = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}$. We use the robust tanh estimator. If the butterfly ballot did not cause any systematic voting errors, we would expect that support for Nelson (D) in a precinct would be *negatively* associated with support for Buchanan and that support for Deckard (Ref.) would be *positively* associated with support for Buchanan. If the butterfly ballot did cause asymmetric voting errors, we would expect support for Nelson (D) to be *positively*—instead of *negatively*—associated with support for Buchanan.

The first line in Table 6 presents the results for election-day precincts across the whole of Palm Beach County. Support for Nelson (D) is *positively* associated with support for Buchanan. Such a pattern is what we would expect to observe if the butterfly ballot caused many Democrats to vote for Buchanan. The pattern supports the claim that Buchanan's votes tended to come from mistaken Gore supporters.

*** Table 6 about here ***

The pattern does not occur for votes for other offices that included a Reform candidate but

[38] It is worth noting that even if Republicans and Democrats were equally likely to vote mistakenly for Buchanan, the net effect of the butterfly ballot would strongly favor Bush since Palm Beach County is heavily Democratic.

did not use the butterfly ballot. In 2000 in Palm Beach County, Reform candidates contested the legislative races in both Florida State Senate District 35 and Florida U.S. Congressional District 16. Only the presidential portion of the 2000 Palm Beach County ballot used the butterfly format, so it is probably safe to assume that the outcomes of the non-presidential races in Palm Beach County were not contaminated by the butterfly ballot. Using this assumption, we are able to examine the relationship between Reform Party support and Democratic Party support both in the presence and in the absence of the butterfly ballot.[39]

Restricting attention to Palm Beach County election-day precincts in State Senate District 35, we robustly estimate two separate models. In the first model y_i is the count of votes received by Buchanan, and in the second y_i is defined as the count of votes received by Reform State Senate candidate Sherree Lowe. We also estimate two parallel models restricting attention to Palm Beach County election-day precincts in Congressional District 16. In the first model y_i is the count of votes received by Buchanan, and in the second model y_i is the count of votes received by Reform candidate John McGuire. Because the butterfly ballot was relevant only for the presidential vote, we expect support for Nelson (D) to be *negatively* associated with support both for Lowe and for McGuire.

The results in Table 6 for District 35, show that support for Nelson (D) remains *positively* associated with support for Buchanan across precincts in the district. But support for Nelson (D) is *negatively* associated with support for Lowe (Ref.). As expected, the coefficient associated with the Nelson (D) vote share switches sign. The results for Congressional District 16 parallel those of State Senate District 35. Across the precincts of Congressional District 16, support for Nelson (D) is *positively* associated with support for Buchanan, but support for Nelson (D) is *negatively* associated with support for McGuire (Ref.). We also find that in both districts support for Deckard (Ref.) is not significantly associated with support for Buchanan, but it is positively associated with support for McGuire (Ref.).

[39]The issue positions stated on the candidates' web pages suggest that all three of the Florida Reform candidates are in sympathy with the Buchanan faction of Reform (for Deckard see `http://www.joeldeckard.com/issues.htm`; for Lowe see, `http://www.sherreelowe.com/` and `http://election.dos.state.fl.us/cgi-bin/CandHtml.exe?account=31167&elecid=20001107-GEN`; for McGuire see, `http://www.ronhoward.org/mcissues.htm`; all pages accessed April 7, 2001). For instance, all are pro-life on the abortion issue and all look askance at free trade. Deckard goes so far as to express concerns about *legal* immigration. Scattered other reports describing the candidates' views support the impressions conveyed by their web pages.

In the absence of claims about the butterfly ballot, the precinct-level findings would be quite counter-intuitive. One would expect precincts that are *more* Democratically-inclined with respect to the U.S. Senate race to be *less* Reform-inclined when it comes to other races. The results presented in Table 6 support the claim that the butterfly ballot caused systematic, biased voter errors that cost Gore more lost votes than Bush. Democratic-inclined precincts (as measured by the Nelson (D) vote proportion) are less likely to vote for Reform candidates in general (e.g., Lowe and McGuire) but are more likely to vote for Buchanan. The difference is the butterfly ballot.

The key features of Table 6 are the contrasts between Buchanan and Lowe and between Buchanan and McGuire. It is possible to explain the positive association between Nelson's (D) vote share and votes for Buchanan by asserting that Reform members in Palm Beach County chose to live among Democrats. The unintuitive nature of such an assertion notwithstanding, the assertion contradicts the finding that the proportion of votes for Nelson (D) is negatively associated with the votes both for Lowe (Ref.) and McGuire (Ref.). Also there is no significant relationship between votes for Deckard, the Reform candidate for the U.S.. Senate, and votes for Buchanan. But there is, as one would expect, a strong positive relationship between votes for Deckard and votes for at least one of the other two Reform candidates.

The results in this section also enable us to examine—and refute—one final class of alternative explanations for Buchanan's anomalous vote total in Palm Beach County. It is conceivable that Buchanan's Palm Beach County vote total was caused by a group of anomalous precincts within the county. Anomalous results concentrated within a few precincts would suggest that excess votes were the result of localized phenomena, rather than the butterfly ballot, which was used uniformly throughout the county. For example malfunctioning vote machines such as the one in Tillman precinct in Jasper County, South Carolina, could have recorded extra votes for Buchanan in a few precincts. Alternatively, intentional fraud in a few precincts could be the source of Buchanan's success. Finally, pockets of intense election-day Reform mobilization could have delivered the extra votes.

Such explanations are, however, quite difficult to reconcile with the precinct-level patterns documented in this section. Given our use of a robust estimator, the coefficient estimates in Table 6 would not be affected by a few precinct-level anomalies produced by irregular voting

processes.[40] Moreover, a localized mobilization effort should affect outcomes in multiple races whereas the peculiar relationship between Nelson (D) vote share and Reform vote is present only in the presidential race, which used the butterfly ballot.

4 Comparisons Using Individual Ballots

We extend our analysis by analyzing ballot data from Palm Beach County punch card readers.[41] These data enable us to compare an individual's presidential vote choice with his or her choices for other offices. In this section we do two things. First we use the ballot-level data to validate the precinct-level regression results reported in Table 6. Then we use a focused replication of the absentee ballot natural experiment to produce another estimate of the magnitude of the butterfly ballot effect.

With ballot-level data the binomial regression model reduces to the familiar binary logistic regression model.[42] In the linear predictor $x_i'\beta = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}$ for ballot i, x_{1i} and x_{2i} are dummy variables that respectively indicate whether the ballot records a vote for Nelson (D) or a vote for Deckard (Ref.). Variable $x_{1i} = 1$ if there is a vote for Nelson (D) and $x_{1i} = 0$ if there is not a vote for Nelson (D). Variable $x_{2i} = 1$ is defined similarly.

The estimates reported for election-day ballots in Table 7 show a positive and significant coefficient on voting for Nelson (D), matching the positive coefficient on the precinct-level proportion of votes for Nelson (D) that appears in Table 6. Among election-day ballots, voting for the Democratic candidate for the U.S. Senate (Nelson) is *positively* associated with voting for Buchanan for president. That is not the case with the absentee ballot. Among the absentee ballots the Nelson (D) coefficient is negative and insignificant. The confidence intervals of the Nelson (D) coefficients for the two ballot forms do not overlap. Hence we may reject the hypothesis that a Nelson (D) voter has the same chance of voting for Buchanan regardless of the form of the ballot.

[40] In the analysis of all precincts, reported in Table 6, 443 of the 515 precincts had $w_i = 1$ and 10 of the 515 precincts had $w_i = 0$.

[41] Ballot data were acquired from the Palm Beach County Supervisors of Elections. The Palm Beach County results do not include returns for 25 precincts that were overwritten by test data when Palm Beach County tested their vote tabulating machines. The loss of Buchanan votes (2.1 percent) is proportional to the number of precincts lost.

[42] In the absence of grouping there cannot be overdispersion (McCullagh and Nelder, 1989, 125) and high-breakdown estimators do not exist. To be precise, Christmann (1994) shows that for non-grouped binary data no estimator exists that has a high breakdown point and produces the estimate $\pi_i = 1$ if $y_i = 1$ for all i and the estimate $\pi_i = 0$ if $y_i = 0$ for all i. M-estimators, including maximum likelihood, have the latter property.

*** Table 7 about here ***

The proportions in Table 8 show that Palm Beach County voters who support the Democratic Senate candidate are significantly more likely to vote for Buchanan on the butterfly ballot than are their counterparts who use the absentee ballot. The table lists the proportion of votes in Palm Beach County going to Buchanan among all ballots that record U.S. Senate votes for either Nelson (D) or Deckard (Ref). Individuals who vote for the Democratic Senate candidate are six times more likely to vote for Buchanan using the butterfly ballot than the absentee ballot. Fewer than two in a thousand absentee voters in Palm Beach County who vote for Nelson (D) also vote for Buchanan, while the proportion of election-day Nelson (D) voters who also vote for Buchanan is ten in a thousand. If we treat the absentee proportion as the proportion of votes truly intended to go to Buchanan, then about 85 of every 1,000 Nelson (D) voters in Palm Beach County—about 2,300 voters—appear to have mistakenly voted for Buchanan.[43]

*** Table 8 about here ***

Table 8 shows that individuals who vote for Deckard (Ref.) are more likely to vote for Buchanan using the absentee ballot. This group of voters was not hypothesized to be affected by the butterfly ballot, and the difference between election day and absentee Buchanan vote proportions is small.

The ballot data add to the evidence that the butterfly ballot caused systematic voting errors in Palm Beach County that cost Gore votes. We gain analytical precision with the ballot data, but this section complements rather than replaces the county and precinct analyses presented above. Ballot-level data are rarely retained or made available after an election, and therefore it is not generally possible to compare these results across states or counties. Without such comparisons, the ballot-level results must be considered with some caution. Moreover, data from two percent of the precincts in Palm Beach County ballot are not available. Our precinct-level analysis does not suffer from missing data. Also, unlike the county-level and precinct-level analysis, the analysis of individual ballots cannot use robust estimation techniques. In the absence of robust—high breakdown point—results, it would have been possible to claim that the aberrations we have found may be limited to a few idiosyncratic precincts and not characteristic of Palm Beach County as a whole. But the aberrations prevail throughout the county.

[43]This number is calculated by $269835 \times 0.0085 = 2294$. The 269,835 number is the total number of votes that Nelson (D) received in the entire county on election day, including the precincts missing from the ballot image data.

5 Discussion

We have examined the source of the seemingly anomalous support for Buchanan in Palm Beach County by focusing on allegations that the county's use of a butterfly ballot caused systematic voting errors that boosted the number of votes for Buchanan.[44] In particular, we presented new robust estimation and outlier detection methods for overdispersed binomial data and showed that, with respect to the Reform vote in 2000, Palm Beach County is the largest outlier among all counties in the United States we are able to examine. We also showed that Palm Beach county was not a Reform vote outlier in 1996, the year of a presidential election in which the county did not use a butterfly ballot.

Having identified Palm Beach County as an outlier, we have sought to determine whether the butterfly ballot was indeed the mechanism that caused this anomaly. A comparison of election day versus absentee results in Palm Beach County shows that Buchanan's success in Palm Beach County did not extend to absentee voters, who did not use the butterfly ballot. We have examined the claim that Democratic presidential candidate Al Gore in particular was harmed by the butterfly ballot. We found that Buchanan's support in Palm Beach County tended to come from more Democratic precincts and from voters who voted for the Democratic candidate for Senate, a pattern that supports the claim that Buchanan's votes tended to come from mistaken Gore supporters.

Was the butterfly ballot pivotal in the 2000 presidential race? The evidence is very strong that it was. Had Palm Beach County used a ballot design in the presidential race that did not lead to systematic biased voting errors, our findings suggest that, other things equal, Al Gore would have won a majority of the officially certified votes in Florida.

Although we have focused here on the butterfly ballot in Palm Beach county, our methods could be used on a regular basis as part of an ongoing effort to identify election anomalies and to develop improvements in the administration of elections to help eliminate such anomalies. Our new outlier detection methods offer an accurate and powerful technology for detecting irregular vote outcomes. But determining *why* a given irregular outcome occurred requires a strategy of triangulation, such as the one we have pursued here. Different models and different types of data need to be marshaled

[44] Another allegation related to the butterfly ballot is that the Palm Beach County ballot led to excessive overvoting in the presidential race. The subject of overvoting is beyond the scope of this paper (recall footnote 21).

to eliminate plausible alternative explanations. In the case of Palm Beach County in the 2000 presidential election, such a strategy leads to the conclusion that "The Butterfly Did It."

6 Appendix: Robust Estimation Methodology

6.1 Background

The classical linear regression model for n observations and k regressors has the form $y_i = x_i'\beta + \epsilon_i$, $i = 1, \ldots, n$, where β is a vector of k unknown coefficients, the data y_i and x_i are observed, and the unobserved disturbance ϵ_i has conditional mean $E(\epsilon_i \mid x_i) = 0$ and variance σ^2. We seek parameter estimates $\hat{\beta}$ and $\hat{\sigma}^2$ that converge in probability to β and σ^2 as n gets large: we want *consistent* estimates.[45] Least squares (LS) chooses $\hat{\beta}$ to minimize the sum of squared residuals $r_i = y_i - x_i\hat{\beta}$ over all $i = 1, \ldots, n$:

$$\hat{\beta}_{\text{LS}} = \underset{\hat{\beta}}{\text{argmin}} \sum_{i=1}^{n} r_i^2. \tag{8}$$

The LS estimate for σ^2 is then the mean (adjusted for the number of coefficients) of the squared residuals $(y_i - x_i\hat{\beta}_{\text{LS}})^2$. Even one contaminated data point can cause $\hat{\beta}_{\text{LS}}$ to take values arbitrarily different from β. Hence LS has a breakdown point of $1/n$ (asymptotically, 0). LS is not robust.

For the regression model the maximum possible breakdown point is, asymptotically, 0.5. One popular estimator for β that achieves that maximum is *least median of squares* (LMS):

$$\hat{\beta}_{\text{LMS}} = \underset{\hat{\beta}}{\text{argmin}} \underset{i}{\text{med}}\, r_i^2 \tag{9}$$

(Rousseeuw, 1984; Rousseeuw and Leroy, 1987).[46] LMS has two important defects. $\hat{\beta}_{\text{LMS}}$ is consistent for β, but the estimator converges at the slow rate of $n^{-1/3}$. LMS also is inefficient when the disturbance e_i is an identically and independently distributed Gaussian random variable (i.e., no ϵ_i outliers) and the model is otherwise correctly specified. One way to achieve greater efficiency is to use LMS estimates as starting values for a redescending M-estimator such as the tanh estimators

[45] To be technically precise, the statistical theory that supports the development of tanh and LQD estimators assumes the existence of a *Fisher-consistent* estimator (Hampel et al., 1986, 83).

[46] Western (1995) presents LMS.

to be described in the next subsection (Hampel et al., 1981).

Other estimators exist that achieve the maximum breakdown point while having a $n^{-1/2}$ convergence rate and better Gaussian efficiency than LMS does. The LQD estimator for β is such an estimator (Croux et al., 1994). For all the elements of β except the intercept, LQD has a Gaussian efficiency of 67.1% (Croux et al., 1994).[47] In addition to an efficiency gain over LMS, LQD provides a superior estimate of the scale (i.e., σ) when ϵ_i has an asymmetric distribution, because LQD does not estimate the scale by measuring a symmetric spread of the residuals around a central location value (Rousseeuw and Croux, 1993).

6.2 Robust Overdispersed Binomial Model Implementation

The point of departure for our estimation method is the fact that if the overdispersed binomial model of (2) and (4) is correctly specified then given a consistent estimate $\hat{\beta}$ for β and hence $\hat{\pi}_i = [1 + \exp(-x_i'\hat{\beta})]^{-1}$, residuals of the form

$$r_i^* = \frac{y_i - m_i\hat{\pi}_i}{\sqrt{m_i\hat{\pi}_i(1-\hat{\pi}_i)}} \tag{10}$$

are approximately normal.[48] Indeed, a good moment estimator for σ^2 may be defined in terms of r_i^* (McCullagh and Nelder, 1989, 126–127, eqn 4.23).

The LQD estimator focuses on the $\binom{h_k}{2}$ order statistic of the set $\{|r_i^* - r_j^*| : i < j\}$ of absolute differences, where $\{|r_i^* - r_j^*| : i < j\}$ has $\binom{n}{2}$ elements and $h_k = \lceil (n+k)/2 \rceil$. Following Croux et al. (1994) we use the notation

$$Q_n^* = \{|r_i^* - r_j^*| : i < j\}_{\binom{h_k}{2}:\binom{n}{2}} \tag{11}$$

to denote that order statistic. To implement LQD we choose estimates $\hat{\beta}$ to minimize Q_n^*.[49] Let $\hat{\beta}_{\text{LQD}^*}$ designate the estimated coefficient vector and let $\hat{Q}_n^*$ designate the corresponding minimized

[47] For the linear regression model, LQD does not estimate the intercept term. If necessary an intercept must be estimated separately.

[48] The approximate normality depends on conditions such as independence across i and large values for m_i and $m_i\pi_i(1-\pi_i)$.

[49] In the overdispersed binomial model there is information about the constant from the factor $[\hat{\pi}_i(1-\hat{\pi}_i)]^{-1/2}$ in r_i^*.

value of Q_n^*. The LQD scale estimate is

$$\hat{\sigma} = \hat{Q}_n^* \frac{1}{\sqrt{2}\Phi^{-1}(5/8)}, \tag{12}$$

where Φ^{-1} is the quantile function for the standard normal distribution (Rousseeuw and Croux, 1993, 1277).

The efficiency of the LQD estimator improves on LMS, but to minimize the chances of falsely identifying counties as outliers we would like to improve further. To do that we use the LQD scale estimate in a tanh estimator for β. In addition to achieving the maximum breakdown point, tanh estimators minimize the asymptotic variance of the estimates for a given upper bound d on how sensitive the variance is to a change in the distribution of the data (Hampel et al., 1986, 125–136). Given a good estimate of the scale, then, tanh estimators are the most efficient possible estimators of location that have the key robustness property of having bounded response to arbitrary changes in parts of the data (Hampel et al., 1986, 166).

The tanh estimator is based on the function

$$\psi(u) = \begin{cases} u, & \text{for } 0 \leq |u| \leq p \\ (A(d-1))^{1/2} \tanh[\frac{1}{2}((d-1)B^2/A)^{1/2}(c-|u|)] \operatorname{sign}(u), & \text{for } p \leq |u| \leq c \\ 0, & \text{for } c \leq |u| \end{cases}$$

where p, c, d, A and B are constants satisfying $0 < p < c$ and other conditions.[50] Given a scale estimate $\hat{\sigma}$ and a vector of trial estimates $\hat{\beta}$, we compute the standardized residuals r_i of (5) and then weights

$$w_i = \begin{cases} \psi(|r_i|)/|r_i|, & \text{for } r_i \neq 0 \\ 1, & \text{for } r_i = 0. \end{cases}$$

Observation i is weighted by w_i in what is otherwise the usual iteratively reweighted least squares algorithm to estimate β. The given scale value remains unchanged but the weights are updated to

[50] For details see Hampel et al. (1981, 645) or Hampel et al. (1986, 160–165).

match the current coefficient estimates. Numerical convergence is required for both the $\hat{\beta}$ values and the weights. Because redescending M-estimators such as the tanh estimator have multiple solutions, starting values affect the results. We use $\hat{\beta}_{\mathrm{LQD}^*}$ to start the coefficients and use the LQD values $(r_i^* - \mathrm{med}_i\, r_i^*)/\hat{\sigma}$ for an initial set of residuals ($\mathrm{med}_i\, r_i^*$ denotes the median of the r_i^* values, $i = 1, \ldots, n$). To estimate the asymptotic variance of the coefficient estimates we use the sandwich estimator (White, 1994, 92) $\mathrm{avar}(\hat{\beta}) = \hat{J}^{-1}\hat{I}\hat{J}^{-1}$ where $\hat{I}$ denotes the outer product of the score and $\hat{J}$ denotes the Hessian matrix.

The particular tanh estimator we use has $c = 4.0$ and $d = 5.0$ which imply values $p = 1.803134$, $A = 0.857044$ and $B = 0.911135$ as given in the indicated row of Table 2 in Hampel et al. (1981, 645).[51] The value of c fixes the threshold for the magnitude of r_i beyond which an observation is completely rejected by assigning it a weight $w_i = 0$.

The LQD objective function is difficult to optimize. Because high breakdown-point estimators are not smooth functions of the data, optimization techniques that are based solely on derivative information, such as Newton-Raphson, are unreliable (Stromberg, 1993). High breakdown-point objective functions have multiple minima. We use a global optimizer that makes local use of derivative information: GENetic Optimization Using Derivatives (GENOUD) (Sekhon and Mebane, 1998). GENOUD combines evolutionary algorithm methods with a derivative-based, quasi-Newton method to solve difficult unconstrained optimization problems.[52]

[51] Hampel et al. (1981) use k for the tuning parameter of ψ that we have denoted by d. The same information about the tuning parameters appears in Table 2 of (Hampel et al., 1986, 163) with notation r and k used for the parameters we have denoted by c and d.

[52] See `http://jsekhon.fas.harvard.edu/rgenoud/` for an **R** version of the GENOUD software and `http://jsekhon.fas.harvard.edu/genoud/` for more information.

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Table 1: Counties with the Largest Positive Studentized Residuals, 2000 National Analysis

State	County Name	Studentized Residual	Vote Proportion Expected	Actual	Order
FL	Palm Beach	36.14	0.0010	0.0079	1
SC	Jasper	28.26	0.0013	0.0379	2
KS	Pottawatomie	19.53	0.0096	0.0528	3
WV	Hancock	18.16	0.0048	0.0289	4
NV	Clark	15.88	0.0037	0.0080	5
WV	Brooke	15.57	0.0046	0.0286	6
OH	Jefferson	14.56	0.0066	0.0303	7
OH	Athens	14.23	0.0049	0.0292	8
MN	Hennepin	13.67	0.0010	0.0040	9
NE	Douglas	12.52	0.0018	0.0038	10
WI	Milwaukee	12.25	0.0010	0.0037	11
MN	Ramsey	11.67	0.0009	0.0057	12
IA	Woodbury	11.49	0.0038	0.0107	13
WV	Marshall	10.35	0.0054	0.0196	14
WV	Ohio	10.25	0.0031	0.0126	15
AZ	Maricopa	9.99	0.0057	0.0080	16
MT	Silver Bow	9.89	0.0100	0.0265	17
KS	Shawnee	9.67	0.0048	0.0097	18
MO	St. Louis	9.66	0.0012	0.0024	19
KS	Sedgwick	9.41	0.0034	0.0062	20
MO	St. Louis City	9.30	0.0005	0.0025	21
AR	Union	8.95	0.0052	0.0186	22
MO	Jackson	8.55	0.0014	0.0030	23
AR	Faulkner	8.50	0.0052	0.0148	24
IA	Plymouth	8.20	0.0043	0.0143	25
OH	Belmont	7.79	0.0074	0.0218	26
CO	Adams	7.60	0.0049	0.0093	27
CO	Jefferson	7.48	0.0034	0.0058	28
FL	Pinellas	7.47	0.0010	0.0025	29
NE	Dakota	7.33	0.0046	0.0144	30
WI	Marathon	7.22	0.0041	0.0116	31
NM	Bernalillo	7.19	0.0011	0.0019	32
SD	Minnehaha	7.13	0.0034	0.0062	33
NY	Erie	6.95	0.0050	0.0079	34

Notes: Results based on 2,998 counties. This table presents all counties with studentized residuals greater than or equal to 4.0.

Table 2: Counties with the Largest Positive Studentized Residuals, 2000 National Analysis (cont'd)

State	County Name	Studentized Residual	Vote Proportion Expected	Actual	Order
NE	Thurston	6.95	0.0063	0.0245	35
MD	Somerset	6.59	0.0025	0.0064	36
KS	Wyandotte	6.42	0.0025	0.0057	37
CO	El Paso	6.39	0.0047	0.0073	38
VA	Warren	6.35	0.0026	0.0088	39
NE	Dixon	6.31	0.0080	0.0233	40
WI	Lincoln	5.99	0.0046	0.0178	41
TX	Harris	5.84	0.0014	0.0017	42
OH	Ashland	5.81	0.0061	0.0176	43
IA	Dubuque	5.79	0.0040	0.0074	44
PA	Allegheny	5.46	0.0024	0.0036	45
KY	Boone	5.41	0.0021	0.0043	46
GA	Whitfield	5.35	0.0045	0.0081	47
WI	Wood	5.34	0.0048	0.0123	48
KY	Kenton	5.10	0.0021	0.0036	49
MS	Leflore	5.09	0.0012	0.0039	50
CO	Arapahoe	4.96	0.0031	0.0048	51
MS	Washington	4.85	0.0012	0.0033	52
IA	Sioux	4.84	0.0018	0.0056	53
KS	Wabaunsee	4.76	0.0098	0.0260	54
MT	Deer Lodge	4.55	0.0101	0.0254	55
MS	Wilkinson	4.54	0.0022	0.0078	56
IN	Allen	4.48	0.0059	0.0083	57
KY	Graves	4.25	0.0032	0.0063	58
WI	Brown	4.22	0.0032	0.0060	59
SC	Abbeville	4.19	0.0022	0.0084	60
SD	Union	4.19	0.0100	0.0189	61
MD	Wicomico	4.09	0.0023	0.0042	62
AL	Lauderdale	4.07	0.0056	0.0092	63
GA	Richmond	4.06	0.0030	0.0044	64
OH	Hocking	4.06	0.0078	0.0202	65
IA	Benton	4.02	0.0038	0.0081	66
TX	Cottle	4.01	0.0023	0.0132	67

Notes: Results based on 2,998 counties. This table presents all counties with studentized residuals greater than or equal to 4.0.

Table 3: Counties with the Largest Negative Studentized Residuals, 2000 National Analysis

State	County Name	Studentized Residual	Vote Proportion Expected	Actual	Order
LA	Orleans	−8.89	0.0112	0.0035	1
LA	Caddo	−6.21	0.0105	0.0035	2
AL	Madison	−5.61	0.0056	0.0028	3
OR	Deschutes	−4.37	0.0074	0.0042	4
LA	East Baton Rouge	−4.37	0.0072	0.0041	5
WY	Teton	−4.35	0.0132	0.0033	6
WY	Albany	−4.12	0.0131	0.0054	7
GA	Fulton	−4.08	0.0028	0.0021	8

Notes: Results based on 2,998 counties. This table presents all counties with studentized residuals less than or equal to −4.0.

Table 4: Contiguous Counties Among Those with the Largest Studentized Residuals

State	County Name	Studentized	State	County Name	Studentized
OH	Hocking	4.06	KY	Kenton	5.10
OH	Athens	14.23	KY	Boone	5.41
MD	Wicomico	4.09	WI	Wood	5.34
MD	Somerset	6.59	WI	Lincoln	5.99
SD	Union	4.19	WI	Marathon	7.22
IA	Sioux	4.84	KS	Wyandotte	6.42
NE	Dixon	6.31	MO	Jackson	8.55
NE	Thurston	6.95	OH	Belmont	7.79
NE	Dakota	7.33	WV	Ohio	10.25
IA	Plymouth	8.20	WV	Marshall	10.35
IA	Woodbury	11.49	OH	Jefferson	14.56
MT	Deer Lodge	4.55	WV	Brooke	15.57
MT	Silver Bow	9.89	WV	Hancock	18.16
KS	Wabaunsee	4.76	MO	St. Louis City	9.30
KS	Shawnee	9.67	MO	St. Louis	9.66
KS	Pottawatomie	19.53	MN	Ramsey	11.67
CO	Arapahoe	4.96	MN	Hennepin	13.67
CO	Jefferson	7.48			
CO	Adams	7.60			

Notes: Results based on 2,998 counties. This table presents all counties with studentized residuals of magnitude greater than or equal to 4.0 that are contiguous.

Table 5: Testing Equality of Election Day and Absentee Buchanan Vote Proportions

County	z-score Difference	County	z-score Difference
Palm Beach	4.84	Pasco	0.10
Santa Rosa	1.36	Sumter	0.10
Escambia	1.29	Gulf	0.05
Okaloosa	1.17	Columbia	0.05
Levy	0.89	Suwannee	0.04
Bay	0.88	Jefferson	0.03
Hamilton	0.86	Sarasota	0.03
Hardee	0.86	Manatee	0.00
Baker	0.69	Brevard	0.00
Nassau	0.68	Wakulla	−0.02
Hernando	0.66	Indian River	−0.03
Liberty	0.64	Desoto	−0.04
Hillsborough	0.64	Okeechobee	−0.08
Gadsden	0.62	Lafayette	−0.08
Lake	0.62	Orange	−0.10
Calhoun	0.60	Madison	−0.10
Bradford	0.56	Polk	−0.13
Union	0.51	St. Lucie	−0.18
Clay	0.47	Highlands	−0.19
Citrus	0.45	Hendry	−0.20
Taylor	0.43	Volusia	−0.26
Flagler	0.38	Leon	−0.32
Holmes	0.37	Miami-Dade	−0.43
Marion	0.35	Alachua	−0.45
Jackson	0.34	Pinellas	−0.48
Washington	0.30	Osceola	−0.50
Gilchrist	0.25	Glades	−0.53
Charlotte	0.23	Broward	−0.66
Monroe	0.17	Collier	−0.69
Franklin	0.16	Dixie	−0.73
St. Johns	0.14	Martin	−0.80
Seminole	0.14	Lee	−0.87
Putnam	0.12	Duval	−1.74
Walton	0.10		

Table 6: Votes for Reform Candidates across Palm Beach County Precincts

Scope	Candidate	Intercept	Nelson	Deckard
All Precincts	Buchanan	−6.17 (0.14)	2.06 (0.20)	−12.74 (13.56)
District 35	Buchanan	−7.48 (0.47)	3.85 (0.66)	13.13 (23.16)
District 35	Lowe	−1.98 (0.32)	−1.86 (0.48)	18.06 (14.19)
District 16	Buchanan	−7.00 (0.27)	3.32 (0.43)	3.95 (14.62)
District 16	McGuire	−3.37 (0.29)	−1.00 (0.49)	25.96 (9.09)

Notes: tanh estimator with precinct-level data from the 2000 election.
Sandwich standard errors in parentheses.

Table 7: Buchanan Votes Regressed on U.S. Senate Vote in Palm Beach County

	Election Day		Absentee	
	β	SE	β	SE
Intercept	−5.18	0.034	−6.11	0.156
Nelson (D)	0.61	0.040	−0.21	0.236
Deckard (Ref)	2.41	0.138	3.68	0.400

Source: Ballot image data.
Note: Excludes ballots with spoiled presidential votes (under and over-votes).

Table 8: Proportion Voting for Buchanan Given U.S. Senate Vote Choice in Palm Beach County

	Election Day		Absentee	
Senate Candidate	Proportion	N	Proportion	N
Bill Nelson (D)	0.0102	228455	0.0017	17779
Joel Deckard (Ref)	0.0590	1000	0.0808	99

Source: Ballot image data.
Note: Excludes ballots with spoiled presidential votes (under and over-votes).

Figure 1: The Palm Beach County Butterfly Ballot

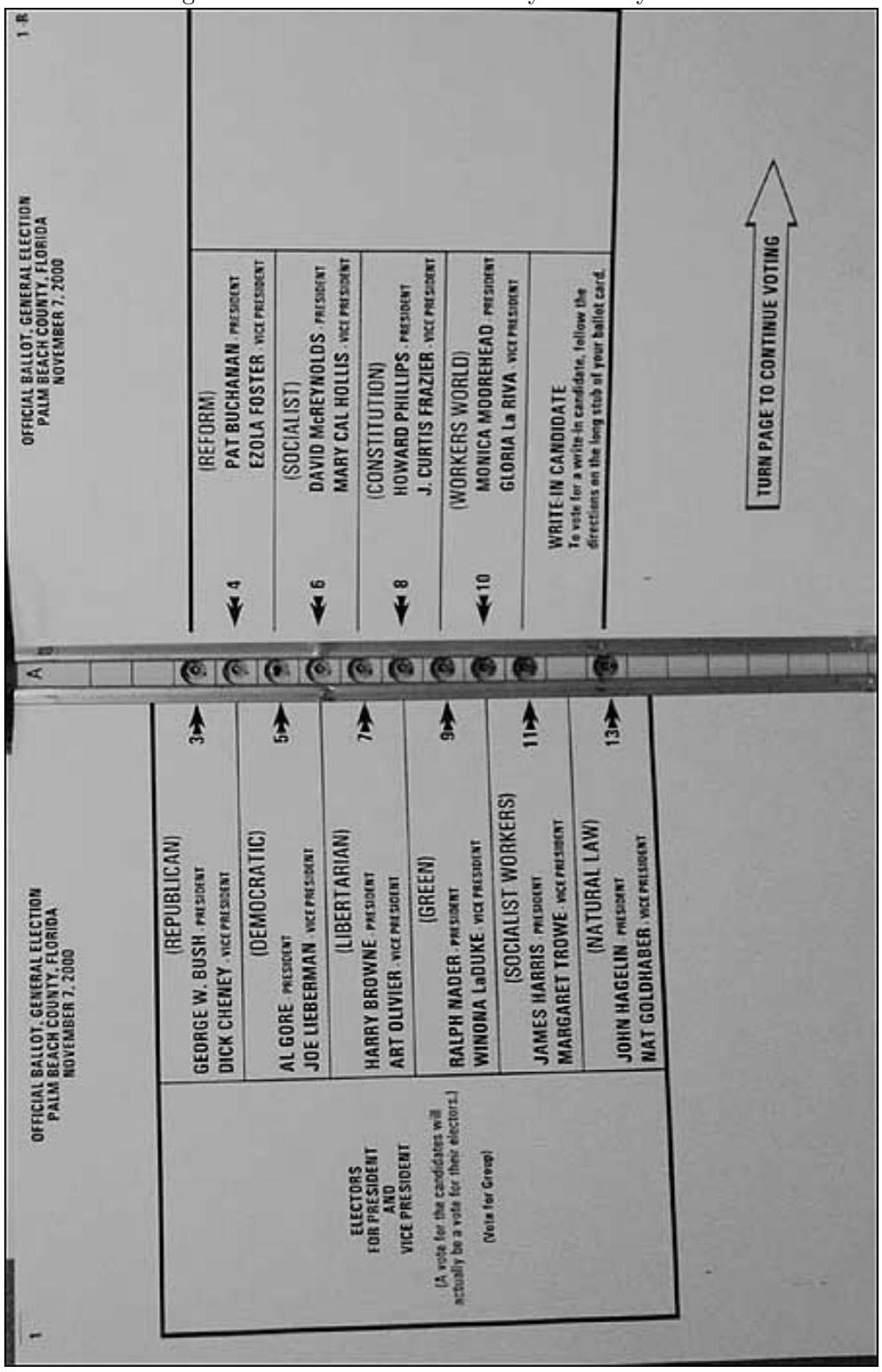

Figure 2: Histogram of Studentized Residuals from United States Counties

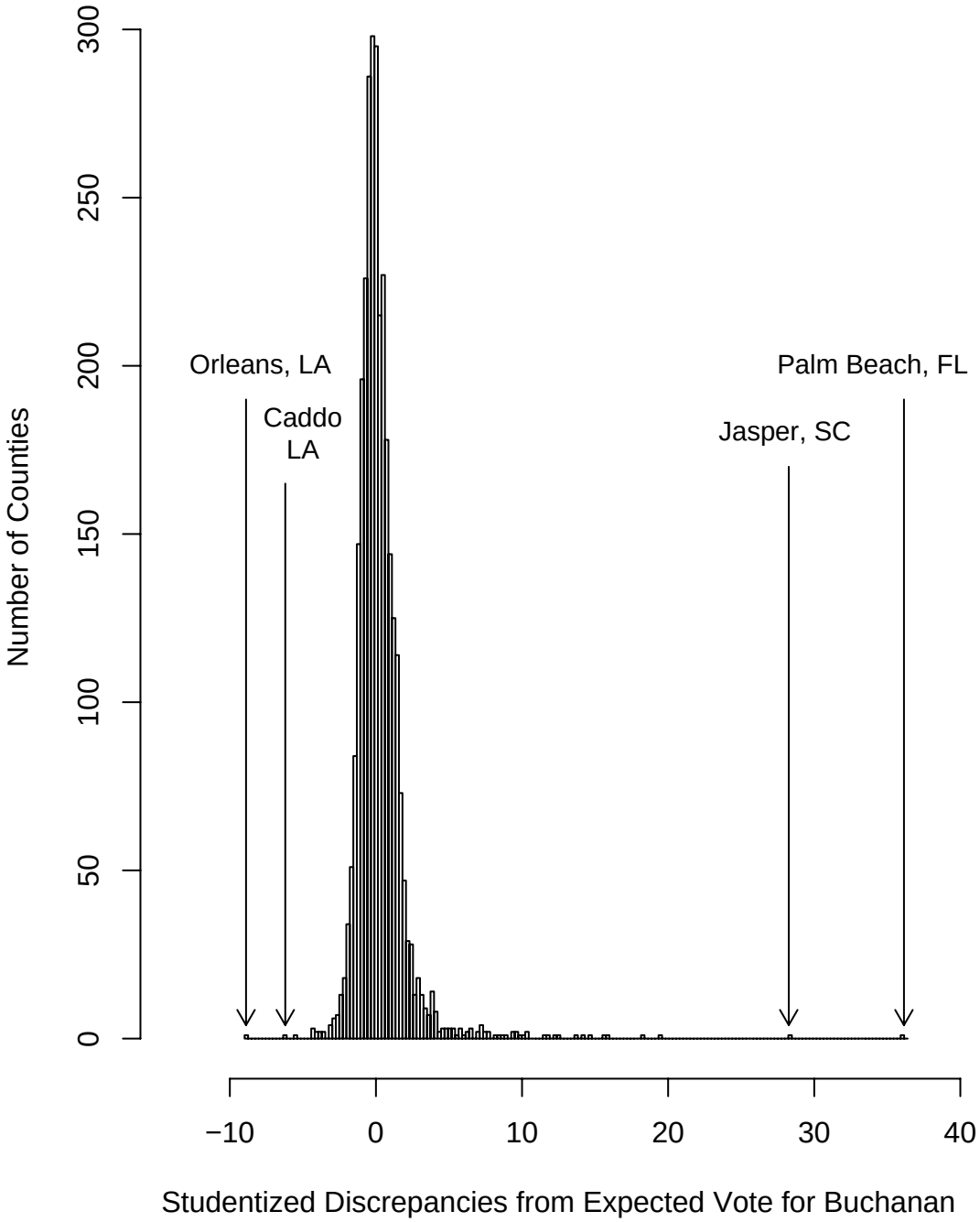

Figure 3: 2000 Election-Day Minus Absentee Support for Buchanan in Florida Counties