Potential Outcomes and Causal Effects

September 16, 2010

Neyman

I will now discuss the design of a field experiment involving plots. I should emphasize that this is a task for an agricultural person however, because mathematics operates only with general designs. In designing this experiment, let us consider a field divided into m equal plots and let

$$U_1, U_2, \cdots, U_m$$

be the true yields of a particular variety on each of these plots. If all the members U_i are equal, each of them may be called the average yield of the field. Otherwise the average yield may be thought of as the arithmetic mean

$$a = \frac{\sum_{i=1}^{m} U_i}{m}.$$

Neyman

In the ith urn, let us put m balls (as many balls as plots of the field), with labels indicating the unknown potential yield of the ith variety on the respective plot, along with the label of the plot. Thus on each ball we have one of the expressions

$$U_{i1},\ U_{i2},\ \cdots,\ U_{ik},\ \cdots,\ U_{im} \tag{13}$$

[29]

where i denotes the number of the urn (variety) and k denotes the plot number, while U_{ik} is the yield of the ith variety on the kth plot.

Disappearing Balls

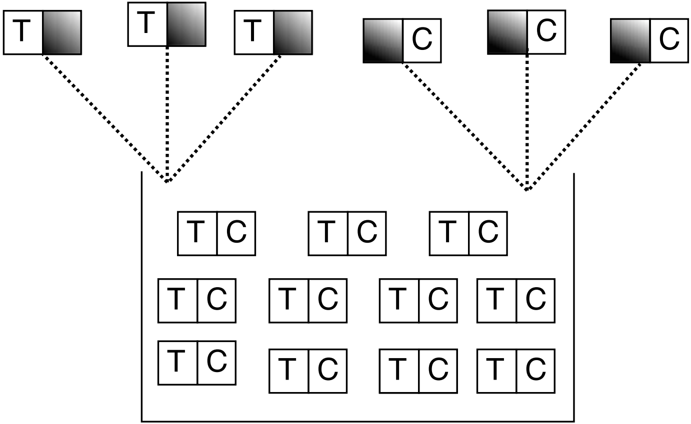
The number

$$a_i = \frac{\sum_{k=1}^{m} U_{ik}}{m}$$

is the average of the numbers (13) and is the best estimate of the yield from the ith variety on the field.

Further suppose that our urns have the property that if one ball is taken from one of them, then balls having the same (plot) label disappear from all the other urns.

Tickets



A Sampling Problem?

The goal of a field experiment which consists of the comparison of ν varieties will be regarded as equivalent to the problem of comparing the numbers

$$a_1, a_2, \cdots, a_\nu$$

or their estimates by way of drawing several balls from an urn.

The simplest way of obtaining an estimate of the number a_i would be by drawing κ balls from the ith urn in such a way that after noting the expressions on the balls drawn, they would be returned to the urn. In this way we would obtain κ independent outcomes of an experiment, and their average X_i would, based on the law of large numbers, be an estimate of the mathematical expectation of the result of our trial. Let x

Average Treatment Effects

It should be emphasized that the problem of determining the difference between the yields of two varieties becomes more complicated in this case. Let us consider the scheme with ν urns. [From now on, x_i and x_j are the averages of κ trials corresponding to varieties i and j, sampled as in the scheme with ν urns.] It is easy to see that

$$\mathbb{E}(x_i - x_j) = a_i - a_j,$$

so that the expected value of the difference of the partial averages of yields from two different varieties is equal to the difference of their expectations. It can also be determined that this difference is an estimate of $a_i - a_j$, but

Where does Uncertainty Come From?

the expression for the standard deviation becomes more complicated:

$$\begin{aligned}
\mu^2_{x_i - x_j} &= \mathbb{E}[x_i - x_j - (a_i - a_j)]^2 \\
&= \mathbb{E}(x_i - a_i)^2 + \mathbb{E}(x_j - a_j)^2 \\
&\quad - 2\mathbb{E}(x_i - a_i)(x_j - a_j) \\
&= \mu^2_{x_i} + \mu^2_{x_j} - 2[\mathbb{E}\, x_i x_j - a_i a_j].
\end{aligned}$$

Defining Causal Effects

Group	Y_{i1}	Y_{i0}
$T = 1$	Observable: $Y_{i1} \| T = 1$	Counterfactual: $Y_{i0} \| T = 1$
$T = 0$	Counterfactual: $Y_{i1} \| T = 0$	Observable: $Y_{i0} \| T = 0$

- Observed Outcome: $Y_i = T_i Y_{i1} + (1 - T_i) Y_{i0}$

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- Estimand is ATE (SATE): $\bar{\tau} = E[Y_{i1} - Y_{i0}]$
- How useful is ATE when effects are heterogeneous?
- Decompose $\bar{\tau}$:

$$\begin{aligned}\bar{\tau} &= \{\pi E[Y_{i1}|T=1] + (1-\pi)E[Y_{i1}|T=0]\} \\ &\quad -\{\pi E[Y_{i0}|T=1] + (1-\pi)E[Y_{i0}|T=0]\}\end{aligned}$$

, where π is the proportion in the treatment group.

Identifying $\bar{\tau}$

- π, $E[Y_{i1}|T=1]$, and $E[Y_{i0}|T=0]$ are estimable from data, but $E[Y_{i1}|T=0]$ and $E[Y_{i0}|T=1]$ are not.

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- Expected bias:

$$\begin{aligned} E[Y_{i1}|T=1]-E[Y_{i0}|T=0] &= \bar{\tau}+\{E[Y_{i0}|T=1]-E[Y_{i0}|T=0]\} \\ &+(1-\delta)\{E[\bar{\tau}|T=1]-E[\bar{\tau}|T=0]\} \end{aligned}$$

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- $\{E[Y_{i0}|T=1]-E[Y_{i0}|T=0]\}$ is *expected baseline bias*, difference in the average outcome in the absence of treatment.
- $(1-\delta)\{E[\bar{\tau}|T=1]-E[\bar{\tau}|T=0]\}$ is *differential treatment effect* bias, i.e. the average difference in the treatment effect between those in the treatment group and those in the control group.

Identifying Assumptions

What assumptions do we need to identify $\bar{\tau}$?

- Assumption 1: $E[Y_{i1}|T = 1] = E[Y_{i1}|T = 0]$
- Assumption 2: $E[Y_{i0}|T = 1] = E[Y_{i0}|T = 0]$

A global assumption that gets us both assumption 1 and assumption 2 is independence between treatment assignment and potential outcomes:

$$\{Y_{i1}, Y_{i0}\} \perp\!\!\!\perp T$$

Beyond ATE

What about other estimands?

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- Average effect of treatment on the treated (ATT): $\bar{\tau}|(T=1) = E[Y_{i1}|T=1] - E[Y_{i0}|T=1]$
- Average effect of treatment on the controls (ATC): $\bar{\tau}|(T=1) = E[Y_{i1}|T=0] - E[Y_{i0}|T=0]$

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- Conditional independence:

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- With this assumption, it follows that:
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- If we are interested in ATC, we only need the following:
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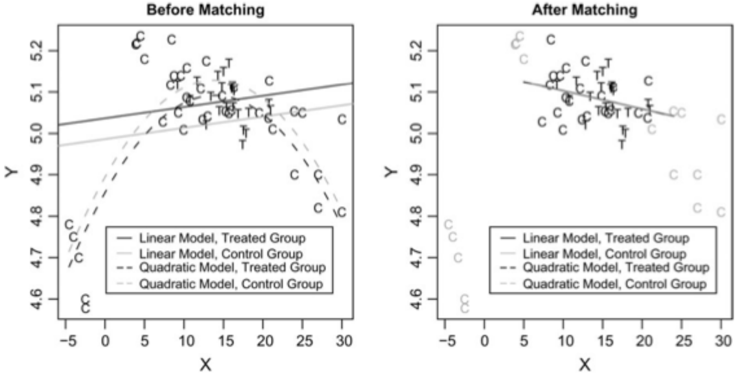
ATT


Before Matching
After Matching
Linear Model, Treated Group
Linear Model, Control Group
Quadratic Model, Treated Group
Quadratic Model, Control Group
X
Y

SUTVA

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- "No interference between units" is "the observation on one unit should be unaffected the particular assignment of treatment to the other units"

No Interference

- Consider a uniform randomized experiment with two strata, four units in the first strata and two units in the first strata and two units in the second strata, for 6 units in total. Half the units in each stratum receive treatment.
- There are 12 possible treatment assignments contained in the set Ω.

$$\Omega = \left\{ \begin{array}{cccccc} \begin{bmatrix}1\\1\\0\\0\\1\\0\end{bmatrix} & \begin{bmatrix}1\\0\\1\\0\\1\\0\end{bmatrix} & \begin{bmatrix}1\\0\\0\\1\\1\\0\end{bmatrix} & \begin{bmatrix}0\\1\\1\\0\\1\\0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\\1\\0\end{bmatrix} & \begin{bmatrix}0\\0\\1\\1\\1\\0\end{bmatrix} \\ \begin{bmatrix}1\\1\\0\\0\\0\\1\end{bmatrix} & \begin{bmatrix}1\\0\\1\\0\\0\\1\end{bmatrix} & \begin{bmatrix}1\\0\\0\\1\\0\\1\end{bmatrix} & \begin{bmatrix}0\\1\\1\\0\\0\\1\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\\0\\1\end{bmatrix} & \begin{bmatrix}0\\0\\1\\1\\0\\1\end{bmatrix} \end{array} \right\}.$$

Causal Effects without SUTVA

- Without SUTVA, a causal effect is defined for every possible combination of the treatment assignment.
- The potential outcome for unit i might be $Y_{i100000000000}$ or $Y_{i010000000000}$, etc.
- How many potential outcomes will each unit have?
- Potential outcomes still well defined!