

Estimation of the Nonaccelerating Inflation Rate of Unemployment (NAIRU)

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The nonaccelerating inflation rate of unemployment (NAIRU) is an important economic and political concept. It is significant not only because most macroeconomists use it when thinking about a variety of issues but also because it plays a pivotal role in guiding policy. In this note I briefly describe how I estimate the NAIRU.

1 NAIRU Estimation

The NAIRU model, the standard model for inflation, is usually based on the expectations-augmented Phillips relation

$$\pi_t - \pi_t^e = \beta(u_t - u_t^*) + \delta X_t + v_t, \quad (1)$$

where π_t is an estimate of the actual inflation rate, π_t^e is the expected inflation rate, u_t is the unemployment rate, u_t^* is the NAIRU, X_t contains additional regressors intended to control for supply shocks, and v_t is an error term.

Like many others, I use a “random walk” model for inflationary expectations (Congressional Budget Office 1994; Fuhrer 1995; Gordon 1990; Staiger, Stock and Watson (SSW) 1997a, 1997b; Tootell 1994; Weiner 1993). According to the “random walk model,” $\pi_t^e = \pi_{t-1}$, so that $\pi_t - \pi_t^e = \Delta\pi_t$. Hence,

$$\Delta\pi_t = \beta(u_t - u_t^*) + \delta X_t + v_t. \quad (2)$$

Equation 2 neglects the possibility of serial correlation in the error term. It is therefore conventional to estimate an autoregressive specification:

$$\Delta\pi_t = \beta(L)(u_t - u_t^*) + \delta(L)\Delta\pi_{t-1} + \gamma(L)X_t + \varepsilon_t, \quad (3)$$

where L is the lag operator, $\beta(L)$, $\delta(L)$, and $\gamma(L)$ are lag polynomials and ε_t is a serially uncorrelated error term.

Equation 3 is difficult to estimate because the model is nonlinear in the parameters. When the NAIRU, u_t^* , does not vary with t , Equation 3 can be rewritten in a form which can be conveniently estimated by ordinary least squares (OLS):

$$\Delta\pi_t = \mu + \beta(L)u_t + \delta(L)\Delta\pi_{t-1} + \gamma(L)X_t + \varepsilon_t. \quad (4)$$

The estimate of the NAIRU is then

$$\hat{u}^* = \frac{-\hat{\mu}}{\hat{\beta}(1)}, \quad (5)$$

where $\beta(1) = \sum_{i=1}^p \beta_i$, with p being the order of the lag polynomial $\beta(L)$. Notice that the NAIRU is a nonlinear function of the coefficients μ and $\beta(1)$.

The estimation approach outlined in Equations 4–5 can be altered for the case where the NAIRU varies with time. One way to accomplish this is to replace $\hat{\mu}$ with $\sum_{j=1}^l \alpha_j H^{j-1}(t)$ where H^i is a Hermite polynomial of order i and where t is the time argument centered around 0.¹ Therefore,

$$\Delta\pi_t = \sum_{j=1}^l \alpha_j H^{j-1}(t) + \beta(L)u_t + \delta(L)\Delta\pi_{t-1} + \gamma(L)X_t + \varepsilon_t, \quad (6)$$

and the estimate of the time-varying NAIRU is

$$\hat{u}_t^* = \frac{-\sum_{j=1}^l \hat{\alpha}_j H^{j-1}(t)}{\hat{\beta}(1)}. \quad (7)$$

Selection of the order of the Hermite polynomial series is done using Mallows' C_p .² Hermite polynomials in particular and orthogonal polynomials in general have many properties which make them useful for flexible-form estimation (de Boor 1978; Hinich and Roll 1981; Szegő 1975)—see Appendix A.³ Less computation is required for Hermite polynomials than for cubic splines.

SSW (1997a, 1997b) use several approaches to estimate the time-varying NAIRU. One of them is similar to the one specified in Equation 6. They do not allow contemporaneous unemployment and supply shocks to affect the current period's inflation rate. SSW (1997a, 1997b) approximate the NAIRU by a cubic spline in time, written as $\bar{\phi}'S_t$, where S_t is a vector of deterministic functions of time.⁴ The SSW estimation equation for the spline approximated time-varying NAIRU is

$$\Delta\pi_t = \phi'S_{t-1} + \beta(L)u_{t-1} + \delta(L)\Delta\pi_{t-1} + \gamma(L)X_{t-1} + \varepsilon_t, \quad (8)$$

where $\phi = -\beta(1)\bar{\phi}$ and $\hat{u}_t^* = -\hat{\phi}'S_t/\hat{\beta}(1)$. SSW estimate Equation 8 by OLS.

Following SSW (1997b) the lag operators begin with u_{t-1} and X_{t-1} instead of u_t and X_t —see Equation 8. Also following SSW (1997b), I use monthly lags over one year for both u_t and $\Delta\pi_t$ and lags over one quarter for the supply shocks, X_t . The supply shocks are specified as the difference between food and energy inflation and overall CPI inflation.⁵ The data are monthly and the price level measure is the full CPI.

¹The argument, t , must be centered around 0 because Hermite polynomials are orthogonal on the symmetric interval $(-\infty, \infty)$ with respect to the weighting function $w(t) = e^{-t^2}$. See Appendix A for details.

²The current model selects a Hermite series of order 6.

³I do not wish to imply that Hermite polynomials are necessarily superior to other polynomials such as Legendre or Chebyshev. The use of Legendre polynomials does not significantly alter my results.

⁴See SSW (1997b, 207) Equation 10.

⁵See King and Watson (1994) footnote 18. I had previously included the NIXON variable that Gordon (1990, 1167) suggests as a supply shock, but it is not significant and is rejected by various model selection criteria—LR test, Mallows' C_p and BIC.

A Appendix: Hermite Polynomial Series

I use the Hermite polynomial series to estimate the NAIRU. The Hermite series is often used to implement cubic splines (de Boor 1978). Hermite polynomials form one of the families of classical orthogonal polynomials (Szegő 1975). The variance-covariance matrix is often well-conditioned (Hinich and Roll 1981). The Hermite polynomials may be defined for $j > 0$ by the recurrence relation

$$H^{j+1}(t) = 2tH^j(t) - 2jH^{j-1}(t).$$

where t is the argument (time), j is the order and where $H^0(t) = 1$ and $H^1(t) = 2t$. The argument, t , must be centered around 0 because the polynomials are orthogonal on the symmetric interval $(-\infty, \infty)$ with respect to the weighting function $w(t) = e^{-t^2}$,

$$\int_{-\infty}^{+\infty} H^j(t)H^k(t)e^{-t^2} dt = 2^k k! \pi^{\frac{1}{2}} \delta^{jk}, \quad j, k \geq 0.$$

A few Hermite polynomials are listed below. Note how quickly the polynomials increase in size. This is one of the reasons why the parameters associated with the polynomials are extremely difficult to interpret.

$$H^2(t) = 4t^2 - 2$$

$$H^3(t) = 8t^3 - 12t$$

$$H^4(t) = 16t^4 - 48t^2 + 12$$

$$H^{16}(t) = 65536t^{16} - 3932160t^{14} + 89456640t^{12} - 984023040t^{10} + 5535129600t^8 - 15498362880t^6 \\ + 19372953600t^4 - 8302694400t^2 + 518918400$$

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