Chicago Price Theory Summer Camp Notes*

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1 Monday, June 25, 2007

1.1 Kevin Murphy

1.1.1 The consumer's problem

The basic utility maximization problem is $\max u(X) \quad \text{s.t.} \quad Xp = M$. The Legrangian results in $\frac{\partial U}{\partial X_n} - \lambda p_n = 0 \quad \forall \quad n = 1, \ldots, k$. These equations imply that marginal utility is proportional to prices. Comparing two goods shows that

$$\frac{\frac{\partial U}{\partial X_i}}{\frac{\partial U}{\partial X_j}} = \frac{p_i}{p_j}$$

To convert from utility to dollars, see that $\frac{\frac{\partial U}{\partial X_i}}{\lambda} \equiv$ dollar cost (where $\lambda =$ marginal utility of income). Conveniently, utility changes over time can be determined solely by looking at changes in consumption:

$$\frac{\partial U}{\partial t} = \sum_i \frac{\partial U}{\partial X_i} \frac{\partial X_i}{\partial t} = \lambda \sum_i p_i \frac{\partial X_i}{\partial t}$$

*In June 2007, the Becker Center on Chicago Price Theory at the University of Chicago prepared a week-long summer camp to acquaint students from other universities with traditional price theory. This is a compilation of my notes from the week. I would like to thank the Becker Center for the opportunity to attend the event.

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1.1.2 Marshallian demand curves and elasticities

Marshallian demand curves hold M constant and are defined by $X_i = X_i(p_1, \ldots, p_n, M)$. The demand for a good is not dependent upon the quantities consumed of other goods. Additionally, optimal adjustment requires that consumption of other goods can change in the long run.

By taking derivatives of both sides of the budget constraint, you can see that the share-weighted average of income elasticities must be 1:

$$\begin{aligned} \eta_i &= \frac{\partial X_i}{\partial M}\frac{M}{X_i}, \quad \sum_i X_i p_i = M \Longrightarrow \sum_i \frac{\partial X_i}{\partial M}\frac{p_i X_i}{M}\frac{M}{X_i} = \sum_i \eta_i s_i = 1 \\ \epsilon_i &= \frac{\partial X_i}{\partial p_i}\frac{p_i}{X_i} \end{aligned}$$

When examining elasticities, it is important to differentiate between broad categories of goods and specific goods. For income elasticities, broad categories of goods are normal goods, though specific goods may not be (e.g., food versus rice in China). Any specific good may be superior and inferior, depending upon the income level. For price elasticities, broad categories are inelastic, but specific goods are elastic. It is important to think about how many substitutes exist (or how broad a category is) when thinking about the magnitude of the price elasticity. We often wonder how ϵ_i compares to 1 because this relationship determines how expenditures on the good changes with price.

Profit maximization among price-taking firms implies that the firms are facing elastic demand. If you change the quality of a good, sales of it can go up or down. The effect on other goods is unambiguous, however. For example, a longer lasting lightbulb may mean that a customer has to buy fewer of those bulbs. But the quality increase made the bulb cheaper relative to its competitors, so the other bulbs will be demanded less. In this example, it may be more useful to think of consumers purchasing light-hours, rather than bulbs.

1.1.3 The cost function and Hicksian demand

The cost function, $C = \min Xp \quad \text{s.t.} \quad U(X) = \bar{U}$, defines Hicksian demand under which utility is held constant. The results of the cost minimization problem are the same as those for utility maximization. This represents an important *duality* in price theory. This result can be interpreted as implying that we can index our problem by U or M and move the budget line to the utility curve or the utility curve to the budget line to find the optimal bundle.

Hicksian demand is written as $\frac{\partial C}{\partial X_i} = X_i^H(p_i, \ldots, p_n)$. This function is homogeneous of degree 1 and concave in p. Taking derivatives yields the following result, known as the *law of demand* or the *Slutsky equation*.

$$\begin{aligned}
\frac{\partial X_i^H}{\partial p_i} &= \frac{\partial X_i^M}{\partial p_i} + X_i \frac{\partial X_i^M}{\partial M} \\
\frac{\partial X_i^M}{\partial p_i} &= \underbrace{\frac{\partial X_i^H}{\partial p_i}}_{\leq 0} - X_i \frac{\partial X_i^M}{\partial M} \Longrightarrow \quad \epsilon_{ii}^M = \epsilon_{ii}^H - s_i \eta_i
\end{aligned}$$

Since s_i is small for most goods, it is very difficult for $\epsilon_{ii}^M \geq 0$, in other words, a Giffen good. Since we assume downward-sloping Hicksian demand, Giffen goods must be inferior (i.e., $\eta_i < 0$). But, if a good is inferior, it must have good substitutes (since you consume other options as your income increases), which increases ϵ_{ii}^H, working against finding a Giffen good.

1.1.4 Price indices

The price index is a linear approximation to the cost function and permits substitution among the goods.

$$\frac{C(p_1^1,\ldots,p_n^1,\bar{U})}{C(p_1^0,\ldots,p_n^0,\bar{U})} \approx \underbrace{\frac{\sum X_i^1 p_i^1}{\sum X_i^0 p_i^0}}_{=(1+g_{exp})} = \underbrace{\frac{\sum X_i^0 p_i^1}{\sum X_i^0 p_i^0}}_{=(1+g_p)} \underbrace{\frac{\sum X_i^1 p_i^1}{\sum X_i^0 p_i^1}}_{=(1+g_q)} \approx \frac{p_i^1}{p_i^0}\frac{X_i^1}{X_i^0}$$

1.2 Gary Becker

1.2.1 Discrimination

If a firm wishes to discriminate, it wants to control the number of undesirable workers, N. Instead of pure profit maximization, its problem can be represented in a utility framework:

$$\max U(\pi, N), \frac{\partial U}{\partial \pi} > 0, \quad \frac{\partial U}{\partial N} < 0$$

2 Tuesday, June 26, 2007

2.1 Kevin Murphy

2.1.1 Elasticity expressions

The law of demand relating two goods is $\epsilon_{ij}^M = \epsilon_{ij}^H - s_j \eta_i$. If good i is consumed by individuals $n = 1 \ldots N$, where $X_i = \sum_n X_i^n$, then the Marshallian elasticity can be decomposed into[1]

$$\underbrace{\frac{\partial X_i}{\partial p_j}\frac{p_j}{X_i}}_{\text{Market elasticity}} = \sum_n \underbrace{\frac{\partial X_i^n}{\partial p_j}\frac{p_j}{X_i^n}}_{\text{Elasticity of } n} \times \underbrace{\frac{X_i^n}{X_i}}_{\text{Share of } n}$$

Under Hicksian demand, $\frac{\partial X_i^H}{\partial p_j} = \frac{\partial^2 C}{\partial p_j \partial p_i} = \frac{\partial X_j^H}{\partial p_i}$. Thus, cross-price elasticities can be related as $s_i \epsilon_{ij} = s_j \epsilon_{ji}$, where s_i is the fraction of income spent on good i. Hence, the change in

[1] If individual demands are independent of one another.

price of a good receiving a high proportion of an individual's budget will have a large effect on the demand of goods marked by lower expenditures.

2.1.2 Elasticity regressions

Regression equations can be created to identify own- and cross-price elasticities from the Marshallian demand system. A two-good system would be

$$d\log X_1 = \epsilon_{11} d\log P_1 + \epsilon_{12} d\log P_2 + \eta_1 d\log M$$
$$d\log X_2 = \epsilon_{21} d\log P_1 + \epsilon_{22} d\log P_2 + \eta_2 d\log M$$

But we also know

$$\begin{aligned} s_1 &= \frac{P_1 X_1}{M}, \\ s_2 &= \frac{P_2 X_2}{M}, \\ s_1 &= 1 - s_2, \\ \epsilon_{12} &= \epsilon_{21}\frac{s_2}{s_1} \end{aligned}$$

Lastly, from homogeneity of demand,[2] $\epsilon_{11} + \epsilon_{12} + \eta_1 = 0$ and $\epsilon_{21} + \epsilon_{22} + \eta_2 = 0$. Hence, we only need to identify s_1, η_1, and any price elasticity parameter to completely solve the system.

If M is denominated in nominal dollars, Marshallian demand is being calculated. If M is in real dollars, use the Hicksian analogs of the previous equations. Let $\hat{M}$ be the real change in income (i.e., $d\log\hat{M} = d\log M - s_1 d\log P_1 + s_2 d\log P_2$) and the resulting equations

[2] If you multiply prices and income by the same scalar, the budget set doesn't change and thus neither does demand.

are

$$d\log X_1 = \underbrace{(\epsilon_{11} + s_1\eta_1)}_{=\epsilon_{11}^H} d\log P_1 + \underbrace{(\epsilon_{12} + s_2\eta_1)}_{=\epsilon_{12}^H} d\log P_2 + \eta_1 d\log \hat{M}$$

$$d\log X_2 = \underbrace{(\epsilon_{21} + s_1\eta_2)}_{=\epsilon_{21}^H} d\log P_1 + \underbrace{(\epsilon_{22} + s_2\eta_2)}_{=\epsilon_{22}^H} d\log P_2 + \eta_2 d\log \hat{M}$$

These equations can be simplified further by substituting $\epsilon_{11}^H = -\epsilon_{12}^H$.[3]

2.1.3 Relating elasticities of supply and demand

When the quantity consumed changes, two equations must hold simultaneously:

$$d\log Q = d\log D + \epsilon^D d\log P$$

$$d\log Q = d\log S + \epsilon^S d\log P$$

From these equations, if $d\log P > 0$, then $d\log D > d\log Q > d\log S$, as a result of the signs of the elasticities. The opposite relation holds if $d\log P < 0$.

2.2 Gary Becker

2.2.1 Peer effects

Unlike other social disciplines, economics is able to use the same underlying model to examine individual and aggregate behavior, namely, utility maximization. This formulation is often devoid of peer effects, a prominent consideration of sociologists. But these factors can be included within the standard framework. Taking X to be a good (possibly) exhibiting peer effects and Y a bundle of other goods, the consumer solves $\max U_i = U(X_a, Y_a; \tilde{X})$, for $\tilde{X}$ being the consumption of all other individuals. The questions of interest are how $\frac{\partial U_a}{\partial X}$ and

[3] A result that follows from Hicksian demand being homogeneous of degree zero.

$\frac{\partial}{\partial X}(\frac{\partial U_a}{\partial X_a})$ compare to 0. Examining how the demand function changes with price yields

$$\begin{aligned}\frac{\partial X}{\partial P_X} &= \sum\left[\frac{\partial X_i}{\partial P_X} + \frac{\partial X_i}{\partial X}\frac{\partial X}{\partial P_X}\right] \\ &= \underbrace{\frac{1}{1-\sum\frac{\partial X_i}{\partial X}}}_{\text{Social multiplier}}\sum\frac{\partial X_i}{\partial P_X}\end{aligned}$$

The social multiplier can be greater than 1, generating explosive results. Also, common shocks will produce large responses. The presence of social effects can generate upward sloping demand (i.e., when the social multiplier is greater than 1, often over only a portion of the demand curve), but this is not the best perspective for this problem. Instead, ask "how much are people willing to pay for a given quantity?"

Imagine a restaurant with a fixed number of seats, A, which lies in range of the demand curve where the social multiplier is greater than one. Since demand is upward sloping at this point, demand will rise until the social multiplier is 1. There will be a shortage—metered by a queue for a table. The equilibrium, then, will be a quantity of A, but the price such that the social multiplier is 1.

Is it bad that there is an "arms race" for tables at a popular restaurant? Since people value being at the popular restaurant, they could be happy to wait (see problem set 1 example).

2.3 Habit formation

Habit formation implies that *past* consumption of a good enters the utility function along with current consumption. Past consumption is like a stock variable that has been accumulated. To derive a utility function examining this property, we will assume separability. Let X_t be the current consumption of a habit good and S_t its stock. Y_t are all other goods: $U = U(X_t, Y_t, S_t)$, $S_t = \sum_i = 1^\infty a_i X_{t-i}$. Habit formation implies that $\frac{\partial}{\partial S_t}\left(\frac{\partial U}{\partial X_t}\right) > 0$. If $\frac{\partial U}{\partial S_t} > 0$, then it is a beneficial habit. It is a detrimental habit if the derivative is less than 0.

The problem can be reconstructed into a dynamic programing framework:

$$\begin{aligned} U_t &= U(X_t, Y_t, X_{t-1}) \\ V &= \sum \beta_t U_t \\ B_0 &= \sum \left(\frac{1}{1+r}\right)^t (M_t - Y_t - p_t X_t) \end{aligned}$$

The habit formation model suggests that past consumption may be complementary to current consumption; stocks and flows are often complementary. If it is examined in a more symmetrical fashion, the *future* can also influence decisions made today.

For a given habit stock, an increase in price today will lower consumption today, reducing the stock in the future, and lowering future consumption as well. The result is that the short-run response to price will be smaller than the long-run response.

3 Wednesday, June 27, 2007

3.1 Kevin Murphy

3.1.1 The demand for goods tied to capital

The approach outlined in 2.1.2 can be used to examine a market containing capital and another good.[4] The Hicksian equations for this model are

$$d \log X_1 = \epsilon_{11}^H d \log P_1 + \epsilon_{1K}^H d \log P_K + \eta d \log \hat{M}$$

$$d \log X_K = \epsilon_{K1}^H d \log P_1 + \epsilon_{KK}^H d \log P_K + \eta d \log \hat{M}$$

[4]Capital can be thought of broadly—ranging from machinery to consumer durables to the supply of doctors. A capital good is defined by a stock with inelastic short-run supply.

The capital stock cannot change in the short run (i.e., $d\log X_K = 0$) and $\eta = 0$.[5] Using these facts and the second equation,

$$\frac{d\log P_K}{d\log P_1} = \frac{-\epsilon^H_{K1}}{\epsilon^H_{KK}}$$

and, from the first equation,

$$\frac{d\log X_1}{d\log P_1} = \epsilon^H_{11} + \frac{-\epsilon^H_{1K}\epsilon^H_{K1}}{\epsilon^H_{KK}}$$

The first term on the right hand side is negative, while the second is positive. This shows that the response to a price change for the good tied to capital is more inelastic in the short-run relative to its long-run response.[6] This stems from the fact that the stock of capital cannot respond to its own price changes in the short-run. Also note that short-run demand is a function of long-run demand. These results hold whether the good is a complement or substitute for capital.

3.1.2 The price of capital

Capital is a stock and investment is a flow. There are two prices for capital: the purchase price (P_t) and the rental price (R_t). The two, obviously, are related. The depreciation and interest rates are also important. These factors combine into[7]

$$\begin{aligned}
K_t &= I_t + (1-\delta) I_{t-1} + (1-\delta)^2 I_{t-2} \ldots \\
P_t &= R_t + \frac{1-\delta}{1+r} R_{t-1} + \left(\frac{1-\delta}{1+r}\right)^2 R_{t-2} + \ldots \\
R_t &= P_t - \frac{(1-\delta) P_{t+1}}{1+r} = \frac{P_t}{1+r}\left[r + \delta\frac{P_{t+1}}{P_t} + \frac{P_t - P_{t+1}}{P_t}\right]
\end{aligned}$$

[5] If the supply of capital is not perfectly inelastic in the short-run, set $d\log X_K = \epsilon^S d\log P_K$.

[6] Since the ϵ matrix is negative semi-definite, $\frac{d\log X_1}{d\log P_1}$ will be negative.

[7] If your data do not go back far enough in the past to record all the investment levels and rental prices, you can assume that these figures were growing at a constant rate prior to the start of your data.

The last term in the final equality represents re-evaluation; the same good may be worth less in the following period.

Depreciation can be imagined in two ways: as a house burning to the ground or as an iceberg slowly melting.[8] The δ term captures both of these occurrences. It is appropriately calculated by comparing the value of a year-old and a new unit of the good in the same time period: $1-\delta=\frac{P_t^{\text{1year old}}}{P_t^{\text{new}}}$. As firms move toward using short-lived capital, there will be an output boom as new short-lived capital is utilized alongside the slow-depreciating variety. But, as the older capital is taken out of service, output will fall. The productivity of capital may also fall faster than the depreciation rate. For example, a laptop computer may continue to function for many years, but it will only provide useful productivity for three or four.

What would happen to computer sales if technology stopped advancing? It may be tempting to believe that sales will fall, but they actually will *rise*. When technology advances, the price of a good today will be lower in the future. But if technology ceases to advance, the future price will not fall. Hence, P_{t+1} will effectively increase, lowering the rental price of capital, which, in turn, lowers the purchase price of capital. A lower price implies higher sales. The intuition is that consumers will wait for improved technology or wait for current technology to become cheaper in the future. Removing these incentives lead to higher purchases today.

3.1.3 Steady-state capital levels

There are four equations that identify the equilibrium level of capital:

$$\begin{aligned}
K_t &= D\left(R_t\right) \quad \text{(demand in the rental market)} \\
P_t &= \int_t^{\infty} e^{(r+\delta)(\tau-t)} R_t \, d\tau \quad \text{(capital pricing equation)} \\
I_t &= I\left(P_t\right) \quad \text{(investment)} \\
\dot{K}_t &= I_t - \delta K_t \quad \text{(law of motion)}
\end{aligned}$$

[8] Even the labor force can be seen as depreciating. Let $L_t = (1-\delta) L_{t-1} + H_t$. Depreciation could represent death, leaving the workforce, or forgetting previously-held knowledge.

Under uncertainty, the expectations of the preceding equations would be used, but expectations are not as important when δ is high. If some amount of K is destroyed, R, P, and I will jump up discontinuously and fall over time. The steady-state will not be overshot due to exponential discounting (cf. the life cycle model). We reach the steady–state fastest when the elasticity of supply is high and elasticity of demand is low. From this proposition, the consumption of durables is smooth and production is volatile. The rental price will be more volatile than the purchase price.

4 Thursday, June 28, 2007

4.1 Kevin Murphy

4.1.1 Steady-state capital levels, continued

Continuing with examples of shocks to the steady-state, imagine increased demand in the rental market. If the change is unanticipated, R, P, and I will jump discontinuously and K will increase over time. If the change is anticipated, I begins increasing and overshoots its steady-state level at time t. R falls leading up to time t because K is higher than its initial steady-state level. It discontinuously jumps at time t, rising above its new steady-state value. Anticipating higher future values of R forces P to jump discontinuously immediately. It rises above its future steady-state value and overshoots it at time t before falling to its final level.

In the preceding example, why can't all the capital be built by time t? Imagine that this is the case. Then, R must be below its steady-state value, then jump to precisely this value at time t. If R follows this pattern, then P will rise to its steady-state value without overshooting. But, if P follows this pattern, I cannot overshoot its steady-state level, rendering it impossible to accumulate the steady-state level of capital prior to time t (since any investment level below the final steady-state cannot maintain the steady-state level of capital, since investment and depreciation offset one another in the steady-state).

4.1.2 Total factor production

Define $Y = TF(L, K)$. Taking derivatives,

$$\begin{aligned} dY &= dTF(L,K) + TF_L dL + TF_K dK \\ &= dTF(L,K)\frac{T}{T} + TF_L dL \frac{L}{L} + TF_K dK \frac{K}{K} \end{aligned}$$

Dividing through by Y yields[9]

$$\begin{aligned} \Delta Y &= \Delta T + s_L \Delta L + s_K \Delta K \\ \Delta T &= \Delta Y - (s_L \Delta L + s_K \Delta K) \end{aligned}$$

Using $PY = WL + RK$ and constant returns to scale[10]

$$\begin{aligned} \Delta P + \Delta Y &= s_L\left(\Delta L + \Delta W\right) + s_K\left(\Delta K + \Delta R\right) \\ \underbrace{\Delta Y - (s_L \Delta L + s_K \Delta K)}_{=\Delta T \text{ from above}} &= \underbrace{s_L \Delta W + s_K \Delta R - \Delta P}_{=\Delta T \text{ calculated from prices}} \\ \Delta T &= s_L\left(\Delta W - \Delta P\right) + s_K\left(\Delta R - \Delta P\right) \end{aligned}$$

This result shows that gains in productivity are divided between labor and capital. In the long-run, $F_K = \rho + \delta$ because K is elastic. This implies that $\Delta R = \Delta P = 0$ and all gains from productivity are allocated to labor. The change in the wage is $\frac{\Delta T}{s_L}$.

4.2 Steve Levitt

When planning a course, you must consider whether you are creating *producers* or *consumers* of the information being presented. Additionally, it is often useful to present bad papers as examples of what *not* to do and to encourage students to read critically and search for

[9] $s_K \equiv \frac{MP_K K}{Y}$, which holds with no assumptions on $F(K)$.

[10] Specifically making use of the fact that, under constant returns, the factor price divided by the output price is equal to the marginal product of the factor.

errors in reasoning. Levitt's course examines the different methodologies underlying reduced-form (e.g., difference-in-difference and instrumental variables) and structural approaches to modeling.

Levitt draws the distinction between "good" structure and "bad" structure. Good structure stems from economic assumptions (e.g., demand curves slope downward), while bad structure arises from necessary, though arbitrary, assumptions that are imposed by the researcher to make the problem tractable (e.g., functional forms and representative agents). Interestingly, schools prefer job market candidates that demonstrate a command of structural methods, while journals and awards committees prefer reduced-form approaches.

Reduced-form	
Pros	Cons
Transparent identification	Can't answer the true question of interest
Easy to read	Doesn't involve much economics
Easy to write	Can't extrapolate/specific to that situation
Example: Bresnahan and Reiss	

Structural	
Pros	Cons
Gets at deep parameters (rewarded by the profession)	Hard to read and write
Doesn't care about identification	No one believes the results
Can examine counterfactuals and changes	Lacks transparency
Example: Pakes	

5 Friday, June 29, 2007

5.1 Gary Becker

5.1.1 Illegal goods: drugs

The costs of punishment under the war on drugs can be viewed as a tax on producers (since consumers often receive much more lenient treatment, though the analysis would be similar). Under a tax, all but the deadweight loss is a transfer to the government, but for punishment, the entire increased cost is lost. If $\epsilon^D < 1$, as is commonly thought, these costs are increasing with strengthening punishments. Beside imposing a cost on suppliers, punishment also has a cost to mete out. These factors combine to make the war on drugs very costly to fight.

If drugs were legalized, these costs could be avoided. If a tax of the same magnitude as the prior punishment was enacted, much of the costs could be captured as a transfer. Also, government could focus its attention on punishing a smaller pool of lawbreakers. The government would have an incentive to end illegal trade in order to increase its tax revenues. Demand for illegal drugs would become elastic due to the presence of legal alternatives. These factors help to ensure that dealing drugs legally was more profitable than doing so in the black market, a necessary condition for success.