PS C236A / Stat C239A
Problem Set 3 - Solutions

1: First we need to show that $Z \perp X|e(X)$, which is equivalent to showing $P(Z|X) = P(Z|e(X))$.

$P(Z|e(X)) = E[Z|e(X)] = E[E[Z|e(x), x]|e(x)] = E[P(Z|X, e(X))|e(x)] = E[e(x)|e(x)] = e(x) = P(Z|X)$

Now we need to show that conditioning on the propensity score and under the stated assumptions, that the ATT is identified. We want to estimate $E((r_1 - r_0)|Z = 1)$ which is the ATT estimand. Note that conditioning on $Z = 1$ is equivalent to conditioning on $X|Z = 1$, which–as we proved above–is equivalent to conditioning on $e(X)|Z = 1$.

We can observe without assumptions: $E(r_1|Z = 1) - E(r_0|Z = 0)$ Because we assume that $r_0 \perp Z$, then $E(r_0|Z = 0)$ can be rewritten as $E(r_0|Z = 1)$. As a result, $E(r_1|Z = 1) - E(r_0|Z = 1) = E(r_1 - r_2|Z = 1)$

2. a.
   b.
   c.
   d.
   e.
3. a.
   b.
   c.
4.