Causal Inference in the Age of Big Data

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Causal Inference and Big Data

- Human activity is generating massive datasets with granular population information:
  - Administrative data: schools, criminal justice, IRS
  - Browsing, search, and purchase data from online platforms
  - Internet of things
  - Electronic medical records, genetic markers
- Big in size and breadth: wide datasets
- Data can be used for personalization of treatments, modeling behavior, creating markets
- Many inferential issues: e.g., heterogeneity, targeting optimal treatments, p-hacking, interpretable results

Machine Learning (ML) versus Causal Inference

- Causal Inference: we're predicting something we don't directly observe and possibly cannot estimate well in a given sample

- ML algorithms are good at prediction, but have issues with causal inference:
  - Interventions imply counterfactuals: response schedule versus model prediction
  - Validation requires estimation in the case of causal inference
  - Identification problems not solved by large data
  - Predicting the outcome mistaken for predicting the causal effect
    - targeting based on the lagged outcome

Classical Justifications Versus ML Pipelines

Two different justifications for statistical procedures:

1. (classical) statistical theory:
   it works because we have **relevant** theory that tells us it should
   Hopefully, this is not simply: "Assume that the data are generated by the following model . . ." (Brieman 2001)

2. Training/test loop:
   it works because we have validated against ground truth and it works

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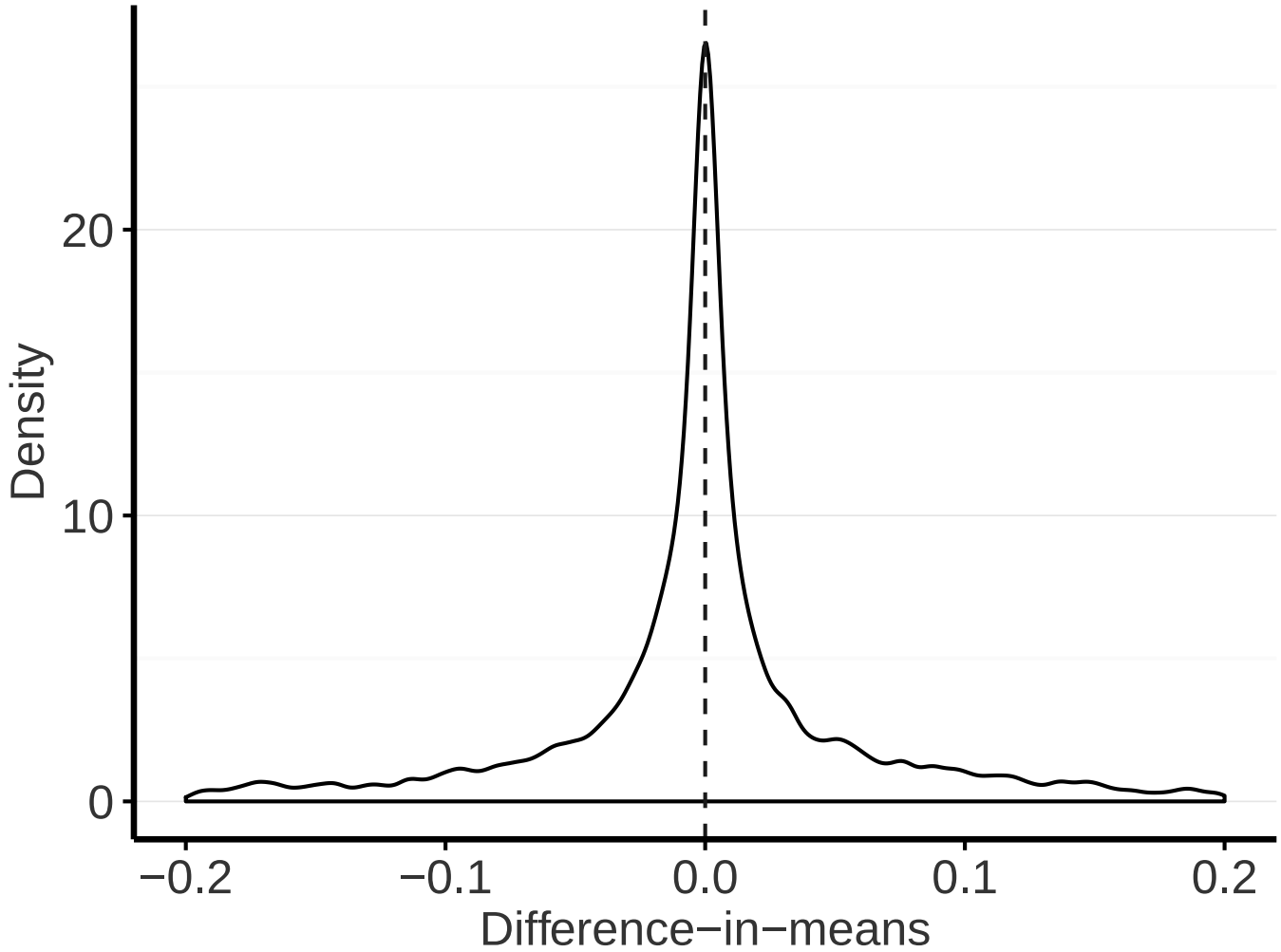
On the **normal distribution**:

> *"Everyone believes in it: experimentalists believing that it is a mathematical theorem, mathematicians believing that it is an empirical fact." — Henri Poincaré (quoted by de Finetti 1975)*

Distribution of Treatment Effects



Sekhon and Shem-Tov (2017)

Neighbors mailing

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Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY — VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	_____
9995 JENNIFER KAY SMITH		Voted	_____
9997 RICHARD B JACKSON		Voted	

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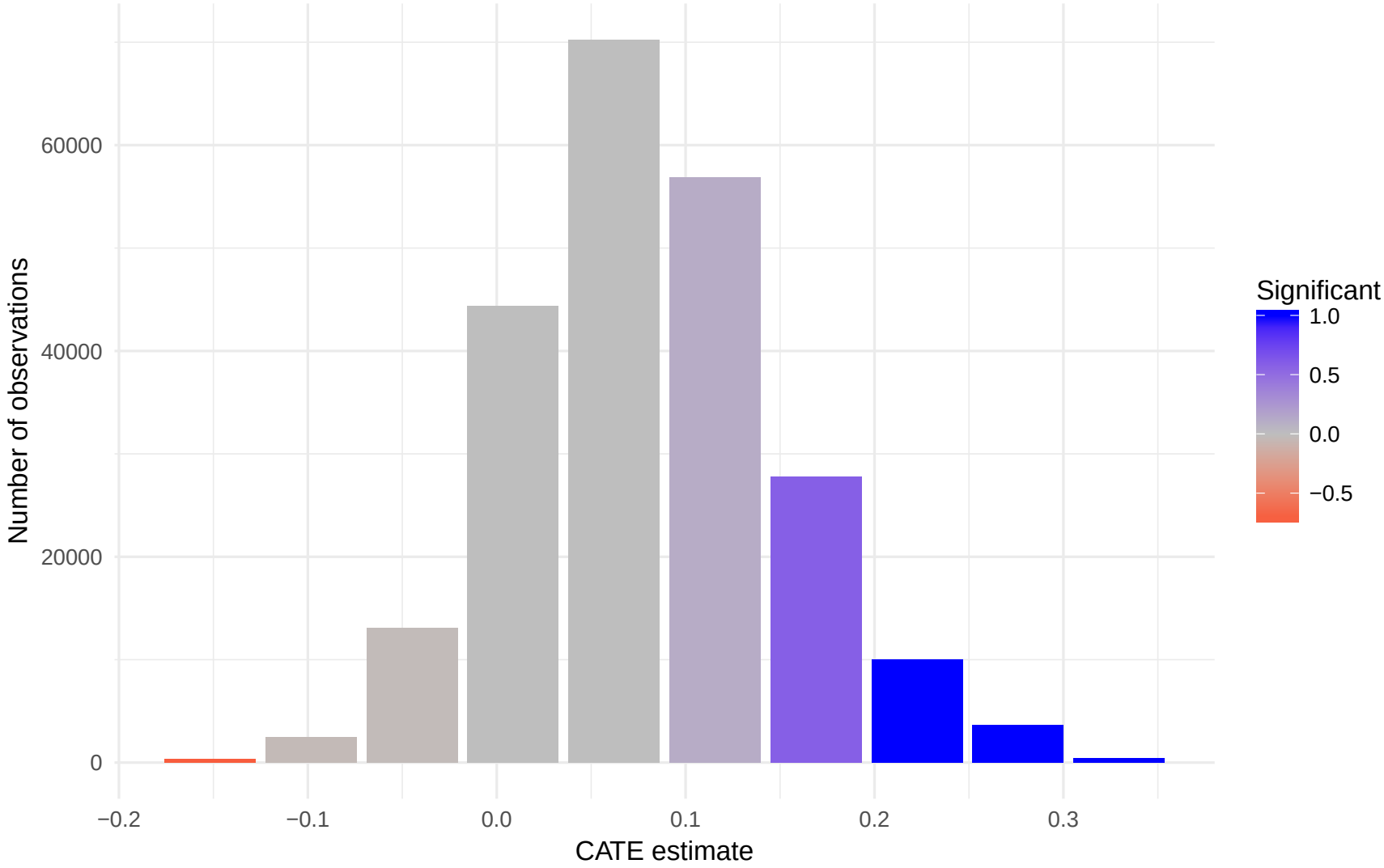
GOTV: Social pressure

Künzel, Sekhon, Bickel, Yu (2019) reanalysis of Gerber, Green, Lairmer (2008)



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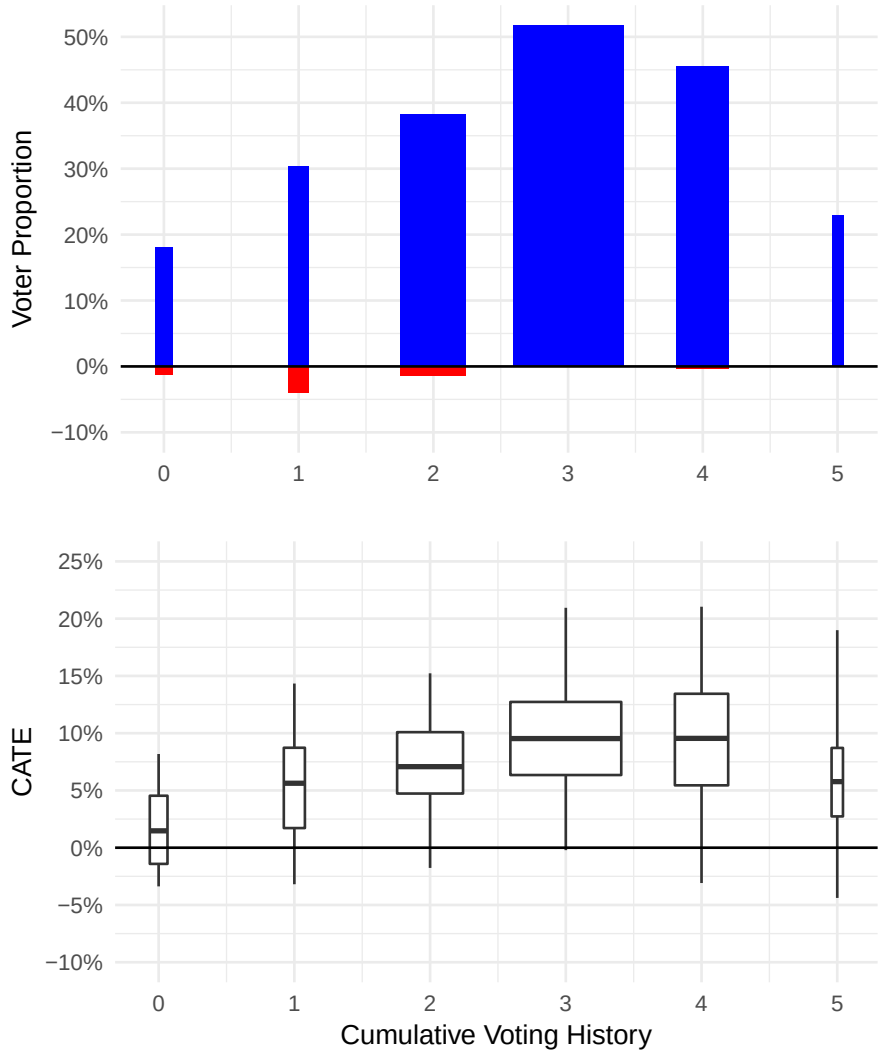
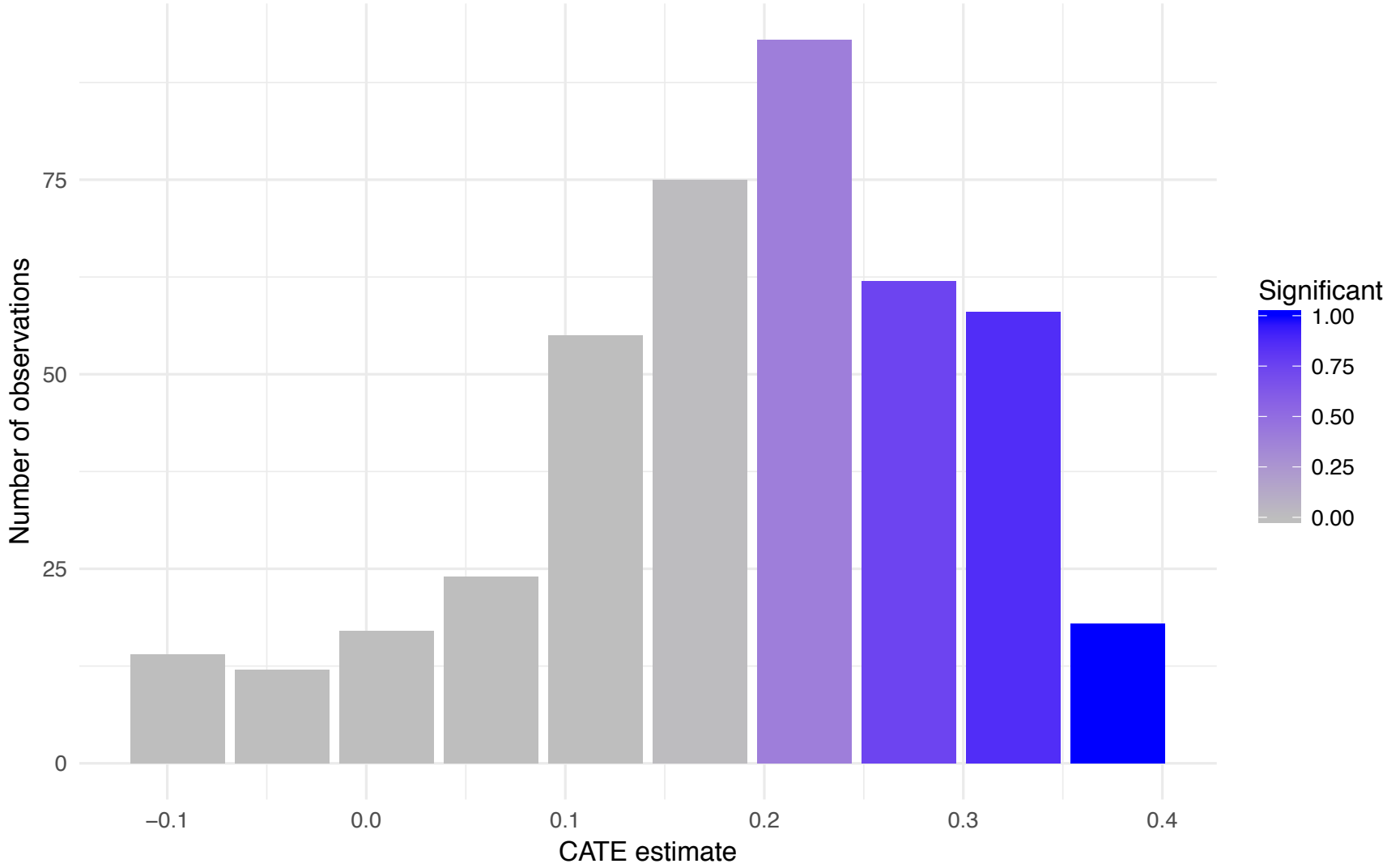


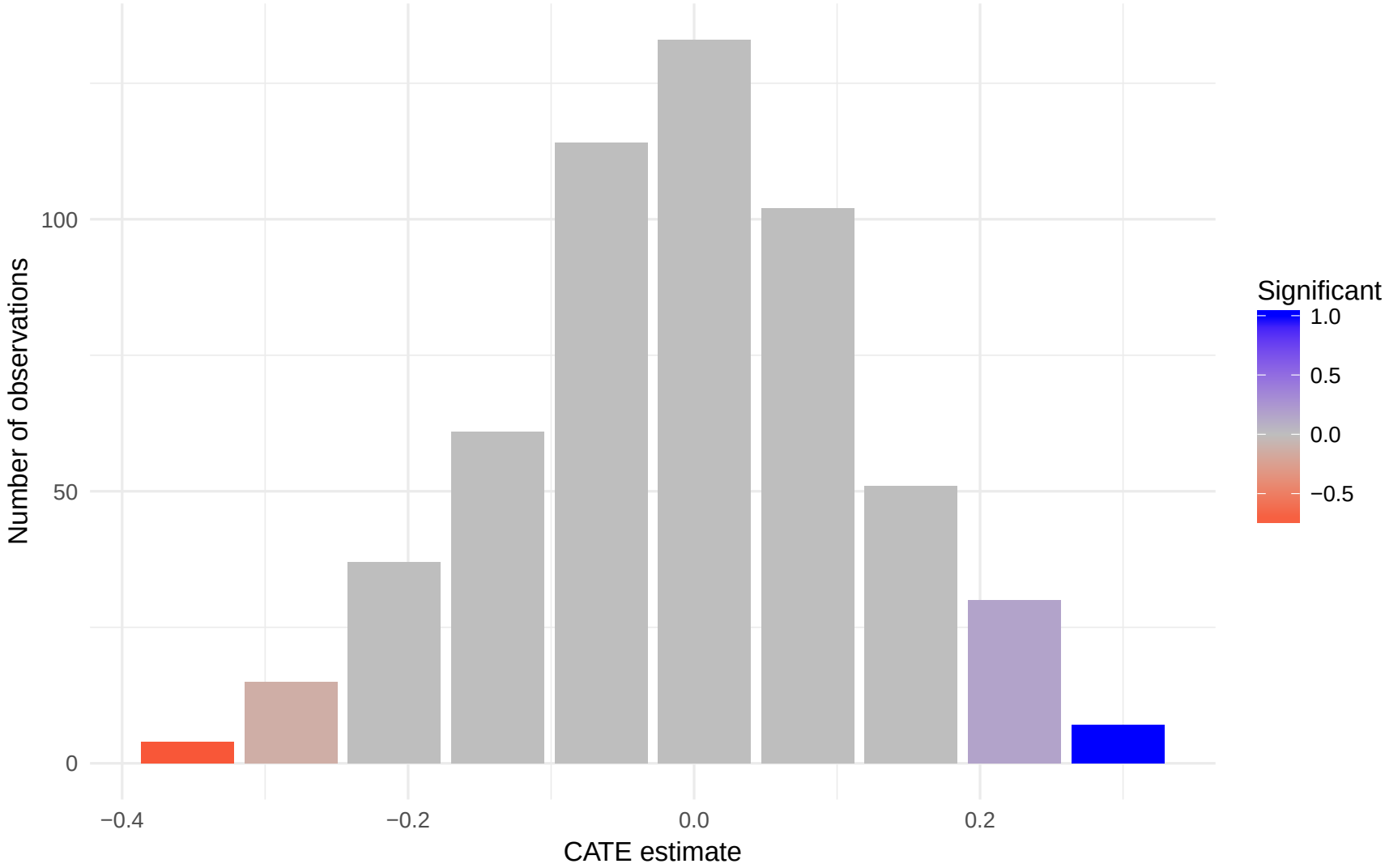
Persuasion: Transphobia

Künzel, Sekhon, Bickel, Yu (2019) reanalysis of Broockman, Kalla (2015)



Persuasion: Abortion stigma

Sekhon (2018) reanalysis of Broockman, Kalla, Sekhon (2017)



Estimating Heterogeneous Treatment Effects

- Estimating heterogenous treatment effects is more difficult than estimating overall average effects
  - p-hacking is a big problem
  - Pre-analysis plans don't really solve the problem
  - Many methods do not have known statistical properties
  - Asymptotic theory may not be a good guide for finite samples
- The problem is easier if we can make simplifying assumptions about the heterogenous treatment effect function

Conditional Average Treatment Effect (CATE)

Individual Treatment Effect (ITE): $D_i := Y_i(1) - Y_i(0)$

Let $\hat{\tau}_i$ be an estimator for D_i

$\tau(x_i)$ is the **CATE** for all units whose covariate vector is equal to x_i:

$$CATE := \tau(x_i) := \mathbb{E}\Big[D\Big|X = x_i\Big] = \mathbb{E}\Big[Y(t) - Y(c)\Big|X_i = x_i\Big]$$

Variance of Conditional Average Treatment Effect

$$CATE := \tau(x_i) := \mathbb{E}\Big[D\Big|X = x_i\Big] = \mathbb{E}\Big[Y(1) - Y(0)\Big|X_i = x_i\Big]$$

Decompose the MSE at x_i:

$$\mathbb{E}\left[(D_i - \hat{\tau}_i)^2 | X_i = x_i\right] =$$
$$\underbrace{\mathbb{E}\left[(D_i - \tau(x_i))^2 | X_i = x_i\right]}_{\text{Approximation Error}} + \underbrace{\mathbb{E}\left[(\tau(x_i) - \hat{\tau}_i)^2 | X_i = x_i\right]}_{\text{Estimation Error}}$$

- Since we cannot estimate D_i, we estimate the CATE at x_i
- But the error for the CATE is not the same as the error for the ITE

Supplementary

How to estimate the CATE?

Meta–learners

A meta–learner decomposes the problem of estimating the CATE into several sub–regression problems. The estimator which solve those sub–problems are called **base–learners**

- Flexibility to choose base–learners which work well in a particular setting
- Neural Networks, (honest) Random Forests, BART, or other machine learning algorithms

Estimators for the CATE

$$\begin{aligned}\tau(x) &= \mathbb{E}[Y(1) - Y(0)|X = x] \\ &= \mathbb{E}[Y(1)|X = x] - \mathbb{E}[Y(0)|X = x]\end{aligned}$$

Estimators for the CATE

$$\begin{aligned}\tau(x) &= \mathbb{E}[Y(1) - Y(0)|X = x] \\ &= \mathbb{E}[Y(1)|X = x] - \mathbb{E}[Y(0)|X = x] \\ &= \mu_1(x) - \mu_0(x)\end{aligned}$$

T–learner

1.) Split the data into control and treatment group,

2.) Estimate the response functions separately,

$$\hat{\mu}_1(x) = \hat{\mathbb{E}}[Y^{obs}|X = x, W = 1]$$
$$\hat{\mu}_0(x) = \hat{\mathbb{E}}[Y^{obs}|X = x, W = 0],$$

3.) $\hat{\tau}(x) := \hat{\mu}_1(x) - \hat{\mu}_0(x)$

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S–learner

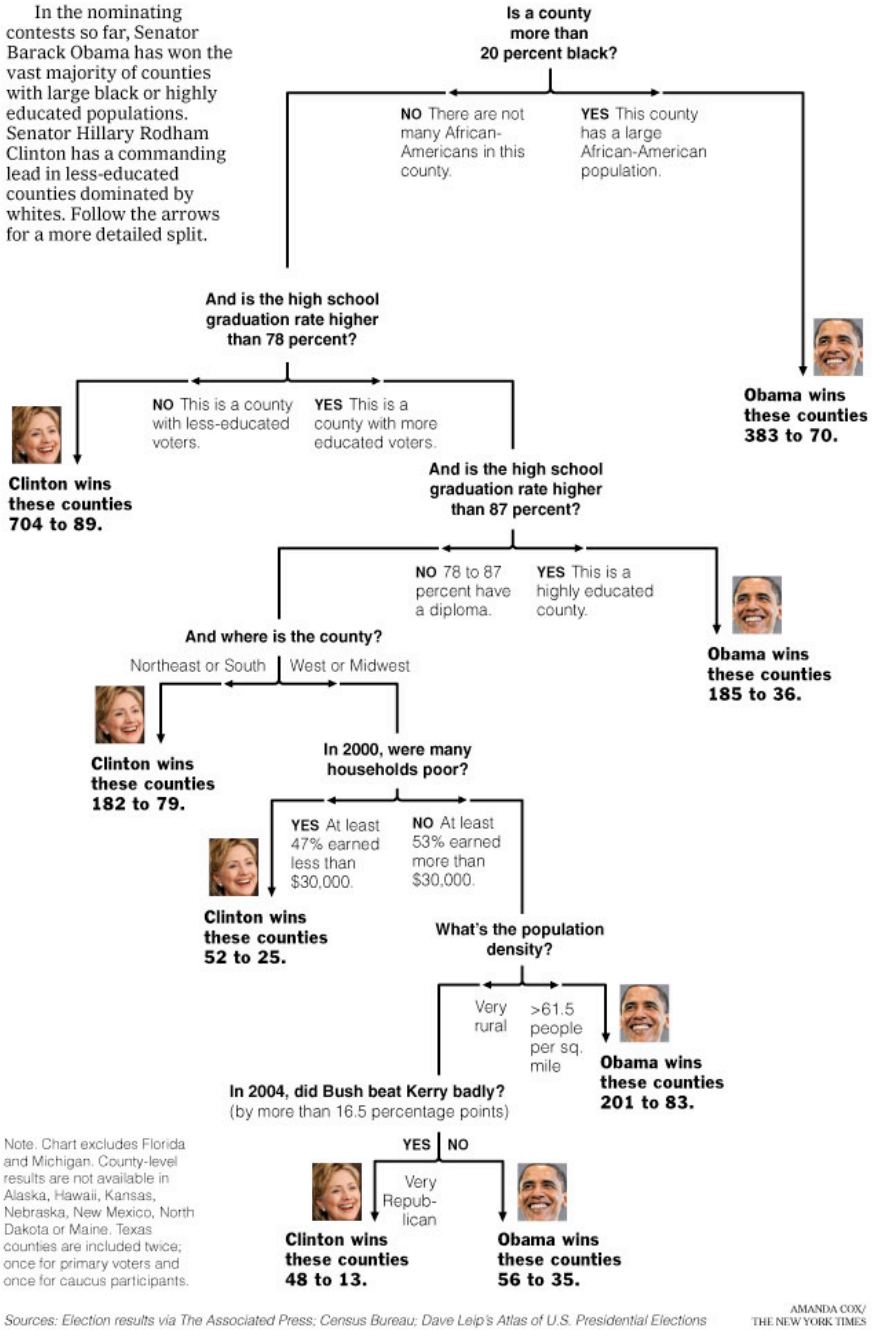
1.) Use the treatment assignment as a usual variable without giving it any special role and estimate

$$\hat{\mu}(x, w) = \hat{\mathbb{E}}[Y^{obs}|X = x, W = w]$$

2.) $\hat{\tau}(x) := \hat{\mu}(x, 1) - \hat{\mu}(x, 0)$

Decision Tree: The Obama-Clinton Divide

In the nominating contests so far, Senator Barack Obama has won the vast majority of counties with large black or highly educated populations. Senator Hillary Rodham Clinton has a commanding lead in less-educated counties dominated by whites. Follow the arrows for a more detailed split.

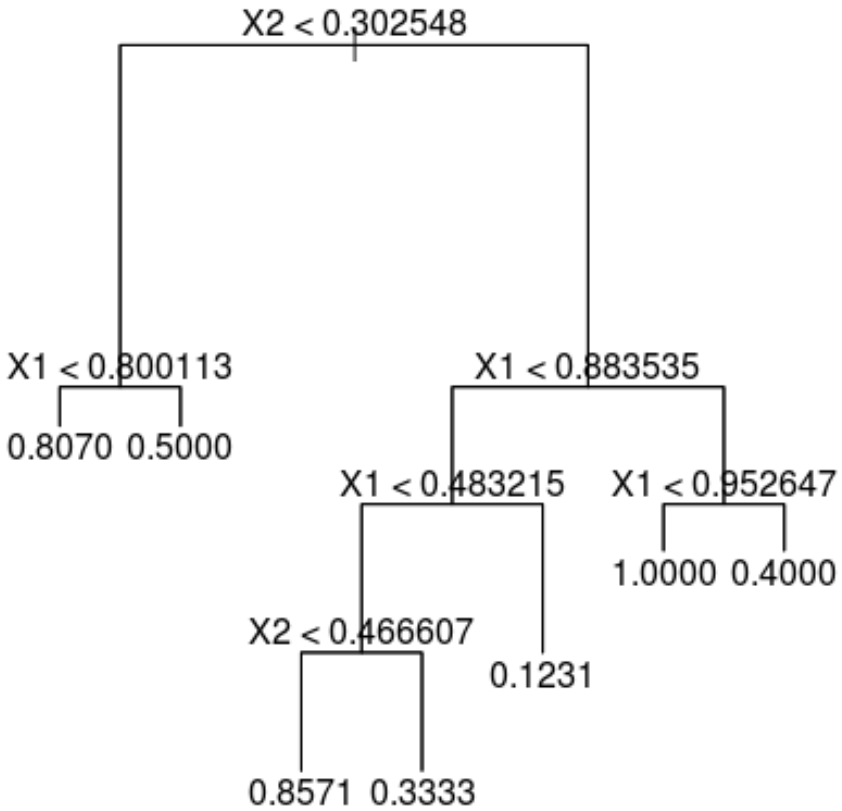


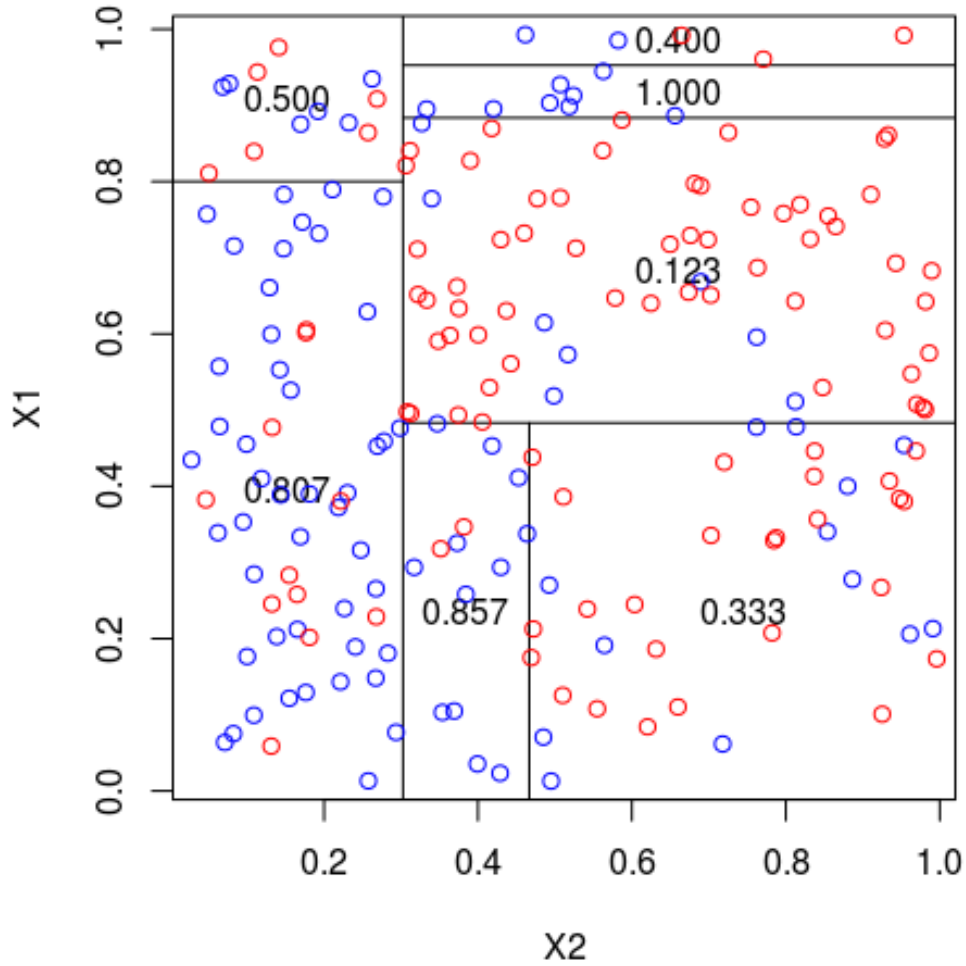
Note. Chart excludes Florida and Michigan. County-level results are not available in Alaska, Hawaii, Kansas, Nebraska, New Mexico, North Dakota or Maine. Texas counties are included twice; once for primary voters and once for caucus participants.

Sources: Election results via The Associated Press; Census Bureau; Dave Leip's Atlas of U.S. Presidential Elections

AMANDA COX/ THE NEW YORK TIMES

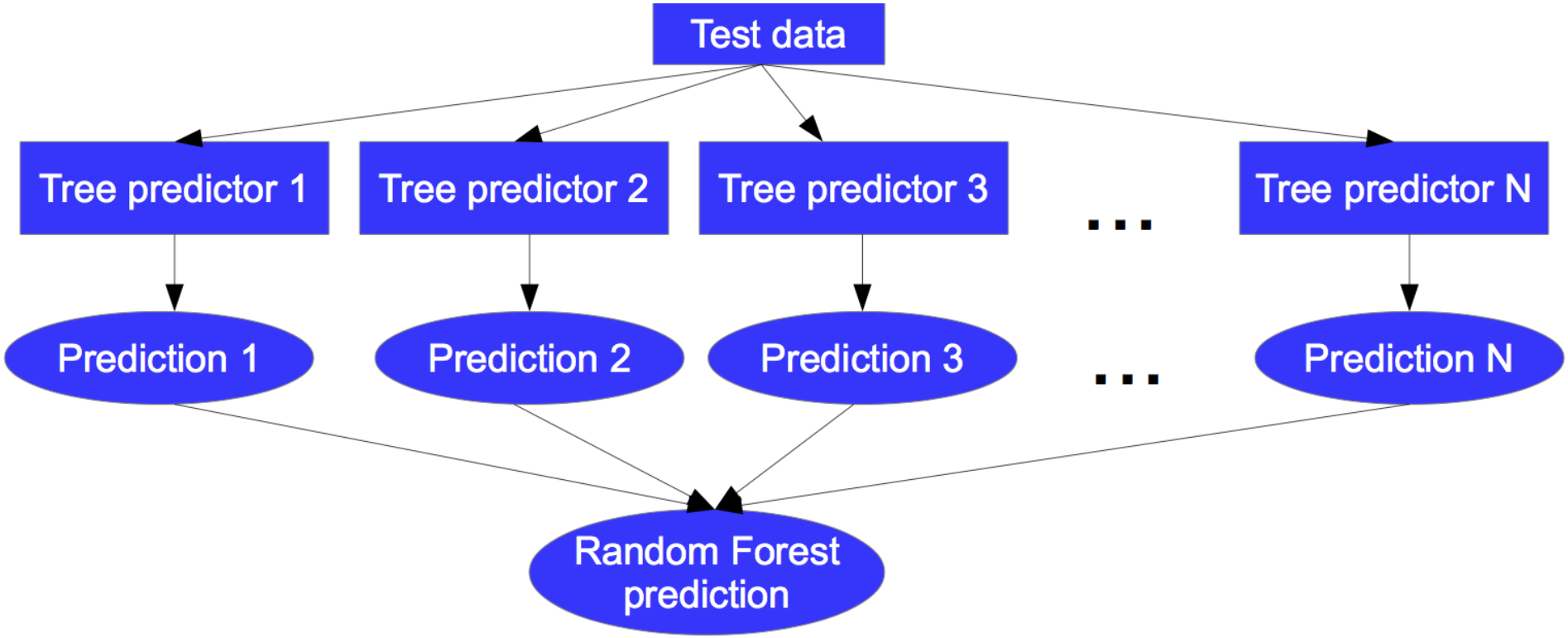
Regression Trees





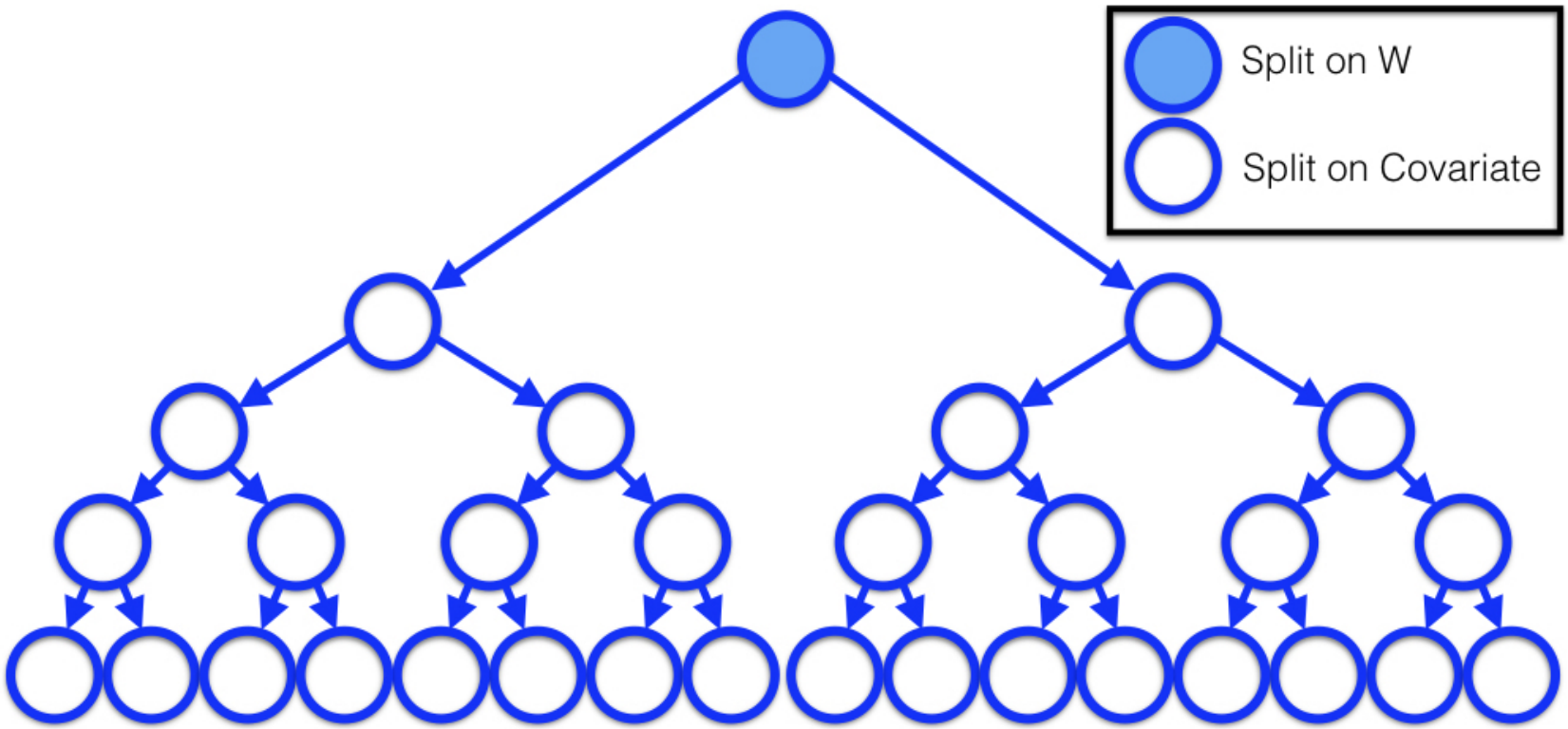
http://freakonometrics.hypotheses.org/1279

Random Forest = Many "Random" Trees



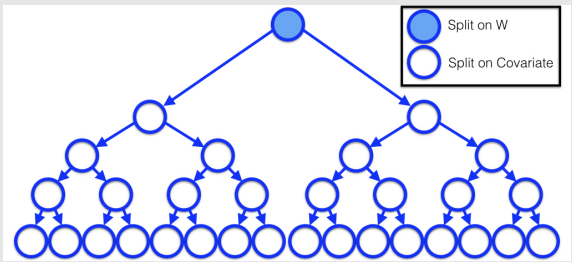
Supplementary

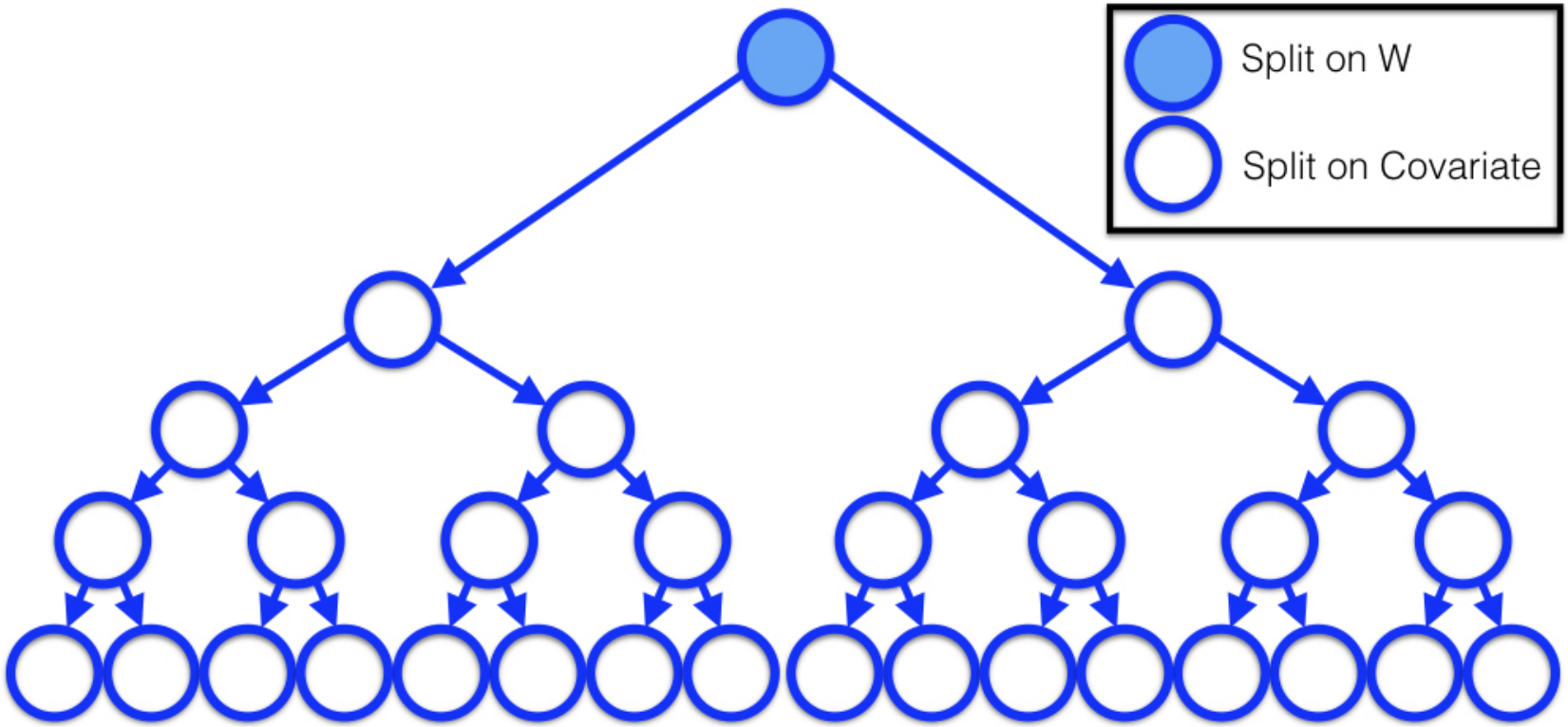
Estimating CATE



Estimating CATE

T–learner





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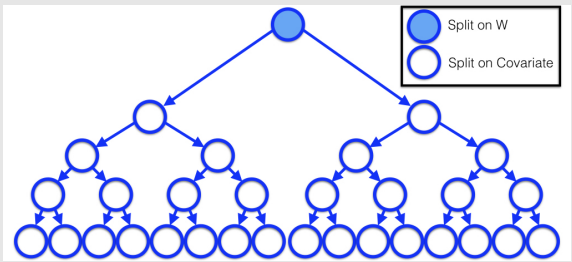

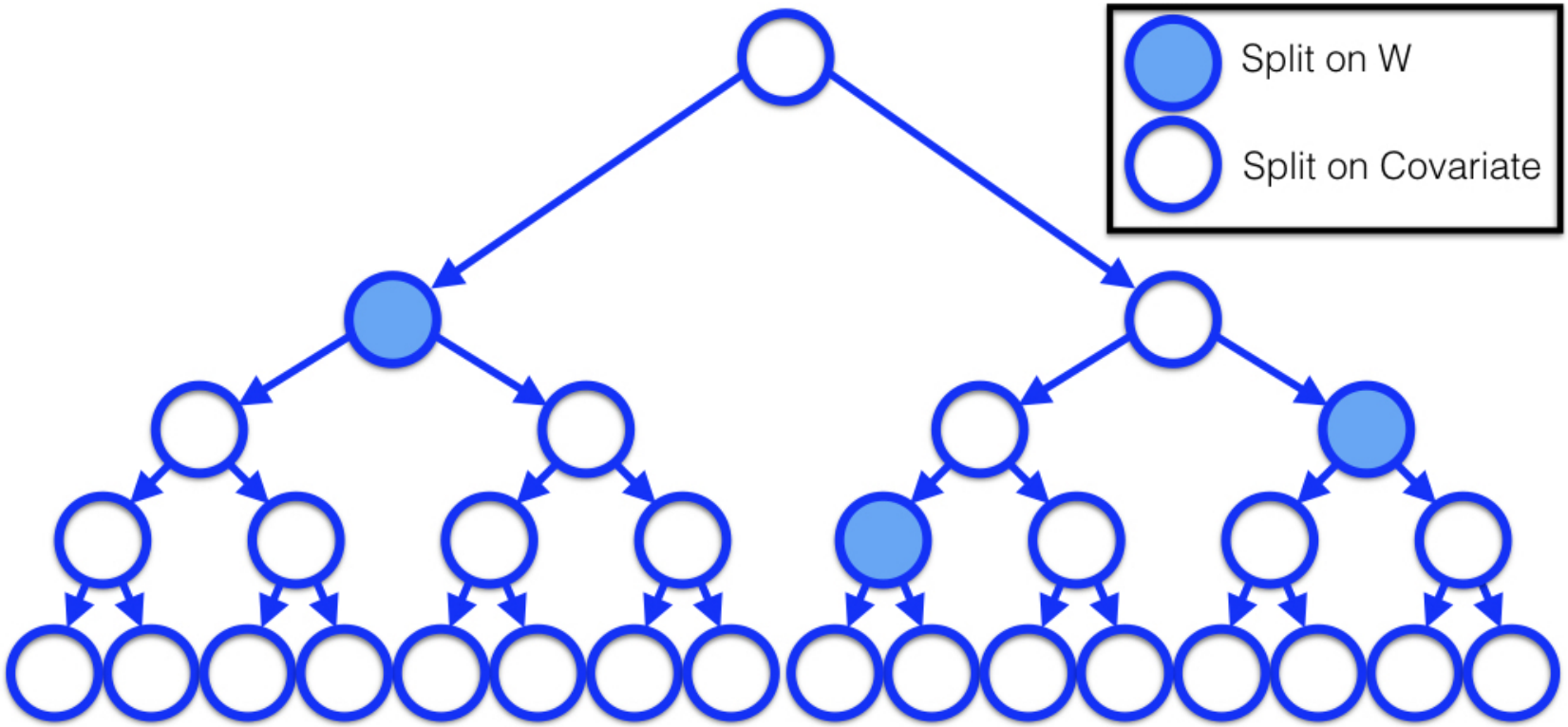

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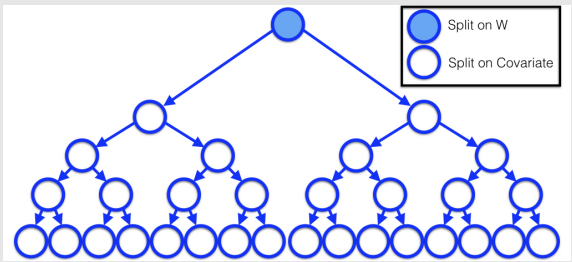
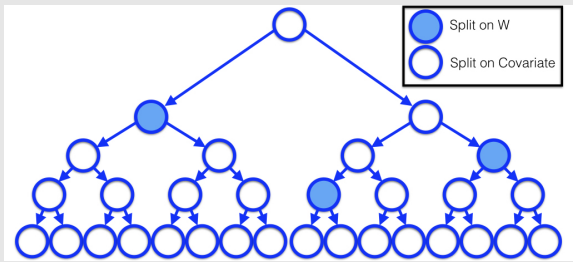
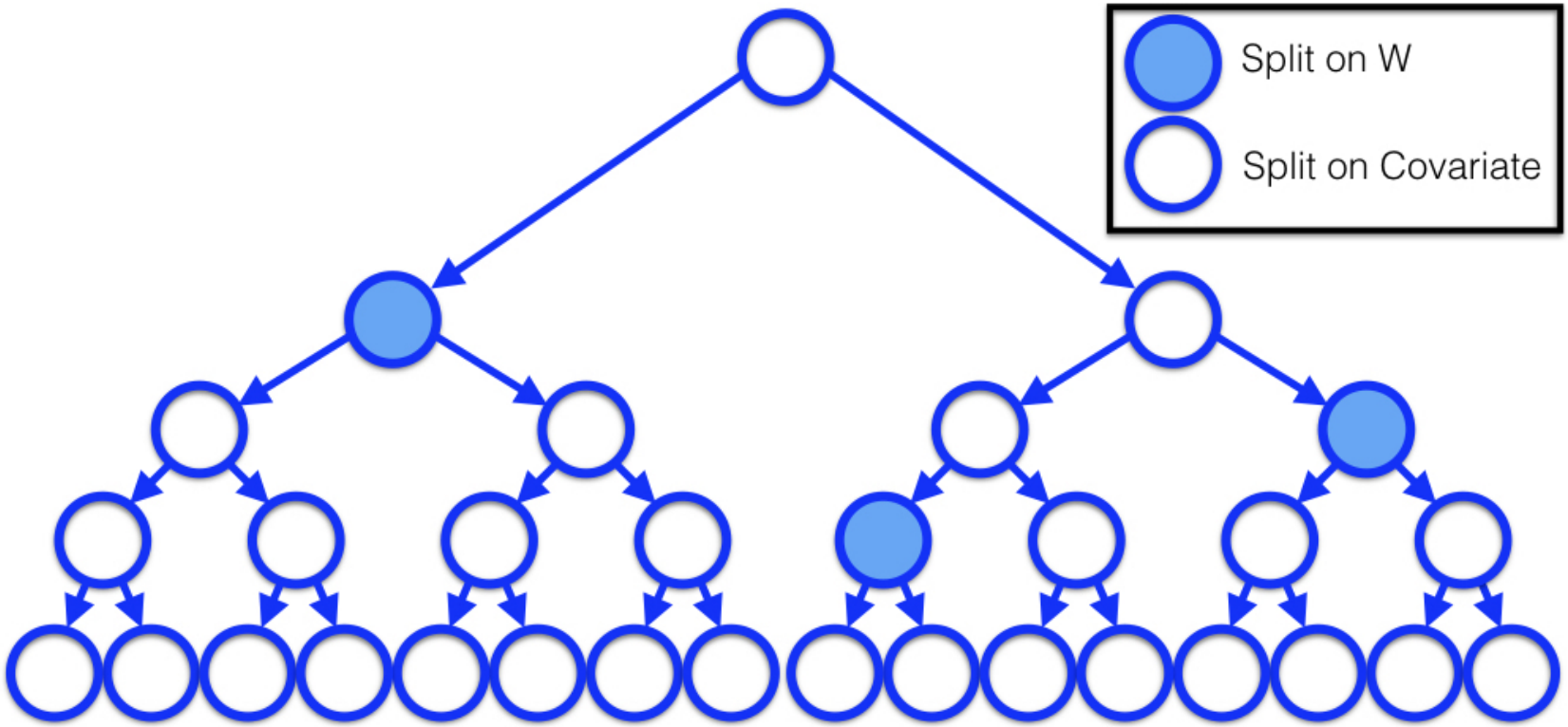


S–learner





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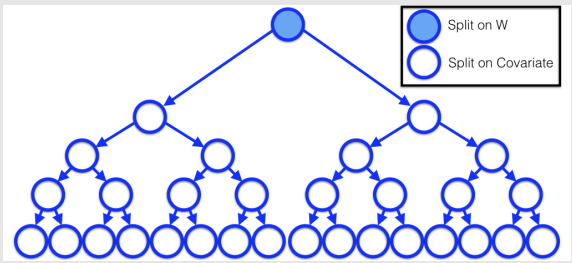

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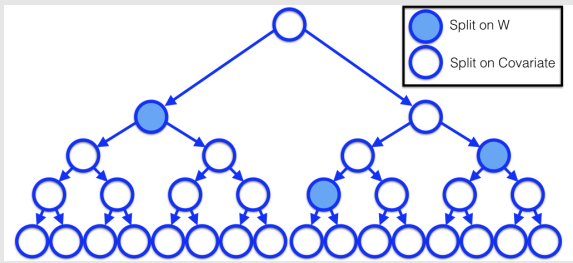
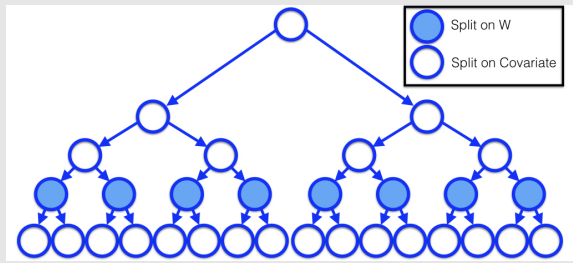
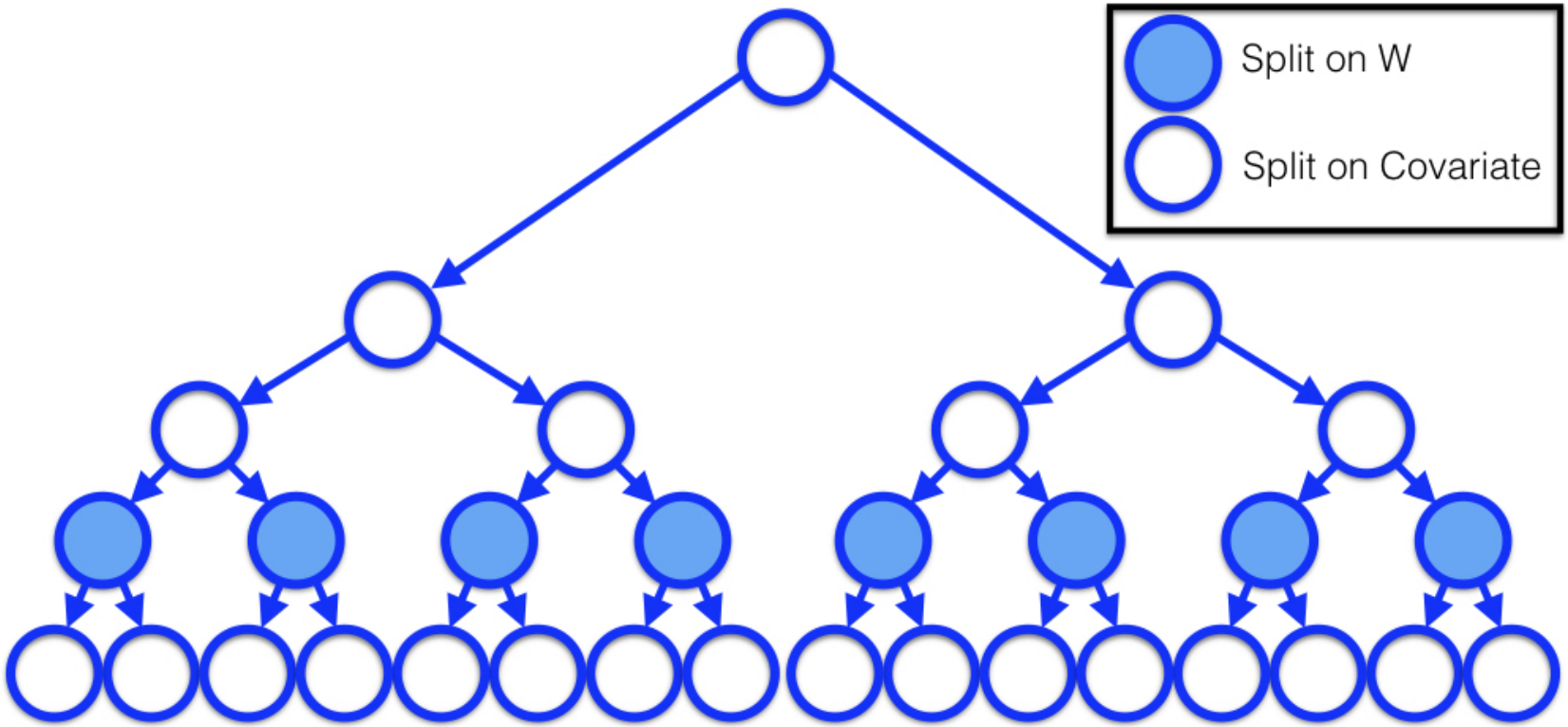


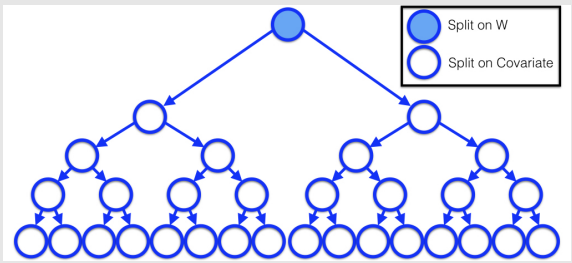
Causal Forest





Estimating CATE

T–learner



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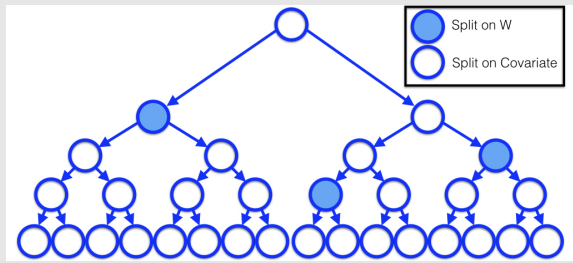

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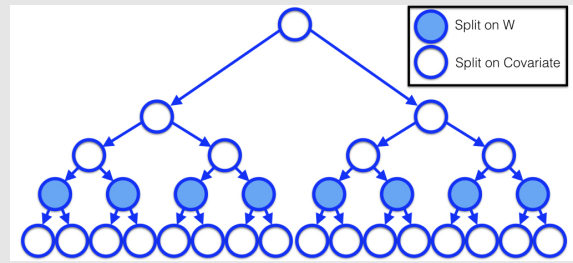
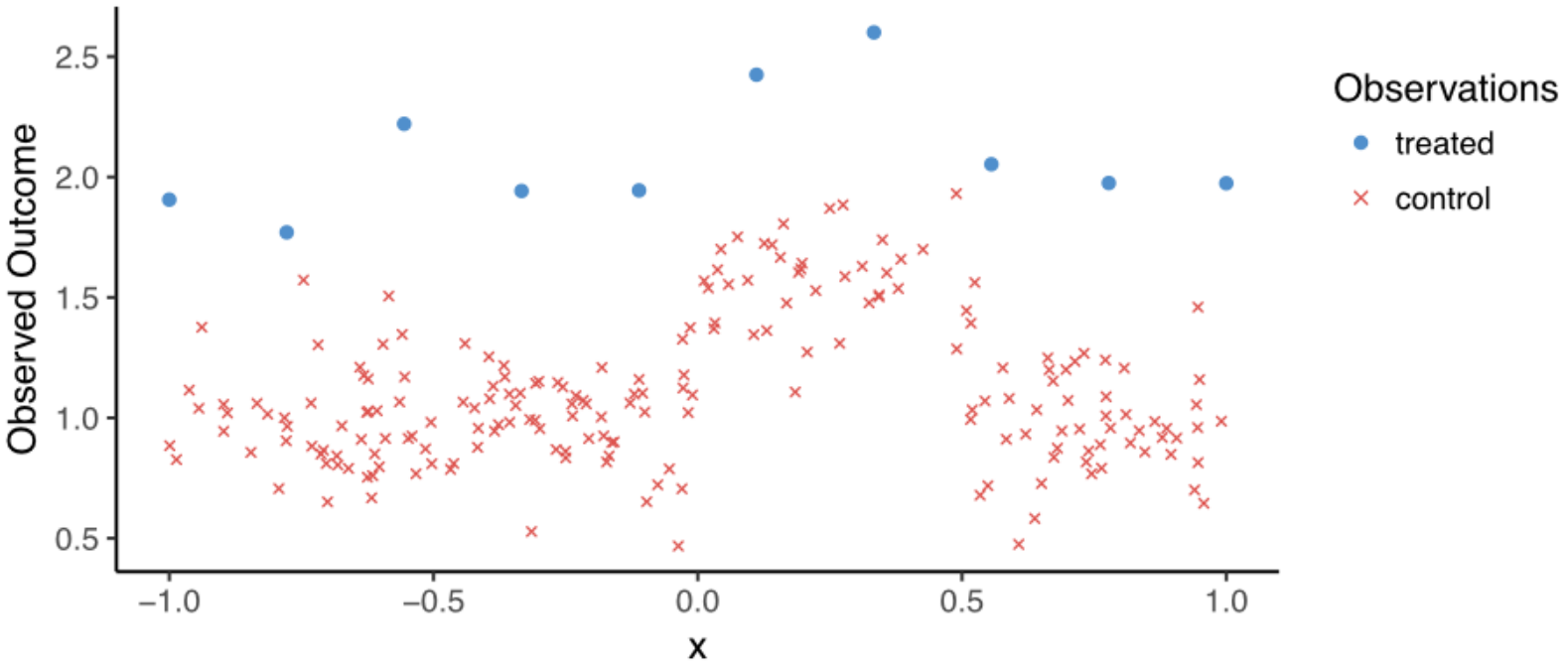


Honesty (Biau and Scornet, 2015; Scornet, 2015)

A tree estimator is **honest** iff the tree structure does not depend on the Y values used for leaf predictions:

- Purely random tree
- Wager and Athey (2017) definition of Causal Forest: Split the data and use half of it to span the tree
- Satisfies sample splitting of Chernozhukov et. al. 2016, etc.

Motivating X



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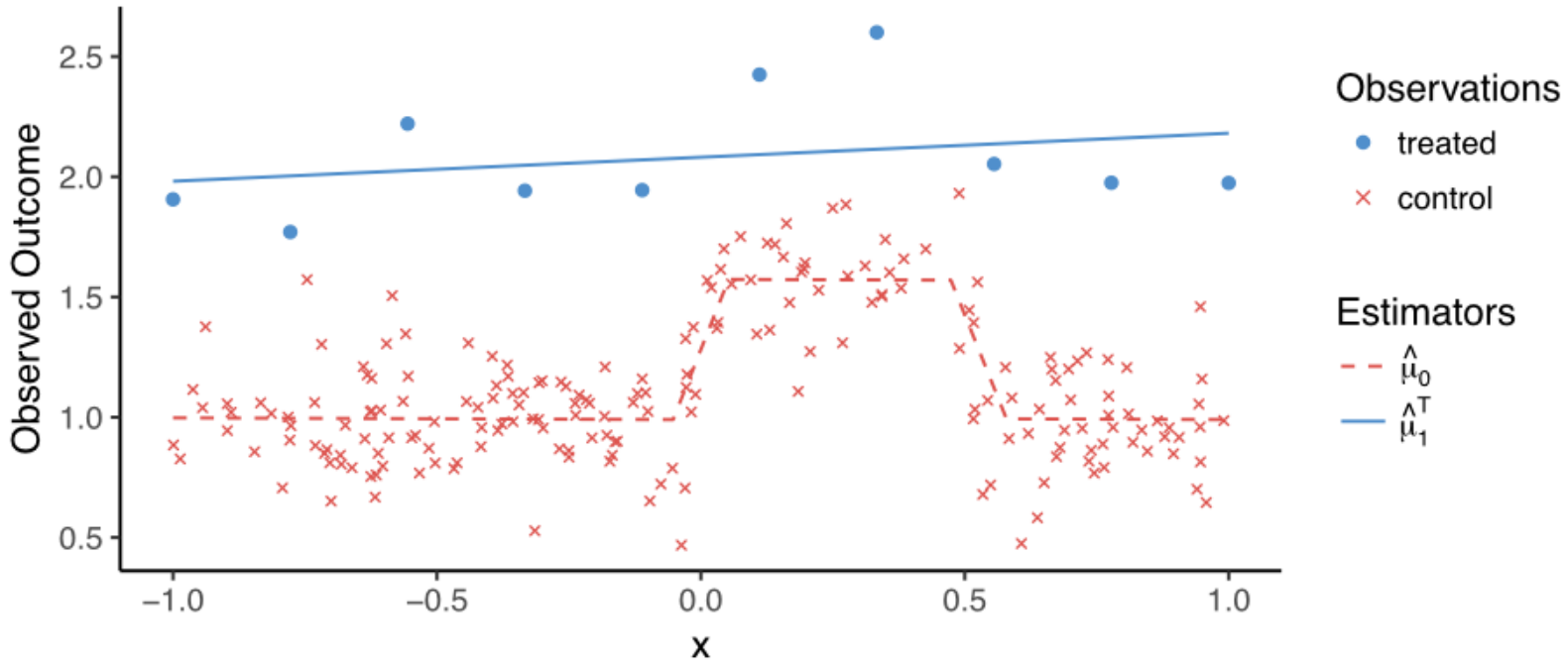

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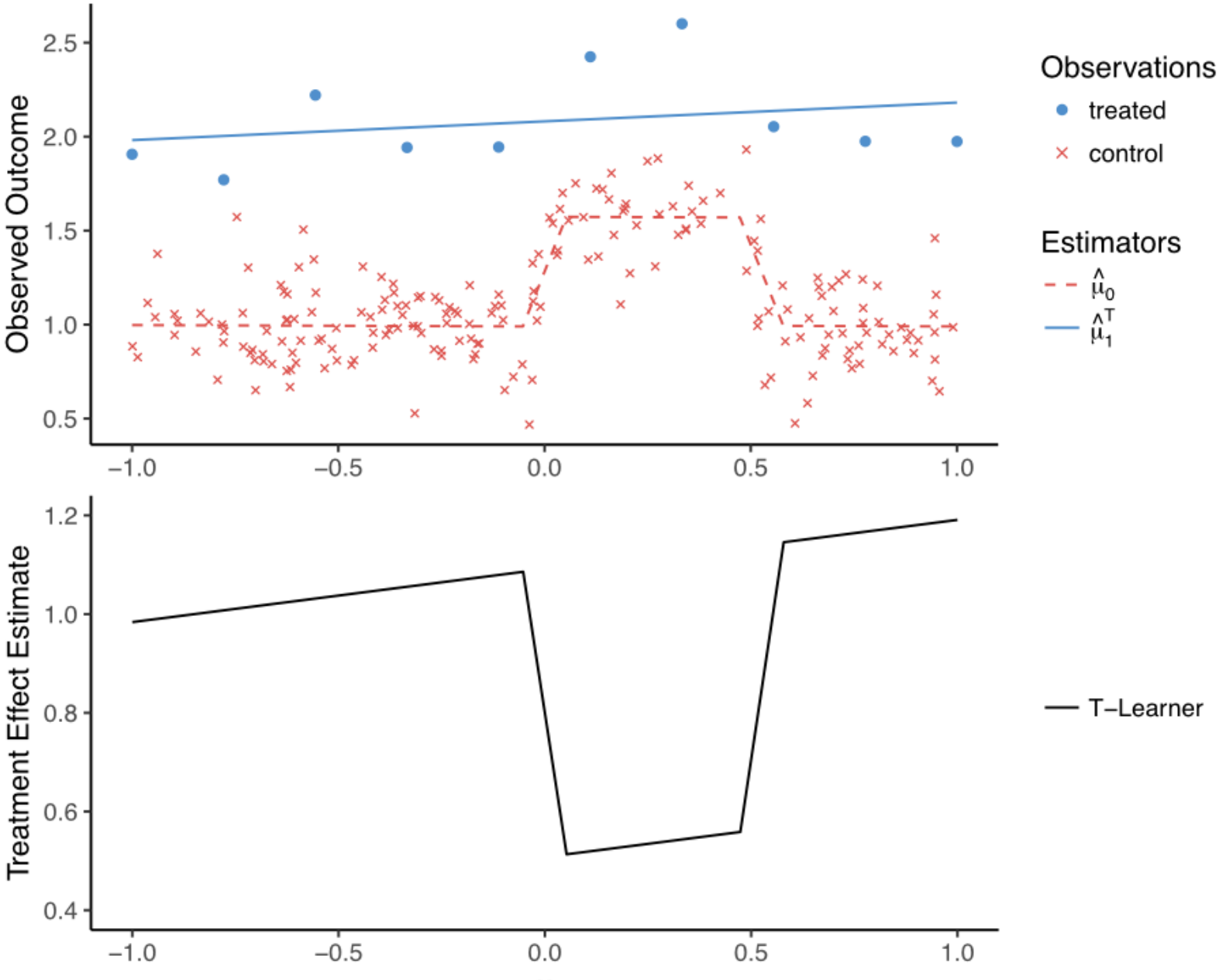

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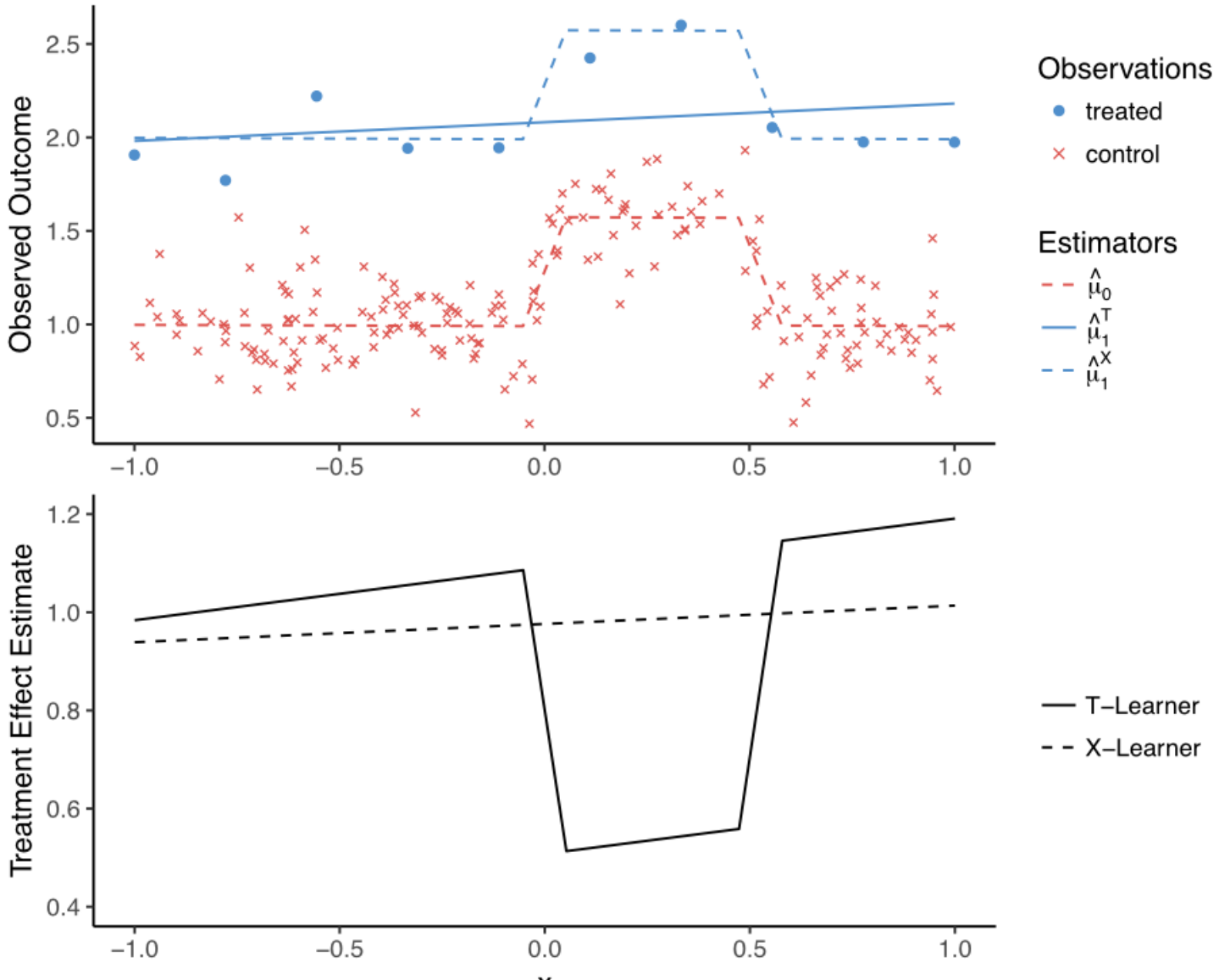


Definition of the X–learner

$$\begin{aligned}\tau(x) &= \mathbb{E}[Y(1) - Y(0)|X = x] \\ &= \mathbb{E}[Y(1) - \mu_c(x)|X = x]\end{aligned}$$

with $\mu_c(x) = \mathbb{E}[Y(0)|X = x]$.

X–learner

1.) Estimate the control response function:

$$\hat{\mu}_c(x) = \hat{\mathbb{E}}[Y(0)|X = x],$$

2.) Define the **imputed ITE**:

$$\tilde{D}_i^1 := Y_i(1) - \hat{\mu}_c(X_i(1)),$$

3.) Estimate the CATE:

$$\hat{\tau}(x) = \hat{\mathbb{E}}[\tilde{D}^1|X = x].$$

Definition of the X–learner

Algorithm 1 X–learner

1: **procedure** $\text{X–Learner}(X, Y, W)$

2: $\quad \hat{\mu}_c = M_1(Y^0 \sim X^0)$ ▷ Estimate response function

4: $\quad \tilde{D}_i^1 := Y_i^1 - \hat{\mu}_c(X_i^1)$ ▷ Impute ITE

6: $\quad \hat{\tau}_1 = M_3(\tilde{D}^1 \sim X^1)$ ▷ Estimate CATE

9: **end procedure**

Definition of the X–learner

Algorithm 2 X–learner

procedure $\text{X–Learner}(X, Y, W)$

$\hat{\mu}_c = M_1(Y^0 \sim X^0)$ ▷ Estimate response function

$\hat{\mu}_t = M_2(Y^1 \sim X^1)$

$\tilde{D}_i^1 := Y_i^1 - \hat{\mu}_c(X_i^1)$ ▷ Impute ITE

$\tilde{D}_i^0 := \hat{\mu}_t(X_i^0) - Y_i^0$

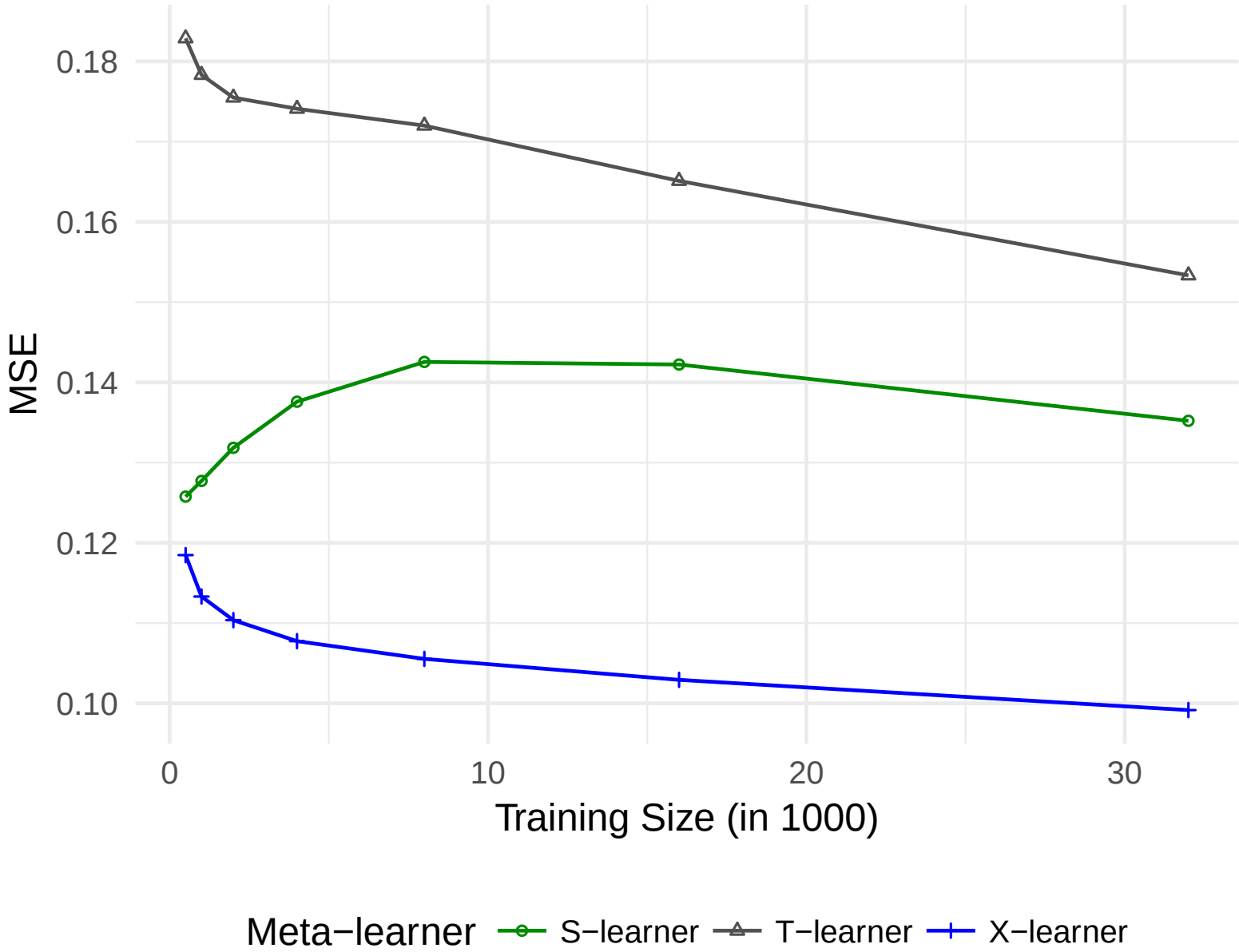
$\hat{\tau}_1 = M_3(\tilde{D}^1 \sim X^1)$ ▷ Estimate CATE

$\hat{\tau}_0 = M_4(\tilde{D}^0 \sim X^0)$

$\hat{\tau}(x) = g(x)\hat{\tau}_0(x) + (1 - g(x))\hat{\tau}_1(x)$ ▷ Average

end procedure

Data Simulation: Social pressure and Voter Turnout



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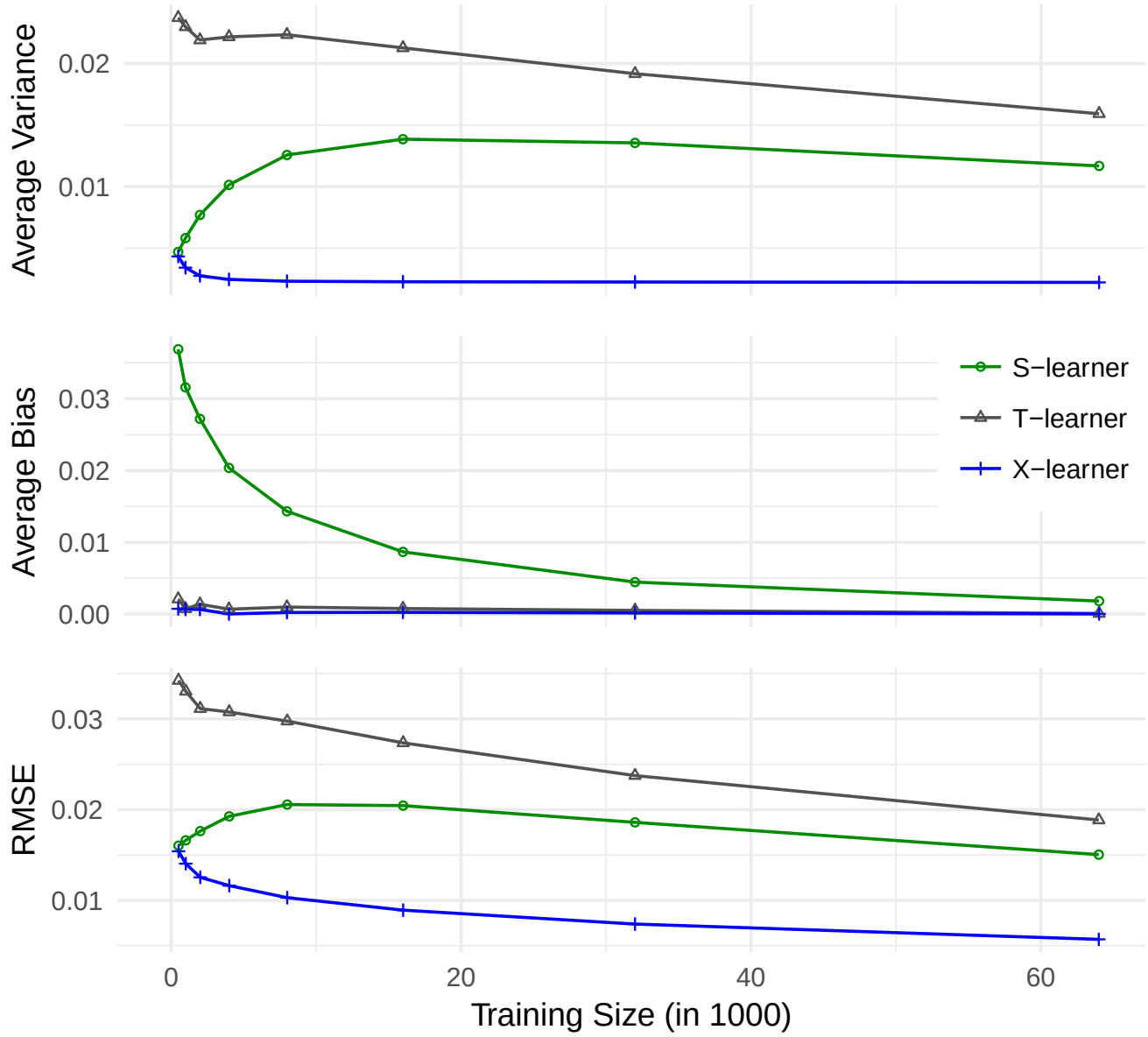
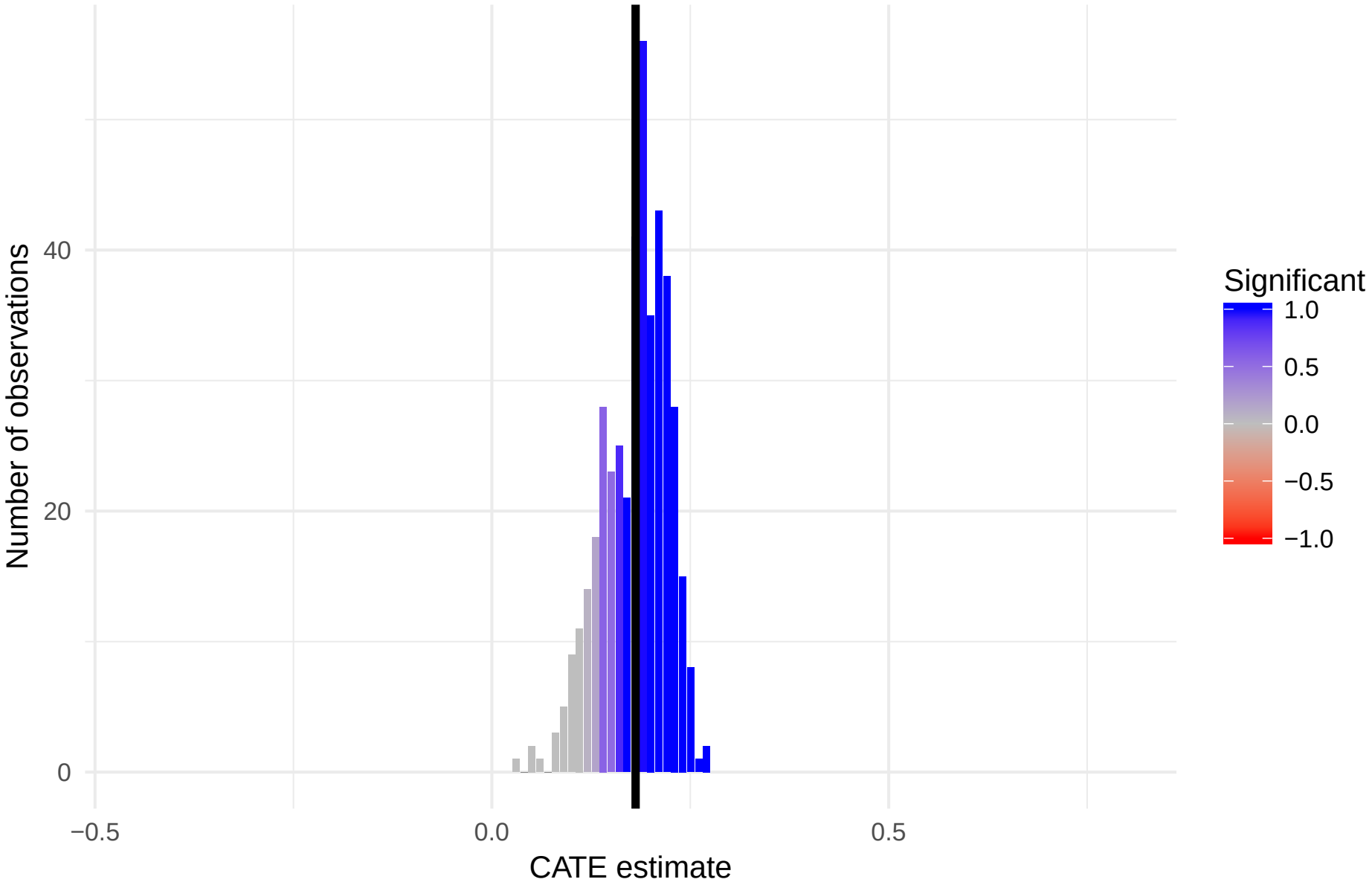
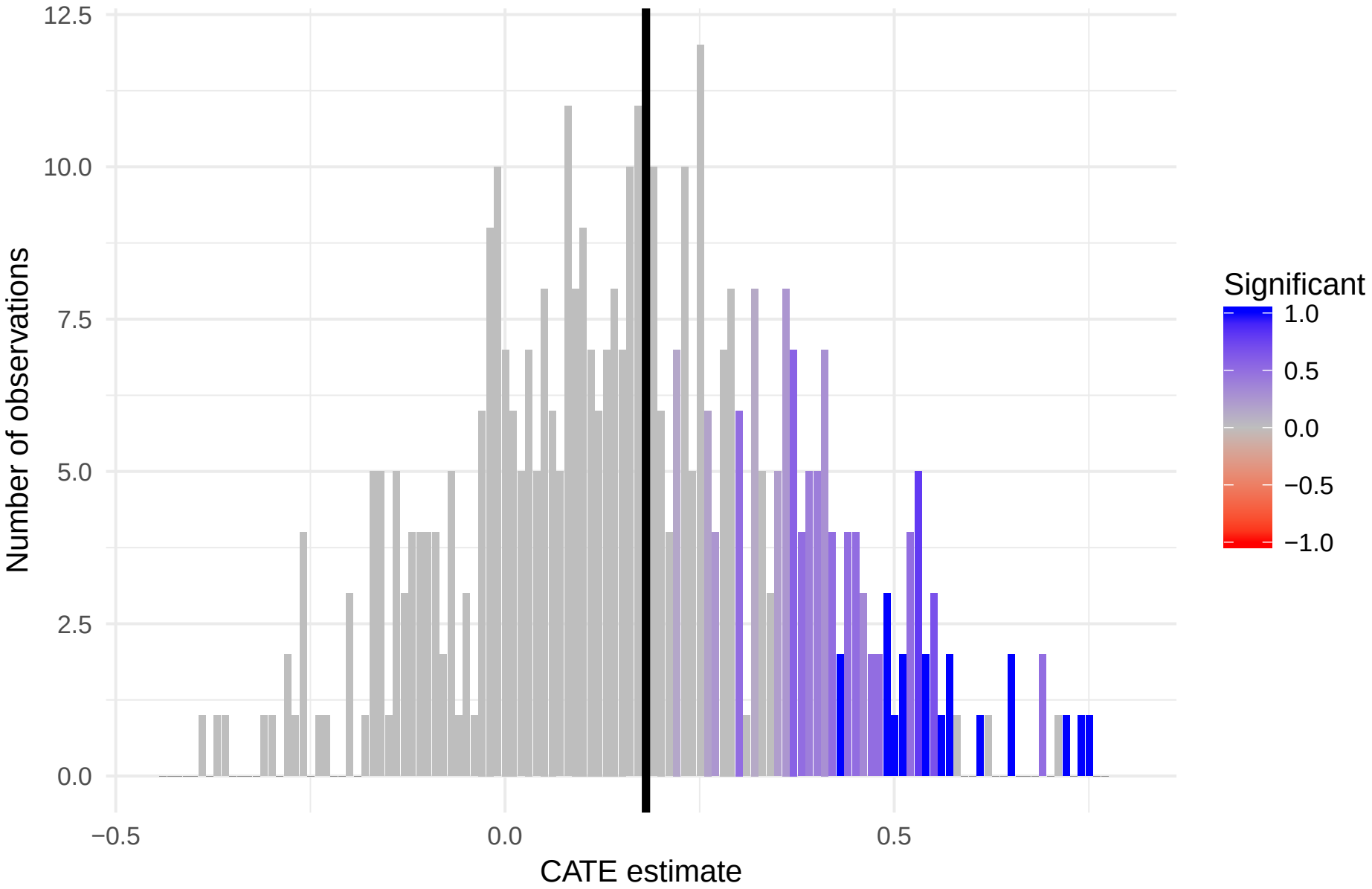


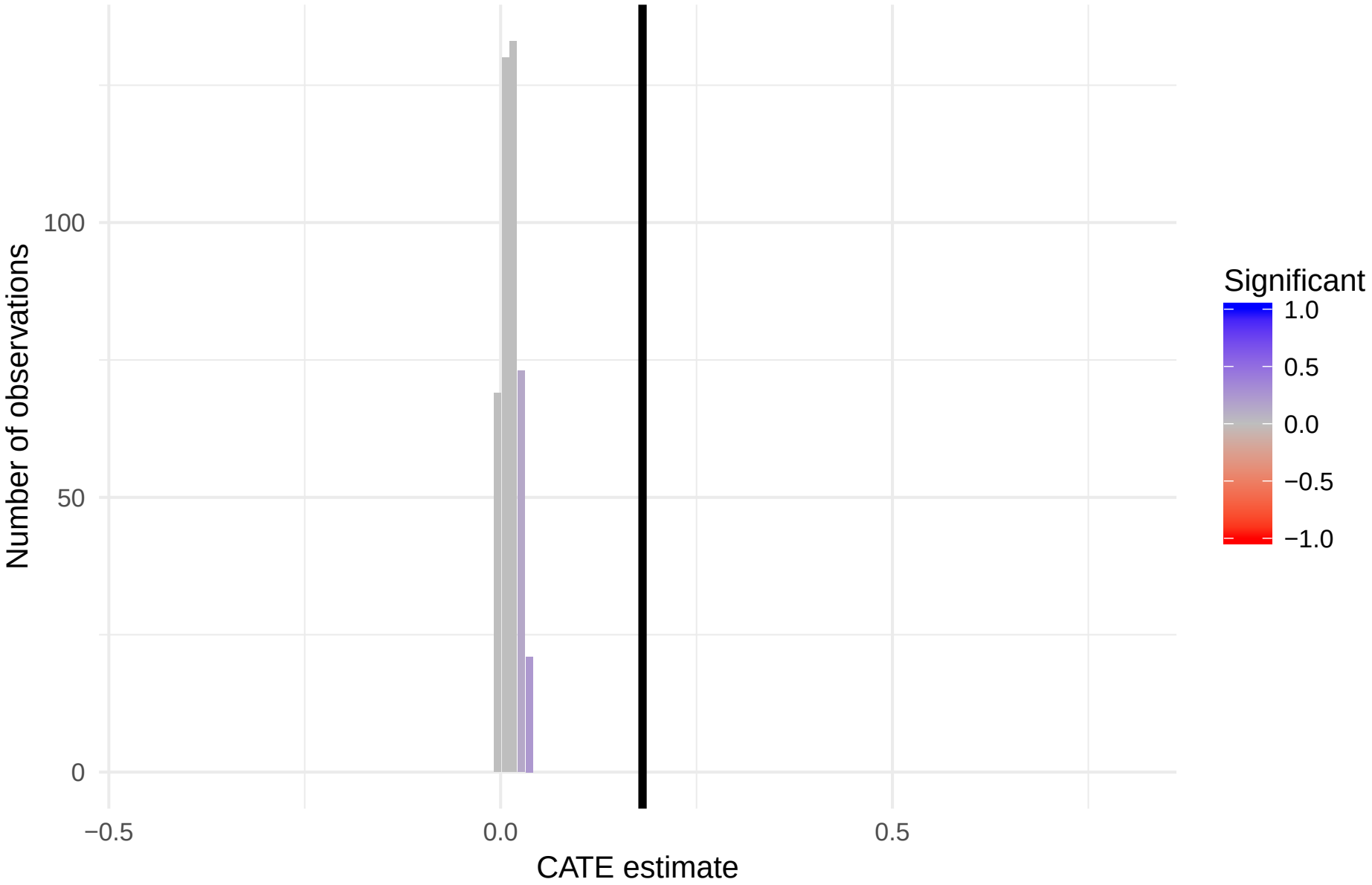
Reducing Transphobia: X–RF



Reducing Transphobia: T–RF



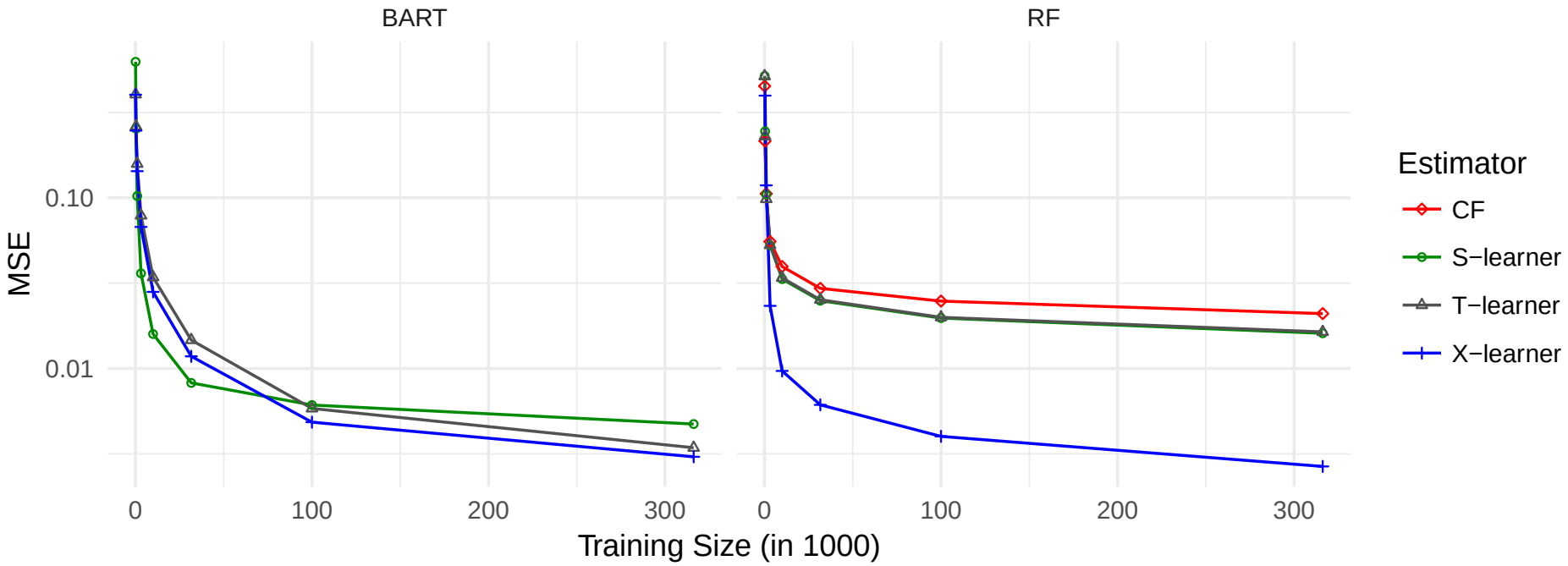
Reducing Transphobia: S–RF



Reducing Transphobia: Simulation

Algorithm	RMSE	Bias
X–RF	1.102	0.0122
T–RF	1.090	0.0110
S–RF	1.207	-0.1073

Complex Treatment Effect



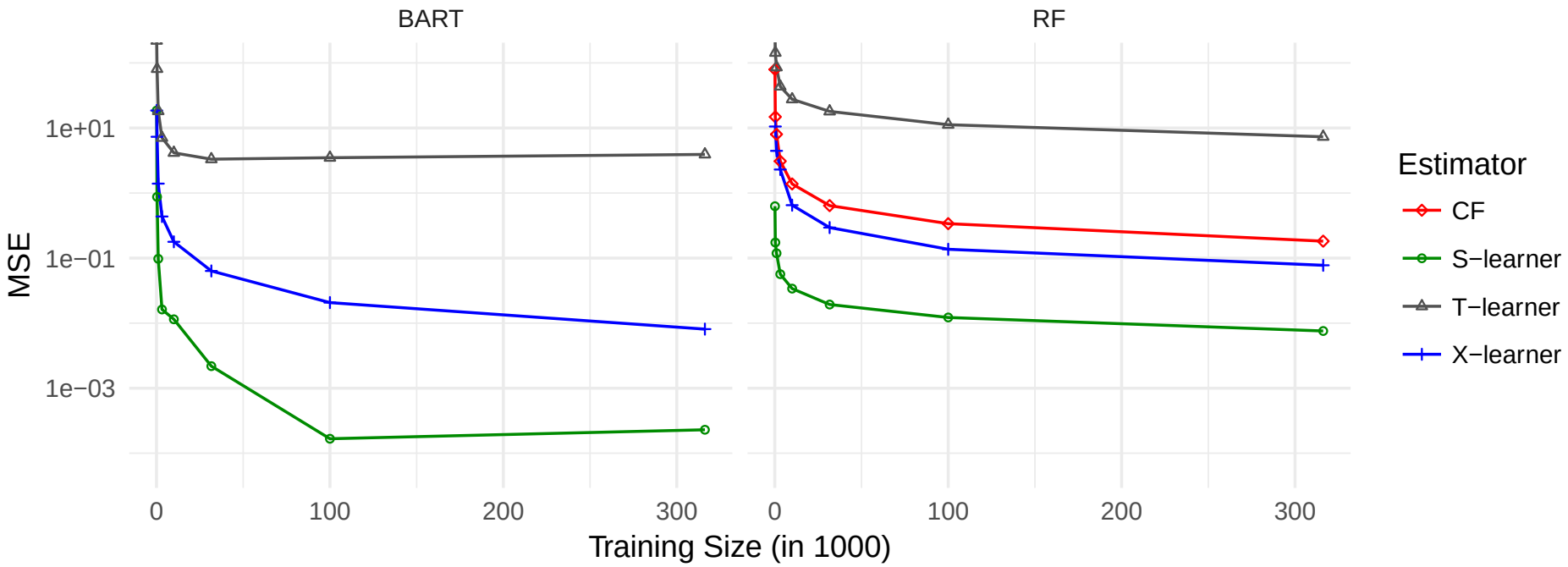
Complex Setting (WA, 2)

$$\mu_1(x) = \frac{1}{2}\eta(x_1)\eta(x_2) \text{ with } \eta(x) = 1 + \frac{1}{1 + e^{-20(x-1/3)}}$$

$$\mu_0(x) = -\mu_1(x)$$

$$e(x) = 0.5$$

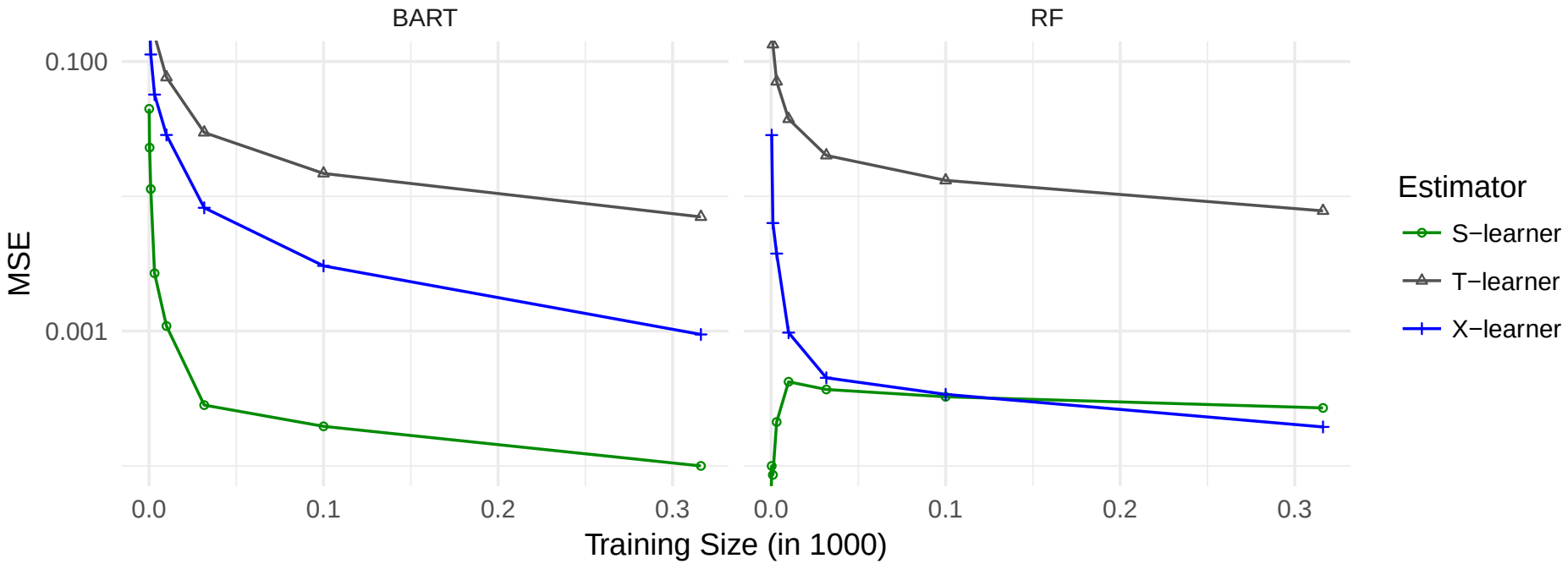
No Treatment Effect



Simple Setting

$$\mu_1(x) = x^T\beta, \text{ with } \beta \sim \text{Unif}([1,30]^d)$$
$$\mu_0(x) = \mu_1(x)$$
$$e(x) = 0.5$$

Resisting Confounding

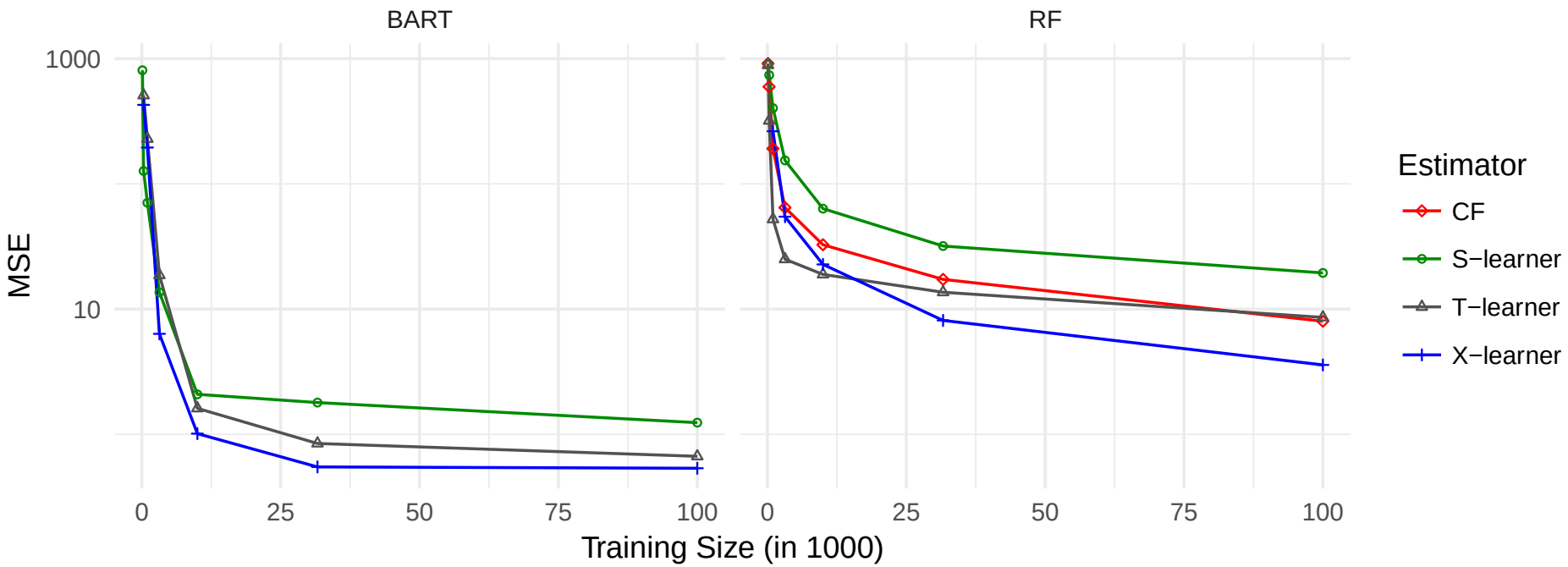


Confounded without TE (WA, 1)

$$\mu_1(x) = 2x_1 - 1,$$
$$\mu_0(x) = 2x_1 - 1,$$
$$e(x) = \frac{1}{4}(1 + \beta_{2,4}(x_1))$$

More Estimators

Flexibility of Base Learners is Needed



Complicated Setting

$$\mu_1(x) = x^T \beta_1, \text{ with } \beta_1 \sim \text{Unif}([1, 30]^d)$$
$$\mu_0(x) = x^T \beta_0, \text{ with } \beta_0 \sim \text{Unif}([1, 30]^d)$$
$$e(x) = .5$$

Properties of the X–learner: Setup for Theory

A model for estimating the CATE

$$W \sim \text{Bern}(e(X))$$
$$Y(0) = \mu_0(X) + \varepsilon(0)$$
$$Y(1) = \mu_1(X) + \varepsilon(1)$$

- If τ satisfies some regularity conditions (e.g. sparsity or smoothness), it can be directly exploited in the second base–learner
- This effect is in particular strong when μ_0 can be estimated very well
- Or when the error when estimating $\mu_0(x_i)$ is uncorrelated from the error when estimating $\mu_0(x_j)$ for $i \neq j$

Theorem 1

Künzel, Sekhon, Bickel, Yu 2019

Assume we observe m control and n treatment units,

1.) Ignorability holds: $(Y(0), Y(1)) \perp W|X$

2.) The treatment effect is linear, $\tau(x) = x^T \beta$

3.) There exists an estimator $\hat{\mu}_0$ with $\mathbb{E}[(\mu_0(x) - \hat{\mu}_0(x))^2] \leq C_x^0 m^{-a}$

Then the X-learner with $\hat{\mu}_0$ in the first stage, OLS in the second stage, achieves the parametric rate in n,

$$\mathbb{E}\left[\|\tau(x) - \hat{\tau}_X(x)\|^2\right] \leq C_x^1 m^{-a} + C_x^2 n^{-1}$$

If there are a many control units, such that $m \asymp n^{1/a}$, then

$$\mathbb{E}\left[\|\tau(x) - \hat{\tau}_X(x)\|^2\right] \leq 2C_x^1 n^{-1}$$

Theorem 2

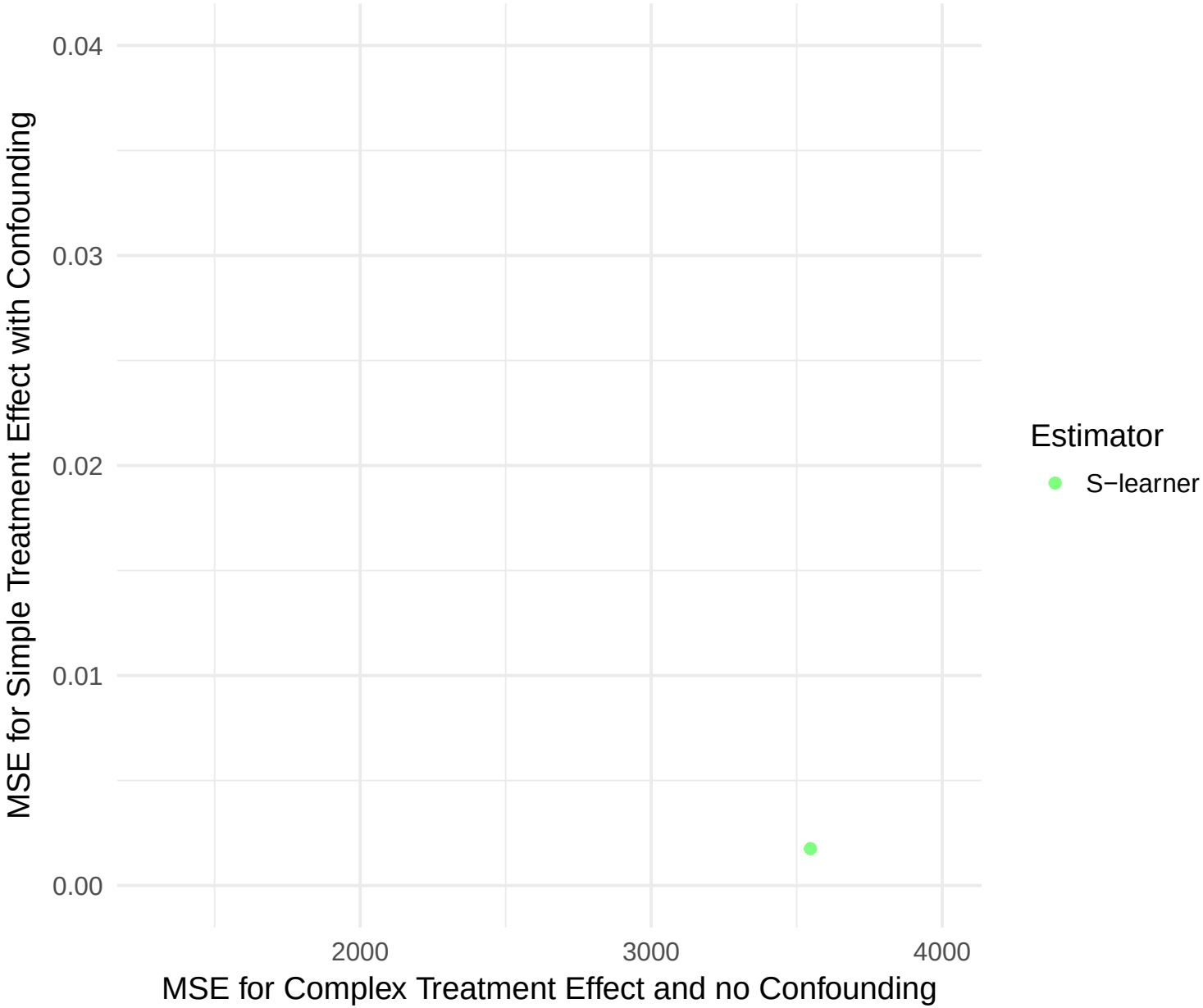
Foster and Syrgkanis 2019

Under Neyman orthogonality, the method with sample splitting achieves a rate of the same order as it would have if it were supplied with the true propensity score and response functions.

Heuristically, small deviations in nuisance functions do not invalidate moment conditions.

Rely on sample splitting to have conditions formulated only in terms of rates and not in terms of complexity of ML estimators.

Adaptivity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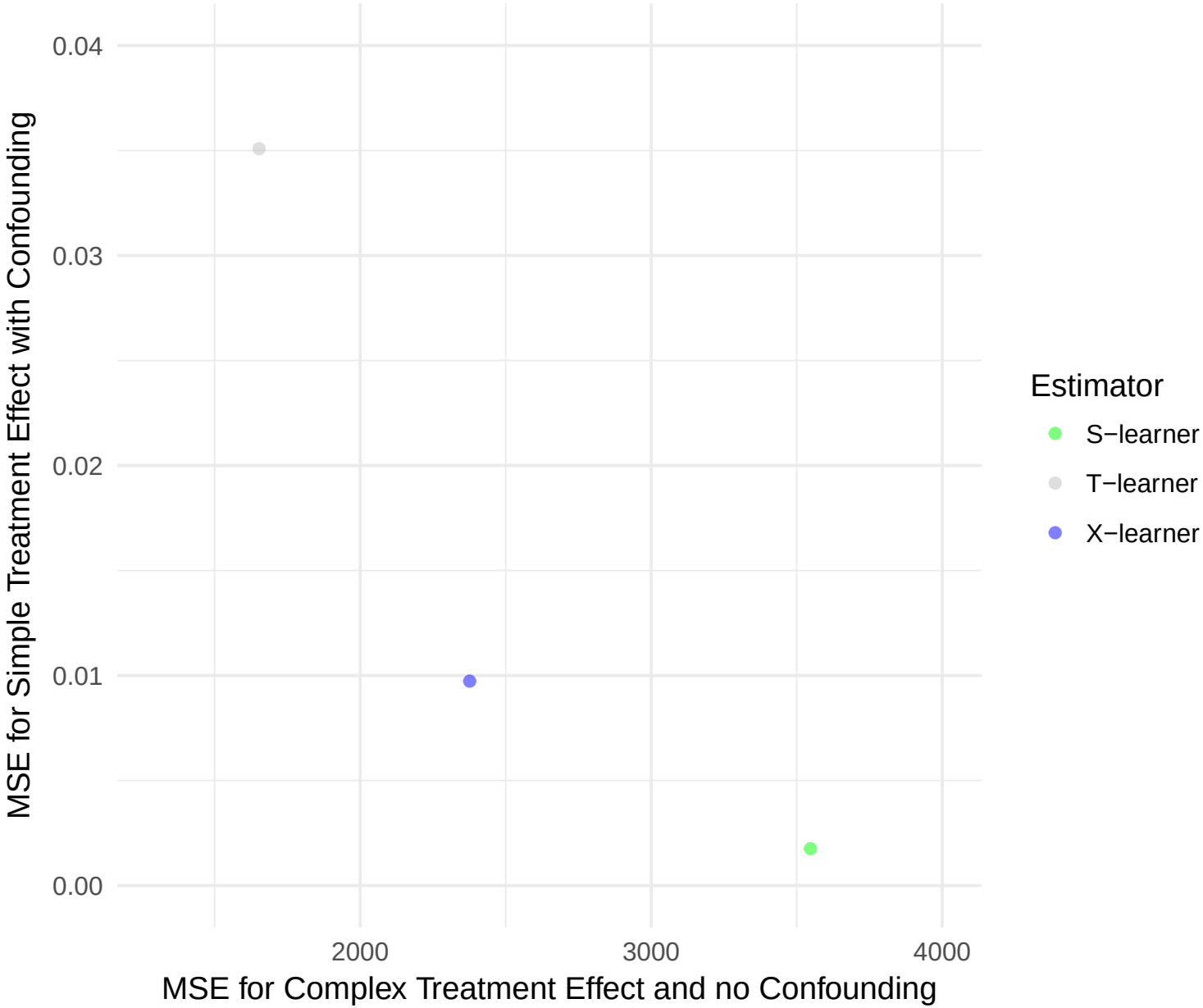
MSE for Complex Treatment Effect and no Confounding
Estimator
S−learner

Adaptivity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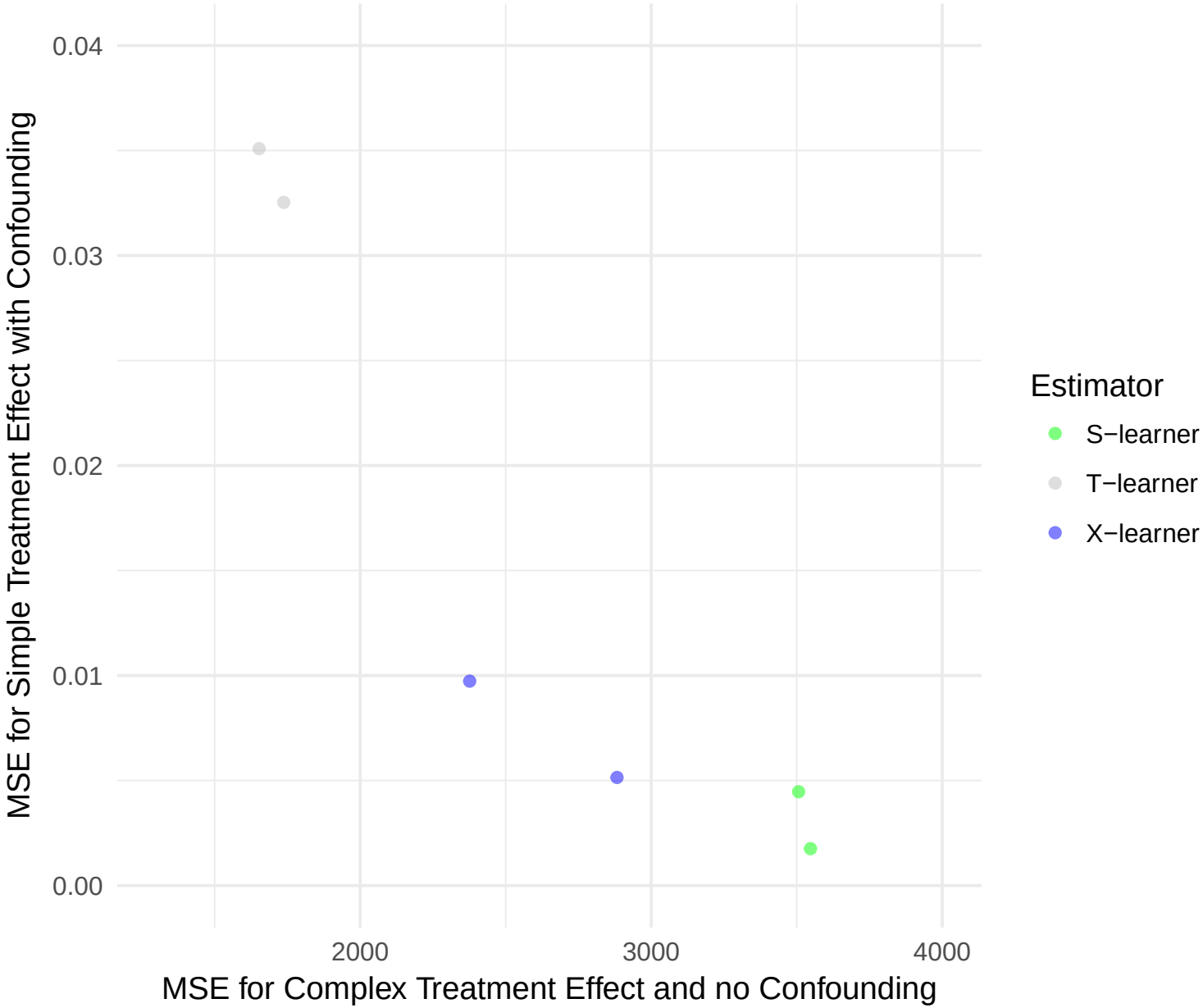
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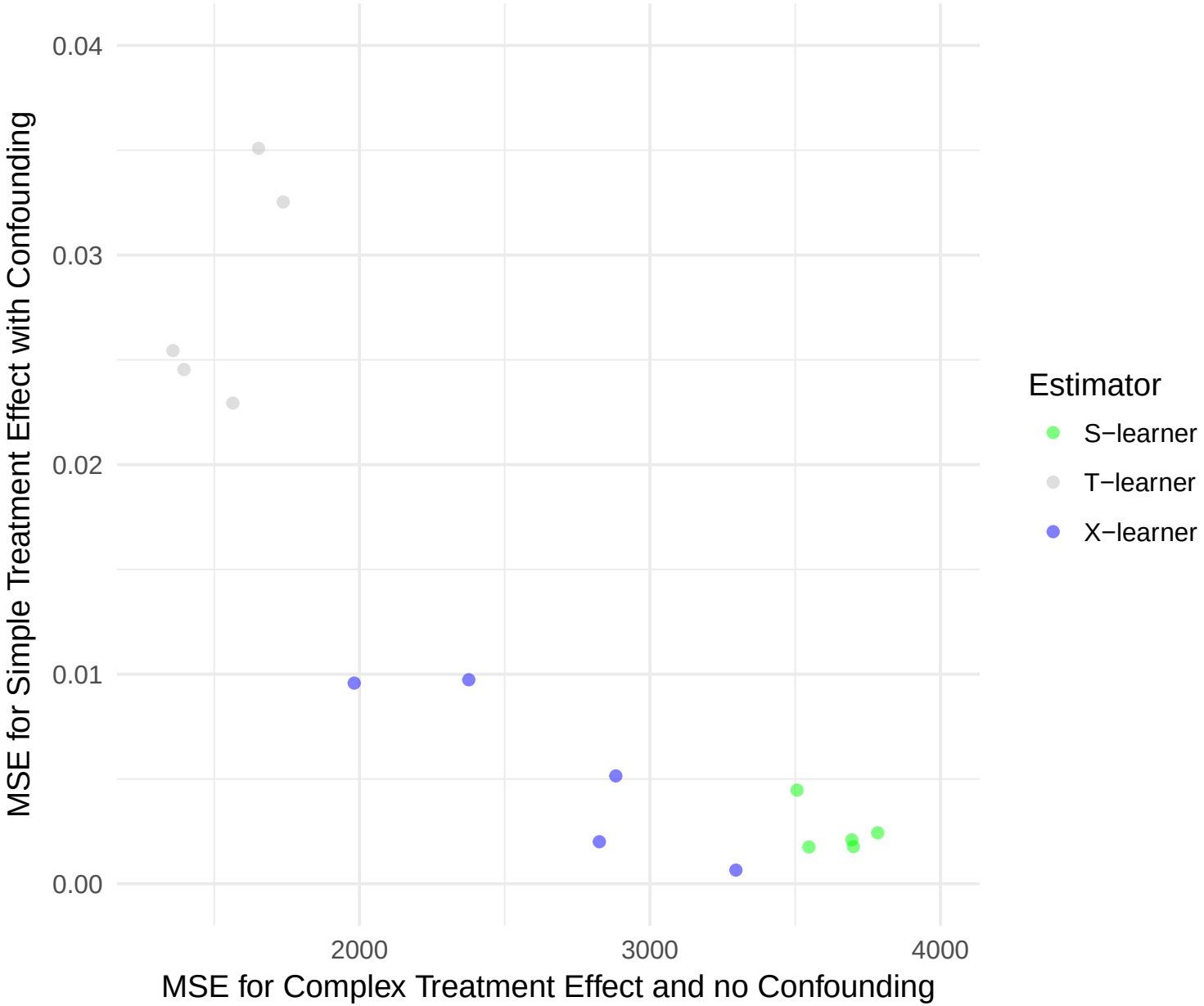
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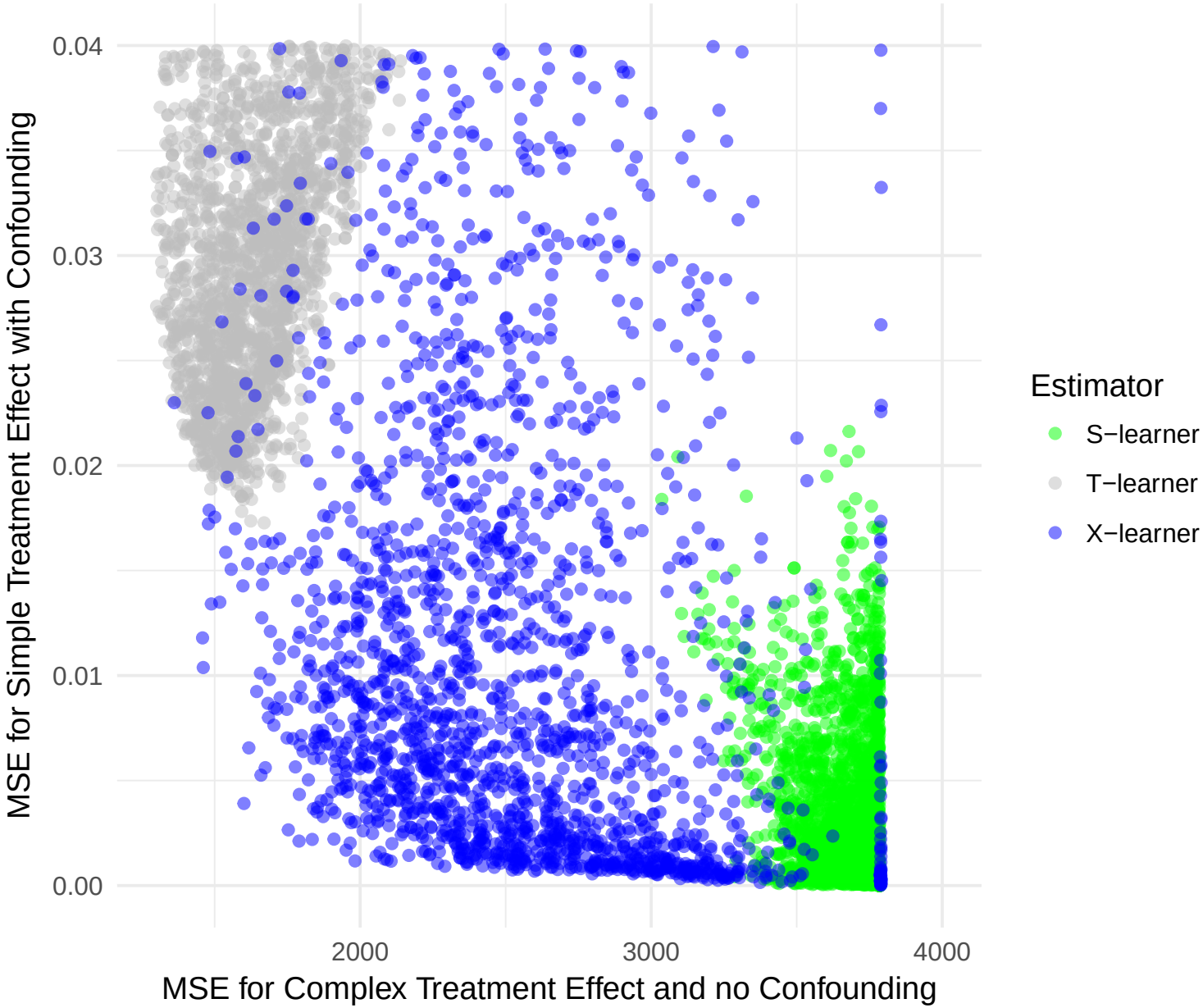
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Conclusion

- Created Neural Network architectures and transfer learning from one study to another: Künzel, Stadie, Vemuri, Ramakrishnan, Sekhon, Abbeel (2018)
- We expect more from our experiments than ever before
- We should protect the Type I error rate—e.g., honest Random Forests, cross-fitting
- Power is a significant concern
- Somethings are easier to validate than others: experiments estimating average sample effects versus CATE
- Observational data?
- Validation, validation, and validation

My Collaborators

Peter Bickel

David Broockman

Joshua Kalla

Sören R. Künzel

Yotam Shem-Tov

Bin Yu

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- David Broockman
- Joshua Kalla
- Sören Künzel
- Varsha Ramakrishnan
- Yotam Shem-Tov
- Nikita Vemuri
- Simon Walters
- Bin Yu

Tuning

All meta–learners can be separated into several small regression problems, and we tune them separately using tuning methods which are specific for each of the learner

We have implemented a package combining the X–learner with honest Random Forests and it currently implements three tuning methods:

1.) Pre-specified tuning

2.) Gaussian Process

3.) Hyperband

Supplementary

Conjecture

Conjecture about the Minimax rates of the X–learner

If the response functions can be estimated at a particular rate a_μ, the CATE can be estimated at a rate of a_τ, the right choice of base learners, and some additional assumptions, then the two parts of the X–learner will achieve the rates of:

$$\hat{\tau}_0 \in \mathcal{O}(m^{-a_\tau} + n^{-a_\mu})$$

$$\hat{\tau}_1 \in \mathcal{O}(m^{-a_\mu} + n^{-a_\tau})$$

Theorem 2

Theorem covers the case when estimating the CATE function is not beneficial

Künzel, Sekhon, Bickel, Yu 2017

X–learner is minimax optimal for a class of estimators using KNN as the base leaner. Assume:

- Outcome functions are Lipschitz continuous
- CATE function has no simplification
- Features are uniformly distributed $[0, 1]^d$

The fastest possible rate of convergence for this class of problems is:

$$\mathcal{O}\left(\min(n_0, n_1)^{-\frac{1}{2+d}}\right)$$

- The speed of convergence is dominated by the size of the smaller assignment group
- In the worst case, there is nothing to learn from the other assignment group

Individual Treatment Effects: Information Theory Bound

$Y_u \sim P = N(\mu, \sigma^2)$, and we want to predict a new Y_i.
Our expected risk with infinite data is:

$$\mathbb{E}(\mu - Y_i)^2 =$$

Individual Treatment Effects: Information Theory Bound

$Y_u \sim P = N(\mu, \sigma^2)$, and we want to predict a new Y_i.
Our expected risk with infinite data is:

$$\mathbb{E}(\mu - Y_i)^2 = \sigma^2 = \alpha$$

With one data point?

Individual Treatment Effects: Information Theory Bound

$Y_u \sim P = N(\mu, \sigma^2)$, and we want to predict a new Y_i.
Our expected risk with infinite data is:

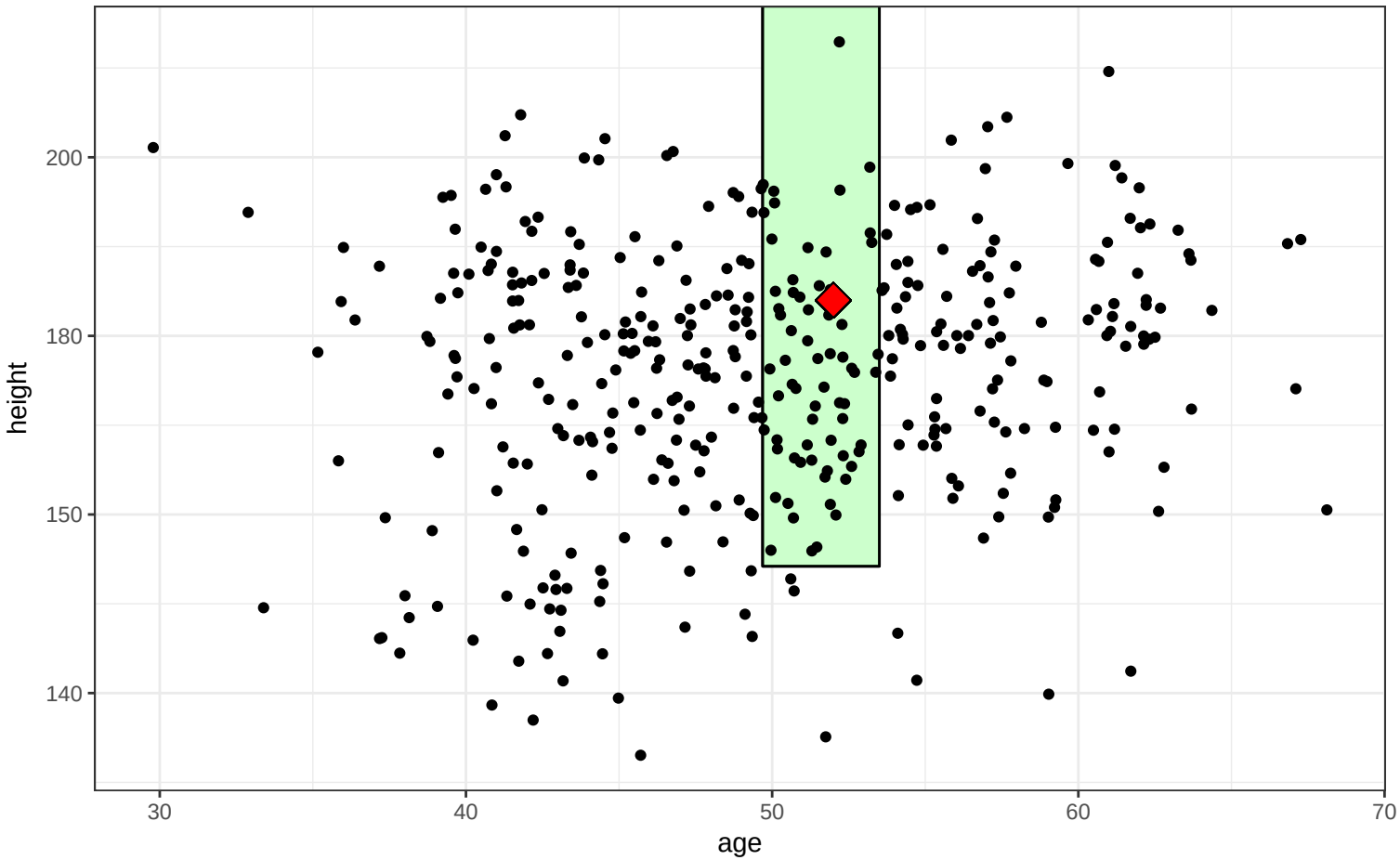
$$\mathbb{E}(\mu - Y_i)^2 = \sigma^2 = \alpha$$

With one data point?

$$\begin{aligned} E(Y_i - Y_u)^2 &= E(Y_i - \mu + Y_u - \mu)^2 \\ &= E(Y_i - \mu)^2 + E(Y_u - \mu)^2 \\ &= 2\sigma^2 \\ &= 2\alpha \end{aligned}$$

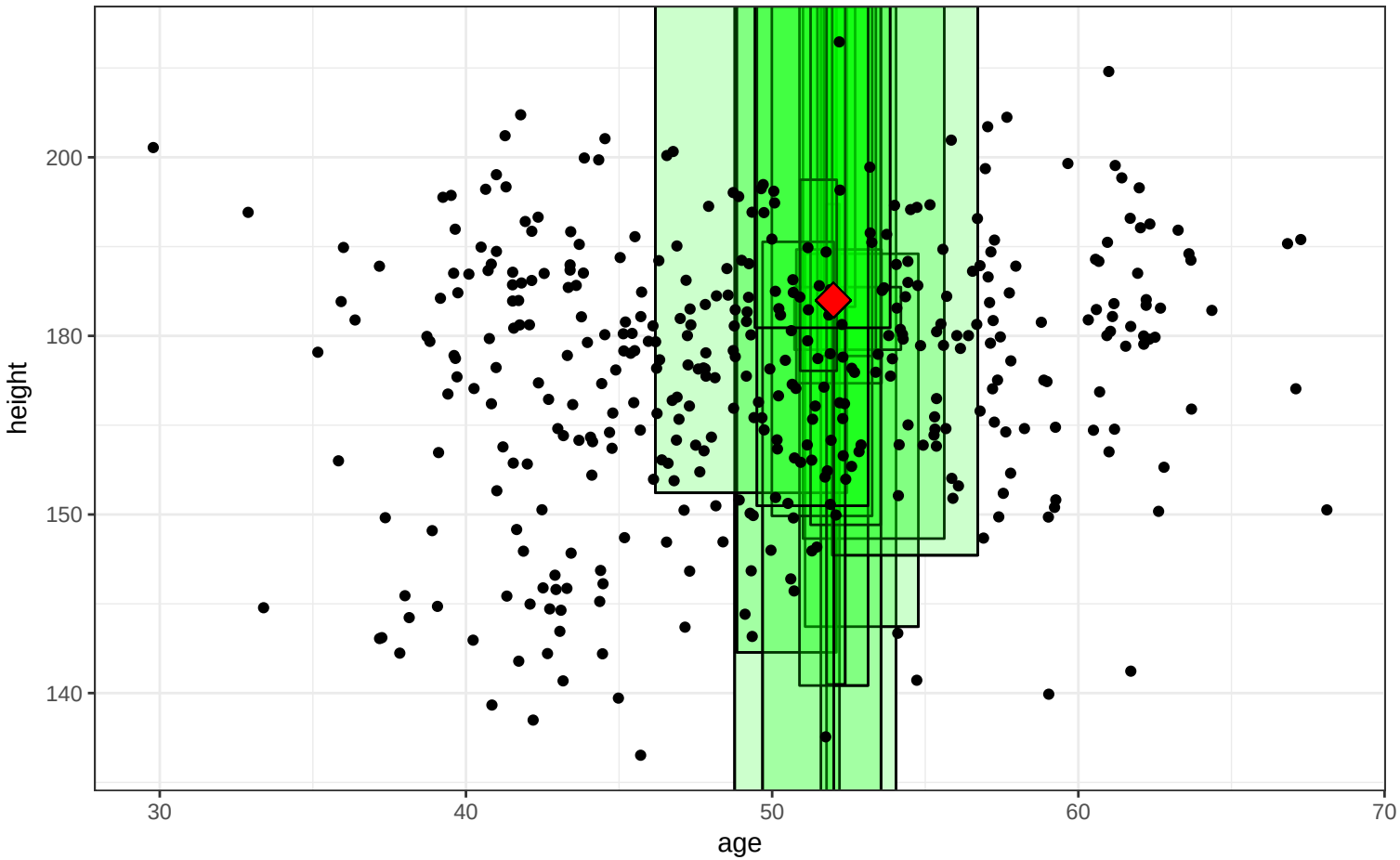
General results for Cover-Hart class, which is a convex cone (Gneiting, 2012)
Back to CATE

The averaging effect of Random Forest



Back to RF

The averaging effect of Random Forest



Back to RF

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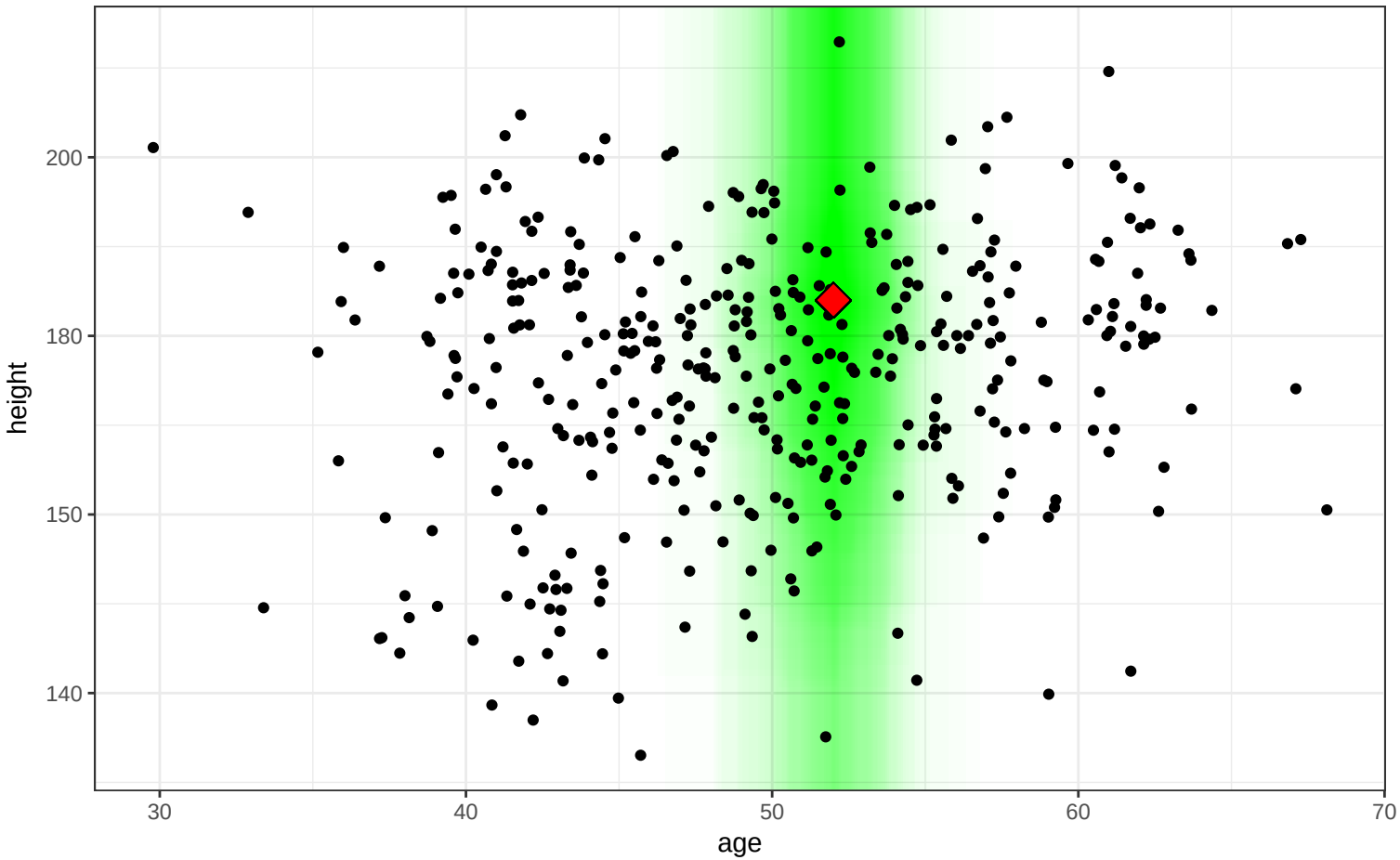
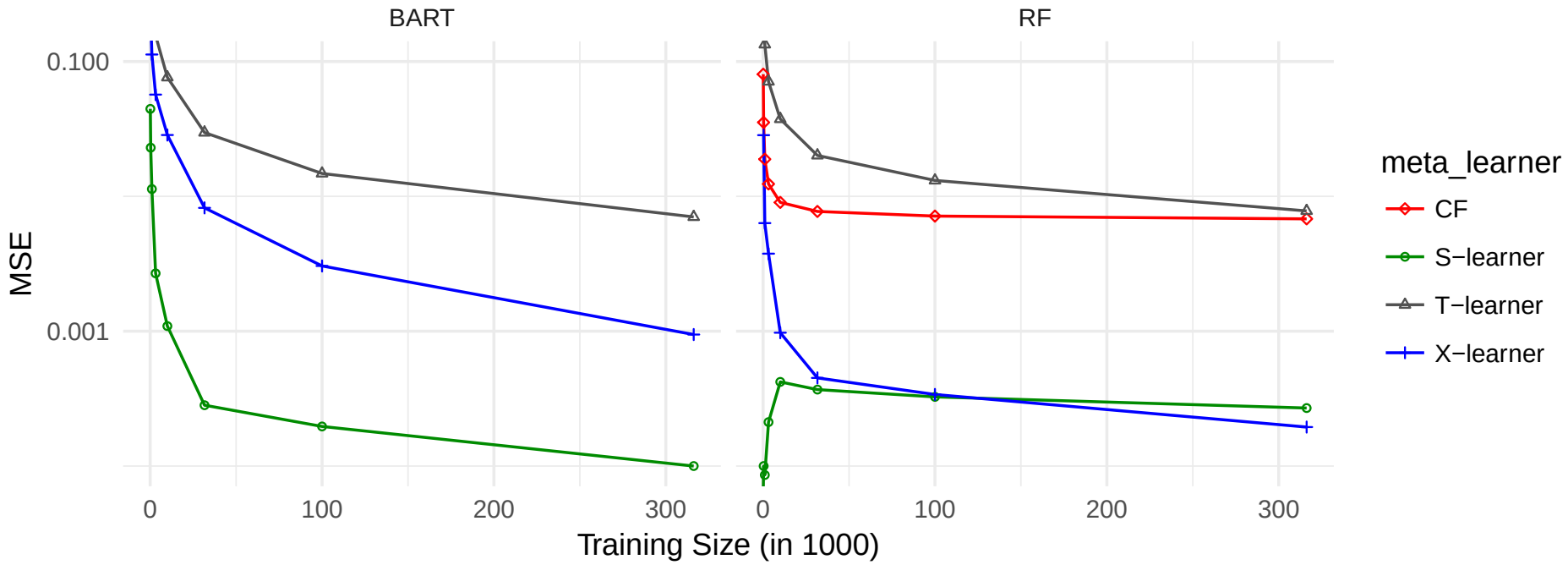


Averaging leaves makes the weighing function of random forest smooth

Back to RF

Resisting Confounding: different base learners, same effect

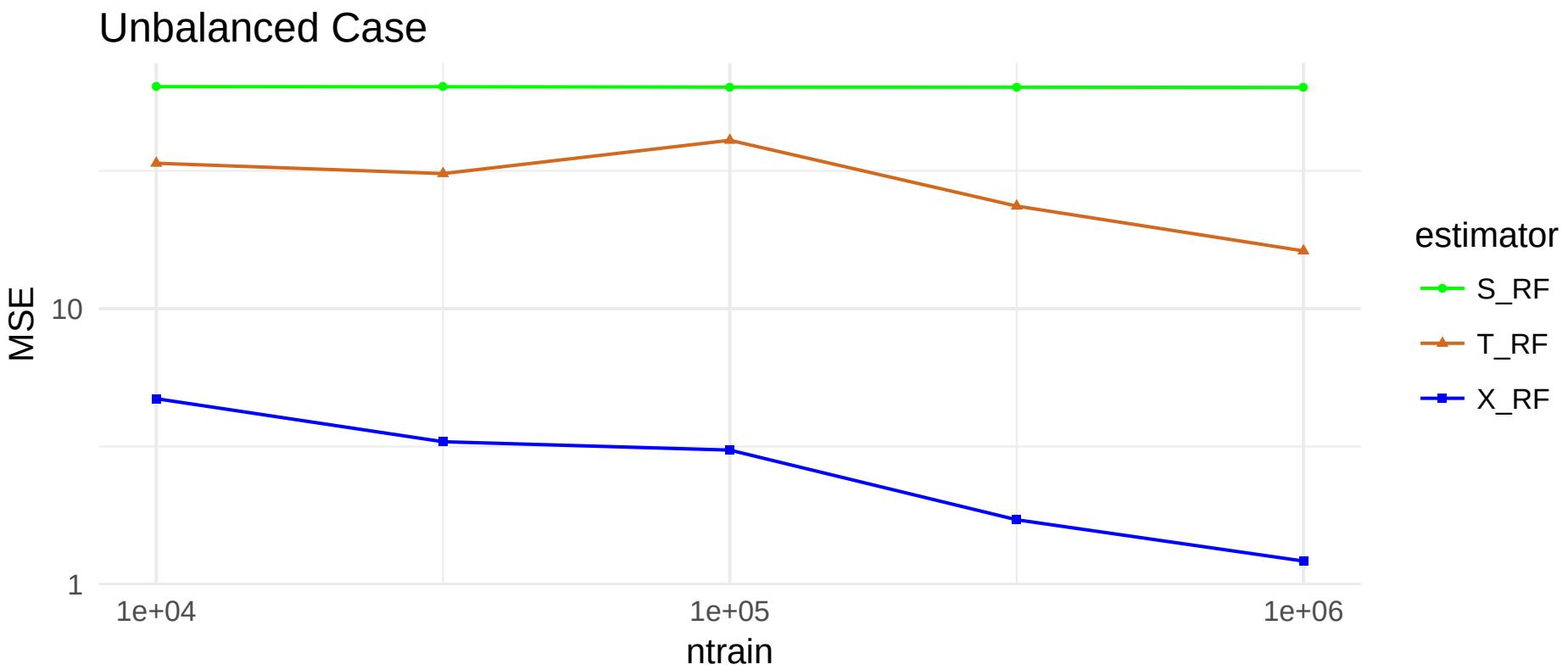


Confounded without TE (WA, 1)

$$\mu_1(x) = 2x_1 - 1,$$
$$\mu_0(x) = 2x_1 - 1,$$
$$e(x) = \frac{1}{4}(1 + \beta_{2,4}(x_1))$$

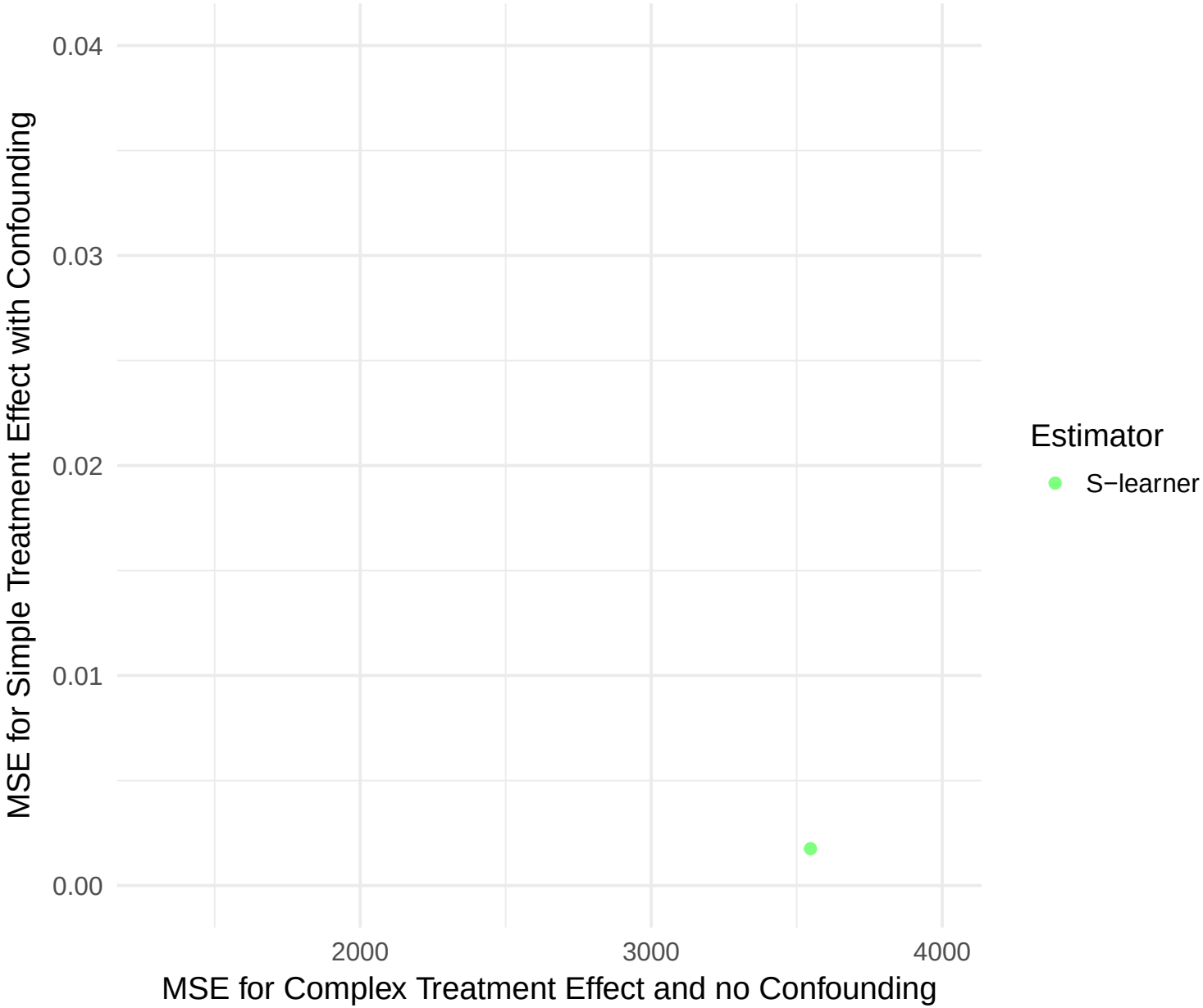
Back

The Unbalanced Case



$$\mu_0(x) = x^T\beta + 5 * 1(x1 > .5), \quad \text{with} \quad \beta \sim \text{Unif}\left([1,5]^d\right)$$
$$\mu_1(x) = \mu_1(x) + 8$$
$$e(x) = 0.01$$

Adaptivity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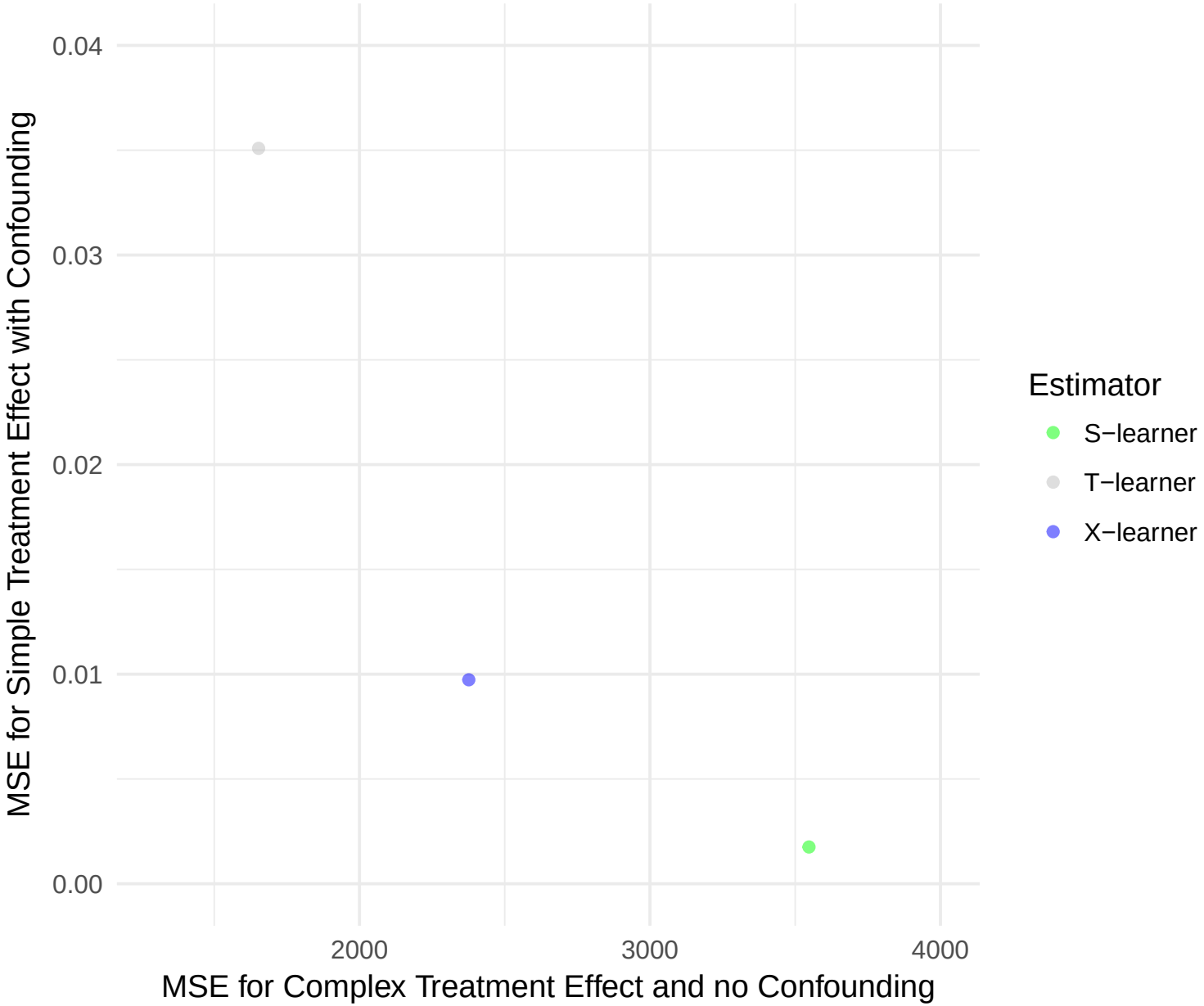
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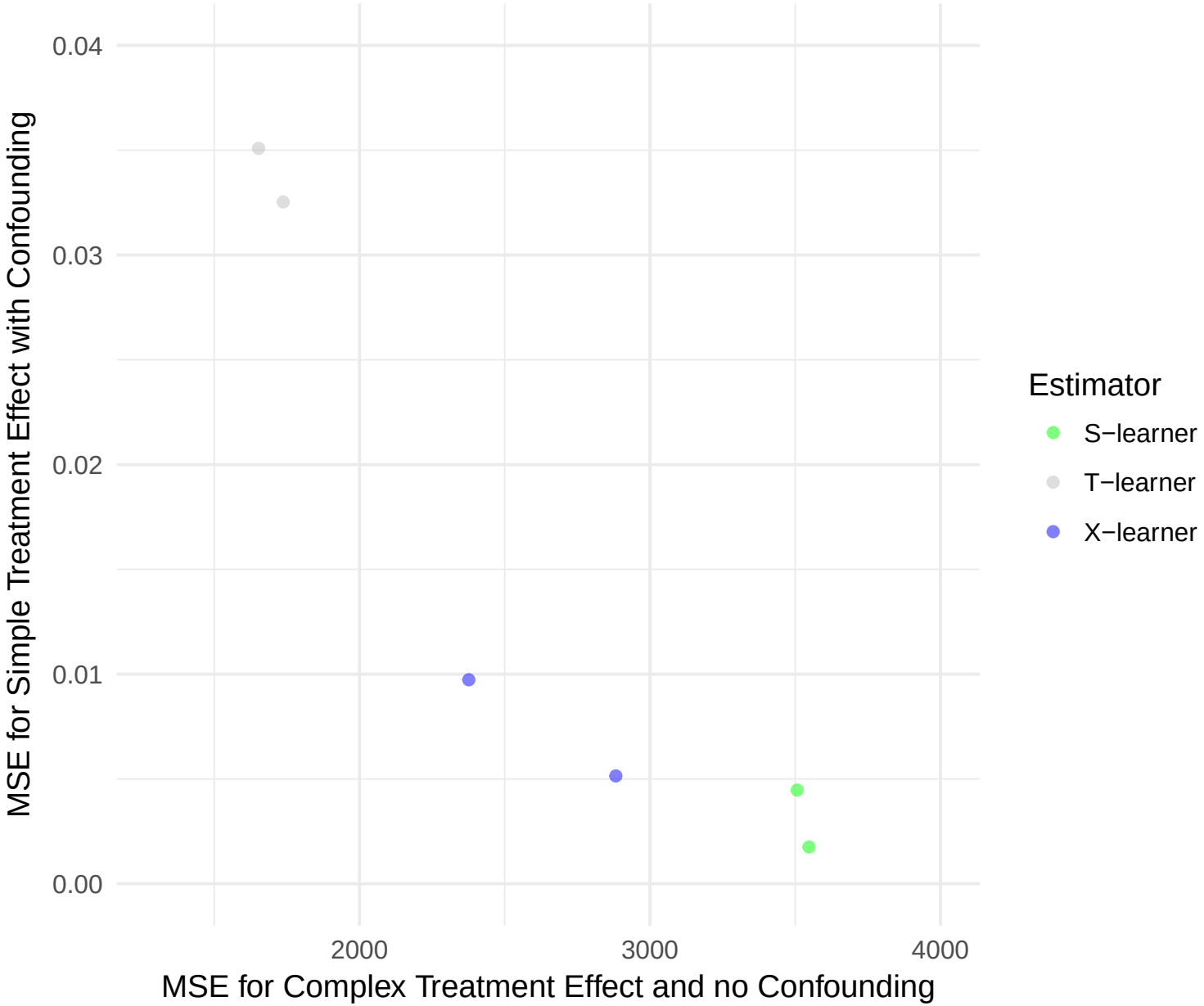
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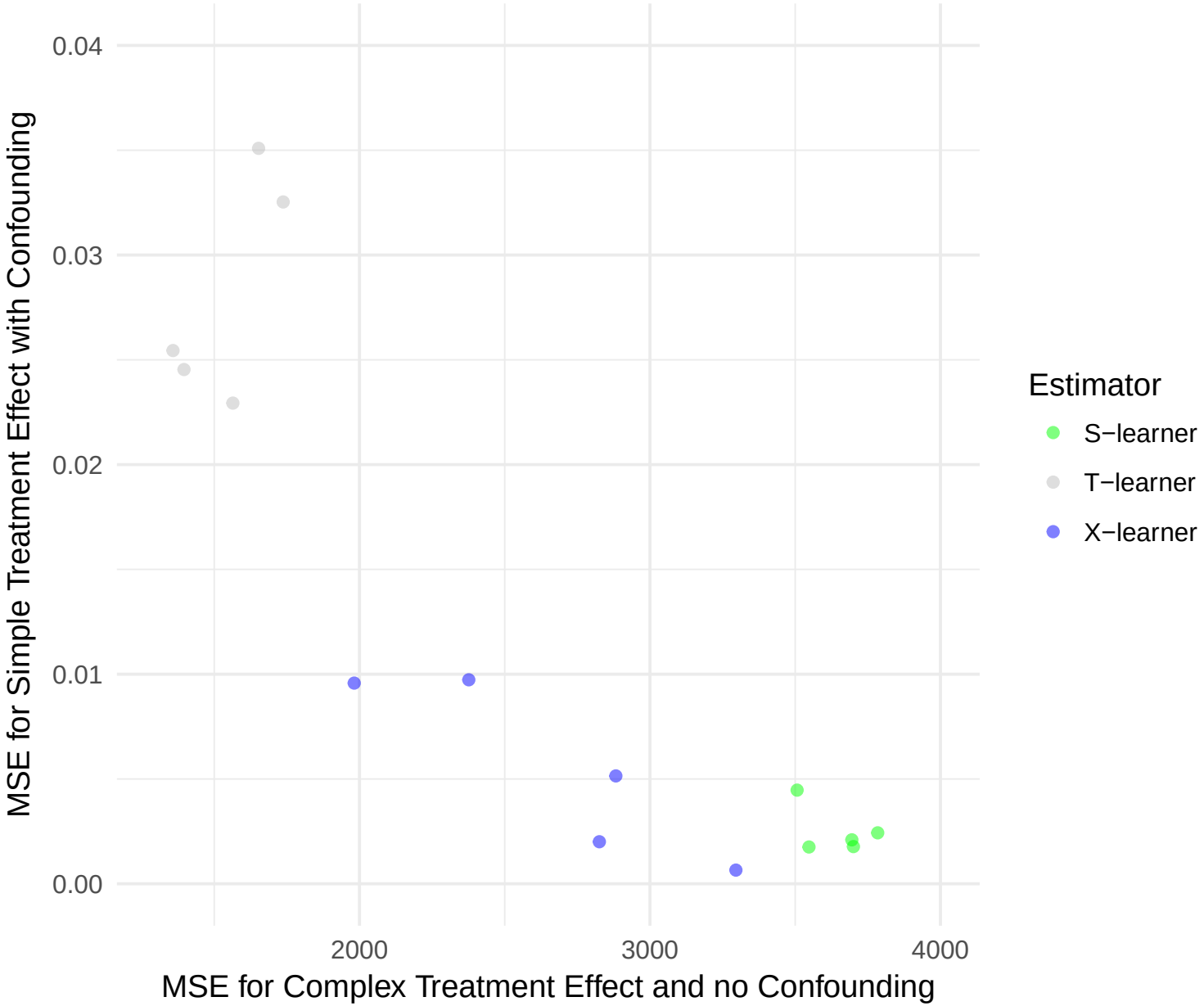
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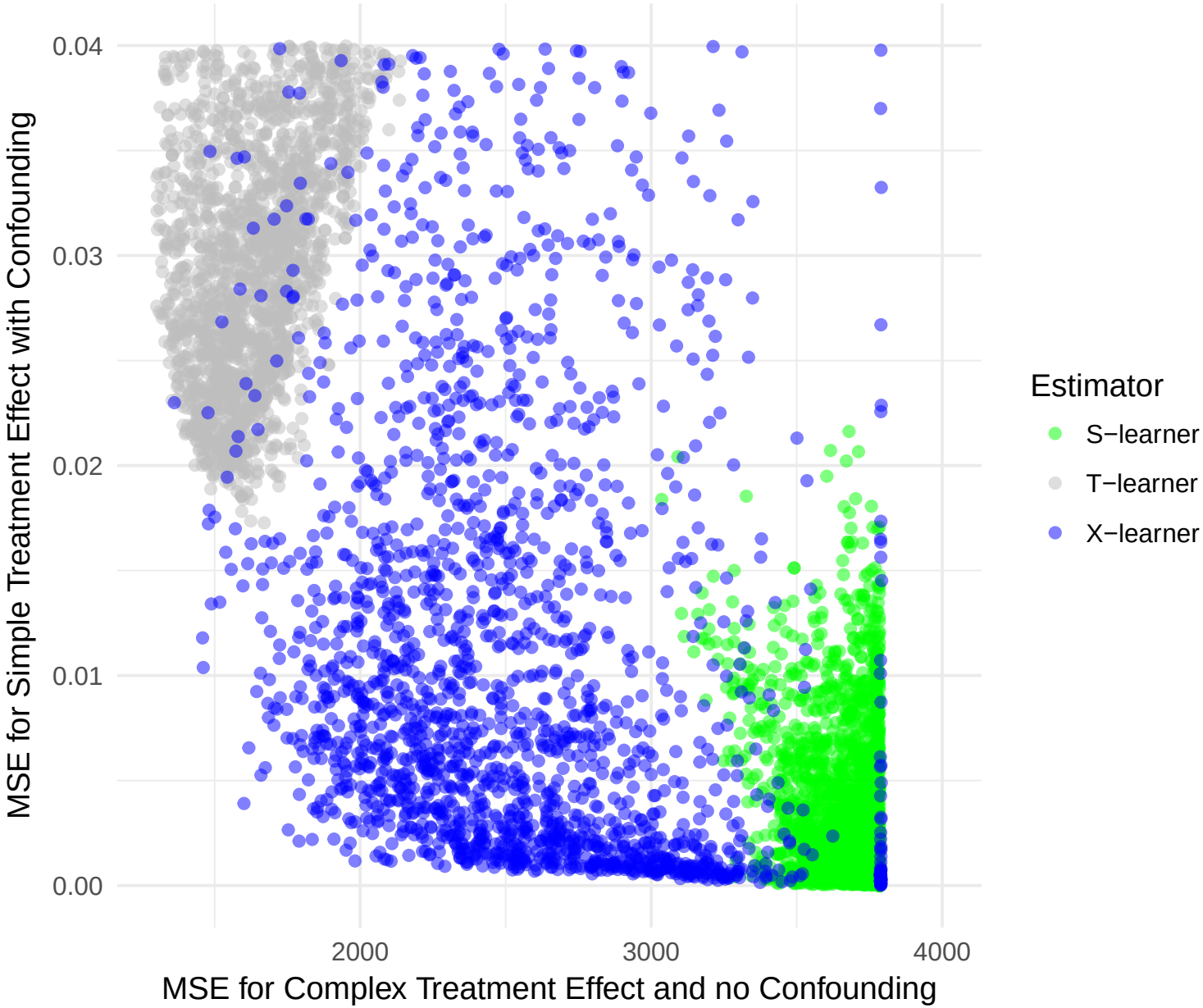
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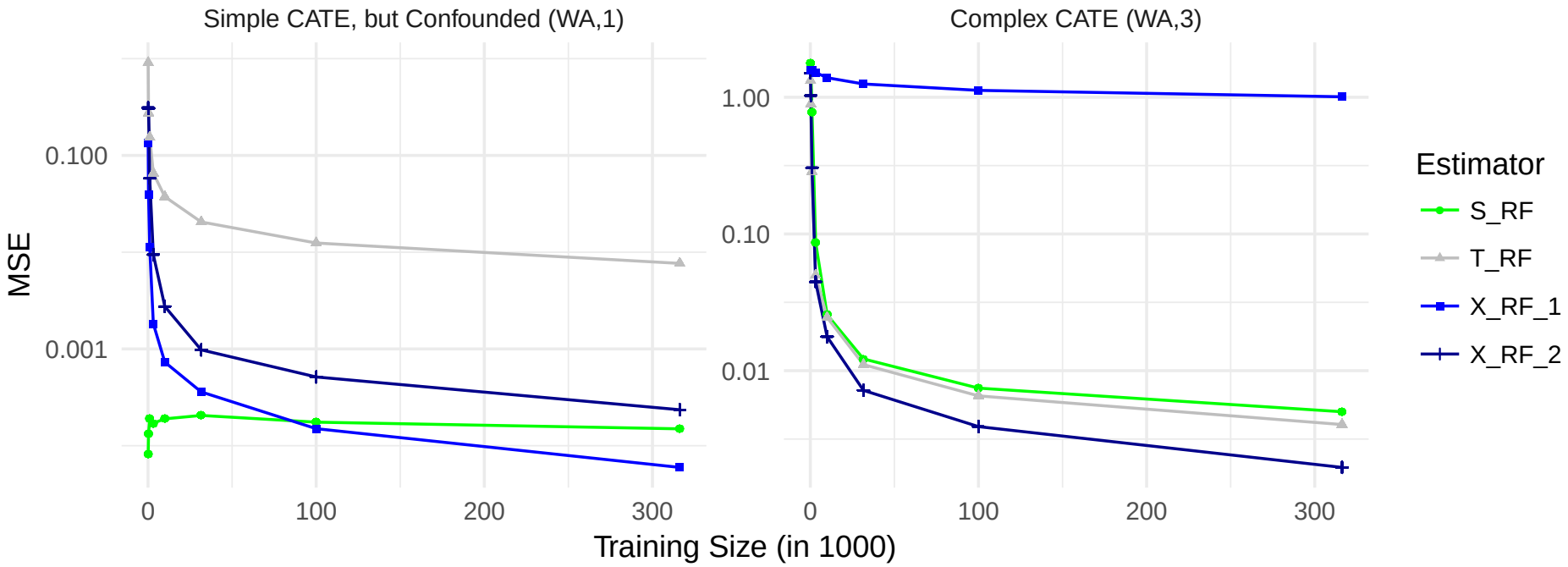

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$$\mu_1(x) = \zeta(X_1)\zeta(X_2),$$
$$\mu_0(x) = -\zeta(X_1)\zeta(X_2),$$
$$e(x) = 0.5,$$
$$\zeta(x) = \frac{2}{1 + e^{-12(x-1/2)}}$$

Even Classical Justifications Should be Validated

- Question: coverage for the population mean. Is $n = 1000$ enough?
- Sometimes, no. Not for many metrics, even when they are bounded
- For some metrics, asking for 95% CI results in only 60% coverage
- Data is very irregular
  Many zeros, IQR: 0

$$\frac{p100 - p99}{p99 - p50} > 10,000$$