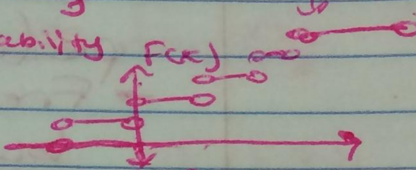
BM Lesson 1

1/29/2018

# Some Elem. Prob. Review

- Let $X$ be a random variable (r.v.)
  [Google (for fun) mixed r.v.'s = Cont. & discr.].
  - Discrete: $\exists\, p(x) := P(X = x)$
    PMF; $X$ = r.v., $x$ = realization
    - $\to \text{Supp}(X) := \{x : p(x) > 0\}$ "Support"
    - $\to p : \text{Supp}(X) \to (0, 1]$
    - $\to X \sim \text{Deg}(c) := \{c \text{ w.p. } 1\}$ — distributed as "degenerate r.v." "with probability"
    - $\to F(x) := P(X \le x)$ CDF "cumulative dist. fn."



discrete has no pdf, by definition

    - $\to |\text{Supp}(X)| \le |\mathbb{N}|$ "is countably infinite or finite"
    - $\to$ Examples:
      - $X \sim \text{Bern}(p) := \underbrace{p^x (1-p)^{1-x}}_{p(x)}$ "Bernoulli"
        $\text{Supp}(X) = \{0, 1\}$
      - $X \sim \text{Bin}(n, p) := \binom{n}{x} p^x (1-p)^{n-x}$ Binomial
  - Continuous R.V
    $f(x) := F'(x)$
    PDF = "prob. density fn."
    - $\to P(X \in [a, b]) = F(b) - F(a) = \int_a^b f(x)\,dx$
    - $\to p(x) = P(X = x) = P(X \in [x, x]) = \int_x^x f(x)\,dx = 0 \Rightarrow$ no PMF exists.
    - $\to |\text{Supp}(X)| = |\mathbb{R}|$ "uncountably infinite" of the continuum
    - $\to$ Examples
      - $X \sim \text{Exp}(\lambda) := \underbrace{\lambda e^{-\lambda x}}_{f(x)}$ exponential; $\text{Supp}(X) = [0, \infty)$
      - $X \sim \mathcal{N}(\mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ Normal; $\text{Sup}(x) = \mathbb{R}$

$\text{Supp}(x) := \{x : f(x) > 0\}$

* Intro Stuff

• $X \sim Bern(x) := p^x(1-p)^{1-x}$

$p(0) = p(X=0) = 1-1 = 0$

$p(1) = p(X=1) = 1$

$X \sim \{ \begin{smallmatrix} 0 & \text{we} & 1-p \\ 1 & \mu_p & p \end{smallmatrix}$

$\sum_{x \in Supp(x)} p(x) = 1$

$\int_{x \in Supp(x)} p(x) dx = 1$

• Parameter(s): Chosen inputs to model

"parameter space": non-degenerate, legal values of the parameter(s).

~~ped(exp)~~ - So for $Bern(x) \Rightarrow p \in (0,1)$.

- From now on, parameters are denoted by $\theta$ & the parameter space by $\Theta$.

- So ~~from~~ we write

→ $X \sim Bern(\theta) := \theta^x(1-\theta)^{1-x}$, $\theta \in \Theta = (0,1)$

→ $X \sim Bin(n, \theta) := \binom{n}{x} \theta^x (1-\theta)^{n-x}$ or

$X \sim Bin(\theta_2, \theta_1) := \binom{\theta_2}{x} \theta_1^x (1-\theta_1)^{\theta_2 - x}$

$Supp(X) = \{0, 1, \dots, n-1, n\}$, $\theta_2 \in \Theta_2 = \{1, 2, \dots\}$

• Parametric Model $\mathcal{F}$

$\mathcal{F} = \{ p(x;\theta) : \theta \in \Theta \}$ and $\dim(\Theta) < \infty$.

- From now on, $p(x)$, $f(x)$ will be denoted $p(x)$.

* Further, $p(x;\theta)$ will be used.

"including the following info".

- For example, $\mathcal{F} := \{ \theta^x(1-\theta)^{1-x} : \theta \in (0,1) \}$ is ~~the~~ a family of Bernoulli r.v.s.

• Independently Identically Distributed (iid)

$p(x_1, x_2, \dots, x_n; \theta) = p(x_1;\theta) p(x_2;\theta) \cdots p(x_n;\theta)$

[joint mass function ~~or~~ joint density fn]

$= \prod_{i=1}^{n} p(x_i, \theta)$, if

if $X_1, X_2, \dots, X_n \overset{iid}{\sim} p(x_i, \theta)$.

• Example

$X_1, X_2, \ldots X_6 \overset{iid}{\sim} \text{Bern}(\theta)$

$\vec{X} : \langle x_1, x_2, \ldots x_6 \rangle$.

If $\vec{x} := \langle 0,0,1,0,1,0 \rangle$,

$p(\vec{x} = \langle 0,0,1,0,1,0 \rangle; \theta) = \prod_{i=1}^{6} \theta^{x_i}(1-\theta)^{1-x_i} = \theta^2(1-\theta)^4$.

+ ~~Statistical inferencing~~

• When $\theta$ is unknown but we want to know it, we'd use Statistical inferencing.

• There are 3 goals:

① We want the best guess for $\theta$; We'll call this $\hat{\theta}$.

② Confidence set - range of likely values of $\theta$.

③ Test theories about $\theta$.

• Example: Consider $X_1, \ldots, X_6 \sim \text{Bern}(0.5)$;

$p(\vec{x} = \langle 0,0,1,0,1,0 \rangle; \theta = 0.5) = 0.5^6 = 0.0156$.

- How about $\theta = 0.25$? $\Rightarrow p(\cdots) = (0.25^2)(0.75)^4 = 0.0196$.