LEC 5 2/14/2018 (P.1)

(H) all the possible data in the set!

$x = \langle 0, 1, 1 \rangle$

$\hat{\theta}_{MLE} =$

$\Theta = \{0, 1, 0.25, 0.5, 0.75, 0.9\}$.

$\theta \sim U(\Theta_0)$

discrete uniform:
principle of indifference
uniformative objection.

$P(x \mid \theta = 0.1) = 0.009$

$P(x \mid \theta = 0.25) = 0.047$

$P(x \mid \theta = 0.5) = 0.125$

$P(x \mid \theta = 0.75) = 0.141$

$P(x \mid \theta = 0.9) = 0.061$

point estimation

$$\hat{\theta}_{MLE} := \underset{\theta \in \Theta}{\operatorname{argmax}} \{P(\theta \mid X)\} = \underset{\theta \in \Theta}{\operatorname{argmax}} \left\{ \frac{P(x \mid \theta) P(\theta)}{P(X)} \right\}.$$

↑ max min position

$$\underset{\theta \in \Theta}{\operatorname{argmax}} \{P(X \mid \theta) P(\theta)\} = \underset{\theta \in \Theta}{\operatorname{argmax}} \{P(x \mid \theta)\} = \mathcal{L}(\theta, x)$$

$\hat{\theta}_{MLE}$

$$P(\theta \mid X) = \frac{P(X \mid \theta) P(\theta)}{P(X)}$$

$$P(y) = \sum_{x} P(x, y)$$

$$\frac{P(x \mid \theta) P(\theta)}{P(\theta) \sum_{\theta \in \Theta} P(x \mid \theta)}$$

$$P(\theta = 0.75 \mid x = \langle 0, 1, 1 \rangle)$$

$$37\% = \frac{0.141}{0.009 + 0.047 + 0.125 + 0.141 + 0.061}$$

$$\frac{P(X \mid \theta)}{\sum_{\theta \in \Theta} P(x \mid \theta)}$$

Support of Prior Θ did not match Θ

$\Theta_0 \neq \Theta$ prior didn't

$\hat{\theta}_{MLE} = 0.75 \neq \hat{\theta}_{MLE} = 0.66$

Does not work, 0.66 is not part of the solution $P(\theta)$ is not continuous, it is discrete.

$\Theta = \{0.25, 0.75\}$ and $\theta \sim u(H_0)$

$x_1 = 0$

$$P(\theta=0.25 \mid x_1=0) = \frac{P(x_1=0 \mid \theta=0.25)}{P(x_1=0\mid\theta=0.25)+P(x_1=0\mid\theta=0.75)}$$

$$= \frac{0.75}{0.75+0.25}$$

$$= 0.75$$

$$\Rightarrow P(\theta=0.75 \mid x_1=0) = 1-0.75 = 0.25 \quad \#.$$

$x_2 = 1$

$$P(\theta=0.25 \mid x_1=0, x_2=1) = \frac{P(x_2\mid\theta)\,P(\theta\mid x_1)}{P(x_1\,x_2)}$$

$$= \frac{\overset{0.25}{P(x_2=1\mid\theta=0.25)}\ \overset{0.75}{P(\theta=0.25\mid x_1=0)}}{\underset{0.25}{P(x_2=1\mid\theta=0.25)}\ \underset{0.75}{P(\theta=0.25\mid x_1=0)} + \underset{0.75}{P(x_2=1\mid\theta=0.75)}\ \underset{0.25}{P(\theta=0.75\mid x_1=0)}}$$

$$= 0.5$$

$$P(\theta=0.25 \mid x_1=0, x_2=1) = 0.5$$

$x_3 = 1$

$$P(\theta=0.25 \mid x=(0,1,1)) = \frac{P(x_3 \mid \theta)\, P(\theta \mid x_1, x_2)}{P(x)}$$

$$= \frac{P(x_3=1 \mid \overset{0.25}{\theta=0.25})\, P(\theta=0.25 \mid x_1=0, x_2=1)}{P(x_3=1 \mid \theta=0.25) + P(x_3=1 \mid \theta=0.75)\, P(\theta=0.75 \mid x_1=0, x_2=1)}$$

$= 0.25$

$$P(\theta \mid x) = \frac{P(x \mid \theta)\, P(\theta)}{P(x)}$$

Bernaulli iid.

$$P(\theta \mid x_1, x_2 \ldots x_n) = \frac{\overbrace{P(x_1, \ldots x_n \mid \theta)}^{P(x_1 \mid \theta) \cdot P(x_2 \mid \theta) \ldots P(x_n \mid \theta)}\, P(\theta)}{\underbrace{P(x_1 \ldots x_n)}_{= P(x_2 \ldots x_n \mid x_1)\, P(x_1)}}$$

Grouping $= \dfrac{P(x_2 \ldots x_n \mid \theta)\, \dfrac{P(x_1 \mid \theta)\, P(\theta)}{P(x_1)}}{P(x_2 \ldots x_n \mid x_1)}$ → this part $= P(\theta \mid x_1)$

$$= \frac{P(x_3 \ldots x_n \mid \theta)\, \dfrac{P(x_2 \mid \theta)\, P(\theta \mid x_1)}{P(x_2 \mid x_1)}}{P(x_3 \ldots x_n \mid x_1, x_2)}$$

(we want to make it look like

$$= \frac{\overbrace{P(x_2 \mid \theta)\, P(x_1 \mid \theta)}^{P(x_1, x_2 \mid \theta)}\, P(\theta)}{\underbrace{P(x_2 \mid x_1)\, P(x_1)}_{P(x_1 x_2)}}$$

$p(x_2|\theta)\, p(x_3 \dots x_n|\theta)$ $\quad$ $p(\theta|x_1)$

$\quad\quad\quad\quad = \quad\quad\quad\quad\quad\quad\quad\quad\quad =$

$p(x_2, \dots x_n|\theta)$ $\quad$ $\frac{p(x_1|\theta)\, p(\theta)}{p(x_1)}$

$p(x_2 \dots x_n|x_1)$

$=$

$$P(AB|C) = P(A|BC)\, P(B|C)$$

$$= p(x_3 \dots x_n|x_2 x_1)\, p(x_2|x_1)$$

this part is how to derive grouping part

$$* \; P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(AB) = P(A|B) \cdot P(B)$$

- We haven't seen x_4 yet, but we have seen x_1, x_2, x_3

What is $(x_4 | x_1, x_2, x_3)$ distribution?

What does it mean?

$P(x) = \sum_\theta P(x, \theta)$

$\theta \mid x_1, x_2, x_3$		x_4		$P(x_4 \theta \mid x_1 x_2 x_3)$
0.25	0.25	0.75	0	0.625
		0.25	1	0.1875
0.75	0.75	0.25	0	0.5625
		0.75	1	0.1875

$H_0 = \{0.25, 0.75\}$

$P(x_4 = 0 | x_1, x_2, x_3) = 0.625$

$P(x_4 = 1 | x_1, x_2, x_3) = 0.375$

$(x_4 | x_1, x_2, x_3) \sim \text{Bernoulli}(0.375)$

Involve the concept $P(x) = \sum_\theta P(x, \theta)$

$P(x_4 | x_1, x_2, x_3) = \sum P(x \theta | x_1, x_2, x_3)$

$= \sum_\theta P(x_4 | \theta \underbrace{x_1 x_2 x_3}_{\text{does not matter}}) P(\theta | x_1, x_2, x_3)$

$= \sum_\theta P(x_4 | \theta) P(\theta | x_1, x_2, x_3)$