Lecture 1

341 - Bayesian Modeling

X is a random variable (r.v)

discrete or continuous

- **Discrete r.v.**

$$\underset{\text{"for every"}}{\exists}\ p(x) \underset{\text{"defined as"}}{:=} \overset{\text{probability}}{P}(\underset{\text{r.v.}}{X} = \underset{\text{"realization"}}{x})$$

- No PDF ; PMF (Probability Mass function)

* $\text{Supp}(X) := \{x : p(x) > 0\}$ $\quad p : \text{supp}[X] \to [0,1]$

support for x

$$\underset{\substack{\text{distribution}\\\text{as}}}{X \sim} \underset{\substack{\text{degenerate}\\\text{brandname}\\\text{r.v}}}{\text{Deg}(c)} := \{c \underset{\substack{\text{with}\\\text{probability}}}{\text{w.p}} 1\}$$

$$F(x) := P(X \leq x)$$

- CDF (cumulative distribution function)

* $|\text{Supp}(X)| \leq |\mathbb{N}|$ · finite · countably infinite

ex: $X \sim \text{Bern}(p) := \underbrace{p^x(1-p)^{1-x}}_{p(x)}$ or $X \sim \text{Bin}(n,p) := \binom{n}{x} p^x (1-p)^{n-x}$

$$\text{Supp}(X) = \{0,1\}$$

- Continuous R.V

$$f(x) := F'(x)$$

- PDF (probability density function)

* $Supp(X) := \{ x : f(x) > 0 \}$

$$P(X \in [a,b]) = F(b) - F(a) = \int_a^b f(x)\,dx$$

$$p(x) = P(X = x) = P(X \in [x,x]) = \int_x^x f(x)\,dx = 0 \quad (\underline{\underline{\text{No PMF}}})$$

- $|Supp(X)| = |\mathbb{R}|$ infinite uncountable

ex: $X \sim Exp(\lambda) := \underbrace{\lambda e^{-\lambda x}}_{f(x)}$

$supp(X) = [0, \infty)$

ex: $X \sim N(\mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(X-\mu)^2}$

$supp(X) = \mathbb{R}$

Note

$\sum_{x \in supp(X)} p(x) = 1$ $\int_{x \in supp(X)} p(x)\,dx = 1$

discrete continuous

- Parameter(s)

defined probability

denoted 'θ'

chosen inputs to model

- Parameter space : θ $(0,1)$

don't include 0 and 1 b/c the make degenerate case

denoted 'Θ'

non degenerate case are the values

ex:

$$X \sim Bern(\theta) = \theta^{x}(1-\theta)^{1-x}$$

$$\theta \in \Theta = (0,1)$$

ex:

$$X \sim Bi(\underset{\text{fixed}}{n}, \theta) = \binom{n}{x}\theta^{x}(1-\theta)^{n-x}$$

$$\theta \in \Theta = (0,1)$$

$$supp(X) = \{0,1,2,\dots,n-1,n\}$$

Note

n can be θ_2, and

$\theta_2 \in \Theta_2 = \{1,2,\dots\}$

- Parametric Model ($\mathcal{F}$)

$$\mathcal{F} := \{P(x;\theta) : \theta \in \Theta\}$$

dimension$[\Theta] < \infty$

$p(x)$ pmf

$f(x)$ pdf

$\Big\}$ denoted by $p(x)$ or $p(x;\theta)$

$$\mathcal{F} := \{\theta^{x}(1-\theta)^{1-x} : \theta \in (0,1)\}$$

- Note. We will work with independent & identically distributed (iid)

Joint mass/density function

$$p(x_1, x_2, \dots, x_n; \theta) = P(x_1; \theta) P(x_2; \theta) \dots P(x_n; \theta)$$

$$(x_1, x_2, \dots, x_n) \overset{iid}{\sim} p(x_i, \theta) = \prod_{i=1}^{n} p(x_i; \theta)$$

ex:

$$(X_1, X_2, \dots, X_6) \overset{iid}{\sim} \text{Bern}(\theta) \quad \text{so} \quad X_i := \langle x_1, \dots, x_6 \rangle$$

find $p(X = \langle 0,0,1,0,1,0 \rangle; \theta) = \prod_{i=1}^{6} \theta^{x_i} (1-\theta)^{1-x_i}$

$$= \underset{X=0}{(1-\theta)^4} \; \underset{X=1}{\theta^2}$$

a. consider $\theta = 0.5$

$$P(X = \langle 0,0,1,0,1,0 \rangle; \theta = 0.5) = (1-0.5)^4 (0.5)^2 = 0.5^6 = 00156$$

b. consider $\theta = 0.25$

$$P(X = \langle 0,0,1,0,1,0 \rangle; \theta = 0.25) = (1-0.25)^4 (0.25)^2 = .75^4 \, .25^2 = 00198$$

- $\mathcal{L}(\theta; x) := p(x; \theta)$

check values of θ

likelihood of x ↑   joint probability of x based on all θ ↑

- Statistical inference: to find θ when it is unknown

① Best guess θ is called $\hat{\theta}_{MLE}$ (max. likelihood estimate)

② confidence test; range of likely value of θ

③ test various theory about θ