Algorithmic Analysis

Big O: $f(n) \in O(g(n)) \longleftrightarrow \exists c \in \mathbb{R}, n_0 \in \mathbb{Z}^{\geq 0}$ s.t. $\forall n \geq n_0, f(n) \leq c \cdot g(n)$

Big Ω: $f(n) \in \Omega(g(n)) \longleftrightarrow \exists c \in \mathbb{R}, n_0 \in \mathbb{Z}^{\geq 0}$ s.t. $\forall n \geq n_0, f(n) \geq c \cdot g(n)$

Big Θ: $f(n) \in \Theta(g(n)) \longleftrightarrow f(n) \in O(g(n)) \wedge f(n) \in \Omega(g(n))$

$\longleftrightarrow \exists c_1, c_2 \in \mathbb{R}, n_0 \in \mathbb{Z}^{\geq 0}$ s.t. $\forall n \geq n_0, c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

Growth Rates: $1 \rightarrow \log(n) \rightarrow n \rightarrow n\log(n) \rightarrow n^2 \rightarrow n^3 \rightarrow c^n \rightarrow n!$

Recursion Analysis

Case 1: work per call follows pattern - $T(n) = T(n-1) + O(1)$
Case 2: work per level is the same - $T(n) = 2T(\frac{n}{2}) + O(n)$
Case 3: work per call is the same - $T(n) = 2T(n-1) + O(1)$

Sorting Algorithms

Sort	Worst	Best	Expected
Bubble $A[i] \overset{swap}{\leftrightarrow} A[i+1]$	$O(n^2)$	$O(n)$ *pre-sorted*	$O(n^2)$
Insertion *by next unsorted*	$O(n^2)$	$O(n)$ *pre-sorted*	$O(n^2)$
Selection *by smallest*	$O(n^2)$	$O(n)$ *pre-sorted*	$O(n^2)$
Merge	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quick *3-partition*	$O(n^2)$	$O(n)$ *uniform list*	$O(n \log n)$
Radix	$O(d(n+N))$	$O(d(n+N))$	$O(d(n+N))$

Basic Data Structures

Amortisation: $T(n) \div n$, where $n =$ no. of operations

	SPACE	*get*	*add*	*remove*
Dynamic Array	$O(n)$	$O(1)$	$O(n)$*	$O(n)$
Linked List	$O(n)$	$O(1)$	$O(n)$	$O(n)$
Stack - LIFO	*SPACE:* $O(n)$		$O(1)^1$	$O(1)$
Queue - FIFO	*SPACE:* $O(n)$		$O(1)$	$O(1)$

[1] *amortised if array-based implementation*

Binary Trees

Proper Binary Tree: internal nodes have exactly 2 children
Complete Binary Tree: levels $0 \rightarrow h-1$ full, level h left-most

Pre-Order	In-Order	Post-Order
self → left → right	left → self → right	left → right → self

Properties:

- Full level l has 2^l nodes (note $l \geq 0$)
- Max no. of nodes = $2^{l_{MAX}} - 1$, max internal nodes = $2^{l_{MAX}-1} - 1$
- h = no. of edges from lowest leaf = $\lfloor \log_2 n \rfloor$

  ⚠️ *care with null leaves in implementation vs. conceptual tree*
- l & d use 0-based index.

Heaps & Priority Queues

Binary Heap: complete binary tree with heap property order

$key(node) \leq key(parent(node))$ OR $key(node) \geq key(parent(node))$

Insertion: insert at last node location then **upHeap** -

1. Check $k < parent \wedge k \neq root$
2. Swap $k \longleftrightarrow parent_{\text{greater than current}}$
3. Repeat until $k \geq parent \vee k = root \longrightarrow O(\log n)$

RemoveMin: swap root $\longleftrightarrow$ last node then **downHeap** -

1. Check $k > child_{\text{left or right}}$
2. Swap $k \longleftrightarrow child_{\text{less than current}}$
3. Repeat until $k > child_{\text{both}} \vee k = leaf \longrightarrow O(\log n)$

Array-Based: left child @ $2i+1$ & right child @ $2i+2$

Heap Sort: for a in $A \rightarrow$ add to heap $\rightarrow$ removeMin back to A

Bottom Up Heap Construction: get $l_{MAX} = \lfloor \log_2 n \rfloor \rightarrow$ get no. of leaves on bottom level (fill remaining with nulls) $\rightarrow$ add nodes to merge heaps

Maps - Hash Tables

General Summary: pre-hash → compress → handle collisions → rehash

Pre-hash to *Hash Code*:

- Component sum: e.g. sum of all char in string (collision risk)
- Polynomial accumulation: $p(z) = a_0 z^0 + a_1 z^1 + ... \ z \in \mathbb{Z}$
- Cyclic shift: replace z with bit-shifted version (e.g. $z \ll 5$)

Compress to *Hash Value*: below - $N =$ table size $\wedge N \in \mathbb{Z}^{\text{prime}}$

- Division: $h(k) = k \bmod N$
- MAD: $h(k) = ((ak+b) \bmod p) \bmod N$
  $p > N \wedge p \in \mathbb{Z}^{\text{prime}}, a \in [1, p-1], b \in [0, p-1]$

Collision Handling:

- 123 test $test12$

ADT Methods

Stack	`push(V)` `pop()` `top()` or `peek()`
Queue	`enqueue(V)` `dequeue()` `front()` or `peek()`
Priority Q	`insert(K, V)` `removeMin()` `min()`
Entry	`getKey()` `getValue()` `compareTo(Entry)`
Map	`get(K)` `put(K, V)` `remove(K)` `entrySet()` `keySet()` `values()`