NAME:

Textbook Material for Test 3 (Wednesday April 29, 2015)

Chapter 6: Normal Curves and Sampling Distributions	Section 6.1: Normal curves (except control charts) Section 6.2: Standard normal distribution Section 6.3: Areas under normal curves (except assessing normality) Section 6.4: Sampling distribution for $\overline{X}$ Section 6.5: Central Limit Theorem Section 6.6: Normal Approximation to binomial distribution
Chapter 7: Estimation	Section 7.1: Estimating μ when σ is known Section 7.2: Estimating μ when σ is unknown Section 7.3: Estimating *p* in the binomial distribution
Chapter 8: Hypothesis Testing	Section 8.1: Introduction to statistical tests Section 8.2: Testing the mean μ Section 8.3: Testing a proportion *p*

---

1) If X has a normal distribution with mean 15 and standard deviation 3, determine the appropriate parameters of the distribution of $\bar{x}$ for sample sizes

a. n = 4 3/2
b. n = 16 3/4
c. n = 100 3/10

2) Given that X is a normal variable with mean 110 and standard deviation 12, find:

a. $P(X \leq 120)$ = 0.7967
b. $P(X \geq 80)$ = 0.9938
c. $P(108 \leq X \leq 117)$ = 0.2865

3) Find *z* such that 5% of the area under the standard normal curve lies to the right of *z*.

1.645

4) Find *z* such that 99% of the area under the standard normal curve lies between -*z* and *z*.

-2.58, +2.58

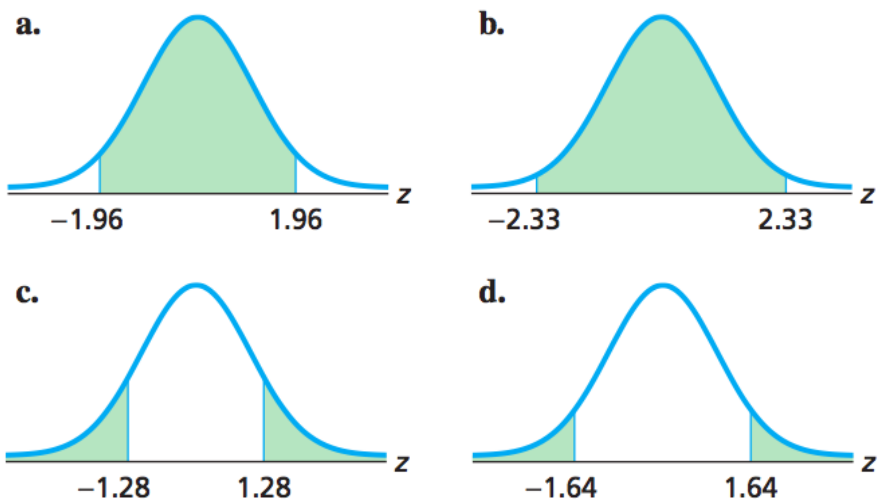
5) Find *z* such that 13.35% of the area under the standard normal curve lies to the left of *z*.

-1.11

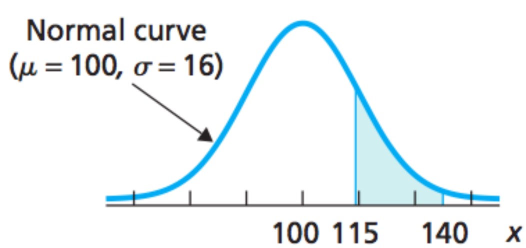
6) The Customer Service Center in a large department store has determined that the amount of time spent with a customer about a complaint is normally distributed with a mean of 9.3 minutes and a standard deviation of 2.5 minutes. What is the probability that for a randomly chosen customer with a complaint, the amount of time spent resolving the complaint will be:

a. less than 10 minutes = 0.6103
b. longer than 5 minutes = 0.9573
c. between 8 and 15 minutes = 0.6872

7) Use the table of the standard normal curve to obtain the following shaded areas:



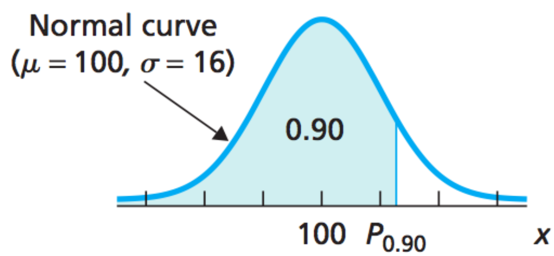
8) What is the area of the shaded region, between 115 and 140, in the following normal curve:



9) The life of of an electronic device is normally distributed with mean 5000 hours and standard deviation 450 hours.

a. Find the probability that the device will wear out in 5000 hours or less.
0.50
b. The manufacturer wants to place a guarantee on the devices so that no more than 5% fail during the guarantee period. How many hours should the guarantee cover?
4260 hours

10) What is the x-score corresponding to P90 as shown in the following figure:



11) Attendance at large exhibition shows in Denver averages about 8000 people per day, with standard deviation of about 500. Assume that the daily attendance figures follow a normal distribution. What is the probability that the daily attendance:

a. will be fewer than 7200 people?
   0.0548
b. will be more than 8900 people?
   0.0359
c. will be between 7200 and 8900 people?
   0.9093

12) Consider a sample from a population having mean 128 and standard deviation 16. Compute the approximate probability that the **sample mean** $\overline{X}$ lie between 124 and 132 when the sample size is:

a. n = 9 0.5468
b. n = 25 0.7888
c. n = 100 0.9876

13) The blood cholesterol levels of a population of workers have mean 202 and standard deviation 14. If a sample of 36 workers is selected, approximate the probability that the sample mean $\overline{X}$ of their blood cholesterol levels will lie between 198 and 206.
0.913

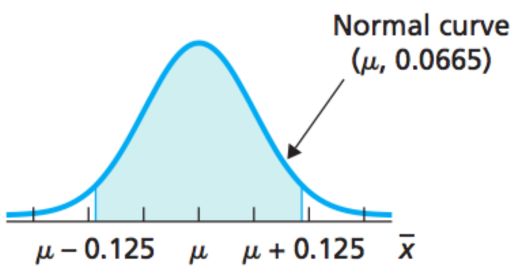
14) Consider a sample size of 16 from a population having mean 100 and standard deviation σ. Approximate the probability that the sample mean $\overline{X}$ lies between 96 and 104 when:

a. σ = 16 0.6826
b. σ = 8 0.9544
c. σ = 4 1

15) The mean living space for single-family detached homes is 1742 sq. ft. Assume a standard deviation of 568 sq. ft.

a. For samples of 25 single-family detached homes, determine the mean and standard deviation of $\bar{x}$.
μ = 1742, σ = 113.6
b. Repeat part (a) for a sample size of 500.
μ = 1742, σ = 25.4

16) Let μ denote the population mean birth weight of all male babies. Assume that the weights are normally distributed with standard deviation σ = 1.33 lb. For samples of size 400, the sample mean birth weight, $\overline{X}$ is approximately normally distributed (see figure below)



What is the percentage of male babies that have mean birth weights within 0.125 lb of the population mean μ?

93.98%

17) A variable of a population has mean μ and standard deviation σ. For a large sample size *n*, fill in the blanks. Justify your answers.

a. Approximately _____% of all possible samples have means within $\sigma/\sqrt{n}$ of the population mean μ.
b. Approximately _____% of all possible samples have means within $2\sigma/\sqrt{n}$ of the population mean μ.
c. Approximately _____% of all possible samples have means within $3\sigma/\sqrt{n}$ of the population mean μ.

18) Suppose X has a normal distribution with σ = 6. A random sample of size 16 has sample mean 50. Find a 90% confidence interval for μ.

47.53 to 52.47

19) Suppose X has a mound-shaped distribution with σ = 9. A random sample of size 36 has sample mean 20. Find a 95% confidence interval for μ.

17.06 to 22.94

20) A small group of hummingbirds has been under study in a ecological reserve. The average weight for these birds is $\bar{x} = 3.15$ grams. Based on previous studies, we can assume that the weights have a normal distribution with σ = 0.33 grams.

a. Find the 80% confidence interval for the average weights of studied hummingbirds.
   3.04gm to 3.26gm
b. What's the margin of error?
   0.11 gm
c. Find the sample size necessary for an 80% confidence level with margin of error 0.08.
   n = 28

21) A random sample of size 36 is drawn from a given distribution. The sample mean is 100.

a. Suppose the distribution has σ = 30. Compute a 90% confidence interval for μ. What is the value of the margin of error?

   Interval: 91.77 to 108.23; margin: 8.23
b. Suppose the distribution has σ = 20. Compute a 90% confidence interval for μ. What is the value of the margin of error?

   Interval: 94.52 to 105.48; margin: 5.48

22) A random sample of size 36 is drawn from a population with σ = 12 and sample mean 30.

a. Compute a 95% confidence interval for μ based on a sample size of 49. What is the value of the margin of error?

   Interval: 26.64 to 33.36; margin: 3.36
b. Compute a 95% confidence interval for μ based on a sample size of 100. What is the value of the margin of error?

   Interval: 27.65 to 32.35; margin: 2.35

23) Suppose X has a mound-shaped distribution. A random sample of size 16 has sample mean 10 and sample standard deviation 2. Find a 90% confidence interval for μ using a Student's *t* distribution.

9.12 to 10.88

24) A random sample of size 81 has sample mean 20 and sample standard deviation 3. Find a 95% confidence interval for μ using a Student's *t* distribution.

19.34 to 20.66

25) For a tree-ring dating study, we have a sample of 8 trees with the following values:
1189, 1271, 1267, 1268, 1316, 1275, 1317, 1275

a. Verify that the sample mean year is $\bar{x} \approx 1272$, with sample standard deviation s ≈ 37.
b. Find a 90% confidence interval for the mean of all three-ring dates from this archaeological site. *Hint:* use a Student's *t* distribution.
   1249 to 1295

26) Adult wild mountain lions captured and released for the first time in the San Andres Mountains (New Mexico) gave the following weights (in pounds):

68	104	128	122	60	64

a. Verify that the sample mean is $\bar{x} = 91$, with sample standard deviation s ≈ 30.7.
b. Find a 75% confidence interval for the population average weight μ of all adult mountain lions in the San Andres Mountains. *Hint:* use a Student's *t* distribution.
   74.7 lb to 107.3 lb

27) Consider n = 100 binomial trials with k = 30 successes.

a. Determine the parameters of a normal distribution to approximate the p-hat distribution
   $np = 30, \ \sqrt{npq} = 4.5825$
b. Find a 90% confidence interval for the population proportion of successes *p*.
   0.225 to 375

28) Consider n = 200 binomial trials with k = 80 successes.

c. Determine the parameters of a normal distribution to approximate the p-hat distribution
   $np = 80, \ \sqrt{npq} = 6.928$
d. Find a 95% confidence interval for the population proportion of successes *p*.
   0.332 to 0.468

29) A random sample of 5792 physicians showed that 3139 provide some charity care.

a. Let *p* represent the proportion of all physicians who provide some charity care. Find a point estimate for *p*.
   p-hat = 0.5420
b. FInd a 99% confidence interval for *p*.
   0.53 to 0.56

30) Case studies showed that out of 10,351 convicts who escaped from U.S. prisons, only 7867 were recaptured.

a. Let *p* represent the proportion of all escaped convicts who will eventually be recaptured. Find a point estimate for *p*.
   p-hat = 0.76

b. Find a 99% confidence interval for *p*.
0.75 to 0.77

31) In a survey of 1000 corporations, 250 said that, given a choice between a job candidate who smokes and an equally qualified non-smoker, the nonsmoker would get the job.

a. Let *p* represent the proportion of all corporations preferring a nonsmoking candidate. Find a point estimate for *p*.
p-hat = 0.25
b. Find a 0.95 confidence interval for *p*.
0.22 to 0.28

32) A random sample of size 20 from a normal distribution has $\sigma = 4$ and $\overline{X} = 8$.

a. Compute that sample test statistic *z* under the null hypothesis H_0: $\mu = 7$.
z = 1.12
b. For $H_1 \neq 7$, estimate the P-value of the test statistic.
0.2628
c. For a level of significance of 0.05 and the hypotheses of parts (a) and (b) do you reject or fail to reject the null hypothesis?
Fail to reject

33) The body weight of a healthy 3-month-old colt should be about $\mu = 60$ kg.

a. If you want to set up a statistical test to challenge the claim that $\mu = 60$ kg, what would you use for the null hypothesis H_0?
H_0: $\mu = 60$ kg
b. Suppose you want to test the claim that the average weight of a wild colt is less than 60 kg. What would you use for the alternative hypothesis H_1?
H_1: $\mu < 60$ kg
c. Suppose you want to test the claim that the average weight of a wild colt is different from 60 kg. What would you use for the alternative hypothesis H_1?
H_1: $\mu \neq 60$ kg

34) Over the past 8 weeks, a veterinarian took the following glucose readings from a horse (in mg/100ml): 93 88 82 105 99 110 84 89
The sample mean is $\bar{x} \approx 93.8$. We may assume that the glucose level has a normal distribution, and we know from past experience that $\sigma = 12.5$. The mean glucose level for horses should be $\mu = 85$ mg/100ml. Do these data indicate that the analyzed horse has an overall average glucose level higher than 85? Use $\alpha = 0.05$.
H_0: $\mu = 85$ mg/100ml
H_1: $\mu > 85$ mg/100ml

Corresponding $z \approx 1.99$

P-value of 0.0233 (area to the right of 1.99)

Since P-value ≤ 0.05, we reject H_0.

It seems that the horse average glucose level is higher than average.