NAME:

1) Consider each of the following distributions. Determine if it is a valid probability distribution or not, and explain your answer.

a) Distribution 1

| Х | P(x) |
|---|------|
| 0 | 0.25 |
| 1 | 0.60 |
| 2 | 0.15 |

b) Distribution 2

| у | P(y) |
|---|------|
| 0 | 0.25 |
| 1 | 0.60 |
| 2 | 0.20 |

2) In the following problems, p(i) stands for P(X = i). Find the expected value E(X) when:

a.
$$p(1) = 0.1$$
, $p(2) = 0.3$, $p(3) = 0.3$, $p(4) = 0.2$, $p(5) = 0.1$

$$(2) = 0.3$$

$$p(3) = 0.3$$

$$p(4) = 0.2,$$

$$p(5) = 0.1$$

b.
$$p(1) = 0.2$$
, $p(2) = 0$, $p(3) = 0.6$, $p(4) = 0$, $p(5) = 0.2$

$$p(2) = 0$$
,

$$p(3) = 0.6$$
,

$$p(4)=0,$$

$$p(5) = 0.2$$

c.
$$p(3) = 1$$

3) Consider the following probability distributions. Compute the standard deviation when:

a.
$$p(1) = 1/3$$
, $p(2) = 1/3$, $p(3) = 1/3$

$$p(2) = 1/3$$

$$p(3) = 1/3$$

b.
$$p(1) = 1/2$$
, $p(2) = 1/3$, $p(3) = 1/6$

$$p(2) = 1/3,$$

$$p(3) = 1/6$$

| 4) | Suppos | se that | а | random | variable | X | can | take | on | any | of | the | values | 1, | 2, | and | 3. | Find | the |
|----|----------|---------|----|-----------|-----------|-----|-------|--------|-----|-------|----|-----|--------|-----|-----|-----|----|------|-----|
| ex | pected v | value E | (X |) and the | e varianc | e \ | /ar(X | () if: | p(1 |) = 0 | .3 | and | p(2) | = 0 |).5 | | | | |

5) If E(X) = 5, and E(Y) = 12, find:

a.
$$E(3X + 4Y) =$$

b.
$$E(4 + Y) =$$

c.
$$E(2 + 5Y + X) =$$

6) If the two teams in a World Series have the same chance of winning each game, independent of the results of the previously played games, then the probabilities that the series will end in 4, 5, 6 or 7 games are:

P(series will end in 4 games) = 1/8

P(series will end in 5 games) = 1/4

P(series will end in 6 games) = 5/16

P(series will end in 7 games) = 5/16

What is the expected number of games played in such a series?

- **7)** If it rains tomorrow, you will earn \$200 by doing some tutoring; if it is dry, you will earn \$300 by doing construction work. If the probability of rain is 1/4, what is the expect amount that you will earn tomorrow?
- **8)** An investment has a 0.4 probability of making a \$30,000 profit, and a 0.6 probability of losing \$15,000. Does this investment have a positive expected gain?

9) Norb and Gary are playing a golf tournament. Their scores are random variables with the following means and standard deviations:

Norb,
$$X_1$$
: $\mu_1 = 115$, $\sigma_1 = 12$
Gary, X_2 : $\mu_2 = 100$, $\sigma_2 = 8$

Assume the scores of Norb and Gary to be independent of each other.

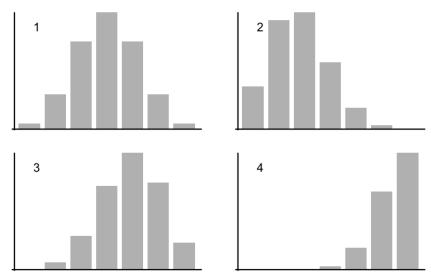
- a. The difference between their scores is $W = X_1 X_2$. Compute the mean, variance and standard deviation for W.
- b. The average of their scores is $A = 0.5X_1 + 0.5X_2$. Compute the mean, variance and standard deviation for A.
- c. The tournament rules have a special handicap system for each player. For Norb, the handicap formula is $L = 0.8X_1$ 2. Compute the mean, variance, and standard deviation for the random variable L.
- d. For Gary, the handicap formula is $H = 0.95X_2 5$. Compute the mean, variance, and standard deviation for the random variable H.

Binomial Probs

- **10)** A student takes a true–false test consisting of 15 questions. Assume that the student guesses at each question. Find the probability that:
 - a. the student gets at least 1 question correct.
 - b. the student gets a 60% or better on the exam.

- **11)** In a state lottery, a player must choose 8 of the numbers from 1 to 40. Assuming that the choice of a player is equally likely to be any of the $_{40}C_8$ combinations, what is the probability that a player has:
 - a. All 8 of the selected numbers?
 - b. Seven of the selected numbers?
 - c. At least six of the selected numbers?
- **12)** An instructor gives her class a set of 10 problems and tells the class that the final exam will consist of a random selection of 5 of the problems. If a student has figured out how to do 7 of the problems, what is the probability that he or she will correctly answer:
 - a. All 5 problems?
 - b. At least 4 of the problems?
- **13)** The quality-control inspector of a production will reject a batch syringes if two or more defective syringes are found in a random sample of eight syringes taken from the batch. Suppose the batch contains 1% defective syringes.
 - a. Identify the values of *n* (number of trials) and *p* (probability of "success")
 - b. Make a plot of the distribution showing the probabilities of $\mathbf{k} = 0, 1, 2, 3, 4, 5, 6, 7$, and 8 defective syringes in a random sample of 8.

- c. What is the expected number of defective syringes the inspector will find?
- d. What is the probability that the batch will be accepted?
- **14)** A recent audit of Los Angeles 911 calls showed that 85% were not emergencies. Suppose the 911 operators in LA have just received four calls.
 - a. What is the probability that all four calls are, in fact, emergencies?
 - b. What is the probability that three or more calls are not emergencies?
 - c. How many calls n would need to be answered to be 96% (or more) sure that at least one call is, in fact, an emergency? *Hint:* you need to find the value n such that $P(k \ge 1) = 0.96$
- **15)** The following figure shows four binomial distributions with n = 6 trials. Match the given probability of success with the corresponding graph.



- a) p = 0.30 goes with graph _____
- b) p = 0.50 goes with graph _____
- c) *p* = 0.65 goes with graph _____
- d) p = 0.90 goes with graph ____