Homework 3; Monday, March 2

NAME:

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1) A box contains three balls—one red, one blue, and one yellow. Consider an experiment that consists of withdrawing a ball from the box, replacing it, and withdrawing a second ball.

a. What is the sample space of this experiment?

b. What is the event that the first ball drawn is yellow?

c. What is the event that the same ball is drawn twice?

2) Repeat the previous problem but now consider that the second ball is drawn without replacement of the first ball.

d. What is the sample space of this experiment?

e. What is the event that the first ball drawn is yellow?

f. What is the event that the same ball is drawn twice?

3) Audrey and her boyfriend Charles must choose which colleges they will attend in the coming fall. Audrey was accepted at UC Davis (UCD), UC San Diego (UCSD), UC Irvine (UCI), and San Francisco State University (SFSU). Charles was accepted at UCI and SFSU. Let the outcome of the experiment consist of the colleges that Audrey and Charles choose to attend.

a. List all the outcomes in sample space S.

b. List all the outcomes in the event that Audrey and Charles attend the same school.

c. List all the outcomes in the event that Audrey and Charles attend different schools.

d. List all the outcomes in the event that Audrey and Charles attend schools in the same state.

4) A cafeteria offers a three-course meal. One chooses a main course, a starch, and a dessert. The possible choices are as follows:

Meal	**Choices**
Main course	Chicken or roast beef
Starch course	Pasta or rice or potatoes
Dessert	Ice cream or gelatin or apple pie

An individual is to choose one course from each category.

a. List all the outcomes in the sample space.

b. Let A be the event that ice cream is chosen. List all the outcomes in A.

c. Let B be the event that chicken is chosen. List all the outcomes in B.

d. List all the outcomes in the event $A \cap B$.

e. Let C be the event that rice is chosen. List all the outcomes in C.

f. List all the outcomes in the event $A \cap B \cap C$

5) The following pairs of events E and F relate to the same experiment. Tell in each case whether E and F are disjoint (i.e. mutually exclusive) events.

a. A die is rolled. Event E is that it lands on an even number, and F is the event that it lands on an odd number.

b. A die is rolled. Event E is that it lands on 3, and F is the event that it lands on an even number.

c. A person is chosen. Event E is that this person was born in the United States, and F is the event that this person is a U.S. citizen.

d. A man is chosen. Event E is that he is over 30 years of age, and F is the event that he has been married for over 30 years.

e. A woman waiting in line to register her car at the department of motor vehicles is chosen. Event E is that the car is made in the United States, and F is the event that it is made in a foreign country.

6) A bowl contains 12 poker chips—3 red, 4 white, and 5 blue. If one of these poker chips is selected at random from the bowl, what is the probability that its color is

a. red?

b. red or white?

c. not white?

7) A student council is made up of four women and six men. One of the women is president of the council. A member of the council is selected at random to report to the dean.

a. What is the probability that a woman is selected?

b. What is the probability that a man is selected?

c. What is the probability that the president of the student council is selected?

8) The dean of a College found that 12% of the female students are majoring in computer science. If 64% of the students at the college are women, what is the probability that a student chosen at random will be a woman majoring in computer science?

9) The following data comes from a Center for Health Statistics. The table shows the number of female hospitalizations for cardiovascular disease, by age group, during one year in Florida.

Age group (yr)	*Number*
0–19	810
20–39	5,029
40–49	10,977
50–59	20,983
60–69	36,884
70–79	65,017
80 and over	69,167

One of these case records is selected at random. Find the probability that the woman was:

a. in her 50s.

b. less than 50 years old.

c. between 40 and 69 years old, inclusive.

d. 70 years old or older.

10) The following frequency distribution reports the highest education level achieved by *Standard and Poor's* top 500 CEOs.

Level	Frequency
No college	14
B.S. / B.A.	164
M.B.A.	191
Doctorate	50

Find the probability that a randomly selected CEO from Standard and Poor's top 500 achieved the educational level of:

a. B.S. / B.A.

b. either M.B.A. or Doctorate

c. at least some college

11) An ordinary deck of playing cards has 52 cards. There are four suits—spades, hearts, diamonds, and clubs—with 13 cards in each suit. Spades and clubs are black; hearts and diamonds are red. If one of these cards is selected at random, what is the probability that it is:

a. spade?

b. not a club?

c. red?

d. a face card (jack, queen, or king)?

12) A hospital administration did a survey of patients regarding satisfaction with care and type of surgery. The results follow:

	Heart	Hip	Knee	*Total*
Not satisfied	7	12	2	*21*
Neutral	15	38	10	*63*
Satisfied	32	16	25	*73*
Very satisfied	4	22	23	*49*
Total	*58*	*88*	*60*	*206*

a) Find the probability that a patient is satisfied:

i) $\frac{32}{206}$ ii) 73 iii) $\frac{122}{206}$ iv) 122 v) $\frac{73}{206}$ _____

b) Find the probability that a patient is very satisfied and had knee surgery:

i) $\frac{109}{206}$ ii) $\frac{11}{206}$ iii) $\frac{23}{206}$ iv) $\frac{23}{60}$ v) $\frac{23}{60}+\frac{23}{49}$ _____

c) Find the probability that a patient is neutral and had hip surgery:

i) $\frac{38}{88}$ ii) $\frac{88}{206}$ iii) $\frac{38}{206}$ iv) $\frac{38}{63}$ v) $\frac{63}{88}$ _____

d) Find the probability that a patient had knee surgery:

i) 60 ii) 206 iii) $\frac{146}{206}$ iv) $\frac{60}{206}$ v) $\frac{2}{206}$ _____

e) Find the probability that a patient is satisfied and had heart surgery:

i) $\frac{73}{206}$ ii) $\frac{58}{73}$ iii) $\frac{32}{58}$ iv) $\frac{32}{206}$ v) $\frac{32}{73}$ _____

f) Find the probability that a patient is not satisfied and had heart surgery:

i) $\frac{7}{58}$ ii) $\frac{7}{206}$ iii) $\frac{7}{21}$ iv) $\frac{21}{206}$ v) $\frac{58}{206}$ _____

13) Explain what is wrong with the following argument: "When two balanced dice are rolled, the sum of the dice can be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12, giving 11 possibilities. Therefore the probability is 1/11 that the sum is 12."

14) Use the following data to compute the following 14 probabilities (listed below the table)

	Democrat	Republican	Independent	*Total*
females	250	300	50	*600*
males	250	100	50	*400*
Total	*500*	*400*	*100*	*1000*

1. P(Democrat)
2. P(Republican)
3. P(Independent)
4. P(Republican and male)
5. P(Independent and male)
6. P(Independent and female)
7. P(female and Republican)
8. P(Republican | male)
9. P(Republican | female)
10. P(male | Democrat)
11. P(female | Democrat)
12. P(female | Republican)
13. P(Democrat | not Independent)
14. P(Republican | not Independent)

15) Two fair dice are tossed:

a. What is the probability of a sum of six?

b. What is the probability of a sum of five?

c. What is the probability of doubles (e.g. 1-1, 2-2, 3-3, etc)?

d. What is the probability of a sum of six or doubles?

e. What is the probability of a sum of six and doubles?

f. What is the probability of a sum of five or doubles?

g. What is the probability of a sum of five and doubles?

16) At a certain university in the United States, 62% of the students are at least bilingual—speaking English and at least one other language. Of these students, 80% speak Spanish and, of the 80% who speak Spanish, 10% also speak French.
Determine the probability that a randomly selected student at this university:

a. does not speak Spanish.

b. speaks Spanish and French.

17) What is wrong with the following argument? “If two coins are tossed, then there are three possible outcomes: (1) 2 heads, (2) 1 head and 1 tail, (3) 2 tails. Hence the probability of two heads is 1/3”

18) Suppose that 25%of a forest consists of trees of species A, 40% of species B, and 35% of species C.

a. What is the probability that a tree selected at random will be of species A?

b. What is the probability that the tree selected will not be of species A?

c. If it is known that the tree is not of species A, what is the probability that it will be of species B?

19) A club has 90 members: 50 are lawyers and 50 are liars. Everyone is either a lawyer or a liar.

a. What is the probability that a randomly selected member is both a lawyer and a liar?

b. What is the probability that a randomly selected member is both a lawyer but not a liar?

c. What is the probability that a lawyer is not a liar?

20) Brain Teasers

Assume A and B are events such that: $0 < P(A) < 1$, and $0 < P(B) < 1$. Select the correct answer:

1. $P(A) / 2 = 0.6$	always true	sometimes true	never true
2. $P(A \text{ and } A^c) = 0$	always true	sometimes true	never true
3. $P(A \text{ or } A^c) = 0$	always true	sometimes true	never true
4. $P(A \text{ and } B) \leq P(A)$	always true	sometimes true	never true
5. $P(A \mid B) + P(A^c \mid B) = 1$	always true	sometimes true	never true
6. $P(A \mid B^c) + P(A^c \mid B^c) = 1$	always true	sometimes true	never true
7. $P(A \mid A^c) = 1$	always true	sometimes true	never true
8. $P(B \mid A) = P(B)$	always true	sometimes true	never true
9. $P(A \text{ or } B) = P(A) + P(B)$	always true	sometimes true	never true
10. $P(A) = P(B)$	always true	sometimes true	never true
11. $P(A \mid B) - P(A) = 0$	always true	sometimes true	never true
12. $P(A \mid B)\, P(B) = P(B \mid A)\, P(A)$	always true	sometimes true	never true
13. $P(A^c \text{ or } B) = P(A^c \text{ and } B^c)$	always true	sometimes true	never true
14. $P(A \text{ and } B) / P(B) = P(A)$	always true	sometimes true	never true