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Assignment #6

Ques #1

① $g_1(x) = (3+x-2x^2)^{1/4}$

$\Rightarrow x = (3+x-2x^2)^{1/4}$ $[g(x) = x]$

$\Rightarrow x^4 = 3+x-2x^2$

$\Rightarrow x^4+2x^2-x-3=0$

$\Rightarrow f(x) = x^4+2x^2-x-3$ $[\because f(x)=0]$

② $g_2(x) = \left(\frac{x+3-x^4}{2}\right)^{1/2}$

$x = \left(\frac{x+3-x^4}{2}\right)^{1/2}$

$\Rightarrow 2x^2 = x+3-x^4$

$\Rightarrow x^4+2x^2-x-3=0$

$\therefore f(x) = x^4+2x^2-x-3$

③ $g_3(x) = \left(\frac{x+3}{x^2+2}\right)^{1/2}$

$\Rightarrow x = \left(\frac{x+3}{x^2+2}\right)^{1/2}$

$\Rightarrow x^2 = \frac{x+3}{x^2+2}$

$\Rightarrow x^4+2x^2 = x+3$

$\Rightarrow x^4+2x^2-x-3=0$

$f(x) = x^4+2x^2-x-3=0$

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④. $g_4(x) = \dfrac{3x^4+2x^2+3}{4x^3+4x+-1.}$

$\Rightarrow x = \dfrac{3x^4+2x^2+3}{4x^3+4x-1.}$

$\Rightarrow 4x^4+4x^2-1 = 3x^4+2x^2+3$

$\Rightarrow 4x^4+4x^2-1-3x^4-2x^2+3=0. \Rightarrow x^4+2x^2-x-3=0.$

$\Rightarrow f(x) = x^4-2x^2-x-3.$

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Ques #2

① $g_1(x) = (3+x-2x^2)^{1/4}$,

Here, $x_0 = 1$ & $x_{k+1} = x_{3+1}$
$= x_4$
$= g(x_3)$

$x_0 = 0.\ ; \ g_1(0) = 1.31607$

$x_1 = 1.31607;\ g_2(1.31607) = 0.96074.$

$x_2 = 0.96074;\ g_3(0.96074)$

$x_0 = 1.$

$g(1) = x_1 = 1.18921.$

$g(1.18921) = x_2 = 1.08006$

$g(1.08006) = x_3 = 1.14967$

$g(1.14967) = x_4 = 1.10782$

② $g_2(x) = \left(\frac{x+3-x^4}{2}\right)^{1/2}$

$x_1 = g(1) = 1.22474$

$x_2 = g(1.22474) = 0.99367.$

$x_3 = g(0.99367) = 1.22857.$

$x_4 = g(1.22857) = 0.98750$

③ $g_3(x) = \left(\frac{x+3}{x^2+2}\right)^{1/2}$

$x_1 = g_1(1) = 1.15470.$

$x_2 = g_2(1.15470) = 1.11643.$

$x_3 = g_3(1.11643) = 1.12605$

$x_4 = g_4(1.12605) = 1.12364$

④ $g_4(x) = \frac{3x^4+2x^2+3}{4x^3+4x-1}.$

$x_1 = g(1) = 1.14286$

$x_2 = g(1.14286) = 1.12448.$

$x_3 = g(1.12448) = 1.12412$

$x_4 = g(1.12412) = 1.12412.$

$\boxed{\text{Ques \#3}}$

We know, error bound $= |x_4 - x_3|$

for ① $g_1(x) = |1.10782 - 1.14967| = |-0.04185| = 0.04185.$

② $g_2(x) = |0.98750 - 1.22857| = |-0.24107| = 0.24107.$

③ $g_3(x) = |1.12364 - 1.12605| = |-2.41\times10^{-3}| = 2.41\times10^{-3}.$

④ $g_4(x) = |1.12412 - 1.12412| = 0$

The 4th $g_1(x) = \frac{3x^4+2x^2+3}{4x^3+4x-1}$ gives the best approximation after four iterations solution. (Ans).

$\boxed{\text{Ques \#4}}$

$f(x) = x^3+4x^2-x-4.$

① $f(x)=0.$

$x^3+4x^2-x-4=0.$

$\Rightarrow x^2(x+4)-1(x+4)=0.$

$\Rightarrow (x+4)(x^2-1)=0.$

$x=-4$

$x=\pm 1.$

Roots of the function are, +1, -1, -4. (Ans)

(2)

$f(x) = x^3+4x^2-x-4$

if $f(x)=0$

(1) $x^3+4x^2-x-4=0.$

$\Rightarrow x^3 = x+4-4x^2$

$\Rightarrow x = (x+4-4x^2)^{1/3} = g_1(x)$

(2) $\cancel{x^3+4x^2-x-4=0.}$

$\Rightarrow x^3+4x^2-x=4.$

$\Rightarrow x(x^2+4x-1)=4.$

$\Rightarrow x = \dfrac{4}{x^2+4x-1}$

$\Rightarrow \cancel{x^3+4x^2=x+4}$

$\Rightarrow x^2(x+4)=(x+4)$

$\Rightarrow x^2 = \dfrac{(x+4)}{(x+4)}.$

(2) $x^3+4x^2-x-4=0.$

$4x^2 = x+4-x^3$

$\Rightarrow x^2 = \dfrac{x+4-x^3}{4}$

$x = \left(\dfrac{x+4-x^3}{4}\right)^{1/2} = g_2(x)$

(3). $x^3+4x^2-x-4=0.$

$x^3+4x^2-4 = x. = g_3(x)$

(Ans).

① $g_1(x) = (x+4-4x^2)^{1/3}$

$\lambda = |g_1'(x)| = \left|\frac{1}{3}(x+4-4x^2)^{\frac{1}{3}-1}(1-8x)\right|$

$= \left|\frac{1}{3}(1-8x)(x+4-4x^2)^{-\frac{2}{3}}\right|$

$= \begin{cases} \text{for } x_* = 1, \lambda = |-2.33| = 2.33 > 1, \text{divergence} \\ \text{for } x_* = -1, \lambda = |3| = 3 > 1 \Rightarrow \text{divergence} \\ \text{for } x_* = -4 \ \lambda = |0.6875| = 0.6875 > 1. \end{cases}$

Lineari convergence

② $g_2(x) = \left(\frac{x+4-x^3}{4}\right)^{1/2} = \frac{\sqrt{x+4-x^3}}{2}$

$\lambda = |g_2'(x)| = \left|\frac{1}{2}(x+4-x^3)^{-\frac{1}{2}}(-3x^2+1)\right|.$

$= \left|\frac{-3x^2+1}{2\sqrt{x+4-x^3}} \times \frac{1}{2}\right|$

$= \frac{-3x^2+1}{4\sqrt{x+4-x^3}}.$

$= \begin{cases} \text{for } x_* = 1., \lambda = |-0.25| = 0.25 < 1 \ \text{Lineari conv} \\ \text{root } x_* = -1, \lambda = |-0.25| = 0.25 < 1. \quad " \\ \text{root } x_* = -4 \ \lambda = |-1.46875| \\ = 1.46875 \rightarrow \text{divergence} \end{cases}$

③ $g_3(x) = x^3+4x^2-4$.

$\lambda = |g_3'(x)| = 3x^2+8x$.

$$= \begin{cases} x_* = 1 \quad \lambda = 11 > 1 \longrightarrow \text{divergence. } \cancel{\text{convergence}} \\ x_* = -1, \ \lambda = |-5| = 5 > 1 \longrightarrow \text{"} \\ x_* = -4, \ \lambda = 16 > 1 \longrightarrow \text{"} \end{cases}$$