Solution: Graded Quiz #8

Q#1: A linear system is defined by the matrix equation $Ax = b$This system

- may have a solution if $A$is a square matrix.
- has a unique solution if and only if $A$is non-singular.
- has a unique solution if $\exists\, x \in \mathrm{R}^n$ for every $b \in \mathrm{R}^n$.
- ✓ All of the above. (Answer)

Q#2: The tranformation matrix $A$of the linear system described by $Ax = b$is of order $n \times n$This matrix $A$is transformed into a lower triangular form by the Gaussian elemination method. How many matrix elements of $A$are changed to zero?

- ✓ $\frac{1}{2}n\,(n-1)$ (Answer)
- $\frac{1}{2}n\,(n+1)$
- $\frac{1}{2}n.$
- $n+1$

Q#3: Which of the following statement(s) is(are) NOT true about the Gaussian elemination method?

- $\det\,(A)$does not change.
- The lower triangular and upper triangular form gives the same solution.
- $\det L = \det U$where $L$and U are the lower and upper triangular forms of A
- ✓ The row operation changes all matrix elements of the matrix A (Answer)

Q#4: Suppose you have a linear system where A is 10×10 square matrix. How many row operations are need to obtain a lower traingular matrix?

- ✓ 9. (Answer)
- ◯ 10.
- ◯ It canot be determined. Need more information.
- ◯ 11.

Q#5: A 8×8 square matrix, A, is changed to an upper triangular form by the row operation in the Gaussian elimination method. After the completion of the 3^{rd} row operation, how many matrix elements of A has been chnaged to zero by the row operations?

- ◯ 22.
- ◯ 12.
- ✓ 18. (Answer)
- ◯ 13.

Q#6: In the video lecture 15, Part-II, we showed an example. What we can say about the solution of the example?

- ◯ Back substitution method is used to solve the problem.
- ◯ There was a typo in the typed lecture slide which was corrected during the video lecture. That typo did not occur in the example part of the lecture.
- ◯ The problem could also been solved by transforming the matrix A into lower triangular form.
- ✓ All of the above. (Answer)

Q#7: In the LU-decomposition method, the matrix A is transformed into

- [x] upper triangular form. (Answer)
- [ ] lower triangular form.
- [ ] singular form.
- [ ] None of them above.

>>Q#8: The lower trangular matrix L is defined as

$$L = \left(F^{(1)}\right)^{-1} \left(F^{(2)}\right)^{-1} \cdots \left(F^{(n-1)}\right)^{-1},$$

where the matrix $F^{(k)}$ are constructed out of the row multipliers and 1's as shown in the lecture. If $n = 15$ how many matrix elements of $F^{(5)}$ will be non-zero?

- [ ] 15.
- [ ] 20.
- [x] 25. (Answer)
- [ ] 16.

Problem: The Augmented matrix of a linear system is given by

$$\mathrm{Aug}(A) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 2 & 12 & -2 & 4 \end{array}\right) \Rightarrow A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{pmatrix} \ \& \ b = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix}$$

Q#9: [1 Mark] Find the matrix $F^{(1)}$. Show calculations.

Q#10: [1 Mark] Find the matrix $F^{(2)}$. Show calculations.

Solution of Q#9 & 10 :

Note that the $F^{(k)}$ matrices are made out of the row multipliers, m_{ik}'s.

$$F^{(k)} = \begin{pmatrix} 1 & 0 & \cdots & & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & 1 & & \vdots \\ \vdots & & -m_{k+1,k} & \ddots & \\ \vdots & & \vdots & & \\ 0 & & -m_{n,k} & \cdots & 1 \end{pmatrix}$$

Q#9 Here : $n=3 \Rightarrow \cancel{k=2,3}$. & $k=1$.

$$\Rightarrow F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{pmatrix}$$

Here: $m_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{1} = 1$ (From the given matrix A)

and $m_{31} = \frac{a_{31}}{a_{11}} = \frac{2}{1} = 2$ (")

Therefore: $\boxed{F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}}$

Q#10 Here $n=3$ and $k=2$.

So, $$F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{pmatrix}$$

Here: $m_{32} = \frac{a_{32}}{a_{22}}$; but here a_{32} and a_{22} are element of $A^{(1)}$.

where $$A^{(1)} \equiv F^{(1)}A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 2 & 12 & -2 & 4 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -4 & 1 & 4 \\ 0 & 8 & -4 & 4 \end{array}\right)$$

Hence: $m_{32} = \frac{a_{32}}{a_{22}} = \frac{8}{-4} = -2$.

Therefore: $\boxed{F^{(2)} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}}$