Paper Source
Subject
Date: Time:

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An # 24.

(1) $f(x) = x^2 + x - 72$ with $x_* \in [5, 10]$.

$f(x) = x^2 \cancel{+8x-7}\ 9x - 8x - 72 = 0$

$\Rightarrow x(x+9) - 8(x+9) = 0$

$\Rightarrow (x+9)(x-8) = 0$

$x = 8.$

$x_0 = 8$; $f(x_0) = 0.$

$x_1 = 9$; $f(x_1) = 18.$

We know,

$$x_{k+2} = x_{k+1} - \frac{f(x_{k+1})(x_{k+1} - x_k)}{f(x_{k+1}) - f(x_k)}$$

$\boxed{\text{For } k=0}$,

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 9 - \frac{18(\cancel{8}-9-8)}{\cancel{18-0}} =$$

$\boxed{k=1}$

For $x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} =$

$\boxed{k=2}$

For $x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)}$.

(2). If $x_0 = 6$, $f(x_0) = -30$. $x_2 = 8.14286$, $f(x_2) = 2.45$.
$x_1 = 7$, $f(x_1) = -16$. ; $x_3 = 7.99$, $f(x_3) = -0.1699$.

From (1) we get,

$\boxed{K=0}$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 7 - \frac{-16(7-6)}{-16+30} = 8.14286.$$

$\boxed{K=1}$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 8.14286 - \frac{2.45 \times (8.14-7)}{2.45+16} = 7.98862$$

$\boxed{K=2}$

$$x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)} = 7.99 - \frac{-0.1699(7.99 - 8.14286)}{-0.1699 - 2.45}$$

$$= 7.99991.$$

(Ans).

Ans #51

①.

$f(x) = 2x^3+x^2-13x+6.$

As, $f(2)=0.$

if $f(x)=0.$

$2x^3+x^2-13x+6=0$

$\Rightarrow 2x^3-4x^2+5x^2-10x-3x+6=0$

$\Rightarrow 2x^2(x-2)+5x(x-2)-3(x-2)=0.$

$\Rightarrow (x-2)(2x^2+5x-3)=0.$

$\Rightarrow (x-2)(2x^2+\cancel{2x+3x}\overset{6x-x}{}-3)=0$

$\Rightarrow (x-2)\ 2x(x+3)-1(x+3)=0.$

$\Rightarrow (x-2)(x+3)(2x-1)=0$

$x=2,-3,\frac{1}{2} \Rightarrow$ roots.

(Ans).

Paper Source
Subject.......
Date:....... Time:.......

(2).

$$f(x) = 2x^3 + x^2 - 13x + 6.$$

if $f(x) = 0$.

$$2x^3 + x^2 - 13x + 6 = 0.$$

$$\Rightarrow 13x = 2x^3 + x^2 + 6.$$

$$\Rightarrow x = \frac{2x^3 + x^2 + 6}{13.} \text{ —— (I).}$$

$$x = g_1(x)$$

Again, $2x^3 + x^2 - 13x + 6 = 0.$

$$x^2 = 13x - 2x^3 - 6.$$

$$x = \sqrt{13x - 2x^3 - 6} \text{ —— (II).}$$

$$= g_2(x)$$

(I) & (II) are the 2 different fixed points.

(3).

For, $g_1(x) = \frac{2x^3 + x^2 + 6}{13.}$

$$g_1'(x) = \frac{1}{13}(6x^2 + 2x)$$

For, $|g_1'(x)| = K.$ = ~~4.62~~ 3.69 for $x = -3$. ; divergence

2.15 for $x = 2$. ; divergence

0.192 for $x = \frac{1}{2}$; converge linearly

(Ans).

$g_2(x) = \sqrt{13x - 2x^3 - 16}$

$g_2'(x) = \frac{1}{2}(13x - 2x^3 - 16)^{-\frac{1}{2}} \times (13 - 6x^2)$

For $g_2'(x) = \cancel{\text{error for } x = 3}$. $2.75 > 1$ for $x = 2 \Rightarrow$ Divergence

$= \cancel{\text{error for } x = \frac{1}{2}}$ $11.5 > 1$ for $x_k = \frac{1}{2} \Rightarrow$ "

$= \cancel{\text{error for } x = 2}$. $6.83 > 1$ for $x_k = -3 \Rightarrow$ "

~~(Ans)~~ (Ans)

Ans no. 62.

Here $x_1 + x_2 - 3x_3 = -9$. —— (i)

$2x_1 + 4x_2 - x_3 = -5$. —— (ii)

$4x_1 + x_2 + 2x_3 = 9$. —— (iii)

(1) Augmented matrix = Aug(A) = $\left(\begin{array}{ccc|c} 1 & 1 & -3 & -9 \\ 2 & 4 & -1 & -5 \\ 4 & 1 & 2 & -9 \end{array}\right)$.

Matrix A = $\begin{pmatrix} 1 & 1 & -3 \\ 2 & 4 & -1 \\ 4 & 1 & 2 \end{pmatrix}$

(2) $A = \left(\begin{array}{ccc|c} 1 & 1 & -3 & -9 \\ 2 & 4 & -1 & -5 \\ 4 & 1 & 2 & 9 \end{array}\right)$

$A_1 = \left(\begin{array}{ccc|c} 1 & 1 & -3 & -9 \\ 0 & 2 & -7 & -23 \\ 0 & -3 & 14 & 45 \end{array}\right)$.

$R_2 = R_2 - \frac{2}{1}R_1$

$R_3 = R_3 - \frac{4}{1}R_1$

$F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -4 & 0 & 1 \end{pmatrix}$.

③ $A_1 = \left(\begin{array}{ccc|c} 1 & 1 & -3 & -9 \\ 0 & 2 & -7 & -23 \\ 0 & -3 & 14 & 45 \end{array}\right)$

$A_2 = \left(\begin{array}{ccc|c} 1 & 1 & -3 & -9 \\ 0 & 2 & -7 & -23 \\ 0 & 0 & 3.5 & 10.5 \end{array}\right)$ $\left[R_3 = R_3 - \left(\frac{-3}{2}\right)R_2\right]$

$A^{(2)} = F^{(1)}(A)$

$= \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -4 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -3 \\ 2 & 4 & -1 \\ 4 & 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 & -3 \\ 0 & 2 & 5 \\ 0 & 3 & 14 \end{pmatrix}$

$F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1.5 & 1 \end{pmatrix}$ (Ans).

$U = A^{(3)} = F^{(2)} . A^{(2)}$

$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1.5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -3 \\ 0 & 2 & 5 \\ 0 & -3 & 14 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 & -3 \\ 0 & 2 & 5 \\ 0 & 0 & 21.5 \end{pmatrix}$

$b = \begin{pmatrix} -9 \\ -23 \\ 10.5 \end{pmatrix}$

$$[U] \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = b.$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & -3 \\ 0 & 2 & 5 \\ 0 & 0 & 21.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -9 \\ -23 \\ 10.5 \end{pmatrix}.$$

$21.5x_3 = 10.5$

$x_3 = 0.488372.$

$2x_2 + 5x_3 = -23$

$x_2 = -12.72093.$

$x_1 + x_2 - 3x_3 = -9.$

$x_1 = 5.186046.$

(5) $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & -1.5 & 1 \end{bmatrix}.$

(6) $LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & -1.5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -3 \\ 0 & 2 & 5 \\ 0 & 0 & 21.5 \end{bmatrix}.$

$= \begin{bmatrix} 1 & 1 & -3 \\ 2 & 4 & -1 \\ 4 & 1 & 2 \end{bmatrix}.$

$A - LU = \begin{pmatrix} 1 & 1 & -3 \\ 2 & 4 & -1 \\ 4 & 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 1 & -3 \\ 2 & 4 & -1 \\ 4 & 1 & 2 \end{pmatrix} = 0$ (Ans)

10

Ans: 142

① $S = \{\vec{V_1}, \vec{V_2}, \vec{V_3}\}$

$V_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, V_2 = \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}, V_3 = \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$

$V_1 \cdot V_2 = 1 \cdot 1 + 0 \cdot \sqrt{2} + (-1) \cdot 1$
$= 1 - 1$
$= 0.$

$V_1 \cdot V_3 = 1 + 0 - 1 = 0$
$V_2 \cdot V_3 = 1 - 2 + 1$
$= 0.$

As, all of them are 0 so S is orthogonal set. (Ans)

② $V_{1norm} = \frac{V_1}{|V_1|} = \frac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{1^2 + 0 + (-1)^2}} = \frac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{2}} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$

$V_{2norm} = \frac{V_2}{|V_2|} = \frac{\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}}{\sqrt{1^2 + (\sqrt{2})^2 + 1^2}} = \frac{\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}}{2} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ 1/2 \end{pmatrix}$

$V_{3norm} = \frac{V_3}{|V_3|} = \frac{\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}}{\sqrt{1 + (-\sqrt{2})^2 + 1}} = \frac{\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}}{2}$

$= \begin{pmatrix} 1/2 \\ -1/\sqrt{2} \\ 1/2 \end{pmatrix}$ (Ans.

Ques #11.

1

$$\text{Truncation} = \frac{h^2}{6} |f'''(\xi)|_{max} + \cancel{\frac{\varepsilon_M}{h} |f(\xi)|_{max}}$$

$$\text{rounding} = \frac{\varepsilon_M}{h} |f(\xi)|_{max}$$

$$\frac{\text{Trun}}{\text{round}} = \frac{h^2 f'''}{} \frac{\text{round}}{\text{Trunc}} = \frac{\varepsilon_M}{h} |f(\xi)|_{max} \times \frac{6}{h^2 f''' f(\xi)_{max}}$$

Extremum = 1

$$= \frac{\varepsilon_M . 6 . f(\xi)_{max}}{h^3 \; f'''(\xi)_{max}}$$

$$h^3 = 6\varepsilon_M \cdot \frac{f(\xi)_{max}}{f'''(\xi)_{max}}$$

3!

$$h = \left(3! \, \varepsilon_M \frac{f(\xi)_{max}}{f'''(\xi)_{max}}\right)^{1/3}$$

[Showed].

Paper Source
Subject.........
Date:.........Time:

(2).

$$h = \left(3\varepsilon_M \frac{f(\xi)_{max}}{f'''(\xi)_{max}}\right)^{1/3}.$$

$f(x) = \sin x$

$f(x)_{max} = \sin\pi = 0.$

$f'(x) = \cos x.$

$f''(x) = -\sin x$

$f'''(x) = -\cos x.$

$f'''(x)_{max} = -\cos\pi$

$= 1.$

$\varepsilon_M = 1.0\times10^{-10}.$

$$\Rightarrow h = \left(3\times 1.0\times10^{-10}\times\frac{0}{1}\right)^{1/3}.$$

$= 0.$

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Paper Source
Subject
Date: Time:

Ques #32:

(1)

For upper triangle

$$l_{11} x_1 = b_1$$

$$l_{21} x_1 + l_{22} x_2 = b_2.$$

$$l_{31} x_1 + l_{32} x_2 + l_{33} x_3 = b_3.$$

$$l_{41} x_1 + l_{42} x_2 + l_{43} x_3 + l_{44} x_4 = b_4.$$

$x_1 = \frac{b_1}{l_{11}} = 1 \text{ div} = j = 1$

$x_2 = \frac{b_2 - l_{21} x_1}{l_{22}} \Rightarrow j = 2.$

$x_3 = \frac{b_3 - l_{31} x_1 - l_{32} x_2}{l_{33}}; \; j = 3$

So $x_j = \frac{b_j - \sum_{k=j+1}^{n} u_{jk} x_k}{u_{jj}}; \; j = n, n-1, \ldots, 1$

[Proved]

(Ans)

2.

We know,

$$N = \sum_{k=1}^{n-1} \left[2(n-k)^2 + (n-k) \right]$$

$n = 18$

$k = 18.$

$$= \sum_{k=1}^{18-1} \left[2(n-k)^2 + (n-k) \right] = \frac{2}{3}n^3 - \frac{1}{2}n^2 - \frac{1}{6}n \quad [n=18]$$

$= 3723.$

3723 operations needed.

no of operation we know $= \sum_{j=1}^{n} \left[1 + 2(j-1) \right]$

$= n^2$

$= 18^2 = 324$

(Ans)