

Practice Sheet Final 1.0

1. (a) Derive formula for Secant method for finding root(s) of a nonlinear equation. Why would you use the Secant method instead of Newton's method for finding root(s) of a nonlinear equation?

(b) Use Secant method to estimate the root of $f(x) = x^3 - 3x^2 + x$ with initial estimates $x_{-1}=0.3$ and $x_0=0.35$. Show your result along with the percentage errors in tabular form for the first three iterations.

2. Discuss the limitations of Newton's Method of solving a root of a nonlinear equation?
3. Using LU decomposition method find the inverse of the matrix given below:

$$\begin{bmatrix} 2 & 5 & 8 & 3 & 7 & 9 & 1 & 4 & 6 \end{bmatrix}$$

4. Using Gauss elimination method solve the below system:

$$3x_1 + 5x_2 + 7x_3 + 9x_4 = 1.4$$

$$7x_1 + 3x_2 + 11x_3 + 4x_4 = 1.8$$

$$2x_1 + 5x_2 + 3x_3 + 2x_4 = 2.7$$

$$8x_1 + 7x_2 + 7x_3 + 4x_4 = 3.4$$

5. Find the root of the below equation using secant method with initial value $x_{-1}=0.3$ and $x_0=0.9$. Do your calculation for the first three iterations and show your results in a tabular form with all the percentage errors.

$$f(x) = \sin \sin(x) + 2x^2 + 5$$

6. (a) Apply LU decomposition method to find the inverse of the given matrix:

$$\begin{bmatrix} 4 & 7 & 8 & 11 & 12 & 7 & 3 & 8 & 13 \end{bmatrix}$$

(b) Use bi-section method to find the root(s) of $f(x) = x^2 - e^{-2x} - (x)$ with $x_l = 0$ and $x_u = -0.8$. Show your results along with the percentage errors for the first three iterations in a tabular form.

7. Solve the system of equations below using LU decomposition:

$$x_1 + 3x_2 + 2x_3 + 4x_4 = 1.4$$

$$2x_1 + x_2 + x_3 + 3x_4 = 1.8$$

$$2x_1 + 5x_2 + x_3 + x_4 = 2.7$$

$$3x_1 + 4x_2 + 2x_3 + 5x_4 = 3.4$$

8. Using Gauss elimination method solve the below system:

$$12x_1 + 10x_2 - 7x_3 = 15$$

$$6x_1 + 5x_2 + 3x_3 = 14$$

$$24x_1 - x_2 + 5x_3 = 28$$

9. Discuss 2 problems of Gaussian elimination.

10.

Determine the real root of $f(x) = -26 + 85x - 91x^2 + 44x^3 - 8x^4 + x^5$ using the bisection method. Employ initial guesses of $x_l = 0.5$ and $x_u = 1.0$. Iterate the process until the approximate error falls below a stopping criterion of $\epsilon_s = 10\%$. Note that you must show the detail calculation of the first iteration.

$$\epsilon_s = \text{tolerance}$$