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Section: 06

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Assignment - 2

(1)

$f(x) = \sin(x)$; $[0, \frac{\pi}{2}, \pi]$

As nodes = 3

$n = 2$.

Here, $x_0 = 0$ | $f(x_0) = \sin(0) = 0$

$x_1 = \frac{\pi}{2}$ | $f(x_1) = \sin(\frac{\pi}{2}) = 1$

$x_2 = \pi$ | $f(x_2) = \sin(\pi) = 0$

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$= f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

Here, $f[x_0] = a_0$; $f[x_0, x_1] = a_1$; $f[x_0, x_1, x_2] = a_2$

$x_0 = 0$	$f[x_0] = 0$	$f[x_0, x_1] = \frac{1-0}{\pi/2 - 0} = \frac{2}{\pi}$
$x_1 = \frac{\pi}{2}$	$f[x_1] = 1$	
$x_2 = \pi$	$f[x_2] = 0$	$f[x_1, x_2] = \frac{0-1}{\pi - \frac{\pi}{2}} = -\frac{2}{\pi}$

$$f[x_0, x_1, x_2] = \frac{-\frac{2}{\pi} - \frac{2}{\pi}}{\pi - 0}$$

$$= -\frac{4}{\pi} \times \frac{1}{\pi}$$

$$= -\frac{4}{\pi^2}$$

$$\boxed{\begin{aligned} \# \; & a_0 = f[x_0] = 0 \\ \# \; & a_1 = f[x_0, x_1] = +\frac{2}{\pi} \\ \# \; & a_2 = f[x_0, x_1, x_2] = -\frac{4}{\pi^2} \end{aligned}}$$

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(2)

$$P_2(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

Here, from (1) we got, $f[x_0] = 0$, $f[x_0,x_1] = +\frac{2}{\pi}$, $f[x_0,x_1,x_2] = -\frac{4}{\pi^2}$ | $x_0 = 0$, $x_1 = \frac{\pi}{2}$, $x_2 = \pi$.

$$P_2(x) = 0 + \left(-\frac{2}{\pi}\right)(x-0) + \left(-\frac{4}{\pi^2}\right)(x-0)\left(x-\frac{\pi}{2}\right)$$

$$= 0 + \frac{2x}{\pi} - \frac{4x}{\pi^2}\left(x-\frac{\pi}{2}\right)$$

$$= +\frac{2x}{\pi} - \frac{4x^2}{\pi^2} + \frac{4x\pi}{\pi^2 \cdot 2}$$

$$\cancel{= -\frac{2x}{\pi} - \frac{4x^2}{\pi^2} + \frac{2x}{\pi}} = \frac{2x}{\pi} - \frac{4x^2}{\pi^2} + \frac{2x}{\pi} = \frac{4x}{\pi} - \frac{4x^2}{\pi^2}$$

$$= \frac{4x}{\pi}\left(1 - \frac{x}{\pi}\right)$$

$$\cancel{= -\frac{4x^2}{\pi^2}}$$

$$\boxed{\cancel{\# \; P_2(x) = -\frac{4x^2}{\pi^2}}} \text{ (Ans).}$$

$$\# \; P_2(x) = \frac{4x}{\pi}\left(1 - \frac{x}{\pi}\right)$$

(3).

from (1), $x_0 = 0$, $x_1 = \pi/2$, $x_2 = \pi$, $x_3 = 3\pi/2$

$f(x_0) = 0$
$f(x_1) = 1$
$f(x_2) = 0$
$f(x_3) = -1$.

$f[x_0,x_1] = +\frac{2}{\pi}$

$f[x_1,x_2] = -\frac{2}{\pi}$

$$f[x_2,x_3] = \frac{-1-0}{\frac{3\pi}{2} - \pi} = \frac{-1}{\frac{3\pi-2\pi}{2}} = \frac{-1}{\frac{\pi}{2}} = -\frac{2}{\pi}$$

$$f[x_0,x_1,x_2] = \frac{-\frac{2}{\pi}-\frac{2}{\pi}}{\pi-0} = \frac{-4}{\pi}\times\frac{1}{\pi} = -\frac{4}{\pi^2}$$

$$f[x_1,x_2,x_3] = \frac{-\frac{2}{\pi}+\frac{2}{\pi}}{\frac{3\pi}{2}-\frac{\pi}{2}} = 0$$

$$f[x_0,x_1,x_2,x_3] = \frac{0+\frac{4}{\pi^2}}{\frac{3\pi}{2}-0} = \frac{4}{\pi^2}\times\frac{2}{3\pi} = \frac{8}{3\pi^3}$$

$$P_3(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2)$$

$f[x_0] = 0$

$f[x_0,x_1] = \frac{2}{\pi}$

$f[x_0,x_1,x_2] = -\frac{4}{\pi^2}$

$f[x_0,x_1,x_2,x_3] = \frac{8}{3\pi^3}$

Again, $P_{n+1}(x) = P_n(x) + g_{n+1}(x)$.

$P_3(x) = P_2(x) + g_3(x)$.

$$g_3(x) = a_3(x-x_0)(x-x_1)(x-x_2). \quad [a_3 = f[x_0,x_1,x_2,x_3]]$$

$$= \frac{8}{3\pi^3}(x-0)\left(x-\frac{\pi}{2}\right)(x-\pi)$$

$$= \frac{8x}{3\pi^3}\left(x^2-\pi x-\frac{\pi x}{2}+\frac{\pi^2}{2}\right)$$

$$= \frac{8x^3}{3\pi^3} - \frac{8\pi x^2}{3\pi^3} - \frac{8\pi x^2}{6\pi^3} + \frac{8x\pi^2}{6\pi^3}$$

$$= \frac{8x^3}{3\pi^3} + \frac{-8x^2}{3\pi^2} - \frac{8x^2}{6\pi^2} + \frac{8x}{6\pi}$$

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$$= \frac{8x^3}{3\pi^3} + \frac{-16x^2-8x^2}{6\pi^2} + \frac{8x}{6\pi}$$

$$= \frac{8}{3}\left(\frac{x}{\pi}\right)^3 \pm \frac{24x^2}{6\pi^2} + \frac{8x}{6\pi}$$

$$= \frac{8}{3}\left(\frac{x}{\pi}\right)^3 \pm \frac{4x^2}{\pi^2} + \frac{8x}{6\pi} = g_3(x)$$

$$P_3(x) = P_2(x) + g_3(x)$$

$$= \frac{4x}{\pi}\left(1-\frac{x}{\pi}\right) + \frac{8x^3}{3\pi^3} + \frac{4x^2}{\pi^2} + \frac{4x}{3\pi}$$

$$= \cancel{\frac{4x}{\pi} - \frac{4x^2}{\pi^2} + \frac{8x^3}{3\pi^3} \pm \frac{4x^2}{\pi^2} + \frac{4x}{3\pi}}$$

$$= \frac{8x^3}{3\pi^3} + \left(\frac{4x}{3\pi} + \frac{4x}{\pi}\right)$$

$$= \frac{8x^3}{3\pi^3} + \frac{16}{3}\frac{x}{\pi}$$

$$= \frac{8x}{3\pi}\left(\frac{x^2}{\pi^2} + 2\right)$$

$$= \frac{4x}{\pi} - \frac{4x^2}{\pi^2} + \frac{8x^3}{3\pi^3} - \frac{4x^2}{\pi^2} + \frac{4x}{3\pi}$$

$$= \frac{4x}{\pi} - \frac{8x^2}{\pi^2} + \frac{8x^3}{\pi^3} + \frac{4x}{3\pi}$$

$$= \frac{12x+4x}{3\pi} + \frac{8x^3}{3\pi^3} - \frac{8x^2}{\pi^2}$$

$$= \frac{16x}{3\pi} + \frac{8x^3}{3\pi^3} - \frac{8x^2}{\pi^2}$$

$$P_3(x) = \frac{8x^3}{3\pi^3} - \frac{8x^2}{\pi^2} + \frac{16x}{3\pi}$$ (Ans).

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4.

We know, $|f(x) - P_n(x)| = \frac{f^{n+1}(\xi)}{(n+1)!} (x-x_0) \cdots (x-x_n)$

$$|f(x) - P_3(x)| = \frac{f^4(\xi)}{4!} \underbrace{(x-0)(x-\frac{\pi}{2})(x-\pi)(x-\frac{3\pi}{2})}_{W(x)}$$

$f(x) = \sin x.$
$f^1(x) = \cos x$
$f^2(x) = -\sin x$
$f^3(x) = -\cos x$
$f^4(x) = +\sin x.$

$\frac{f^4(\xi)}{4!} = \frac{\sin x}{24}.$

$W(x) = x(x-\frac{\pi}{2})(x-\pi)(x-\frac{3\pi}{2}).$

5.

$$W(x) = x(x-\pi)(x-\frac{\pi}{2})(x-\frac{3\pi}{2})$$
$$= (x^2-\pi x)(x^2 - \frac{3\pi x}{2} - \frac{\pi x}{2} + \frac{3\pi^2}{4})$$
$$= (x^2-\pi x)(x^2 - 2\pi x + \frac{3\pi^2}{4})$$
$$= (x^4 - 2\pi x^3 + \frac{3\pi^2 x^2}{4} - \pi x^3 + 2\pi^2 x^2 - \frac{3\pi^3 x}{4})$$
$$= x^4 - 3\pi x^3 + \frac{11}{4}\pi^2 x^2 - \frac{3}{4}\pi^3 x$$

$$W'(x) = 4x^3 - 9\pi x^2 + \frac{11}{2}\pi^2 x - \frac{3}{4}\pi^3.$$

first derivative has to be 0 for the maximum value,

$W'(x) = 0$

$\Rightarrow 4x^3 - 9\pi x^2 + \frac{11}{2}\pi^2 x - \frac{3}{4}\pi^3 = 0$

by calculating we got, $W(x) = x^4 - 3\pi x^3 + \frac{11}{4}\pi^2 x^2 - \frac{3}{4}\pi^3 x$

$x_1 = 0.6$	$W(x_1) = \cancel{23.092} = -6.088$.
$x_2 = 4.11$	$W(x_2) = \cancel{4} - 6.088$
$x_3 = 2.36$	$W(x_3) = \cancel{13.35}\ 3.42$.

$\frac{\sin(x)}{24} = \frac{\sin(90^\circ)}{24} = \frac{1}{24}$.

$W(x_3) = 3.42$.

$$\boxed{\#\ \text{Maximum error} = \frac{1}{24} \times 3.42 = 0.1425.}$$

(Ans)