Name: Farah Jasmin Khan.
ID: 1910239
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Quiz-3.

(5)

**Q#1** $x_0 = 3;\ x_1 = 5.$

$f(x_0) = 225;\ f(x_1) = 383$

$f'(x_0) = 77;\ f'(x_1) = 80.$

$$l_0(x) = \frac{x-x_1}{x_0-x_1} = \frac{x-5}{3-5} = \frac{x-5}{-2}$$
$$l_1(x) = \frac{x-x_0}{x_1-x_0} = \frac{x-3}{5-3} = \frac{x-3}{2}$$
} Langrange basis.

**Q#2** We know,

if $n = 1$,

$deg = 2n+1 = 3$:

$$P_3(x) = f(x_0)h_0(x) + f(x_1)h_1(x) + f'(x_0)\hat{h}_0(x) + f'(x_1)\hat{h}_1(x)$$
$$= 225\,h_0(x) + 383\,h_1(x) + 77\,\hat{h}_0(x) + 80\,\hat{h}_1(x)$$

$$h_0(x) = 1 - 2(x-x_0)\,l_0'(x)\,l_0^2(x)$$
$$= 1 - 2(x-3)\left(-\frac{1}{2}\right)\left(\frac{x-5}{-2}\right)^2.$$

$$h_1(x) = 1 - 2(x-x_1)\,l_1'(x)\,l_1^2(x)$$
$$= 1 - 2(x-5)\left(\frac{1}{2}\right)\left(\frac{x-3}{2}\right)^2.$$

$$\hat{h}_0(x) = (x-x_0)\,l_0^2(x) = (x-3)\left(\frac{x-5}{-2}\right)^2.$$
$$\hat{h}_1(x) = (x-x_1)\,l_1^2(x) = (x-5)\left(\frac{x-3}{2}\right)^2.$$
} Hermite Basis

**Q#3**
$$P_3(x) = 225\left\{\left(1 - 2(x-3)\left(-\frac{1}{2}\right)\right)\left(\frac{x-5}{-2}\right)^2\right\} + 383\left\{\left(1 - 2(x-5)\frac{1}{2}\right)\left(\frac{x-3}{2}\right)^2\right\}$$
$$+ 77(x-3)\left(\frac{x-5}{-2}\right)^2 + 80 \times (x-5)\left(\frac{x-3}{2}\right)^2.$$

$$P_3\left(\underset{x}{4}\right) = 225 \times \frac{(x-2)(x-5)^2}{4} + 383 \times \frac{(x-3)^2(x+6)}{4} + 77 \times \frac{(x-3)(x-5)^2}{4}$$
$$+ 80 \times \frac{(x-5)(x-3)^2}{4}$$

$$\boxed{P_3(4) = \frac{1213}{4} = 303.25} \quad \text{(Ans).}$$