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Assignment-8

$\boxed{\text{Ans \# 1}}$

~~Given, $4x_1$~~

Given, $4x_1 - x_2 + x_3 = 8$ — (I).
$2x_1 + 5x_2 + 2x_3 = 3$ — (II).
$x_1 + 2x_2 + 4x_3 = 11$ — (III).

(I) − (2 × (II)).

$$4x_1 - x_2 + x_3 = 8$$
$$(-)\ 4x_1 + 10x_2 + 4x_3 = 6$$
$$\overline{\quad -11x_2 - 3x_3 = 2 \quad}\ \text{— (IV)}$$

(I) − (4 × (III)).

$$4x_1 - x_2 + x_3 = 8.$$
$$(-)\ -4x_1 + 8x_2 + 16x_3 = 44$$
$$\overline{\quad -9x_2 - 15x_3 = -36 \quad}\ \text{— (V).}$$

(IV) − $\left(\frac{11}{9} \times \text{(V)}\right)$.

$$-11x_2 - 3x_3 = 2.$$
$$(-)\ -9 \times \frac{11}{9}x_2 - 15 \times \frac{11}{9}x_3 = -36 \times \frac{11}{9}$$
$$\overline{\quad \frac{46}{3}x_3 = 46. \quad}$$
$$\therefore x_3 = 3$$

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From (iv) $-11x_2-3x_3=2$

Or $-11x_2-3\times 3=2$

$\therefore x_2=-1$

From (iii) $x_1+2x_2+4x_3=11$

or, $x_1+2\times(-1)+4\times 3=11$

$\Rightarrow x_1=1$.

Therefore, $x_1=1$, $x_2=-1$, $x_3=3$

Therefore, this system have a unique solution.

$\boxed{\text{Ans \#2}}$

Augmented matrix, $Aug(A)=\left(\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 2 & 5 & 2 & 3 \\ 1 & 2 & 4 & 11 \end{array}\right) \xrightarrow[a_{31},\ R_3=R_3-\frac{1}{4}R_1]{a_{21},\ R_2=R_2-\frac{2}{4}R_1}$

$\left(\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 11/2 & 3/2 & -1 \\ 0 & 9/4 & 15/4 & 9 \end{array}\right)$ $\left[\begin{array}{l} R_2=R_2-m_{21}\cdot R_1; \\ m_{21}=\frac{a_{21}}{a_{11}}=\frac{2}{4}; \\ R_3=R_3-m_{31}R_1; \\ m_{31}=\frac{a_{31}}{a_{11}}=\frac{1}{4} \end{array}\right]$ $\xrightarrow{a_{32},\ R_3=R_3-\frac{9}{22}R_2}$

$\left(\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 11/2 & 3/2 & -1 \\ 0 & 0 & 69/22 & 207/22 \end{array}\right)$ $\left[\begin{array}{l} R_3=R_3-m_{32}\cdot R_2, \\ m_{32}=\frac{a_{32}}{a_{12}}=\frac{9/4}{11/2}=\frac{9}{22} \end{array}\right]$

Hence, $\frac{69}{22} x_3 = \frac{207}{22}$

$\therefore x_3 = 3$

$\frac{11}{2} x_2 + \frac{3}{2} x_3 = -1$

or $\frac{11}{2} x_2 + \frac{3}{2} \times 3 = -1$

$\therefore x_2 = -1$

$4x_1 - x_2 + x_3 = 8$

or, $4x_1 - (-1) + 3 = 8$

$x_1 = 1$

Therefore, $\left.\begin{aligned} x_1 &= 1 \\ x_2 &= -1 \\ x_3 &= 3 \end{aligned}\right\} \rightarrow$ same as 1.

$\boxed{\text{Ans \#3}}$

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 4 \end{pmatrix}$$

$$F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -2/4 & 1 & 0 \\ -1/4 & 0 & 1 \end{pmatrix} \left[\begin{aligned} & a_{21},\ r_2 = r_2 - \tfrac{2}{4} r_1,\ -m_{21} = -\tfrac{2}{4}; \\ & a_{31},\ r_3 = r_3 - \tfrac{1}{4} r_1,\ -m_{31} = -\tfrac{1}{4} \end{aligned} \right].$$

$$F^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{4} & 0 & 1 \end{pmatrix}$$

Now, $A^{(2)} = F^{(1)} A = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -1/4 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 4 & -1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 4 \end{pmatrix}$.

$$A^{(2)} = \begin{pmatrix} 4 & -1 & 1 \\ 0 & 11/2 & 3/2 \\ 0 & 9/4 & 15/4 \end{pmatrix}; \quad F^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -9/22 & 1 \end{pmatrix} \left[\begin{aligned} & a_{32} \\ & r_3 = r_3 - \tfrac{[illegible]}{22} r_2 \\ & -m_{32} = -\tfrac{[illegible]}{[illegible]} \end{aligned} \right]$$

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Ans #4

$$L = (F^{(1)})^{-1} \cdot (F^{(2)})^{-1}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2/4 & 1 & 0 \\ 1/4 & 9/22 & 1 \end{pmatrix} \left[\begin{array}{c} m_{21} = \frac{2}{4}, m_{31} = \frac{1}{4}, \\ m_{32} = \frac{9}{22} \end{array} \right].$$

$L.U = A.$

$$U = A^{(3)} = F^{(2)} . A^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -9/22 & 1 \end{pmatrix} \times \begin{pmatrix} 4 & -1 & 1 \\ 0 & -11/2 & 3/2 \\ 0 & 9/4 & 15/4 \end{pmatrix}.$$

$$U = \begin{pmatrix} 4 & -1 & 1 \\ 0 & +1/2 & 3/2 \\ 0 & 0 & 69/22 \end{pmatrix}.$$

$$\text{NOW, } L.U = \begin{pmatrix} 1 & 0 & 0 \\ 2/4 & 1 & 0 \\ 1/4 & 9/22 & 1 \end{pmatrix} \times \begin{pmatrix} 4 & -1 & 1 \\ 0 & 11/2 & 3/2 \\ 0 & 0 & 69/22 \end{pmatrix}.$$

$$= \begin{pmatrix} 4 & -1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 4 \end{pmatrix} = A.$$

∴ Since, $L.U = A$ as it follows the formula so. L & U are correct.

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Ans #5

$$A = \begin{pmatrix} 4 & -1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 4 \end{pmatrix} . \quad b = \begin{pmatrix} 8 \\ 3 \\ 11 \end{pmatrix}$$

Now, $L.y = b.$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 2/4 & 1 & 0 \\ 1/4 & 9/22 & 1 \end{pmatrix} \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \\ 11 \end{pmatrix}.$$

$$\Rightarrow \begin{pmatrix} y_1 \\ \frac{2}{4}y_1 + y_2. \\ \frac{1}{4}y_1 + \frac{9}{22}y_2 + y_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \\ 11 \end{pmatrix}.$$

$\therefore \ y_1 = 8.$

$\frac{2}{4}y_1 + y_2 = 3.$

$\Rightarrow \ y_2 = -1.$

$\frac{1}{4}y_1 + \frac{9}{22}y_2 + y_3 = 11.$

$\Rightarrow \ y_3 = \frac{207}{22}$

Now, $V.x = y.$

$$\Rightarrow \begin{pmatrix} 4 & -1 & 1 \\ 0 & 11/2 & 3/2 \\ 0 & 0 & 69/22 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \\ \frac{207}{22} \end{pmatrix}.$$

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$$\Rightarrow \begin{pmatrix} 4x_1 - x_2 + x_3 \\ \frac{11}{2}x_2 + \frac{3}{2}x_3 \\ \frac{69}{22}x_3 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \\ \frac{207}{22} \end{pmatrix}.$$

$\therefore\ x_3 = 3.$

$\frac{11}{2}x_2 + \frac{3}{2}x_3 = -1$

$\Rightarrow\ x_2 = -1.$

$4x_1 = x_2 + x_3 = 8$

$\Rightarrow x_1 = 1.$

Therefore, $x = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}.$

$x_1 = 1$

$x_2 = -1$

$x_3 = 3.$

As values of ~~1 & 5~~ ~~1, 2, 5 are the same~~ Ques 1, 5 are the same. The solution agrees with answer found in Question-2