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Section: 06

Ques #1

(1)

$x = 0.5$; $h = 0.32$

$f(x) = 6e^{-3x}$

We know, central difference $= \dfrac{f(x+h) - f(x-h)}{2h} = f'(x)$

$$f'(x) = \frac{6e^{-3(x+h)} - 6e^{-3(x-h)}}{2h}$$

$$\Rightarrow f'(0.5) = \frac{6}{2 \times 0.32}\left\{e^{-3(0.5+0.32)} - e^{-3(0.5-0.32)}\right\}$$

$$= 9.375 \times (-0.4973)$$

$$\Rightarrow f'(0.5) = -4.6623 \text{ (Ans)}$$

(2)

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} = \frac{6}{2h}\left\{e^{-3(x+h)} - e^{-3(x-h)}\right\} \quad [h = 0.16]$$

$$f'(0.5) = \frac{6}{2 \times 0.16}\left\{e^{-3(0.5+0.16)} - e^{-3(0.5-0.16)}\right\}$$

$$\Rightarrow f'(0.5) = 18.75 \times (-0.2225) = -4.1724$$

(Ans)

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(3)

We know, $D_n^{(1)} = \dfrac{2^2 D_{n/2} - D_n}{2^2 - 1} = f'(x_1) + \dfrac{(\frac{1}{2^2} - 1)}{(2^2-1)5!} f^{(5)}(x_1) h^4 + O(h^6)$

$= \dfrac{\cancel{2^2 f'(0.16) - f'(0.32)}}{\cancel{2^2 - 1}} = \dfrac{2^2 D_{0.16} - D_{0.32}}{2^2 - 1}$

$= \dfrac{2^2 \times (-4.1724) - (-4.6623)}{(2^2 - 1)}$

$D_{0.32}^{(1)} = -4.0091$ (Ans)

(4)

Error $= \left| \dfrac{-4.01634 + 4.0091}{-4.01634} \right| \times 100$

$= 0.18026\%$

$f'(0.5)$
$= -4.01634$

Ques ② ①.

$$D_h = \frac{f(x_1+h) - f(x_1-h)}{2h}$$

$$f(x_1+h) = f(x_1) + f'(x_1)h + \frac{f^{(2)}(x_1)h^2}{2!} + \frac{f^{(3)}(x_1)}{3!}h^3 + \frac{f^{(4)}(x_1)h^4}{4!} + \frac{f^{(5)}(x_1)}{5!}h^5 + \frac{f^{(6)}(x_1)}{6!}h^6 + \frac{f^{(7)}(x_1)}{7!}h^7 + \theta(h^8)$$

$$f(x_1-h) = f(x_1) - f'(x_1)h + \frac{f^{(2)}(x_1)h^2}{2!} - \frac{f^{(3)}(x_1)}{3!}h^3 + \frac{f^{(4)}(x_1)}{4!}h^4 - \frac{f^{(5)}(x_1)}{5!}h^5 + \frac{f^{(6)}(x_1)}{6!}h^6 - \frac{f^{(7)}(x_1)}{7!}h^7 + \theta(h^8) + \theta(h^9)$$

$$D_h = \frac{1}{2h}\left\{2hf'(x_1) + \frac{2h^3 f^{(3)}(x_1)}{3!} + \frac{2h^5 f^{(5)}(x_1)}{5!} + \frac{2h^7 f^{7}(x_1)}{7!} + \theta(h^9)\right\}$$

$$= f'(x_1) + \frac{f^3(x_1)}{3!}h^2 + \frac{h^4 f^5(x_1)}{5!} + \frac{h^6 f^7(x_1)}{7!} + \theta(h^8)$$

$$D_{h/2} = f'(x_1) + \frac{f^3(x_1)}{3!}\left(\frac{h}{2}\right)^2 + \frac{h^4 f^5(x_1)}{5!}\left(\frac{h}{2}\right)^4 + \frac{h^6 f^7(x_1)}{7!}\left(\frac{h}{2}\right)^6 + \theta(h^8)$$

$$2^2 D_{h/2} - D_h = 2^2 f'(x_1) + \cancel{2^2}\cdot\cancel{\frac{f^3(x_1)}{3!}\cdot\frac{h^2}{2^2}} + \cancel{2^2}\cdot\frac{f^5(x_1)}{5!}\cdot\frac{h^4}{\cancel{2^4}\,2^2} + 2^2\cdot\frac{h^6 f^7(x_1)}{7!\,2^6} + 2^2\theta(h^8) - \left\{f'(x_1) - \cancel{\frac{f^3(x_1)}{3!}h^2} \pm \frac{h^4 f^5(x_1)}{5!} \pm \frac{h^6 f^7(x_1)}{7!} - \theta(h^8)\right\}$$

$$= (2^2-1)f'(x_1) + \left(\frac{1}{2^2}-1\right)\frac{h^4 f^5(x_1)}{5!} + \left(\frac{1}{2^4}-1\right)\frac{h^6 f^7(x_1)}{7!} + (2^2-1)\theta(h^8)$$

$$\frac{2^2 D_{h/2} - D_h}{2^2-1} = f'(x_1) + \frac{\left(\frac{1}{2^2}-1\right)}{(2^2-1)} \frac{h^4 f^5(x_1)}{5!} + \frac{\left(\frac{1}{2^4}-1\right)}{(2^2-1)} \frac{h^6 f^7(x_1)}{7!}$$

$$+ \theta(h^8)$$

$$= D_h^{(1)} \quad \text{(Ans)}$$

$$D_{h/2}^{(1)} = f'(x_1) + \frac{\left(\frac{1}{2^2}-1\right)}{(2^2-1)} \frac{f^5(x_1)}{5!}\left(\frac{h}{2}\right)^4 + \frac{\left(\frac{1}{2^4}-1\right)}{(2^2-1)} \frac{f^7(x_1)}{7!}\left(\frac{h}{2}\right)^6$$

$$+ \theta(h^8) \quad \text{(Ans)}$$

2

$$2^4 . D_{h/2}^{(1)} - D_h^{(1)} = 2^4 . f'(x_1) + \cancel{\frac{\left(\frac{1}{2^2}-1\right)}{(2^2-1)} \frac{f^5(x_1)}{5!} \frac{h^4}{2^4} \times 2^4} + \frac{\left(\frac{1}{2^4}-1\right)}{(2^2-1)} \frac{f^7(x_1)}{7!}$$

$$\frac{h^6}{2^6} \times 2^4 + 2^4\theta(h^8) - f'(x_1) - \cancel{\frac{\left(\frac{1}{2^2}-1\right)}{(2^2-1)} \frac{h^4 f^5(x_1)}{5!}}$$

$$- \frac{\left(\frac{1}{2^4}-1\right)}{(2^2-1)} \frac{h^6 f^7(x_1)}{7!} - \theta(h^8).$$

$$= (2^4-1) f'(x_1) + \frac{\left(\frac{1}{2^4}-1\right)\left(\frac{1}{2^2}-1\right)}{(2^2-1)} . \frac{f^7(x_1)}{7!} h^6$$

$$+ (2^4-1)\theta(h^8).$$

$$\frac{2^4 D_{h/2}^{(1)} - D_h^{(1)}}{2^4-1} = f'(x_1) + \frac{\left(\frac{1}{2^4}-1\right)\left(\frac{1}{2^2}-1\right)}{(2^2-1)(2^4-1)} \frac{f^7(x_1)}{7!} h^6 + \theta(h^8).$$

$$= f'(x_1) + \frac{1}{322560} f^7(x_1) h^6 + \theta(h^8)$$

→ 6th approximation (Ans)

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Ques#3

$f(x) = 3x^3 + 12x - 20 \; ; [0,2]$

(1)

K	a_k	m_k	b_k	$f(a_k)$	$f(m_k)$	$f(b_k)$	$x \in [\,,\,]$
0	0	1	2	-20	-5	28	[1,2]
1	1	1.5	2	-5	8.125	28	[1,1.5]
2	1	1.25	1.5	-5	0.859	8.125	[1,1.25]
3	1	1.125	1.25	-5	-2.228	0.859	[1.125, 1.25]

appoximate root is = 1.25 (three iterations).

(2)

Actual root = $X_* = 1.2165$

$m_3 = 1.25$

error = $\left| \dfrac{\cancel{1.25 +} 1.2165 - 1.25}{\cancel{1.25}\ 1.2165} \right| \times 100 = \cancel{2.68}\% \cdot 2.7538\%$

(Ans)

③

$\varepsilon_M = 1.6 \times 10^{-8}$

we know, $n \geq \dfrac{\log(|b_0 - a_0|) - \log(\varepsilon)}{\log(2)} - 1.$

$n \geq \dfrac{\log|2-0| - \log(1.6 \times 10^{-8})}{\log(2)} - 1.$

$n \geq 25.89$

$n \geq 26$

26 or greater iterations (Ans). are needed to find the root.