

## **ASSIGNMENT-2**

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## Assignment-2

(1)

$$f(x) = \sin(x) ; [0, \frac{\pi}{2}, \pi]$$

As nodes = 3

$$n = 2$$

$$\text{Here, } x_0 = 0 \quad \left| \quad f(x_0) = \sin(0) = 0$$

$$x_1 = \frac{\pi}{2} \quad \left| \quad f(x_1) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$x_2 = \pi \quad \left| \quad f(x_2) = \sin(\pi) = 0$$

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$= f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$\text{Here, } f[x_0] = a_0 ; f[x_0, x_1] = a_1 ; f[x_0, x_1, x_2] = a_2$$

$$x_0 = 0 \quad \left| \quad f[x_0] = 0$$

$$x_1 = \frac{\pi}{2} \quad \left| \quad f[x_1] = 1$$

$$x_2 = \pi \quad \left| \quad f[x_2] = 0$$

$$f[x_0, x_1] = \frac{1 - 0}{\frac{\pi}{2} - 0} = \frac{2}{\pi}$$

$$f[x_1, x_2] = \frac{0 - 1}{\pi - \frac{\pi}{2}} = -\frac{2}{\pi}$$

$$f[x_0, x_1, x_2] = \frac{-\frac{2}{\pi} - \frac{2}{\pi}}{\pi - 0}$$

$$= -\frac{4}{\pi} \times \frac{1}{\pi}$$

$$= -\frac{4}{\pi^2}$$

$$\# a_0 = f[x_0] = 0$$

$$\# a_1 = f[x_0, x_1] = \frac{2}{\pi}$$

$$\# a_2 = f[x_0, x_1, x_2] = -\frac{4}{\pi^2}$$

(Ans)

②

$$P_2(x) = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1)$$

Here, from ① we got,  $f[x_0] = 0$

$$\left. \begin{aligned} f[x_0, x_1] &= +\frac{2}{\pi} \\ f[x_0, x_1, x_2] &= -\frac{4}{\pi^2} \end{aligned} \right\} \begin{aligned} x_0 &= 0 \\ x_1 &= \frac{\pi}{2} \\ x_2 &= \pi \end{aligned}$$

$$P_2(x) = 0 + \left(-\frac{2}{\pi}\right)(x-0) + \left(-\frac{4}{\pi^2}\right)(x-0)\left(x-\frac{\pi}{2}\right)$$

$$= 0 + \frac{2x}{\pi} - \frac{4x}{\pi^2}\left(x-\frac{\pi}{2}\right)$$

$$= +\frac{2x}{\pi} - \frac{4x^2}{\pi^2} + \frac{4x\pi}{\pi^2 \cdot 2}$$

$$= -\frac{2x}{\pi} - \frac{4x^2}{\pi^2} + \frac{2x}{\pi} = \frac{2x}{\pi} - \frac{4x^2}{\pi^2} + \frac{2x}{\pi} = \frac{4x}{\pi} - \frac{4x^2}{\pi^2}$$

$$= -\frac{4x^2}{\pi^2}$$

$$= \frac{4x}{\pi} \left(1 - \frac{x}{\pi}\right)$$

$$\boxed{\# P_2(x) = -\frac{4x^2}{\pi^2}} \quad (\text{Ans.})$$

$$\# P_2(x) = \frac{4x}{\pi} \left(1 - \frac{x}{\pi}\right)$$

③

from ①,  $x_0 = 0$ 

$$x_1 = \pi/2$$

$$x_2 = \pi$$

$$x_3 = 3\pi/2$$

$$f(x_0) = 0$$

$$f(x_1) = 1$$

$$f(x_2) = 0$$

$$f(x_3) = -1$$

$$f[x_0, x_1] = +\frac{2}{\pi}$$

$$f[x_1, x_2] = -\frac{2}{\pi}$$

$$f[x_2, x_3] = \frac{-1-0}{\frac{3\pi}{2}-\pi} = \frac{-1}{\frac{3\pi-2\pi}{2}} = \frac{-1}{\frac{\pi}{2}} = -\frac{2}{\pi}$$

(Ans)



$$f[x_0, x_1, x_2] = \frac{-\frac{2}{\pi} - \frac{2}{\pi}}{\pi - 0} = -\frac{4}{\pi} \times \frac{1}{\pi} = -\frac{4}{\pi^2}$$

$$f[x_1, x_2, x_3] = \frac{-\frac{2}{\pi} + \frac{2}{\pi}}{\frac{3\pi}{2} - \frac{\pi}{2}} = 0$$

$$f[x_0, x_1, x_2, x_3] = \frac{0 + \frac{4}{\pi^2}}{\frac{3\pi}{2} - 0} = \frac{4}{\pi^2} \times \frac{2}{3\pi} = \frac{8}{3\pi^3}$$

$$P_3(x) = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) + f[x_0, x_1, x_2, x_3](x-x_0)(x-x_1)(x-x_2)$$

$$f[x_0] = 0$$

$$f[x_0, x_1] = \frac{2}{\pi}$$

$$f[x_0, x_1, x_2] = -\frac{4}{\pi^2}$$

$$f[x_0, x_1, x_2, x_3] = \frac{8}{3\pi^3}$$

$$\text{Again, } P_{n+1}(x) = P_n(x) + g_{n+1}(x).$$

$$P_3(x) = P_2(x) + g_3(x).$$

$$g_3(x) = a_3(x-x_0)(x-x_1)(x-x_2)$$

$$= \frac{8}{3\pi^3}(x-0)\left(x-\frac{\pi}{2}\right)(x-\pi) \quad [a_3 = f[x_0, x_1, x_2, x_3]]$$

$$= \frac{8x}{3\pi^3} \left( x^2 - \pi x - \frac{\pi x}{2} + \frac{\pi^2}{2} \right)$$

$$= \frac{8x^3}{3\pi^3} - \frac{8\pi x^2}{3\pi^3} - \frac{8\pi x^2}{6\pi^3} + \frac{8x\pi^2}{6\pi^3}$$

$$= \frac{8x^3}{3\pi^3} + \frac{-8x^2}{3\pi^2} - \frac{8x^2}{6\pi^2} + \frac{8x\pi}{6\pi}$$

$$\begin{aligned}
 &= \frac{8x^3}{3\pi^3} + \frac{-16x^2 - 8x^2}{6\pi^2} + \frac{8x}{6\pi} \\
 &= \frac{8}{3} \left( \frac{x}{\pi} \right)^3 + \frac{24x^2}{6\pi^2} + \frac{8x}{6\pi} \\
 &= \frac{8}{3} \left( \frac{x}{\pi} \right)^3 + \frac{4x^2}{\pi^2} + \frac{8x}{6\pi} = g_3(x)
 \end{aligned}$$

$$P_3(x) = P_2(x) + g_3(x)$$

$$= \frac{4x}{\pi} \left( 1 - \frac{x}{\pi} \right) + \frac{8x^3}{3\pi^3} + \frac{4x^2}{\pi^2} + \frac{4x}{3\pi}$$

$$= \frac{4x}{\pi} - \frac{4x^2}{\pi^2} + \frac{8x^3}{3\pi^3} + \frac{4x^2}{\pi^2} + \frac{4x}{3\pi}$$

$$= \frac{8x^3}{3\pi^3} + \left( \frac{4x}{3\pi} + \frac{4x}{\pi} \right) = \frac{4x}{\pi} - \frac{4x^2}{\pi^2} + \frac{8x^3}{3\pi^3}$$

$$= \frac{8x^3}{3\pi^3} + \frac{16x}{3\pi} - \frac{4x^2}{\pi^2} + \frac{4x}{3\pi}$$

$$= \frac{8x}{3\pi} \left( \frac{x^2}{\pi^2} + 2 \right) = \frac{4x}{\pi} - \frac{8x^2}{\pi^2} + \frac{8x^3}{\pi^3} + \frac{4x}{3\pi}$$

$$= \frac{12x + 4x}{3\pi} + \frac{8x^3}{3\pi^3} - \frac{8x^2}{\pi^2}$$

$$\Rightarrow \frac{16x}{3\pi} + \frac{8x^3}{3\pi^3} - \frac{8x^2}{\pi^2}$$

$$P_3(x) = \frac{8x^3}{3\pi^3} - \frac{8x^2}{\pi^2} + \frac{16x}{3\pi} \quad (\text{Ans.})$$



(4).

We know,  $|f(x) - P_n(x)| = \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-x_0) \dots (x-x_n)$

$$|f(x) - P_3(x)| = \frac{f^4(\xi)}{4!} (x-0)(x-\frac{\pi}{2})(x-\pi)(x-\frac{3\pi}{2})$$

$$f(x) = \sin x$$

$$f^1(x) = \cos x$$

$$f^2(x) = -\sin x$$

$$f^3(x) = -\cos x$$

$$f^4(x) = +\sin x$$

$$\# \frac{f^4(\xi)}{4!} = \frac{\sin \xi}{24}$$

$$\# W(x) = x(x-\frac{\pi}{2})(x-\pi)(x-\frac{3\pi}{2})$$

(5).

$$W(x) = x(x-\pi)(x-\frac{\pi}{2})(x-\frac{3\pi}{2})$$

$$= (x^2 - \pi x) \left( x^2 - \frac{3\pi x}{2} - \frac{\pi x}{2} + \frac{3\pi^2}{4} \right)$$

$$= (x^2 - \pi x) \left( x^2 - 2\pi x + \frac{3\pi^2}{4} \right)$$

$$= \left( x^4 - 2\pi x^3 + \frac{3\pi^2 x^2}{4} - \pi x^3 + 2\pi^2 x^2 - \frac{3\pi^3 x}{4} \right)$$

$$= x^4 - 3\pi x^3 + \frac{11}{4}\pi^2 x^2 - \frac{3}{4}\pi^3 x$$

$$W'(x) = 4x^3 - 9\pi x^2 + \frac{11}{2}\pi^2 x - \frac{3}{4}\pi^3$$

first derivative has to be 0 for the maximum value,

$$W'(x) = 0$$

$$\Rightarrow 4x^3 - 9\pi x^2 + \frac{11}{2}\pi x - \frac{3}{4}\pi^2 = 0$$

by calculating we got,  $W(x) = x^4 - 3\pi x^3 + \frac{11}{4}\pi x^2 - \frac{3}{4}\pi^2 x$

$$x_1 = 0.6$$

$$x_2 = 4.11$$

$$x_3 = 2.36$$

$$W(x_1) = \cancel{23.092} = -6.088$$

$$W(x_2) = 4 - 6.088$$

$$W(x_3) = \cancel{13.35} = 3.42$$

$$\# \frac{\sin(x)}{24} = \frac{\sin(90^\circ)}{24} = \frac{1}{24}$$

$$\# W(x_3) = 3.42$$

$$\# \text{Maximum error} = \frac{1}{24} \times 3.42 = 0.1425 \quad (\text{Ans})$$