Final Exam

Read the following instructions/rules before you start answering the questions:

- **You MUST keep a Backup Data plan, in case your internet service is interrupted. NO EXCUSE.**
- **The exam time is 1 hour 15 minutes and an additional 15 minutes is for submission of answer script and uploading. So total is 1 hour 30 minutes. No more time time extension, and NO EMAIL SUBMISSION will be accepted.**
- Prepare a Title page containing a) Your Name, b) Your BRACU ID #, c) Your Theory class Section # and d) Date and e) Exam Name.
- Prepare the solution file in orderly fashion: first question first, then maintain the serial.
- Always start an answer to a question from a fresh page.
- Write legibly and neatly, FOLLOW the significant rule for all your calculations.
- YOU MUST WORK ALONE. INVOLVEMENT IN UNFAIR MEANS WILL BE REPORTED TO THE AUTHORITY.

Question # 11: The numerical derivative involves both the truncation and the rounding errors. Let ϵ_M be the machine epsilon of a system. The total error of a derivative of a function $f(x)$ for $x \in [x - h, x + h]$ is given by

$$\text{Error} \leq \frac{h^2}{6} |f'''(\xi)|_{\max} + \frac{\epsilon_M}{h} |f(\xi)|_{\max},$$

where $\xi \in [x - h, x + h]$. The first term on the right-hand side is the contribution from truncation and the second term is the contribution from rounding.

1. [2 Marks] Show that the error is extremum if $h = \left(3\epsilon_M \frac{|f(\xi)|_{\max}}{|f'''(\xi)|_{\max}}\right)^{1/3}$.
2. [2 Marks] Compute h upto five decimal places if $\epsilon_M = 1.0 \times 10^{-10}$, $f(x) = \sin(x)$ for the interval $I = [-\pi, \pi]$.

Problem # 24: Consider the function $f(x) = x^2 + x - 72$ with $x_\star \in [5, 10]$.

1. [2 Marks] Using the function $f(x)$ and according to the Secant method, find the expression for the iteration formula $x_{k+2} = g(x_k, x_{k+1})$, where $k = 0, 1, 2, \cdots$.
2. [2 Marks] Starting from $x_0 = 6$ and $x_1 = 7$, compute up to x_4 using the iteration formula found in the previous part. Express the values up to five decimal places.

Question # 32: An upper-triangular system $Ux = b$ can be solved by backward substitution formula

$$x_j = \frac{b_j - \sum_{k=j+1}^{n} u_{jk} x_k}{u_{jj}}, \quad j = n, ..., 1 .$$

1. [3 marks] Derive the formula for the number of floating-point operations required to find the upper triangular matrix.
2. [1 mark] If the matrix U is of order 18×18, what would be the total number of operations needed to solve the system.

Question # 42: Consider a set of three vectors, $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ in R^3, where

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{v}_3 = \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}.$$

(Show ALL the steps)

1. [2 Marks] Show that the set S is an orthogonal set.
2. [2 Marks] Normalize the vectors in the set S.

Question # 51: Consider the function $f(x) = 2x^3 + x^2 - 13x + 6$ which has real roots.

1. [2 Marks] Find the roots of $f(x)$.
2. [4 Marks] Construct 2 different fixed point function $g(x)$ such that $f(x) = 0$.
3. [6 Marks] Compute the convergence rate of each fixed point function $g(x)$, found in the previous part, and state which root it is converging to or diverging.

Question # 62: A linear system is described by the following linear equations,

$$\begin{aligned} x_1 + x_2 - 3x_3 &= -9 \\ 2x_1 + 4x_2 - x_3 &= -5 \\ \text{and} \quad 4x_1 + x_2 + 2x_3 &= 9 \end{aligned}.$$

Assume that the system has an unique solution. All questions below are related to the Gaussian elimination and LU-decomposition methods. All the symbols in this question paper has the usual meaning unless otherwise stated. Now, answer the following:

1. [1 Marks] Identify the matrices A and write down the augmented matrix of A, *i.e.,* Aug(A).
2. [3 Marks] Now apply the 1st row operation on Aug(A) and denote the result as matrix A_1. Also construct the matrix $F^{(1)}$.
3. [3 Marks] Now apply the 2nd row operation on the matrix A_1 obtained in the previous question and denote the result as matrix A_2. Identify the matrices U and b from A_2. Also

question,and denote the result as matrix A_2. Identify the matrices C and b from A_2. Also construct the matrix $F^{(2)}$.

4. [3 Marks] Using the matrix U and b, obtain the solution of the linear system.
5. [1 Marks] Construct the matrix L from the matrices $F^{(1)}$ and $F^{(2)}$.
6. [1 Marks] Compute the product LU and find $A - LU$.

FINAL EXAM SUBMISSION

Status

You have completed this assignment. Your final grade will be available when the assessments of your response are complete.

▸ Your Response due Sep 22, 2021 17:00 +06 (in 0 minutes) ✔ COMPLETE

Staff Grade NOT AVAILABLE

Waiting for a Staff Grade

Check back later to see if a course staff member has assessed your response. You will receive your grade after the assessment is complete.

▾ Your Grade: Waiting for Assessments

You have completed your steps in the assignment, but some assessments still need to be done on your response. When the assessments of your response are complete, you will see feedback from everyone who assessed your response, and you will receive your final grade.

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