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Section: 06.

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① Taylor expanded forms of D_h & $D_{h/3}$ upto order of $O(h^8)$

is $D_h = f^{(1)}(x) + f^{(3)}(x)\frac{h^2}{3!} + \frac{f^{(5)}(x)}{5!}h^4 + \frac{f^{(7)}(x)}{7!}h^6 + O(h^8)$.

$D_{h/3} = f^{(1)}(x) + \frac{f^{(3)}(x)}{3}\left(\frac{h}{3}\right)^2 + \frac{f^{(5)}(x)}{5!}\left(\frac{h}{3}\right)^4 + \frac{f^{7}(x)}{7!}\left(\frac{h}{3}\right)^6 \cancel{+ O\left(\frac{h}{3}\right)^8} + O(h^8)$.

(Ans).

②

② $D_h = f'(x) + ch^n + \theta(h^{n+1})$

~~2 D~~ $3^n D_{h/3} = \underset{3^n}{f'(x_1)} + c\left(\frac{h}{3}\right)^n + \theta(h^{n+1})$

$$D_h^{(1)} = \frac{3^n D_{h/3} - D_h}{3^n - 1}$$

② $D_h = \frac{f(x_1+h) - f(x_1-h)}{2h}$

$$f(x_1+h) = f(x_1) + f'(x_1)h + \frac{f^{(2)}(x_1)h^2}{2!} + \frac{f^{(3)}(x_1)}{3!}h^3 + \frac{f^{4}(x_1)}{4!}h^4 + \frac{f^{(5)}(x_1)}{5!}h^5 + \frac{f^{(6)}(x_1)}{6!}h^6 + \frac{f^{(7)}(x_1)}{7!}h^7 + \theta(h^8) + \theta(h^9)$$

$$f(x_1-h) = f(x_1) - f'(x_1)h + \frac{f^{(2)}(x_1)h^2}{2!} - \frac{f^{(3)}(x_1)}{3!}h^3 + \frac{f^{4}(x_1)}{4!}h^4 - \frac{f^{(5)}(x_1)}{5!}h^5 + \frac{f^{(6)}(x_1)}{6!}h^6 - \frac{f^{(7)}(x_1)}{7!}h^7 + \theta(h^8) + \theta(h^9)$$

$$D_h = \frac{1}{2h}\left\{ 2hf'(x_1) + \frac{2h^3 f^{(3)}(x_1)}{3!} + \frac{2h^5 f^{(5)}(x_1)}{5!} + \frac{2h^7 f^{(7)}(x_1)}{7!} + \theta(h^9)\right\}$$

$$= f'(x_1) + \frac{f^{(3)}(x_1)}{3!}h^2 + \frac{h^4 f^5(x_1)}{5!} + \frac{h^6 f^7(x_1)}{7!} + \theta(h^8)$$

$$D_{h/3} = f'(x_1) + \frac{f^3(x_1)}{3!}\left(\frac{h}{3}\right)^2 + \frac{f^5(x_1)}{5!}\left(\frac{h}{3}\right)^4 + \frac{f^7(x_1)}{7!}\left(\frac{h}{3}\right)^6 + \theta(h^8)$$

(A)

$$3^2 D_{h/3} - D_h = 3^2 f'(x_1) + \cancel{\frac{3^2 f^3(x_1)}{3!}\frac{h^2}{3^2}} + \frac{f^5(x_1)}{5!}\cdot\frac{h^4}{3^4}\cdot 3^2$$

$$+ \frac{f^7(x_1)}{7!}\,\frac{h^6}{3^6}\cdot 3^2 + 3^2(h^8)$$

$$-\left\{f'(x_1) + \cancel{\frac{f^3(x_1)}{3!}h^2} + \frac{h^4 f^5(x_1)}{5!} + \frac{h^6 f^7(x_1)}{7!}\right.$$

$$\left. + O(h^8)\right\}.$$

$$=(3^2-1)f'(x_1) + \left(\frac{1}{3^2}-1\right)\frac{h^4 f^5(x_1)}{5!} + \left(\frac{1}{3^4}-1\right)\frac{h^6 f^7(x_1)}{7!}.$$

$$+ (3^2-1)h^8.$$

$$\frac{3^2 D_{h/3}-D_h}{3^2-1} = f'(x_1) + \underbrace{\frac{\left(\frac{1}{3^2}-1\right)}{3^2-1}\,\frac{h^4 f^5(x_1)}{5!} + \frac{\left(\frac{1}{3^4}-1\right)}{3^2-1}\,\frac{h^6 f^7(x_1)}{7!}}_{\text{new term.}}$$

$$+O(h^8).$$

$$D_h^{(1)} = f'(x_1) + \underbrace{\frac{\left(\frac{1}{3^2}-1\right)}{(3^2-1)}\,\frac{h^4 f^5(x_1)}{5!} + \frac{\left(\frac{1}{3^4}-1\right)}{3^2-1}\,\frac{h^6 f^7(x_1)}{7!}}_{\text{new term}}$$

$$+O(h^8).$$

(B) # My result is 4th order accurcate.

new term $= -9.26\times10^{-4}\, h^4 f^5(x_1) + -2.45\times10^{-5} h^6 f^7(x_1)$

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$$\cancel{n > \frac{\log(|b_0 - a_0|) - \log(2)}{\log(2)} - 1}.$$

$$\cancel{E} \quad D_h^{(1)} = f'(x) - 9.26\times10^{-4} h^4 f^5(x_1) - 2.45\times10^{-5} h^6 f^7(x_1) + O(h^8).$$

(Ans).

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