

Q#1

We know,

$$P_1(x_0) = a_0 + a_1 x_0.$$

$$\Rightarrow a_0 + a_1 x_0 = 1.$$

$$P_1(x_1) = a_0 + a_1 x_1$$

$$\Rightarrow a_0 + a_1 x_1 = 1.82$$

$$VA = F.$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 0.6 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1.82 \end{pmatrix} \left[\begin{pmatrix} 1 & x_0 \\ 1 & x_1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} f(x_0) \\ f(x_1) \end{pmatrix} \right]$$

$$V = \begin{pmatrix} 1 & 0 \\ 1 & 0.6 \end{pmatrix} \text{ (Ans)} \quad \begin{bmatrix} x_0 = 0 \\ x_1 = 0.6 \end{bmatrix}$$

Q#2

$$V^{-1} = \frac{1}{\det V} \text{Adj } |V|.$$

$$= \frac{1}{(0.6-0)} \begin{vmatrix} 0.6 & 0 \\ -1 & 1 \end{vmatrix}$$

$$= \frac{1}{0.6} \begin{pmatrix} 0.6 & 0 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{0.6}{0.6} & \frac{0}{0.6} \\ \frac{-1}{0.6} & \frac{1}{0.6} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -1.6667 & 1.6667 \end{pmatrix} \text{ (Ans)}$$

$$\left[\begin{array}{l} \det |V| = (0.6-0) \\ V_{11} = (-1)^{1+1} (0.6) = 0.6 \\ V_{12} = (-1)^{1+2} (0) = 0 \\ V_{21} = (-1)^{2+1} (-1) = -1 \\ V_{22} = (-1)^{2+2} (1) = 1 \end{array} \right]$$

$$\textcircled{Q\#3} \quad a_0 = \frac{x_0 f(x_1) - x_1 f(x_0)}{x_0 - x_1} = \frac{0 - 0.6 \times 1}{0 - 0.6} = 1$$

$$a_1 = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = \frac{1 - 1.8221}{0 - 0.6} = 1.3702$$

We know

$$\begin{array}{l|l} x_0 = 0 & f(x_0) = 1 \\ x_1 = 0.6 & f(x_1) = 1.8221 \end{array}$$

$$\# \quad a_0 = 1, \quad a_1 = 1.3702 \quad (\text{Ans})$$

$\textcircled{Q\#4}$

$$\textcircled{1} \quad \text{We know, } P_1(x) = a_0 + a_1 x \\ = 1 + 1.3702x \quad (\text{Ans})$$

$$\textcircled{2} \quad P_1(0.75) = 1 + (1.3702 \times 0.75) = 2.02765 \quad (\text{Ans})$$

$$\textcircled{3} \quad f(x) = e^x = e \\ f(0.75) = e^{0.75} = 2.117$$

$$P_1(x) = 1 + 1.3702x$$

$$P_1(0.75) = 2.02765$$

$$\# \text{ Error} = |f(x) - P_1(x)| = |2.117 - 2.02765| \\ = 0.08935$$

(Ans)

(4).

If we would like to reduce the error in the previous part we need to include more node points to avoid get a higher approximation. By adding more degrees/nodes the answer will be more precise, according to weierstrass approximation (Ans)