ASSIGNMENT-3

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Assignment-4

(a) $\boxed{\text{Ques \#1}}$

(1)

Given $f(x) = 2x^3 - e^x$.

At $x_0 = 2$, step sizes $= 0.1, 0.01 \& 0.001$.

For forward difference,

We know, $f'(x_0) = \dfrac{f(x_0+h) - f(x_0)}{h}$.

at $h = 0.1$,

$$f'(2) = \frac{\{2(x_0+h)^3 - e^{(x_0+h)}\} - \{2(x_0)^3 - e^{x_0}\}}{h}$$

$$= \frac{2(2+0.1)^3 - e^{(2+0.1)} - \{2(2)^3 - e^2\}}{0.1}$$

$$= \frac{10.35 - 8.61}{0.1} = 17.46.$$

at $h = 0.01$,

$$f'(2) = \frac{2(2+0.01)^3 - e^{(2+0.01)} - 8.61}{0.01}$$

$$= \frac{8.78 - 8.61}{0.01} = 16.79.$$

at $h = 0.001$;

$$f'(2) = \frac{2(2+0.001)^3 - e^{(2+0.001)} - 8.61}{0.001} = \frac{8.63 - 8.61}{0.001}$$

$$= 16.62$$

(2)

We know, Truncation error = $\frac{f''(\xi)}{2}(-h)$

$f(x) = 2x^3 - e^x$.

$f'(x) = 6x^2 - e^x$.

$f''(x) = 12x - e^x$

Truncation error, at $x_0 = 2$.

$\boxed{\text{at } h = 0.1}$; ~~error =~~ $\cancel{\frac{12(0.1)}{2}} \times (-0.$

~~at h = 0.01;~~ $f''(2) = 12(2) - e^2$

~~at h = 0.001;~~ $= 16.61$.

error = $\frac{16.61}{2} \times (-0.1) = -0.83$

$\boxed{\text{at } h = 0.01}$

error = $\frac{16.61}{2} \times (-0.01) = -0.083$

$\boxed{\text{at. } h = 0.001}$

error = $\frac{16.61}{2} \times (-0.001) = -0.0083$.

(3).

Relationship between the truncation error & step size is,

$\boxed{\text{truncation error} \propto h}$

(Ans).

(4)

Given, $f(x) = 2x^3 - e^x$.

We know, central difference method

$$f'(x_0) = \frac{f(x_0+2h) - f(x_0)}{2h}.$$

Here, $x_0 = 2$.

$$f'(x_0) = \frac{f(2+2h) - f(2)}{2h.}$$

$$f'(2) = \frac{2(2+2h)^3 - e^{(2+2h)} - \{2.2^3 - e^2\}}{2h.}$$

at $h = 0.1$;

$$f'(2) = \frac{2(2+0.1\times2)^3 - e^{(2+2.0.1)} - (16-e^2)}{2\times0.1}$$

$$= \frac{12.27 - 8.61}{0.2} = 18.3.$$

at $h = 0.01$;

$$f'(2) = \frac{2(2+0.01\times2)^3 - e^{(2+2\times0.01)} - (16-e^2)}{(2\times0.01)}$$

$$= 16.7773$$

at $h = 0.001$;

$$f'(2) = \frac{2(2+0.001\times2)^3 - e^{(2+0.001\times2)} - (16-e^2)}{(2\times0.001)}$$

$$= 16.63$$

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(5).

We know, Truncation error for central difference method,

$$\frac{f^3(\xi)}{3!}(h^2)$$

$f''(x) = 12x - e^x$ [from 2].

$f'''(x) = 12 - e^x$.

for $x_0 = 2$;

$f'''(2) = 12 - e^2 = 4.61$.

$\boxed{\text{at } h = 0.1}$,

$$\text{error} = \frac{f'''(2)}{3!}(\cancel{0.01}\ (0.1)^2$$

$$= \frac{4.61}{3!}(0.1)^2$$

$$= 0.00768 = 7.68 \times 10^{-3}.$$

$\boxed{\text{at } h = 0.01}$

$$\text{error} = \frac{4.61}{3!}(0.01)^2$$

$$= 7.68 \times 10^{-5}.$$

$\boxed{\text{at } h = 0.001}$

$$\text{error} = \frac{4.61}{3!}(0.001)^2.$$

$$= 7.68 \times 10^{-7}.$$

Here, $\boxed{\text{Truncation error} \propto h^2}$

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Ques #2

(1)

$\beta = 2$, $m = 4$, $e_{min} = -1$, $e_{max} = 2$

We know for denormalized form,

$$|x| = \beta^{-1}\beta^{e}$$
$$= 2^{-1}2^{-1}$$
$$= \frac{1}{4} = 0.25$$

$\boxed{\text{minimum } |x| = 0.25}$

(2).

We know for denormalized form,

ϵ_M = Machine Epsilon $= \frac{1}{2} \times \beta^{-m}$

$$= 0.5 \times 2^{-4}$$
$$= \frac{1}{32} = \boxed{0.03125 = \epsilon_m}$$

(3).

We know,
For denormalized form,

$$|fl(x)-x| = \frac{1}{2}\beta^{-m-1}\beta^{e}.$$

$$|x| = \beta^{-1}.\beta^{e}.$$

$$\text{Machine epsilon} = \mathcal{E}_m = \frac{|f(x)-x|}{|x|} = \frac{\frac{1}{2}\beta^{-m-1}\beta^{e}}{\beta^{-1}\beta^{e}} = \frac{1}{2}\beta^{-m}.$$

Machine epsilon ($\mathcal{E}_m$) doesn't depend on its exponent (Ans)

(4).

We know, For normalized form $\mathcal{E}_m = \frac{1}{2}\beta^{-m}$

$$= \frac{1}{2}\times\beta^{-4}$$

$$\boxed{\mathcal{E}_m = \frac{1}{2}\times\frac{1}{2^4} = \frac{1}{32} = 0.03125}$$

(Ans)