# ASSIGNMENT-3

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# Assignment-3

## Ques 1

(1)

$x_0 = 0.1$ | $f(x_0) = -0.62049958$ | $f'(x_0) = 3.58502082$

$x_1 = 0.2$ | $f(x_1) = -0.2898668$ | $f'(x_1) = 3.14033271$

$x_2 = 0.3$ | $f(x_2) = 0.00660095$ | $f'(x_2) = 2.66668043$.

We know, $l_k(x_k) = \dfrac{(x-x_1)\cdots(x-x_n)}{(x_k-x_{n1})\cdots(x_k-x_n)}$

# Langrange Basis.

$l_0(x) = \dfrac{(x-0.2)(x-0.3)}{(0.1-0.2)(0.1-0.3)} = \dfrac{(x-0.2)(x-0.3)}{0.02} = 50x^2 - 25x + 3$
$= \cancel{x^2 - 0.5x + 0.06}$

$l_1(x) = \dfrac{(x-0.1)(x-0.3)}{(0.2-0.1)(0.2-0.3)} = \dfrac{(x-0.1)(x-0.3)}{-0.01} = -100x^2 + 40x - 3$
$= \cancel{-x^2 + 0.4x - 0.03}$

$l_2(x) = \dfrac{(x-0.1)(x-0.2)}{(0.3-0.1)(0.3-0.2)} = \dfrac{(x-0.1)(x-0.2)}{0.02} = 50x^2 - 15x + 1$
$= \cancel{x^2 - 0.3x + 0.02}$

(2).

$x_0 = 0.1$ | $l_0(x) = x^2 - 0.5x + 0.06$ | $l_0'(x) = 2x - 0.5$

$x_1 = 0.2$ | $l_1(x) = -x^2 + 0.4x - 0.03$ | $l_1'(x) = -2x + 0.4$

$x_2 = 0.3$ | $l_2(x) = x^2 - 0.3x + 0.02$ | $l_2'(x) = 2x - 0.3$

$l_0'(x_0) = 2\times(0.1) - 0.5 = -0.3$

$l_1'(x_1) = -2\times(0.2) + 0.4 = 0$.

$l_2'(x_2) = 2(0.3) + 0.3 = 0.9$.

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(2).

| | | | |
|---|---|---|---|
| $x_0 = 0.1$ | $l_0(x) = 50x^2 - 25x + 3$ | $l_0'(x) = 100x - 25$ | $l_0'(0.1) = -15$ |
| $x_1 = 0.2$ | $l_1(x) = -100x^2 + 40x - 3$ | $l_1'(x) = -200x + 40$ | $l_1'(0.2) = 0$ |
| $x_2 = 0.3$ | $l_2(x) = 50x^2 - 15x + 1$ | $l_2'(x) = \cancel{20}\ 100x - 15$ | $l_2'(0.3) = 15$ |

$$h_0(x) = \{1 - 2(x - x_0)\, l_0'(x_0)\}\, l_0^2(x)$$

$$= \{1 - 2(x - 0.1)\times(-15)\}\{50x^2 - 25x + 3)^2$$

$$= \cancel{(1 - 2x +}\ \{1 + 30(x - 0.1)\}(2500x^4 - 2500x^3 + 925x^2 - 150x + 9)$$

$$= (1 + 30x - 3)(2500x^4 - 2500x^3 + 925x^2 - 150x + 9)$$

$$= (30x - 2)(2500x^4 - 2500x^3 + 925x^2 - 150x + 9)$$

$$= 75000x^5 - 80000x^4 + 32750x^3 - 6350x^2 + 570x - 18$$

$$h_1(x) = \{1 - 2(x - 0.2)\times 0\}\{-100x^2 + 40x - 3)^2$$

$$= 1\times(10000x^4 - 8000x^3 + 2200x^2 - 240x + 9)$$

$$h_2(x) = \{1 - 2(x - 0.3)\times 15\}\{50x^2 - 15x + 1)^2$$

$$= \{1 - 30(x - 0.3)\}(\cancel{50x^2 - 15}\ \cancel{2500x^4 -}\ (50x^2 - 15x + 1)(50x^2 - 15x + 1)$$

$$= (1 - 30x + 9)(2500x^4 - 750x^3 + 50x^2 - 750x^3 + 225x^2 - 15x + 50x^2 - 15x + 1)$$

$$= (-30x + 10)(2500x^4 - 1500x^3 + 325x^2 - 30x + 1)$$

$$= -75000x^5 - \cancel{45000}\ 70000x^4 - 24750x^3 + 4150x^2$$

$$- 330x + 10$$

$$\hat{h}_0(x) = (x-x_0)\, l_0^2(x)$$
$$= (x-0.1)(50x^2-25x+3)^2$$
$$= 2500x^5 - 2750x^4 + 1175x^3 - 242.5x^2 + 24x - 0.9$$

$$\hat{h}_1(x) = (x-0.2)(-100x^2+40x-3)^2$$
$$= 10000x^5 - 10000x^4 + 3800x^3 - 680x^2 + 57x - 1.8$$

$$\hat{h}_2(x) = (x-0.3)(50x^2-15x+1)^2$$
$$= 2500x^5 - 2250x^4 + 775x^3 - 127.5x^2 + 10x - 0.3$$

(3)

Hermite polynomial $P_5(x)$.

$$= f(x_0)h_0(x) + f(x_1)h_1(x) + f(x_2)h_2(x) + f'(x_0)\hat{h}_0(x) + f'(x_1)\hat{h}_1(x) + f'(x_2)\hat{h}_2(x)$$

$$= (-0.62049958)(75000x^5 - 80000x^4 + 32750x^3 - 6350x^2 + 570x - 18) + (-0.28398668)(10000x^4 - 8000x^3 + 2200x^2 - 240x + 9) + (0.00660095)(-75000x^5 - 70000x^4 - 24750x^3 + 4150x^2 - 330x + 10) + (3.58502082)(2500x^5 - 2750x^4 + 1175x^3 - 242.5x^2 + 24x - 0.9) + (3.14033271)(10000x^5 - 10000x^4 + 3800x^3 - 680x^2 + 57x - 1.8) + (2.66668043)(2500x^5 - 2250x^4 + 775x^3 - 127.5x^2 + 10x - 0.3)$$ (Ans)

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Ques 2

$\pm(0.1d_1d_2d_3 \cdots d_m)_\beta \beta^e$ with $\beta=2$, $-2\le e\le 5$
$m=4$

1

# Maximum number with sign value $=(0.1111)_2 \cdot 2^5$
$=\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)\cdot 2^5$
$=+30$

2

# Minimum number with sign value $=-30$.

3

# Total numbers generated from the exponent.
$e_{num}=\{-2,-1,0,1,2,3,4,5\}=8$.

Total $=e_{num}\times$ Possibility
$=8\times 2^{m-1}$
$=8\times 2^3$
$=8\times 8=64$

[As there are 4 numbers, 1 is fixed but the 3; $d_1, d_2, d_3$ have possibility of $2^3$]

So there 8 sets each set will have 8 different points.

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(4).

# Maximum value without the use of negative number = 30.

(5)

# Minimum value with out sign bit is = $(0.1000) \times 2^{-2}$
$= \frac{1}{2} \times 2^{-2}$
$= 0.125$

(6).

For $e=5$,
There will be 8 points. They are.

$0.1000 \times 2^5 = 3.2$
$0.1001 \times 2^5 = 3.2032$
$0.1010 \times 2^5 = 3.232$
$0.1011 \times 2^5 = 3.2352$
$0.1100 \times 2^5 = 3.52$
$0.1101 \times 2^5 = 3.5232$
$0.1110 \times 2^5 = 3.552$
$0.1111 \times 2^5 = 3.5552$

Line (Real).