Practice Problems

1. The Runge function $f(x) = \frac{1}{(1+x^2)}$ is between the interval -1 to +1.
   a. Write down the Chebyshev points and the nodes for **n=5**.
   b. Write down the expression for the **Lagrange basis** $L_5(x)$ using the Chebyshev nodes.

2. The Runge function $f(x) = e^{-12x^2}$ is between the interval -1 to +1.
   a. Calculate the Chebyshev nodes for order 5 and order 12 respectively.
   b. If you would have plotted the polynomial functions of **order 5** and **12** respectively on top of the original function $f(x) = e^{-12x^2}$, what could you have deciphered by looking at the plots of the graphs?

1. The upward velocity of a rocket is given as a function of time (t) as:

s	**m/s**
0	**0**
7	**101**
15	**197**
22	**280**
27	**360**
32	**460**

a. Determine the value of the velocity at t=16 seconds with fourth order polynomial interpolation using Newton's divided difference polynomial method.

b. Using the third order polynomial interpolant for velocity, find the distance covered by the rocket from t=11s to t=16s.

c. Using the third order polynomial interpolant for velocity, find the acceleration of the rocket at t=16s.

2. Find the value of y for $x=2.1$ using a 2^{nd} order Lagrange polynomial with the appropriate data sets from the table below. Also find the change of Y between X=1.5 and 2..

Sl.	**x**	**Y**
1	-1	2.2
2	0	10.6
3	1	17.0
4	2	22.4
5	3	25.8

3. Find the value of y for $x=1.8$ using a *3rd* order Newton's divided difference polynomial with the appropriate data sets from the table below.

Sl.	x	Y
1	-1	2.5
2	0	12.6
3	1	19.0
4	2	22.4
5	3	27.8

4. What is Round off error and Truncation true error in numerical method?

5. Let's assume you have a dataset as given below. Perform second order Lagrange interpolation and Newton's divided difference interpolation to find the f(x) for x=1.5. Also comment on the results that you are getting from the two methods.

x	1	1.3	1.6	1.9	2.2
f(x)	0.1411	−0.6878	−0.9962	−0.5507	0.3115

(b) For the given data, fit a Lagrange interpolating polynomial of order four. Use it to estimate the value of the function at $x = 0.65$.

x	−1	−0.5	0	0.5	1
$f(x)$	9	0.625	−5	0	−5

6.

1.

(a) Given the data

x	1.6	2	2.5	3.2	4	4.5
$f(x)$	2	8	14	15	8	2

Calculate $f(2.8)$ using Newton's interpolating polynomials of order 1 and 3. Choose the sequence of the points for your estimates to attain the best possible accuracy.

7. What is the advantage and disadvantage of using Lagrange interpolating polynomial? And how can we solve that problem?

1. Let $f(x)$ is a function. Given nodes are {0, 0.7, 1.4, 2.1} and the values of the function of the nodes are {2, 2.80, 3.431, 7.05}.
   a) Find the Vadermonde matrix for a linear polynomial passing through (0,2) and (0.7, 2.80).
   b) Find the Reverse Vadermonde matrix of the matrix in (a).
   c) Find the value of a_0 and a_1 .

2. In the previous questions, you have computed the coefficients, a0 and a1 of the linear polynomial that passes the points (0,2) and (0.7,2.80) . Using these results, compute the following:
   a) Write down the linear polynomial p1(x).
   b) Compute p1(0.50).
   c) If $f(x)=e^x$, find the error $|f(x)-p1(x)|$ at x=0.50 .
   d) If we would like to reduce the error in the previous part, what we need to do.