Name: Farah Jasmin Khan
ID: 19101239
Section: 06.

Paper Source
Subject.......
Date:....... Time

①.

1. $f(x) = 2x^2 - 77x + 1$.

we know, $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x_1 = \frac{-(-77) + (\sqrt{(77)^2 - 4.2.1})}{2.2} = 38.4870.$$

$$x_2 = \frac{-(-77) - \sqrt{(-77)^2 - 4.2.1}}{2.2} = 0.01299$$

~~a^2~~ ($x^2 - (\alpha+\beta)x + \alpha\beta$.

$x_1 + x_2 = 38.4870 + 0.01299 = 38.4999$

$x_1 . x_2 = 38.4870 \times 0.01299 = 0.4999.$

loss ~~of significance happens when undergo~~ floating
~~point arithmetic~~ operation on two
numbers increas

②

bypass $x_1 + x_2 = 38.4870 + 0.01299$
$= 38.4999.$

$x_1 . x_2 = 38.4870 \times 0.01299$
$= 0.4999$

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Paper Source
Subject
Date Time

loss of significance is whe 2 numbers operation increase. relative error substantialy more than

abs error

(2).

$$x_1 + x_2 = 38.4870 + 0.01297$$

$$x_1 \cdot x_2 = 38.4870 \times 0.01293$$
$$= 0.4999$$

For $2^{11} = 2043$. numbers can be represented with binarys.

but with normalisation exponent

bias = will be lessened to e-1023

so the range is smaller & more accurate.

* This method reduces redundant data.

* data consistency is given.

* flexibility in database design.

we can get equally spaced sets.

with value of m, n & emin. emax.

but all the numbers aren't equally spaced.

(3).

(1) $f(x) = \cos(x)$.

$$x_0 = -\frac{\pi}{4} \quad f(x_0) = 0.707 =$$
$$x_1 = 0 \quad f(x_1) = 1.$$
$$x_2 = \frac{\pi}{4}. \quad f(x_2) = 0.707$$

$$L_0(x_0) = \frac{(x-0)(x-\frac{\pi}{4})}{(-\frac{\pi}{4}-0)(-\frac{\pi}{4}-\frac{\pi}{4})} = \frac{x(x-\pi/4)}{1.2337} = \frac{x(x-\frac{\pi}{4})}{\frac{\pi^2}{8}}$$

$$L_1(x_1) = \frac{(x+\frac{\pi}{4})(x-\pi/4)}{(0+\pi/4)(0-\frac{\pi}{4})} = \frac{(x-\frac{\pi}{4})(x+\frac{\pi}{4})}{-\frac{\pi^2}{16}}$$

$$L_2(x_2) = \frac{(x+\frac{\pi}{4})(x-0)}{(\frac{\pi}{4}+\frac{\pi}{4})(\frac{\pi}{4}-0)} = \frac{x(x+\frac{\pi}{4})}{\frac{2\pi}{4} \times \frac{\pi}{4}} = \frac{x(x+\frac{\pi}{4})}{\frac{\pi^2}{8}}$$

(2).

we know, $P_2(x) = a_0 + a_1 x + a_2 x$.

$$= f(x_0) \cdot l_0(x) + f(x_1) . l_1(x) + f(x_2) l_2(x)$$

$$= 0.707 \times \frac{x(x-\pi/4)}{\frac{\pi^2}{8}} + 1 \times \frac{x^2-(\frac{\pi}{4})^2}{-\frac{\pi^2}{16}} +$$

$$0.707 \times \frac{(x+\pi/4)x}{\frac{\pi^2}{8}}$$

(Ans)

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(i). Round off error is error caused by approximate representation of numbers

For example: $\frac{200}{3} = 66.6667$ here the last digit is rounded ~~to 7~~ from 6 to 7. The diff. between $\frac{200}{3}$ & 66.6667 is round off error.

$\frac{200}{3} - 66.6667$ = round off error.

Truncation error is error caused by truncating a mathmatical procedure.

Ex: In exact differentiation, we need dx approaching zero, in numerical diff we can only choose dx = finite.

(ii).

newton divided difference method.

⑤ (4)

① .

$x_0 = 0.0$ | $f(x_0) = 1.01$ | $f[x_0, x_1] = \dfrac{1.22 - 1.01}{0.2 - 0.0} = 1.05$

$x_1 = 0.2$ | $f(x_1) = 1.22$ | $f[x_1, x_2] = \dfrac{1.49 - 1.22}{0.4 - 0.2} = 1.35$

$x_2 = 0.4$ | $f(x_2) = 1.49.$ |

$$f[x_0, x_1; x_2] = \frac{1.35 - 1.05}{0.4 - 0.0} = 0.75.$$

We know,

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$
$$= f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$
$$= 1.01 + 1.05(x - 0.0) + 0.75(x - 0)(x - 0.2)$$
$$= 1.01 + 1.05x + 0.75x(x - 0.2) = 1.01 + 0.9x + 0.75x^2.$$

②.

$$f(0.25) = P_2(0.25) = 1.01 + 1.05 \times 0.25 + 0.75 \times 0.25(0.25 - 0.2)$$
$$= 1.28219.$$

(Ans)

③. $x_3 = 0.6$ | $f(x_3) = 1.82.$

$$f[x_2, x_3] = \frac{1.82 - 1.49}{0.6 - 0.4} = 1.65$$

Paper Source
Subject
Date Time

$$f[x_1, x_2, x_3] = \frac{1.65-1.35}{0.6-0.2} = 0.75$$

$$f[x_0, x_1, x_2, x_3] = \frac{0.75-0.75}{0.6-0} = 0.$$

$$g_3(x) = f[x_0, x_1, x_2, x_3].(x-x_0)(x-x_1)(x-x_2) = 0.$$

$$
\begin{aligned}
P_3(x) &= P_2(x) + g_3(x) \\
&= 1.01 + 1.05x + 0.75x(x-0.2) \\
&= 1.01 + 1.05x + 0.75x^2 - 0.15x. \\
&= 1.01 + 0.9x + 0.75x^2 \quad \text{(Ans)}
\end{aligned}
$$

5.

1.

$$x_0 = -0.3 \quad f(x_0) = -0.27652$$
$$x_1 = -0.2 \quad f(x_1) = -0.25074$$
$$x_2 = -0.1 \quad f(x_2) = -0.16134.$$

We know, $f'(x_0) = \dfrac{f(x_0+2h) - f(x_0)}{2h}$

$$f'(-0.2) = \frac{f(-0.2+2h) - f(-0.2)}{2h}$$

$$f(x) = e^{2x} - \cos(2x)$$

$$h = \cancel{(-0.3+0.2) = -0.1}. \ (-0.2+0.3) = 0.1$$

$$f'(-0.2) = \frac{e^{2(-0.2+2\times0.1)} - \cos\{2(-0.2+2\times0.1)\} - \{e^{2(0-0.2)} - \cos(2.0.2)\}}{2\times0.1}$$

$$= \frac{\cancel{1} - \cancel{1} - (0.67 - 0.921)}{2\times0.1}$$

$$= \frac{-0.251}{0.2} = \cancel{-1.255}. \ 2.05376.$$

Paper Source
Subject
Date. Time

(2) $[-0.3, -0.1]$

$h = -0.1 + 0.3 = 0.2$

$f(x) = e^{2x} - \cos(2x)$

$f'(x) = 2e^{2x} + \cancel{2\cos(2x)} + 2\sin(2x)$

$f'(-0.2) = 0.56180$, $f'(-0.3) = -0.032$ $f'(-0.1) = 2.84$

For $h = 0.2$!

$$f'(-0.3) = \frac{e^{2(-0.3+2\times 0.2)} - \cos\{2(-0.3+2\times 0.2)\} - \{e^{2\cdot -0.3} - \cos 2\cdot(-0.3)\}}{2\times 0.2}$$

$$= \frac{0.8187 - 0.9801 - (\cancel{0.6488 - 0.8253})}{2\cdot 0.2} = \cancel{-7.6115} = 1.2947$$

$$f'(-0.2) = \frac{e^{2(-0.2+2\times 0.1)} - \cos(2(-0.3+2\times 0.2)) - \{e^{2\times 0.2} - \cos 2(-0.2)\}}{2\times 0.1}$$

$= \cancel{1.255}$ 2.05376

$$f'(-0.1) = \frac{e^{2(-0.1+2\cdot 0.2)} - \cos(2(-0.1+2\cdot 0.2)) - \{e^{2\times 0.1} - \cos 2\cdot(-0.1)\}}{2\times 0.1}$$

$$= \frac{1.221 + 0.77857 - (-0.8187 + 0.1987)}{0.2}$$

$= \cancel{13.09785}$ 2.8953.

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Truncation error for

$x = -0.1$, err $= \cancel{2.8953}\ 2.84 - 2.8953 = -0.0553$

$x = -0.2$ err $= \cancel{2}\cdot 0.56180 - 2.05376 = -1.492$

$x = -0.3$, err $= -0.032 - 1.2947 = -1.3267$.

(3).

We know, Truncation error $= \frac{f^3(x)}{3!}(h^2)$

$f'(x) = 2e^{2x} - 2\sin(2x)$

$f''(x) = 4e^{2x} + 4\cos(2x)$

$f'''(x) = \cancel{4}\ 8e^{2x} \cancel{+} - 8\sin(2x)$

For $h = 0.2$, $f'''(x) = \frac{8e^{2x} - 8\sin(2x)}{3!}(0.2)^2$

$= (8e^{2x} - 8\sin 2x) \times 6.67 \times 10^{-3}$

$f'''(-0.1) = 0.05426$

$f'''(-0.2) = 0.4600$

$f'''(\cancel{9}\ -0.3) = 4.097$

(Ans)

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