ASSIGNMENT-1

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Assignment-1

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Section: 06

(1)

If the function $f(x)$ is interpolated by a polynomial for $x \in [0.5, 1.5]$ the points to be chosen as nodes are. 0, 0.6 & 1.2 as all of them follow this condition $0.5 \leq x \leq 1.5$.
Chosen nodes = {0, 0.6, 1.2}. So points will be (0, 1), (0.6, 1.82211), (1.2, 3.3201)

(2)

First 3 nodes & functions are,

$x_0 = 0$	$f(x_0) = 1$
$x_1 = 0.6$	$f(x_1) = 1.82211$
$x_2 = 1.2$	$f(x_2) = 3.3201$

For. $P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0.6)(x-1.2)}{(0-0.6)(0-1.2)} = \frac{(x-0.6)(x-1.2)}{0.72}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-0)(x-1.2)}{(0.6-0)(0.6-1.2)} = \frac{-x(x-1.2)}{0.36}$$

$$l_2(x) = \frac{(x-x_1)(x-x_0)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-0)(x-0.6)}{(1.2-0)(1.2-0.6)} = \frac{x(x-0.6)}{0.72}$$

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Langrange basis for interpolating function $P_2(x)$.

$$l_0(x) = \frac{(x-0.6)(x-1.2)}{0.72}$$

$$l_1(x) = \frac{-x(x-1.2)}{0.36}$$

$$l_2(x) = \frac{x(x-0.6)}{0.72} \quad \text{(Ans)}$$

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(3).

Algebraic expression of $P_2(x)$

$= l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$

From (2) we got,

$$l_0(x) = \frac{(x-0.6)(x-1.2)}{0.72}$$

$$l_1(x) = -\frac{x(x-1.2)}{0.36}$$

$$l_2(x) = \frac{x(x-0.6)}{0.72}$$

functions:

$f(x_0) = 1$; $f(x_1) = 1.8221$; $f(x_2) = 3.3201$

From putting the values in the formula we get,

$$= \frac{x(x-0.6)(x-1.2)}{0.72} \times 1 + \frac{-x(x-1.2)}{0.36} \times 1.8221 + \frac{x(x-0.6)}{0.72} \times 3.3201$$

$$= x\,\frac{x^2-1.2x-0.6x+0.72}{0.72} + (-x^2+1.2x) \times 5.061 + (x^2-0.6x) \times 4.611$$

$$= \frac{25}{18}x^2 - \frac{5}{2}x + 1 - 5.061x^2 + 6.0732x + 4.611x^2 - 2.767x$$

$$= \frac{169}{180}x^2 + \frac{4031}{5000}x + 1$$

$$= 1 + 0.81x + 0.94x^2 = P_2(x)$$

(Ans)

④

We know,

$f(x) = e^x$

$P_2(x) = 1 + 0.81x + 0.94x^2$

For $x=0$,

$f(0) = e^0 = 1$

$P_2(0) = 1 + (0.81 \times 0 + 0.94 \times 0^2) = 1.$

$|f(x) - P_2(x)| = |1-1| = 0 \quad [x=0]$

For $x = 0.6$

$f(0.6) = e^{0.6} = 1.8221$

$P_2(0.6) = 1 + (0.81 \times 0.6) + (0.94 \times 0.6^2) = 1.8221.$

$|f(x) - P_2(x)| = |1.822 - 1.8221| = 0. \quad [x=0.6]$

For $x = 1.2$

$f(1.2) = e^{1.2} = 3.32$

$P_2(1.2) = 1 + (0.81 \times 1.2) + (0.94 \times 1.2^2) = 3.32.$

$|f(1.2) - P_2(1.2)| = |3.32 - 3.32| = 0.$

For 0, 0.6 & 1.2 po nodal points $|f(x) - P_2(x)| = 0$ [verified]

(5).

We know, $f(x) = e^x$

$P_2(x) = 1 + 0.81x + 0.94x^2$.

For $x = 0.75$

$f(0.75) = e^{0.75} = 2.117$

$P_2(0.75) = 1 + (0.81 \times 0.75) + \{0.94 \times (0.75)^2\} = 2.1536$.

$\# \quad |f(0.75) - P_2(0.75)| = |2.117 - 2.136|$

$= |-0.0191|$

$= 0.019.$

≈ 0.02 (Ans).