Name: Farah Jasmin khan
ID: 19101239.
Section: CSE06

Paper Source
Subject.........
Date:.........Time

Q #4.

(1) The chebyshev points will be defined:

$$\boxed{\theta_j = \frac{(2j+1)\pi}{2(n+1)}}$$ where $j = 0, 1, \dots n$
n = number of nodes.

(2) $f(x) = \frac{1}{1+25x^2}$; $[-2,2]$. for $n=7$.

$$\theta_j = \frac{(2j+1)\pi}{2(n+1)} = \frac{(2j+1)\pi}{16}$$

$\theta_0 = \frac{\pi}{16}$; $\theta_1 = \frac{3\pi}{16}$; $\theta_2 = \frac{5\pi}{16}$; $\theta_3 = \frac{7\pi}{16}$; $\theta_4 = \frac{9\pi}{16}$; $\theta_5 = \frac{11\pi}{16}$.

$\theta_6 = \frac{13\pi}{16}$; $\theta_7 = \frac{15\pi}{16}$ → $\boxed{\text{Chebyshev points}}$

(3). $x_j = 2\cos\theta_j$ as $[-2,2]$. The $\boxed{\text{chebyshev nodes}}$ are–

so, $x_0 = 2\cos\frac{\pi}{16} = \cancel{0.393}$; x, 1.96.

$x_1 = 2\cos\frac{3\pi}{16} = 1.66$

$x_2 = 2\cos\frac{5\pi}{16} = 1.11$

$x_3 = 2\cos\frac{7\pi}{16} = 0.39$

$x_4 = 2\cos\frac{9\pi}{16} = -0.39$

$x_5 = 2\cos\frac{11\pi}{16} = -1.11$

$x_6 = 2\cos\frac{13\pi}{16} = -1.66$

$x_7 = 2\cos\frac{15\pi}{16} = -1.96$

Paper Source
Subject
Date Time

(4) We know,

$$L_7(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)}{(x_7-x_0)(x_7-x_1)(x_7-x_2)(x_7-x_3)(x_7-x_4)(x_7-x_5)(x_7-x_6)}$$

$$= \frac{(x-1.96)(x-1.66)(x-1.11)(x-0.39)(x+0.39)(x+1.11)(x+1.66)\cancel{(x+1.96)}}{(-1.96-1.96)(-1.96-1.66)(-1.96-1.11)(-1.96-0.39)(-1.96+0.39)(-1.96+1.11)(-1.96+1.66)}$$ [From 3]

$$= \frac{(x-1.96)(x-1.66)(x-1.11)(x-0.39)(x+0.39)(x+1.11)(x+1.66)\cancel{(x+1.96)}}{(-3.92)\times(-3.62)\times(-3.07)\times(-2.35)\times(-1.57)(-0.85)(-0.3)}$$

$$= \frac{(x-1.96)(x-1.66)(x-1.11)(x-0.39)(x+0.39)(x+1.11)(x+1.66)}{-40.98}$$

$$= \frac{(x^2-2.75)(x^2-1.23)(x^2-0.152)(x-1.96)}{-40.98}$$ (Ans)

$= L_7(x)$

= Langrange basis.