Solution: Assignment #7

Consider the following two equations:
Equation-1: $f(x) = x^2 + 3x - 7$
Equation-2: $f(x) = x^3 + 3x^2 + 5x + 7$

Question # 1: [8 Marks] Solve the 1^{st} equation $f(x) = 0$ using Newton's Method. Show your calculation for at least 4 iterations, and also express the numerical values up to five decimal places.

Question # 2: [8 Marks] Solve the 2^{nd} equation $f(x) = 0$ using Secant Method. Show your calculation up to at least 4 iterations, and also express the numerical values up to five decimal places.

Question # 3: [4 Marks] Explain which technique used in the previous two questions to find the root is better to find x_*. Give at least two reasons.

Q#1: Here $f(x) = x^2 + 3x - 7$

So, $f'(x) = 2x + 3$

$$\therefore g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^2+3x-7}{2x+3}$$

$$= \frac{2x^2 + \cancel{3x} - x^2 - \cancel{3x} + 7}{2x+3}$$

$$\Rightarrow g(x) = \frac{x^2+7}{2x+3}$$

So, the iteration formula is:

$x_{k+1} = g(x_k)$; $k = 0, 1, \cdots$

Let's start with: $x_0 = 3$.

$x_0 = 3 \Rightarrow x_1 = g(x_0) = \frac{3^2+7}{2(3)+3} = 1.77778$

$x_1 = 1.77778 \Rightarrow g(x_1) = 1.54991$

$x_2 = 1.54991 \Rightarrow g(x_2) = 1.54139$

$x_3 = 1.54139 \Rightarrow g(x_3) = 1.54138$

$\boxed{x_4 = 1.54138} \Rightarrow g(x_4) = 1.54138$

$\Rightarrow$ After 4 iterations: $\boxed{x_* \approx x_4 = 1.54138}$

Q#2 $f(x) = x^3 + 3x^2 + 5x + 7$.

Here the iteration formula is

$$x_{k+2} = x_{k+1} - \frac{f(x_{k+1})(x_{k+1} - x_k)}{f(x_{k+1}) - f(x_k)}$$

with $k = 0, 1, 2, \cdots$, etc.

Let's chose: $x_0 = 0$ and $x_1 = 1$

$$\therefore x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 0.77778.$$

$$x_3 = x_2 - \frac{f(x_2)\cdot(x_2 - x_1)}{f(x_2) - f(x_1)} = 2.25825.$$

$$x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)} = 1.38857.$$

$$x_5 = x_4 - \frac{f(x_4)(x_4 - x_3)}{f(x_4) - f(x_3)} = 3.98621.$$

After 4 iterations:

$\boxed{x_* \approx x_5 \approx 3.98621}$

Q#3 There are two advantages using the secant method:

(i) No need to worry about the turning point because $f'(x)$ is not involved.

(ii) Only one function to be calculated per iteration.