Assignment 6

Question #1

1)

a) $x = (3 + x - 2x^2)^{\frac{1}{4}}$

$\Rightarrow x^4 = 3 + x - 2x^2$

$\Rightarrow x^4 + 2x^2 - x - 3 = 0$

$\therefore f(x) = 0$

b) $x = \left(\frac{x + 3 - x^4}{2}\right)^{\frac{1}{2}}$

$\Rightarrow 2x^2 = x + 3 - x^4$

$\Rightarrow x^4 + 2x^2 - x - 3 = 0$

$\therefore f(x) = 0$

c) $x = \left(\frac{x+3}{x^2+2}\right)^{\frac{1}{2}}$

$\Rightarrow x^2(x^2+2) = x + 3$

$\Rightarrow x^4 + 2x^2 - x - 3 = 0$

$\therefore f(x) = 0$

d) $x = \frac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1}$

$\Rightarrow 4x^4 + 4x^2 - x = 3x^4 + 2x^2 + 3$

$\Rightarrow x^4 + 2x^2 - x - 3 = 0$

$\therefore f(x) = 0$

2)

a) $p_1 = g_1(p_0) = 1.18921$

$p_2 = g_1(p_1) = 1.08006$

$p_3 = g_1(p_2) = 1.14967$

$p_4 = g_1(p_3) = 1.10782$

b) $p_1 = g_2(p_0) = $ ~~1.09605~~ 1.22474

$p_2 = g_2(p_1) = 0.99367$

$p_3 = g_2(p_2) = 1.22857$

$p_4 = g_2(p_3) = 0.98750$

c) $p_1 = g_3(p_0) = 1.15470$

$p_2 = g_3(p_1) = 1.11643$

$p_3 = g_3(p_2) = 1.12605$

$p_4 = g_3(p_3) = 1.12364$

d) $p_1 = g_4(p_0) = 1.14286$

$p_2 = g_4(p_1) = 1.12498$

$p_3 = g_4(p_2) = 1.12912$

$p_4 = g_4(p_3) = 1.12412$

3) Part (d) gives the best answer since $|p_4 - p_3|$ is the smallest For d

Question # 2 $f(x) = x^3 + 4x^2 - x - 4$

1) $f(x) = 0$

$x^3 + 4x^2 - x - 4 = 0$

$\Rightarrow x^2(x+4) - 1(x+4) = 0$

$\therefore x = \pm 1, -4$

2)

Case - I

$x = x^3 + 4x^2 - 4 \Rightarrow g(x) = x \quad \cdots (i)$

Case - II

$x(x^2 + 4x - 1) = 4$

$\therefore x = \dfrac{4}{x^2 + 4x - 1} \Rightarrow g(x) = x \quad (ii)$

Case - III

$4x^2 = -x^3 + x + 4$

$\therefore x = \pm \dfrac{1}{2}\sqrt{-x^3 + x + 4} \Rightarrow g(x) = x \quad (iii)$

3) convergence rate $\lambda = g'(x) = \left| \frac{dg}{dx} \Big|_{x=x^*} \right|$

case-I

$g(x) = x^3 + 4x^2 - 4$

$g'(x) = 3x^2 + 8x$

$\therefore \lambda = |g'(x_*)| = \begin{cases} -5 & \text{for } x_* = -1 \Rightarrow \text{divergence} \\ 11 & \text{For } x_* = 1 \Rightarrow \text{"} \\ 16 & \text{For } x_* = -4 \Rightarrow \text{"} \end{cases}$

case-II

$g(x) = \frac{4}{x^2 + 4x - 1}$

$g'(x) = -4(x^2+4x-1)^{-2} \times 2x + 4$

$= -\frac{8x - 16}{(x^2+4x-1)^2}$

$\therefore \lambda = |g'(x_*)| = \begin{cases} 0.5 & \text{For } x_* = -1 \Rightarrow \text{linear convergence} \\ 1.5 & \text{For } x_* = 1 \Rightarrow \text{divergence} \\ 16 & \text{For } x_* = -4 \Rightarrow \text{"} \end{cases}$

case-III

$g(x) = \frac{1}{2}(-x^3 + x + 4)^{\frac{1}{2}}$

$g'(x) = \frac{1}{4}(-x^3 + x + 4)^{-\frac{1}{2}} \times (-3x^2 + 1)$

$= \frac{-3x^2 + 1}{4\sqrt{-x^3 + x + 4}}$

$\therefore \lambda = |g'(x_*)| = \begin{cases} 0.25 & \text{for } x_* = -1 \Rightarrow \text{linear convergence} \\ 0.25 & \text{for } x_* = 1 \Rightarrow \text{"} \\ 1.47 & \text{for } x_* = -4 \Rightarrow \text{divergence} \end{cases}$