Lab 1: The Apollo Mission Report

I. Introduction

The Apollo missions are the first attempts of humans reaching the Moon and in order to get there safely, the gravitational potential and the forces the Apollo 11 command module will face must be understood. Gravitational potential is the work done to move an object from infinity towards a point. When the rocket launches, it will face the gravitational potential of the Earth and Moon with varying strengths. Gravitational potential can be found with the equation below.

$$\Phi(r) \; =- \frac{GM}{r} \qquad [1]$$

Where G is the gravitational constant, M is the mass of the body creating the potential, and r is the separation between the objects. Potential is a negative quantity that approaches zero.

Anything with mass will exert a gravitational force on nearby objects, which is the pull those objects feel from one another. As the command module takes off, it will feel a gravitational force from the Earth, and as it approaches the Moon, it will feel a different gravitational force from the Moon. The gravitational force is shown in Equation 2.

$$\vec{F}_{21} =- \; G\frac{M_1 m_2}{\left|\vec{r}_{21}\right|^2}\hat{r}_{21} \qquad [2]$$

Where $\vec{r}_{21}$ is the distance between the two masses, and $\hat{r}_{21}$, the unit vector, is the direction vector divided by the magnitude of the vector itself.

II. The Gravitational Potential of the Earth-Moon System

Using Equation 1, the gravitational potential of the Earth-Moon system was found. This is the potential that Apollo 11 travels through to land safely on the Moon. Figure 1 shows the combined system with a logarithmically scaled colorbar to better visualize what this means.

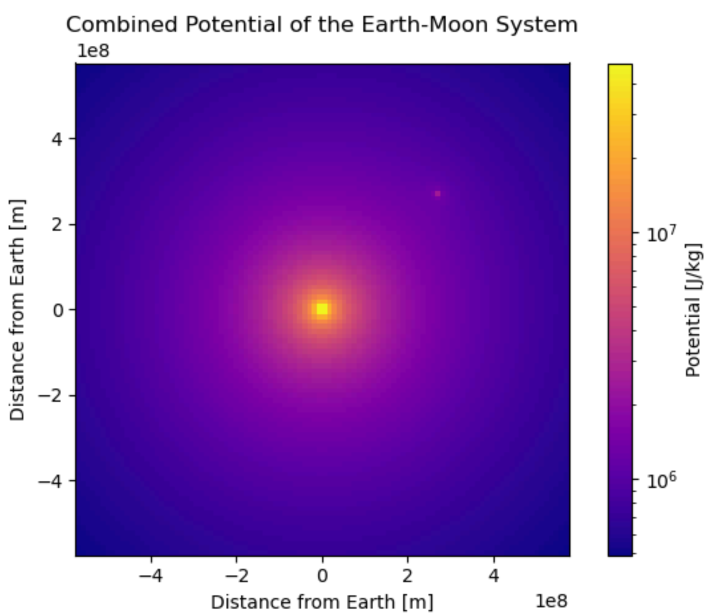

Figure 1: Mesh Plot of the combined gravitational potential of the Earth-Moon system. The colorbar is logarithmically scaled.

This was made by calculating the values of the gravitational potential created by the Earth and the Moon separately using Equation 1 and summing them together. The potential was evaluated in a grid in every direction of the chosen (x,y) plane. These values were then plotted in a mesh grid, which attributes a specific color to the strength of the potential due to the surrounding objects.

The gravitational potential of the Earth is shown at the center and the potential for the Moon is at a distance of 1.5 Earth-Moon distances away (in meters). As shown in the plots and the equation, the Earth has a more influence on Apollo 11.

III. The Gravitational Force of the Earth-Moon System

The gravitational force of the Earth-Moon system on the command module can be found using Equation 2. As the module travels to the Moon, it will feel the gravitational force exerted by the Earth decrease and the force of the Moon increases. Below in Figure 2, a plot of the gravitational force of the Earth-Moon System on the rocket is shown.

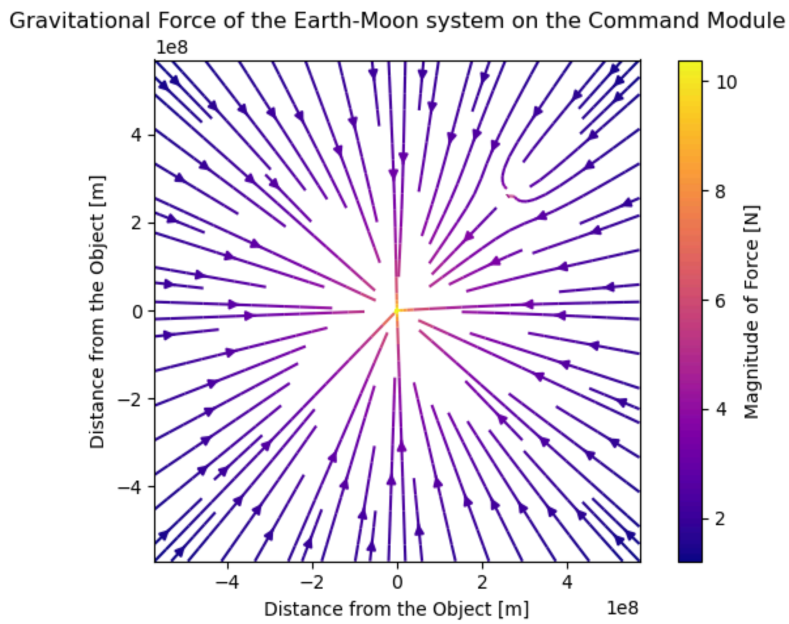

Figure 2: The gravitational force of the Earth and the Moon on the Command Module with a logarithmic color scale.

This was plotted using Equation 2 to calculate the gravitational force due to each body in the x and y-direction. This equation intakes two masses: the primary and secondary mass. The primary mass is the Earth or Moon and the secondary mass is the command module. Each component of force from the Earth and Moon are summed together respectively to find the combined force in each direction. This was then graphed on the same grid points as Figure 1 to characterize the force on the same scale. A logarithmic colorbar was also added to this plot to provide meaning to the lines.

As shown in the plot, the origin is where the Earth resides and about 3.8 on both axes is where the Moon is. The Earth exerts a larger gravitational force on the rocket.

IV. Projected Performance of the Saturn V Stage I

As the Saturn V rocket takes off it will undergo three stages, none of which can happen unless stage I is completed. For a successful mission it is important to note the total burn time, the acceleration the rocket faces, and the altitude it reaches by the end of the first stage. This can be experimentally calculated to determine the projected performance of this process. The burn time, acceleration, and altitude can be calculated using Equation 3, 4, and 5, respectively.

$$T = \frac{m_0 - m_f}{\dot{m}} \qquad [3]$$

Where m_0 is the initial wet mass (fuel, rocket parts, payload), m_f is the mass of the rocket once the fuel is used completely, and $\dot{m}$ is the fuel burn rate, which we assume to be constant.

$$\Delta v(t) = v_e ln(\frac{m_0}{m(t)}) - gt \qquad [4]$$

Where v_e is the fuel exhaust velocity, g is the gravitational acceleration, t is time, and $m(t) = m_0 - \dot{m}t$.

$$h = \int_0^T \Delta v(t)dt \qquad [5]$$

Using these equations, I found the projected total burn time and altitude to be $157.7\ s$ and $74094 \pm 5 \times 10^{-8}\ m$ respectively.

V. Discussion and Future Work

In doing these calculations, we assumed the Moon and the Earth to be in the same plane, despite there being a five degree difference, and approximated these bodies to be point masses so we could use Equation 1. To make these calculations more realistic in future work, we could calculate the potentials in part II to start from the surface of the Earth and Moon because the rocket can never be inside of those bodies so those potentials never need to be considered. We could also treat these bodies as non-uniform density spheres rather than point sources due to our close proximity to them.

From the test results that NASA received with the prototype of Saturn V, the burn time was $160\ s$ and the altitude was about $70\ km$. This is 1.01 and 0.94 times greater than what was projected. This overestimate could be because in the projected calculations we did not consider drag force on the module, which would act against the rocket causing it to use more fuel.