# Lab 1 – Bunny – Algorithm

$r_i(t)$: The location of the *i*th particle in world space at time *t*

$$r_i' r_i'^T = \begin{pmatrix} r_{ix}' \\ r_{iy}' \\ r_{iz}' \end{pmatrix} \begin{pmatrix} r_{ix}' & r_{iy}' & r_{iz}' \end{pmatrix} = \begin{pmatrix} {r_{ix}'}^2 & r_{ix}' r_{iy}' & r_{ix}' r_{iz}' \\ r_{iy}' r_{ix}' & {r_{iy}'}^2 & r_{iy}' r_{iz}' \\ r_{iz}' r_{ix}' & r_{iz}' r_{iy}' & {r_{iz}'}^2 \end{pmatrix}$$

$$r_i'^T r_i' = {r_{ix}'}^2 + {r_{iy}'}^2 + {r_{iz}'}^2$$

$I_{ref}$ is specified in body-space and is constant over the simulation. So, it is a state for a body before simulation begins.

$$I_{ref} = \sum m_i (r_i^T r_i \mathbf{1} - r_i r_i^T) = \sum \begin{pmatrix} m_i\left({r_{iy}'}^2 + {r_{iz}'}^2\right) & -m_i r_{ix}' r_{iy}' & -m_i r_{ix}' r_{iz}' \\ -m_i r_{iy}' r_{ix}' & m_i\left({r_{ix}'}^2 + {r_{iz}'}^2\right) & -m_i r_{iy}' r_{iz}' \\ -m_i r_{iz}' r_{ix}' & -m_i r_{iz}' r_{iy}' & m_i\left({r_{ix}'}^2 + {r_{iy}'}^2\right) \end{pmatrix}$$

where **1** is identity matrix.

Since

$$a \times b = \begin{pmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{pmatrix} = \begin{pmatrix} a_y b_z - b_y a_z \\ -a_x b_z + b_x a_z \\ a_x b_y - b_x a_y \end{pmatrix}$$

and

$$a^* b = \begin{pmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{pmatrix} \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a \times b$$

so, $a^* = \begin{pmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{pmatrix}$.

The algorithm of Impulse method is shown in *Algorithm I*.

| ***Algorithm I*** |
|---|
| $R \leftarrow$ Matrix.Rotate($q$)<br>$x_i \leftarrow x + Rr_i$<br>$\phi(x) \leftarrow (x-p)\cdot n$<br>**if** $\phi(x) > 0$<br> **then** done<br>**else**<br> find all positions ($r_i$) where collision happened<br> compute the number of collisions<br> **if** there is no collision<br>  done<br> **else**<br>  $r_i \leftarrow \frac{sum\ of\ r_i}{number\ of\ collisions}$<br>  $Rr_i \leftarrow R\cdot r_i$<br>  $v_i \leftarrow v + \omega \times Rr_i$<br>  **if** $v_i\cdot N$ is not less than 0<br>   **then** done<br>  **else**<br>   $v_{N,i} \leftarrow (v_i\cdot N) * N$<br>   $v_{T,i} \leftarrow v_i - v_{N,i}$<br>   $a \leftarrow \max(1-\mu_T(1+\mu_N)\|v_{N,i}\| / \|v_{r,i}\|, 0)$<br>   $v_{N,i}^{new} \leftarrow -1.0f * \mu_N * v_N$<br>   $v_{T,i}^{new} \leftarrow a * v_T$<br>   $v_i^{new} \leftarrow v_{N,i}^{new} + v_{T,i}^{new}$<br>   $I \leftarrow RI_{ref}R^T$<br>   compute $I^{-1}$<br>   compute $(Rr_i)^*$<br>   $K \leftarrow \frac{1}{M}\mathbf{1} - (Rr_i) * I^{-1}(Rr_i) *$ |

$J \leftarrow K^{-1}(v_i^{new} - v_i)$

**update**

$v \leftarrow v + \frac{1}{M}J$

$\omega \leftarrow \omega + I^{-1}(Rr_i \times J)$

$q \leftarrow q + \left[0 \quad \frac{\Delta t}{2}\omega\right] \times q$

$x$