Notes: Notes on literature of Labour Economics

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Chapter 1

Intensive and extensive margins of female labour supply

1.1 The baseline model

The utility function is defined as:

$$u(c_{h,t}, l_{h,t}) = \frac{M_{h,t}^{1-\gamma}}{1-\gamma} \exp\left(\pi z_{h,t} + \zeta_{h,t}\right).$$

The preference aggregator (why aggregator) for hours of leisure and consumption is

$$M_{h,t}(c_{h,t}, l_{h,t}; z_{h,t}, \chi_{h,t}) = \left(\frac{(c_{h,t}^{1-\phi} - 1)}{1-\phi} + (\alpha_{h,t}(z_{h,t}, \chi_{h,t})) \frac{(l_{h,t}^{1-\theta} - 1)}{1-\theta} \right)$$

$\alpha_{h,t} = \exp\left(\psi_0 + \psi_z z_{h,t} + \chi_{h,t}\right)$

Suppose for now there is no random discrete choice, the inter-temporal budget constraint:

$$A_{h,t+1} = (1 + r_{t+1}) \left(A_{h,t} + (w_{h,t}^f (H - l_{h,t}) + y_{h,t}^m - c_{h,t} \right)$$

$A_{h,t}$ is the beginning asset holding, r_t is the risk-free interest rate, F is the fixed cost of work, dependent on the age of the youngest child, $a_{h,t}$. Female wages are given by $w_{h,t}^f$ and husband earning $y_{h,t}^m$.

The maximization is done with respect to the following payoffs:

$$\max_{c_t, l_t} u(c_t, l_t) + \beta V(A_{t+1})$$
$$\text{subject to } A_{t+1} = (1 + r_{t+1}) \left(A_t + w_{h,t}^f (H - l_t) + y_{h,t}^m - c_t \right)$$

The first order condition with respect to $c_t, l_t and A_t$ are

$$
\begin{aligned}
u_c(c_t, l_t) + \beta(1 + r_{t+1})V'(A_{t+1}) &= 0 \\
u_l(c_t, l_t) + \beta w_{h,t}(1 + r_{t+1})V'(A_{t+1}) &= 0 \\
V'(A_t) &= \beta(1 + r_{t+1})V'(A_{t+1})
\end{aligned}
$$

From the model we can have two conditions

MRS condition: $w_{ht}u_{c,ht} = l_{c,ht}$

Euler Equation: $u_{c,ht} = \beta u_{c,ht+1}$, in addition: $u_{l,ht} = u_{l,ht+1}\dfrac{w_{h,t}}{w_{h,t+1}}$

The regression from MRS Therefore from the MRS condition, we have that

$$
\begin{aligned}
w_{h,t}c_{h,t}^{-\phi} &= \alpha_{h,t}(z_{h,t}, \chi_{h,t}))l_{h,t}^{-\theta} \\
\log(w_{h,t}) + \phi\log(c_{h,t}) &= \psi_0 + \psi_1 z_{h,t} + \xi_{h,t} - \theta\log(l_{h,t})
\end{aligned}
$$

The parameters can be identified from MRS:

- We observe the wages, consumptions, leisure time and the exogenous variables: $\{w, c, l, z\}$
- From the reduced form regression, we have the preference shocks identified (ψ_0, ψ_1) with the assumption that $\xi_{h,t}$ is normal.
- We also identify the structural preference parameter ϕ and θ.
- After we obtain $\hat{\phi}, \hat{\theta}, \hat{\psi}_0, \hat{\psi}_1$ we cam back out $\hat{M}(c_{h,t}, l_{h,t})$
- Then $\log M(c_{h,t}, l_{h,t}) - \log \hat{M}(c_{h,t}, l_{h,t})$ follows the assumed distribution

The Euler equation and what can be identified from it? The Euler equation condition is that

$$
\begin{aligned}
&\exp(\pi z_{h,t} + \zeta_{h,t})M(c_{h,t}, l_{h,t})^{-\gamma}c_{h,t}^{-\phi} \\
&= \beta(1 + r_{t+1})\exp(\pi z_{h,t+1} + \zeta_{h,t+1})M(c_{h,t+1}, l_{h,t+1})^{-\gamma}c_{h,t+1}^{-\phi}
\end{aligned}
$$

$$
M(c_{h,t}, l_{h,t}) = \left(\frac{(c_{h,t}^{1-\phi} - 1)}{1-\phi} + (\alpha_{h,t}(z_{h,t}, \chi_{h,t}))\frac{(l_{h,t}^{1-\theta} - 1)}{1-\theta}\right)
$$

Log-linearize and plug in the estimated utility to have that

$$
\begin{aligned}
&\pi z_{h,t} + \zeta_{h,t} - \phi c_{h,t} - \gamma\log(\hat{M}(c_{h,t}, l_{h,t})) \\
&\quad = \log(\beta) + \log(1 + r_{t+1}) + \pi z_{h,t+1} + \zeta_{h,t+1} - \gamma\log(\hat{M}(c_{h,t+1}, l_{h,t+1})) - \phi c_{h,t+1}
\end{aligned}
$$

Then we have $\pi\Delta z_{h,t} + \Delta\zeta_{h,t} - \hat{\phi}\Delta c_{h,t} - \gamma\Delta\log\hat{M}(c_{h,t}, l_{h,t}) + \beta + \log(1 + r_t) = 0$

What parameters we can estimate from the Euler equation

- It feels wierd that no transition of $z_{h,t}$ is assumed? But may be we don't need this assumption.
- We can estimate the parameters π, γ, β from the data?
- The r_t is observed?

1.2 Section with discrete choice

The additional parameters are ξ and $F(a_{h,t})$ where $F(a_{h,t})$ If there is a discrete choice to participate in labour market, then the utility function is defined as:

$$u(c_{h,t}, l_{h,t}, P_{h,t}) = \frac{M_{h,t}^{1-\gamma}}{1-\gamma} \exp\left(\xi P_{h,t} + \pi z_{h,t} + \zeta_{h,t}\right).$$

$$M_{h,t}(c_{h,t}, l_{h,t}; z_{h,t}, \chi_{h,t}) = \left(\frac{(c_{h,t}^{1-\phi} - 1)}{1-\phi} + (\alpha_{h,t}(z_{h,t}, \chi_{h,t})) \frac{(l_{h,t}^{1-\theta} - 1)}{1-\theta} \right)$$

$\alpha_{h,t} = \exp\left(\psi_0 + \psi_z z_{h,t} + \chi_{h,t}\right)$ The intertemporal budget constraint is defined as:

$$A_{h,t+1} = (1 + r_{t+1}) \left(A_{h,t} + \left(w_{h,t}^f (H - l_{h,t}) - F(a_{h,t}) \right) P_{h,t} + y_{h,t}^m - c_{h,t} \right)$$

The parameters that we can get from MRS and Euler Equation

- Similarly, estimate the parameters within $M(c_{h,t}, l_{h,t})$ function and obtain $\hat{M}(c_{h,t}, l_{h,t})$.
- The Euler equation is slightly different:

$$\xi \Delta P_{h,t} \pi \Delta z_{h,t} + \Delta \zeta_{h,t} - \hat{\phi} \Delta c_{h,t} - \gamma \Delta \log \hat{M}(c_{h,t}, l_{h,t}) + \beta + \log(1 + r_t) = 0$$

- ξ is the entry cost?

Random shocks: what's confusing to me.

- No entry cost of labour market.
- The shock ζ induce increase of current period consumption/labour.
- Are ζ and χ correlated?

1.3 My estimation strategy?

I would propose a 3-step estimation strategy. In the first step, in addition to estimating MRS, I will need to estimate the transition of z_{it} as well.(So that the agents have a prior about the future events.)

In the second step, estimate the entry decision of the labour market.

In the third step, estimate the forward looking continuous choice.

Attanasio et al. (2015) Related

- Keane, M. P. and R. Rogerson (2012): Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom," Journal of Economic Literature, 50, 464-476.
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- Blundell, R., A. Bozio, and G. Laroque (2011): Labor Supply and the Extensive Margin," American Economic Review, 101, 482-486.
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- Rogerson, R. and J. Wallenius (2009): Micro and Macro Elasticities in a Life Cycle Model with Taxes," Journal of Economic Theory, 144, 2277-2292.
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- Imai, S. and M. P. Keane (2004): Intertemporal Labor Supply and Human Capital Accumulation," International Economic Review, 45, 601-641.
- Keane, M. P. and N. Wasi (2016): Labor Supply: the Roles of Human Capital and the Extensive Margin," The Economic Journal, 126, 578-617.

1.4 Intro

The paper estimates labour supply elasticities at the micro level and show that we can learn from very heterogeneous elasticities for aggregate behavior. The paper use US CEX DATA. The paper shows there is substantial heterogeneity in how individuals respond to wage changes at all margins, both due to observables,(age, wage, hours worked and the wage level) as well as unobservables(tastes for leisure.)

The paper estimate the distribution of Marshallian eslaticity to be 0.54 and Frisch elasticity to be 0.87.

The estimation is based on percentile.

1.4.1 Background

- The size of elasticity of labour supply to changes in wages has been studied for a long time.
- Recent debates focused on the perceived discrepancy between estimates coming from micro studies. Keane and Rogerson (2015) and Keane and Rogerson (2012) survey some of these issues and the papers by Blundell et al. (2011), Ljungqvist and Sargent (2011) and Rogerson and Wallenius (2009) contain some alternative views on the debate.
- Preferences for consumption and leisure are affected in family composition, health status, fertility, and unobserved taste.
- Therefore elasticity vary across section, even across business cycle.
- **Problem**: how big is the variance.
- Whole economic environment is important in aggregate labour supply, Chang and Kim(2006) the aggregate response depends on the distribution of reservation wages.
- In macro, assume consumption and leisure additive separable.

1.4.2 The paper

- The paper make assumption about shape of utility function. The flexible specification allow for observed and unobserved heterogeneity in taste.
- Estimate the full life-cycle model and aggregate individual behavior similar to Erosa et all(2016).
- Take way imply future labour supply: Keane and Wasi (2016), Imain and Keane(2004).
- Keane and Wasi(2016) introduce human capital, and find labour supply elasticities are highly heterogeneous and vary substantiallywith age, education and tax structure.

1.5 Life cycle model of female labour supply

In our model, both the intensive and extensive margins are meaningful because of the presence of fixed costs of going to work related to family composition and because of preference costs specifically related to participation.

$$\max_{c,l} E_t \sum_{j=0}^{T} \beta^j u(c_{h,t+j}, l_{h,t+j}, P_{h,t+j}; z_{h,t+j}, \chi_{h,t+j}, \zeta_{h,t+j}),$$

where c is the consumption, l is the female leisure, P is indicator of women's labour force participation, z is demographic pattern(education, age, family composition). χ and ξ are taste shifters. z are observables, χ, ξ are unobservables.

$$u(c_{h,t}, l_{h,t}, P_{h,t}) = \frac{M_{h,t}^{1-\gamma}}{1-\gamma} \exp\left(\xi P_{h,t} + \pi z_{h,t} + \zeta_{h,t}\right).$$

The preference aggregator for hours of leisure and consumption is

$$M_{h,t}(c_{h,t}, l_{h,t}; z_{h,t}, \chi_{h,t}) = \left(\frac{(c_{h,t}^{1-\phi} - 1)}{1-\phi} + (\alpha_{h,t}(z_{h,t}, \chi_{h,t})) \frac{(l_{h,t}^{1-\theta} - 1)}{1-\theta} \right)$$

$\alpha_{h,t} = \exp\left(\psi_0 + \psi_z z_{h,t} + \chi_{h,t}\right)$

Parameters $(\phi, \theta, \psi_0, \psi_z, \gamma, \xi, \pi)$.

Inter-temporal budget constraint:

$$A_{h,t+1} = (1 + r_{t+1}) \left(A_{h,t} + \left(w_{h,t}^f (H - l_{h,t}) - F(a_{h,t}) \right) P_{h,t} + y_{h,t}^m - c_{h,t} \right)$$

$A_{h,t}$ is the beginning asset holding, r_t is the risk-free interest rate, F is the fixed cost of work, dependent on the age of the youngest child, $a_{h,t}$. Female wages are given by $w_{h,t}^f$ and husband earning $y_{h,t}^m$. No explicit borrowing constraint but households cannot go bankrupt.

The fixed cost $F(a_{h,t}) = pG(a_{h,t}) + \bar{F}$. Female wages $\log(w_{h,t}^f) = \log(w_{h,0}^f) + \log(e_{h,t}^f) + \nu_{h,t}^f$.

Each period of the model is one quarter. Household choose typical hours of work each week. Within the dynamic problem just described, individuals make decisions taking stochastic process as given.

1.5.1 Model basics

- Female wages are given by the following process: $\log w_{h,t} = \log w_{h,0}^f + \log e_{h,t}^f + \nu_{h,t}^f$, $e_{h,t}^f$ is the level of female human capital, at time t. Consistent with observations with US-based studies(Hirsch(2005), Aaronson and French(2004)).

- Human capital does not depend on the history of labour supply and assumed to evolve exogenously $\log(e_t^f) = \iota_1^f t + \iota_2^f t^2$.

- Men always work and male earnings are given by :$\log y_{h,t}^m = \log y_{h,0}^m + \iota_1^m t + \iota_2^m t^2 + \nu_{h,t}^m$.

- Initial distribution for women wage:$w^f_{h,0}$, earnings for men $y^m_{h,0}$.
- Female and male wage changes:(subject to shocks and positively correlated)

$$
\begin{aligned}
\nu_{h,t} &= \nu_{h,t-1} + \xi_{h,t} \\
\xi_{h,t} &= (\xi^f_{h,t}, \xi^m_{h,t}) \sim N(\mu_\xi, \sigma^2_\xi) \\
\mu_\xi &= (-\sigma^2_{\xi^f}/2, -\sigma^2_{\xi^m}/2), \sigma^2_\xi = ()
\end{aligned}
$$

1.5.2 MRS, Marshallian and Hicksan Elasticities

- Within period resource not earned by women as : $y_t = (A_{h,t} + y^m_{h,t} - F(a_{h,t})P_{h,t}) - \frac{A_{h,t+1}}{1+r_{t+1}}$, is the resources saved into the next period.
- Within period budget constraint: $c_t + w_t l_t = y_t + w_t H$.
- Suppose the solution is interior with strictly positive number of working hours. F.O.C. for within-period optimality implies that the ratio of the marginal utility of leisure to that of consumption, $w_{h,t} = \frac{u_{l_{h,t}}}{u_{c_{h,t}}} = \alpha_{h,t}\frac{l^{-\theta}_{h,t}}{c^{-\phi}_{h,t}}$.
- The Marshallian elasticities for female hours of work and consumption

$$
\begin{aligned}
\epsilon^M_h &= \frac{\partial \log h}{\partial \log w} = -\left(\frac{\phi w(H-l) - c}{\theta c + \phi wl}\right)\frac{l}{H-l} \\
\epsilon^M_c &= \frac{\partial \log c}{\partial \log w} = \frac{\phi w(H-l) + wl}{\theta c + \phi wl}
\end{aligned}
$$

- For the balanced growth, we would require $\phi = 1$. If preferences ware standard CES, $\phi = \theta$.
- If $\theta, \phi > 1$, $\epsilon^M_c < 1, \epsilon^M_h < 0$.
- The static Hicksian response net off the increase in within0period resources due to the wage increase is

$$
\begin{aligned}
\epsilon^H_h &= \left(\epsilon^M_l - \frac{\partial \log l}{\partial \log(c+wl)}\frac{w(H-l)}{(c+wl)}\right)\frac{-l}{H-l} = \frac{-wl^2}{(\theta c + \phi wl)(H-l)} \\
\epsilon^H_c &= \epsilon^M_c + \frac{\partial \log c}{\partial \log(c+wl)}\frac{wl}{c+wl} = \frac{-c}{\theta c + \phi wl}
\end{aligned}
$$

1.5.3 Frisch Elasticities

The size of changes in labour supply induced by permanent shifts to the wage structure can be approximated by Hicksian and Marshallian elasticities.

The changes induced by expected changes in wages over time are captured by Frisch(marginal utility of wealth constant) elasticity.

- The Frisch elasticity captures the change over time in hours worked in response to the anticipated evolution of wages, with the marginal utility of wealth unchanged.
- The marginal utility unchanged because the wage change conveys no new information or because the wage change is temporary and life time approximately unchanged.
- The expression for Frisch elasticity

$$\epsilon_h^F = -\frac{u_c u_{cc}}{u_{cc}u_{ll} - u_{cl}^2}\frac{w}{h}$$

- Frisch intertemporal elasticities must be at least as large as Hicks elasticities.
- To compute the Frisch elasticity we need the parameters that characterise intertemporal allocation,

1.6 Empirical strategy

The identification assumptions. The whole model in its various components is essential to evaluate the size and aggregation properties of the elasticity of lbour supply to changes in wages. Type of variabilities.

1.6.1 Equilibrium conditions

- The model uses equilibrium conditions. MRS or those that enters Euler equation.
- $E[h(X;\theta)\mathcal{Z}] = 0$ where $h(\cdot)$ is a function of data X and parameters θ, and is linear in the vector of parameters.
- 2LS and GMM to estimate the structural parameter θ.
- what is the $h(\cdot)$ function ?

1.6.2 Intratemporal margins

- Estimate the parameters of the within period utility function: θ ϕ and α.
- The equations we obtain that : $\log(w_{h,t}) = \phi\log(c_{h,t}) - \phi\log(l_{h,t}) + \psi_z z_{h,t} + \psi_0 + \xi_{h,t}$.

- Papers use differential changes in wages and hours across education groups to identify labour supply elasticities. MaCurdy(1983) Ziliak and Kniesner(1999) use age-education interaction as instrument for wages and hours.
- Use 10 year birth cohort and education dummies with a quintic time trend(???).

1.6.3 Euler equation estimation

The second step uses Euler equation to estimate the preference parameters that govern the inetrtemporal substitutability and non-seperability

- The evolution of marginal utility of consumption:

$$
\begin{aligned}
&\beta(1+r_{t+1})u_{c_{h,t+1}}(\cdot) = u_{c_{h,t}}(\cdot)\epsilon_{h,t+1} \\
&\qquad \eta_{h,t+1} = \kappa_{h,t} + \log\beta + \log(1+r_{t+1}) - \phi\Delta\log(c_{h,t+1})0\gamma\Delta log(M_{h,t+1}) + \phi\Delta P_{h,t+1} + \pi\Delta z_{h,t+1} \\
&\qquad \eta_{h,t+1} = \log(\epsilon_{h,t+1}) - E[\log(\epsilon_{h,t+1})]
\end{aligned}
$$

Bibliography

Attanasio, O., Levell, P., Low, H., and Sánchez-Marcos, V. (2015). Aggregating Elasticities: Intensive and Extensive Margins of Female Labour Supply.