The estimators

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1 Estimators

The estimators assume that the conditional choice probabilities are correctly estimated in the first step. Therefore, we obtain the consistent estimator of F and P: $\hat{F}$ and $\hat{P}$ from the data. We define the mappings as $\Phi : \mathcal{B}_P \to \mathcal{B}_V$ and $\Lambda : \mathcal{B}_V \to \mathcal{B}_P$. The estimators construct different Φ mappings.

- Hotz-Miller estimator

$$\begin{aligned}
&\Phi_{HM}(P;\theta) = (I - \beta F^e(P))^{-1}(u^e(P;\theta) + e^e(P)),\\
&\text{where } F^e(P) = \left[\sum_{d\in\mathcal{D}} p(d,z)f(z'|z,d)\right]_{z,z'\in\mathcal{Z}} \text{ and } u^e(P;\theta) = \left[\sum_{d\in\mathcal{D}} p(d,z)u(d,z;\theta)\right]_{z\in\mathcal{Z}}\\
&\Psi_{HM}(P;\theta) = \Lambda(\Phi_{HM}(P;\theta))
\end{aligned} \tag{1}$$

- Euler equation estimator

$$\begin{aligned}
\Phi_{EE}(P;\theta) &= (I - \beta F(0))^{-1}(u(0;\theta) + e(0,P))\\
\Psi_{EE}(P;\theta) &= \Lambda(\Phi_{EE}(P;\theta))
\end{aligned} \tag{2}$$

- Finite Dependence

$$\Psi_{FD}(d,x;\theta,P) = \frac{u(d,x;\theta) + \beta\sum_{x'|x} f(x'|x,d)(u(0,x';\theta) + e(0,x,P))}{\sum_{d'} u(d',x;\theta) + \beta\sum_{x'|x} f(x'|x,d')(u(0,x';\theta) + e(0,x,P))}$$

2 Two-step estimators

- In the first step, obtain non-parametric estimator of the conditional choice probabilities $\hat{p}(d,x)$.
- In the second step, use the maximum likelihood estimator and obtain $\hat{\theta} = \arg\max_\theta \sum_{i=1}^N \sum_{t=1}^T \Psi(d_{it}, x_{it}, \theta, \hat{P})$.