

# Using Euler Equation to Estimate Non-Finite-Dependent Dynamic Discrete Choice Model with Unobserved Heterogeneity

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# Motivation

# Dynamic Discrete Choice Model

## Model priors

- The agents are forward looking and maximize expected inter-temporal payoffs.
- Structural functions : agents' preferences and beliefs about uncertain events.
- Estimated under principle of revealed preference, use micro-data on individuals' choices and outcomes.

Empirical applications includes

- Industrial organization Aguirregabiria and Ho (2012), Berry (1992), Yakovlev (2016), Sweeting (2013);
- Health economics Beauchamp (2015), Gaynor and Town (2012), Gowrisankaran and Town (1997), Gowrisankaran et al. (2011);
- Marketing Dubé et al. (2005), Doraszelski and Pakes (2007), Doganoglu and Klapper (2006);
- Labor economics Todd and Wolpin (2006), Fang and Wang (2009), Keane et al. (2011);
- Other Schivardi and Schneider (2008), Rust and Rothwell (1995).

The difficulties in incorporating unobserved heterogeneity :

- Computational heavy : value function iteration or Hotz-Miller inversion
- EM algorithm : more iterations account for unobserved heterogeneity.
- Existing methods relies on "Finite Dependence"(Arcidiacono and Ellickson (2011)).

The contribution of this project :

- Conceptually redefine the problem as a stochastic problem.
- Propose an alternative faster estimation strategy.
- Incorporate unobserved heterogeneity and EM algorithm in dynamic discrete choice.

# Model

# Baseline entry exit model

Now consider the baseline model of dynamic choice model

- Time is discrete and indexed by  $t$ .
- Firms have preferences defined states of the world between periods 0 and  $T$  finite / infinite.
- A state of the world has two component : predetermined  $s_t$  and discrete action  $d_t \in \mathcal{D} = \{0, 1\}$ .
- Time-separable utility function  $\sum_{t=0}^T \beta^t U_t(d_t, s_t)$ , where  $\beta \in [0, 1)$  is the discount factor.
- Let  $d_t^*(s_t)$  denote optimal decision rule,  $V_t(s_t)$  be the value function at period  $t$ .

$$V_t(s_t) = \max_d \left\{ U_t(d, s_t) + \beta \int V_{t+1}(s_{t+1}) dF_t(s_{t+1}|a, s_t) \right\}. \quad (1)$$



# Key assumptions

Here are the key assumptions made in estimating the models :

- Assumption 1(Additive separable) :  $s = (x_t, \epsilon_t)$ ,  $\epsilon_t = [\epsilon_t(0), \epsilon_t(1)]$  ,  
 $U(d_t, s_t) = u(d_t, x_t; \theta) + \epsilon_t(d)$ .  $x_t$  is observed by the economist,  $\epsilon_t$  is not observed by the economist.
- Assumption 2(Finite domain of  $x$ ) :  $x \in \mathcal{X}$ ,  $|\mathcal{X}|$  is finite.
- Assumption 3(Conditional independence) :  
 $F(s_{t+1}|a_t, s_t) = G_\epsilon(\epsilon_{t+1}|x_{t+1})F_x(x_{t+1}|x_t, d_t)$ .
- Assumption 4(Distribution of  $\epsilon$ ) :  $\epsilon_t = \{\epsilon_t(d) : d \in \mathcal{D}\} \sim_{i.i.d} T1EV$  .

# Motivating Example : Entry Exit Problem

For example, take Consider a stationary infinite time horizon entry-exit problem :

- The firm observe the state  $x_t = (y_t, z_t)$ . The profitability  $z_t \in \mathcal{Z}$ , where  $|\mathcal{Z}| = N$  is finite, and operation state  $y_t = d_{t-1} \in \{0, 1\}$ .
- The firm makes entry decision  $d_t \in \mathcal{D} = \{0, 1\}$ .
- $z_t$  follows a first order Markov process  $f(x_{t+1}|x_t, d_t)$ ;
- The firm's flow payoff  $u(x_t, d_t; \theta)$ .

# Entry Exit Problem : Continue

- The ex-ante value function :

$$\begin{aligned}\bar{V}(x_t) &= E_{\epsilon} V(x_t, \epsilon) \\ &= E_{\epsilon} \left\{ \max_{d \in \mathcal{D}} \{ u(d, x_t; \theta) + \epsilon_t(d) + \beta E_{x_{t+1}|x_t, d} \bar{V}(x_{t+1}) \} \right\} \quad (2) \\ &= E_{\epsilon} \left\{ \max_{d \in \mathcal{D}} \{ v(d, x_t; \theta) + \epsilon_t(d) \} \right\}\end{aligned}$$

- The firm's strategy  $d_t^* = \arg \max_{d \in \mathcal{D}} \{ v(x_t, d_t) + \epsilon_t(d) \}$ , where  $v(x_t, d)$  is the sum of future payoffs.
- $v(x_t, d; \theta) = u(x_t, d; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} f(x_{t+1}|x_t, d) \bar{V}(x_{t+1})$ .
- Estimation technique : use the distribution of  $\epsilon$ , form the logit likelihood function :  $l(d_t, x_t; \theta) = \frac{\exp(v_{d_t}(s_t))}{\sum_{d \in \mathcal{D}} \exp(v_d(s_t))}$ .

# Important Result in OCP Mapping

For an arbitrary conditional choice probability  $p(s_t) = [p_1(s_t), p_0(s_t)]$ , define the expected payoff

$$\begin{aligned} W(s_t, P(s_t)) &= \left\{ \sum_{d=0}^1 p_d(s_t) \{ v_d(s_t) + e_d(p(s_t)) \} \right\} \\ &= p_1(s_t) \{ \tilde{v}(s_t) + \tilde{e}_1(p(s_t)) \} + v_0(s_t) + e_0(p(s_t)). \end{aligned}$$

## Proposition

Aguirregabiria and Magesan (2016) has shown that it is equivalent to choose the conditional choice probability before observing the shock

$$P^*(s_t) = \arg \max_{p(s_t)} W(s_t, p(s_t)).$$

# Optimal choice probability(OCP) mapping

Rewrite the value function

$$\begin{aligned} V(s_t) &= \sum_{d=0}^1 p_d^*(s_t) \{v_d(s_t) + e_d(p_t^*(s_t))\}, \\ &= \sum_{d=0}^1 p_d^*(s_t) \{\tilde{v}_d(s_t) + \tilde{e}_d(p_t^*(s_t))\} + v_0(s_t) + e_0(p_0^*(s_t)), \end{aligned} \tag{3}$$

where

- $\tilde{u}_1(s_t) = u_1(s_t) - u_0(s_t), \tilde{v}_1(s_t) = v_1(s_t) - v_0(s_t),$   
 $\tilde{e}_1(P^*(s_t)) = e_1(P^*(s_t)) - e_0(P^*(s_t)).$
- $\tilde{e}_1(P^*(s_t)) = \log(P^*(0, s_t)/P^*(1, s_t)).$

# Decision in probability space

Now consider an optimization problem defined in the probability space instead of the action space :

- The firm chooses the sequence of  $\left\{ \{P_t(x_t)\}_{x_t \in \mathcal{X}} \right\}_{t=0}^{\infty} = \left\{ \{[p_t(x_t, 0), p_t(x_t, 1)]\}_{x_t \in \mathcal{X}} \right\}_{t=0}^{\infty}$  for all possible future states to maximize the discounted utility ;
- The sequence of choices will affect the ex-ante distribution of  $x_{t+1}$ .
- $\kappa_t(x_t|x_0) \in [0, 1]$  and  $\sum_{s_t} \kappa_t(x_t|x_0) = 1$ .
- $\kappa_t(x_t|x_0) = \begin{cases} \mathbf{1}(x_t = x_0) & \text{if } t = 0 \\ \sum_{x_{t-1}} \kappa_{t-1}(x_{t-1}|x_0) \sum_{d=0}^1 p_t(d)(x_{t-1}) f_d(x_t|x_{t-1}) & \text{if } t \geq 1 \end{cases}$

# Euler Equation

Redefine the agent's problem as an analogue to a continuous optimization problem

$$\max_{P_t(x_t)} \left\{ \sum_{t=0}^{\infty} \beta^t \kappa_t(x_t|x_0) \left[ \sum_{d=0}^1 p_t(x_t, d) (u(x_t, d) + e(P_t(x_t), d)) \right] \right\} \quad (4)$$

$$\text{subject to } \kappa_{t+1}(x_{t+1}|x_0) = \sum_{x_t \in \mathcal{X}} \sum_{d \in \mathcal{D}} \kappa_t(x_t|x_0) p(x_t, d) f(x_{t+1}|x_t, d).$$

The first order condition of this problem :  $P_t(s_t)$  and  $\kappa_{t+1}(x_{t+1}|x_0)$  :

- $u(x_t, 1) + e(P_t(s_t), 1) + \beta \sum_{x_{t+1}} f(x_{t+1}|x_t, d) \bar{V}(x_{t+1}) = u(x_t, 0) + e(P_t(x_t), 0) + \beta \sum_{x_{t+1}} f(x_{t+1}|x_t, 0) \bar{V}(x_{t+1}),$
- $\bar{V}_t(s_t) = u_0(s_t) + e_0(s_t) + \sum_{s_{t+1}} f_0(s_{t+1}|s_t) \bar{V}(s_{t+1}).$

## Proposed Estimators



## EE(direct inversion) estimator

In a stationary game,  $P_t(x) = P^*(x) \forall x \in \mathcal{X}, t = 0, \dots, T$ .

### Value function estimator

$$V = (I - \beta F_0)^{-1}(u_0 + e_0),$$

where  $F_0 = [f(x_i|x_j, 0)]_{ij}$  is the transition density matrix,

$V = [V(x) : x \in \mathcal{X}]$ ,  $u_0 = [u(x, 0) : x \in \mathcal{X}]'$ ,  $e_0 = [e(x) : x \in \mathcal{X}]$ , can be estimated from the sample transition density.

With this property :

- Only estimate the transition density  $F_0$ , instead of all  $F_d$  for  $d \in \mathcal{D}$ .
- Saves time compare to  $V = (I - \beta F^P)^{-1}(u^P + e^P)$ , where  $F^P$  is weighted transition density weighted by  $P$ ,  $u^P, e^P$  are the weighted payoffs.
- Therefore it is possible to add an unobserved states variable even if the model does not need to display finite-dependence property as in Arcidiacono and Ellickson (2011).

# Finite Dependence Property

## Finite Dependence(Arcidiacono and Ellickson (2011))

The model exhibit Finite Dependence if there exist  $p_{t+1}(s_t), \dots, p_{t+\rho}(s_t)$ , such that the choice at time  $t$  is obliterated after  $\rho$  periods. It is called  $\rho + 1$  dependence.

- In the entry/exit decision, the model shows 2 period dependence if  $f_1(s_{t+1}|s_t) = f_0(s_{t+1}|s_t)$ .
- With the property, reduce computational complexity.
- The property is restrictive : firms past entry does not impact future profitability.

# Finite dependent estimator(*FD*)

*Step 1* : In each iteration  $k$ , the likelihood is defined as

$$l(d, z; \theta) = \frac{\exp \left( u(d, z; \theta) + \beta \sum_{z' \in \mathcal{Z}} f(z'|d, z)(u(0, z'; \theta) + e(0, z', p^{(k-1)})) \right)}{\sum_{d' \in \mathcal{D}} \exp \left( u(d', z; \theta) + \beta \sum_{z' \in \mathcal{Z}} f(z'|d', z)u(0, z'; \theta) + e(0, z', p^{(k-1)}) \right)},$$

where  $p^{(k-1)}$  is the CCP in last round.

*Step 2* : Update the CCPs

$$p^{(k)}(d, z) = l(d, z; \theta^{(k)})$$

## 2-Step Finite Dependent Estimator(*FD2*)

*Step 1* : Given the estimator  $\theta^{(k-1)}$ ,  $V^{(k-1)}$ , define the likelihood function as :

$$\begin{aligned} l(d, z; \theta) &= \Lambda(V^{(k-1)}, \theta) \\ &= \frac{\exp \left( u(d, z; \theta) + \beta \sum_{z' \in \mathcal{Z}} f(z'|d, z) V^{(k-1)}(z') \right)}{\sum_{d' \in \mathcal{D}} \exp \left( u(d', z; \theta) + \beta \sum_{z' \in \mathcal{Z}} f(z'|d', z) V^{(k-1)}(z') \right)}. \end{aligned}$$

Estimate  $\theta$  :  $\theta^{(k)} = \arg \max_{\theta} \sum_{i=1}^N \sum_{t=0}^T l(d_{it}, z_{it}; \theta)$ .

*Step 2* : Update the CCPs and value function

$$\begin{aligned} p^{(k)}(d, z) &= l(d, z; \theta^{(k)}) \\ \mathbf{V}^{(k)} &= \Gamma^{FD2}(V^{(k-1)}, p^{(k)}, \theta^{(k-1)}) \\ &= \gamma - \log(1 - p^{(k)}) + u_0(\theta^{(k-1)}) + \beta \mathbf{F}_0 \mathbf{V}^{(k-1)}. \end{aligned}$$

# Almost Finite Dependent Estimator(*AFD*)

Both *AFD* and *AFD2* estimator is similar to *FD* and *FD2* except for the value function :

## Contraction Mapping

$$\begin{aligned} \mathbf{V} = & \gamma + \omega(-\log(\mathbf{p}) + u_1(\theta) + \beta \mathbf{F}_1 \mathbf{V}) \\ & + (1 - \omega)(-\log(1 - \mathbf{p}) + u_0(\theta) + \beta \mathbf{F}_0 \mathbf{V}), \end{aligned}$$

where  $\omega$  is the weight.

# Comparisons between contraction mappings

Method	Contraction Mapping
NFXP	$V(s_t) = E_\epsilon \left\{ \max_{d \in \mathcal{D}} [u_d(s_t) + \epsilon_d + \beta \sum_{s_{t+1}   s_t} V(s_{t+1})] \right\}$
Hotz-Miller	$V = (I - \beta F^P)^{-1} (u^P + e^P)$
EE	$V = (I - \beta F_0)^{-1} (u_0 + e_0)$

- NFXP and SEQ-full use the value function contraction mapping.
- FD and SEQ-EE estimator use the contraction mapping in the probability space.

# Monte Carlo Experiments

# Data generating process : Homogeneous agent model

Table – Parameters in DGP

<i>Flow-Payoff Parameters</i>	$\theta_0^{VP} = 0.5$ $\theta_1^{VP} = 1.0$ $\theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5$ $\theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0$ $\theta_1^{EC} = 1.0$
<i>State Variable Transition</i>	$z_{kt}$ is AR(1), $\gamma_0^k = 0$ , $\gamma_1^k = 0.6$
<i>Productivity Transition</i>	$\omega_t$ is AR(1), $\gamma_0^\omega = 0$ , $\gamma_1^\omega = 0.9$
<i>Past action on productivity</i>	$\gamma_a \in [0, 5]$
<i>Discount Factor</i>	$\beta = 0.95$



# Finite Dependent Model

Table – Two-step : Finite dependent models

	<i>FD</i>	<i>FD2</i>	<i>AFD</i>	<i>AFD2</i>	<i>HM</i>	<i>EE</i>
<i>Market = 200, Time = 20, <math>\gamma_a = 0</math></i>						
$\theta_0^{VP}$	0.4845 (0.0706)	0.4845 (0.0706)	0.4845 (0.0706)	0.4845 (0.0706)	0.5016 (0.0350)	0.4845 (0.0706)
$\theta_0^{FC}$	0.5447 (0.0904)	0.5447 (0.0904)	0.5447 (0.0904)	0.5447 (0.0904)	0.5098 (0.0627)	0.5447 (0.0904)
<i>Market = 200, Time = 120, <math>\gamma_a = 0</math></i>						
$\theta_0^{VP}$	0.4963 (0.0189)	0.4963 (0.0189)	0.4963 (0.0189)	0.4963 (0.0189)	0.4983 (0.0140)	0.4963 (0.0189)
$\theta_0^{FC}$	0.4990 (0.0301)	0.4990 (0.0301)	0.4990 (0.0301)	0.4990 (0.0301)	0.4954 (0.0279)	0.4990 (0.0301)

DGP :  $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$ .

# Two-step : Non-finite dependent models

Table – Non-finite Dependent two-step estimators

	<i>FD</i>	<i>FD2</i>	<i>AFD</i>	<i>AFD2</i>	<i>HM</i>	<i>EE</i>
<i>Market = 200, Time = 20, <math>\gamma_a = 5</math></i>						
$\theta_0^{VP}$	0.3434 (0.0790)	0.5679 (0.1457)	0.4925 (0.0860)	0.5067 (0.0908)	0.5307 (0.0800)	0.5691 (0.1460)
$\theta_0^{FC}$	-0.0155 (0.2228)	0.7095 (0.3321)	0.4432 (0.2402)	0.4751 (0.2518)	0.5833 (0.2209)	0.7134 (0.3330)
<i>Market = 200, Time = 120, <math>\gamma_a = 5</math></i>						
$\theta_0^{VP}$	0.3058 (0.0333)	0.4965 (0.0484)	0.4829 (0.0436)	0.4954 (0.0453)	0.4982 (0.0395)	0.4975 (0.0485)
$\theta_0^{FC}$	-0.1239 (0.0845)	0.4920 (0.1237)	0.4583 (0.1096)	0.4860 (0.1140)	0.4977 (0.1036)	0.4953 (0.1239)

DGP :  $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$ .

# Sequential Estimation

Table – The mean and standard deviation of sequential estimators

	<i>FD</i>	<i>FD2</i>	<i>AFD</i>	<i>AFD2</i>
<i>Market = 200, Time = 20, <math>\gamma_a = 0</math></i>				
$\theta_0^{VP}$	0.5163 (0.0376)	0.5079 (0.0369)	0.5163 (0.0376)	0.4799 (0.0672)
$\theta_0^{FC}$	0.4203 (0.0635)	0.5146 (0.0591)	0.4203 (0.0635)	0.5516 (0.0804)
<i>Market = 200, Time = 20, <math>\gamma_a = 5</math></i>				
$\theta_0^{VP}$	0.3128 (0.0661)	0.5084 (0.0925)	-0.1775 (0.1838)	0.4940 (0.1151)
$\theta_0^{FC}$	-0.2597 (0.1703)	0.5167 (0.2506)	-1.9434 (0.6454)	0.4391 (0.3056)
DGP : $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5.$				

# Continue : Sequential Estimation

Table – The mean and standard deviation of sequential estimators

	<i>HM</i>	<i>EE</i>	<i>SEQ(1)</i>	<i>SEQ(2)</i>	<i>SEQ(5)</i>
<i>Market = 200, Time = 20, <math>\gamma_a = 0</math></i>					
$\theta_0^{VP}$	0.5080 (0.0368)	0.5079 (0.0369)	0.5079 (0.0369)	0.5079 (0.0369)	0.5079 (0.0369)
$\theta_0^{FC}$	0.5148 (0.0593)	0.5146 (0.0591)	0.5146 (0.0591)	0.5146 (0.0591)	0.5146 (0.0591)
<i>Market = 200, Time = 20, <math>\gamma_a = 5</math></i>					
$\theta_0^{VP}$	0.5096 (0.0938)	0.5084 (0.0925)	0.5043 (0.0921)	0.5084 (0.0925)	0.5084 (0.0925)
$\theta_0^{FC}$	0.5207 (0.2567)	0.5167 (0.2506)	0.5034 (0.2493)	0.5167 (0.2506)	0.5167 (0.2506)

# Data generating process : Heterogeneous agent model

Table – Parameters in DGP

<i>Flow-Payoff Parameters <math>\theta^1</math></i>	$\theta_0^{VP} = 0$ $\theta_1^{VP} = 1.0$ $\theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5$ $\theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0$ $\theta_1^{EC} = 1.0$
<i>Flow-Payoff Parameters <math>\theta^2</math></i>	$\theta_0^{VP} = 1$ $\theta_1^{VP} = 1.0$ $\theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5$ $\theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0$ $\theta_1^{EC} = 1.0$
<i>Mixing Probability</i>	(0.5, 0.5)
<i>State Variable Transition</i>	$z_{kt}$ is AR(1), $\gamma_0^k = 0$ , $\gamma_1^k = 0.6$
<i>Productivity Transition</i>	$\omega_t$ is AR(1), $\gamma_0^\omega = 0$ , $\gamma_1^\omega = 0.9$
<i>Past action on productivity</i>	$\gamma_a = 2$
<i>Discount Factor</i>	$\beta = 0.95$

# Time and iteration

Table – Median Time and Iteration when increase state space

<i>Algorithms</i>	<i>nGrid</i>	2	3	4	5	6
	$ \mathcal{X} $	64	486	2048	6250	15552
	<i>Market</i>	100				
	<i>Time</i>	20				
FD2	<i>Time</i>	11.2472	13.9627	27.9147	390.0466	3103.6867
	<i>Iteration</i>	40.5	48.5	37	47.5	32.5
FD2(FV)	<i>Time</i>	9.7783	14.1659	42.0155	612.4756	3097.6401
	<i>Iteration</i>	32.5	47	70	118	32
EE	<i>Time</i>	12.1462	21.3075	18.6141	181.0266	1039.5331
	<i>Iteration</i>	38.5	69.5	43	80.5	52
HM	<i>Time</i>	30.3638	35.6079	982.0085	-	-
	<i>Iteration</i>	91.5	59.5	53	-	-
SEQ(1)	<i>Time</i>	6.0499	17.2884	24.1402	100.8548	509.4910
	<i>Iteration</i>	22.5	64.5	55	43.5	35.5

† The results shows the time and iteration used in the estimation based on 12 Monte Carlo simulations of different state space.



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