Using Euler Equation to Estimate Non-Finite-Dependent Dynamic Discrete Choice Model with Unobserved Heterogeneity

CEA Meeting 2019

Jasmine Hao and Hiro Kasahara

Vancouver School of Economics

Table of contents

Motivation

Dynamic Discrete Choice Model

Model priors

- The agents are forward looking and maximize expected inter-temporal payoffs.
- Structural functions : agents' preferences and beliefs about uncertain events.
- Estimated under principle of revealed preference, use micro-data on individuals' choices and outcomes.

4 / 31

Empirical applications includes

- Industrial organization Aguirregabiria and Ho (2012), Berry (1992), Yakovlev (2016),Sweeting (2013) ;
- Health economics Beauchamp (2015),Gaynor and Town (2012), Gowrisankaran and Town (1997),Gowrisankaran et al. (2011) ;
- Marketing Dubé et al. (2005),Doraszelski and Pakes (2007), Doganoglu and Klapper (2006) ;
- Labor economics Todd and Wolpin (2006),Fang and Wang (2009),Keane et al. (2011) ;
- Other Schivardi and Schneider (2008), Rust and Rothwell (1995).

The difficulties in incorporating unobserved heterogeneity :

- Computational heavy : value function iteration or Hotz-Miller inversion
- EM algorithm : more iterations account for unobserved heterogeneity.
- Existing methods relies on "Finite Dependence"(Arcidiacono and Ellickson (2011)).

The contribution of this project :

- Conceptually redefine the problem as a stochastic problem.
- Propose an alternative faster estimation strategy.
- Incorporate unobserved heterogeneity and EM algorithm in dynamic discrete choice.

Model

Baseline entry exit model

Now consider the baseline model of dynamic choice model

- Time is discrete and indexed by t.
- Firms have preferences defined states of the world between periods 0 and T finite / infinite.
- A state of the world has two component : predetermined s_t and discrete action $d_t \in \mathcal{D} = \{0, 1\}$.
- Time-separable utility function $\sum_{t=0}^{T} \beta^t U_t(d_t, s_t)$, where $\beta \in [0, 1)$ is the discount factor.
- Let $d_t^*(s_t)$ denote optimal decision rule, $V_t(s_t)$ be the value function at period t.

$$V_t(s_t) = \max_d \left\{ U_t(d, s_t) + \beta \int V_{t+1}(s_{t+1}) dF_t(s_{t+1}|a, s_t) \right\}. \quad (1)$$

Key assumptions

Here are the key assumptions made in estimating the models :

- Assumption 1(Additive separable) : $s = (x_t, \epsilon_t)$, $\epsilon_t = [\epsilon_t(0), \epsilon_t(1)]$, $U(d_t, s_t) = u(d_t, x_t; \theta) + \epsilon_t(d)$. x_t is observed by the economist, ϵ_t is not observed by the economist.
- Assumption 2(Finite domain of x) : $x \in \mathcal{X}$, $|\mathcal{X}|$ is finite.
- Assumption 3(Conditional independence) : $F(s_{t+1}|a_t, s_t) = G_\epsilon(\epsilon_{t+1}|x_{t+1})F_x(x_{t+1}|x_t, d_t)$.
- Assumption 4(Distribution of ϵ) : $\epsilon_t = \{\epsilon_t(d) : d \in \mathcal{D}\} \sim_{i.i.d} T1EV$.

Motivating Example : Entry Exit Problem

For example, take Consider a stationary infinite time horizon entry-exit problem :

- The firm observe the state $x_t = (y_t, z_t)$. The profitability $z_t \in \mathcal{Z}$, where $|\mathcal{Z}| = N$ is finite, and operation state $y_t = d_{t-1} \in \{0, 1\}$.
- The firm makes entry decision $d_t \in \mathcal{D} = \{0, 1\}$.
- z_t follows a first order Markov process $f(x_{t+1}|x_t, d_t)$;
- The firm's flow payoff $u(x_t, d_t; \theta)$.

Entry Exit Problem : Continue

- The ex-ante value function :

$$\begin{aligned} \bar{V}(x_t) &= E_\epsilon V(x_t, \epsilon) \\ &= E_\epsilon \Big\{ \max_{d \in \mathcal{D}} \big\{ u(d, x_t; \theta) + \epsilon_t(d) + \beta E_{x_{t+1}|x_t, d} \bar{V}(x_{t+1}) \big\} \Big\} \\ &= E_\epsilon \Big\{ \max_{d \in \mathcal{D}} \big\{ v(d, x_t; \theta) + \epsilon_t(d) \big\} \Big\} \end{aligned} \quad (2)$$

- The firm's strategy $d_t^* = \arg\max_{d \in \mathcal{D}} \big\{ v(x_t, d_t) + \epsilon_t(d) \big\}$, where $v(x_t, d)$ is the sum of future payoffs.
- $v(x_t, d; \theta) = u(x_t, d; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} f(x_{t+1}|x_t, d) \bar{V}(x_{t+1})$.
- Estimation technique : use the distribution of ϵ, form the logit likelihood function : $l(d_t, x_t; \theta) = \frac{\exp(v_{d_t}(s_t))}{\sum_{d \in \mathcal{D}} \exp(v_d(s_t))}$.

Important Result in OCP Mapping

For an arbitrary conditional choice probability $p(s_t) = [p_1(s_t), p_0(s_t)]$, define the expected payoff

$$
\begin{aligned}
W(s_t, P(s_t)) &= \Big\{ \sum_{d=0}^{1} p_d(s_t)\big\{v_d(s_t) + e_d(\rho(s_t))\big\}\Big\} \\
&= p_1(s_t)\Big\{\tilde{v}(s_t) + \tilde{e}_1(p(s_t))\Big\} + v_0(s_t) + e_0(p(s_t)).
\end{aligned}
$$

Proposition

Aguirregabiria and Magesan (2016) has shown that it is equivalent to choose the conditional choice probability before observing the shock

$$P^*(s_t) = \arg\max_{p(s_t)} W(s_t, p(s_t)).$$

Optimal choice probability(OCP) mapping

Rewrite the value function

$$
\begin{aligned}
V(s_t) &= \sum_{d=0}^{1} p_d^*(s_t)\{v_d(s_t) + e_d(p_t^*(s_t))\}, \\
&= \sum_{d=0}^{1} p_d^*(s_t)\{\tilde{v}_d(s_t) + \tilde{e}_d(p_t^*(s_t))\} + v_0(s_t) + e_0(p_0^*(s_t)),
\end{aligned} \tag{3}
$$

where

- $\tilde{u}_1(s_t) = u_1(s_t) - u_0(s_t)$, $\tilde{v}_1(s_t) = v_1(s_t) - v_0(s_t)$, $\tilde{e}_1(P^*(s_t)) = e_1(P^*(s_t)) - e_0(P^*(s_t))$.
- $\tilde{e}_1(P^*(s_t)) = \log(P^*(0, s_t)/P^*(1, s_t))$.

Decision in probability space

Now consider an optimization problem defined in the probability space instead of the action space :

- The firm chooses the sequence of $\left\{\{P_t(x_t)\}_{x_t\in\mathcal{X}}\right\}_{t=0}^{\infty} = \left\{\{[p_t(x_t,0), p_t(x_t,1)]\}_{x_t\in\mathcal{X}}\right\}_{t=0}^{\infty}$ for all possible future states to maximize the discounted utility ;
- The sequence of choices will affect the ex-ante distribution of x_{t+1}.
- $\kappa_t(x_t|x_0) \in [0,1]$ and $\sum_{s_t} \kappa_t(x_t|x_0) = 1$.
- $\kappa_t(x_t|x_0) =$
$$\begin{cases} \mathbf{1}(x_t = x_0) & \text{if } t = 0 \\ \sum_{x_{t-1}} \kappa_{t-1}(x_{t-1}|x_0) \sum_{d=0}^{1} p_t(d)(x_{t-1}) f_d(x_t|x_{t-1}) & \text{if } t \geq 1 \end{cases}.$$

Euler Equation

Redefine the agent's problem as an analogue to a continuous optimization problem

$$\max_{P_t(x_t)} \Big\{ \sum_{t=0}^{\infty} \beta^t \kappa_t(x_t|x_0) \Big[\sum_{d=0}^{1} p_t(x_t,d)(u(x_t,d) + e(P_t(x_t),d) \Big] \Big\} \tag{4}$$
$$\text{subject to } \kappa_{t+1}(x_{t+1}|x_0) = \sum_{x_t \in \mathcal{X}} \sum_{d \in \mathcal{D}} \kappa_t(x_t|x_0) p(x_t,d) f(x_{t+1}|x_t,d).$$

The first order condition of this problem : $P_t(s_t)$ and $\kappa_{t+1}(x_{t+1}|x_0)$:

- $u(x_t,1) + e(P_t(s_t),1) + \beta \sum_{x_{t+1}} f(x_{t+1}|x_t,d)\bar{V}(x_{t+1}) = u(x_t,0) + e(P_t(x_t),0) + \beta \sum_{x_{t+1}} f(x_{t+1}|x_t,0)\bar{V}(x_{t+1})$,
- $\bar{V}_t(s_t) = u_0(s_t) + e_0(s_t) + \sum_{s_{t+1}} f_0(s_{t+1}|s_t)\bar{V}(s_{t+1})$.

Proposed Estimators

EE(direct inversion) estimator

In a stationary game, $P_t(x) = P^*(x) \forall x \in \mathcal{X}, t = 0, \dots, T$.

Value function estimator

$$V = (I - \beta F_0)^{-1}(u_0 + e_0),$$

where $F_0 = [f(x_i|x_j, 0)]_{ij}$ is the transition density matrix, $V = [V(x) : x \in \mathcal{X}]$, $u_0 = [u(x, 0) : x \in \mathcal{X}]'$, $e_0 = [e(x) : x \in \mathcal{X}]$, can be estimated from the sample transition density.

With this property :

- Only estimate the transition density F_0, instead of all F_d for $d \in \mathcal{D}$.
- Saves time compare to $V = (I - \beta F^P)^{-1}(u^P + e^P)$, where F^P is weighted transition density weighted by P, u^P,e^P are the weighted payoffs.
- Therefore it is possible to add an unobserved states variable even if the model does not need to display finite-dependence property as in Arcidiacono and Ellickson (2011).

Finite Dependence Property

Finite Dependence(Arcidiacono and Ellickson (2011))

The model exhibit Finite Dependence if there exist $p_{t+1}(s_t), \ldots, p_{t+\rho}(s_t)$, such that the choice at time t is obliviated after ρ periods. It is called $\rho + 1$ dependence.

- In the entry/exit decision, the model shows 2 period dependence if $f_1(s_{t+1}|s_t) = f_0(s_{t+1}|s_t)$.
- With the property, reduce computational complexity.
- The property is restrictive : firms past entry does not impact future profitability.

Finite dependent estimator(*FD*)

Step 1 : In each iteration k, the likelihood is defined as

$$l(d,z;\theta) = \frac{\exp\left(u(d,z;\theta)+\beta\sum_{z'\in\mathcal{Z}} f(z'|d,z)(u(0,z';\theta)+e(0,z',p^{(k-1)}))\right)}{\sum_{d'\in\mathcal{D}}\exp\left(u(d',z;\theta)+\beta\sum_{z'\in\mathcal{Z}} f(z'|d',z)u(0,z';\theta)+e(0,z',p^{(k-1)})\right)},$$

where $p^{(k-1)}$ is the CCP in last round.
Step 2 : Update the CCPs

$$p^{(k)}(d,z) = l(d,z;\theta^{(k)})$$

2-Step Finite Dependent Estimator($FD2$)

Step 1 : Given the estimator $\theta^{(k-1)}$, $V^{(k-1)}$, define the likelihood function as :

$$
\begin{aligned}
l(d, z; \theta) &= \Lambda(V^{(k-1)}, \theta) \\
&= \frac{\exp\left(u(d, z; \theta) + \beta \sum_{z' \in \mathcal{Z}} f(z'|d, z) V^{(k-1)}(z')\right)}{\sum_{d' \in \mathcal{D}} \exp\left(u(d', z; \theta) + \beta \sum_{z' \in \mathcal{Z}} f(z'|d', z) V^{(k-1)}(z')\right)}.
\end{aligned}
$$

Estimate θ : $\theta^{(k)} = \arg\max_\theta \sum_{i=1}^{N} \sum_{t=0}^{T} l(d_{it}, z_{it}; \theta)$.
Step 2 : Update the CCPs and value function

$$
\begin{aligned}
p^{(k)}(d, z) &= l(d, z; \theta^{(k)}) \\
\boldsymbol{V}^{(k)} &= \Gamma^{FD2}(V^{(k-1)}, p^{(k)}, \theta^{(k-1)}) \\
&= \gamma - \log(1 - p^{(k)}) + u_0(\theta^{(k-1)}) + \beta \boldsymbol{F}_0 \boldsymbol{V}^{(k-1)}).
\end{aligned}
$$

Almost Finite Dependent Estimator(*AFD*)

Both *AFD* and *AFD*2 estimator is similar to *FD* and *FD*2 except for the value function :

Contraction Mapping

$$\boldsymbol{V} = \gamma + \boldsymbol{\omega}(-\log(\boldsymbol{p}) + u_1(\theta) + \beta \boldsymbol{F}_1 \boldsymbol{V}) + (1-\boldsymbol{\omega})(-\log(1-\boldsymbol{p}) + u_0(\theta) + \beta \boldsymbol{F}_0 \boldsymbol{V}),$$

where ω is the weight.

Comparisons between contraction mappings

Method	Contraction Mapping
NFXP	$V(s_t) = E_\epsilon \left\{ \max_{d \in \mathcal{D}} [u_d(s_t) + \epsilon_d + \beta \sum_{s_{t+\mathbf{1}} \mid s_t} V(s_{t+1})] \right\}$
Hotz-Miller	$V = (I - \beta F^P)^{-1}(u^P + e^P)$
EE	$V = (I - \beta F_0)^{-1}(u_0 + e_0)$

- NFXP and SEQ-full use the value function contraction mapping.
- FD and SEQ-EE estimator use the contraction mapping in the probability space.

Monte Carlo Experiments

Data Generating Process

Table – Parameters in DGP

Flow-Payoff Parameters	$\theta_0^{VP} = 0.5 \quad \theta_1^{VP} = 1.0 \quad \theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5 \quad \theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0 \quad \theta_1^{EC} = 1.0$
State Variable Transition	z_{kt} is AR(1), $\gamma_0^k = 0$, $\gamma_1^k = 0.6$
Productivity Transition	ω_t is AR(1), $\gamma_0^\omega = 0$, $\gamma_1^\omega = 0.9$
Past action on productivity	$\gamma_a \in [0, 5]$
Discount Factor	$\beta = 0.95$

Finite Dependent Model

Table – Finite dependent two-step estimators

	FD	*FD2*	*AFD*	*AFD2*	*HM*	*EE*
	$Market = 200$, $Time = 20, \gamma_a = 0$					
θ_0^{VP}	0.4845	0.4845	0.4845	0.4845	0.5016	0.4845
	(0.0706)	(0.0706)	(0.0706)	(0.0706)	(0.0350)	(0.0706)
θ_0^{FC}	0.5447	0.5447	0.5447	0.5447	0.5098	0.5447
	(0.0904)	(0.0904)	(0.0904)	(0.0904)	(0.0627)	(0.0904)
	$Market = 200$, $Time = 120, \gamma_a = 0$					
θ_0^{VP}	0.4963	0.4963	0.4963	0.4963	0.4983	0.4963
	(0.0189)	(0.0189)	(0.0189)	(0.0189)	(0.0140)	(0.0189)
θ_0^{FC}	0.4990	0.4990	0.4990	0.4990	0.4954	0.4990
	(0.0301)	(0.0301)	(0.0301)	(0.0301)	(0.0279)	(0.0301)

DGP : $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$.

Non-Finite Dependent Model

Table – Non-finite Dependent two-step estimators

	FD	*FD2*	*AFD*	*AFD2*	*HM*	*EE*
$Market = 200, Time = 20, \gamma_a = 5$						
θ_0^{VP}	0.3434	0.5679	0.4925	0.5067	0.5307	0.5691
	(0.0790)	(0.1457)	(0.0860)	(0.0908)	(0.0800)	(0.1460)
θ_0^{FC}	-0.0155	0.7095	0.4432	0.4751	0.5833	0.7134
	(0.2228)	(0.3321)	(0.2402)	(0.2518)	(0.2209)	(0.3330)
$Market = 200, Time = 120, \gamma_a = 5$						
θ_0^{VP}	0.3058	0.4965	0.4829	0.4954	0.4982	0.4975
	(0.0333)	(0.0484)	(0.0436)	(0.0453)	(0.0395)	(0.0485)
θ_0^{FC}	-0.1239	0.4920	0.4583	0.4860	0.4977	0.4953
	(0.0845)	(0.1237)	(0.1096)	(0.1140)	(0.1036)	(0.1239)

DGP : $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$.

Sequential Estimation I

Table – The mean and standard deviation of sequential estimators

	FD	*FD2*	*AFD*	*AFD2*
$Market = 200, Time = 20, \gamma_a = 0$				
θ_0^{VP}	0.5163	0.5079	0.5163	0.4799
	(0.0376)	(0.0369)	(0.0376)	(0.0672)
θ_0^{FC}	0.4203	0.5146	0.4203	0.5516
	(0.0635)	(0.0591)	(0.0635)	(0.0804)
$Market = 200, Time = 20, \gamma_a = 5$				
θ_0^{VP}	0.3128	0.5084	-0.1775	0.4940
	(0.0661)	(0.0925)	(0.1838)	(0.1151)
θ_0^{FC}	-0.2597	0.5167	-1.9434	0.4391
	(0.1703)	(0.2506)	(0.6454)	(0.3056)

DGP : $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$.

27 / 31

Sequential Estimation II

Table – The mean and standard deviation of sequential estimators

	HM	*EE*	*SEQ*(1)	*SEQ*(2)	*SEQ*(5)
$Market = 200, Time = 20, \gamma_a = 0$					
θ_0^{VP}	0.5080	0.5079	0.5079	0.5079	0.5079
	(0.0368)	(0.0369)	(0.0369)	(0.0369)	(0.0369)
θ_0^{FC}	0.5148	0.5146	0.5146	0.5146	0.5146
	(0.0593)	(0.0591)	(0.0591)	(0.0591)	(0.0591)
$Market = 200, Time = 20, \gamma_a = 5$					
θ_0^{VP}	0.5096	0.5084	0.5043	0.5084	0.5084
	(0.0938)	(0.0925)	(0.0921)	(0.0925)	(0.0925)
θ_0^{FC}	0.5207	0.5167	0.5034	0.5167	0.5167
	(0.2567)	(0.2506)	(0.2493)	(0.2506)	(0.2506)

DGP

Table – Parameters in DGP

Flow-Payoff Parameters θ^1	$\theta_0^{VP} = 0 \quad \theta_1^{VP} = 1.0 \quad \theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5 \quad \theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0 \quad \theta_1^{EC} = 1.0$
Flow-Payoff Parameters θ^2	$\theta_0^{VP} = 1 \quad \theta_1^{VP} = 1.0 \quad \theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5 \quad \theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0 \quad \theta_1^{EC} = 1.0$
Mixing Probability	$(0.5, 0.5)$
State Variable Transition	z_{kt} is AR(1), $\gamma_0^k = 0$, $\gamma_1^k = 0.6$
Productivity Transition	ω_t is AR(1), $\gamma_0^\omega = 0$, $\gamma_1^\omega = 0.9$
Past action on productivity	$\gamma_a = 2$
Discount Factor	$\beta = 0.95$

29 / 31

Time and iteration

Table – Median Time and Iteration when increase state space

Algorithms	nGrid	2	3	4	5	6
	$\lvert\mathcal{X}\rvert$	64	486	2048	6250	15552
	Market			100		
	Time			20		
FD2	*Time*	11.2472	13.9627	27.9147	390.0466	3103.6867
	Iteration	40.5	48.5	37	47.5	32.5
FD2(FV)	*Time*	9.7783	14.1659	42.0155	612.4756	3097.6401
	Iteration	32.5	47	70	118	32
EE	*Time*	12.1462	21.3075	18.6141	181.0266	1039.5331
	Iteration	38.5	69.5	43	80.5	52
HM	*Time*	30.3638	35.6079	982.0085	-	-
	Iteration	91.5	59.5	53	-	-
SEQ(1)	*Time*	6.0499	17.2884	24.1402	100.8548	509.4910
	Iteration	22.5	64.5	55	43.5	35.5

† The results shows the time and iteration used in the estimation based on 12 Monte Carlo simulations of different state space.

Thank You

tenki
hvala
ขอขอบคุณคุณ
takk
спасибо
kam sah hamnida
je
дзякуй
dziękuję
bedankt
תודה
dhanyavadagalu
tack
gracias
blagodaram
mèsi
xièxie
tanemirt
dank je
rahmet
enkosi
mochchakkeram
trugarez
arigatô
danke
kop khun krap
faafetai lava
manana
diolch
akun
shukriya
ありがとう
kia ora
dankon
děkuji
ačiū
grazzi
mamnun
gràçie
kiitos
spas
thank you
barka
शुक्रिया
sulpáy
tapadh leat
chnorakaloutioun
dhanyavad
gratias ago
tau
dankie
gràcies
teşekkür ederim
bayarlalaa
σας ευχαριστώ
obrigada
sagolun
murakoze
taiku
mahalo
didi madloba
sukriya
obrigado
chokrane
rahmat
dakujem
misaotra
welalin
mercé
najis tuke
شكرا
terima kasih
謝謝
köszönöm
asante
grazie
nandri
mersi
sobodi
maururu
merci
nanni
vinaka
matondo
cảm ơn bạn
go raibh maith agat
paldies
ngiyabonga

Aguirregabiria, V. and Ho, C. Y. (2012). A dynamic oligopoly game of the US airline industry : Estimation and policy experiments. *Journal of Econometrics*, 168(1) :156–173.

Aguirregabiria, V. and Magesan, A. (2016). Solution and Estimation of Dynamic Discrete Choice Structural Models Using Euler Equations.

Arcidiacono, P. and Ellickson, P. (2011). Practical Methods for Estimation of Dynamic Discrete Choice Models. *Annual Review of Economics*, 3(1).

Beauchamp, A. (2015). Regulation, Imperfect competition, and the U.S. abortion market. Technical Report 3.

Berry, S. (1992). Estimation of a model of entry in the airline industry. *Econometrica*, 60(4) :889–917.

Doganoglu, T. and Klapper, D. (2006). Goodwill and dynamic advertising strategies. *Quantitative Marketing and Economics*, 4(1) :5–29.

Doraszelski, U. and Pakes, A. (2007). A Framework for Applied Dynamic Analysis in IO. *Handbook of Industrial Organization*, 3(December 2007) :1887–1966.

Dubé, J. P., Hitsch, G. J., and Manchanda, P. (2005). An empirical model of advertising dynamics. *Quantitative Marketing and Economics*, 3(2) :107–144.

Fang, H. and Wang, Y. (2009). Estimating Dynamic Discrete Choice Models with Hyperbolic Discounting, with an Application to Mammography Decisions.

Gaynor, M. and Town, R. (2012). Competition in Health Care Markets. *Handbook of Health Economics*, (2) :499–637.

Gowrisankaran, G., Lucarelli, C., Schmidt-Dengler, P., and Town, R. (2011). Government policy and the dynamics of market structure : Evidence from Critical Access Hospitals.

Gowrisankaran, G. and Town, R. J. (1997). Dynamic equilibrium in the hospital industry. *Journal of Economics and Management Strategy*, 6(1) :45–74.

Keane, M., Todd, P., and Wolpin, K. (2011). *The Structural Estimation of Behavioral Models : Discrete Choice Dynamic Programming Methods and Applications*, volume 4.

Rust, J. and Rothwell, G. (1995). Optimal response to a shift in regulatory regime : The case of the US nuclear power industry. *Journal of Applied Econometrics*, 10(1 S) :S75–S118.

Schivardi, F. and Schneider, M. (2008). Strategic experimentation and disruptive technological change. *Review of Economic Dynamics*, 11(2) :386–412.

Sweeting, A. (2013). Dynamic product positioning in differentiated product markets : The effect of fees for musical performance rights on the commercial radio industry. *Econometrica*, 81(5) :1763–1803.

Todd, P. E. and Wolpin, K. I. (2006). Assessing the Impact of a School Subsidy Program in Mexico : Using a Soc Experiment to Validate a Dynam Fertility Behavioral Model of Child Schooling. *The American Economic Review*, 96(5) :1384–1417.

Yakovlev, E. (2016). Demand for Alcohol Consumption and Implication for Mortality : Evidence from Russia.