Using Euler Equation to Estimate Non-Finite-Dependent Dynamic Discrete Choice Model with Unobserved Heterogeneity

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Jasmine Hao and Hiro Kasahara

Vancouver School of Economics

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Background

Dynamic Discrete Choice Model

Model priors

- The agents are forward looking and maximize expected inter-temporal payoffs.
- Structural functions: agents' preferences and beliefs about uncertain events.
- Estimated under principle of revealed preference, use micro-data on individuals' choices and outcomes.

Empirical applications includes

- ◇ Industrial organization Aguirregabiria and Ho (2012), Berry (1992), Yakovlev (2016),Sweeting (2013);
- ◇ Health economics Beauchamp (2015),Gaynor and Town (2012), Gowrisankaran and Town (1997),Gowrisankaran et al. (2011);
- ◇ Marketing Dubé et al. (2005),Doraszelski and Pakes (2007), Doganoglu and Klapper (2006);
- ◇ Labor economics Todd and Wolpin (2006),Fang and Wang (2009),Keane et al. (2011);
- ◇ Other Schivardi and Schneider (2008), Rust and Rothwell (1995).

The difficulties in incorporating unobserved heterogeneity:

- ◇ Computational heavy: value function iteration or Hotz-Miller inversion
- ◇ EM algorithm: more iterations account for unobserved heterogeneity.
- ◇ Existing methods relies on "Finite Dependence"(Arcidiacono and Ellickson (2011)).

The contribution of this project:

- ◇ Conceptually redefine the deterministic problem as a stochastic problem.
- ◇ Propose alternative estimator and incorporate unobserved heterogeneity and EM algorithm in dynamic discrete choice.
- ◇ Demonstrate the performance using Monte Carlo simulation.

Baseline Model

Baseline entry exit model

Now consider the baseline model of dynamic choice model

- ◇ Time is discrete and indexed by t.
- ◇ Firms have preferences defined states of the world between periods 0 and T finite / infinite.
- ◇ A state of the world has two component: predetermined s_t and discrete action $d_t \in \mathcal{D} = \{0, 1\}$.
- ◇ Time-separable utility function $\sum_{t=0}^{T} \beta^t U_t(d_t, s_t)$, where $\beta \in [0, 1)$ is the discount factor.
- ◇ Let $d_t^*(s_t)$ denote optimal decision rule, $V_t(s_t)$ be the value function at period t.

$$V_t(s_t) = \max_d \left\{ U_t(d, s_t) + \beta \int V_{t+1}(s_{t+1}) dF_t(s_{t+1}|a, s_t) \right\}. \quad (1)$$

Key assumptions

Here are the key assumptions made in estimating the models:

- Assumption 1(Additive separable): $s = (x_t, \epsilon_t)$, $\epsilon_t = [\epsilon_t(0), \epsilon_t(1)]$, $U(d_t, s_t) = u(d_t, x_t; \theta) + \epsilon_t(d)$. x_t is observed by the economist, ϵ_t is not observed by the economist.
- Assumption 2(Finite domain of x): $x \in \mathcal{X}$, $|\mathcal{X}|$ is finite.
- Assumption 3(Conditional independence): $F(s_{t+1}|a_t, s_t) = G_\epsilon(\epsilon_{t+1}|x_{t+1})F_x(x_{t+1}|x_t, d_t)$.
- Assumption 4(Distribution of ϵ): $\epsilon_t = \{\epsilon_t(d) : d \in \mathcal{D}\} \sim_{i.i.d} T1EV$.

Motivating Example: Entry Exit Problem

For example, take Consider a stationary infinite time horizon entry-exit problem:

- ◇ The firm observe the state $x_t = (y_t, z_t)$. The profitability $z_t \in \mathcal{Z}$, where $|\mathcal{Z}| = N$ is finite, and operation state $y_t = d_{t-1} \in \{0, 1\}$.
- ◇ The firm makes entry decision $d_t \in \mathcal{D} = \{0, 1\}$.
- ◇ z_t follows a first order Markov process $f(x_{t+1}|x_t, d_t)$;
- ◇ The firm's flow payoff $u(x_t, d_t; \theta)$.

Entry Exit Problem: Bellman Value Function

The ex-ante value function:

$$\begin{aligned}\bar{V}(x_t) &= E_\epsilon V(x_t, \epsilon) \\ &= E_\epsilon \Big\{ \max_{d \in \mathcal{D}} \big\{ v(d, x_t; \theta) + \epsilon_t(d) \big\} \Big\}\end{aligned} \tag{2}$$

The firm's strategy $d_t^* = \arg\max_{d \in \mathcal{D}} \big\{ v(x_t, d_t) + \epsilon_t(d) \big\}$, where

$$v(x_t, d; \theta) = u(x_t, d; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} f(x_{t+1} | x_t, d) \bar{V}(x_{t+1}).$$

Estimation technique: use the distribution of ϵ, form the logit likelihood function: $l(d_t, x_t; \theta) = \frac{\exp(u(x_t, d_t; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} f(x_{t+1} | x_t, d_t) \bar{V}(x_{t+1}))}{\sum_{d \in \mathcal{D}} \exp(u(x_t, d; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} f(x_{t+1} | x_t, d) \bar{V}(x_{t+1}))}$.

Bellman Equation in Probability Space

Decision and state in probability space

Decision:

Now consider an optimization problem defined in the probability space. The firm chooses the sequence of $\left\{\left\{\mathbf{P}_t(x_t)\right\}_{x_t\in\mathcal{X}}\right\}_{t=0}^{\infty}$ for all possible future states to maximize the discounted utility;

State:

The ex-ante distribution of x_{t+1}. $\boldsymbol{\kappa}_t(x_t|x_0) \in [0,1]$ and $\sum_{s_t} \boldsymbol{\kappa}_t(x_t|x_0) = 1$.

$$\boldsymbol{\kappa}_t(x_t|x_0) = \begin{cases} \mathbf{1}(x_t = x_0) & \text{if } t = 0 \\ \sum_{x_{t-1}} \boldsymbol{\kappa}_{t-1}(x_{t-1}|x_0) \sum_{d=0}^{1} p_t(d)(x_{t-1}) f_d(x_t|x_{t-1}) & \text{if } t \geq 1 \end{cases}.$$

Bellman Operator

Define the Bellman operator as

$$\boldsymbol{W}^*(\boldsymbol{\kappa}_t) = \max_{\tilde{P}_t} \boldsymbol{\kappa}_t^T \boldsymbol{U}^{\boldsymbol{P}_t} + \beta \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+1})$$
$$\text{subject to } \boldsymbol{\kappa}_{t+1} = \boldsymbol{F}^{\boldsymbol{P}_t} \boldsymbol{\kappa}_t,$$

where

- ◇ $\boldsymbol{\kappa}_t$, $\boldsymbol{U}^{\boldsymbol{P}_t}$ are vectors of length $|\mathcal{X}|$.
- ◇ $\boldsymbol{U}^{\boldsymbol{P}_t} = [\boldsymbol{U}^{\boldsymbol{P}_t}(x^{(1)}, \dots, \boldsymbol{U}^{\boldsymbol{P}_t}(x^{(|\mathcal{X}|)})]^\top$.
- ◇ $\boldsymbol{U}^{\boldsymbol{P}_t}(x) = \boldsymbol{P}_t(x)^\top \Big(\boldsymbol{u}(x) + \boldsymbol{e}^{\boldsymbol{P}_t}(x)\Big)$.
  - $\boldsymbol{u}(x) = [u(d, x)]_{d \in \mathcal{D}}$
  - $\boldsymbol{e}^{\boldsymbol{P}_t}(x) = [\gamma - log(P_t(d, x))]_{d \in \mathcal{D}}$
- ◇ $\boldsymbol{F}^{\boldsymbol{P}_t}$ is the $\boldsymbol{P}_t$-weighted transition matrix.

Approach I: Envelop Theorem

$$\frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_t)}{\partial \boldsymbol{\kappa}_t} = \boldsymbol{U}^{\boldsymbol{P}_t} + \beta \boldsymbol{F}^{\boldsymbol{P}_t} \frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+1})}{\partial \boldsymbol{\kappa}_{t+1}}, \tag{3}$$

$$\left(diag(\boldsymbol{\kappa}_t) \otimes I_{|\mathcal{D}|-1}\right) \tilde{\boldsymbol{u}}^{\boldsymbol{P}_t} + \beta \left(diag(\boldsymbol{\kappa}_t) \otimes I_{|\mathcal{D}|-1}\right) \tilde{\boldsymbol{F}} \frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+1})}{\partial \boldsymbol{\kappa}_{t+1}} = 0, \tag{4}$$

where

- $\tilde{\boldsymbol{U}}^{\boldsymbol{P}_t}$ is the derivative vector: $\tilde{\boldsymbol{U}}^{\boldsymbol{P}_t} = \tilde{\boldsymbol{u}} + \tilde{\boldsymbol{e}}^{\boldsymbol{P}_t}$ where $\tilde{\boldsymbol{u}} = [u(d,x) - u(0,x)]_{d \in \mathcal{D}/\{0\}, x \in \mathcal{X}}$ and $\tilde{\boldsymbol{e}}^{\boldsymbol{P}_t(x)} = -[log(P_t(d,x)) - log(P_t(0,x))]_{d \in \mathcal{D}/\{0\}, x \in \mathcal{X}}$.
- $\tilde{\boldsymbol{F}} = [\boldsymbol{f}(d,x) - \boldsymbol{f}(0,x)]_{d \in \mathcal{D}/\{0\}}$, $\boldsymbol{f}(d,x)$ the Markov transition probability of x_{t+1} given the state and decision.

Approach I: Envelop Theorem

Combine equation(3),(4):

$$\frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_t)}{\partial \boldsymbol{\kappa}_t} = \boldsymbol{U}_0^{\boldsymbol{P}_t} + \beta \boldsymbol{F}_0 \frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+1})}{\partial \boldsymbol{\kappa}_{t+1}}, \tag{5}$$

Combine equation (4),(5) to get

$$\tilde{\boldsymbol{u}} + \tilde{\boldsymbol{e}}^{\boldsymbol{P}_t} + \beta \tilde{\boldsymbol{F}} \left(\boldsymbol{u}_0 + \boldsymbol{e}_0^{\boldsymbol{P}_t} + \beta \boldsymbol{F}_0 \frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+1})}{\partial \boldsymbol{\kappa}_{t+1}} \right) = 0. \tag{6}$$

- ◇ $\boldsymbol{U}_0^{\boldsymbol{P}_t} = \boldsymbol{u}_0 + \boldsymbol{e}_0^{\boldsymbol{P}_t}$,
- ◇ $\boldsymbol{u}_0 = [u(0, x^{(1)}), \ldots, u(0, x^{(|\mathcal{X}|)})]^\top$.
- ◇ $\boldsymbol{e}_0^{\mathcal{P}_t} = [\gamma - \log(P_t(0, x^{(1)})), \ldots, \gamma - \log(P_t(0, x^{(|\mathcal{X}|)}))]^\top$.
- ◇ In addition,

$$\frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_t)}{\partial \boldsymbol{\kappa}_t} = [\bar{V}(x^{(1)}), \ldots, \bar{V}(x^{(|\mathcal{X}|)})]^\top.$$

Proposition 1

In a stationary model,

$$\frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_t)}{\partial \boldsymbol{\kappa}_t} = (I - \beta \boldsymbol{F}_0)^{-1}\left(\boldsymbol{u}_0 + \boldsymbol{e}_0^{\boldsymbol{P}_t}\right).$$

The logit likelihood function from equation (6):

$$l(d_t, x_t; \theta) = \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))},$$

$$v(d, x_t; \theta) = u(d, x_t; \theta) + \beta \tilde{\boldsymbol{f}}(d_t, x_t) \frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+1})}{\partial \boldsymbol{\kappa}_{t+1}}.$$

Likelihood Function(FD)

Proposition 2 (Finite Dependence)

If the model display the finite dependence property, there exists an arbitrary action $d^{\dagger}$ such that $\tilde{\boldsymbol{F}}\boldsymbol{F}_{d^{\dagger}} = \boldsymbol{0}$.

Proposition 3 (Characterization of Bellman Equation)

$$\frac{\partial \boldsymbol{W}^{*}(\boldsymbol{\kappa}_t)}{\partial \boldsymbol{\kappa}_t} = \boldsymbol{u}_{d^{\dagger}} + \boldsymbol{e}_{d^{\dagger}}^{\boldsymbol{P}_t} + \boldsymbol{F}_{d^{\dagger}} \frac{\partial \boldsymbol{W}^{*}(\boldsymbol{\kappa}_t)}{\partial \boldsymbol{\kappa}_t}.$$

The logit likelihood function for finite dependence:

$$l(d_t, x_t; \theta) = \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))},$$

$$v(d, x_t; \theta) = u(d, x_t; \theta) + \beta \tilde{\boldsymbol{f}}(d_t, x_t)\left(\boldsymbol{u}_{d_{t+1}^{\dagger}, t+1} + \boldsymbol{e}_{d_{t+1}^{\dagger}, t+1}^{\boldsymbol{P}_{t+1}}\right)$$

Likelihood Function(AFD)

Proposition 4 (Almost Finite Dependent Estimator)

If the model does not exhibit finite dependence, we can find $d^{\dagger}_{t+1}$ to minimize the norm of $|\tilde{\boldsymbol{F}}\boldsymbol{F}_{d^{\dagger}_{t+1}}|$.

$$
\begin{aligned}
l(d_t, x_t; \theta) &= \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))}, \\
v(d, x_t; \theta) &= u(d, x_t; \theta) + \beta \tilde{\boldsymbol{f}}(d_t, x_t)\Big(\boldsymbol{u}_{d^{\dagger}_{t+1}, t+1} + \boldsymbol{e}^{\boldsymbol{P}_{t+1}}_{d^{\dagger}_{t+1}, t+1} \\
&\qquad + \boldsymbol{F}_{d^{\dagger}_{t+1}, t+1} \frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+2})}{\partial \boldsymbol{\kappa}_{t+2}}.\Big)
\end{aligned}
$$

Approach II: Calculus of Variation

Define the optimal objective function as

$$\boldsymbol{W}_t^* = \boldsymbol{\kappa}_t^T \max_{\tilde{\boldsymbol{P}}_t, \tilde{\boldsymbol{P}}_{t+1}} \boldsymbol{U}^{\tilde{\boldsymbol{P}}_t} + \beta \boldsymbol{F}^{\tilde{\boldsymbol{P}}_t} \left(\boldsymbol{U}^{\tilde{\boldsymbol{P}}_{t+1}} + \beta \boldsymbol{F}^{\tilde{\boldsymbol{P}}_{t+1}} \boldsymbol{W}_{t+2}^* \right)$$

Subject to $\left(\tilde{\boldsymbol{P}}_{\boldsymbol{t}}^* \tilde{\boldsymbol{F}} + \boldsymbol{F}_0\right)\left(\tilde{\boldsymbol{P}}_{\boldsymbol{t+1}}^* \tilde{\boldsymbol{F}} + \boldsymbol{F}_0\right) = \left(\tilde{\boldsymbol{P}}_{\boldsymbol{t}} \tilde{\boldsymbol{F}} + \boldsymbol{F}_0\right)\left(\tilde{\boldsymbol{P}}_{\boldsymbol{t+1}} \tilde{\boldsymbol{F}} + \boldsymbol{F}_0\right)$

Proposition 5 (No Solution to Calculus of Variation)

If the model does not exhibit finite dependence, there does not exist a pair of $(\tilde{\boldsymbol{P}}_t, \tilde{\boldsymbol{P}}_{t+1}) \neq (\boldsymbol{P^*}_t, \boldsymbol{P^*}_{t+1})$ such that
$\left(\tilde{\boldsymbol{P}}_{\boldsymbol{t}}^* \tilde{\boldsymbol{F}} + \boldsymbol{F}_0\right)\left(\tilde{\boldsymbol{P}}_{\boldsymbol{t+1}}^* \tilde{\boldsymbol{F}} + \boldsymbol{F}_0\right) = \left(\tilde{\boldsymbol{P}}_{\boldsymbol{t}} \tilde{\boldsymbol{F}} + \boldsymbol{F}_0\right)\left(\tilde{\boldsymbol{P}}_{\boldsymbol{t+1}} \tilde{\boldsymbol{F}} + \boldsymbol{F}_0\right)$.

Estimators for Heterogeneous Agent Model

EM Algorithm

M types of agent, $\theta = (\theta^1, \ldots, \theta^M)$.
Let π^m denote the probability of being type m.
$l(d_{it}, z_{it}; \theta_m)$ is the likelihood function.

$$\{\hat{\theta}, \hat{\pi}\} = \arg\max_{\theta,\pi} = \sum_{n=1}^{N} \log \left\{ \sum_{m=1}^{M} \pi^m \Pi_{t=1}^{T} l(d_{it}, z_{it}, s; \theta^m) \right\}, \tag{7}$$

where $\hat{P} = (\hat{P}^1, \ldots, \hat{P}^M)$ is an estimator for CCPs,
$\hat{V} = (\hat{V}^1, \ldots, \hat{V}^M)$ is an estimator of the value function.
$\hat{q}_{is}$, the probability n is type m

$$\hat{q}_{im} = \frac{\hat{\pi}^m \Pi_{t=1}^{T} l(d_{it}, z_{it}, \hat{P}^m, \hat{V}^m, \hat{\theta}^m)}{\sum_{s'=1}^{S} \hat{\pi}^{m'} \Pi_{t=1}^{T} l(d_{it}, z_{it}, \hat{P}^{m'}, \hat{V}^{m'}, \hat{\theta}^{m'})}. \tag{8}$$

EM Algorithm

Step 1: Compute $\hat{q}_{is}^{(k)}$ as

$$\hat{q}_{im}^{(k)} = \frac{\hat{\pi}^{m,(k-1)} \prod_{t=1}^{T} l(d_{it}, z_{it}, \hat{P}^{m,(k-1)}, \hat{V}^{m,(k-1)}, \hat{\theta}^{m,(k-1)})}{\sum_{m' \in \mathcal{M}} \hat{\pi}^{m',(k-1)} \prod_{t=1}^{T} l(d_{it}, z_{it}, \hat{P}^{m,(k-1)}, \hat{V}^{m,(k-1)}, \hat{\theta}^{m',(k-1)})}.$$

Step 2: Using $\hat{q}_{im}^{(k)}$ to compute $\hat{\pi} m, (k)$: $\hat{\pi}^{m,(k)} = \frac{1}{N} \sum_{i=1}^{N} \hat{q}_{im}^{(k)}$.
Step 3: Update the CCPs $\hat{P}^{(k)}$, and the value function $\hat{V}^{(k)}$.
Step 4: Update estimator of θ with the equation

$$\hat{\theta}_k = \arg\max_{\theta} \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{s \in \mathcal{S}} \hat{\pi}_{s,k-1} \log l(d_{it}, x_{it}, s, \hat{P}_{k-1}, \hat{\theta}_{k-1}). \tag{9}$$

Likelihood Function

$$l(d_t, x_t; \theta) = \frac{\exp(\tilde{v}(d_t, x_t))}{1+\sum_{d\in\mathcal{D}/\{0\}}\exp(\tilde{v}(d, x_t))}$$

Table: Likelihood function comparison

Method	diff in continuation value ($\tilde{v}(d,x)$
NFXP, HM, EE, SEQ(q)	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+\mathbf{1}}\in\mathcal{X}} \tilde{f}(x_{t+1}\|x_t, d_t) V$
FD	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+\mathbf{1}}\in\mathcal{X}} \tilde{f}(x_{t+1}\|x_t, d_t) \left(u_0 + \gamma - \log(p_0)\right)$
AFD	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+\mathbf{1}}\in\mathcal{X}} \tilde{f}(x_{t+1}\|x_t, d_t) \left(\sum_d \omega(d)\left(u_d + \gamma - \log(p_d)\right)\right)$
FD2 AFD2	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+\mathbf{1}}\in\mathcal{X}} \tilde{f}(x_{t+1}\|x_t, d_t) \left(u_0 + \gamma - \log(p_0) + V\right)$ $\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+\mathbf{1}}\in\mathcal{X}} \tilde{f}(x_{t+1}\|x_t, d_t) \left(\sum_d \omega(d)\left(u_d + \gamma - \log(p_d) + V\right)\right)$

Value function

Table: Comparisons between value function computation

Method	Contraction Mapping
NFXP	$V(x_t) = E_\epsilon \left\{ \max_{d \in \mathcal{D}} [u_d(x_t) + \epsilon_d + \beta \sum_{x_{t+1} \mid x_t} V(x_{t+1})] \right\}$ till convergence
SEQ(q)	$V(x_t) = E_\epsilon \left\{ \max_{d \in \mathcal{D}} [u_d(x_t) + \epsilon_d + \beta \sum_{x_{t+1} \mid x_t} V(x_{t+1})] \right\}$ for q times
Hotz-Miller	$V = (I - \beta F^P)^{-1}(u^P + e^P)$
EE	$V = (I - \beta F_0)^{-1}(u_0 + \gamma - log(p_0))$
FD2	$V = u_0 + \gamma - \log(p_0) + \beta F_0 V$
AFD2	$V = sum_d \omega(d)\,(u_d + \gamma - \log(p_d) + V)$

Monte Carlo Experiments

Data generating process: Homogeneous agent model

Table: Parameters in DGP

Flow-Payoff Parameters	$\theta_0^{VP} = 0.5 \quad \theta_1^{VP} = 1.0 \quad \theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5 \quad \theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0 \quad \theta_1^{EC} = 1.0$
State Variable Transition	z_{kt} is AR(1), $\gamma_0^k = 0$, $\gamma_1^k = 0.6$
Productivity Transition	ω_t is AR(1), $\gamma_0^\omega = 0$, $\gamma_1^\omega = 0.9$
Past action on productivity	$\gamma_a \in [0, 5]$
Discount Factor	$\beta = 0.95$

Finite Dependent Model

Table: Two-step: Finite dependent models

	FD	*FD2*	*AFD*	*AFD2*	*HM*	*EE*
			$Market = 200, Time = 20, \gamma_a = 0$			
θ_0^{VP}	0.4845	0.4845	0.4845	0.4845	0.5016	0.4845
	(0.0706)	(0.0706)	(0.0706)	(0.0706)	(0.0350)	(0.0706)
θ_0^{FC}	0.5447	0.5447	0.5447	0.5447	0.5098	0.5447
	(0.0904)	(0.0904)	(0.0904)	(0.0904)	(0.0627)	(0.0904)
			$Market = 200, Time = 120, \gamma_a = 0$			
θ_0^{VP}	0.4963	0.4963	0.4963	0.4963	0.4983	0.4963
	(0.0189)	(0.0189)	(0.0189)	(0.0189)	(0.0140)	(0.0189)
θ_0^{FC}	0.4990	0.4990	0.4990	0.4990	0.4954	0.4990
	(0.0301)	(0.0301)	(0.0301)	(0.0301)	(0.0279)	(0.0301)

DGP: $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$.

Two-step: Non-finite dependent models

Table: Non-finite Dependent two-step estimators

	FD	*FD2*	*AFD*	*AFD2*	*HM*	*EE*
			$Market = 200$, $Time = 20, \gamma_a = 5$			
θ_0^{VP}	0.3434	0.5679	0.4925	0.5067	0.5307	0.5691
	(0.0790)	(0.1457)	(0.0860)	(0.0908)	(0.0800)	(0.1460)
θ_0^{FC}	-0.0155	0.7095	0.4432	0.4751	0.5833	0.7134
	(0.2228)	(0.3321)	(0.2402)	(0.2518)	(0.2209)	(0.3330)
			$Market = 200$, $Time = 120, \gamma_a = 5$			
θ_0^{VP}	0.3058	0.4965	0.4829	0.4954	0.4982	0.4975
	(0.0333)	(0.0484)	(0.0436)	(0.0453)	(0.0395)	(0.0485)
θ_0^{FC}	-0.1239	0.4920	0.4583	0.4860	0.4977	0.4953
	(0.0845)	(0.1237)	(0.1096)	(0.1140)	(0.1036)	(0.1239)

DGP: $\theta_0^{VP} = 0.5$,$\theta_0^{FC} = 0.5$.

Sequential Estimation

Table: The mean and standard deviation of sequential estimators

	FD	*FD2*	*AFD*	*AFD2*
$Market = 200, Time = 20, \gamma_a = 0$				
θ_0^{VP}	0.5163	0.5079	0.5163	0.4799
	(0.0376)	(0.0369)	(0.0376)	(0.0672)
θ_0^{FC}	0.4203	0.5146	0.4203	0.5516
	(0.0635)	(0.0591)	(0.0635)	(0.0804)
$Market = 200, Time = 20, \gamma_a = 5$				
θ_0^{VP}	0.3128	0.5084	-0.1775	0.4940
	(0.0661)	(0.0925)	(0.1838)	(0.1151)
θ_0^{FC}	-0.2597	0.5167	-1.9434	0.4391
	(0.1703)	(0.2506)	(0.6454)	(0.3056)

DGP: $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$.

Continue: Sequential Estimation

Table: The mean and standard deviation of sequential estimators

	HM	*EE*	*SEQ*(1)	*SEQ*(2)	*SEQ*(5)
	$Market = 200$, $Time = 20, \gamma_a = 0$				
θ_0^{VP}	0.5080	0.5079	0.5079	0.5079	0.5079
	(0.0368)	(0.0369)	(0.0369)	(0.0369)	(0.0369)
θ_0^{FC}	0.5148	0.5146	0.5146	0.5146	0.5146
	(0.0593)	(0.0591)	(0.0591)	(0.0591)	(0.0591)
	$Market = 200$, $Time = 20, \gamma_a = 5$				
θ_0^{VP}	0.5096	0.5084	0.5043	0.5084	0.5084
	(0.0938)	(0.0925)	(0.0921)	(0.0925)	(0.0925)
θ_0^{FC}	0.5207	0.5167	0.5034	0.5167	0.5167
	(0.2567)	(0.2506)	(0.2493)	(0.2506)	(0.2506)

Data generating process: Heterogeneous agent model

Table: Parameters in DGP

Flow-Payoff Parameters θ^1	$\theta_0^{VP}=0$ $\theta_1^{VP}=1.0$ $\theta_2^{VP}=-1.0$ $\theta_0^{FC}=0.5$ $\theta_1^{FC}=1.0$ $\theta_0^{EC}=1.0$ $\theta_1^{EC}=1.0$
Flow-Payoff Parameters θ^2	$\theta_0^{VP}=1$ $\theta_1^{VP}=1.0$ $\theta_2^{VP}=-1.0$ $\theta_0^{FC}=0.5$ $\theta_1^{FC}=1.0$ $\theta_0^{EC}=1.0$ $\theta_1^{EC}=1.0$
Mixing Probability	$(0.5, 0.5)$
State Variable Transition	z_{kt} is AR(1), $\gamma_0^k=0$, $\gamma_1^k=0.6$
Productivity Transition	ω_t is AR(1), $\gamma_0^\omega=0$, $\gamma_1^\omega=0.9$
Past action on productivity	$\gamma_a=2$
Discount Factor	$\beta=0.95$

Time and iteration

Table: Median Time and Iteration when increase state space

Algorithms	nGrid	2	3	4	5	6
	$\lvert\mathcal{X}\rvert$	64	486	2048	6250	15552
	Market			100		
	Time			20		
FD2	*Time*	11.2472	13.9627	27.9147	390.0466	3103.6867
	Iteration	40.5	48.5	37	47.5	32.5
FD2(FV)	*Time*	9.7783	14.1659	42.0155	612.4756	3097.6401
	Iteration	32.5	47	70	118	32
EE	*Time*	12.1462	21.3075	18.6141	181.0266	1039.5331
	Iteration	38.5	69.5	43	80.5	52
HM	*Time*	30.3638	35.6079	982.0085	-	-
	Iteration	91.5	59.5	53	-	-
SEQ(1)	*Time*	6.0499	17.2884	24.1402	100.8548	509.4910
	Iteration	22.5	64.5	55	43.5	35.5

† The results shows the time and iteration used in the estimation based on 12 Monte Carlo simulations of different state space.

Thank You


tenki
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dank je
дзякуй
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bedankt
תודה
dhanyavadagalu
tack
gracias
blagodaram
mèsi
xièxie
tanemirt
rahmet
enkosi
mochchakkeram
trugarez
djere dieuf
arigatô
danke
kop khun krap
faafetai lava
manana
diolch
akun
shukriya
ありがとう
kia ora
dankon
děkuji
ačiū
grazzi
mamnun
gràçie
kiitos
spas
thank you
barka
शुक्रिया
sulpäy
tapadh leat
chnorakaloutioun
dhanyavad
gratias ago
tau
dankie
gràcies
bayarlalaa
σας ευχαριστώ
obrigada
teşekkür ederim
sagolun
murakoze
taiku
mahalo
didi madloba
sukriya
obrigado
chokrane
rahmat
dakujem
misaotra
welalin
mercé
najis tuke
terima kasih
謝謝
شكرا
asante
grazie
nandri
mersi
köszönöm
sobodi
merci
nanni
vinaka
mauruuru
matondo
cảm ơn bạn
go raibh maith agat
paldies
ngiyabonga

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