Building up Trust in a Dynamic Game: A study on Collusive Price-fixing in the Chilean Pharmaceutical Retail industry

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Abstract

This paper provides a tractable model that separates firms' *incentive problems* and *coordination problems* during the initiation of collusion. In the Chilean pharmaceutical industry, firms collude through price leadership. Collusion gradually diffuses among markets: firms collusively raise prices in a couple of markets per week. We propose a model of price leadership under the dynamic pricing game framework to incorporate the *coordination problems* by allowing firms' beliefs about competitors' conduct to be biased towards a competitive equilibrium. As firms observe supra-competitive prices, they adaptively learn that competitors are willing to collude. The market characteristics explain firms' willingness to collude: those markets with lower cross-firm elasticities, the collusive price leadership costs lower. We show that the gradualism is explained by the heterogeneous market characteristics as well as firms' learning to coordinate.
Keywords: collusion; pharmaceutical retail; dynamic game; biased belief

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1 Introduction

Collusion has been a central topic of the industrial organization since the work by Bain (1959). It harms consumer welfare and also jeopardizes fair market competition (Harrington (1987)). Understanding the initiation of collusion and evaluating policy intervention has been a widely discussed issue.

The theory is abundant in modelling the *implementation* of collusion but often assumes away the *initiation* stage. See, for example, Fudenberg and Maskin (1986). This paper studies how collusions are initiated, specifically, how firms learn to collude through collusive price leadership. Oligopolistic firms without communication trying to achieve supracompetitive outcomes face two problems, the *incentive problem* and the *coordination problem*. The *incentive problem* is that collusive profits need to be high enough so that firms are willing to start the collusion. We model this incentive problem by requiring any credible agreement must be a subgame perfect Nash equilibrium. The *coordination problem* arises because, typically, there are many possible subgame perfect equilibria, and firms may be uncertain which equilibrium they are in.

The theory of collusion is relatively rich in the *incentive problems* during the initiation stage. This paper accounts for the incentive problems by using market characteristics such as price elasticities and market size to explain a market-specific likelihood of collusion. Besides the market factors (Ivaldi et al. (2003)), coordinating practices such as price leadership explain firms' incentives to collude (Byrne and De Roos (2019)). Contrarily, few theoretical works focus on the *coordination problem* in the initiation stage. Firms risk mis-communications and disagreements while transitioning to a collusive price leadership equilibrium. (See, e.g. Byrne and De Roos (2019) and Wang (2009).)

The paper makes two contributions to the existing literature. First, this paper is the first empirical work to address the coordination problems during the initiation of a price-fixing cartel. The gradual increase in prices across markets observed in the data cannot be explained by a rational expectations price leadership model. Our model with firms' learning can explain this observed pattern. Secondly, this paper proposes a model to account for firms' nonequilibrium beliefs while allowing the beliefs to be determined semi-endogenously. In a dynamic collusive price leadership game, we allow firms to hold beliefs biased by firm-specific "belief parameters" towards competitive equilibrium beliefs. This method allows us to conduct counterfactual analysis on collusion initiations, given changes in incentive problems and coordination problems separately.

This paper analyzes firms' initiation of collusion based on a case of price-fixing in Chile's pharmaceutical retail market during 2006 to 2008. The smallest chain, Salcobrand, was acquired by a local business group, Juan Yarur, in August 2007. The change of ownership causes Salcobrand to change business strategies. The other two chains, Cruz Verde and FASA adapt to the new "atmosphere" and form their beliefs after observing Salcobrand's business strategy changes. The collusion episode has been studied by Chilet (2016, 2018), who examine the firms' willingness to collude using reduced-form designs focusing on the incentive problems. This paper uses a structural model to account for the coordination problems as well as incentive problems. Chilet (2016) finds that when firms face multi-market contact, they collude gradually: they collude on a few products each week using collusive price leadership strategies and continue the process, and the process lasts a few months. The firms collude on more differentiated markets first (Chilet (2018)). However, the market characteristics alone are not sufficient to explain the gradualism. We show that firms' learning can better explain the gradualism of collusion. Firms learn the coordination experience in some markets and exercise the conduct on other markets later.

To explain this gradualism of cooperation and the firms' learning, we propose a model under the framework of Maskin and Tirole (1988). To account for the collusive price leadership, firms decide whether and when to be the price leader/ the price follower. The conventional Markov Perfect Equilibrium (MPE) often assumes that firms' strategies are functions of payoff relevant variables and that firms' beliefs are in equilibrium everywhere. In contrast, our model allows firms' beliefs to be not necessarily rational. In our models, firms' strategies are functions of payoff relevant variables and an estimable function of the game's history, summarized by the number of successful cumulated collusions. The actual profits given all firms' actions are independent of the game history, and firms' beliefs evolve given the game history. This condition provides the exclusion restriction to identify the ratios of beliefs across different values of game history. The idea of conditioning the firms' strategies on the game history is similar to that by Fershtman and Pakes (2000), where they assume firms' strategies are conditional on whether they observed other firms' deviations. We model firms' *incentive problems* by including fixed cost structural parameters in firms' preferences. The fixed cost parameters reflect firms' expected government penalties, and the expected penalties grow with the length of collusion. The *coordination problem* is modelled by assuming firms' beliefs are initially biased towards a competitive equilibrium. The firms' beliefs evolve and converge to rational beliefs under the price leadership equilibrium as the number of colluded markets increase.

We incorporate firm-specific "belief parameters" to measure the convergence to rational equilibrium, which are functions of the number of colluded markets. The proposed model nests a rational expectation model for some value of the belief parameters. The biasness of the beliefs are testable. In contrast to other learning models such as adaptive learning and fictitious play, our proposed method is simpler to implement and offers clear identification results. Furthermore, the estimated value of belief parameters is easy to interpret: it measures how far away firms' actual beliefs are from the rational expectation beliefs. Our model also allows us to account for counterfactual policy analysis on how the market will respond, given firms' learning processes. The identification of ratios of the belief parameters relies on two exclusion restrictions: (1) one firm's lagged pricing decision affects his own payoff through adjustment costs while other firms' lagged pricing decisions do not. (2) The payoffs on a given market are not affected by the market outcomes on other markets.

We find that the proposed model with relaxed beliefs explains the firms' gradualism in cooperation much better than the rational model. The estimated rational model fails to replicate what we observe in the data. In particular, the rational model predicts that the price leader will start the price increase for most of the products at the beginning of the coordination episode. The rational model predicts that the price leader, Salcobrand, is 10.35 times more likely to start the collusive price leadership as compared to the nonparametrically estimated probabilities. The incentive to collude can be partly explained by market differentiation and cost associated with leadership and partly by firms' failures or successes in coordination on other markets. We demonstrate the importance of coordination problems by considering two counterfactual policies. The first is to impose a price cap that reduces the firms' profits from collusion. The second is a divestiture of the firms and form a fourth chain store. We show that with the divesture to form the fourth chain store, the price leader Salcobrand is 78 % less likely to lead the price increase, and therefore, the collusion will be less alleged to happen. The presence of the fourth player aggravates the coordination problems by introducing more uncertainties to the price leader. Henceforth, the coordination issue could hinder firms' intentions to continue cooperating when more agents are involved in the dynamic pricing game.

Related Literature

This paper relates to empirical literature measuring the incentive among firms to collude, for example, Igami and Sugaya (2016), Clark and Houde (2013) and Wang (2009). In the theory of microeconomics, the willingness to cooperate is a central concept, and cartels are among the main problems in industrial organizations. Despite the rich theoretical literature, few empirical works quantify the dynamics of the incentive of cartel members to collude.

A market's likelihood to collusion can be explained by market characteristics such as market concentration [1], firm asymmetry [2], frequency of interaction, the existence of a new market entrant(Igami and Sugaya (2016)), market transparency and product differentiation. One noticeable trait is the *degree of market differentiation* [3]. In this paper, we use estimated elasticity to proxy the market differentiation. A similar measure is used by the work by Chilet (2016) on the Chilean pharmacy price-fixing. The work finds that firms collude on more differentiated markets first, which incurs smaller losses if deviated. Firms' asymmetries in product differentiation reduce the benefit of cheating(Rhee and Thomadsen (2004) and Martin (2002)). This paper also considers the cost asymmetries by controlling firms' fixed effects in the cost estimations.

[1] See Bain (1951), Bain (1956), Bain (1959), Demsetz (1973) and Tirole (1988)

[2] See Rhee and Thomadsen (2004), Martin (2002)

[3] See, for example, Deneckere and Kovenock (1992), Chang (1991), Ross (1992) and Thomadsen and Rhee (2007). Empirical evidence by Symeonidis (2003) studies cartels in the UK in the 1950s found cartels to be more likely in "low-advertising industries", which is associated with low product differentiation.

Another relevant stream in collusion literature is on collusive price leadership in facilitating collusion. Theories of collusive leadership show that a price leader's existence facilitates collusion(Rotemberg and Saloner (1990) and Mouraviev and Rey (2011)). With the presence of cost asymmetry, firms achieve collusion by redistributing the market share. The incentive is weaker for the leader due to the demand persistence from loyalty, brand image, or habits; therefore, leaders are usually the less efficient firm [4]. Empirical research documents that less efficient firms serve as the price leader in the coordinated price increases(Clark and Houde (2013) and Marshall et al. (2008)). The chain with the smallest operation(Salcobrand) scale leads to the price increase in the Chilean pharmacy case; this corresponds to the theoretical prediction. We propose a dynamic game that is compatible with the theoretical collusive price leadership. We allow for nonequilibrium belief in a subset of the data and show that the other two firms need time to learn to cooperate in the collusive price leadership.

Lastly, this paper relates to the discussions on collusion in the industry with multi-market contacts. The pharmaceutical retail industry often involves multi-product firms. The incentives for collusion on one market can be affected by other markets. [5] Empirical findings have shown that the presence of multi-market contact[6] Past researches show that demand and cost linkage and "pooled incentive constraints" (Bulow et al. (1985) and Bernheim and Whinston (1990)). This paper provides a potential explanation of the linkage between a multi-market contract and exercising market power through firms' learning: firms learn the coordination experience in some markets and exercise the conduct on other markets later.

This paper is also related to an active research area in modelling a firm's beliefs in dynamic games, allowing for beliefs' endogeneity but relaxing the assumption of rational expectations. Aguirregabiria and Jeon (2019) provides an extensive review of existing literature on this topic. After the seminal papers by Bresnahan and Reiss (1990, 1991), the empirical discrete game with incomplete information is widely applied in IO topics, especially oligopolistic markets. Examples include firms' entry/exit decisions [7] and price competition[8]. Firms form beliefs of uncertainty in demand, future uncertainties and strategies by other firms. The heterogeneous firms' ability to form expectations and the implications on market outcomes have been long recognized in economics, at least since the work of Simon (1959) and Muth (1960). Firms need to acquire information, and empirical studies in various industries show the learning process (e.g. Goldfarb

[4] See, for example, Röller and Steen (2006), Mouraviev and Rey (2011) and Miklós-Thal (2011)

[5] Few researches study the collusion in retail industries except for Chilet (2016), Chilet (2018), Clark and Houde (2013) and Genakos et al. (2018). Genakos et al. (2018) studies fruits and vegetables in Greece) and Clark and Houde (2013) studies gasoline price-fixing in Quebec.

[6] See Evans and Kessides (1994), Parker and Roller (1997) and Ciliberto and Williams (2014).

[7] Berry (1992), Toivanen and Waterson (2002), Pesendor and Schmidt-Dengler (2003), Bajari et al. (2007) and Aguirregabiria and Ho (2012).

[8] For example, Fershtman and Pakes (2000) propose a framework of firms competing in price; Kano (2013) propose a model that allows firms to consider the menu costs in a dynamic price-setting game.

and Xiao (2011), Huang et al. (2018) and Doraszelski et al. (2018)). [9] It is restrictive to assume that firms will instantly form rational expectations when building partnerships and familiarize themselves when following the price leadership mechanism to coordinate. Similarly, in laboratory experiments, evidence suggests players beliefs are nonequilibrium(Salz and Vespa (2020)).

Dynamic games with general beliefs are strongly under-identified(See Aguirregabiria (2020) Section 4.2 for discussion), and therefore, researchers impose restrictions on the belief formation. These restrictions include Bayesian Nash Equilibrium where firms hold rational beliefs assumption [10], Cognitive Hierarchy, Level-K models [11], fictitious play and adaptive learning.[12] This project uses a different method to account for the firms' convergence to rational equilibrium. We assume the firms' strategies eventually convergence to equilibrium during the last episodes of the data, and we use the assumption to identify the biased belief in the earlier episodes. The purpose of the paper is to analyze the market outcome instead of studying how firms acquire the proficiency to cooperate. Therefore instead of learning, we allow firms to form a biased belief of other firms' strategies that can be captured by the "belief parameters".

The rest of the paper is organized as follows. Section 2 gives an overview of the Chilean pharmacy retailing industry. We provide empirical analysis on firms' incentives to follow a price leadership mechanism and find that the incentives are explained by the market differentiation and successful history of the coordinated price increase. Section 3 presents the structural model with nonequilibrium beliefs. Section 4 discusses the identification of beliefs and structural parameters. We provide a Monte Carlo experiment to demonstrate the performance of the proposed estimator. Section 5 presents the estimation results and counterfactuals experiments.

2 Background and Data

2.1 Price Evolution of Chilean Retail Pharmacy

This section provides an overview of the Chilean pharmaceutical retail market and empirical analysis that shows the changing of firms' behaviour patterns from 2006 to 2008. The review on the market largely borrows from Chilet (2018) and Chilet (2016)'s case background. The review bases on the report requested by the National Economic Prosecutor(NEP) of Chile for the investigation

[9] For example, Goldfarb and Xiao (2011) shows that after the Telecommunications Act of 1996. The data suggest that more experienced, better-educated managers tend to enter markets with fewer competitors; Huang et al. (2018) document firms' learning about demands after the privatization of the Washington State liquor market; Doraszelski et al. (2018) document the convergence of market outcomes to a rest point after the deregulation in the UK electricity market.

[10] Rational expectation assumption has attractive features that beliefs are endogenously determined in the equilibrium of the model. However, rational overlooks the process of information acquisition, for example, Pesaran (1987) and Manski (2004). Armantier and Richard (2003) assumes firms' strategy is Bayesian Nash Equilibrium based on analysis of the airline industry. Aryal and Zincenko (2019) propose a framework of Cournot competition that relies on equilibrium strategies.

[11] See Goldfarb and Xiao (2018), Hortaçsu et al. (2019) and Brown et al. (2013).

[12] Fudenberg and Levine (2009) review nonequilibrium learning in games.

of this case. The three large chains are Cruz Verde, FASA and Salcobrand. Cruz Verde was the largest chain, with 512 stores, while FASA and Salcobrand had 347 and 295. Cruz Verde's market share increased steadily from 2004 - 2007 from 32% to 41%.[13] FASA became an international drugstore chain with Chile, Mexico, and Peru, 37% of its revenue comes from Chile. Salcobrand was formed from the merger of two chains, Salco and Brand, in 2000. The three drugstore chains have a joint market share of 92 percent of the retail market as 2006 of the branded drugs. The rest of the markets consists of the other drugstores that carry generic brands and therefore are not compatible with the branded drugs. It is fair to state that the data include almost every retail purchase of the included drugs during the time period. The share of the population with a drug insurance plan was extremely low at the time, and therefore, the transaction price should be viewed as an out-of-pocket expense. [14] The medicines' prices are not controlled or regulated, and the health system does not usually reimburse drug expenditure. Branding of medicines is essential for a substantial premium that sells the purchase decision and leading brands. However, medicines are sold only in drugstores, and advertising of prescription drugs is illegal. Physicians prescribe brands, and the consumers purchase from the drugstore given their prescriptions. The consumers are restricted to purchase only the prescribed drug. Switching to a different brand of the same molecule is forbidden by the law. If a drugstore were to adjust the price of a listed product, the company determines a price and then uploads it into a central database. Each company renews its database daily. Prices may be slightly different among branches. Additionally, drugstores offer "loyalty discounts" to shoppers. [15] During 2006, stores offer weekly discounts. The information exchange happens very often among retail chains. The drugstores monitor competitors' prices quite often by regularly purchasing drugs in their competitors' stores, and the drugstores compare prices of top-selling medications more frequently. The pharmaceutical manufacturers also monitor prices regularly and may inform the drugstores if they find significant differences.

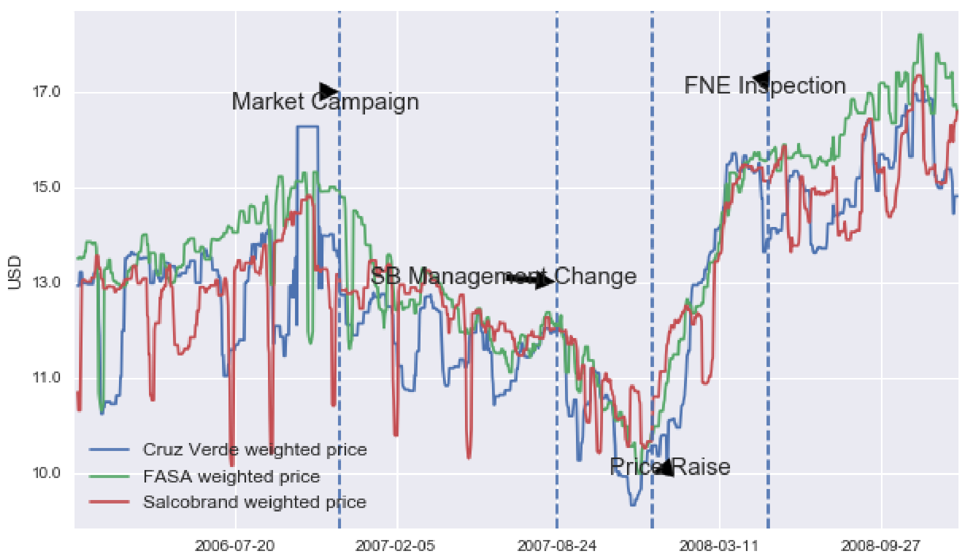
During the observations from the year 2006 to 2008, the price movement contains three distinctive episodes. The competition period observed in the data is from January 2006 to November 2006. The price war period is from March 2006 to November 2007, and the coordinated price increase episode is from December 2007 to March 2008. According to the expert report requested by NEP, the three chain drug stores had been relying heavily on a loss-leading strategy since 2005. During the competition episode, the firms offered regular sales on Tuesdays and Thursdays, but overall the weighted average prices were stable. Starting from November 2006, the firms began to undercut the prices. The price war escalates in August 2007 due to Cruz Verde's marketing cam-

[13]Figures from December 2008. Investors Conference presentation. FASA March 2009. Accessed online July 2012.

[14]According to the survey of Chilet (2016) and the expert report Núñez et al. (2008).

[15]Observations to the evidence. NEP, p. 120. Reply to FASA to the indictment. Usually, customers are asked before paying for their identification number to know whether a discount applies to them. FASA claims that it does not have a loyalty program, as opposed to the other two chains. These claims are confirmed by the data, which show a substantial difference between the list and actual purchase price in Cruz Verde's and Salcobrand's prices, and no difference in FASA's prices

Figure 1: Price trend of Chilean major pharmaceutical retailers



paign ”Low Prices without Competitors” that openly compared prices between itself and FASA, claiming to have the lowest prices in the market. In August 2007, Salcobrand’s 100% ownership was sold to Juan Yarur Companies for 130 million dollars. In November 2007, a court deemed Cruz Verde’s advertising campaign to be unfair competition and halted it. A few weeks later, the pharmacies started coordinating price increases. An antitrust investigation started in May 2008, which led to an average price increase of almost 50% in 222 best-selling brands. The firms’ prices remain steady after the investigation started.

This paper focuses on the transition from the price war episode to the coordination episode. During the coordination episode, from December 2007 to April 2008, the firms coordinate to increase the price of a particular product in a gradual method. The pharmacies raised prices of a small number of drugs every week to or above the pre-price war level through the price leadership mechanism. [16] The price leader increases the price for a certain product; then, the followers match the price increase. The price leader may be different firms in different markets. In the observations, most markets’ price increase is led by Salcobrand. [17] Salcobrand’s change of ownership

[16]Chilet (2018) documents the gradualism in the Chilean pharmacy market. One potential explanation of gradualism in partnership building among firms is by Watson (1999, 2002). The paper relies on the assumption that the payoff from coordination increases along with the level of coordination.

[17]The report Núñez et al. (2008) provides an extensive analysis of the firms’ action patterns during the coordination.

explains the change of other firms' business strategy since the acquisition introduced uncertainty regarding the new owner's willingness to continue the price war. The recruitment of executives facilitated communication among the pharmacies. Therefore, firms had a chance to rebuild coordination. [18]. The manufacturers acted as the channel of communication among the drugstores. Accordingly, internal emails show the pharmacies referring to medicines in groups according to their manufacturer. After the indictment, prices did not drop in the post-coordination period. The observation coincides with the findings of post-cartel tacit collusions(Harrington (2004)). One common conjecture is that demand shocks or supply shocks induce price change. The pharmacies coordinated price increases on 222 brands, almost all prescription-only medicine manufactured by 37 different pharmaceutical companies. The prices of other drugs have practically not changed, and the wholesale prices of these drugs only changed slightly. Another anecdotal evidence to argue against supply-side shocks is that the weighted average price level of Chilean drug prices was different from that of other South American countries, according to the report by Vasallo (2010). See Table 18 in the Appendix for details.

[18] As quoted by Chilet (2016), The change of business strategy dynamics is noted by a former Cruz Verde board member of who stated: " Salcobrand's [new administration] came to change this dynamic of big emotional aggressiveness between the companies because, in fact, Salcobrand present[ed] itself as a neutral competitor that [made] its decisions mostly based on economic principles. " (Deposition of Fernándo Suárez Laureda. Observations to the evidence. NEP, p. 224.)

Table 1: Price Increase Descriptives

Panel A: The Size and Timing given Product Characteristics					
	Size of Price Increase			Time of Price Increase	
	Salcobrand	Cruz Verde	FASA	First Inc	Second Inc
By Treatment					
Non-chronic	0.040	0.041	0.031	69.052	112.778
	(0.030)	(0.022)	(0.023)	(31.528)	(33.345)
Chronic	0.062	0.061	0.045	87.062	126.371
	(0.076)	(0.034)	(0.029)	(34.281)	(32.663)
By Patent					
Non-patent	0.041	0.048	0.036	84.488	121.914
	(0.034)	(0.023)	(0.026)	(39.731)	(34.596)
Patent	0.063	0.057	0.042	78.604	120.042
	(0.076)	(0.036)	(0.029)	(30.719)	(33.155)
By Prescription					
Non-prescription	0.042	0.049	0.036	75.811	116.882
	(0.028)	(0.021)	(0.019)	(34.629)	(35.433)
Prescription	0.057	0.055	0.041	81.788	121.367
	(0.070)	(0.034)	(0.029)	(34.324)	(33.259)
Whether there is second price increase					
With Second Price Inc	0.053	0.052	0.039	65.346	120.654
	(0.069)	(0.035)	(0.030)	(26.160)	(33.481)
No Second Price Inc	0.056	0.055	0.042	95.281	-
	(0.060)	(0.028)	(0.026)	(34.931)	-
Panel B: The Timing of The Second Price Increase					
With Second Price Inc				65.346	120.654
				(26.160)	(33.481)
No Second Price Inc				95.281	-
				(34.931)	-

[1] The size of price increase is computed by taking the 95 % quantile of the price change during the coordination period. [2] The wait time is the days before the price increase starting from October 30, 2007.

The firms' coordinate on the price increase for one product twice. Table 1 describes the relationship between product characteristics and the time to collude. We observe that the percentages of price increases are higher for drugs that treat chronic diseases, patented and prescription only.

Firms' market power can explain the Scale of the price increase. The other thing worth noticing is that firms raise prices for non-chronic disease treatment earlier than those for chronic diseases. The last section of the table shows the coordination time difference between the drugs that the firms coordinated twice on the price increase, and those the firms only increased the price once. For those drugs that the firms coordinated twice, the time for the first coordination is earlier. The observation indicates that firms' may adjust their pricing policy at some point during the coordination episode.

Works in economic theory show that firms are more likely to collude if there exists a price leader, and the decisions become similar to sequential pricing decisions(for example, Mouraviev and Rey (2011)). The price leader faces costs resulting from the uncertainty of other firms' willingness to cooperate and the forsaken market shares. The price was raised by a price leadership mechanism(1-2-3 mechanism or staggered mechanism, as documented by the NEP expert reportNúñez et al. (2008)). The chains raised prices of a given brand by taking turns in the price increases. A witness, a FASA executive, stated that Salcobrand conveyed messages through the manufacturers indicating that they were ready to be the first chain to raise the prices [19]. According to the National Economic Prosecutor and declarations of FASA's executives, the procedure most used to increase prices was the following: Every week, Salcobrand raised the price of a group of drugs, while the other two chains wait a few days and then take turns as the second firm to raise the price. The other firms will increase their prices for the same product a few days afterward. Henceforth, in one week, all three chains would have shows the dynamics of the price changes.

Figure 2 shows the frequency of the documented 1-2-3 price increase during the year 2006 to 2008. Note that there is a significant increase in the 1-2-3 price increase pattern during the coordination episode compare to other times. Define a price increase as a positive price change of more than 15 %, and define a coordinated price increase as follow: The increase in price ($> 15\%$ or more than 1500 peso) happens for a particular product for three firms; One firm initiates the price increase, while the other two firms follow within four days; The price levels before and after increases should be reasonably close($< 15\%$). Firms maintain the post increase price level for at least three days. Figure 3 shows an example of the firms' 1-2-3 price increase for the product *FOLISANIN 5 MG. CAJA 30 COMP*. The price increase is initiated by Salcobrand, following by FASA and then Cruz Verde. The price leader faces a temporary decrease in market share during the price increase. The loss of market share serves as a mechanism to transfer payment to the followers(Chilet (2018)). Similar patterns have been documented in the gasoline retail market collusion(Clark and Houde (2013)).

[19] Testimony of an executive of FASA. Observations to the evidence. NEP, p. 116. The translation of the quotes is by Chilet (2016).

Figure 2: Coordinated Price Increase Frequency and Characteristics

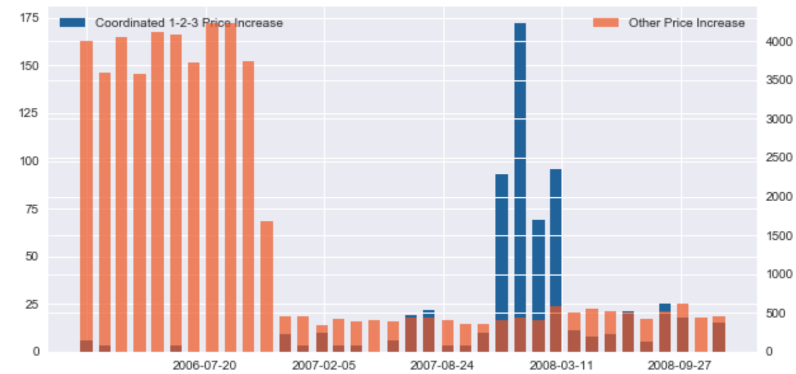


Figure 3: Strategic Price Increase for FOLISANIN 5 MG. CAJA 30 COMP.

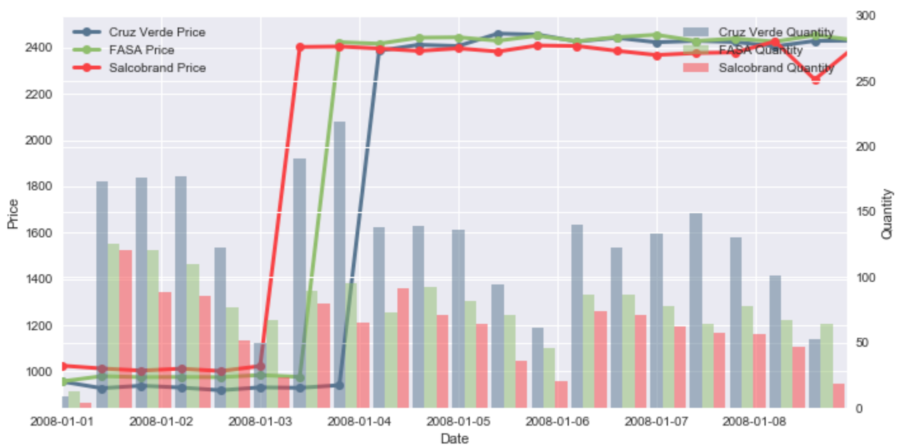


Table 2: The 1-2-3 Price Increase Frequency

Panel A: Frequency of Coordinated Price Increase			
Time periods	Jan,2006 - Nov, 2007	Dec,2007 - Apr, 2008	May,2008 - Dec, 2008
Frequency	32	162	21
Percentage	14.9%	75.3%	9.8%
Monthly average	1.39	33.40	2.62

Panel B: Frequency by Sequence of Price Increase				
Sequence	Total	Jan,2006 - Nov, 2007	Dec,2007 - Apr, 2008	May,2008 - Dec, 2008
SB-CV-FASA	74	7	65	2
SB-FASA-CV	88	3	85	0
FASA-SB-CV	23	8	5	10
FASA-CV-SB	22	10	6	6
CV-FASA-SB	4	3	0	1
CV-SB-FASA	4	1	1	2
Total	32	162	21	215

The table is reported in the expert report Núñez et al. (2008). This table shows that the frequency of 1-2-3 price increase is significantly higher in the time period of Dec 2007 to April 2008.

The table 2 shows the frequency of the coordinated price increase. Panel A shows the count and monthly average number of increase before, during and after the coordination episode. One noticeable fact is that the average number of coordinated price increase in the post-coordination period is higher than the pre-coordination period. Panel B shows the sequence of coordination. Most of the price increase was lead by the smallest chain store Salcobrand, Cruz Verde and FASA take turns to be the second to increase the price. FASA also leads part of the price increase while Cruz Verde is always the follower during the coordination episode.

2.2 Data

We use the compiled transaction data from the Competition Tribunal of Chile. The transaction dataset includes daily national level sales and quantity by the three drugstore chains of the 222 brands that the chains were accused of being colluding on for the years 2006-2008. The data contain the name of the purchased drug, the daily nationwide revenue-weighted average price, and daily units sold nationwide by each store chain. From the expert report, we have the manufacturer's information on the products. We combine the drug attribute data from Catalog.md, Drugbank.com, and Farmazon.cl. Catalog.md contains information on drugs' active ingredients and all the producers that produce the drug to the brand level. Farmazon.cl contains the prescription information of a certain drug in Chile. Drugbank.com contains the treatment information of a certain molecule.

2.3 Time Varying Incentive to Collude

We estimate a survival model and study the particular ordering of products the pharmacies chose to collude on every week over time. Chilet (2016) has also estimated the survival model and show that firms collude on the more differentiated markets first. The survival analysis is different in the sense that we show that past collusion success will increase future incentives to collude, controlling for the market differentiation.

In the survival models, failure is defined as the first coordinated price increase. The aim of the survival analysis is to study how the collusive scheme starts and develops over time. The model should allow for the probability of occurrence to vary over time. Therefore, in addition to the facilitating factors, we also include their interactions with log time. Time interactions allow relaxing the proportional-hazards assumption introducing time-varying effects.[20] In the survival model, failure is defined as the first coordinated price increase in the market. The identification of the effect comes from the variation of product characteristics in the same industry, as opposed to comparing cases of collusions in different industries. We analyze the factors that the literature has identified as making collusion easier (see, for example, Levenstein and Suslow (2006); Ivaldi et al. (2007); Motta (2003)). These facilitating factors are many times supported by the theory, but it is difficult to provide empirical support for them because of the lack of variation within an industry. The analysis includes the market characteristics as the explanatory variable. The cross elasticity is the estimated cross elasticity shown in Section 4 in the market-level demand system. The market size the median of a weekly sales volume. The share dispersion is the dispersion in the market shares across the three firms. Higher share dispersions indicate that the firms' market shares are more asymmetric on the market. [21] Besides the market characteristics, we also include the past events such as successes in coordination(the number of successful coordination achieved by the three firms); failures in coordination(the number of unsuccessful coordination attempted by the firms); the number of price decrease happened in the last two week period for Cruz Verde, Salcobrand and FASA, respectively. A failed collusion is defined as the case that within 5 day period, there is a price increase of more than 15 % percent in the price level and not all the firms increeased the price increase within the 5-day period.

[20]See discussion in Hosmer et al. (2011). If the interaction coefficient is not zero, the effects of the covariates vary over time, and the impact of treatment on hazard is non-proportional.

[21]We assume that the firms enter the risk set in November 2007 and exit it either when their price was increased or in April 2008.

Table 3: Time of Collusion - Survival Model

	Dependent variable: Time to the First Coordinated Price Increase					
	Market Characteristics	Cumulative Past Events		Non-cumulative Past Events		
	(1)	(2)	(3)	(4)	(5)	(6)
$Cross\ Elasticity$	0.0248	0.0357	0.035	0.0244	0.0244	0.0247
	(0.0246)	(0.0315)	(0.0314)	(0.0246)	(0.0245)	(0.0246)
$Cross\ Elasticity * Ln(t)$	-0.0037	-0.0053	-0.0052	-0.0036	-0.0036	-0.0037
	(0.0037)	(0.0047)	(0.0047)	(0.0037)	(0.0037)	(0.0037)
$Market\ Size$	10.1006***	9.3913*	9.7513*	10.297***	9.8346***	10.1665***
	(2.553)	(5.257)	(5.2558)	(2.5748)	(2.5483)	(2.5561)
$Market\ size * Ln(t)$	-1.5065***	-1.4001*	-1.4538*	-1.5359***	-1.4664***	-1.5165***
	(0.3826)	(0.7894)	(0.7893)	(0.3859)	(0.3819)	(0.3831)
$Share\ Dispersion$	45.3541	52.9556	70.103	49.4483	45.4013	45.3579
	(56.7315)	(80.71)	(80.0564)	(57.1709)	(56.432)	(56.7494)
$Share\ Dispersion * Ln(t)$	-6.774	-7.8864	-10.4655	-7.3866	-6.7774	-6.7748
	(8.481)	(12.0943)	(11.9964)	(8.5473)	(8.4364)	(8.4836)
$Sucess\ Coord$		-0.0035	-0.0028			
		(0.0048)	(0.0048)			
$Fail\ Coord$		0.0109***				
		(0.0037)				
$Price\ Dec\ CV$				0.0084		
				(0.0176)		
$Price\ Dec\ FA$					-0.0626*	
					(0.0381)	
$Price\ Dec\ SB$						0.0142
						(0.0242)
N	16493	15270	15270	16493	16493	16493
log-likelihood	-3232.0	-3101.0	-3122.0	-3232.0	-3225.0	-3232.0

The table 3 presents the results of various specifications of Cox models. The interpretation of the estimates is that the hazard of a coordinated price increase increase over time in products in which the firms' cross elasticity is higher and decreases in products where the asymmetry of market shares is higher. Column (1) shows the results of non-proportional hazards over time by market characteristics. Column (2) and (3) present the results where we include the cumulative past events as explanatory variables. Column (4),(5) and (6) present the effect of price decrease in the past five days on future probability of coordination. By comparing columns (2)-(6) with Column (1), including indicators of past events does not change the impact of market characteristics on the timing of collusion.

The interpretation of the Column (1) shows that the firms collude on markets with smaller cross-elasticity, smaller market size and smaller share dispersion. The sign of the results coincide with the expectation that firms will start to collude on a more *differentiated* market since differentiation grants a certain monopoly power to the firms and thus limits consumer poaching(Chilet (2016)). Column (2) and (3) show that the past number of success/failure in coordination attempts will affect the hazard of a coordinated price increase. Column (4) - (6) shows the price decrease will affect the probability of a coordinated price increase. The results indicate that past success/failure of firms' attempts to collude will affect the future success of firms' attempts to collude.

3 Model

This section introduces a model under the dynamic game framework with a relaxed belief, similar to that by Aguirregabiria and Magesan (2019). The industry consists of N firms and M markets. Firms are indexed by $i \in \mathcal{I}$. Firms compete on markets $m \in \mathcal{M}$ simultaneously. Define a single market as the product of a specific dosage—for example, *Maltofer Gts. Frasco 30 Ml* and *Maltofer 100 Mg* are treated as different products. If a consumer is prescribed with a certain brand, she cannot purchase the other without a change of prescription, even if they have the same molecule(Chilet (2016)). Time is discrete and indexed by t, where $t = 1, \ldots, T$. In the data, t corresponds to a day in the observation period. The decision variable $\mathbf{a}_i$ is a vector of the decisions on each market: $\mathbf{a}_i = (a_{im} : m \in \mathcal{M})$, where a_{im} corresponds to firm i's pricing decision on market m. The dimension of $\mathbf{a} = (\mathbf{a}_i : i \in \mathcal{I}) \in \mathcal{A} = [0, 1]^{NM}$. Let a_{imt} be a binary indicator that firm i is setting high/low price on market m at time t. The state variable $x \in \mathcal{X}$ is a vector of variables that are known by all firms. The state variable $\mathbf{x} = (\mathbf{y}, \mathbf{z}, h)$, where $\mathbf{y}$ is the last period lagged pricing decisions $\mathbf{y}_t = \mathbf{a}_{t-1}$, $\mathbf{z}$ is a vector of exogeneous variables and h is a state variable that reflect the game history. On each period(day), the firms set the prices for the M markets simultaneously taken account last period pricing decision $\mathbf{y}_t$, exogensous state variable $\mathbf{z}_t$ and a function of history h_t. The payoff to firm i Π_i given the states and the actions is defined by

$$\Pi_i(\mathbf{a}_t, \mathbf{x}_t, \boldsymbol{\epsilon}_{it}) = \sum_{m \in \mathcal{M}} \mathrm{R}_{im}(\mathbf{a}_{mt}, \mathbf{x}_{mt}) - \sum_{m \in \mathcal{M}} \left(\mathrm{F}_{im}(\mathbf{a}_{mt}, \mathbf{x}_{mt}) + \epsilon_{imt}(a_{imt})\right),$$

where $\mathrm{R}_{im}(\mathbf{a}_{mt}, \mathbf{x}_{mt})$ is the variable profit of firm i on market m at time t, and $\mathrm{F}_{im}(\mathbf{a}_{mt}, \mathbf{x}_{mt})$ is the unobserved cost associated with the pricing decisions on market m. The term $\boldsymbol{\epsilon}_{it} = (\epsilon_{imt} : m \in \mathcal{M})$ is private information of firm i, identically distributed over firms, markets and over time with cumulative distribution function G_ϵ.

The firms' beliefs are probabilistic distributions of other firms' behaviour in each possible state. Let $\mathbf{B}_{it} \equiv \{\mathrm{B}_{it}(\mathbf{a}_{-i}, \mathbf{x}) : \mathbf{a}_{-i} \in \mathcal{A}_{-i}, \mathbf{x} \in \mathcal{X},\ \sum_{\mathbf{a}_{-i} \in \mathcal{A}_{-i}} \mathrm{B}_{it}(\mathbf{a}_{-i}, \mathbf{x}) = 1\} \in [0, 1]^{A|\mathcal{X}|}$ denote player i's belief at period t. The price leadership is modelled by allowing firms to decide given the lagged pricing decision. The set up is similar to that by Rotemberg and Saloner (1990) because firms can make different decision observing the other firms' lagged action. If a firm starts a price increase, the other firms decide whether to follow the price increase. This model accounts for the price leader transfers part of the market shares during the price leading to the followers.

Firms are forward-looking and maximize inter-temporal profits, considering the implications of their current pricing choice and competitors' behaviours. Let $\sigma_i(\mathbf{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{it}) \equiv \{\sigma_{im}(\mathbf{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{it}) : i \in \mathcal{I}\}$ be the vector of strategy functions of firm i, with each element corresponds to decision on one market. Firms maximize their profit given their belief of other firms $\mathbf{B}_{it}$, where each element of the belief function corresponds to a conditional probability of other firms' action. A strategy function σ_{it} (and the associated conditional choice probability(CCP) function P_{it}) is *rational* for every possible value of $(\mathbf{x}_t, \epsilon_{it}) \in \mathcal{X} \times \mathbb{R}^A$ the action maximizes player i's expected and discounted value given his beliefs on the opponent's strategy. Given the beliefs at time t, $\mathbf{B}_{it}$, player i's best response at time t is the optimal solution to a single-agent dynamic programming problem. The value function is defined by the following Bellman equation:

$$V_i^{\mathbf{B}_{it}}(\mathbf{x}_t, \boldsymbol{\epsilon}_{it}) = \max_{\mathbf{a}_{it}} \mathbb{E}_{\mathbf{a}_{-it}} \Big\{ v_{it}^{\mathbf{B}_{it}}(\mathbf{x}_t, \mathbf{a}_{it}) + \boldsymbol{\epsilon}_{it}(\mathbf{a}_{it}) \Big\},$$

where $v_{it}^{\mathbf{B}_{it}}(\mathbf{x}_t, \mathbf{a}_{it})$ is the *conditional choice value function*

$$v_{it}^{\mathbf{B}_{it}}(\mathbf{x}_t, \mathbf{a}_{it}) = \pi_i^{\mathbf{B}_{it}}(\mathbf{x}_t, \mathbf{a}_{it}) + \sum_{\mathbf{x}_{t+1} \in \mathcal{X}} f_i^{\mathbf{B}_{it}}(\mathbf{x}_{t+1} | \mathbf{x}_t, \mathbf{a}_{it}) V_i^{\mathbf{B}_{i,t+1}}(\mathbf{x}_{t+1}, \boldsymbol{\epsilon}_{i,t+1}),$$

and $\beta \in (0, 1)$ is the discount factor. The belief-weighted flow payoff function and belief-weighted transition probability are

$$\pi_i^{\mathbf{B}_{it}}(\mathbf{x}_t, \mathbf{a}_{it}) = \sum_{a_{-i} \in \mathcal{A}_{-i}} \pi_{it}(\mathbf{x}_t, \mathbf{a}_{it}, \mathbf{a}_{-i}) \mathrm{B}_{it}(\mathbf{a}_{-i}, \mathbf{x}_t),$$

and

$$f_{it}^{\mathbf{B}_{it}}(\mathbf{x}_{t+1} | \mathbf{x}_t, \mathbf{a}_{it}) = \sum_{a_{-i} \in \mathcal{A}_{-i}} f(\mathbf{x}_{t+1} | \mathbf{x}_t, \mathbf{a}_{it}, \mathbf{a}_{-i}) \mathrm{B}_{it}(\mathbf{a}_{-i}, \mathbf{x}_t).$$

Formally, the best response function of firm i can be represented using the threshold condition:

$$\sigma_i(\mathbf{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{it}) = \mathbf{a} \text{ i.f.f. } \Big\{ \boldsymbol{\epsilon}_{it}(\mathbf{a}') - \boldsymbol{\epsilon}_{it}(\mathbf{a}) \leq v_{it}^{\mathbf{B}_{it}}(\mathbf{x}_t, \mathbf{a}) - v_{it}^{\mathbf{B}_{it}}(\mathbf{x}_t, \mathbf{a}') \text{ for any } \mathbf{a}' \neq \mathbf{a} \Big\}.$$

Following the previous work by Hotz and Miller (1993), we construct the conditional choice probability(CCP) representation, or optimal conditional probability (OCP) function, by integrating the best response function $\sigma_i(\mathbf{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{it})$ over $\boldsymbol{\epsilon}_{it}$:

$$\mathbf{P}_{it}(\mathbf{a}, \mathbf{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{i,t}) = \int \mathbb{1}\Big\{ \sigma_i(\mathbf{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{it}) = a \Big\} dG_{\epsilon}(\boldsymbol{\epsilon}_{it}) = \Lambda\Big(\mathbf{a}; \mathrm{v}_{it}^{\mathbf{B}_{it}}(\mathbf{x}_t)\Big),$$

where $\Lambda(\mathbf{a};\cdot)$ is the CDF of the vector of $\{\boldsymbol{\epsilon}_{it}(\mathbf{a}') - \boldsymbol{\epsilon}_{it}(\mathbf{a}) : \mathbf{a}' \neq \mathbf{a}\}$ and $\mathrm{v}^{\mathbf{B}_{it}}(\mathbf{x})$ is $(A-1)\times 1$ vector of continuation value differences: $\mathrm{v}^{\mathbf{B}_{it}}(\mathbf{x}) = \{\tilde{v}^{\mathbf{B}_{it}}(\mathbf{x},\mathbf{a}) : \mathbf{a} \in \mathcal{A} \setminus \{\mathbf{0}\}\}$, where $\tilde{v}^{\mathbf{B}_{it}}(\mathbf{x},\mathbf{a})$ is the difference in the continuation value $v^{\mathbf{B}_{it}}(\mathbf{x},\mathbf{a}) - v^{\mathbf{B}_{it}}(\mathbf{x},\mathbf{0})$.

The model is strongly under-identified; therefore, we need to impose restrictions on the beliefs. Empirical literature assumes rational expectations by imposing the restriction that firms' strategies are represented by Markov Perfect Equilibrium(MPE). To capture the *coordination problems* faced by the firms, we impose the following assumption to capture firms' adapting process. Firms gradually learn the true transition density after successful coordinations. Let h_t be the number of successful coordination history of the metagame at time t. Assume that firms update beliefs with the successful coordination in raising prices for products. Therefore, the belief is a function of the equilibrium choice probability as well as the number of successful coordination: $\mathbf{B}_{i,t} = \mathbf{B}_i(h_t)$. The optimal choice mapping is

$$\mathbf{P}_{it}(\mathbf{a}_{it}, \mathbf{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_i(h_t)) = \Lambda\Big(\mathbf{a}; \mathrm{v}_{it}^{\mathbf{B}_i(h_{t+1})}(\mathbf{x}_t)\Big).$$

The model implies that with price leadership, under the subgame perfect equilibrium, the followers are likely to follow the price leader if there exists a sustainable collusive equilibrium.

3.1 Demand model

We consider a simple logit demand model for each market separately. Define a market as the consumers of a single product, where a product is defined as a brand in a given dosage sold. The product can be described in the following characteristics: (1) the product fixed effect utility, representing the average willingness to pay from consuming such product, (2) the product-store fixed effect, which captures the consumer's loyalty to the store and (3) the product characteristics valued by consumers but unobserved to the researchers. Assume that all the consumers are homogeneous. Therefore the indirect utility of a consumer purchasing the product m from chain store i at time t at period t is

$$U_{imt} = u_{imt} - \alpha_m p_{imt} + \nu_{imt}, \quad i \in \mathcal{I} \cup \{0\},$$

where p_{imt} is the price of the product m sold at store i time t, u_{imt} is the willingness to pay for the product of the average consumer in the market, and ν_{imt} is a consumer-specific component that captures consumer heterogeneity in preferences, α_m is the price coefficient on market m. Besides buying from the three chain stores ($i \in \mathcal{I}$), the consumer can also choose the outside option, denoted with $i = 0$. The outside option can be not buying the product or buying the generic version of the product. Product m's quality depends on the product characteristics mentioned above. Consider the following specification of product quality: $u_{imt} = u_m + \xi_{im}^{(1)} + \xi_{imt}^{(2)}$. $\xi_{im}^{(1)}$ is the store-product fixed effect that captures the service quality difference between stores. $\xi_{imt}^{(2)}$

represent the demand shock that is store-product specific, and follows an $AR(1)$-process.

A consumer purchases product m from store i if and only if U_{imt} is greater than the utilities of the product sold by any other stores. This condition characterizes the unit demand of an individual consumer. Therefore, firm i's aggregate market share s_{imt} can be obtained by integrating individual demands over the consumer's idiosyncratic shock variable ν_{imt}.

$$s_{imt} = \int \mathbb{1}\Big\{U_{imt} \geq U_{i'mt} \quad \forall i' \in \mathcal{I} \cup \{0\}\Big\} dP_\nu(\nu),$$

where s_{imt} is the market share of firm i on market m, the P_ν is the population distribution function of ν. We can derive a closed form market share from equation (3.1). The identification is discussed in section 4.1.

3.2 Variable Cost

The variable profit of firm i on product m is $\mathrm{R}_{imt} \equiv (p_{imt} - c_{imt})q_{imt}$, where c_{imt} is the unit cost of product m of firm i. The average unit cost is assumed to be constant with respect to the unit sold. The retailers negotiate the purchase from the labs, and the marginal cost negotiated are similar across the chained stores(Núñez et al. (2008)). Specify the marginal cost as follow:

$$c_{imt} = c_m + \omega_{im}^{(1)} + \omega_{imt}^{(2)},$$

where c_m is the average unit cost of the product m, $\omega_{im}^{(1)}$ captures the store-product fixed effect of the marginal cost and $\omega_{imt}^{(2)}$ captures the marginal cost shock.

3.3 Fixed Cost

The firms compete on M markets, simultaneously taking the demand as given. The pricing decisions are equilibrium market outcomes. Firms decide whether to start or follow a collusive price leadership considering the associated fixed cost. The firm endogenizes the fixed cost for each collusive price increase, where the menu cost is denoted using F_{imt} in equation (3). Each fixed cost has three components, the menu cost, the fixed cost and the leadership cost. The endogenized cost can be written as follow:

$$\mathrm{F}_{imt} = MC_{im}\mathbb{1}(a_{imt} \neq x_{imt}) + a_{imt}FC_{im} + a_{imt}\mathbb{1}(\mathbf{a}_{-imt} = \mathbf{0})LC_{im},$$

where MC_{im}, FC_{im} and LC_{im} represent the menu cost, fixed cost and leadership cost respectively of setting the price to certain level as defined above. Each chain store uploads its price to a central system, and the price will be informed in the locations across the nation(Núñez et al. (2008)). The stores also offer discounts and loyalty rewards to their customers. Therefore small fluctuations of prices are not due to the price adjustments. The fixed costs are common knowledge for all the firms. The menu costs associates with each time that a firm is posting a price change. Since a price change is made in the system, assume that a firm's menu cost is the same across products. The fixed cost is the firm's cost if it charges a high price on a certain market. The fixed cost account for the firms' subjective loss, given the probability and the potential fines if the anti-trust authorities prosecute them. The fixed cost also grows with the length of collusive pricing. Firms endogenize the potential penalties when deciding whether to collude. The leadership cost captures the price leader's potential loss, such as future market share loss due to consumers' inertia. The price leader pays the leadership costs if they are the first to raise the price.

The specification of the components are as follow:

$$
\begin{aligned}
&MC_{im} = \gamma_i^{MC,0},\\
&FC_{im} = \gamma_i^{FC,0} + \gamma_i^{FC,Profit}\widehat{Profit\ Diff}_{im} + \gamma_i^{FC,Size}\overline{Market\ Size}_m,\\
&LC_{im} = \gamma_i^{LC,Profit}\widehat{Profit\ Diff}_{im} + \gamma_i^{LC,Size}\overline{Market\ Size}_m,
\end{aligned}
$$

where the menu cost parameter is the same across markets, the fixed cost and leadership costs are assumed to be proportional to the abnormal profit and market size. The abnormal profit is defined as the difference between collusive profit and the competitive profit. In this specification, $\widehat{Profit\ Diff}_{im}$ is the computed abnormal profit, $\overline{Market\ Size}_m$ is the log of average daily sales volume of product m. To account for firms' unobserved cost, we estimate the structural parameters of $\{\gamma_i^{MC,0}, \gamma_i^{FC,0}, \gamma_i^{FC,Size}, \gamma_i^{FC,Profit}, \gamma_i^{LC,Size}, \gamma_i^{LC,Profit}\}$ are menu cost, fixed cost and leadership costs coefficients.

3.4 Reducing the dimensionality of the dynamic game

From a computational point of view, the solution and the estimation of the dynamic game of network competition in the section is extremely challenging. Solving the dynamic game requires integrating the value functions over the space of the state variables $\{\mathbf{x}_t, \boldsymbol{\epsilon}_{it}\}$. Given the number of markets in the empirical analysis, the dimensionality of the state is huge($2^{(3*200)} \approx 4 * 10^{180}$). Solving for an equilibrium of a dynamic game with this state space is intractable. To reduce computational complexity, we introduce several assumptions to reduce the dynamic game dimension and make the estimation tractable. Besides the common identification assumptions of dynamic discrete choice models, we also introduce three main assumptions to ease the dimensionality problem: (1) The decisions of prices are discrete: the firm-market specific collusive price level and the competitive price level; (2) A firm's price decisions are made locally by each market manager without knowing the realization of private shocks on other markets; (3) Discretize the history into arbitrary intervals, and beliefs are updated accordingly.

Assumption DIM-1 (Discrete market price level)**.** *For each market, the firm chooses between two distinctive price levels of collusive price and competitive price at each time.*

Assumption DIM-1 states that the firms can choose from two distinctive price levels on a given market m. Although the price decision for the firms is continuous for each market, there are two reasons that we use discrete pricing decisions. First, the cost shocks happen at a low frequency comparing to daily price adjustment, and therefore, we can assume the costs to be constant. Second is that the optimal pricing decision is unique, given the firm's decision to collude, the market power and the marginal costs. The discretization is a standard dynamic-static decomposition of firms' decisions(for example, Fershtman and Pakes (2000) and Aguirregabiria and Ho (2012)). In the observed data, the price fluctuates on a small scale around two distinct price levels. The price level adjustments infrequently happen from one price level to another. We use the notation of a_{imt} to represent firm's pricing decision. Let $a_{imt} = 1$ be that the firm i's pricing decision on market m at time t is a high(collusive) price level, and $a_{imt} = 0$ denotes the low(competitive) price level.

Assumption DIM-2 (Local decisions)**.** *(A) The market manager of the market* (i, m) *chooses from two distinctive price levels to maximize the total discounted value of*

$$\max_{\mathbf{a}_{imt}\in\{0,1\}} \mathbb{E}_t\{\sum_{s=0}^{T} \beta^s \pi_{im}(\mathbf{x}_{m,t+s}, \mathbf{a}_{m,t+s}) + \epsilon_{im,t+s}(a_{im,t+s})\},$$

where $\pi_{im}(\mathbf{x}_{mt}, \mathbf{a}_{mt}, \epsilon_{imt}) = \mathrm{R}_{im}(\mathbf{x}_{mt}, \mathbf{a}_{mt}) - \mathrm{F}_{im}(\mathbf{x}_{mt}, \mathbf{a}_{mt}))$.

(B) The shock ϵ_{imt} *is private information of market manager* (i, m). ■

The assumption DIM-2 states that for each firm i, all the markets make independent decisions conditional on the market state variable and the beliefs of other firms' actions on the same market. The assumption DIM-2(A) states that for each firm-market manager (i, m), the pricing decision a_{im} is based on market level information $\mathbf{x}_{mt}$. Note that a_{im} is a component of the vector $\mathbf{a}_i$ and $\mathbf{x}_{mt}$ is a sub-vector of $\mathbf{x}_t$ as defined previously in the metagame. The market level information is a subset of the firm-level information and the union of the market information contains the firm-level information: $\mathbf{x}_t \in \mathcal{X}, \mathbf{x}_{mt} \in \mathcal{X}_m$ and $\mathcal{X} \subseteq \cup_{m\in\mathcal{M}}\mathcal{X}_m$. Define the market level information as $\mathbf{x}_{mt} = \{\mathbf{y}_{mt}, \mathbf{z}_{mt}, h_t\}$ where $\mathbf{y}_{mt} = \mathbf{a}_{m,t-1}$ is the past pricing decision on market m, $\mathbf{z}_{mt}$ is the exogeneous state variables on market m and h_t is the history variable of the metagamem, which in our case is the number of successful collusion. The assumption DIM-2(B) states that the objective function for every markets are independent. The idiosyncratic shocks on market m is not correlated with the decision of market m' if $m' \neq m$. The assumption also implies that the belief function on market m can be written as $\mathbf{B}_{im}(h_t)$ and the firm-level belief is the collection of beliefs on all markets: $\mathbf{B}_i(h_t)(\mathbf{a}_{-i}) = \Pi_{m\in\mathcal{M}}\mathbf{B}_{im}(h_t)(\mathbf{a}_{-im})$. The strategy function of local manager (i, m)

is $\sigma_{im}(\mathbf{x}_{mt}, \boldsymbol{\epsilon}_{imt}, \mathbf{B}_{im}(h_t))$. Each manager is solving a single-agent dynamic decision problem, except they incorporate dynamic strategic interactions between markets by allowing beliefs to depend on other market managers' past decisions. The learning process is similar to the pattern documented by Byrne and De Roos (2016), where firms repeatedly experimented before reaching a mutual understanding of the price-setting mechanism in the Australian gasoline market.

By imposing the assumption, we can re-write the firms' decision to firm-market level decisions and derive the market-level optimal choice probability: $\mathbf{P} \equiv \{\mathbf{P}_{im}(\mathbf{a}_{imt}, \mathbf{x}_{mt}, \mathbf{B}_{im}(h_t)) : i \in \mathcal{I},\ m \in \mathcal{M}\}$, where $a_{-im} = \{a_{i'm} : i' \in \mathcal{I}, i' \neq i\} \in \mathcal{A}_{-im}$.

Assumption DIM-3 (Belief update)**.** *The belief of firm i is dependent of the history. For a given history h, $B_{im}(h)(\mathbf{x}_m) = B_{im'}(h)(\mathbf{x}_{m'})$ if $\mathbf{x}_m = \mathbf{x}_{m'}$.*

Define the history of the firm level meta-game as the number of successful collusions happened at time t. Furthermore, to make the estimation tractable, we discretize the value of the history h_t into four discete values: $\{[0, 30], [31, 90], [90, 150], [150, \infty)\}$.

4 Identification and Estimation

This section discusses the identification of beliefs and firms' structural parameters. Suppose the researcher observes the panel data of $\{\mathbf{a}_{mt}, \mathbf{x}_{mt}\}$ over periods $t \in \{1, 2, \ldots, T_{data}\}$ and on market $m \in \mathcal{M}$, where $\mathbf{a}_{mt} = (\mathbf{a}_{imt} : i \in \mathcal{I})$ and the set of players are $\mathcal{I} = \{CV, FA, SB\}$. CV denotes Cruz Verde, FA denotes FASA and SB denotes Salcobrand. Let $\mathbf{P}^0$ be the vector of CCPs with the true(population) conditional probabilities: $\mathbf{P}_{im}(a_{imt}|\mathbf{x}_{mt})$ for player i in market m at period t. We want to use the samples to estimate the structural "parameters", i.e. the payoffs $\{\Pi_{it}, \beta\}$, transition probabilities $\{f_t\}$, distributions of the unobservables, and beliefs parameters $\{\mathbf{B}_i(h)\}$ for $i \in \{CV, FA, SB\}$ and history variable $h \in \mathcal{H}$. The belief parameters are allowed to evolve arbitrarily for each $h \in \mathcal{H}$. For primitives other than players' beliefs, we assume the unknown variable's distribution is $\Lambda(\cdot)$ is known to the researchers up to a scale parameter. Assume that the discount factor β is known, and the transition probability functions $\{f_t\}$ are non-parametrically identified. This section focuses on identifying the beliefs and the payoff functions assuming $\{f_t, \Lambda_{it}, \beta\}$ are known. The identification of the payoff parameters relies on the standard identification of dynamic games under revealed preference(Bajari et al. (2007)). The identification of beliefs relies on two additional exclusion restrictions. Firstly, the firm i's payoff is only affected by its own lagged pricing decision through menu cost but not other firms' lagged pricing decisions. Second, the market outcomes on a given market m given firms' decisions and state variables are not affected by the market outcomes on other markets.

In this section, the discussion is as follows. In subsection 4.1, we discuss the identification of the payoff parameters assuming the identification of the conditional choice probabilities $\mathbf{B}_i$ and the belief functions $\mathbf{B}_i$. In the subsection 4.2, we discuss the identification of beliefs under the following exclusion restrictions: (1) one firm's lagged pricing decision affects his payoff through adjustment costs while other firms' lagged pricing decisions do not. (2) The profits on a given market are not affected by the market outcomes in other markets.

4.1 Identification of payoff parameters

The identification of the payoff functions consists of two parts, the variable profit function R_{im} and the unobserved fixed cost function F_{im}. The variable profit function is estimated using the competition episode data, from January 2006 to November 2006. The fixed costs are estimated under revealed preference using the coordination episode's decisions, from December 2007 to April 2008. Assume the demands are invariant from the competition episode to the coordination episode.

4.1.1 Variable Profits

The variable profit is $\mathrm{R}_{im} \equiv (p_{imt} - c_{imt})q_{imt}$, where q_{imt} is the equilibrium quantity sold given the prices, p_{imt} is the price and c_{imt} is the marginal cost of firm i product m. During the competition episode, firms practice on regular discounts through the weekdays. Overall, the weekly average prices are stable over time. Assume that firms compete under Nash-Bertrand equilibrium.

Assume that the idiosyncratic shock of consumers ν follows Type 1 Extreme Value distribution. Use the notation s_{imt} to denote the market share of firm i in product m at time t. Assume the number of potential customers on the market at a given date is fixed, denoted by M_m. Derive the closed-form market share from equation (3.1):

$$\ln(s_{imt}) - \ln(s_{0mt}) = u_{imt} - \alpha_m p_{imt} = u_m - \alpha_m p_{imt} + \xi_{im}^{(1)} + \xi_{imt}^{(2)} - \alpha_m p_{imt}, \tag{1}$$

where $\ln(s_{0mt})$ is the share of outside product of product m, and $s_{0mt} \equiv 1 - \sum_{i\in\mathcal{I}} s_{imt}$. The demand shock creates a serial correlation problem. In order to estimate the price coefficient $\hat{\alpha}_m$, take a partial first-difference of equation (1):

$$\log(s_{imt}/s_{0mt}) - \rho_m \log(s_{im,t-1}/s_{0m,t-1}) = u_m - \alpha u_m - \alpha_m(p_{imt} - \rho_m p_{imt-1}) + (1-\rho)\xi_{im}^{(1)} + \epsilon_{imt}^{D}. \tag{2}$$

For each market m, we estimate the persistence of demand shock ρ_m and the price elasticities α_m by running regression of equations 2. There is endogeneity issue such that $cov(p_{imt}, \epsilon_{imt}^{D}) \neq 0$. The current demand shock is independent of the previous period of price, and therefore $cov(p_{imt}, p_{im,t-s}) \neq 0$ or $s \geq 2$. The demand system equations (1) are estimated using Arellano and Bond (1991) instrument variables: the lagged price more than two periods ago $\{p_{im,t-s},\ s \geq 2\}$.

Given the price coefficient α_m and marginal cost c_{im} for the products $m \in \mathcal{M}$ of retailer $i \in \mathcal{I}$, the Nash-Bertrand equilibrium is characterized by the system of price equations

$$p_{imt} - c_{imt} = (1 - s_{imt})^{-1} / \alpha_m, \tag{3}$$

To obtain $\hat{c}_{imt}$, plug the estimated $\hat{\alpha}_m$ in the equation (3) and estimate the equation using ordinary least square. The firms' marginal cost on the market m is estimated by averaging the costs throughout the eleven months from January 2006 to November 2006. The estimated profits given the price decisions are $R_{im}(\mathbf{a}_{mt}) = M_m \hat{s}_{imt}(\mathbf{a}_{mt}) \left(p_{im}(a_{imt}) - \hat{c}_{im}\right)$, where M_m is the time-invariant market size for market m, $\mathbf{a}_{mt}$ is the pricing decisions on market m and $\hat{s}_{imt}(\mathbf{a}_{mt})$ is the estimated market share given the actions,$p_{im}(a_{imt})$ is the corresponding pricing level of firm i given the decision of whether to set collusive price a_{imt}.

4.1.2 Fixed Costs

The estimation of the fixed costs relies on revealed preference. Firms' decisions are assumed determined by payoff-relevant state variables. An firm's payoff-relevant information at time t is $\{\mathbf{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{it}(h_t)\}$. Assume for now that the beliefs of the firms $\mathbf{B}_{it}$ are consistently estimated. The vector of common knowledge state variables is $\mathbf{x}_t$, and it evolves over time according to the transition function $f(\mathbf{x}_{t+1}|\mathbf{a}_t, \mathbf{x}_{t+1})$ where $\mathbf{a}_t = \{\mathbf{a}_{it} : i \in \mathcal{I}\}$ represents the vector of current actions by all players.

Assumption 1 (Best Response)**.** *Assume the follows hold: (A) Firms' strategy functions depend only on payoff relevant state variables: $\mathbf{x}_t$ and ϵ_{it}. Also, a firm's belief about the strategy of rival firms is a function of only common knowledge payoff relevant state variables $\mathbf{x}_t$. (B) For every player i, $\mathbf{P}^0_{im}$ is his best response at period t given his beliefs $\mathbf{B}_{im}$ and the payoff functions π_{im}. (C) A firm's beliefs about his own actions in the future are unbiased expectations of his actual actions in the future. (D) It is common knowledge that players' private information ϵ is independently distributed across players.* ■

Assumption 1 is the critical assumption for identifying the structural parameters. Assumption 1 (A) assumes that the players' decisions are conditional on the payoff related state variables and the beliefs only. The payoff-relevant information set is $\{\mathbf{x}_t, \boldsymbol{\epsilon}_{it}\}$. This assumption is similar to the assumption by Maskin and Tirole (1987) except that we allow the firms' to form biased belief given non-payoff-related state variables. Assumption 1 (B) assumes players are rational in the sense that their actual behaviour is the best response given their beliefs. Assumption 1 (C) assumes a firm has rational belief regarding his behaviour. This assumption 1 (D) implies that a player's beliefs should satisfy the restriction that other players' actions are independent conditional on common knowledge state variables.

Assumption 2. *Assume the following: (A) a_{imt}, x_{mit} are finitely supported; (B) $\epsilon_{imt}(a_{imt})$ is additive* *seperable ; (C) The transition of ϵ_{imt} is conditionally independent of $\mathbf{x}_{mt}|\mathbf{x}_{mt-1}$. (D) Firms' private information ϵ_{imt} are drawn from a $T1EV$ distribution $G_i(\cdot)$.* ■

The assumption 2 follows the standard assumption of the dynamic game framework(see, for example, Magnac and Thesmar (2002)). The assumption assumes independency of the players' actions.

With Assumption 2, we have

$$\mathrm{q}(a_{imt}, \mathbf{x}_t, \mathbf{B}_{im}(h_t)) = \log(\mathrm{P}_{im}(a_{imt}, \mathbf{x}_t, \mathbf{B}_{im}(h_t))/\mathrm{P}_{im}(0, \mathbf{x}_t, \mathbf{B}_{im}(h_t))). \tag{4}$$

Theorem 1 (Hotz–Miller inversion theorem)**.** *Under the assumption 2, for any $(i, m, t, \boldsymbol{x})$, the mapping $\mathbf{P}_{im}(\mathbf{a}_{im}, \boldsymbol{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{i,t}) = \Lambda\Big(\mathbf{a}_{im}; \mathrm{v}_{im}^{\mathbf{B}_{im}(h)}(\boldsymbol{x}_{mt})\Big)$ is invertible such that there is a one-to-one relationship between the $(A_m - 1) \times 1$ vector of CCPs $\mathbf{P}_{im}(\boldsymbol{x}_t, \boldsymbol{\epsilon}_{it}, \mathbf{B}_{im}(h))$ and the $(A_m - 1) \times 1$ vector of value differences $\tilde{\mathbf{v}}_{im}^{\mathbf{B}_{im}(h)}(\boldsymbol{x}_{mt})$.* ■

For any market level policy function $\sigma_{im}(\mathbf{x}_m, \boldsymbol{\epsilon}_{im})$, define the market level *conditional choice probability*(CCP) by integrating the policy function over ϵ_{imt}:

$$\mathrm{P}_{im}(a|\mathbf{x}_m) = \int \mathbb{1}\Big\{\sigma_{im}(\mathbf{x}_m, \epsilon_{im}) = a\Big\} dG_{\epsilon}(\epsilon_{im}).$$

It is convenient to represent players' behavior using the *Conditional Choice Probability* (CCP) functions. When the state variable $\mathbf{x} \in \mathcal{X}$ has a finite support, we can represent the CCP function $\mathbf{P}_{it}(\cdot)$ using a finite-dimensional vector $\mathbf{P}_{im} = \{\mathrm{P}_{im}(\mathbf{a}_{im}|\mathbf{x}_m) : a_{im} \in \mathcal{A}_{im}, x_m \in \mathcal{X}_m\}$.

The parameters of interest in the dynamic game are the paramters for menu cost, leadership costs, and fixed costs: $\boldsymbol{\theta}_i = \{\gamma_i^{MC,0}, \gamma_i^{FC,0}, \gamma_i^{FC,Size}, \gamma_i^{FC,Profit}, \gamma_i^{LC,Size}, \gamma_i^{LC,Profit}\}$ for $i = CV, FA, SB$. Let $\mathbf{P}^*(h)$ be the equilibrium probability at time h and let $\mathbf{V}^{\mathbf{P}^*(h)}$ be the firms' value function associated with $\mathbf{P}^*(h)$.

As a result of firms' making optimal decision given their beliefs and the payoff parameter, the outcome of the dynamic game can be described as a vector $\mathbf{P}$ of *condition choice probabilities*(CCPs) that solves the equilibrium fixed point problem $\mathbf{P}^*(h) = \boldsymbol{\Psi}(\mathbf{P}^*(h), \mathbf{B}(h))$. Following the *Representation Lemma* in Aguirregabiria and Mira (2007), we can represent a MPE of the dynamic game as a fixed point of the alternative mapping, which is convenient for estimation. Write the profit function as a linear function of parameters $\boldsymbol{\theta}$:

$$\begin{aligned}\pi_{im}(\mathbf{x}_{mt}, \mathbf{a}_{mt}) &= (1 - a_{imt})\mathbf{w}_{imt}^{\top}(0, \mathbf{a}_{-imt}, \mathbf{x}_{mt})\boldsymbol{\theta}_i + a_{imt}\mathbf{w}_{imt}^{\top}(1, \mathbf{a}_{-imt}, \mathbf{x}_{mt})\boldsymbol{\theta}_i, \\ \Pi_{im}(\mathbf{x}_{mt}, \mathbf{a}_{mt}) &= \pi_{im}(\mathbf{x}_{mt}, \mathbf{a}_{mt}) + \epsilon_{imt}(a_{imt}),\end{aligned} \tag{5}$$

where $\boldsymbol{\theta}_i$ is a column vector with the dimension 7×1 that contains the structural parameters characterizing the fixed cost:

$$\boldsymbol{\theta}_i = (1, \{\gamma_i^{MC,0}\}, \{\gamma_i^{FC,0}\}, \{\gamma_i^{FC,Profit}\}, \{\gamma_i^{FC,Size}\}, \{\gamma_i^{LC,Profit}\}, \{\gamma_i^{LC,Size}\})^\top. \tag{6}$$

$\mathbf{w}_{imt}(0, \mathbf{a}_{-imt}, \mathbf{x}_{mt})$ and $\mathbf{w}_{imt}(1, \mathbf{a}_{-imt}, \mathbf{x}_{mt})$ are column vectors with the dimension 16×1 defined by:

$$\begin{aligned} &\mathbf{w}_{imt}(0, \mathbf{a}_{-imt}, \mathbf{x}_{mt}) \equiv \Big(R_{im}\big((0, \mathbf{a}_{-imt})\big), x_{imt}, \mathbf{0}\Big)^\top, \\ &\mathbf{w}_{imt}(1, \mathbf{a}_{-imt}, \mathbf{x}_{mt}) \equiv \Big(R_{im}\big((1, \mathbf{a}_{-imt})\big), 1 - x_{imt}, 1, \mathbb{1}(\mathbf{x}_{mt} = \mathbf{0}), \overline{Rev}_{im} \times [1, \mathbb{1}(\mathbf{x}_{mt} = \mathbf{0})]\Big)^\top, \end{aligned} \tag{7}$$

where $\overline{Rev}_{im}$ is the average daily revenue obtained from the market. Define

$$\mathrm{e}_{imt}^{\mathbf{P},\mathbf{B}}(a_{imt}, \mathbf{x}_{mt}) = \gamma - \log\Big(P_{im}\big(a_{imt}, \mathbf{x}_{mt}, \mathbf{B}_{im}(h)\big)\Big) \tag{8}$$

as the expected value of $\epsilon(a_{imt})$ conditional on that the market manager (i, j) chooses action a_{imt}, where γ is the Euler constant. We can represent a best response as a vector of *CCPs* $\mathbf{P} = \{\mathrm{P}_{im}(\mathbf{a}_{im}, \mathbf{x}_t, \mathbf{B}_{im}) : \mathbf{a}_{im} \in \mathcal{A}_{im}, \mathbf{x}_{mt} \in \mathcal{X}_m, i \in \mathcal{I}\}$. Let the vector

$$\begin{aligned} \tilde{\mathbf{w}}_{imt}^{\mathbf{B}_{im},\mathbf{P}_{im}}(a_{imt}, \mathbf{x}_{mt}) = \sum_{\mathbf{a}_{-im}} \mathrm{B}_{im}(h)(\mathbf{a}_{-im}, x_{mt})\Big(&\mathbf{w}_{imt}(1, \mathbf{a}_{-im}, \mathbf{x}_{mt}) - \mathbf{w}_{imt}(0, \mathbf{a}_{-im}, \mathbf{x}_{mt}) \\ &+\beta \sum_{\mathbf{x}_{m,t+1}} \tilde{f}^{\mathbf{B}_{im},\mathbf{P}_{im}}(\mathbf{w}_{m,t+1} | \mathbf{a}_{imt}, \mathbf{a}_{-im}, \mathbf{w}_{m,t+1}) \mathbf{V}_{\mathrm{w},im}^{\mathbf{B}_{im},\mathbf{P}_{im}}(\mathbf{w}_{m,t+1})\Big) \end{aligned} \tag{9}$$

be the difference in discounted flow payoff weighed by the belief $\mathbf{B}_{im}(h)$ and

$$\begin{aligned} &\tilde{\mathrm{e}}_{imt}^{\mathbf{B}_{im},\mathbf{P}_{im}}(a_{imt}, \mathbf{x}_{mt}) \\ &= \sum_{\mathbf{a}_{-im}} \mathrm{B}_{im}(h)(\mathbf{a}_{-im}, x_{mt})\Big(\beta \sum_{\mathbf{x}_{m,t+1}} \tilde{f}^{\mathbf{B}_{im},\mathbf{P}_{im}}(\mathbf{w}_{m,t+1} | \mathbf{a}_{imt}, \mathbf{a}_{-im}, \mathbf{w}_{m,t+1}) \mathbf{V}_{\mathrm{e},im}^{\mathbf{B}_{im},\mathbf{P}_{im}}(\mathbf{w}_{m,t+1})\Big) \end{aligned} \tag{10}$$

be the difference in the expected idiosyncratic shocks. The matrix valuation $\mathbf{V}_{\mathrm{w},im}^{\mathbf{B}_{im},\mathbf{P}_{im}}(\mathbf{w}_{m,t+1})$

and $\mathbf{V}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{\mathrm{e},im}(\mathbf{w}_{m,t+1})$ are written as

$$\begin{aligned}
\mathbf{V}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{\mathrm{w},im}(\mathbf{w}_{m,t+1}) &= (\mathbf{I}-\beta\mathbf{F}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im})^{-1}\mathbf{w}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im}, \mathbf{V}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{\mathrm{e},im}(\mathbf{w}_{m,t+1}) = (\mathbf{I}-\beta\mathbf{F}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im})^{-1}\mathbf{e}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im},\\
\mathbf{w}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im} &= \Big[\sum_{a_{im}}\sum_{\mathbf{a}_{-im}} P_{im}(a_{im},\mathbf{x}_m,\mathbf{B}_{im})B_{im}(\mathbf{a}_{-im},\mathbf{x}_m)\mathrm{w}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im}(a_{im},\mathbf{a}_{-im},\mathbf{x}_m,):\mathbf{x}_m\in\mathcal{X}_m\Big]^\top,\\
\mathbf{e}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im} &= \Big[\sum_{a_{im}}\sum_{\mathbf{a}_{-im}} P_{im}(a_{im},\mathbf{x}_m,\mathbf{B}_{im})B_{im}(\mathbf{a}_{-im},\mathbf{x}_m)\mathrm{e}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im}(a_{im},\mathbf{a}_{-im},\mathbf{x}_m):\mathbf{x}_m\in\mathcal{X}_m\Big]^\top.
\end{aligned} \tag{11}$$

Following the *Representation Lemma* in Aguirregabiria and Mira (2007), we can write the fixed point as a solution to the the *policy iteration mapping*:

$$\begin{aligned}
\mathbf{P}_{im}(a_{im},\mathbf{x}_m,\mathbf{B}_{im}) &= \mathbf{\Psi}(\mathbf{P}_{im},\boldsymbol{\theta}_i)(a_{im},\mathbf{x}_m,\mathbf{B}_{im})\\
&= \frac{\exp\left\{\left(\tilde{\mathbf{w}}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{imt}(a_{imt},\mathbf{x}_{mt})\right)^\top\boldsymbol{\theta}_i+\tilde{\mathrm{e}}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{imt}(a_{imt},\mathbf{x}_{mt})/\sigma_\epsilon\right\}}{\sum_{\tilde{a}_{im}}\exp\left\{\left(\tilde{\mathbf{w}}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{imt}(\tilde{a}_{im},\mathbf{x}_{mt})\right)^\top+\tilde{\mathrm{e}}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{imt}(\tilde{a}_{im},\mathbf{x}_{mt})/\sigma_\epsilon\right\}}.
\end{aligned} \tag{12}$$

For a fixed value of $\mathbf{P}$, the evaluation of $\Psi(\mathbf{P}_{im},\boldsymbol{\theta}_i)$ can be written as a function that is linear to $\boldsymbol{\theta}_i$ because $\mathbf{w}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im}$ and $\mathrm{e}^{\mathbf{B}_{im},\mathbf{P}_{im}}_{im}$ are fixed.

Write $\mathbf{P}_{im}(h)$ as an abbreviation for $\mathbf{P}_{im}(h)(a_{im},\mathbf{x}_m,\mathbf{B}_{im}(h))$. With estimates for $\mathbf{P}_{im}(h)$ and $\mathbf{B}_{im}(h)$ for all the markets and histories $m\in\mathcal{M},h\in\mathcal{H}$, write a pseudo maximum likelihood method following that of Aguirregabiria and Mira (2002); Aguirregabiria and Magesan (2019) to estimate the structural parameter of $\boldsymbol{\theta}_i$. The pseudo likelihood function is derived from the i.i.d. extreme value ϵ's and can be written as:

$$Q_i(\boldsymbol{\theta}_i,\mathbf{B}_i,\mathbf{P}_i)=\sum_{m,t}\log\left(\frac{\exp\left\{\left(\tilde{\mathbf{w}}^{\mathbf{B}_{im}(h_t),\mathbf{P}_{im}(h_t)}_{imt}(a_{imt},\mathbf{x}_{mt})\right)^\top\boldsymbol{\theta}_i+\tilde{\mathrm{e}}^{\mathbf{B}_{im}(h_t),\mathbf{P}_{im}(h_t)}_{imt}(a_{imt},\mathbf{x}_{mt})/\sigma_\epsilon\right\}}{\sum_{\tilde{a}_{im}}\exp\left\{\left(\tilde{\mathbf{w}}^{\mathbf{B}_{im}(h_t),\mathbf{P}_{im}(h_t)}_{imt}(\tilde{a}_{im},\mathbf{x}_{mt})\right)^\top+\tilde{\mathrm{e}}^{\mathbf{B}_{im}(h_t),\mathbf{P}_{im}(h_t)}_{imt}(\tilde{a}_{im},\mathbf{x}_{mt})/\sigma_\epsilon\right\}}\right), \tag{13}$$

where h_t is the value of h at time t. Therefore the estimates for $\hat{\boldsymbol{\theta}}_i=\arg\max Q_i(\boldsymbol{\theta}_i,\mathbf{B}_i,\mathbf{P}_i)$.

4.2 Identification of Beliefs

The dynamic game with unconstrained beliefs is under-identified, as discussed in Aguirregabiria and Magesan (2019). Assume a firm's belief is affected by two components, a firm-specific "belief parameter" $\lambda_i(h)$ and the other firms' choice probabilities. The above specification disentangles firms' processes in information acquisition and the equilibrium effect of other firms' actions through strategic interactions. The beliefs are updated with the number of successful price increases on other markets such that $\lambda_i(h)$ is a function of history.

Assumption 3 (Belief formation)**.** *Assume that at the meta-game history $h_t = h$, we have the belief of firm i satisfying the following form:*

$$\begin{aligned}\mathbf{B}_{im}(h)(a_{-im}, \mathbf{x}_{mt}) = \Pi_{i' \neq i}\Bigg(& a_{i'm}\lambda_i(h)\mathbf{P}_{i'm}(a_{i'm}, \mathbf{x}_{mt}, \mathbf{B}_{i'm}(h)) \\ & +(1 - a_{i'm})(1 - \lambda_i(h)\mathbf{P}_{i'm}(a_{i'm}, \mathbf{x}_{mt}, \mathbf{B}_{i'm}(h)))\Bigg),\end{aligned} \tag{14}$$

where $\boldsymbol{a}_{-im} = \{a_{i'm} : i' \in \mathcal{I}, i' \neq i\}$ and $\lambda_i(h) > 0$.

Assume that the firms will form their belief of other players based on a parameter $\lambda_i(h) \in (0, 1)$. The parameter measures the process of firms' belief before eventually reaching a subgame perfect equilibrium under the price leadership. If $\lambda_i(h)$ is close to zero, the firm i thinks firms are competing under a static Nash equilibrium. For a given h, the conditional choice probability is the best response to the beliefs as specified in equation (14).

Note that a player has the same beliefs in two markets with the same observable characteristics, that is, for every two markets m and m' with $\mathbf{x}_{mt} = \mathbf{x}_{m't}$, we have that $\mathbf{B}_{im} = \mathbf{B}_{im'}$. The beliefs at a given history h_t are determined through the biased belief equilibrium, and firms' biases in beliefs are only captured by the parameters $\{\lambda_i(h_t)\}_{i\in\mathcal{I}}$. Beliefs are updated once the firms achieve a certain number of price increases. For primitives other than players' beliefs, we make some assumptions that are standard in previous research on the identification of static games and of dynamic structural models with rational or equilibrium beliefs. We assume that the distribution of the unobservables, ϵ_{imt}, is known to the researcher up to a scale parameter. Let $\mathbf{q}_{im}(\mathbf{P}_{im}) = \{\mathrm{q}(a_{imt}, \mathbf{P}_{im}) : a \in \mathcal{A}_{im}\}$ be the inverse mapping of $\boldsymbol{\Lambda}$ such that if $\mathbf{P}_{im} = \boldsymbol{\Lambda}(\tilde{\mathbf{v}}_{im})$ then $\tilde{\mathbf{v}}_{im} = \boldsymbol{\Lambda}^{-1}(\mathbf{P}_{im})$. Therefore $\tilde{\mathbf{v}}_{im}^{\mathbf{B}_{im}(h)}(\mathbf{x}_{mt}) = \mathbf{q}_{im}(\mathbf{P}_{im}(\mathbf{x}_{mt}, \mathbf{B}_{im}(h)))$. By the definition of $\tilde{\mathbf{v}}_{im}^{\mathbf{B}_{im}(h)}(\mathbf{x}_{mt})$, each element of $\mathbf{q}_{im}(\mathbf{P}_{im}(\mathbf{x}_{mt}, \mathbf{B}_{im}(h)))$ can be written as the vector of

$$\mathrm{q}(a_{imt}, \mathrm{P}_{im}(\boldsymbol{a}_{im}, \mathbf{x}_t, \mathbf{B}_{im}(h))) = \Big(\mathbf{B}_{im}(h)(\mathbf{x}_{mt})\Big)^\top \Big[\tilde{\boldsymbol{\pi}}_{im}(a_{imt}, \mathbf{x}_{mt}) + \tilde{\boldsymbol{c}}_i^{\mathbf{B}_{im}(h)}(a_{imt}, \mathbf{x}_{mt})\Big]. \tag{15}$$

$\mathbf{B}_{im}(h)(\mathbf{x}_{mt}), \tilde{\boldsymbol{\pi}}_{im}(a_{imt}, \mathbf{x}_{mt}), \tilde{\mathbf{c}}_i(a_{imt}, \mathbf{x}_{mt})$ are $A_{-im} \times 1$ vector. $\{\mathbf{B}_{im}(h)(\mathbf{x}_{mt}) = \{b_{im}(a_{imt}, \mathbf{x}_{mt}) : \boldsymbol{a}_{-im} \in \mathcal{A}_{-im}\}$ is the belief vector of firm i on market j given state $\mathbf{x}_{mt}$, $\tilde{\boldsymbol{\pi}}_{im}(a_{imt}, \mathbf{x}_{mt}) = \{\tilde{\pi}_{im}(a_{imt}, \mathbf{x}_{mt}) : \boldsymbol{a}_{-im} \in \mathcal{A}_{-im}\}$ is the *differences in flow payoffs* and $\tilde{\boldsymbol{c}}_i^{\mathbf{B}_{im}(h)}(a_{imt}, \mathbf{x}_{mt})$ is the *continuation value function* that provides the expectation of discounted future payoffs given future beliefs, current state, and current choices of all players:

$$\tilde{\boldsymbol{c}}_i^{\mathbf{B}_{im}(h)}(a_{imt}, \mathbf{x}_{mt}) = \beta \sum_{a_{-im}} f_m(\mathbf{x}_{m,t+1}|(a_{imt}, a_{-im}), \mathbf{x}_{mt})\bar{V}_{im}^{\mathbf{B}_{im}(h)}(\mathbf{x}_{mt+1}).$$

With slight abuse of notations, write $\mathrm{q}(a_{imt}, \mathrm{P}_{im}(\boldsymbol{a}_{im}, \mathbf{x}_t, \mathbf{B}_{im}(h)))$ as $\mathrm{q}(a_{imt}, \mathbf{x}_t, \mathbf{B}_{im}(h))$. Note that the players ignore the marginal effect market-level decision of the change of history.

To identify the biased belief, follow the exclusion restrictions(*Section 3.2.3, Assumption ID-3*) in Aguirregabiria (2019) and assume the following Assumption 4.

Assumption 4 (Exclusion restriction). *The vector of state variables $\mathbf{x}_{mt}$ can be partitioned into two subvectors, $\mathbf{x}_t = (\boldsymbol{y}_{mt}, \boldsymbol{z}_{mt})$. The vectors $\boldsymbol{y}_{mt}$ and $\boldsymbol{z}_{mt}$ satisfy the following conditions:*

(A) $\boldsymbol{y}_{mt} = \{y_{imt} : i \in \mathcal{I}\}$ where $\boldsymbol{y}_{imt}$ represents past pricing decision of firm i on market j at time t that enter into the payoff function of player i on market j but not the payoff function of any of the other players or any other markets, $\pi_{im}(a_{imt}, a_{-imt}, y_{imt}, y_{-imt}, \boldsymbol{z}_{mt}) = \pi_{im}(a_{imt}, a_{-imt}, y_{imt}, y'_{-imt}, \boldsymbol{z}_{mt})$.

(B) The transition probability of the state variable y_{imt} is such that the value of $y_{i,t+1}$ does not depend on (y_{imt}, y_{-imt}) once we condition on a_{it} and z_t, i.e.,
$f_m(\mathbf{x}_{m,t+1}|(a_{imt}, a_{-im}), \mathbf{x}_{mt}) = f_{z,m}(\boldsymbol{z}_{m,t+1}|\boldsymbol{z}_{mt})\Pi_{i\in\mathcal{I}} f_{y,m}(\boldsymbol{y}_{im,t+1}|a_{imt})$.

(C) The flow payoff functions $\boldsymbol{\pi}_{im}(a_{imt}, a_{-im}, y_{im}, \boldsymbol{z}_m)$ is invariant across history $h \in \mathcal{H}$. ∎

Assume that the joint distribution of $y_{imt}, y_{-imt}, \boldsymbol{z}_{mt}$, over the population of M markets where we observe these variables, has a strictly positive probability at every point in the joint support set $\mathcal{X}_m$. We can write the quantile function as:

$$\mathrm{q}(a_{im}, y_{im}, y_{-im}, \boldsymbol{z}_m, \mathbf{B}_{im}(h)) = \Big(\mathbf{B}_{im}(h)(y_{im}, y_{-im}, \boldsymbol{z}_m)\Big)^\top \tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, y_{im}, \boldsymbol{z}_m) \tag{16}$$

where $\tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, y_{im}, \boldsymbol{z}_m) = \{\tilde{\mathrm{g}}_{im}(a_{im}, a_{-im}, y_{im}, \boldsymbol{z}_m) : a_{-im} \in \mathcal{A}_{-im}\}$, and

$$\begin{aligned}\tilde{\mathrm{g}}_{im}(a_{im}, a_{-im}, y_{im}, \boldsymbol{z}_m) = {} & \tilde{\boldsymbol{\pi}}_{im}(a_{im}, y_{im}, \boldsymbol{z}_m) \\ & + \beta \sum_{a_{-im}} \tilde{f}_m\big(\boldsymbol{y}_{im,t+1}|(a_{im}, a_{-im})\big) f_{z,m}(\boldsymbol{z}_{m,t+1}|\boldsymbol{z}_m)\bar{V}_{im}^{\mathbf{B}_{im}(h)}(\mathbf{x}_{j+1}),\end{aligned} \tag{17}$$

where $\tilde{f}_m\big(\boldsymbol{y}_{im,t+1}|(a_{im}, a_{-im})\big) = f_m\big(\boldsymbol{y}_{im,t+1}|(a_{im}, a_{-im})\big) - f_m\big(\boldsymbol{y}_{im,t+1}|(0, a_{-im})\big)$ is the difference in the transition density. For any $(a_{im}, y_{im}, \boldsymbol{z}_m)$, the equation (16) holds. In addition, the continuation value of player i given the collection of all players, $\tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, y_{im}, \boldsymbol{z}_m)$, does not depends on other players' lagged pricing decisions y_{-im}. Therefore, following the Aguirregabiria and Magesan (2019) Proposition 2, with $|\mathcal{Y}_{-im}| \geq |\mathcal{A}_{-im}|$, we can identify the continuation value function given the collection of all players $\tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, y_{im}, \boldsymbol{z}_m)$ if we know the belief function $\mathbf{B}_{im}(h)$. The belief assumed to be non-equilibrium in this model and therefore unknown. Therefore we cannot identify $\tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, y_{im}, \boldsymbol{z}_m)$.

Lemma 2 (Partial identification of continuation value). *For each $h \in \mathcal{H}$, we can identify the belief-weighted continuation value associated a given action $\mathrm{M}_{\lambda_i(h)}\tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{imt}, y_{imt}, \boldsymbol{z}_{mt})$ for each history h and each action-state combination $(a_{imt}, y_{imt}, \boldsymbol{z}_{mt})$, where $\mathrm{M}_{\lambda_i(h)} = \Big(\otimes_{i'\neq i} \begin{bmatrix} 1 & -\lambda_i(h) \\ 0 & \lambda_i(h) \end{bmatrix} \Big)$.*

With the assumption 4, the continuation value given the actions by the players can be written as:

$$\tilde{\mathrm{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, a_{-im}, y_{im}, \boldsymbol{z}_m) = \tilde{\boldsymbol{\pi}}_{im}(a_{imt}, a_{-im}, y_{im}, \boldsymbol{z}_m) + \tilde{\mathbf{c}}_i^{\mathbf{B}_{im}(h)}(a_{im}, a_{-im}, \boldsymbol{z}_m). \tag{18}$$

Therefore, for state y_{im} and y'_{im} where $y_{im} \neq y'_{im}$, we have the following equation holds:

$$\begin{aligned}
&\tilde{\mathrm{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, a_{-im}, y_{im}, \boldsymbol{z}_m) - \tilde{\mathrm{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, a_{-im}, y'_{im}, \boldsymbol{z}_m) \\
&\quad = \tilde{\boldsymbol{\pi}}_{im}(a_{imt}, a_{-im}, y_{im}, \boldsymbol{z}_m) - \tilde{\boldsymbol{\pi}}_{im}(a_{imt}, a_{-im}, y'_{im}, \boldsymbol{z}_m).
\end{aligned} \tag{19}$$

Assumption 5 (Unbiased belief in the last episode)**.** *For the last episode in the observed periods $\bar{h} = \max(\mathcal{H})$, the players beliefs are unbiased everywhere: $\mathbf{B}_{im}(\boldsymbol{a}_{-i,m}, \mathbf{x}_m, \bar{h}) = \Pi_{i' \neq i}\mathbf{P}_{i'm}(a_{i'm}, \mathbf{x}_m, \mathbf{B}_{im}(\bar{h}))$, where $\boldsymbol{a}_{-im} = \{a_{i'm} : i' \neq i\}$ for every player $i \in \mathcal{I}$, every markets $m \in \mathcal{M}$ and every possible states $\mathbf{x}_m \in \mathcal{X}_m$.* ■

Because $\mathrm{M}_{\lambda_i(h)}\tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{imt}, y_{imt}, \boldsymbol{z}_{mt})$ is identified for every $i \in \mathcal{I}, m \in \mathcal{M}, y_{imt} \in \mathcal{Y}_{im}, \boldsymbol{z}_{mt} \in \mathcal{Z}_m$, the following equation holds.

$$\begin{aligned}
&\mathrm{M}_{\lambda_i(h)}\Big(\tilde{\mathrm{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, a_{-im}, y_{im}, \boldsymbol{z}_m) - \tilde{\mathrm{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{im}, a_{-im}, y'_{im}, \boldsymbol{z}_m)\Big) \\
&\quad = \mathrm{M}_{\lambda_i(h)}\Big(\tilde{\boldsymbol{\pi}}_{im}(a_{imt}, a_{-im}, y_{im}, \boldsymbol{z}_m) - \tilde{\boldsymbol{\pi}}_{im}(a_{imt}, a_{-im}, y'_{im}, \boldsymbol{z}_m)\Big).
\end{aligned} \tag{20}$$

Therefore, $\mathrm{M}_{\lambda_i(h)}$ is identified for all $h \in \mathcal{H}$. Assume that firms' beliefs are in equilibrium after successfully colluding on more than two hundred drugs, and $\lambda(\bar{h}) = 1$. Since the payoff function is invariant, we can identify $\mathrm{M}_{\lambda_i(h)}$ for all histories $h \in \mathcal{H}$.

4.3 Estimation

The two-step method has two critical limitations that are relevant to this application. First, to achieve the consistency of $\hat{\boldsymbol{\theta}}$, the initial nonparametric estimator of $\mathbf{P}_0$ should be consistent. The first stage, nonparametric estimation of $\mathbf{P}_0$, is not plausible in dynamic models with serially correlated or time-invariant unobserved heterogeneity. Our model allows a time-invariant store and brand fixed effect. Secondly, the nonparametric specification of $\mathbf{P}_0$ does not consider this structure of the profit function. In the nonparametric estimation, every local manager (i, m) has its own unrestricted CCP function $\mathbf{P}_{im}(\cdot)$. The CCPs of interest is the probability of each firm leading a price increase, stopping being the price leader, following and not following a price increase. We only observe the price increase once per product; therefore, we cannot claim to have consistent nonparametric estimates of the CCP functions $\mathbf{P}_{im}$. In the first stage, the nonparametric estimation of CCPs can be very noisy even without unobserved heterogeneity. The two-step estimator

θ_0 is inconsistent. This noisy estimation of CCPs implies large biases in the two-step estimator of the structural parameters. We propose to use a recursive estimation method to overcome the finite sample bias. In order to account for the bias in the first stage nonparametric estimation, we modify the Nested Pseudo Likelihood (NPL) estimator Aguirregabiria and Mira (2007) to deal with these limitations of the two-step method. The NPL mapping $\mathbf{\Phi}(\cdot)$ is the composition of the equilibrium or best response mapping $\mathbf{\Psi}$ and the mapping that provides the pseudo maximum likelihood estimator of θ for a given arbitrary vector of CCPs $\mathbf{P}$. That is, the NPL mapping is defined as $\Phi(\mathbf{P})$.

In the estimation, we consider a recursive estimator with an updated probability of leading the price increase. We followed the following steps in order to obtain the structural parameters $\{\lambda_i, \boldsymbol{\theta}_i\}_{i=CV,FA,SB}$.

Step 1: Obtain the nonparametric CCP estimations $\mathbf{P}_{im}^0$ for each player on each market using the logit estimation. The explanatory variables include dummies for the intervals of history, dummies of the lagged pricing decision, market size and estimated elasticities.

Step 2: Estimate λ_i using sample analogue estimator and compute the belief $\mathbf{B}_i^0$ estimation using the estimation strategy specified by (20).

Step 3: Given the estimator $\mathbf{P}_i^k$ and $\mathbf{B}_i^0$, estimate $\hat{\boldsymbol{\theta}}_i$ with the estimator specified in equation (13).

Step 4: Update the probability of initializing a price increase for all the players $\mathbf{P}_i^{(k+1)}(1,\mathbf{0}) = \mathbf{\Psi}(\mathbf{P}_i^{(k)}, \hat{\boldsymbol{\theta}}_i, \mathbf{B}_i)(1,\mathbf{0})$ with the fixed point mapping as (12).

For the estimation, repeat *Step 3* and *Step 4* recursively.

4.4 Monte Carlo Experiment

In this experiment, we consider a two-player pricing game. In the first phase, the players' beliefs are not rational. The players play their best response, given their beliefs about rivals' strategy. The experiment contains two stages. In the first stage, firms hold the biased belief and in the second stage firm's beliefs converge to rational beliefs. The specifications of the data generating process are as shown in the table 4. For each simulation, we estimate the parameters use the method discussed in section 4.3.

In the table 5, we present the monte carlo experiment under the third specification with a maximum iteration of 10. Panel A presents the bootstrapped error and 95% CI coverage for the parameters. The coverage is close to 95 %, which indicates the estimators are valid, given the correct specification. Panel B presents the prediction of CCPs of the model that impose the equilibrium belief constraint(the constrained model) and the unconstrained model. The unconstrained model generates CCP predictions that have lower Mean Absolute Bias(MAB) but higher variance. This indicates that the proposed estimator reduces the bias in CCP prediction at the cost of efficiency.

For comparison, I consider the specification to let the maximum iteration be 1, 5 and 10. In the Monte Carlo experiment, we show the estimation using three specifications: the first specification assuming the players to be rational at both period; the second specification uses only the second phase of the data to estimate; the third specification using both episodes of the data while assuming the firms are only rational in the second phase. The constrained model generates biased CCP predictions. The results are presented in the Appendix C.1.

Table 4: Dynamic Game Structural Parameters

Number of markets	200	Number of Time Periods	50
Number of Player	2	Discount Factor	0.99
Market Size		15	
High Price	(1,1)	Lower Price	(0.8,0.8)
Menu Cost	(2,2)	Leading Cost	(-5,-5)
Fixed Cost		(0,0)	
Biased belief model	$\lambda_1(1) = \lambda_2(1) = 0.5$		$\lambda_1(2) = \lambda_2(2) = 1.0$
Unbiased belief model	$\lambda_1(1) = \lambda_2(1) = 1.0$		$\lambda_1(2) = \lambda_2(2) = 1.0$

Table 5: Monte Carlo Experiment: Biased Belief Model

Panel A: Coverage

Parameter	Bootstraped Std.Err	Estiamted Std.Err	95 % CI Coverage
θ_1^{MC}	0.0939	0.0881	0.9100
θ_1^{FC}	0.0939	0.0906	0.9700
θ_1^{LC}	0.9174	0.7927	0.8200
θ_2^{MC}	0.0934	0.0817	0.9400
θ_2^{FC}	0.0939	0.0900	0.9700
θ_2^{LC}	0.9304	0.6859	0.8600

Panel B: Predicted CCPs

	Update λ				No Update λ			
	MAB		Std		MAB		Std	
	Con	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*
P_1^{lead}	0.1483	0.0652	0.0414	0.0798	0.1480	0.0552	0.0401	0.0434
$\mathrm{P}_1^{collude}$	0.0035	0.0007	0.0004	0.0005	0.0035	0.0006	0.0004	0.0000
P_2^{lead}	0.1469	0.0681	0.0418	0.0852	0.1446	0.0540	0.0415	0.0464
$\mathrm{P}_2^{collude}$	0.0036	0.0007	0.0004	0.0006	0.0035	0.0006	0.0005	0.0000

5 Results and Counterfactual

5.1 Estimation Results

In the estimation, we split the metagame history into four distinctive grids to reduce the estimation's dimensionality. Let h be the number of markets that the firms have successfully cooperated to raise the collusion price level. We split the history into the following four grids: $\{[0, 30], [31, 90], [90, 150], [150, \infty)\}$. A firm i's probability of leading is determined by the firm's belief regarding the other firms' behaviour of following. If a firm believes the other two firms will follow closely with the price increase, then the price leader's incentive is higher. The data from transition is considered starting from October 31st, 2007 to June 19th, 2008, with 282 days.

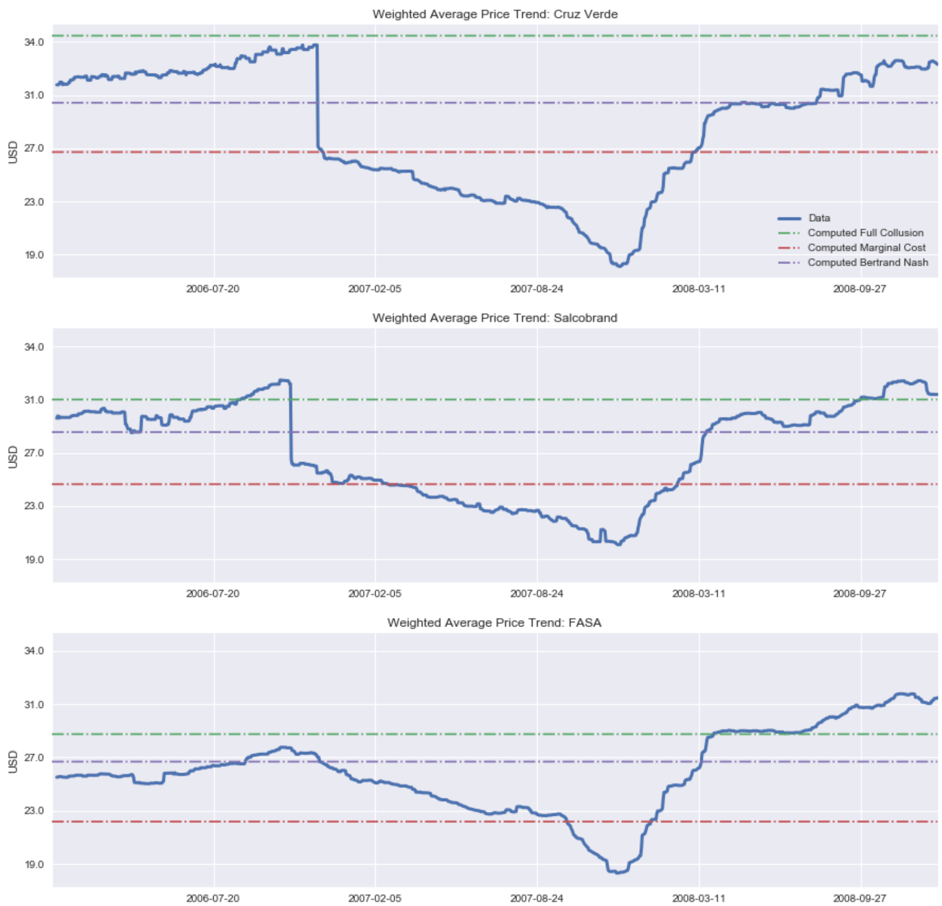
To estimate the demand model, we use the price and quantity data from January 1st, 2006, to November 1st, 2006. We consider an Arrelano-Bond type of instrument to account for the endogeneity of price and quantity. Table 6 reports the estimated price coefficients using the IV and OLS regression models. The OLS tends to over-estimate the price-coefficient, and therefore underestimate the market power. Figure 4 shows the solved equilibrium price level of IV demand estimation. The collusion price is computed by solving the optimal price level under full collusion, where firms maximize joint profit when setting prices. The Bertrand price level is computed, assuming firms are competing without taking other firms' profit in their objectives. The marginal cost reflects the estimated marginal cost.

To estimate the dynamic game, first, we start with a non-parametric conditional choice probability estimation. We estimate the conditional choice probability for each history and market $h \in \mathcal{H}, m \in \mathcal{M}$. The estimation of the CCP is based on the estimated price coefficient in the demand system $\hat{\alpha}_m$, the market size, which is defined as the median of one week's sale volume, and whether the drug is prescription required. After the first stage estimation of the profit, we use the median price before/ after the drug is deemed colluding as the action. We can back out the variable profits from the first stage estimation of demand. For the identification of belief, we make the assumption that firms' beliefs are unbiased during the last episode defined by history in the cooperation. The discount belief parameter $\lambda(h)$ is identified from equation (20).

We obtain the first stage non-parametric CCP estimator $\mathbf{P}_0(\boldsymbol{y}_{mt}, \boldsymbol{z}_m, h_t)$ by considering a logit regression with the explanatory variable of $\boldsymbol{y}_{mt}$, h_t, market size, estimates of demand price coefficients and, whether the drug is patented, whether the drug is a prescription and whether the drug is a treatment for chronic disease. In this representation, the the decision variable $\boldsymbol{a}_{imt}$ is the pricing decision for firm i on market m at time t. Let $\boldsymbol{a}_{imt} = 1$ be the decision of firm i charge the high price on market m at tie t . The endogenous state variable $\boldsymbol{y}_{mt} = \boldsymbol{a}_{m,t-1}$ represent the previous period $t-1$ pricing decision. The exogenous variable $\boldsymbol{z}_m$ denote the market characteristics of market m including market size, estimates of price coefficients in demand system, dummies for patent, prescription and chronic disease treatment. The model assumes the firm's beliefs are in equilibrium in the last episode of the coordination period. The belief is estimated using the exogenous restriction in equation (20). We follow the estimation technique in section 5.2 and estimate the model recursively. Table 7 describes the estimated menu cost, leadership cost and fixed cost using the non-equilibrium belief model and equilibrium belief model, respectively. The equilibrium belief model over-estimate Leadership costs and fixed costs. Table 8 the first column shows the estimation results of menu costs and fixed cost when not imposing the rational belief assumption, and the second column shows the estimation results when we assume the firm's belief is non-rational at the beginning. The estimation is done recursively by estimating the cost parameters for each round and then update the probability of leading a price increase using the OCP mapping. Figure 5 shows the model prediction for the equilibrium belief and non-equilibrium belief, respectively. If we impose an equilibrium belief assumption, the model predicts that firms will raise the price earlier than the actual data.

The model prediction suggests that without taking into accounts of the non-equilibrium beliefs, we may fail to explain why firms start the collusion gradually.

Figure 4: Price Level Predicted with Estimated Demand System



1 The predicted collusion price level is computed, assuming that firms' are colluding given the demand system. The predicted price war level is the estimated marginal cost.

Table 6: Estimated Demand Price Coefficients

$\hat{\alpha}_m$	IV	OLS
$\hat{\alpha}_m$	0.8236	1.1828
	$[0.2257, 1.6108]$	$[0.2508, 2.6102]$
$s.e.(\hat{\alpha}_m)$	0.1835	0.0630
	$[0.0385, 0.1134]$	$[0.0239, 0.1103]$
R-square	0.3271	0.4931
	$[0.0178, 0.7848]$	$[0.2608, 0.6614]$
Durbin Test Stats	54.8629	-
	$[7.6387, 109.1056]$	-
No. $\hat{\alpha}_m$ negative	4	6
No. of Markets	214	214

[1] The first row shows the mean of the statistics averaged across markets. [2] The second row shows the 10 %th and 90 %th quantiles of the statistics.

Figure 5: The Model Prediction

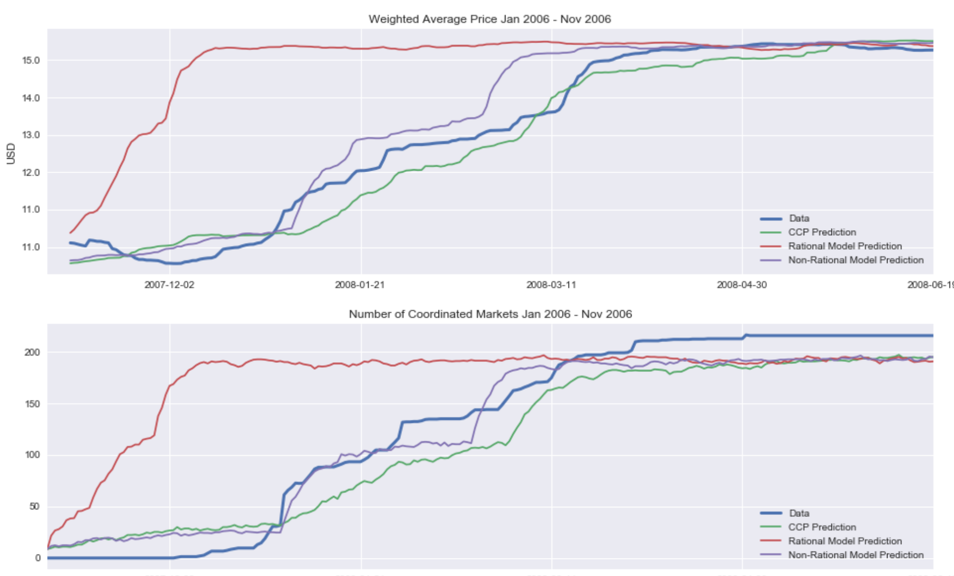


Table 7: Estimated Structural Parameters

Panel A: Estimation of Belief Parameters $\lambda(h)$			
h	*Cruz Verde*	*FASA*	*Salcobrand*
0 - 30	0.5187	0.3176	0.4699
	(0.0651)	(0.0468)	(0.0392)
0 - 90	0.6107	0.6291	0.4304
	(0.0646)	(0.0417)	(0.0396)
90 - 150	0.6183	0.6513	0.4791
	(0.0508)	(0.0491)	(0.0381)
150 +	1.	1.	1.

Panel B: Estimation of Strucatural Costs			
		Rational Model	Non-rational Model
Menu Cost	Cruz Verde	-232.4682	-7.6522
	FASA	-730.8975	-276.4451
	Salcobrand	-22.3094	-298.0671
Fixed Cost	Cruz Verde	-329.8713	-1.4162
		[-671.2018, 4.2168]	[-3.96 , 1.19]
	FASA	-645.5794	-114.1933
		[-1260.4551, -70.0513]	[-201.21, -32.75]
	Salcobrand	-74.6131	-31.8427
		[-135.4597, -0.0099]	[-56.29, -1.87]
Leader Cost	Cruz Verde	-9447.4493	-6884.5454
		[-16557.9705, 17.1637]	[-12219.71, -137.79]
	FASA	-12843.0407	-7683.2954
		[-25449.8779, 206.1243]	[-14242.44, -591.13]
	Salcobrand	-349.9771	-2667.0397
		[-834.9016, -10.2718]	[-4457.68, 40.50]

[1] In panel A, the estimation of $\hat{\lambda}_i(h)$ is based on the first stage non-parametric CCP estimations. [2] In panel A, we report the standard deviation with parametric Bootstrap of 99 in the bracket. [3] In panel B, we compute the forecasted fixed cost and leader cost for each drug. In the bracket, we show the 10th and 90th quantile of the computed costs.

Table 8: Estimation of entry cost and fixed cost for the dynamic pricing game

Parameters	Firm	Rational Belief	Biased Belief
Menu Cost*(thousands Pesos)*			
	Cruz Verde	-232.4269	-7.5694
		(132.1622)	(95.4579)
	FASA	-731.2133	-276.4821
		(277.5601)	(167.7036)
	Salcobrand	-22.3106	-297.6240
		(38.1729)	(119.5720)
Fixed Cost			
	Cruz Verde	-1.0375	0.0150
		(0.1449)	(0.3055)
	FASA	-0.9747	-0.1379
		(0.1192)	(0.0833)
	Salcobrand	-0.3387	-0.1364
		(0.1850)	(0.1070)
(thousands Pesos)			
	Cruz Verde	20.0057	-4.0629
		(16.8951)	(14.4915)
	FASA	20.6453	-19.8784
		(27.7783)	(15.3428)
	Salcobrand	1.0101	-1.4620
		(3.1708)	(3.1489)
Leadership Cost			
	Cruz Verde	44.1761	8.9671
		(55.3574)	(19.8923)
	FASA	33.5448	1.8640
		(13.4868)	(5.6158)
	Salcobrand	-7.3078	23.8538
		(4.7534)	(12.8438)
Market Size			
	Cruz Verde	-11.8094	-4.7683
		(17.3570)	(5.0383)
	FASA	-8.9274	-2.2334
		(3.6205)	(1.2510)
	Salcobrand	0.9324	-5.8287
		(0.9563)	(2.2761)

[1] Data: 202 markets x 232 days = 46864 observations.
[2] The specification: menu cost is in Pesos, while fixed cost is proportional to the daily average revenue.

5.2 Counterfactual

By having had a cartel, a market has revealed itself to be predisposed to this illegal activity. If the market structure is left unimpaired, collusion could reappear, either again as explicit collusion or as tacit collusion. Although fines and damages are a deterrent, neither modifies the market to make future collusion less likely and is a remedy for the market structure.

For the counterfactuals, Consider two types of policy intervention. Firstly, the Government regulates the medicine prices by imposing a price cap for the increase (such as 10%); Secondly, as proposed by Harrington (2018)(pp.234), divest the industry. The policy encourages each chain to divest 25% of its stores and create a new firm with the stripped assets. The price cap affects the incentive through two channels. 1. The profit from the collusion is lower compare to the competition. Therefore, the price leader has less incentive to lead the price; 2. A similar agreement affects followers. The additional profit from colluding is low, and therefore, the leader expects the followers to have little incentive to follow. The structural remedy of divesture lowers the likelihood of collusion and prevents the post-cartel tacit collusion. The remedy makes coordination harder because it is more difficult to achieve

The counterfactuals outcomes are simulated using an equilibrium outcome of the best response given the alternative payoffs. For the first experiment, the collusion price that the firms are allowed to charge is capped by ten percent. The probability of leading considers the probability that followers match the price. For the second experiment, Assume that each firm divests its asset and forms the fourth chain. The profit for each market is computed with the estimated price-coefficient and market size. Assume that the market size for each drug has not changed after the divesture. The simulated prediction for the model with a non-equilibrium belief is shown in figure 6. If we limit the price cap to 10 %, the firms will still achieve collusion, but the weighted average price will be lower. For the second counterfactual, the firms divest their assets and form the fourth chain. The coordinated price increase will still happen but will take longer and involve fewer markets.

6 Conclusion

This work is the first to study the initiation problem of collusion. This paper provides a structural model for firms' decisions when switching to a new and more profitable equilibrium using a gradual approach. The model captures firms' learning-to-coordinate behaviour without imposing structures on the learning process compared to the "traditional" learning models, such as fictitious play, Bayesian learning and adaptive learning. This paper estimates a "belief parameter" to account for the non-rational behaviour when firms start to switch pricing strategy to an anti-competitive level. The "belief parameter" eventually converge to a rational belief equilibrium. The non-rational model generates predictions that are compatible with the gradual transition of

Figure 6: The Model Counterfactual With Non-Equilibrium Belief

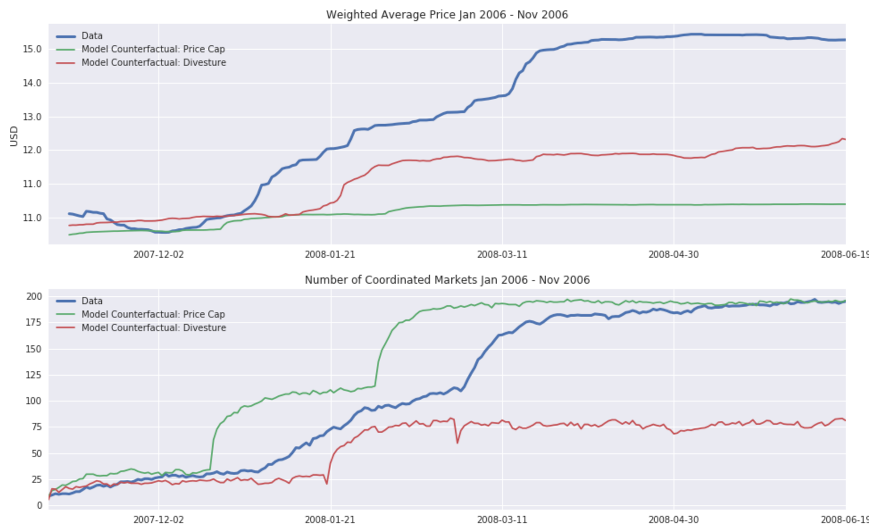
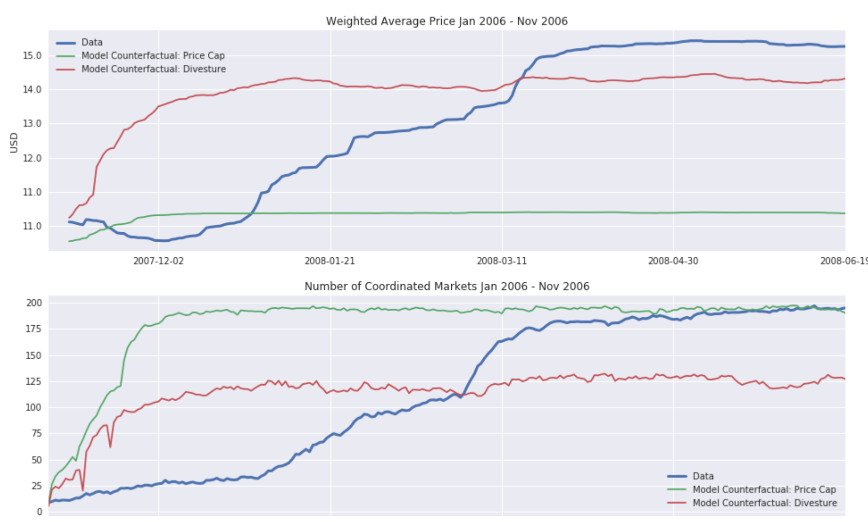


Figure 7: The Model Counterfactual With Equilibrium Belief



the market outcome compared to the Markov Perfect equilibrium model. The benefit of imposing such an assumption is that belief is determined endogenously but is not sensitive to the initial priors firms hold. The partially endogenized belief can be used to evaluate policies that change firms' payoffs. The policies change firms' beliefs through strategic interactions.

This paper discusses the decision-making process when firms have multi-market contact. Firms in the retail industries often face multi-market contacts. Similar examples can be found in car manufacturers and airline industries. Literature suggests that multi-market contacts facilitate collusions. Potential explanations include the mixed incentive to collude(harsher punishment if deviated), supply and demand linkage. The initiation of collusion with the presence of multi-market contact usually involves diffusion of collusive outcomes from markets to markets. This paper explains that the diffusion of collusion is because firms learn to coordinate. The design is similar to the experimentation in single market contact, where firms experiment with strategies to signal their incentive to collude (Wang (2009)).

In the Counterfactual experiments, we consider two potential policy interventions. One intuitive one is that the government imposes restrictions such that the price increase cannot exceed a certain percentage, for example, ten percents. The second Counterfactual experiment follows the structural remedy suggested by Harrington and Harker (2017): divest the firms and form the fourth chain as a competitor. The counterfactual experiment shows that the divesture policy can prevent firms from reaching the collusion's subgame perfect equilibrium. The price cap policy can curb the price increase but cannot stop firms from reaching the subgame perfect equilibrium.

The model has several limitations in predicting the market outcome. First, this model can explain firms' incentives to lead the price increase but cannot account for firms' incentives to stay in the collusion. The "belief parameters" is a function of the number of collisions that happened in the market. As in contrast to Fershtman and Pakes (2000), where the belief is a function of whether deviation happened, this model does not feature that firms' beliefs update with deviations. In this dataset, we do not observe firms' deviations after successful collusions, therefore it is not possible to evaluate firms' incentive given deviations. Second, the belief parameter is a parsimonious way of modelling firms' non-rational behaviour. The belief parameter is not fully endogenized in the model, as done by Bayesian learning or adaptive learning. Third, the model assumes that buyers' do not switch to other products in response to the price increase, which is a restrictive assumption. However, given that switching to other brands will require prescriptions, assume that buyers do not switch in a four-month time frame seem reasonable. Lastly, unlike that dynamic game model in an oligopoly market structure, this model does not account for a new entrant.

A Proofs

Proof. To show the results hold, first show that for each $h \in \mathcal{H}$, we can write the belief as $\mathbf{B}_{im}(h)(\boldsymbol{x}) = \mathrm{M}_{\lambda_i(h)}\Big(\otimes_{i'\neq i} \begin{bmatrix} 1 \\ \mathrm{P}_{i'j}(1,\boldsymbol{x}) \end{bmatrix} \Big)$, where $\mathrm{M}_{\lambda_i(h)} = \Big(\otimes_{i'\neq i} \begin{bmatrix} 1 & -\lambda_i(h) \\ 0 & \lambda_i(h) \end{bmatrix} \Big)$.

Since the firms only have two choices, $\mathbf{P}_{im}(\boldsymbol{x}_m) = \begin{bmatrix} \mathrm{P}_{im}(0,\boldsymbol{x}_m) \\ \mathrm{P}_{im}(1,\boldsymbol{x}_m) \end{bmatrix}$, where $\mathrm{P}_{im}(0,\boldsymbol{x}_m)+\mathrm{P}_{im}(1,\boldsymbol{x}_m) = 1$. Therefore $\mathbf{P}_{im}(\boldsymbol{x}_m)$ can be written as a linear function of $\mathrm{P}_{im}(1,\boldsymbol{x}_m)$: $\mathbf{P}_{im}(\boldsymbol{x}_m) = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \mathrm{P}_{im}(1,\boldsymbol{x}_m) \end{bmatrix}$. Need to show that we can write the belief of player i as a linear product of lambda and other players true beliefs: $\mathbf{B}_{im}(h) = \mathbf{M}_{\lambda_i(h)}\mathbf{P}_{im}(h)$.

For player i', at history h, player i's belief about his probability of follow can be written as: $\big(\mathbf{B}_{im}(h)(\boldsymbol{x}_m)\big)_{i'} = \begin{bmatrix} 1 & -\lambda_i(h) \\ 0 & \lambda_i(h) \end{bmatrix} \begin{bmatrix} 1 \\ \mathrm{P}_{im}(1,\boldsymbol{x}_m) \end{bmatrix}$.

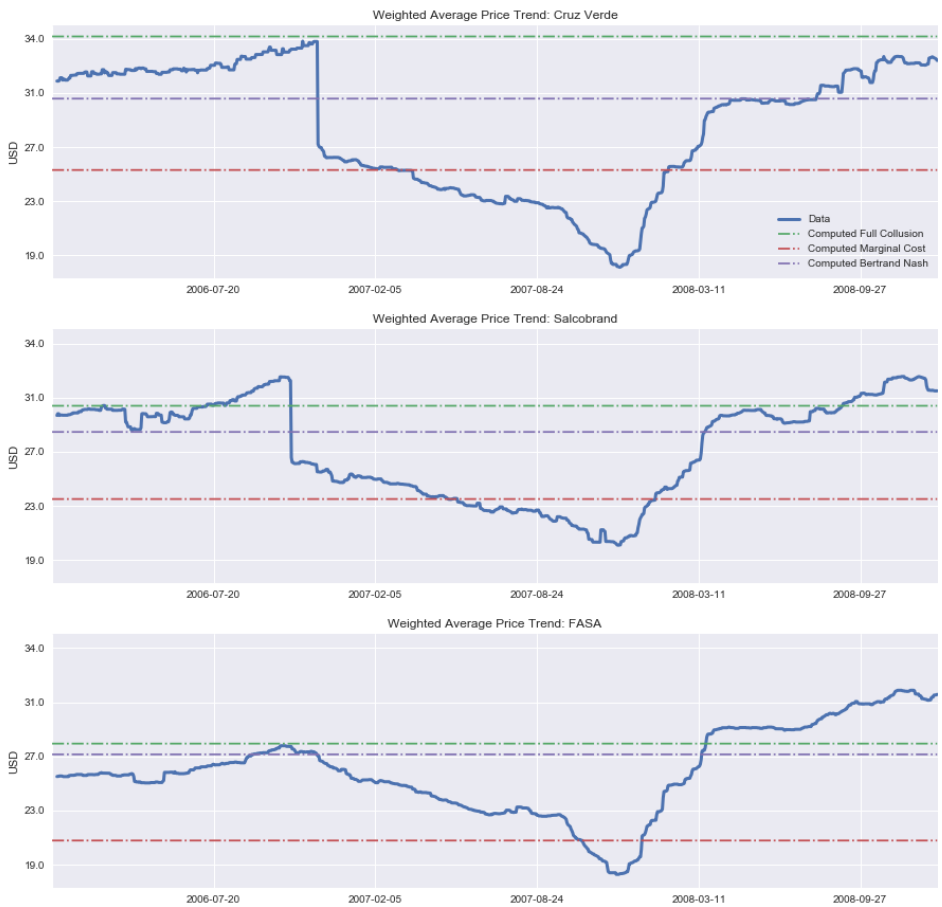
The belief of player i at history h is formed as a Kronecker product of other players due given assumption 3. Organize the equation and get $\mathbf{B}_{im}(h)(\boldsymbol{x}_m) = \otimes_{i'\neq i}\big(\mathbf{B}_{im}(h)(\boldsymbol{x}_m)\big)_{i'}$. By the mixed-product property of the Kronecker product, $\mathbf{B}_{im}(h)(\boldsymbol{x}_m) = \Big(\otimes_{i'\neq i} \begin{bmatrix} 1 & -\lambda_i(h) \\ 0 & \lambda_i(h) \end{bmatrix} \Big)\Big(\otimes_{i'\neq i} \begin{bmatrix} 1 \\ \mathrm{P}_{im}(1,\boldsymbol{x}_m) \end{bmatrix} \Big)$. $\mathrm{M}_{\lambda_i(h)}\tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{imt}, y_{imt}, \boldsymbol{z}_{mt})$ for each h and $(a_{imt}, y_{imt}, \boldsymbol{z}_{mt})$. Therefore, we can identify the combination of $\mathrm{M}_{\lambda_i(h)}\tilde{\mathbf{g}}_{im}^{\mathbf{B}_{im}(h)}(a_{imt}, y_{imt}, \boldsymbol{z}_{mt})$ for each h and $(a_{imt}, y_{imt}, \boldsymbol{z}_{mt})$.

□

B Alternative Model

B.1 OLS estimated demand

Figure 8: Price Level Predicted with Estimated Demand System



1 The predicted collusion price level is computed assuming that firms' are colluding given the demand system. The predicted price war level is the estimated marginal cost.

B.2 Alternative Dynamic Model

Table 9: Estimated Structural Parameters

Panel A: Estimation of Belief Parameters $\lambda(h)$			
h	*Cruz Verde*	*FASA*	*Salcobrand*
0 - 30	0.5781	0.3341	0.5273
	(0.1407)	(0.1527)	(0.1037)
0 - 90	0.6630	0.6506	0.4929
	(0.1858)	(0.1776)	(0.1049)
90 - 150	0.7033	0.6297	0.5369
	(0.1658)	(0.1727)	0.1029
150 +	1.	1.	1.

Panel B: Estimation of Strucatural Costs			
Menu Cost	Cruz Verde	-217.7112	-166.4192
	FASA	-703.5729	-321.0671
	Salcobrand	-40.0416	-43.4309
Fixed Cost	Cruz Verde	-356.9581	-61.0117
		[-186.76, -5.42]	[-688.58, -14.85]
	FASA	-624.1926	-64.6952
		[-253.58, -10.19]	[-1193.34, -69.4882]
	Salcobrand	-58.4083	-17.7217
		[-97.42, -1.03]	[-163.39, -1.83]
Leader Cost	Cruz Verde	-29132.9758	-5442.1026
		[-13157.71, -2.12]	[-93820.91, 19.41]
	FASA	-13065.5601	-7488.7094
		[-14651.65, -432.22]	[-28087.57, -58.16]
	Salcobrand	-492.3985	-3524.9935
		[-6520.78,-12.31]	[-1210.74, -4.30]

[1] In panel A, the estimation of $\hat{\lambda}_i(h)$ is based on the first stage non-parametric CCP estimations. [2] In panel A, we report the standard deviation with parametric Bootstrap of 99 in the bracket. [3] In panel B, we compute the forecasted fixed cost and leader cost for each drug. In the bracket, we show the 10th and 90th quantile of the computed costs.

Table 10: Estimation of entry cost and fixed cost for the dynamic pricing game

Parameters	Firm	Non-equilibrium Belief	Equilibrium Belief
Menu Cost*(thousands Pesos)*			
	Cruz Verde	-166.4192	-217.7112
	FASA	-321.0671	-703.5729
	Salcobrand	-43.4309	-40.0416
Fixed Cost			
	Cruz Verde	-2.1194	-0.8853
	FASA	-1.2078	-0.3344
	Salcobrand	-1.7647	-2.1351
Market Size			
	Cruz Verde	0.3241	-0.0384
	FASA	0.2071	-0.0888
	Salcobrand	0.2891	0.3195
Chronic			
	Cruz Verde	-0.0783	0.1122
	FASA	-0.1981	-0.1150
	Salcobrand	-0.1717	-0.1701
Leadership Cost			
	Cruz Verde	-0.6876	-125.9161
	FASA	2.4430	10.4390
	Salcobrand	7.4891	-3.3116
Market Size			
	Cruz Verde	-3.7784	-7.4356
	FASA	-2.4587	-5.9604
	Salcobrand	-4.1547	-0.0027
Chronic			
	Cruz Verde	13.9883	155.4510
	FASA	1.9981	10.6152
	Salcobrand	4.2040	2.0519

[1] Data: 202 markets x 232 days = 46864 observations. [2] The specification: menu cost is in Pesos while fixed cost is proportional to the daily average revenue.

C Bootstrap

C.1 Biased Belief Model

Table 12 shows the estimation results of different model specifications. The maximum iteration is the number of iteration allowed in the recursive estimation. With an increased number of iterations, the constrained model generates a biased estimation of the structural parameters. Although the estimation using the unbiased phase data only always generate an unbiased estimate, the variance is large.

Table 11 shows the prediction of CCPs of the model that impose the equilibrium belief constraint(the constrained model) and the unconstrained model. The unconstrained model generates CCP predictions that have lower Mean Absolute Bias(MAB) but higher variance. This indicates that the proposed estimator reduces the bias in CCP prediction at the cost of efficiency.

Table 11: Monte Carlo Experiment: Biased Belief Model 100 Monte Carlos

	Max Iteration = 1							
	Update λ				No Update λ			
	MAB		Std		MAB		Std	
	Con	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*
P_1^{lead}	0.1458	0.0566	0.0399	0.0418	0.1397	0.0615	0.0377	0.0383
$P_1^{collude}$	0.0035	0.0006	0.0004	0.0000	0.0035	0.0006	0.0004	0.0000
P_2^{lead}	0.1434	0.0588	0.0367	0.0403	0.1427	0.0593	0.0377	0.0405
$P_2^{collude}$	0.0036	0.0006	0.0004	0.0000	0.0035	0.0006	0.0005	0.0000
	Max Iteration = 5							
	Update λ				No Update λ			
	MAB		Std		MAB		Std	
	Con	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*
P_1^{lead}	0.1421	0.0694	0.0394	0.0780	0.1514	0.0505	0.0418	0.0324
$P_1^{collude}$	0.0035	0.0008	0.0004	0.0004	0.0035	0.0006	0.0004	0.0000
P_2^{lead}	0.1434	0.0681	0.0428	0.0766	0.1467	0.0551	0.0418	0.0290
$P_2^{collude}$	0.0035	0.0008	0.0005	0.0005	0.0035	0.0006	0.0004	0.0000
	Max Iteration = 10							
	Update λ				No Update λ			
	MAB		Std		MAB		Std	
	Con	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*
P_1^{lead}	0.1483	0.0652	0.0414	0.0798	0.1480	0.0552	0.0401	0.0434
$P_1^{collude}$	0.0035	0.0007	0.0004	0.0005	0.0035	0.0006	0.0004	0.0000
P_2^{lead}	0.1469	0.0681	0.0418	0.0852	0.1446	0.0540	0.0415	0.0464
$P_2^{collude}$	0.0036	0.0007	0.0004	0.0006	0.0035	0.0006	0.0005	0.0000

[1] The *Con* denotes the constrained estimation and *Uncon* denotes the model without the rational constraint. [2] Unstrained models are estimated with the restriction that $\lambda_1 = \lambda_1 = 1$. [3] P_i^{lead} is the probability of player i leading a price increase and $P_i^{collude}$ is the probability that player i chooses to remain in collusion.
[4] The probaiblity is forecasted with the models' best response function.

Table 12: Monte Carlo Experiment: The Comparison Between Models

	Max Iteration = 1			Max Iteration = 5			Max Iteration = 10		
DGP	**I**	**II**	**III**	**I**	**II**	**III**	**I**	**II**	**III**
-2.0000	-2.2646	-1.9959	-2.3239	-2.2709	-1.9973	-2.0075	-2.2580	-1.9937	**-1.9966**
-	(0.0889)	(0.0992)	(0.0897)	(0.0886)	(0.0989)	(0.0812)	(0.0884)	(0.0987)	**(0.0848)**
0.0000	0.1057	0.0189	0.0302	0.0915	0.0096	-0.0031	0.0992	0.0091	**0.1213**
-	(0.0919)	(0.1077)	(0.0939)	(0.0910)	(0.1067)	(0.0740)	(0.0910)	(0.1067)	**(0.0819)**
-5.0000	-4.4831	-5.1404	-4.2518	-4.3661	-5.0459	-4.6050	-4.5243	-5.1361	**-5.3256**
-	(0.6224)	(0.7812)	(0.6547)	(0.6243)	(0.7864)	(0.4855)	(0.6251)	(0.7848)	**(0.5642)**
-2.0000	-2.2531	-1.9808	-2.3101	-2.2595	-1.9862	-1.9920	-2.2555	-1.9841	**-2.0009**
-	(0.0883)	(0.0990)	(0.0891)	(0.0888)	(0.0993)	(0.0814)	(0.0881)	(0.0987)	**(0.0846)**
0.0000	0.1032	0.0211	0.0336	0.1069	0.0229	0.0146	0.1024	0.0212	**0.1079**
-	(0.0916)	(0.1080)	(0.0936)	(0.0921)	(0.1084)	(0.0753)	(0.0913)	(0.1074)	**(0.0812)**
-5.0000	-4.4885	-5.1497	-4.2652	-4.4454	-5.0687	-4.6692	-4.5126	-5.1640	**-5.3533**
-	(0.6264)	(0.7877)	(0.6587)	(0.6270)	(0.7900)	(0.4945)	(0.6263)	(0.7865)	**(0.5717)**

[1] Model **I** impose the assumption that the players are rational.

[2] Model **II** uses only the data from equilibrium belief time period.

[3] Model **III** is estimated with the proposed method.

Table 13: Monte Carlo Experiment: Biased Belief Model 100 Monte Carlos

	Max Iteration = 1					
	Update λ			No Update λ		
	Con	*Unbiased Only*	*Uncon*	*Con*	*Unbiased Only*	*Uncon*
	MSE	MSE	MSE	MSE	MSE	MSE
θ_1^{MC}	0.0768	0.0086	0.1126	0.0732	0.0084	0.1094
θ_1^{FC}	0.0228	0.0146	0.0150	0.0192	0.0099	0.0097
θ_1^{LC}	0.9350	0.7783	1.2705	0.7338	0.5838	1.0941
θ_2^{MC}	0.0697	0.0077	0.1021	0.0830	0.0116	0.1195
θ_2^{FC}	0.0193	0.0114	0.0114	0.0196	0.0116	0.0108
θ_2^{LC}	0.7827	0.6833	1.0853	0.9093	0.7421	1.1933
	Max Iteration = 5					
	Update λ			No Update λ		
	Con	*Unbiased Only*	*Uncon*	*Con*	*Unbiased Only*	*Uncon*
	MSE	MSE	MSE	MSE	MSE	MSE
θ_1^{MC}	0.0792	0.0087	0.0063	0.0861	0.0094	0.0056
θ_1^{FC}	0.0169	0.0116	0.0063	0.0163	0.0122	0.0052
θ_1^{LC}	1.0285	0.8902	0.6166	0.8036	0.6463	1.0716
θ_2^{MC}	0.0751	0.0108	0.0080	0.0727	0.0088	0.0064
θ_2^{FC}	0.0233	0.0154	0.0112	0.0201	0.0108	0.0059
θ_2^{LC}	1.0326	0.9711	0.7412	0.8617	0.8347	0.9816
	Max Iteration = 10					
	Update λ			No Update λ		
	Con	*Unbiased Only*	*Uncon*	*Con*	*Unbiased Only*	*Uncon*
	MSE	MSE	MSE	MSE	MSE	MSE
θ_1^{MC}	0.0746	0.0111	0.0120	0.0797	0.0096	0.0066
θ_1^{FC}	0.0202	0.0147	0.0329	0.0186	0.0117	0.0059
θ_1^{LC}	0.8730	0.9063	1.3037	0.8497	0.7167	0.9541
θ_2^{MC}	0.0729	0.0105	0.0112	0.0790	0.0085	0.0075
θ_2^{FC}	0.0198	0.0125	0.0271	0.0181	0.0122	0.0071
θ_2^{LC}	0.7962	0.8469	1.5497	0.8973	0.7139	1.6359

C.2 Unbiased Belief Model

Table 14: Monte Carlo Experiment: Unbiased Belief Model 100 Monte Carlos

	Max Iteration = 1							
	Update λ				No Update λ			
	MAB		Std		MAB		Std	
	Con	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*
P_1^{lead}	0.0287	0.1373	0.0349	0.1042	0.0254	0.1308	0.0314	0.1058
$\mathrm{P}_1^{collude}$	0.0005	0.0047	0.0006	0.0026	0.0005	0.0046	0.0006	0.0028
P_2^{lead}	0.0235	0.1337	0.0314	0.0990	0.0303	0.1287	0.0381	0.0997
$\mathrm{P}_2^{collude}$	0.0005	0.0047	0.0007	0.0025	0.0005	0.0046	0.0006	0.0028

	Max Iteration = 5							
	Update λ				No Update λ			
	MAB		Std		MAB		Std	
	Con	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*
P_1^{lead}	0.0308	0.1433	0.0370	0.0862	0.0283	0.1184	0.0363	0.0953
$\mathrm{P}_1^{collude}$	0.0005	0.0016	0.0006	0.0019	0.0005	0.0017	0.0006	0.0012
P_2^{lead}	0.0282	0.1529	0.0362	0.0828	0.0280	0.1218	0.0347	0.0947
$\mathrm{P}_2^{collude}$	0.0005	0.0015	0.0006	0.0018	0.0005	0.0017	0.0006	0.0012

	Max Iteration = 5							
	Update λ				No Update λ			
	MAB		Std		MAB		Std	
	Con	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*	*Con*	*Uncon*
P_1^{lead}	0.0313	0.1441	0.0381	0.0912	0.0248	0.1022	0.0306	0.0901
$\mathrm{P}_1^{collude}$	0.0004	0.0030	0.0006	0.0028	0.0005	0.0017	0.0006	0.0013
P_2^{lead}	0.0275	0.1481	0.0349	0.0957	0.0274	0.1039	0.0353	0.0893
$\mathrm{P}_2^{collude}$	0.0005	0.0029	0.0006	0.0028	0.0005	0.0016	0.0005	0.0011

[1] The *Con* denotes the constrained estimation and *Uncon* denotes the model without the rational constraint. [2] Unstrained models are estimated with the restriction that $\lambda_1 = \lambda_1 = 1$. [3] P_i^{lead} is the probability of player i leading a price increase and $\mathrm{P}_i^{collude}$ is the probability that player i chooses to remain in collusion.
[4] The probaiblity is forecasted with the models' best response function.

Table 15: Monte Carlo Experiment: Unbiased Belief Model 100 Monte Carlos

	Max Iteration = 1					
	Update λ			No Update λ		
	Con	*Unbiased Only*	*Uncon*	*Con*	*Unbiased Only*	*Uncon*
	MSE	MSE	MSE	MSE	MSE	MSE
θ_1^{MC}	0.0768	0.0086	0.1126	0.0732	0.0084	0.1094
θ_1^{FC}	0.0228	0.0146	0.0150	0.0192	0.0099	0.0097
θ_1^{LC}	0.9350	0.7783	1.2705	0.7338	0.5838	1.0941
θ_2^{MC}	0.0697	0.0077	0.1021	0.0830	0.0116	0.1195
θ_2^{FC}	0.0193	0.0114	0.0114	0.0196	0.0116	0.0108
θ_2^{LC}	0.7827	0.6833	1.0853	0.9093	0.7421	1.1933
	Max Iteration = 5					
	Update λ			No Update λ		
	Con	*Unbiased Only*	*Uncon*	*Con*	*Unbiased Only*	*Uncon*
	MSE	MSE	MSE	MSE	MSE	MSE
θ_1^{MC}	0.0792	0.0087	0.0063	0.0861	0.0094	0.0056
θ_1^{FC}	0.0169	0.0116	0.0063	0.0163	0.0122	0.0052
θ_1^{LC}	1.0285	0.8902	0.6166	0.8036	0.6463	1.0716
θ_2^{MC}	0.0751	0.0108	0.0080	0.0727	0.0088	0.0064
θ_2^{FC}	0.0233	0.0154	0.0112	0.0201	0.0108	0.0059
θ_2^{LC}	1.0326	0.9711	0.7412	0.8617	0.8347	0.9816
	Max Iteration = 10					
	Update λ			No Update λ		
	Con	*Unbiased Only*	*Uncon*	*Con*	*Unbiased Only*	*Uncon*
	MSE	MSE	MSE	MSE	MSE	MSE
θ_1^{MC}	0.0746	0.0111	0.0120	0.0797	0.0096	0.0066
θ_1^{FC}	0.0202	0.0147	0.0329	0.0186	0.0117	0.0059
θ_1^{LC}	0.8730	0.9063	1.3037	0.8497	0.7167	0.9541
θ_2^{MC}	0.0729	0.0105	0.0112	0.0790	0.0085	0.0075
θ_2^{FC}	0.0198	0.0125	0.0271	0.0181	0.0122	0.0071
θ_2^{LC}	0.7962	0.8469	1.5497	0.8973	0.7139	1.6359

C.3 Bootstrap Experiment With Belief Estimation

In this bootstrap experiment, we consider a dynamic game with biased belief. In the estimation procedure, we follow the following recursive estimation procedure.

Step 1: Obtain the non-parametric CCP estimations $\mathbf{P}_i^0$.

Step 2: Compute the belief $\mathbf{B}_i^0$ estimation using the estimation strategy specified by Aguirregabiria and Magesan (2019).

Step 3: Given the estimator $\mathbf{P}_i^k$ and $\mathbf{B}_i^0$, estimate $\hat{\boldsymbol{\theta}}_i$ with the estimator specified in equation (13).

Step 4: Update $\mathbf{P}_i^{(k+1)} = \boldsymbol{\Psi}(\mathbf{P}_i^{(k)}, \hat{\boldsymbol{\theta}}_i, \mathbf{B}_i)$ with the fixed point mapping as (12).

Table 16: Bootstrap coverage for menu cost of 2 player dynamic pricing game

	Simulated Mean	Coverage	5% quantile	95% quantile	Boot mean	Boots std
			Panel A: Menu cost of player 1			
Mean	-2.0023	**0.9300**	-2.1104	-1.9034	-2.0044	0.0646
Std	0.0575	0.2564	0.0614	0.0586	0.0581	0.0108
Min	-2.1448	-	-2.2873	-2.0534	-2.1561	0.0374
25%	-2.0490	-	-2.1582	-1.9461	-2.0484	0.0561
50%	-1.9995	-	-2.1072	-1.9072	-2.0026	0.0657
75%	-1.9619	-	-2.0638	-1.8603	-1.9593	0.0720
Max	-1.8653	-	-1.9853	-1.7742	-1.8670	0.0904
			Panel B: Menu cost of player 2			
mean	-2.0098	**0.8800**	-2.1174	-1.9084	-2.0109	0.0657
std	0.0663	0.3266	0.0732	0.0671	0.0674	0.0120
min	-2.2310	-	-2.3594	-2.1171	-2.2337	0.0356
25%	-2.0508	-	-2.1660	-1.9416	-2.0515	0.0579
50%	-1.9953	-	-2.1092	-1.9039	-1.9939	0.0673
75%	-1.9638	-	-2.0684	-1.8623	-1.9641	0.0749
max	-1.8563	-	-1.9411	-1.7623	-1.8560	0.0897

[1] The table is based on 100 Monte Carlo simulation and 99 Bootstraps each.

[2] Each simulation, we simulate data on 100 markets and 50 time periods.

[3] The true parameter is $\theta_1^{menu} = \theta_2^{menu} = -2$.

[4] The demand parameters, market size is 10, marginal cost is 0.7, price elasticity is -1.

Table 17: Bootstrap coverage for menu cost of 3 player dynamic pricing game

	Simulated Mean	Coverage	5% quantile	95% quantile	Boot mean	Boots std
			Panel A: Menu cost of player 1			
mean	-1.9710	**0.8800**	-2.0809	-1.8702	-1.9726	0.0658
std	0.0565	0.3266	0.0611	0.0581	0.0563	0.0130
min	-2.1534	-	-2.2770	-2.0459	-2.1613	0.0355
25%	-2.0055	-	-2.1182	-1.9012	-2.0084	0.0563
50%	-1.9756	-	-2.0833	-1.8763	-1.9761	0.0669
75%	-1.9315	-	-2.0460	-1.8380	-1.9355	0.0755
max	-1.8101	-	-1.9107	-1.7274	-1.8125	0.0922
			Panel B: Menu cost of player 2			
mean	-1.9840	**0.8300**	-2.0900	-1.8797	-1.9838	0.0658
std	0.0811	0.3775	0.0901	0.0814	0.0811	0.0149
min	-2.1646	-	-2.2877	-2.0892	-2.1664	0.0320
25%	-2.0310	-	-2.1440	-1.9271	-2.0312	0.0559
50%	-1.9774	-	-2.0905	-1.8794	-1.9775	0.0669
75%	-1.9467	-	-2.0393	-1.8406	-1.9471	0.0768
max	-1.6782	-	-1.8003	-1.5388	-1.6805	0.1018
			Panel C: Menu cost of player 3			
Mean	-1.9711	**0.8300**	-2.0747	-1.8733	-1.9713	0.0629
std	0.0683	0.3775	0.0793	0.0665	0.0673	0.0160
min	-2.1125	-	-2.3046	-2.0188	-2.1200	0.0322
25%	-2.0163	-	-2.1177	-1.9167	-2.0146	0.0493
50%	-1.9728	-	-2.0729	-1.8805	-1.9712	0.0619
75%	-1.9284	-	-2.0306	-1.8302	-1.9261	0.0752
max	-1.8097	-	-1.8850	-1.7150	-1.8106	0.1021

D Miscellaneous

Table 18: Drug Price in Latin America in year 2006 - 2008

Country	2006 (USD)	2007 (USD)	2008 (USD)	2006 - 2007 (%)	2007 - 2008 (%)
Argentina	5.93	6.36	7.3	7.4	14.7
Bolivia	4.73	4.9	5.98	3.6	22
Brazil	6.86	8.03	8.97	17.1	11.7
Chile	4.15	4.12	4.73	-0.6	14.8
Colombia	4.4	5.41	5.93	23.1	9.5
Ecuador	4.35	4.57	4.77	5.2	4.3
Paraguay	3.65	4.17	4.73	14.2	13.4
Peru	5.81	6.34	7.22	9	14
Uruguay	3.3	3.47	4.05	5	16.8
Venezuela	6.14	7.4	9.42	20.5	27.4

[1] Data source: IMS, Vasallo C. The medicine market in Chile: characterization and recommendations for economic regulation. Final report for the Ministry of Health Economics of MINSAL, Chile. 2010 Jun.

Table 19: inflation Rate of Latin America in year 2005 - 2009

Country	2004	2005	2006	2007	2008	2009
Bolivia	4.4374	5.3932	4.2824	8.7056	14.0068	3.3465
Brazil	6.5972	6.8695	4.1836	3.6413	5.6786	4.8880
Chile	1.0547	3.0526	3.3920	4.4078	8.7163	0.3530
Colombia	5.9013	5.0514	4.2925	5.5451	6.9986	4.2010
Ecuador	2.7422	2.4078	3.2987	2.2762	8.4001	5.1600
Peru	3.6625	1.6163	2.0023	1.7800	5.7859	2.9362
Paraguay	4.3233	6.8074	9.5893	8.1305	10.1548	2.5919
Uruguay	9.1576	4.6993	6.3976	8.1146	7.8771	7.0622

[1] Data source: World bank, International Monetary Fund, International Financial Statistics and data files.

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