

# Using Euler Equation to Estimate Non-Finite-Dependent Dynamic Discrete Choice Model with Unobserved Heterogeneity

CEA Meeting 2019

Jasmine Hao and Hiro Kasahara

Vancouver School of Economics

May 29, 2019

# Table of contents

1. Background
2. Baseline Model
3. Bellman Equation in Probability Space
  - 3.1 Redefine the problem
  - 3.2 Approach: Envelop Theorem
4. Estimators for Heterogeneous Agent Model
  - 4.1 Identification of unobserved heterogeneity
  - 4.2 Proposed estimator
5. Monte Carlo Experiments
  - 5.1 Homogeneous agent model
  - 5.2 Finite mixture model

# Background

# Dynamic Discrete Choice Model

## Model Priors

- ◇ The agents are forward looking and maximize expected inter-temporal payoffs.
- ◇ Structural functions: agents' preferences and beliefs about uncertain events.
- ◇ Estimated under principle of revealed preference, use micro-data on individuals' choices and outcomes.

## Empirical applications includes

- ◇ Industrial organization Aguirregabiria and Ho (2012), Berry (1992), Yakovlev (2016), Sweeting (2013);
- ◇ Health economics Beauchamp (2015), Gaynor and Town (2012),  
Gowrisankaran and Town (1997), Gowrisankaran et al. (2011);
- ◇ Marketing Dubé et al. (2005), Doraszelski and Pakes (2007), Doganoglu and Klapper (2006);
- ◇ Labor economics Todd and Wolpin (2006), Fang and Wang (2009), Keane et al. (2011);
- ◇ Other Schivardi and Schneider (2008), Rust and Rothwell (1995).

The difficulties in incorporating unobserved heterogeneity:

- ◇ Computational heavy: value function iteration or Hotz-Miller inversion
- ◇ EM algorithm: more iterations account for unobserved heterogeneity.
- ◇ Existing methods relies on "Finite Dependence"(Arcidiacono and Ellickson (2011)).

The contribution of this project:

- ◇ Conceptually redefine the deterministic problem as a stochastic problem.
- ◇ Propose alternative estimator and incorporate unobserved heterogeneity and EM algorithm in dynamic discrete choice.
- ◇ Demonstrate the performance using Monte Carlo simulation.

Baseline

# Baseline entry exit model

Now consider the baseline model of dynamic choice model

- ◇ Time is discrete and indexed by  $t \in [0, T]$  and  $T$  finite / infinite.
- ◇ predetermined  $s_t$  and discrete action  $d_t \in \mathcal{D} = \{0, \dots, D\}$ .
- ◇ Time-separable utility function  $\sum_{t=0}^T \beta^t U_t(d_t, s_t)$ ,



# Baseline Model

$$V_t(s_t) = \max_d \left\{ U_t(d, s_t) + \beta \int V_{t+1}(s_{t+1}) dF_t(s_{t+1}|a, s_t) \right\}. \quad (1)$$

- ◇ Assumption 1(Additive separable):  $s = (x_t, \epsilon_t)$ ,  
 $\epsilon_t = [\epsilon_t(0), \epsilon_t(1)]$  ,  $U(d_t, s_t) = u(d_t, x_t; \theta) + \epsilon_t(d)$ .
- ◇ Assumption 2(Finite domain of  $x$ ):  $x \in \mathcal{X}$ ,  $|\mathcal{X}|$  is finite.
- ◇ Assumption 3(Conditional independence):  
 $F(s_{t+1}|a_t, s_t) = G_\epsilon(\epsilon_{t+1}|x_{t+1})F_x(x_{t+1}|x_t, d_t)$ .
- ◇ Assumption 4(T1EV):  $\epsilon_t = \{\epsilon_t(d) : d \in \mathcal{D}\} \sim_{i.i.d} T1EV$  .

## Motivating Example: Entry Exit Problem

For example, take Consider a stationary infinite time horizon entry-exit problem:

- ◇ The firm observe the state  $x_t = (y_t, z_t)$ . The profitability  $z_t \in \mathcal{Z}$ , where  $|\mathcal{Z}| = N$  is finite, and operation state  $y_t = d_{t-1} \in \{0, 1\}$ .
- ◇ The firm makes entry decision  $d_t \in \mathcal{D} = \{0, 1\}$ .
- ◇  $z_t$  follows a first order Markov process  $f(x_{t+1}|x_t, d_t)$ ;
- ◇ The firm's flow payoff  $u(x_t, d_t; \theta)$ .

# Entry Exit Problem: Bellman Value Function

- ◇ Likelihood function:

$$l(d_t, x_t; \theta) = \frac{\exp(v(x_t, d_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))}. \quad (2)$$

- ◇ The strategy  $d_t^* = \arg \max_{d \in \mathcal{D}} \{v(x_t, d_t) + \epsilon_t(d)\}$ , where  $v(x_t, d; \theta) = u(x_t, d; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} f(x_{t+1} | x_t, d) \bar{V}(x_{t+1})$ .
- ◇ The ex-ante value function:

$$\begin{aligned} \bar{V}(x_t) &= E_\epsilon V(x_t, \epsilon) \\ &= E_\epsilon \left\{ \max_{d \in \mathcal{D}} \{v(d, x_t; \theta) + \epsilon_t(d)\} \right\} \end{aligned} \quad (3)$$

# Redefinition



# Decision and state in probability space

Redefine the agent's problem as an analogue to a continuous optimization problem

$$\max_{P_t(x_t)} \left\{ \sum_{t=0}^{\infty} \beta^t \kappa_t(x_t|x_0) \left[ \sum_{d=0}^1 p_t(x_t, d) (u(x_t, d) + e(P_t(x_t), d)) \right] \right\}$$

$$\text{subject to } \kappa_{t+1}(x_{t+1}|x_0) = \sum_{x_t \in \mathcal{X}} \sum_{d \in \mathcal{D}} \kappa_t(x_t|x_0) p(x_t, d) f(x_{t+1}|x_t, d).$$

(4)

# Bellman Operator

Define the Bellman operator as

$$\mathbf{W}(\boldsymbol{\kappa}_t) = \max_{\tilde{\mathbf{P}}_t} \boldsymbol{\kappa}_t^\top \mathbf{U}^{\mathbf{P}_t} + \beta \mathbf{W}(\boldsymbol{\kappa}_{t+1})$$

$$\text{subject to } \boldsymbol{\kappa}_{t+1} = \mathbf{F}^{\mathbf{P}_t} \boldsymbol{\kappa}_t,$$

where

- ◇ Note  $\mathbf{W}^* = \boldsymbol{\kappa}^\top \bar{\mathbf{V}}$ .
- ◇  $\mathbf{U}^{\mathbf{P}_t} = [\mathbf{U}^{\mathbf{P}_t}(x^{(1)}), \dots, \mathbf{U}^{\mathbf{P}_t}(x^{(|\mathcal{X}|)})]^\top$ .
- ◇  $\mathbf{U}^{\mathbf{P}_t}(x) = \mathbf{P}_t(x)^\top \left( \mathbf{u}(x) + \mathbf{e}^{\mathbf{P}_t}(x) \right)$ .
  - ▶  $\mathbf{u}(x) = [u(d, x)]_{d \in \mathcal{D}}$
  - ▶  $\mathbf{e}^{\mathbf{P}_t}(x) = [\gamma - \log(P_t(d, x))]_{d \in \mathcal{D}}$
- ◇  $\mathbf{F}^{\mathbf{P}_t}$  is the  $\mathbf{P}_t$ -weighted transition matrix.

# Approach: Envelop Theorem

$$\frac{\partial W^*(\kappa_t)}{\partial \kappa_t} = U_0^{P_t} + \beta F_0 \frac{\partial W^*(\kappa_{t+1})}{\partial \kappa_{t+1}}, \quad (5)$$

$$\tilde{u} + \tilde{e}^{P_t} + \beta \tilde{F} \left( u_0 + e_0^{P_t} + \beta F_0 \frac{\partial W^*(\kappa_{t+1})}{\partial \kappa_{t+1}} \right) = 0. \quad (6)$$

- ◇  $U_0^{P_t} = u_0 + e_0^{P_t},$
- ◇  $u_0 = [u(0, x^{(1)}), \dots, u(0, x^{(|\mathcal{X}|)})]^\top.$
- ◇  $e_0^{P_t} = [\gamma - \log(P_t(0, x^{(1)})), \dots, \gamma - \log(P_t(0, x^{(|\mathcal{X}|)}))]^\top.$

# Approach: Envelop Theorem

$$\bar{\mathbf{V}}_t = \mathbf{U}_0^{P_t} + \beta \mathbf{F}_0 \bar{\mathbf{V}}_{t+1}, \quad (7)$$

$$\tilde{\mathbf{u}} + \tilde{\mathbf{e}}^{P_t} + \beta \tilde{\mathbf{F}} \left( \mathbf{u}_0 + \mathbf{e}_0^{P_t} + \beta \mathbf{F}_0 \bar{\mathbf{V}}_{t+1} \right) = 0. \quad (8)$$

- ◇  $\mathbf{U}_0^{P_t} = \mathbf{u}_0 + \mathbf{e}_0^{P_t},$
- ◇  $\mathbf{u}_0 = [u(0, x^{(1)}), \dots, u(0, x^{(|\mathcal{X}|)})]^\top.$
- ◇  $\mathbf{e}_0^{P_t} = [\gamma - \log(P_t(0, x^{(1)})), \dots, \gamma - \log(P_t(0, x^{(|\mathcal{X}|)}))]^\top.$



# Likelihood Function(EE)

## Proposition 1

In a stationary model,

$$\bar{\mathbf{V}}_t = (I - \beta \mathbf{F}_0)^{-1} (\mathbf{u}_0 + \mathbf{e}_0^{P_t}).$$

The logit likelihood function from equation (6):

$$l(d_t, x_t; \theta) = \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))},$$

$$v(d, x_t; \theta) = u(d, x_t; \theta) + \beta \tilde{\mathbf{f}}(d_t, x_t) \bar{\mathbf{V}}_{t+1}.$$

# Likelihood Function(FD)

## Proposition 2 (Finite Dependence)

If the model display the finite dependence property, there exists an arbitrary action  $d^\dagger$  such that  $\tilde{\mathbf{F}}\mathbf{F}_{d^\dagger} = \mathbf{0}$ .

The logit likelihood function for finite dependence:

$$l(d_t, x_t; \theta) = \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))},$$

$$v(d, x_t; \theta) = u(d, x_t; \theta) + \beta \tilde{\mathbf{f}}(d_t, x_t) \left( \mathbf{u}_{d_{t+1}^\dagger, t+1} + \mathbf{e}_{d_{t+1}^\dagger, t+1}^{P_{t+1}} \right)$$

# Likelihood Function(AFD)

## Proposition 3 (Almost Finite Dependent Estimator)

If the model does not exhibit finite dependence, we can find  $d_{t+1}^\dagger$  to minimize the norm of  $|\tilde{\mathbf{F}}\mathbf{F}_{d_{t+1}^\dagger}|$ .

$$l(d_t, x_t; \theta) = \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))},$$

$$v(d, x_t; \theta) = u(d, x_t; \theta) + \beta \tilde{\mathbf{f}}(d_t, x_t) \left( \mathbf{u}_{d_{t+1}^\dagger, t+1} + \mathbf{e}_{d_{t+1}^\dagger, t+1}^{P_{t+1}} + \mathbf{F}_{d_{t+1}^\dagger, t+1} \bar{\mathbf{V}}_{t+1} \right)$$

Estimator

# EM Algorithm

- ◇  $M$  types of agent,  $\theta = (\theta^1, \dots, \theta^M)$ .
- ◇  $\pi^m$  denote the probability of being type  $m$ .
- ◇  $l(d_i, z_i; \theta_m)$  is the likelihood function.

$$\{\hat{\theta}, \hat{\pi}\} = \arg \max_{\theta, \pi} = \sum_{n=1}^N \log \left\{ \sum_{m=1}^M \pi^m l(d_i, z_i, \theta^m) \right\}, \quad (9)$$

# EM algorithm in dynamic discrete choice

The posterior:

$$\hat{q}_{im} = \frac{\hat{\pi}^m l(d_i, z_i, \hat{P}^m, \hat{V}^m, \hat{\theta}^m)}{\sum_{m' \in \mathcal{M}} \hat{\pi}^{m'} l(d_i, z_i, \hat{P}^{m'}, \hat{V}^{m'}, \hat{\theta}^{m'})}. \quad (10)$$

- ◇ where  $\hat{P} = (\hat{P}^1, \dots, \hat{P}^M)$  are unbiased estimators for CCPs,
- ◇  $\hat{V} = (\hat{V}^1, \dots, \hat{V}^M)$  is an estimator of the value function,
- ◇  $\hat{\pi} = (\hat{\pi}^1, \dots, \hat{\pi}^M)^\top$  is an estimator of mixing probability,
- ◇  $\hat{q}_{im}$ , the probability  $n$  is type  $m$ .

# Modified EM Algorithm

Given the estimators from last round  $\{\hat{\mathbf{P}}^{(k-1)}, \hat{\mathbf{V}}^{(k-1)}, \hat{\pi}^{(k-1)}\}$ .

*Step 1:* Compute  $\hat{q}_{im}^{(k)}$  as

$$\hat{q}_{im}^{(k)} = \frac{\hat{\pi}^{m,(k-1)} l(d_i, z_i, \hat{\mathbf{P}}^{m,(k-1)}, \hat{\mathbf{V}}^{m,(k-1)}, \hat{\theta}^{m,(k-1)})}{\sum_{m' \in \mathcal{M}} \hat{\pi}^{m',(k-1)} l(d_i, z_i, \hat{\mathbf{P}}^{m',(k-1)}, \hat{\mathbf{V}}^{m',(k-1)}, \hat{\theta}^{m',(k-1)})}.$$

*Step 2:* Using  $\hat{q}_{im}^{(k)}$  to compute  $\hat{\pi}^{m,(k)}$ :  $\hat{\pi}^{m,(k)} = \frac{1}{N} \sum_{i=1}^N \hat{q}_{im}^{(k)}$ .

*Step 3:* Update estimator of  $\theta$  with the equation

$$\hat{\theta}_k = \arg \max_{\theta} \sum_{i=1}^N \hat{q}_{im}^{(k)} \log l(d_{it}, x_{it}, s, \hat{\mathbf{P}}^{m,(k-1)}, \hat{\mathbf{V}}^{m,(k-1)}, \theta). \quad (11)$$

*Step 4:* Update the CCPs  $\hat{\mathbf{P}}^{(k)}$ , and the value function  $\hat{\mathbf{V}}^{(k)}$ .

# Likelihood Function

$$l(d_t, x_t; \mathbf{P}, \mathbf{V}, \theta) = \frac{\exp(\tilde{v}(d_t, x_t))}{1 + \sum_{d \in \mathcal{D} / \{0\}} \exp(\tilde{v}(d, x_t))}$$

**Table:** Likelihood function comparison

Method	diff in continuation value ( $\tilde{v}(d, x)$ )
NFXP, HM, EE, SEQ(q)	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} \tilde{f}(x_t, d_t) \mathbf{V}$
FD2	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} \tilde{f}(x_t, d_t) (u_0 + e_0^P + \gamma - \log(p_0) + \mathbf{F}_0 \mathbf{V})$
AFD2	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} \tilde{f}(x_t, d_t) \left( \sum_d \omega(d) (u_d + e_d^P \gamma - \log(p_d) + \mathbf{F}_d \mathbf{V}) \right)$



# Value function

Table: Comparisons between value function computation

Method	Contraction Mapping
NFXP	$\mathbf{V}(x_t) = E_\epsilon \left\{ \max_{d \in \mathcal{D}} [u_d(x_t) + \epsilon_d + \beta \tilde{\mathbf{f}}(x_t, d_t) \mathbf{V}] \right\}$ till convergence
SEQ(q)	$\mathbf{V}(x_t) = E_\epsilon \left\{ \max_{d \in \mathcal{D}} [u_d(x_t) + \epsilon_d + \beta \tilde{\mathbf{f}}(x_t, d_t) \mathbf{V}] \right\}$ for $q$ times
Hotz-Miller	$\mathbf{V} = (\mathbf{I} - \beta \mathbf{F}^P)^{-1} (\sum_p p(d) (u_d + \mathbf{e}_d^P))$
EE	$\mathbf{V} = (\mathbf{I} - \beta \mathbf{F}_0)^{-1} (u_0 + \mathbf{e}_0^P)$
FD2	$\mathbf{V} = u_0 + \mathbf{e}_0^P + \beta \mathbf{F}_0 \mathbf{V}$
AFD2	$\mathbf{V} = \sum_d \omega(d) (u_d + \mathbf{e}_d^P + \mathbf{F}_d \mathbf{V})$

# Simulation

# Data generating process: Homogeneous agent model

Table: Parameters in DGP

<i>Flow-Payoff Parameters</i>	$\theta_0^{VP} = 0.5$ $\theta_1^{VP} = 1.0$ $\theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5$ $\theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0$ $\theta_1^{EC} = 1.0$
<i>State Variable Transition</i>	$z_{kt}$ is AR(1), $\gamma_0^k = 0$ , $\gamma_1^k = 0.6$
<i>Productivity Transition</i>	$\omega_t$ is AR(1), $\gamma_0^\omega = 0$ , $\gamma_1^\omega = 0.9$
<i>Past action on productivity</i>	$\gamma_a \in [0, 5]$
<i>Discount Factor</i>	$\beta = 0.95$

# Finite Dependent Model

Table: Two-step: Finite dependent models

	<i>FD</i>	<i>FD2</i>	<i>AFD</i>	<i>AFD2</i>	<i>HM</i>	<i>EE</i>
<i>Market = 200, Time = 20, <math>\gamma_a = 0</math></i>						
$\theta_0^{VP}$	0.4845 (0.0706)	0.4845 (0.0706)	0.4845 (0.0706)	0.4845 (0.0706)	0.5016 (0.0350)	0.4845 (0.0706)
$\theta_0^{FC}$	0.5447 (0.0904)	0.5447 (0.0904)	0.5447 (0.0904)	0.5447 (0.0904)	0.5098 (0.0627)	0.5447 (0.0904)
<i>Market = 200, Time = 120, <math>\gamma_a = 0</math></i>						
$\theta_0^{VP}$	0.4963 (0.0189)	0.4963 (0.0189)	0.4963 (0.0189)	0.4963 (0.0189)	0.4983 (0.0140)	0.4963 (0.0189)
$\theta_0^{FC}$	0.4990 (0.0301)	0.4990 (0.0301)	0.4990 (0.0301)	0.4990 (0.0301)	0.4954 (0.0279)	0.4990 (0.0301)
DGP: $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$ .						

## Two-step: Non-finite dependent models

Table: Non-finite Dependent two-step estimators

	<i>FD</i>	<i>FD2</i>	<i>AFD</i>	<i>AFD2</i>	<i>HM</i>	<i>EE</i>
<i>Market = 200, Time = 20, <math>\gamma_a = 5</math></i>						
$\theta_0^{VP}$	0.3434 (0.0790)	0.5679 (0.1457)	0.4925 (0.0860)	0.5067 (0.0908)	0.5307 (0.0800)	0.5691 (0.1460)
$\theta_0^{FC}$	-0.0155 (0.2228)	0.7095 (0.3321)	0.4432 (0.2402)	0.4751 (0.2518)	0.5833 (0.2209)	0.7134 (0.3330)
<i>Market = 200, Time = 120, <math>\gamma_a = 5</math></i>						
$\theta_0^{VP}$	0.3058 (0.0333)	0.4965 (0.0484)	0.4829 (0.0436)	0.4954 (0.0453)	0.4982 (0.0395)	0.4975 (0.0485)
$\theta_0^{FC}$	-0.1239 (0.0845)	0.4920 (0.1237)	0.4583 (0.1096)	0.4860 (0.1140)	0.4977 (0.1036)	0.4953 (0.1239)

DGP:  $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$ .

# Sequential Estimation

**Table:** The mean and standard deviation of sequential estimators

	<i>FD2</i>	<i>AFD2</i>	<i>HM</i>	<i>EE</i>	<i>SEQ(1)</i>
	<i>Market = 200, Time = 20, <math>\gamma_a = 5</math></i>				
$\theta_0^{VP}$	0.5084 (0.0925)	0.4940 (0.1151)	0.5096 (0.0938)	0.5084 (0.0925)	0.5043 (0.0921)
$\theta_0^{FC}$	0.5167 (0.2506)	0.4391 (0.3056)	0.5207 (0.2567)	0.5167 (0.2506)	0.5034 (0.2493)

The DGP parameters are:  $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$ .

# Data generating process: Heterogeneous agent model

Table: Parameters in DGP

<i>Flow-Payoff Parameters <math>\theta^1</math></i>	$\theta_0^{VP} = 0$ $\theta_0^{FC} = 0.5$ $\theta_0^{EC} = 1.0$	$\theta_1^{VP} = 1.0$ $\theta_1^{FC} = 1.0$ $\theta_1^{EC} = 1.0$	$\theta_2^{VP} = -1.0$
<i>Flow-Payoff Parameters <math>\theta^2</math></i>	$\theta_0^{VP} = 1$ $\theta_0^{FC} = 0.5$ $\theta_0^{EC} = 1.0$	$\theta_1^{VP} = 1.0$ $\theta_1^{FC} = 1.0$ $\theta_1^{EC} = 1.0$	$\theta_2^{VP} = -1.0$
<i>Mixing Probability</i>	(0.5, 0.5)		
<i>State Variable Transition</i>	$z_{kt}$ is AR(1), $\gamma_0^k = 0$ , $\gamma_1^k = 0.6$		
<i>Productivity Transition</i>	$\omega_t$ is AR(1), $\gamma_0^\omega = 0$ , $\gamma_1^\omega = 0.9$		
<i>Past action on productivity</i>	$\gamma_a = 2$		
<i>Discount Factor</i>	$\beta = 0.95$		

# Time and iteration

Table: Median Time and Iteration when increase state space

Algorithms	<i>nGrid</i>	2	3	4	5	6
	$ \mathcal{X} $	64	486	2048	6250	15552
	<i>Market</i>	100				
	<i>Time</i>	20				
FD2	<i>Time</i>	11.2472	13.9627	27.9147	390.0466	3103.6867
	<i>Iteration</i>	40.5	48.5	37	47.5	32.5
EE	<i>Time</i>	12.1462	21.3075	18.6141	181.0266	1039.5331
HM	<i>Iteration</i>	38.5	69.5	43	80.5	52
	<i>Time</i>	30.3638	35.6079	982.0085	-	-
	<i>Iteration</i>	91.5	59.5	53	-	-
SEQ(1)	<i>Time</i>	6.0499	17.2884	24.1402	100.8548	509.4910
	<i>Iteration</i>	22.5	64.5	55	43.5	35.5

† The results shows the time and iteration used in the estimation based on 12 Monte Carlo simulations of different state space.



# Summary of Contributions

1. Reformulation of the Bellman equation of discrete choice by continuous choice and states, to derive the restrictions of the model. Particular useful in non-finite-dependent(NFD) models.
2. Propose an alternative to Arcidiacono Miller algorithm that can be applied to NFD models.
3. Show computation gain to using this estimator in Monte Carlo simulations.

heterogeneous

Thank You



- Aguirregabiria, V. and Ho, C. Y. (2012). A dynamic oligopoly game of the US airline industry: Estimation and policy experiments. *Journal of Econometrics*, 168(1):156–173.
- Arcidiacono, P. and Ellickson, P. (2011). Practical Methods for Estimation of Dynamic Discrete Choice Models. *Annual Review of Economics*, 3(1).
- Beauchamp, A. (2015). Regulation, Imperfect competition, and the U.S. abortion market. Technical Report 3.
- Berry, S. (1992). Estimation of a model of entry in the airline industry. *Econometrica*, 60(4):889–917.
- Doganoglu, T. and Klapper, D. (2006). Goodwill and dynamic advertising strategies. *Quantitative Marketing and Economics*, 4(1):5–29.
- Doraszelski, U. and Pakes, A. (2007). A Framework for Applied Dynamic Analysis in IO. *Handbook of Industrial Organization*, 3(December 2007):1887–1966.

- Dubé, J. P., Hitsch, G. J., and Manchanda, P. (2005). An empirical model of advertising dynamics. *Quantitative Marketing and Economics*, 3(2):107–144.
- Fang, H. and Wang, Y. (2009). Estimating Dynamic Discrete Choice Models with Hyperbolic Discounting, with an Application to Mammography Decisions.
- Gaynor, M. and Town, R. (2012). Competition in Health Care Markets. *Handbook of Health Economics*, (2):499–637.
- Gowrisankaran, G., Lucarelli, C., Schmidt-Dengler, P., and Town, R. (2011). Government policy and the dynamics of market structure : Evidence from Critical Access Hospitals.
- Gowrisankaran, G. and Town, R. J. (1997). Dynamic equilibrium in the hospital industry. *Journal of Economics and Management Strategy*, 6(1):45–74.
- Keane, M., Todd, P., and Wolpin, K. (2011). *The Structural Estimation of Behavioral Models: Discrete Choice Dynamic Programming Methods and Applications*, volume 4.

## heterogeneous

- Rust, J. and Rothwell, G. (1995). Optimal response to a shift in regulatory regime: The case of the US nuclear power industry. *Journal of Applied Econometrics*, 10(1 S):S75–S118.
- Schivardi, F. and Schneider, M. (2008). Strategic experimentation and disruptive technological change. *Review of Economic Dynamics*, 11(2):386–412.
- Sweeting, A. (2013). Dynamic product positioning in differentiated product markets: The effect of fees for musical performance rights on the commercial radio industry. *Econometrica*, 81(5):1763–1803.
- Todd, P. E. and Wolpin, K. I. (2006). Assessing the Impact of a School Subsidy Program in Mexico : Using a Soc Experiment to Validate a Dynam Fertility Behavioral Model of Child Schooling. *The American Economic Review*, 96(5):1384–1417.
- Yakovlev, E. (2016). Demand for Alcohol Consumption and Implication for Mortality: Evidence from Russia.