Using Euler Equation to Estimate Non-Finite-Dependent Dynamic Discrete Choice Model with Unobserved Heterogeneity

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Background

Dynamic Discrete Choice Model

Model Priors

- The agents are forward looking and maximize expected inter-temporal payoffs.
- Structural functions: agents' preferences and beliefs about uncertain events.
- Estimated under principle of revealed preference, use micro-data on individuals' choices and outcomes.

Empirical applications includes

- Industrial organization Aguirregabiria and Ho (2012), Berry (1992), Yakovlev (2016),Sweeting (2013);
- Health economics Beauchamp (2015),Gaynor and Town (2012),
  Gowrisankaran and Town (1997),Gowrisankaran et al. (2011);
- Marketing Dubé et al. (2005),Doraszelski and Pakes (2007), Doganoglu and Klapper (2006);
- Labor economics Todd and Wolpin (2006),Fang and Wang (2009),Keane et al. (2011);
- Other Schivardi and Schneider (2008), Rust and Rothwell (1995).

The difficulties in incorporating unobserved heterogeneity:

- Computational heavy: value function iteration or Hotz-Miller inversion
- EM algorithm: more iterations account for unobserved heterogeneity.
- Existing methods relies on "Finite Dependence"(Arcidiacono and Ellickson (2011)).

The contribution of this project:

- Conceptually redefine the deterministic problem as a stochastic problem.
- Propose alternative estimator and incorporate unobserved heterogeneity and EM algorithm in dynamic discrete choice.
- Demonstrate the performance using Monte Carlo simulation.

Baseline

Baseline entry exit model

Now consider the baseline model of dynamic choice model

- Time is discrete and indexed by t.
- Firms have preferences defined states of the world between periods 0 and T finite / infinite.
- A state of the world has two component: predetermined s_t and discrete action $d_t \in \mathcal{D} = \{0, 1\}$.
- Time-separable utility function $\sum_{t=0}^{T} \beta^t U_t(d_t, s_t)$, where $\beta \in [0, 1)$ is the discount factor.
- Let $d_t^*(s_t)$ denote optimal decision rule, $V_t(s_t)$ be the value function at period t.

$$V_t(s_t) = \max_d \left\{ U_t(d, s_t) + \beta \int V_{t+1}(s_{t+1}) dF_t(s_{t+1}|a, s_t) \right\}. \tag{1}$$

Baseline Model

$$V_t(s_t) = \max_d \left\{ U_t(d, s_t) + \beta \int V_{t+1}(s_{t+1}) dF_t(s_{t+1}|a, s_t) \right\}. \quad (2)$$

- Assumption 1(Additive separable): $s = (x_t, \epsilon_t)$, $\epsilon_t = [\epsilon_t(0), \epsilon_t(1)]$, $U(d_t, s_t) = u(d_t, x_t; \theta) + \epsilon_t(d)$.
- Assumption 2(Finite domain of x): $x \in \mathcal{X}$, $|\mathcal{X}|$ is finite.
- Assumption 3(Conditional independence): $F(s_{t+1}|a_t, s_t) = G_\epsilon(\epsilon_{t+1}|x_{t+1}) F_x(x_{t+1}|x_t, d_t)$.
- Assumption 4(T1EV): $\epsilon_t = \{\epsilon_t(d) : d \in \mathcal{D}\} \sim_{i.i.d} T1EV$.

Motivating Example: Entry Exit Problem

For example, take Consider a stationary infinite time horizon entry-exit problem:

- The firm observe the state $x_t = (y_t, z_t)$. The profitability $z_t \in \mathcal{Z}$, where $|\mathcal{Z}| = N$ is finite, and operation state $y_t = d_{t-1} \in \{0, 1\}$.
- The firm makes entry decision $d_t \in \mathcal{D} = \{0, 1\}$.
- z_t follows a first order Markov process $f(x_{t+1}|x_t, d_t)$;
- The firm's flow payoff $u(x_t, d_t; \theta)$.

Entry Exit Problem: Bellman Value Function

The ex-ante value function:

$$\begin{aligned}\bar{V}(x_t) &= E_\epsilon V(x_t, \epsilon) \\ &= E_\epsilon \Big\{ \max_{d \in \mathcal{D}} \{ v(d, x_t; \theta) + \epsilon_t(d) \} \Big\}\end{aligned} \tag{3}$$

The firm's strategy $d_t^* = \arg\max_{d \in \mathcal{D}} \{ v(x_t, d_t) + \epsilon_t(d) \}$, where

$$v(x_t, d; \theta) = u(x_t, d; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} f(x_{t+1} | x_t, d) \bar{V}(x_{t+1}).$$

Likelihood function:

$$l(d_t, x_t; \theta) = \frac{\exp(v(x_t, d_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))}.$$

Redefinition

Decision and state in probability space

Redefine the agent's problem as an analogue to a continuous optimization problem

$$\max_{P_t(x_t)} \left\{ \sum_{t=0}^{\infty} \beta^t \kappa_t(x_t|x_0) \Big[\sum_{d=0}^{1} p_t(x_t, d)(u(x_t, d) + e(P_t(x_t), d)) \Big] \right\}$$

$$\text{subject to } \kappa_{t+1}(x_{t+1}|x_0) = \sum_{x_t \in \mathcal{X}} \sum_{d \in \mathcal{D}} \kappa_t(x_t|x_0) p(x_t, d) f(x_{t+1}|x_t, d). \tag{4}$$

Bellman Operator

Define the Bellman operator as

$$\boldsymbol{W}(\boldsymbol{\kappa}_t) = \max_{\tilde{P}_t} \boldsymbol{\kappa}_t^T \boldsymbol{U}^{\boldsymbol{P}_t} + \beta \boldsymbol{W}(\boldsymbol{\kappa}_{t+1})$$

$$\text{subject to } \boldsymbol{\kappa}_{t+1} = \boldsymbol{F}^{\boldsymbol{P}_t} \boldsymbol{\kappa}_t,$$

where

- Note $\boldsymbol{W}^* = \boldsymbol{\kappa}^\top \bar{\boldsymbol{V}}$.
- $\boldsymbol{\kappa}_t$, $\boldsymbol{U}^{\boldsymbol{P}_t}$ are vectors of length $|\mathcal{X}|$.
- $\boldsymbol{U}^{\boldsymbol{P}_t} = [\boldsymbol{U}^{\boldsymbol{P}_t}(x^{(1)}, \ldots, \boldsymbol{U}^{\boldsymbol{P}_t}(x^{(|\mathcal{X}|)})]^\top$.
- $\boldsymbol{U}^{\boldsymbol{P}_t}(x) = \boldsymbol{P}_t(x)^\top \Big(\boldsymbol{u}(x) + \boldsymbol{e}^{\boldsymbol{P}_t}(x) \Big)$.
  - $\boldsymbol{u}(x) = [u(d, x)]_{d \in \mathcal{D}}$
  - $\boldsymbol{e}^{\boldsymbol{P}_t}(x) = [\gamma - log(P_t(d, x))]_{d \in \mathcal{D}}$
- $\boldsymbol{F}^{\boldsymbol{P}_t}$ is the $\boldsymbol{P}_t$-weighted transition matrix.

Approach: Envelop Theorem

$$\frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_t)}{\partial \boldsymbol{\kappa}_t} = \boldsymbol{U}_0^{\boldsymbol{P}_t} + \beta \boldsymbol{F}_0 \frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+1})}{\partial \boldsymbol{\kappa}_{t+1}}, \tag{5}$$

$$\tilde{\boldsymbol{u}} + \tilde{\boldsymbol{e}}^{\boldsymbol{P}_t} + \beta \tilde{\boldsymbol{F}} \left(\boldsymbol{u}_0 + \boldsymbol{e}_0^{\boldsymbol{P}_t} + \beta \boldsymbol{F}_0 \frac{\partial \boldsymbol{W}^*(\boldsymbol{\kappa}_{t+1})}{\partial \boldsymbol{\kappa}_{t+1}} \right) = 0. \tag{6}$$

- $\boldsymbol{U}_0^{\boldsymbol{P}_t} = \boldsymbol{u}_0 + \boldsymbol{e}_0^{\boldsymbol{P}_t}$,
- $\boldsymbol{u}_0 = [u(0, x^{(1)}), \ldots, u(0, x^{(|\mathcal{X}|)})]^\top$.
- $\boldsymbol{e}_0^{\boldsymbol{P}_t} = [\gamma - \log(P_t(0, x^{(1)})), \ldots, \gamma - \log(P_t(0, x^{(|\mathcal{X}|)}))]^\top$.

Approach: Envelop Theorem

$$\bar{\boldsymbol{V}}_t = \boldsymbol{U}_0^{\boldsymbol{P}_t} + \beta \boldsymbol{F}_0 \bar{\boldsymbol{V}}_{t+1}, \tag{7}$$

$$\tilde{\boldsymbol{u}} + \tilde{\boldsymbol{e}}^{\boldsymbol{P}_t} + \beta \tilde{\boldsymbol{F}} \left(\boldsymbol{u}_0 + \boldsymbol{e}_0^{\boldsymbol{P}_t} + \beta \boldsymbol{F}_0 \bar{\boldsymbol{V}}_{t+1} \right) = 0. \tag{8}$$

- $\boldsymbol{U}_0^{\boldsymbol{P}_t} = \boldsymbol{u}_0 + \boldsymbol{e}_0^{\boldsymbol{P}_t}$,
- $\boldsymbol{u}_0 = [u(0, x^{(1)}), \ldots, u(0, x^{(|\mathcal{X}|)})]^\top$.
- $\boldsymbol{e}_0^{\boldsymbol{P}_t} = [\gamma - \log(P_t(0, x^{(1)})), \ldots, \gamma - \log(P_t(0, x^{(|\mathcal{X}|)}))]^\top$.

Likelihood Function(EE)

Proposition 1

In a stationary model,

$$\bar{\boldsymbol{V}}_t = (I - \beta \boldsymbol{F}_0)^{-1} \left(\boldsymbol{u}_0 + \boldsymbol{e}_0^{\boldsymbol{P}_t} \right).$$

The logit likelihood function from equation (6):

$$l(d_t, x_t; \theta) = \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))},$$

$$v(d, x_t; \theta) = u(d, x_t; \theta) + \beta \tilde{\boldsymbol{f}}(d_t, x_t) \bar{\boldsymbol{V}}_{t+1}.$$

Likelihood Function(FD)

Proposition 2 (Finite Dependence)

If the model display the finite dependence property, there exists an arbitrary action $d^\dagger$ such that $\tilde{\boldsymbol{F}}\boldsymbol{F}_{d^\dagger} = \boldsymbol{0}$.

The logit likelihood function for finite dependence:

$$l(d_t, x_t; \theta) = \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))},$$

$$v(d, x_t; \theta) = u(d, x_t; \theta) + \beta \tilde{\boldsymbol{f}}(d_t, x_t) \left(\boldsymbol{u}_{d^\dagger_{t+1}, t+1} + \boldsymbol{e}^{\boldsymbol{P}_{t+1}}_{d^\dagger_{t+1}, t+1} \right)$$

Likelihood Function(AFD)

Proposition 3 (Almost Finite Dependent Estimator)

If the model does not exhibit finite dependence, we can find $d^{\dagger}_{t+1}$ to minimize the norm of $|\tilde{\boldsymbol{F}}\boldsymbol{F}_{d^{\dagger}_{t+1}}|$.

$$
\begin{aligned}
l(d_t, x_t; \theta) &= \frac{\exp(v(d_t, x_t; \theta))}{\sum_{d \in \mathcal{D}} \exp(v(x_t, d; \theta))}, \\
v(d, x_t; \theta) &= u(d, x_t; \theta) + \beta \tilde{\boldsymbol{f}}(d_t, x_t)\Big(\boldsymbol{u}_{d^{\dagger}_{t+1}, t+1} + \boldsymbol{e}^{\boldsymbol{P}_{t+1}}_{d^{\dagger}_{t+1}, t+1} \\
&\qquad + \boldsymbol{F}_{d^{\dagger}_{t+1}, t+1} \bar{\boldsymbol{V}}_{t+1}.\Big)
\end{aligned}
$$

Estimator

EM Algorithm

- M types of agent, $\theta = (\theta^1, \ldots, \theta^M)$.
- π^m denote the probability of being type m.
- $l(d_i, z_i; \theta_m)$ is the likelihood function.

$$\{\hat{\theta}, \hat{\pi}\} = \arg\max_{\theta,\pi} = \sum_{n=1}^{N} \log\left\{\sum_{m=1}^{M} \pi^m l(d_i, z_i, \theta^m)\right\}, \tag{9}$$

EM algorithm in dynamic discrete choice

The posterior:

$$\hat{q}_{im} = \frac{\hat{\pi}^m l(d_i, z_i, \hat{P}^m, \hat{V}^m, \hat{\theta}^m)}{\sum_{m' \in \mathcal{M}} \hat{\pi}^{m'} l(d_i, z_i, \hat{P}^{m'}, \hat{V}^{m'}, \hat{\theta}^{m'})}. \tag{10}$$

- where $\hat{\boldsymbol{P}} = (\hat{\boldsymbol{P}}^1, \dots, \hat{\boldsymbol{P}}^M)$ are unbiased estimators for CCPs,
- $\hat{\boldsymbol{V}} = (\hat{\boldsymbol{V}}^1, \dots, \hat{\boldsymbol{V}}^M)$ is an estimator of the value function,
- $\hat{\boldsymbol{\pi}} = (\hat{\pi}^1, \dots, \hat{\pi}^M)^\top$ is an estimator of mixing probability,
- $\hat{q}_{im}$, the probability n is type m.

Modified EM Algorithm

Given the estimators from last round $\{\hat{\boldsymbol{P}}^{(k-1)}, \hat{\boldsymbol{V}}^{(k-1)}, \hat{\boldsymbol{\pi}}^{(k-1)}\}$.
Step 1: Compute $\hat{q}_{im}^{(k)}$ as

$$\hat{q}_{im}^{(k)} = \frac{\hat{\pi}^{m,(k-1)} l(d_i, z_i, \hat{\boldsymbol{P}}^{m,(k-1)}, \hat{\boldsymbol{V}}^{m,(k-1)}, \hat{\theta}^{m,(k-1)})}{\sum_{m' \in \mathcal{M}} \hat{\pi}^{m',(k-1)} l(d_i, z_i, \hat{\boldsymbol{P}}^{m,(k-1)}, \hat{\boldsymbol{V}}^{m,(k-1)}, \hat{\theta}^{m',(k-1)})}.$$

Step 2: Using $\hat{q}_{im}^{(k)}$ to compute $\hat{\pi} m, (k)$: $\hat{\pi}^{m,(k)} = \frac{1}{N} \sum_{i=1}^{N} \hat{q}_{im}^{(k)}$.
Step 3: Update estimator of θ with the equation

$$\hat{\theta}_k = \arg\max_{\theta} \sum_{i=1}^{N} \hat{q}_{im}^{(k)} \log l(d_{it}, x_{it}, s, \hat{\boldsymbol{P}}^{m,(k-1)}, \hat{\boldsymbol{V}}^{m,(k-1)}, \theta). \tag{11}$$

Step 4: Update the CCPs $\hat{\boldsymbol{P}}^{(k)}$, and the value function $\hat{\boldsymbol{V}}^{(k)}$.

Likelihood Function

$$l(d_t, x_t; \boldsymbol{P}, \boldsymbol{V}, \theta) = \frac{\exp(\tilde{v}(d_t, x_t))}{1 + \sum_{d \in \mathcal{D}/\{0\}} \exp(\tilde{v}(d, x_t))}$$

Table: Likelihood function comparison

Method	diff in continuation value ($\tilde{v}(d, x)$
NFXP, HM, EE, SEQ(q)	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} \tilde{\boldsymbol{f}}(x_t, d_t) \boldsymbol{V}$
FD2	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} \tilde{\boldsymbol{f}}(x_t, d_t) \left(u_0 + e_0^{\boldsymbol{P}} + \gamma - \log(p_0) + \mathbf{F}_0 \boldsymbol{V} \right)$
AFD2	$\tilde{u}(x_t, d_t; \theta) + \beta \sum_{x_{t+1} \in \mathcal{X}} \tilde{\boldsymbol{f}}(x_t, d_t) \Big(\sum_d \omega(d) (u_d + e_d^{\boldsymbol{P}} \gamma - \log(p_d) + \mathbf{F}_d \boldsymbol{V}) \Big)$

Value function

Table: Comparisons between value function computation

Method	Contraction Mapping
NFXP	$\boldsymbol{V}(x_t) = E_\epsilon \left\{ \max_{d\in\mathcal{D}}[u_d(x_t) + \epsilon_d + \beta\tilde{\boldsymbol{f}}(x_t, d_t)\boldsymbol{V}] \right\}$ till convergence
SEQ(q)	$\boldsymbol{V}(x_t) = E_\epsilon \left\{ \max_{d\in\mathcal{D}}[u_d(x_t) + \epsilon_d + \beta\tilde{\boldsymbol{f}}(x_t, d_t)\boldsymbol{V}] \right\}$ for q times
Hotz-Miller	$\boldsymbol{V} = (I - \beta F^P)^{-1}(\sum_p p(d)(u_d + e_d^P))$
EE	$\boldsymbol{V} = (I - \beta F_0)^{-1}(u_0 + e_0^P)$
FD2	$\boldsymbol{V} = u_0 + e_0^P + \beta\boldsymbol{F}_0\boldsymbol{V}$
AFD2	$\boldsymbol{V} = \sum_d \omega(d)\left(u_d + e_d^P + \boldsymbol{F}_d\boldsymbol{V}\right)$

Simulation

Data generating process: Homogeneous agent model

Table: Parameters in DGP

Flow-Payoff Parameters	$\theta_0^{VP} = 0.5$ $\theta_1^{VP} = 1.0$ $\theta_2^{VP} = -1.0$ $\theta_0^{FC} = 0.5$ $\theta_1^{FC} = 1.0$ $\theta_0^{EC} = 1.0$ $\theta_1^{EC} = 1.0$
State Variable Transition	z_{kt} is AR(1), $\gamma_0^k = 0$, $\gamma_1^k = 0.6$
Productivity Transition	ω_t is AR(1), $\gamma_0^\omega = 0$, $\gamma_1^\omega = 0.9$
Past action on productivity	$\gamma_a \in [0, 5]$
Discount Factor	$\beta = 0.95$

Finite Dependent Model

Table: Two-step: Finite dependent models

	FD	*FD2*	*AFD*	*AFD2*	*HM*	*EE*
$Market = 200, Time = 20, \gamma_a = 0$						
θ_0^{VP}	0.4845	0.4845	0.4845	0.4845	0.5016	0.4845
	(0.0706)	(0.0706)	(0.0706)	(0.0706)	(0.0350)	(0.0706)
θ_0^{FC}	0.5447	0.5447	0.5447	0.5447	0.5098	0.5447
	(0.0904)	(0.0904)	(0.0904)	(0.0904)	(0.0627)	(0.0904)
$Market = 200, Time = 120, \gamma_a = 0$						
θ_0^{VP}	0.4963	0.4963	0.4963	0.4963	0.4983	0.4963
	(0.0189)	(0.0189)	(0.0189)	(0.0189)	(0.0140)	(0.0189)
θ_0^{FC}	0.4990	0.4990	0.4990	0.4990	0.4954	0.4990
	(0.0301)	(0.0301)	(0.0301)	(0.0301)	(0.0279)	(0.0301)

DGP: $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$.

Two-step: Non-finite dependent models

Table: Non-finite Dependent two-step estimators

	FD	*FD2*	*AFD*	*AFD2*	*HM*	*EE*
	$Market = 200$, $Time = 20, \gamma_a = 5$					
θ_0^{VP}	0.3434	0.5679	0.4925	0.5067	0.5307	0.5691
	(0.0790)	(0.1457)	(0.0860)	(0.0908)	(0.0800)	(0.1460)
θ_0^{FC}	-0.0155	0.7095	0.4432	0.4751	0.5833	0.7134
	(0.2228)	(0.3321)	(0.2402)	(0.2518)	(0.2209)	(0.3330)
	$Market = 200$, $Time = 120, \gamma_a = 5$					
θ_0^{VP}	0.3058	0.4965	0.4829	0.4954	0.4982	0.4975
	(0.0333)	(0.0484)	(0.0436)	(0.0453)	(0.0395)	(0.0485)
θ_0^{FC}	-0.1239	0.4920	0.4583	0.4860	0.4977	0.4953
	(0.0845)	(0.1237)	(0.1096)	(0.1140)	(0.1036)	(0.1239)

DGP: $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$.

Sequential Estimation

Table: The mean and standard deviation of sequential estimators

	$FD2$	$AFD2$	HM	EE	$SEQ(1)$
	$Market = 200$, $Time = 20, \gamma_a = 5$				
θ_0^{VP}	0.5084	0.4940	0.5096	0.5084	0.5043
	(0.0925)	(0.1151)	(0.0938)	(0.0925)	(0.0921)
θ_0^{FC}	0.5167	0.4391	0.5207	0.5167	0.5034
	(0.2506)	(0.3056)	(0.2567)	(0.2506)	(0.2493)

The DGP parameters are: $\theta_0^{VP} = 0.5, \theta_0^{FC} = 0.5$.

Data generating process: Heterogeneous agent model

Table: Parameters in DGP

Flow-Payoff Parameters θ^1	$\theta_0^{VP} = 0$	$\theta_1^{VP} = 1.0$	$\theta_2^{VP} = -1.0$
	$\theta_0^{FC} = 0.5$	$\theta_1^{FC} = 1.0$	
	$\theta_0^{EC} = 1.0$	$\theta_1^{EC} = 1.0$	
Flow-Payoff Parameters θ^2	$\theta_0^{VP} = 1$	$\theta_1^{VP} = 1.0$	$\theta_2^{VP} = -1.0$
	$\theta_0^{FC} = 0.5$	$\theta_1^{FC} = 1.0$	
	$\theta_0^{EC} = 1.0$	$\theta_1^{EC} = 1.0$	
Mixing Probability	$(0.5, 0.5)$		
State Variable Transition	z_{kt} is AR(1), $\gamma_0^k = 0$, $\gamma_1^k = 0.6$		
Productivity Transition	ω_t is AR(1), $\gamma_0^\omega = 0$, $\gamma_1^\omega = 0.9$		
Past action on productivity	$\gamma_a = 2$		
Discount Factor	$\beta = 0.95$		

Time and iteration

Table: Median Time and Iteration when increase state space

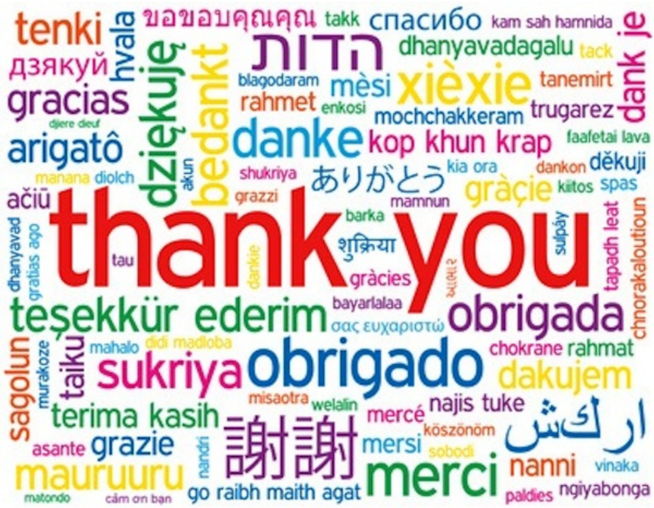
Algorithms	*nGrid*	2	3	4	5	6
	$\lvert\mathcal{X}\rvert$	64	486	2048	6250	15552
	Market			100		
	Time			20		
FD2	*Time*	11.2472	13.9627	27.9147	390.0466	3103.6867
	Iteration	40.5	48.5	37	47.5	32.5
EE	*Time*	12.1462	21.3075	18.6141	181.0266	1039.5331
	Iteration	38.5	69.5	43	80.5	52
HM	*Time*	30.3638	35.6079	982.0085	-	-
	Iteration	91.5	59.5	53	-	-
SEQ(1)	*Time*	6.0499	17.2884	24.1402	100.8548	509.4910
	Iteration	22.5	64.5	55	43.5	35.5

† The results shows the time and iteration used in the estimation based on 12 Monte Carlo simulations of different state space.

Summary of Contributions

1. Reformuation of the Bellman equation of discrete choice by continuous choice and states, to derive the restrictions of the model. Particular useful in non-finite-dependent(NFD) models.
2. Propose an alternative to Arcidiacono Miller algorithm that can be applied to NFD models.
3. Show computation gain to using this estimator in Monte Carlo simulations.

Thank You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