RGE-improved SM Effective Potential

Ref: Sher, Phys.Rept. 179, 273 (1989); Langacker, Phys.Rept. 72, 185 (1981)

The RGE-improved effective potential in the SM

$$V(\phi_c) = -\frac{1}{2}\mu^2(t)G^2(t)\phi_c^2 + \frac{1}{4}\lambda(t)G^4(t)\phi_c^4$$

$t \equiv \ln\frac{\phi_c}{\mu_R}$, ϕ_c is the classical Higgs field, μ_R is a renormalization scale

$$\frac{d\lambda}{dt} = \beta_\lambda(t), \quad \frac{d\mu^2}{dt} = \mu^2(t)\beta_{\mu^2}(t), \quad \frac{dg_i}{dt} = \beta_{g_i}(g_i(t),\lambda(t)), \quad \frac{dy_t}{dt} = \beta_{y_t}(t)$$

$$G(t) \equiv \exp\left[-\int_0^t dt'\,\gamma(t')\right], \quad \beta_\lambda = 4\lambda\gamma + \frac{1}{8\pi^2}(12\lambda^2 + B), \quad B = \frac{3}{16}(3g^4 + 2g^2g'^2 + g'^4) - 3y_t^4$$

$$\beta_{\mu^2} = 2\gamma + \frac{3\lambda}{4\pi^2}, \quad \gamma = \frac{1}{64\pi^2}(-9g^2 - 3g'^2 + 12y_t^2), \quad \beta_{y_t} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 - 8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right)$$

In an SU(N) gauge theory with $n_D(r)$ Dirac fermions, $n_W(r)$ Weyl fermions, and $n_S(r)$ complex scalars in representations r,

$$\beta_g = -\frac{g^3}{16\pi^2}\frac{1}{3}\left\{11N - \sum_r [4n_D(r) + 2n_W(r) + n_S(r)]C(r)\right\}, \quad \mathrm{Tr}(t^at^b) = C(r)\delta^{ab}, \quad C(N) = \frac{1}{2}$$

In a U(1) gauge theory with $n_D(Q)$ Dirac fermions, $n_W(Q)$ Weyl fermions, and $n_S(Q)$ complex scalars with charges Q,

$$\beta_e = \frac{e^3}{16\pi^2}\frac{1}{3}\sum_Q [4n_D(Q) + 2n_W(Q) + n_S(Q)]Q^2$$

$\mathrm{SU(3)_C}$: 6 quarks $\rightarrow$ 6 color triplets

$$\beta_{g_s} = -\frac{g_s^3}{16\pi^2}\frac{1}{3}\left(11\cdot 3 - 4\cdot 6\cdot\frac{1}{2}\right) = -\frac{7g_s^3}{16\pi^2} = b_s g_s^3, \quad b_s = -\frac{7}{16\pi^2}$$

$\mathrm{SU(2)_L}$: 3 fermion generations $\left(\text{each}\begin{cases}\text{1 left-handed lepton doublet}\\ \text{1 left-handed quark doublet with 3 colors}\end{cases}\right)$, 1 Higgs doublet

$$\beta_g = -\frac{g^3}{16\pi^2}\frac{1}{3}\left[11\cdot 2 - 3\cdot 2\cdot(1+3)\cdot\frac{1}{2} - \frac{1}{2}\right] = -\frac{g^3}{16\pi^2}\frac{1}{3}\left(22 - 12 - \frac{1}{2}\right) = -\frac{19g^3}{96\pi^2} = b_2g^3, \quad b_2 = -\frac{19}{96\pi^2}$$

$\mathrm{U(1)_Y}$: $Y_{L_{iL}} = -\frac{1}{2}$, $Y_{\ell_{iR}} = -1$, 3 colors$\left(Y_{Q_{iL}} = \frac{1}{6}, \ Y_{u_{iR}} = \frac{2}{3}, \ Y_{d_{iR}} = -\frac{1}{3}\right)$, $Y_H = \frac{1}{2}$

$$\beta_{g'} = \frac{g'^3}{16\pi^2}\frac{1}{3}\left\{3\cdot 2\cdot\left[2\cdot\left(-\frac{1}{2}\right)^2 + (-1)^2 + 3\cdot 2\cdot\left(\frac{1}{6}\right)^2 + 3\cdot\left(\frac{2}{3}\right)^2 + 3\cdot\left(-\frac{1}{3}\right)^2\right] + 2\cdot\left(\frac{1}{2}\right)^2\right\} = \frac{41g'^3}{96\pi^2} = b'g'^3, \quad b' = \frac{41}{96\pi^2}$$

$\mathrm{U(1)_{EM}}$: $Q_{\ell_i} = -1$, 3 colors$\left(Q_{u_i} = \frac{2}{3}, \ Q_{d_i} = -\frac{1}{3}\right)$, $Q_{\phi^+} = 1$ (charged Goldstone boson)

$$\beta_e = \frac{e^3}{16\pi^2}\frac{1}{3}\left\{3\cdot 4\cdot\left[(-1)^2 + 3\cdot\left(\frac{2}{3}\right)^2 + 3\cdot\left(-\frac{1}{3}\right)^2\right] + 1^2\right\} = \frac{11e^3}{16\pi^2} = be^3, \quad b = \frac{11}{16\pi^2}$$

$$\alpha_s = \frac{g_s^2}{4\pi}, \quad \alpha_2 = \frac{g^2}{4\pi}, \quad \alpha' = \frac{g'}{4\pi}, \quad \alpha = \frac{e^2}{4\pi}, \quad g_1 = \sqrt{\frac{5}{3}}g', \quad \alpha_1 = \frac{5}{3}\alpha'$$

$$\alpha_s^{-1}(Q^2) = \alpha_s^{-1}(\mu_R^2) + 4\pi b_s\ln\frac{\mu_R^2}{Q^2}, \quad \alpha_2^{-1}(Q^2) = \alpha_2^{-1}(\mu_R^2) + 4\pi b_2\ln\frac{\mu_R^2}{Q^2}, \quad \alpha'^{-1}(Q^2) = \alpha'^{-1}(\mu_R^2) + 4\pi b'\ln\frac{\mu_R^2}{Q^2}$$

$$\alpha^{-1}(Q^2) = \alpha^{-1}(\mu_R^2) + 4\pi b\ln\frac{\mu_R^2}{Q^2}, \quad \alpha_1^{-1}(Q^2) = \alpha_1^{-1}(\mu_R^2) + 4\pi b_1\ln\frac{\mu_R^2}{Q^2}, \quad b_1 = \frac{3}{5}b'$$

Boundary conditions: $y_t(\phi_c = 2m_t) = \dfrac{\sqrt{2}m_t}{v}, \quad \left.\dfrac{dV}{d\phi_c}\right|_{\phi_c=v} = 0, \quad \left.\dfrac{d^2V}{d\phi_c^2}\right|_{\phi_c=v} = m_h^2$

$$\frac{dt}{d\phi_c} = \frac{1}{\phi_c}, \quad \frac{d}{d\phi_c} = \frac{1}{\phi_c}\frac{d}{dt}, \quad \frac{d\mu^2}{d\phi_c} = \frac{1}{\phi_c}\beta_{\mu^2}\mu^2, \quad \frac{dG}{d\phi_c} = -\frac{1}{\phi_c}\gamma G, \quad \frac{d\lambda}{d\phi_c} = \frac{1}{\phi_c}\beta_\lambda$$

$$\frac{d}{d\phi_c}(\mu^2 G^2) = \frac{1}{\phi_c}\beta_{\mu^2}\mu^2 G^2 - \frac{1}{\phi_c}2\gamma\mu^2 G^2 = \frac{1}{\phi_c}(\beta_{\mu^2} - 2\gamma)\mu^2 G^2$$

$$\frac{d}{d\phi_c}\left(\frac{1}{2}\mu^2 G^2 \phi_c^2\right) = \mu^2 G^2 \phi_c + \frac{1}{2}(\beta_{\mu^2} - 2\gamma)\mu^2 G^2 \phi_c = \left(1 - \gamma + \frac{\beta_{\mu^2}}{2}\right)\mu^2 G^2 \phi_c$$

$$\frac{d}{d\phi_c}\left(\frac{1}{4}\lambda G^4 \phi_c^4\right) = \lambda G^4 \phi_c^3 + \frac{1}{4}\beta_\lambda G^4 \phi_c^3 - \gamma\lambda G^4 \phi_c^3 = \left[(1-\gamma)\lambda + \frac{\beta_\lambda}{4}\right] G^4 \phi_c^3$$

$$\frac{dV}{d\phi_c} = -\left(1 + \frac{\beta_{\mu^2}}{2} - \gamma\right)\mu^2 G^2 \phi_c + \left[(1-\gamma)\lambda + \frac{\beta_\lambda}{4}\right] G^4 \phi_c^3 = \phi_c(-A_1\mu^2 G^2 + A_2 G^4 \phi_c^2)$$

$$A_1 \equiv 1 + \frac{1}{2}\beta_{\mu^2} - \gamma, \quad A_2 \equiv (1-\gamma)\lambda + \frac{1}{4}\beta_\lambda,$$

$$t = \ln\frac{\phi_c}{v}, \quad G(0) = 1 \qquad [\phi_c = v \quad \Rightarrow \quad t = 0]$$

$$0 = \left.\frac{dV}{d\phi_c}\right|_{\phi_c=v} = -A_1(0)\mu^2(0) + A_2(0)v^2 \quad \Rightarrow \quad \underline{\mu^2(0) = \frac{A_2(0)}{A_1(0)}v^2}$$

$$\frac{d\beta_{\mu^2}}{dt} = 2\frac{d\gamma}{dt} + \frac{3}{4\pi^2}\beta_\lambda, \quad \frac{1}{2}\frac{d\beta_{\mu^2}}{dt} - \frac{d\gamma}{dt} = \frac{3}{8\pi^2}\beta_\lambda$$

$$\frac{d}{d\phi_c}\left[-\left(1 + \frac{\beta_{\mu^2}}{2} - \gamma\right)\mu^2 G^2 \phi_c\right] = \left(1 + \frac{\beta_{\mu^2}}{2} - \gamma\right)\mu^2 G^2 + \left(1 + \frac{\beta_{\mu^2}}{2} - \gamma\right)(\beta_{\mu^2} - 2\gamma)\mu^2 G^2 + \left(\frac{1}{2}\frac{d\beta_{\mu^2}}{dt} - \frac{d\gamma}{dt}\right)\mu^2 G^2$$

$$= -\left[1 + \frac{3}{2}\beta_{\mu^2} - 3\gamma + \frac{1}{2}(\beta_{\mu^2} - 2\gamma)^2 + \frac{3}{8\pi^2}\beta_\lambda\right]\mu^2 G^2$$

$$\frac{d\beta_\lambda}{dt} = 4\beta_\lambda\gamma + 4\lambda\frac{d\gamma}{dt} + \frac{3}{\pi^2}\lambda\beta_\lambda + \frac{1}{8\pi^2}\frac{dB}{dt}$$

$$\frac{d}{dt}\left[(1-\gamma)\lambda + \frac{\beta_\lambda}{4}\right] = -\lambda\frac{d\gamma}{dt} + (1-\gamma)\beta_\lambda + \frac{1}{4}\frac{d\beta_\lambda}{dt} = \beta_\lambda + \frac{3}{4\pi^2}\lambda\beta_\lambda + \frac{1}{32\pi^2}\frac{dB}{dt}$$

$$\frac{d}{d\phi_c}\left\{\left[(1-\gamma)\lambda + \frac{\beta_\lambda}{4}\right]G^4\phi_c^3\right\} = 3\left[(1-\gamma)\lambda + \frac{\beta_\lambda}{4}\right]G^4\phi_c^2 - 4\gamma\left[(1-\gamma)\lambda + \frac{\beta_\lambda}{4}\right]G^4\phi_c^2 + \left[\beta_\lambda + \frac{3}{4\pi^2}\lambda\beta_\lambda + \frac{1}{32\pi^2}\frac{dB}{dt}\right]G^4\phi_c^2$$

$$= \left[(3-4\gamma)(1-\gamma)\lambda + \left(\frac{7}{4} - \gamma\right)\beta_\lambda + \frac{3}{4\pi^2}\lambda\beta_\lambda + \frac{1}{32\pi^2}\frac{dB}{dt}\right]G^4\phi_c^2$$

$$\frac{d^2V}{d\phi_c^2} = -A_3\mu^2 G^2 + A_4 G^4 \phi_c^2$$

$$A_3 \equiv 1 + \frac{3}{2}\beta_{\mu^2} - 3\gamma + \frac{1}{2}(\beta_{\mu^2} - 2\gamma)^2 + \frac{3}{8\pi^2}\beta_\lambda, \quad A_4 \equiv (3-4\gamma)(1-\gamma)\lambda + \left(\frac{7}{4} - \gamma\right)\beta_\lambda + \frac{3}{4\pi^2}\lambda\beta_\lambda + \frac{1}{32\pi^2}\frac{dB}{dt}$$

$$\frac{dB}{dt} = \frac{3}{4}[(3g^3 + gg'^2)\beta_g + (g^2 g' + g'^3)\beta_{g'}] - 12 y_t^3 \beta_{y_t}$$

$$m_h^2 = \left.\frac{d^2V}{d\phi_c^2}\right|_{\phi_c=v} = -A_3(0)\mu^2(0) + A_4(0)v^2 = \left[-A_3(0)\frac{A_2(0)}{A_1(0)} + A_4(0)\right]v^2$$

$$\Rightarrow \quad \underline{-\frac{A_2(0)}{A_1(0)}A_3(0) + A_4(0) = \frac{m_h^2}{v^2} \text{ determines } \lambda(0)}$$

1-Loop Coleman-Weinberg Effective Potential

Ref: Sher, Phys.Rept. 179, 273 (1989); Quiros, hep-ph/9901312; Degrassi et al., 1205.6497

Sher's expression for the standard model

$$V(\phi_c) = V_0 + V_V + V_F + \mathrm{Re}(V_S)$$

$$V_0 = -\frac{1}{2}\mu^2\phi_c^2 + \frac{1}{4}\lambda\phi_c^4, \quad V_V = \frac{3[2g^4 + (g^2 + g'^2)^2]}{1024\pi^2}\phi_c^4 \ln\frac{\phi_c^2}{\mu_R^2}, \quad V_F = -\frac{3y_t^4}{64\pi^2}\phi_c^4 \ln\frac{\phi_c^2}{\mu_R^2}$$

$$V_S = \frac{1}{64\pi^2}(-\mu^2 + 3\lambda\phi_c^2)^2 \ln\frac{-\mu^2 + 3\lambda\phi_c^2}{\mu_R^2} + \frac{3}{64\pi^2}(-\mu^2 + \lambda\phi_c^2)^2 \ln\frac{-\mu^2 + \lambda\phi_c^2}{\mu_R^2}$$

$$V_V + V_F = \frac{B}{64\pi^2}\phi_c^4 \ln\frac{\phi_c^2}{\mu_R^2}, \quad B \equiv \frac{3}{16}(3g^4 + 2g^2g'^2 + g'^4) - 3y_t^4$$

Adopt $\mu_R = v$ and boundary conditions $\left.\frac{dV}{d\phi_c}\right|_{\phi_c = v} = 0$ and $\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi_c = v} = m_h^2$

$$\frac{d}{d\phi_c}\ln\frac{\phi_c^2}{v^2} = \frac{v^2}{\phi_c^2}\frac{2\phi_c}{v^2} = \frac{2}{\phi_c}, \quad \frac{d}{d\phi_c}\ln\frac{-\mu^2 + 3\lambda\phi_c^2}{v^2} = \frac{v^2}{-\mu^2 + 3\lambda\phi_c^2}\frac{6\lambda\phi_c}{v^2} = \frac{6\lambda\phi_c}{-\mu^2 + 3\lambda\phi_c^2}$$

$$\begin{aligned}
\frac{dV}{d\phi_c} &= -\mu^2\phi_c + \lambda\phi_c^3 + \frac{B}{16\pi^2}\phi_c^3 \ln\frac{\phi_c^2}{v^2} + \frac{B}{32\pi^2}\phi_c^3 + \frac{3\lambda}{16\pi^2}(-\mu^2 + 3\lambda\phi_c^2)\phi_c \ln\frac{-\mu^2 + 3\lambda\phi_c^2}{v^2} + \frac{3\lambda}{32\pi^2}(-\mu^2 + 3\lambda\phi_c^2)\phi_c \\
&\quad + \frac{3\lambda}{16\pi^2}(-\mu^2 + \lambda\phi_c^2)\phi_c \ln\frac{-\mu^2 + \lambda\phi_c^2}{v^2} + \frac{3\lambda}{32\pi^2}(-\mu^2 + \lambda\phi_c^2)\phi_c \\
&= F_1(\phi_c)\phi_c
\end{aligned}$$

$$F_1(\phi_c) \equiv -\mu^2 + \lambda\phi_c^2 + \frac{B}{16\pi^2}\phi_c^2\left(\ln\frac{\phi_c^2}{v^2} + \frac{1}{2}\right) + \frac{3\lambda}{16\pi^2}\left[(-\mu^2 + 3\lambda\phi_c^2)\left(\ln\frac{-\mu^2 + 3\lambda\phi_c^2}{v^2} + \frac{1}{2}\right) + (-\mu^2 + \lambda\phi_c^2)\left(\ln\frac{-\mu^2 + \lambda\phi_c^2}{v^2} + \frac{1}{2}\right)\right]$$

$$\left.\frac{dV}{d\phi_c}\right|_{\phi_c = v} = 0 \quad \Rightarrow \quad F_1(v) = 0$$

$$\Rightarrow \quad \underline{\mu^2 = \lambda v^2 + \frac{B}{32\pi^2}v^2 + \frac{3\lambda}{16\pi^2}\left[(-\mu^2 + 3\lambda v^2)\left(\ln\frac{-\mu^2 + 3\lambda v^2}{v^2} + \frac{1}{2}\right) + (-\mu^2 + \lambda v^2)\left(\ln\frac{-\mu^2 + \lambda v^2}{v^2} + \frac{1}{2}\right)\right]}$$

$$\frac{d^2V}{d\phi_c^2} = F_1(\phi_c) + \phi_c F_1'(\phi_c)$$

$$\begin{aligned}
F_1'(\phi_c) &= 2\lambda\phi_c + \frac{B}{8\pi^2}\phi_c\left(\ln\frac{\phi_c^2}{v^2} + \frac{1}{2}\right) + \frac{B}{8\pi^2}\phi_c + \frac{3\lambda}{16\pi^2}\left[6\lambda\phi_c\left(\ln\frac{-\mu^2 + 3\lambda\phi_c^2}{v^2} + \frac{1}{2}\right) + 6\lambda\phi_c + 2\lambda\phi_c\left(\ln\frac{-\mu^2 + \lambda\phi_c^2}{v^2} + \frac{1}{2}\right) + 2\lambda\phi_c\right] \\
&= \phi_c\left[2\lambda + \frac{B}{8\pi^2}\left(\ln\frac{\phi_c^2}{v^2} + \frac{3}{2}\right) + \frac{3\lambda^2}{8\pi^2}\left(3\ln\frac{-\mu^2 + 3\lambda\phi_c^2}{v^2} + \ln\frac{-\mu^2 + \lambda\phi_c^2}{v^2} + 6\right)\right]
\end{aligned}$$

$$\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi_c = v} = m_h^2 \quad \Rightarrow \quad m_h^2 = F_1(v) + vF_1'(v) = vF_1'(v)$$

$$\Rightarrow \quad \underline{\frac{m_h^2}{v^2} = 2\lambda + \frac{3B}{16\pi^2} + \frac{3\lambda^2}{8\pi^2}\left(3\ln\frac{-\mu^2 + 3\lambda v^2}{v^2} + \ln\frac{-\mu^2 + \lambda v^2}{v^2} + 6\right)}$$

$\overline{\text{MS}}$ scheme result for the standard model

$$V(\phi_c) = V_0 + V_V + V_F + \mathrm{Re}(V_S)$$

$$V_0 = -\frac{1}{2}\mu^2\phi_c^2 + \frac{1}{4}\lambda\phi_c^4, \quad V_V = \frac{6}{64\pi^2}\overline{m}_W^4(\phi_c)\left[\ln\frac{\overline{m}_W^2(\phi_c)}{\mu_R^2} - \frac{5}{6}\right] + \frac{3}{64\pi^2}\overline{m}_Z^4(\phi_c)\left[\ln\frac{\overline{m}_Z^2(\phi_c)}{\mu_R^2} - \frac{5}{6}\right]$$

$$V_F = -\frac{12}{64\pi^2}\overline{m}_t^4(\phi_c)\left[\ln\frac{\overline{m}_t^2(\phi_c)}{\mu_R^2} - \frac{3}{2}\right], \quad V_S = \frac{1}{64\pi^2}\overline{m}_h^4(\phi_c)\left[\ln\frac{\overline{m}_h^2(\phi_c)}{\mu_R^2} - \frac{3}{2}\right] + \frac{3}{64\pi^2}\overline{m}_G^4(\phi_c)\left[\ln\frac{\overline{m}_G^2(\phi_c)}{\mu_R^2} - \frac{3}{2}\right]$$

$$\overline{m}_W^2(\phi_c) = \frac{1}{4}g^2\phi_c^2, \quad \overline{m}_Z^2(\phi_c) = \frac{1}{4}(g^2 + g'^2)\phi_c^2, \quad \overline{m}_t^2(\phi_c) = \frac{1}{2}y_t^2\phi_c^2, \quad \overline{m}_h^2(\phi_c) = -\mu^2 + 3\lambda\phi_c^2, \quad \overline{m}_G^2(\phi_c) = -\mu^2 + \lambda\phi_c^2$$

Adopt $\mu_R = v$ and boundary conditions $\left.\frac{dV}{d\phi_c}\right|_{\phi_c=v} = 0$ and $\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi_c=v} = m_h^2$ to determine μ^2 and λ

$$\frac{d\overline{m}_W^4}{d\phi_c} = \left(\frac{1}{4}g^2\right)^2 4\phi_c^3 = \frac{4\overline{m}_W^4}{\phi_c}, \quad \frac{d\overline{m}_h^4}{d\phi_c} = 2\overline{m}_h^2\cdot 6\lambda\phi_c = 12\lambda\overline{m}_h^2\phi_c, \quad \frac{d\overline{m}_G^4}{d\phi_c} = 2\overline{m}_G^2\cdot 2\lambda\phi_c = 4\lambda\overline{m}_G^2\phi_c$$

$$\frac{d}{d\phi_c}\ln\frac{\overline{m}_W^2}{v^2} = \frac{v^2}{\overline{m}_W^2}\frac{g^2}{4v^2}2\phi_c = \frac{2}{\phi_c}, \quad \frac{d}{d\phi_c}\ln\frac{\overline{m}_h^2}{v^2} = \frac{v^2}{\overline{m}_h^2}\frac{6\lambda\phi_c}{v^2} = \frac{6\lambda\phi_c}{\overline{m}_h^2}, \quad \frac{d}{d\phi_c}\ln\frac{\overline{m}_G^2}{v^2} = \frac{2\lambda\phi_c}{\overline{m}_G^2}$$

$$\frac{dV_0}{d\phi_c} = -\mu^2\phi_c + \lambda\phi_c^3$$

$$\frac{dV_V}{d\phi_c} = \frac{6}{64\pi^2}\frac{4\overline{m}_W^4}{\phi_c}\left(\ln\frac{\overline{m}_W^2}{v^2} - \frac{5}{6}\right) + \frac{6}{64\pi^2}\overline{m}_W^4\frac{2}{\phi_c} + \frac{3}{64\pi^2}\frac{4\overline{m}_Z^4}{\phi_c}\left(\ln\frac{\overline{m}_Z^2}{v^2} - \frac{5}{6}\right) + \frac{3}{64\pi^2}\overline{m}_Z^4\frac{2}{\phi_c}$$

$$= \frac{3}{8\pi^2}\frac{\overline{m}_W^4}{\phi_c}\left(\ln\frac{\overline{m}_W^2}{v^2} - \frac{1}{3}\right) + \frac{3}{16\pi^2}\frac{\overline{m}_Z^4}{\phi_c}\left(\ln\frac{\overline{m}_Z^2}{v^2} - \frac{1}{3}\right)$$

$$\frac{dV_F}{d\phi_c} = -\frac{12}{64\pi^2}\frac{4\overline{m}_t^4}{\phi_c}\left(\ln\frac{\overline{m}_t^2}{v^2} - \frac{3}{2}\right) - \frac{12}{64\pi^2}\overline{m}_t^4\frac{2}{\phi_c} = -\frac{3}{4\pi^2}\frac{\overline{m}_t^4}{\phi_c}\left(\ln\frac{\overline{m}_t^2}{v^2} - 1\right)$$

$$\frac{dV_S}{d\phi_c} = \frac{1}{64\pi^2}12\lambda\overline{m}_h^2\phi_c\left(\ln\frac{\overline{m}_h^2}{v^2} - \frac{3}{2}\right) + \frac{1}{64\pi^2}\overline{m}_h^4\frac{6\lambda\phi_c}{\overline{m}_h^2} + \frac{3}{64\pi^2}4\lambda\overline{m}_G^2\phi_c\left(\ln\frac{\overline{m}_G^2}{v^2} - \frac{3}{2}\right) + \frac{3}{64\pi^2}\overline{m}_G^4\frac{2\lambda\phi_c}{\overline{m}_G^2}$$

$$= \frac{3\lambda\phi_c}{16\pi^2}\left[\overline{m}_h^2\left(\ln\frac{\overline{m}_h^2}{v^2} - 1\right) + \overline{m}_G^2\left(\ln\frac{\overline{m}_G^2}{v^2} - 1\right)\right]$$

$$\frac{1}{\phi_c}\frac{dV}{d\phi_c} = -\mu^2 + \lambda\phi_c^2 + \frac{3g^4}{128\pi^2}\phi_c^2\left(\ln\frac{g^2\phi_c^2}{4v^2} - \frac{1}{3}\right) + \frac{3(g^2+g'^2)^2}{256\pi^2}\phi_c^2\left[\ln\frac{(g^2+g'^2)\phi_c^2}{4v^2} - \frac{1}{3}\right] - \frac{3y_t^4}{16\pi^2}\phi_c^2\left(\ln\frac{y_t^2\phi_c^2}{2v^2} - 1\right)$$

$$+ \frac{3\lambda}{16\pi^2}\left[(-\mu^2 + 3\lambda\phi_c^2)\left(\ln\frac{-\mu^2 + 3\lambda\phi_c^2}{v^2} - 1\right) + (-\mu^2 + \lambda\phi_c^2)\left(\ln\frac{-\mu^2 + \lambda\phi_c^2}{v^2} - 1\right)\right]$$

$$\left.\frac{dV}{d\phi_c}\right|_{\phi_c=v} = 0 \quad \Rightarrow \quad \left.\left(\frac{1}{\phi_c}\frac{dV}{d\phi_c}\right)\right|_{\phi_c=v} = 0$$

$$\Rightarrow \quad \underline{\mu^2 = v^2\left\{\lambda + \frac{3g^4}{128\pi^2}\left(\ln\frac{g^2}{4} - \frac{1}{3}\right) + \frac{3(g^2+g'^2)^2}{256\pi^2}\left[\ln\frac{(g^2+g'^2)}{4} - \frac{1}{3}\right] - \frac{3y_t^4}{16\pi^2}\left(\ln\frac{y_t^2}{2} - 1\right)\right\}}$$

$$\underline{+ \frac{3\lambda}{16\pi^2}\left[(-\mu^2 + 3\lambda v^2)\left(\ln\frac{-\mu^2 + 3\lambda v^2}{v^2} - 1\right) + (-\mu^2 + \lambda v^2)\left(\ln\frac{-\mu^2 + \lambda v^2}{v^2} - 1\right)\right]}$$

$$\frac{d}{d\phi_c}\left[(-\mu^2+3\lambda\phi_c^2)\left(\ln\frac{-\mu^2+3\lambda\phi_c^2}{v^2}-1\right)\right]$$

$$=6\lambda\phi_c\left(\ln\frac{-\mu^2+3\lambda\phi_c^2}{v^2}-1\right)+(-\mu^2+3\lambda\phi_c^2)\frac{6\lambda\phi_c}{-\mu^2+3\lambda\phi_c^2}=6\lambda\phi_c\ln\frac{-\mu^2+3\lambda\phi_c^2}{v^2}$$

$$\frac{d}{d\phi_c}\left[(-\mu^2+\lambda\phi_c^2)\left(\ln\frac{-\mu^2+\lambda\phi_c^2}{v^2}-1\right)\right]$$

$$=2\lambda\phi_c\left(\ln\frac{-\mu^2+\lambda\phi_c^2}{v^2}-1\right)+(-\mu^2+\lambda\phi_c^2)\frac{2\lambda\phi_c}{-\mu^2+\lambda\phi_c^2}=2\lambda\phi_c\ln\frac{-\mu^2+\lambda\phi_c^2}{v^2}$$

$$\begin{aligned}\frac{d}{d\phi_c}\left(\frac{1}{\phi_c}\frac{dV}{d\phi_c}\right)&=2\lambda\phi_c+\frac{3g^4}{128\pi^2}\left[2\phi_c\left(\ln\frac{g^2\phi_c^2}{4v^2}-\frac{1}{3}\right)+\phi_c^2\frac{2}{\phi_c}\right]\\&\quad+\frac{3(g^2+g'^2)^2}{256\pi^2}\left\{2\phi_c\left[\ln\frac{(g^2+g'^2)\phi_c^2}{4v^2}-\frac{1}{3}\right]+\phi_c^2\frac{2}{\phi_c}\right\}\\&\quad-\frac{3y_t^4}{16\pi^2}\left[2\phi_c\left(\ln\frac{y_t^2\phi_c^2}{2v^2}-1\right)+\phi_c^2\frac{2}{\phi_c}\right]\\&\quad+\frac{3\lambda}{16\pi^2}\left(6\lambda\phi_c\ln\frac{-\mu^2+3\lambda\phi_c^2}{v^2}+2\lambda\phi_c\ln\frac{-\mu^2+\lambda\phi_c^2}{v^2}\right)\\&=2\lambda\phi_c+\frac{3g^4}{64\pi^2}\phi_c\left(\ln\frac{g^2\phi_c^2}{4v^2}+\frac{2}{3}\right)+\frac{3(g^2+g'^2)^2}{128\pi^2}\phi_c\left[\ln\frac{(g^2+g'^2)\phi_c^2}{4v^2}+\frac{2}{3}\right]\\&\quad-\frac{3y_t^4}{8\pi^2}\phi_c\ln\frac{y_t^2\phi_c^2}{2v^2}+\frac{3\lambda^2}{8\pi^2}\phi_c\left(3\ln\frac{-\mu^2+3\lambda\phi_c^2}{v^2}+\ln\frac{-\mu^2+\lambda\phi_c^2}{v^2}\right)\end{aligned}$$

$$\frac{d^2V}{d\phi_c^2}=\frac{1}{\phi_c}\frac{dV}{d\phi_c}+\phi_c\frac{d}{d\phi_c}\left(\frac{1}{\phi_c}\frac{dV}{d\phi_c}\right)$$

$$\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi_c=v}=m_h^2,\quad\left.\left(\frac{1}{\phi_c}\frac{dV}{d\phi_c}\right)\right|_{\phi_c=v}=0\quad\Rightarrow\quad\left.\left[\phi_c\frac{d}{d\phi_c}\left(\frac{1}{\phi_c}\frac{dV}{d\phi_c}\right)\right]\right|_{\phi_c=v}=m_h^2$$

$$\begin{aligned}\Rightarrow\quad&\underline{\frac{m_h^2}{v^2}=2\lambda+\frac{3g^4}{64\pi^2}\left(\ln\frac{g^2}{4}+\frac{2}{3}\right)+\frac{3(g^2+g'^2)^2}{128\pi^2}\left(\ln\frac{g^2+g'^2}{4}+\frac{2}{3}\right)}\\&\qquad\underline{-\frac{3y_t^4}{8\pi^2}\ln\frac{y_t^2}{2}+\frac{3\lambda^2}{8\pi^2}\left(3\ln\frac{-\mu^2+3\lambda v^2}{v^2}+\ln\frac{-\mu^2+\lambda v^2}{v^2}\right)}\end{aligned}$$

Finite temperature effects for the standard model

$$V_{\rm T}(\phi_{\rm c},T)=V(\phi_{\rm c})+\frac{T^4}{2\pi^2}\{6J_{\rm B}(\overline{m}_W^2/T^2)+3J_{\rm B}(\overline{m}_Z^2/T^2)+{\rm Re}[J_{\rm B}(\overline{m}_h^2/T^2)]+3{\rm Re}[J_{\rm B}(\overline{m}_G^2/T^2)]+12J_{\rm F}(\overline{m}_t^2/T^2)\}$$

$$J_{\rm B}(r)\equiv\int_0^\infty dx\,x^2\ln\left\{1-\exp\left[-\sqrt{x^2+r}\right]\right\},\quad J_{\rm F}(r)\equiv-\int_0^\infty dx\,x^2\ln\left\{1+\exp\left[-\sqrt{x^2+r}\right]\right\}$$

$$J_{\rm B}(0)=-\frac{\pi^4}{45},\quad J_{\rm F}(0)=-\frac{7\pi^4}{360}$$

Subtract $\phi_{\rm c}$-independent terms to keep $V_{\rm eff}(\phi_{\rm c}=0,T)=0$:

$$V_{\rm eff}(\phi_{\rm c},T)\equiv V_{\rm T}(\phi_{\rm c},T)-{\rm Re}[V_{\rm S}(\phi_{\rm c}=0)]-\frac{T^4}{2\pi^2}\left\{9J_{\rm B}(0)+4{\rm Re}\left[J_{\rm B}\left(\frac{-\mu^2}{T^2}\right)\right]-12J_{\rm F}(0)\right\}$$

For $T^2\gg\overline{m}^2$,

$$J_{\rm B}\left(\frac{\overline{m}^2}{T^2}\right)\simeq-\frac{\pi^4}{45}+\frac{\pi^2}{12}\frac{\overline{m}^2}{T^2}-\frac{\pi}{6}\left(\frac{\overline{m}^2}{T^2}\right)^{3/2}-\frac{1}{32}\frac{\overline{m}^4}{T^4}\ln\frac{\overline{m}^2}{a_{\rm B}T^2}-2\pi^{7/2}\sum_{\ell=1}^{\infty}(-1)^\ell\frac{\zeta(2\ell+1)}{(\ell+1)!}\Gamma\left(\ell+\frac{1}{2}\right)\left(\frac{\overline{m}^2}{4\pi^2T^2}\right)^{\ell+2}$$

$$J_{\rm F}\left(\frac{\overline{m}^2}{T^2}\right)\simeq-\frac{7\pi^4}{360}+\frac{\pi^2}{24}\frac{\overline{m}^2}{T^2}+\frac{1}{32}\frac{\overline{m}^4}{T^4}\ln\frac{\overline{m}^2}{a_{\rm F}T^2}+\frac{\pi^{7/2}}{4}\sum_{\ell=1}^{\infty}(-1)^\ell\frac{\zeta(2\ell+1)}{(\ell+1)!}(1-2^{-2\ell-1})\Gamma\left(\ell+\frac{1}{2}\right)\left(\frac{\overline{m}^2}{\pi^2T^2}\right)^{\ell+2}$$

ζ is the Riemann ζ-function, $a_{\rm B}\equiv16\pi^2e^{3/2-2\gamma_{\rm E}}$, $\ln a_{\rm B}=5.4076$, $a_{\rm F}\equiv\pi^2e^{3/2-2\gamma_{\rm E}}$, $\ln a_{\rm F}=2.6351$

$$\frac{T^4}{2\pi^2}[6J_{\rm B}(\overline{m}_W^2/T^2)+3J_{\rm B}(\overline{m}_Z^2/T^2)]$$

$$\simeq6\frac{T^4}{2\pi^2}\left[J_{\rm B}(0)+\frac{\pi^2}{12}\frac{\overline{m}_W^2}{T^2}-\frac{\pi}{6}\left(\frac{\overline{m}_W^2}{T^2}\right)^{3/2}-\frac{1}{32}\frac{\overline{m}_W^4}{T^4}\ln\frac{\overline{m}_W^2}{a_{\rm B}T^2}\right]+3\frac{T^4}{2\pi^2}\left[J_{\rm B}(0)+\frac{\pi^2}{12}\frac{\overline{m}_Z^2}{T^2}-\frac{\pi}{6}\left(\frac{\overline{m}_Z^2}{T^2}\right)^{3/2}-\frac{1}{32}\frac{\overline{m}_Z^4}{T^4}\ln\frac{\overline{m}_Z^2}{a_{\rm B}T^2}\right]$$

$$\simeq\frac{9J_{\rm B}(0)}{2\pi^2}T^4+\frac{1}{8v^2}(2m_W^2+m_Z^2)T^2\phi_{\rm c}^2-\frac{1}{4\pi v^3}(2m_W^3+m_Z^3)T\phi_{\rm c}^3-\frac{3}{64\pi^2v^4}\phi_{\rm c}^4\left(2m_W^4\ln\frac{m_W^2}{a_{\rm B}T^2}+m_Z^4\ln\frac{m_Z^2}{a_{\rm B}T^2}\right)$$

$$\frac{T^4}{2\pi^2}[12J_{\rm F}(\overline{m}_t^2/T^2)]\simeq12\frac{T^4}{2\pi^2}\left(J_{\rm F}(0)+\frac{\pi^2}{24}\frac{\overline{m}_t^2}{T^2}+\frac{1}{32}\frac{\overline{m}_t^4}{T^4}\ln\frac{\overline{m}_t^2}{a_{\rm F}T^2}\right)\simeq\frac{12J_{\rm F}(0)}{2\pi^2}T^4+\frac{m_t^2}{4v^2}T^2\phi_{\rm c}^2+\frac{3}{16\pi^2v^4}\phi_{\rm c}^4m_t^4\ln\frac{m_t^2}{a_{\rm F}T^2}$$

Neglecting 1-loop zero-temperature corrections and finite temperature effects from scalars,

$$\begin{aligned}V_{\rm eff}(\phi_{\rm c},T)&\simeq-\frac{1}{4}m_h^2\phi_{\rm c}^2+\frac{1}{4}\lambda\phi_{\rm c}^4+\frac{1}{8v^2}(2m_W^2+m_Z^2+2m_t^2)T^2\phi_{\rm c}^2-\frac{1}{4\pi v^3}(2m_W^3+m_Z^3)T\phi_{\rm c}^3\\&\quad-\frac{3}{64\pi^2v^4}\phi_{\rm c}^4\left(2m_W^4\ln\frac{m_W^2}{a_{\rm B}T^2}+m_Z^4\ln\frac{m_Z^2}{a_{\rm B}T^2}-4m_t^2\ln\frac{m_t^2}{a_{\rm F}T^2}\right)\\&=D_2(T^2-T_{\rm o}^2)\phi_{\rm c}^2-D_3T\phi_{\rm c}^3+\frac{1}{4}\lambda(T)\phi_{\rm c}^4\end{aligned}$$

$$T_{\rm o}^2\equiv\frac{m_h^2}{4D_2},\quad D_2\equiv\frac{1}{8v^2}(2m_W^2+m_Z^2+2m_t^2),\quad D_3\equiv\frac{1}{4\pi v^3}(2m_W^3+m_Z^3)$$

$$\lambda(T)\equiv\lambda-\frac{3}{16\pi^2v^4}\left(2m_W^4\ln\frac{m_W^2}{a_{\rm B}T^2}+m_Z^4\ln\frac{m_Z^2}{a_{\rm B}T^2}-4m_t^2\ln\frac{m_t^2}{a_{\rm F}T^2}\right)$$

Analysis for $V_{\text{eff}}(\phi_{\text{c}},T)=D_2(T^2-T_{\text{o}}^2)\phi_{\text{c}}^2-D_3T\phi_{\text{c}}^3+\frac{1}{4}\lambda(T)\phi_{\text{c}}^4$

$$V'_{\text{eff}}(\phi_{\text{c}},T)\equiv\frac{\partial V_{\text{eff}}}{\partial\phi_{\text{c}}}=\phi_{\text{c}}[2D_2(T^2-T_{\text{o}}^2)-3D_3T\phi_{\text{c}}+\lambda(T)\phi_{\text{c}}^2]$$

$$V''_{\text{eff}}(\phi_{\text{c}},T)\equiv\frac{\partial^2 V_{\text{eff}}}{\partial^2\phi_{\text{c}}}=2D_2(T^2-T_{\text{o}}^2)-6D_3T\phi_{\text{c}}+3\lambda(T)\phi_{\text{c}}^2$$

$D_3=0$: 2nd order phase transition

$$V'_{\text{eff}}(\phi_{\text{c}},T)=0\quad\Rightarrow\quad\phi_{\text{c1}}(T)=0,\quad\phi_{\text{c2}}(T)=\sqrt{\frac{2D_2(T_{\text{o}}^2-T^2)}{\lambda(T)}}$$

$$V''_{\text{eff}}(\phi_{\text{c1}},T)=2D_2(T^2-T_{\text{o}}^2),\quad V''_{\text{eff}}(\phi_{\text{c2}},T)=4D_2(T_{\text{o}}^2-T^2)$$

$a)\ T>T_{\text{o}}$: $V''_{\text{eff}}(0,T)>0$, $V_{\text{eff}}(\phi_{\text{c1}},T)$ is a minimum; ϕ_{c2} does not exist
$b)\ T=T_{\text{o}}$: $\phi_{\text{c1}}=\phi_{\text{c2}}=0$, $V''_{\text{eff}}(0,T_{\text{o}})=0$; T_{o} is the critical temperature
$c)\ T<T_{\text{o}}$: $V''_{\text{eff}}(0,T)<0$, $V_{\text{eff}}(\phi_{\text{c1}},T)$ becomes a maximum; $V''_{\text{eff}}(0,T)>0$, $V_{\text{eff}}(\phi_{\text{c2}},T)$ is a minimal

$D_3\neq 0$: 1st order phase transition

$$\frac{dV_{\text{eff}}}{d\phi_{\text{c}}}=0\quad\Rightarrow\quad\phi_{\text{c1}}(T)=0,\quad\phi_{\text{c2}}(T)=\frac{3D_3T-\sqrt{9D_3^2T^2-8\lambda(T)D_2(T^2-T_{\text{o}}^2)}}{2\lambda(T)},\quad\phi_{\text{c3}}(T)=\frac{3D_3T+\sqrt{9D_3^2T^2-8\lambda(T)D_2(T^2-T_{\text{o}}^2)}}{2\lambda(T)}$$

$$V''_{\text{eff}}(\phi_{\text{c1}},T)=2D_2(T^2-T_{\text{o}}^2)$$

$$9D_3^2T_{\text{i}}^2-8\lambda(T_{\text{i}})D_2(T_{\text{i}}^2-T_{\text{o}}^2)=0\quad\Rightarrow\quad T_{\text{i}}=T_{\text{o}}\sqrt{\frac{8\lambda(T_{\text{i}})D_2}{8\lambda(T_{\text{i}})D_2-9D_3^2}}>T_{\text{o}}$$

$$V(\phi_{\text{c3}},T_{\text{c}})=0\quad\Rightarrow\quad T_{\text{c}}=T_{\text{o}}\sqrt{\frac{\lambda(T_{\text{c}})D_2}{\lambda(T_{\text{c}})D_2-D_3^2}}>T_{\text{o}}$$

$a)\ T>T_{\text{i}}$: $V_{\text{eff}}(\phi_{\text{c1}},T)$ is a minimum; ϕ_{c2} and ϕ_{c3} do not exist
$b)\ T=T_{\text{i}}$: $V_{\text{eff}}(\phi_{\text{c1}},T_{\text{i}})$ is a minimum; $\phi_{\text{c2}}=\phi_{\text{c3}}=\dfrac{3D_3T_{\text{i}}}{2\lambda(T_{\text{i}})}$ corresponds to an inflection point
$c)\ T_i<T<T_{\text{c}}$: $V_{\text{eff}}(\phi_{\text{c1}},T)$ and $V_{\text{eff}}(\phi_{\text{c3}},T)$ are two minima between which there is a barrier maximized at ϕ_{c2}
$d)\ T=T_{\text{c}}$: $V(\phi_{\text{c1}},T_{\text{c}})=V(\phi_{\text{c3}},T_{\text{c}})$, the two minima are degenerate; $V_{\text{eff}}(\phi_{\text{c2}},T_{\text{c}})$ is the maximum of the barrier
$e)\ T_{\text{o}}<T<T_{\text{c}}$: $V(\phi_{\text{c1}},T_{\text{c}})>V(\phi_{\text{c3}},T_{\text{c}})$, $V_{\text{eff}}(\phi_{\text{c1}},T)$ becomes metastable
$f)\ T=T_{\text{o}}$: $\phi_{\text{c1}}=\phi_{\text{c3}}$, $V''_{\text{eff}}(0,T_{\text{o}})=0$, the barrier disappears; $\phi_{\text{c3}}=\dfrac{3D_3T_{\text{o}}}{\lambda(T_{\text{o}})}$
$g)\ T<T_{\text{o}}$: $V_{\text{eff}}(\phi_{\text{c1}},T)$ becomes a maximum

Introducing a fermionic DM particle

Ref: Dimopoulos, Esmailzadeh, Hall, Tetradis, PLB 247, 601 (1990)

$\phi_{\rm c}$-dependent DM particle mass $\bar{m}_X(\phi_{\rm c}) = \frac{1}{2} g_X \phi_{\rm c}$

$$V_{\rm DM}(\phi_{\rm c}) = -\frac{4}{64\pi^2}\bar{m}_X^4(\phi_{\rm c})\left[\ln\frac{\bar{m}_X^2(\phi_{\rm c})}{\mu_{\rm R}^2} - \frac{3}{2}\right]$$

$$V_{\rm T,DM}(\phi_{\rm c},T) = \frac{T^4}{2\pi^2}\cdot 4J_{\rm F}(\bar{m}_X^2/T^2)$$

$$\left.\frac{dV}{d\phi_{\rm c}}\right|_{\phi_{\rm c}=v} = 0 \quad\Rightarrow\quad \mu^2 = v^2\left\{\lambda + \frac{3g^4}{128\pi^2}\left(\ln\frac{g^2}{4}-\frac{1}{3}\right) + \frac{3(g^2+g'^2)^2}{256\pi^2}\left[\ln\frac{(g^2+g'^2)}{4}-\frac{1}{3}\right] - \frac{3y_t^4}{16\pi^2}\left(\ln\frac{y_t^2}{2}-1\right) - \frac{g_X^4}{16\pi^2}\left(\ln\frac{g_X^2}{2}-1\right)\right\}$$
$$+\frac{3\lambda}{16\pi^2}\left[(-\mu^2+3\lambda v^2)\left(\ln\frac{-\mu^2+3\lambda v^2}{v^2}-1\right) + (-\mu^2+\lambda v^2)\left(\ln\frac{-\mu^2+\lambda v^2}{v^2}-1\right)\right]$$

$$\left.\frac{d^2V}{d\phi_{\rm c}^2}\right|_{\phi_{\rm c}=v} = m_h^2 \quad\Rightarrow\quad \frac{m_h^2}{v^2} = 2\lambda + \frac{3g^4}{64\pi^2}\left(\ln\frac{g^2}{4}+\frac{2}{3}\right) + \frac{3(g^2+g'^2)^2}{128\pi^2}\left(\ln\frac{g^2+g'^2}{4}+\frac{2}{3}\right) - \frac{3y_t^4}{8\pi^2}\ln\frac{y_t^2}{2}$$
$$-\frac{g_X^4}{8\pi^2}\ln\frac{g_X^2}{2} + \frac{3\lambda^2}{8\pi^2}\left(3\ln\frac{-\mu^2+3\lambda v^2}{v^2} + \ln\frac{-\mu^2+\lambda v^2}{v^2}\right)$$

Introducing a scalar DM particle

Ref: Sher, PLB 263, 255 (1991)

$\phi_{\rm c}$-dependent DM particle mass-squared $\bar{m}_X^2(\phi_{\rm c}) = m_0^2 + h^2\phi_{\rm c}^2$

$$V_{\rm DM}(\phi_{\rm c}) = \frac{1}{64\pi^2}\bar{m}_X^4(\phi_{\rm c})\left[\ln\frac{\bar{m}_X^2(\phi_{\rm c})}{\mu_{\rm R}^2} - \frac{3}{2}\right]$$

$$V_{\rm T,DM}(\phi_{\rm c},T) = \frac{T^4}{2\pi^2}\cdot J_{\rm B}(\bar{m}_X^2/T^2)$$

$$\left.\frac{dV}{d\phi_{\rm c}}\right|_{\phi_{\rm c}=v} = 0 \quad\Rightarrow\quad \mu^2 = v^2\left\{\lambda + \frac{3g^4}{128\pi^2}\left(\ln\frac{g^2}{4}-\frac{1}{3}\right) + \frac{3(g^2+g'^2)^2}{256\pi^2}\left[\ln\frac{(g^2+g'^2)}{4}-\frac{1}{3}\right] - \frac{3y_t^4}{16\pi^2}\left(\ln\frac{y_t^2}{2}-1\right)\right\}$$
$$+\frac{3\lambda}{16\pi^2}\left[(-\mu^2+3\lambda v^2)\left(\ln\frac{-\mu^2+3\lambda v^2}{v^2}-1\right) + (-\mu^2+\lambda v^2)\left(\ln\frac{-\mu^2+\lambda v^2}{v^2}-1\right)\right]$$
$$+\frac{h^2}{16\pi^2}(m_0^2+h^2v^2)\left(\ln\frac{m_0^2+h^2v^2}{v^2}-1\right)$$

$$\left.\frac{d^2V}{d\phi_{\rm c}^2}\right|_{\phi_{\rm c}=v} = m_h^2 \quad\Rightarrow\quad \frac{m_h^2}{v^2} = 2\lambda + \frac{3g^4}{64\pi^2}\left(\ln\frac{g^2}{4}+\frac{2}{3}\right) + \frac{3(g^2+g'^2)^2}{128\pi^2}\left(\ln\frac{g^2+g'^2}{4}+\frac{2}{3}\right) - \frac{3y_t^4}{8\pi^2}\ln\frac{y_t^2}{2}$$
$$+\frac{3\lambda^2}{8\pi^2}\left(3\ln\frac{-\mu^2+3\lambda v^2}{v^2} + \ln\frac{-\mu^2+\lambda v^2}{v^2}\right) + \frac{h^2}{8\pi^2}\ln\frac{m_0^2+h^2v^2}{v^2}$$

Ref: Dimopoulos, Esmailzadeh, Hall, Tetradis, PLB 247, 601 (1990)

$$V^0(\phi_\mathrm{c}) = \frac{1}{2}\mu^2\phi_\mathrm{c}^2 + \frac{\lambda}{4!}\phi_\mathrm{c}^4 + B\phi_\mathrm{c}^4\left(\ln\frac{\phi_\mathrm{c}^2}{\sigma^2} - \frac{25}{6}\right), \quad \text{cut-off regularization}$$

$$B = \frac{3\alpha^2}{64}\frac{2+\sec^4\theta_\mathrm{W}}{\sin^4\theta_\mathrm{W}} - \frac{1}{16\pi^2}\sum_i g_i^4$$

$$\frac{dV^0(\phi_\mathrm{c})}{d\phi_\mathrm{c}} = \mu^2\phi_\mathrm{c} + \frac{\lambda}{6}\phi_\mathrm{c}^3 + 4B\phi_\mathrm{c}^3\ln\frac{\phi_\mathrm{c}^2}{\sigma^2} + 2B\phi_\mathrm{c}^3 - \frac{50B}{3}\phi_\mathrm{c}^3 = \mu^2\phi_\mathrm{c} + \left(\frac{\lambda}{6} - \frac{44B}{3}\right)\phi_\mathrm{c}^3 + 4B\phi_\mathrm{c}^3\ln\frac{\phi_\mathrm{c}^2}{\sigma^2}$$

$$\sigma = \phi_\mathrm{min} = 246\ \mathrm{GeV}$$

$$0 = \left.\frac{dV^0(\phi_\mathrm{c})}{d\phi_\mathrm{c}}\right|_{\phi_\mathrm{c}=\sigma} = \sigma\left[\mu^2 + \left(\frac{\lambda}{6} - \frac{44B}{3}\right)\sigma^2\right] = \sigma\left[\mu^2 - B\left(-\frac{\lambda}{6B} + \frac{44}{3}\right)\sigma^2\right]$$

$$\sigma^2 = \frac{\mu^2}{B}\left(-\frac{\lambda}{6B} + \frac{44}{3}\right)^{-1}, \quad \mu^2 = \left(-\frac{\lambda}{6B} + \frac{44}{3}\right)B\sigma^2$$

$$m_{H,0}^2 = \left.\frac{d^2V^0(\phi_\mathrm{c})}{d\phi_\mathrm{c}^2}\right|_{\phi_\mathrm{c}=\sigma} = \mu^2 + \left(\frac{\lambda}{2} - 44B\right)\sigma^2 + 8B\sigma^2 = 2B\sigma^2\left(\frac{\lambda}{6B} - \frac{32}{3}\right)$$

$$V_\mathrm{B}^T(\phi_\mathrm{c},T) = \frac{T}{2\pi^2}\int_0^\infty dp\, p^2 \ln\left[1 - e^{-\sqrt{p^2+m_\mathrm{B}^2(\phi_\mathrm{c})}/T}\right], \quad V_\mathrm{F}^T(\phi_\mathrm{c},T) = -\frac{T}{2\pi^2}\int_0^\infty dp\, p^2 \ln\left[1 + e^{-\sqrt{p^2+m_\mathrm{F}^2(\phi_\mathrm{c})}/T}\right]$$

$$V(\phi_\mathrm{c},T) = V^0(\phi_\mathrm{c}) + 6V_\mathrm{W}^T(\phi_\mathrm{c},T) + 3V_Z^T(\phi_\mathrm{c},T) + 4V_X^T(\phi_\mathrm{c},T)$$

$$m_W = \frac{e\phi_\mathrm{c}}{2\sin\theta_\mathrm{W}}, \quad m_Z = \frac{e\phi_\mathrm{c}}{2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W}}, \quad m_X = g_X\phi$$