粒子物理标准模型拉氏量和费曼规则

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目 录

1 约定

本文各种约定主要遵从文献 [1], 推导和计算参考文献 [1, 2, 3, 4].

度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{1}$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \tag{2}$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \tag{3}$$

手征表示中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}. \tag{4}$$

左右手投影算符

$$P_{\mathrm{L}} \equiv \frac{1}{2}(1-\gamma_5) = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \quad P_{\mathrm{R}} \equiv \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}. \tag{5}$$

Levi-Civita 张量约定取

$$\varepsilon^{123} = +1. \tag{6}$$

费曼规则约定:

- 对于指向相互作用顶点的动量 p, 时空偏导数 ∂_μ 在动量空间费曼规则里贡献一个 $-ip_\mu$ 因子.
- 实线表示费米子, 实线上的箭头表示费米子数流动的方向.
- 虚线表示标量玻色子, 虚线上的箭头表示电荷数流动的方向.
- 螺旋线表示胶子; 波浪线表示其它规范玻色子, 波浪线上的箭头表示电荷数流动的方向.
- 点线表示鬼粒子, 点线上的箭头表示鬼粒子数流动的方向.
- 如果没有额外箭头标记, 动量方向与粒子线上的箭头方向一致; 否则与额外箭头方向一致.

2 标准模型概述

粒子物理标准模型是一个 $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ 规范理论. 模型中有三代费米子, 包括三代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$, 三代带电轻子 $\ell_i = e, \mu, \tau$, 三代上型夸克 $u_i = u, c, t$ 和三代下型夸克 $d_i = d, s, b$ $(i = 1, 2, 3)$. 规范玻色子传递费米子间相互作用.

$\mathrm{SU(3)_C}$ 部分描述夸克的强相互作用, 称为量子色动力学 (Quantum Chromodynamics, QCD), 相应的规范玻色子是胶子. $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ 部分统一描述夸克和轻子的电磁和弱相互作用, 称为电弱统一理论. 理论中有一个 Higgs 二重态, 通过 Brout–Englert–Higgs 机制引发规范群的自发对称性破缺, 使 $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ 群破缺为 $\mathrm{U(1)_{EM}}$ 群. $\mathrm{U(1)_{EM}}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED).

破缺前, 理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度; 左手费米子和右手费米子都没有质量, 具有不同量子数.

破缺后, 3 个规范玻色子与 3 个 Higgs 自由度结合, 从而获得质量, 成为 $W^\pm$ 和 Z^0 玻色子, 传递弱相互作用; 剩下的 1 个无质量规范玻色子是光子, 即是 $\mathrm{U(1)_{EM}}$ 群的规范玻色子, 传递电磁相互作用; 剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子; 与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量, 组合成 Dirac 费米子.

理论中的中微子没有右手分量, 因而没有获得质量. 1998 年实验发现中微子振荡, 证明中微子具有质量, 所以需要扩充标准模型才能正确描述中微子物理.

3 QCD 拉氏量和费曼规则

QCD 的拉氏量可表达成

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \cdots, 8, \tag{7}$$

其中

$$D_\mu = \partial_\mu - ig_{\mathrm{s}}G^a_\mu t^a, \quad G^{a\mu\nu} \equiv \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_{\mathrm{s}} f^{abc} G^{b\mu} G^{c\nu}. \tag{8}$$

$\mathrm{SU(3)_C}$ 群基础表示生成元 $t^a = \lambda^a/2$, 其中 λ^a 为 Gell-Mann 矩阵. 生成元对易关系为 $[t^a, t^b] = if^{abc}t^c$. 结构常数 f^{abc} 是全反对称的, 其非零分量为

$$f_{123} = 1, \quad f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \tag{9}$$

由

$$\begin{aligned} -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} &= -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_{\mathrm{s}} f^{abc} G^b_\mu G^c_\nu)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_{\mathrm{s}} f^{ade} G^{d\mu} G^{e\nu}) \\ &= -\frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu}) - (\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu})] - g_{\mathrm{s}} f^{abc} (\partial_\mu G^a_\nu) G^{b\mu} G^{c\nu} \\ &\quad -\frac{1}{4} g_{\mathrm{s}}^2 f^{abc} f^{ade} G^b_\mu G^c_\nu G^{d\mu} G^{e\nu}, \end{aligned} \tag{10}$$

可得

$$\begin{aligned} \mathcal{L}_{\mathrm{QCD}} &= \sum_q [\bar{q}(i\gamma^\mu \partial_\mu - m_q)q + g_{\mathrm{s}} G^a_\mu \bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu}) - (\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu})] \\ &\quad - g_{\mathrm{s}} f^{abc} (\partial_\mu G^a_\nu) G^{b\mu} G^{c\nu} - \frac{1}{4} g_{\mathrm{s}}^2 f^{abc} f^{ade} G^b_\mu G^c_\nu G^{d\mu} G^{e\nu}. \end{aligned} \tag{11}$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G^a_\mu(x) - \omega^a(x)$, 其中 $\omega^a(x)$ 是某个任意函数, 规范固定条件是 $G^a(x) = 0$. 这是 Lorenz 规范的推广, $\omega^a(x) = 0$ 对应于 Lorenz 规范. 在路径积分量子化中, 以中心为 $\omega^a(x) = 0$ 的高斯权重对 $\omega^a(x)$ 作泛函积分, 有

$$\int \mathcal{D}\omega^a \exp\left[-i\int d^4x \frac{1}{2\xi}(\omega^a)^2\right] \delta(G^a) = \exp\left[-i\int d^4x \frac{1}{2\xi}(\partial^\mu G^a_\mu)^2\right]. \tag{12}$$

可见, 拉氏量中的规范固定项为

$$\mathcal{L}_{\mathrm{QCD,GF}} = -\frac{1}{2\xi}(\partial^\mu G^a_\mu)^2. \tag{13}$$

ξ 的任何一个取值对应于一种规范. $\xi = 1$ 称为 Feynman-'t Hooft 规范, $\xi = 0$ 称为 Landau 规范. 于是, 胶子传播子相关拉氏量为

$$\begin{aligned}\mathcal{L}_{\mathrm{QCD,prop}} &= \frac{1}{2}\left[(\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu}) - (\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu}) - \frac{1}{\xi}(\partial^\mu G^a_\mu)^2\right] \\ &\to \frac{1}{2}G^a_\mu\left[g^{\mu\nu}\partial^2 - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]G^a_\nu.\end{aligned} \tag{14}$$

变换到动量空间, 得

$$-g^{\mu\nu}p^2 + \left(1-\frac{1}{\xi}\right)p^\mu p^\nu, \tag{15}$$

它的逆矩阵是

$$-\frac{1}{p^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right], \tag{16}$$

这是因为

$$\begin{aligned}&-\frac{1}{p^2}\left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}(1-\xi)\right]\left[-g^{\mu\nu}p^2 + \left(1-\frac{1}{\xi}\right)p^\mu p^\nu\right] \\ &= \delta^\nu_\rho - \frac{p_\rho p^\nu}{p^2}\left(1-\frac{1}{\xi}\right) - \frac{p_\rho p^\nu}{p^2}(1-\xi) + \frac{p_\rho p^\nu}{p^2}(1-\xi)\left(1-\frac{1}{\xi}\right) = \delta^\nu_\rho.\end{aligned} \tag{17}$$

从而, 胶子传播子的形式为

$$\frac{-i\delta^{ab}}{p^2+i\varepsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right]. \tag{18}$$

$\mathrm{SU}(3)_\mathrm{C}$ 定域规范变换为

$$q \to Uq, \quad G^a_\mu t^a \to UG^a_\mu t^a U^\dagger + \frac{i}{g_\mathrm{s}}U\partial_\mu U^\dagger, \tag{19}$$

其中 $U(x) = \exp[i\alpha^a(x)t^a]$. 胶子场的无穷小规范变换形式是

$$\begin{aligned}G^a_\mu t^a &\to (1+i\alpha^a t^a)G^b_\mu t^b(1-i\alpha^c t^c) + \frac{i}{g_\mathrm{s}}(1+i\alpha^a t^a)\partial_\mu(1-i\alpha^c t^c) \\ &= G^b_\mu t^b + i\alpha^a G^b_\mu[t^a,t^b] + \frac{1}{g_\mathrm{s}}(\partial_\mu\alpha^c)t^c + \mathcal{O}(\alpha^2) = G^a_\mu t^a - f^{abc}\alpha^a G^b_\mu t^c + \frac{1}{g_\mathrm{s}}(\partial_\mu\alpha^a)t^a + \mathcal{O}(\alpha^2) \\ &= \left(G^a_\mu + f^{abc}G^b_\mu\alpha^c + \frac{1}{g_\mathrm{s}}\partial_\mu\alpha^a\right)t^a + \mathcal{O}(\alpha^2),\end{aligned} \tag{20}$$

即

$$\delta G^a_\mu = \frac{1}{g_\mathrm{s}}\partial_\mu\alpha^a + f^{abc}G^b_\mu\alpha^c = \left(\frac{1}{g_\mathrm{s}}\delta^{ac}\partial_\mu + f^{abc}G^b_\mu\right)\alpha^c, \tag{21}$$

因而规范固定函数 G^a 的无穷小规范变换为

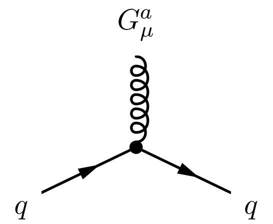
$$\delta G^a = \partial^\mu\delta G^a_\mu = \frac{1}{g_\mathrm{s}}\delta^{ac}\partial^2\alpha^c + f^{abc}\partial^\mu G^b_\mu\alpha^c, \quad g_\mathrm{s}\frac{\delta G^a}{\delta\alpha^c} = \delta^{ab}\partial^2 + g_\mathrm{s}f^{abc}\partial^\mu G^b_\mu. \tag{22}$$

Faddeev-Popov 鬼场的拉氏量是

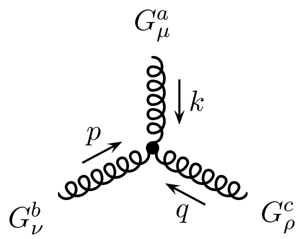
$$\mathcal{L}_{\text{QCD,FP}} = -\bar{\eta}_g^a\left(g_{\text{s}}\frac{\delta G^a}{\delta\alpha^c}\right)\eta_g^c = -\bar{\eta}_g^a(\delta^{ac}\partial^2 + g_{\text{s}}f^{abc}\partial^\mu G_\mu^b)\eta_g^c \to -\bar{\eta}_g^a\delta^{ab}\partial^2\eta_g^b + g_{\text{s}}f^{abc}(\partial^\mu\bar{\eta}_g^a)G_\mu^b\eta_g^c. \quad (23)$$

下面列出 QCD 费曼规则.

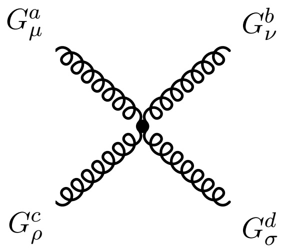
QCD 顶点:



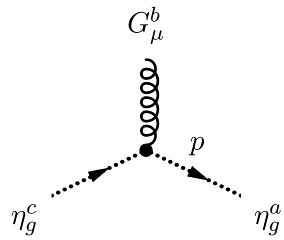
$$= ig_{\text{s}}\gamma^\mu t^a$$



$$= g_{\text{s}}f^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu]$$



$$\begin{aligned} = -ig_{\text{s}}^2[&f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})] \end{aligned}$$

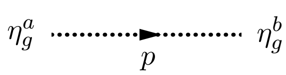


$$= -g_{\text{s}}f^{abc}p^\mu$$

胶子传播子:

$$= \frac{-i\delta^{ab}}{p^2 + i\varepsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right]$$

鬼粒子传播子:



$$= \frac{i\delta^{ab}}{p^2 + i\varepsilon}$$

4 费米子电弱规范相互作用拉氏量和费曼规则

标准模型费米子的量子数列于表 1. 左手费米子场构成 $\mathrm{SU(2)_L}$ 二重态

$$L_{i\mathrm{L}} = \begin{pmatrix} P_\mathrm{L}\nu_i \\ P_\mathrm{L}\ell_i \end{pmatrix} = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}_\mathrm{L}, \quad Q_{i\mathrm{L}} = \begin{pmatrix} P_\mathrm{L}u_i \\ P_\mathrm{L}d'_i \end{pmatrix} = \begin{pmatrix} u_i \\ d'_i \end{pmatrix}_\mathrm{L}. \tag{24}$$

下型夸克的质量本征态 d_j 与规范本征态 d'_j 通过 CKM 矩阵 V_{ij} 联系起来:

$$d'_i = V_{ij} d_j. \tag{25}$$

右手费米子场 $\ell_{i\mathrm{R}} = P_\mathrm{R}\ell_i$, $u_{i\mathrm{R}} = P_\mathrm{R}u_i$ 和 $d'_{i\mathrm{R}} = P_\mathrm{R}d'_i$ 是 $\mathrm{SU(2)_L}$ 单态.

电荷数 Q, 弱同位旋第 3 分量 T^3 和弱超荷 Y 存在如下关系:

$$Q = T^3 + Y. \tag{26}$$

表 1: 标准模型费米子的量子数.

统一记号	第一代	第二代	第三代	Q	T^3	Y	B	$L_{e,\mu,\tau}$
$L_{i\mathrm{L}} = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}_\mathrm{L}$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_\mathrm{L}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_\mathrm{L}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_\mathrm{L}$	0 -1	$1/2$ $-1/2$	$-1/2$ $-1/2$	0 0	1 1
$Q_{i\mathrm{L}} = \begin{pmatrix} u_i \\ d'_i \end{pmatrix}_\mathrm{L}$	$\begin{pmatrix} u \\ d' \end{pmatrix}_\mathrm{L}$	$\begin{pmatrix} c \\ s' \end{pmatrix}_\mathrm{L}$	$\begin{pmatrix} t \\ b' \end{pmatrix}_\mathrm{L}$	$2/3$ $-1/3$	$1/2$ $-1/2$	$1/6$ $1/6$	$1/3$ $1/3$	0 0
$\ell_{i\mathrm{R}}$	e_R	μ_R	τ_R	-1	0	-1	0	1
$u_{i\mathrm{R}}$	u_R	c_R	t_R	$2/3$	0	$2/3$	$1/3$	0
$d'_{i\mathrm{R}}$	d'_R	s'_R	b'_R	$-1/3$	0	$-1/3$	$1/3$	0

$\mathrm{SU(2)_L \times U(1)_Y}$ 规范不变的费米子协变动能项为

$$\mathcal{L}_\mathrm{EWF} = \bar{Q}_{i\mathrm{L}} i \not{D} Q_{i\mathrm{L}} + \bar{u}_{i\mathrm{R}} i \not{D} u_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} i \not{D} d'_{i\mathrm{R}} + \bar{L}_{i\mathrm{L}} i \not{D} L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}} i \not{D} \ell_{i\mathrm{R}}, \tag{27}$$

其中协变导数

$$D_\mu = \partial_\mu - ig' B_\mu Y - ig W^a_\mu T^a, \quad T^a = \frac{\sigma^a}{2}. \tag{28}$$

规范场 $W^a_\mu(x)$ 和 $B_\mu(x)$ 跟左手费米子场的相互作用与右手费米子场不同, 而在 QED 中, 电磁场 $A_\mu(x)$ 跟左手费米子场的相互作用却与右手费米子场相同. 为了回到 QED 的情况, 需要把 $W^3_\mu(x)$ 和 $B_\mu(x)$ 混合起来, 得到电磁场 $A_\mu(x)$ 和另一个中性规范场 $Z_\mu(x)$, 即定义

$$A_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}(g' W^3_\mu + g B_\mu) = s_\mathrm{W} W^3_\mu + c_\mathrm{W} B_\mu, \tag{29}$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}(g W^3_\mu - g' B_\mu) = c_\mathrm{W} W^3_\mu - s_\mathrm{W} B_\mu, \tag{30}$$

$$
W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \tag{31}
$$

或

$$
B_\mu = c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu, \quad W_\mu^3 = s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu, \tag{32}
$$

$$
W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \tag{33}
$$

参数间有如下关系,

$$
s_\mathrm{W} \equiv \sin\theta_\mathrm{W} = \frac{g'}{\sqrt{g^2+g'^2}}, \quad c_\mathrm{W} \equiv \cos\theta_\mathrm{W} = \frac{g}{\sqrt{g^2+g'^2}}, \quad e \equiv \frac{gg'}{\sqrt{g^2+g'^2}} = gs_\mathrm{W} = g'c_\mathrm{W}. \tag{34}
$$

这里 θ_W 称为 Weinberg 角.

利用

$$
\begin{aligned}
g'YB_\mu + gT^3W_\mu^3 &= g'Y(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) + gT^3(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu) \\
&= e(Y+T^3)A_\mu + \left(gc_\mathrm{W}T^3 - \frac{gs_\mathrm{W}}{c_\mathrm{W}}s_\mathrm{W}Y\right)Z_\mu = QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3c_\mathrm{W}^2 - Ys_\mathrm{W}^2)Z_\mu \\
&= QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu,
\end{aligned} \tag{35}
$$

有

$$
\begin{aligned}
D_\mu Q_{i\mathrm{L}} &= (\partial_\mu - ig'B_\mu Y - igW_\mu^aT^a)Q_{i\mathrm{L}} = \partial_\mu Q_{i\mathrm{L}} - i\begin{pmatrix} g'YB_\mu + gT^3W_\mu^3 & \frac{1}{2}g(W_\mu^1 - iW_\mu^2) \\ \frac{1}{2}g(W_\mu^1 + iW_\mu^2) & g'YB_\mu + gT^3W_\mu^3 \end{pmatrix} Q_{i\mathrm{L}} \\
&= \partial_\mu Q_{i\mathrm{L}} - i\begin{pmatrix} QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu & \frac{1}{\sqrt{2}}gW_\mu^+ \\ \frac{1}{\sqrt{2}}gW_\mu^- & QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu \end{pmatrix} Q_{i\mathrm{L}} \\
&= \partial_\mu Q_{i\mathrm{L}} - i\begin{pmatrix} \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]u_{i\mathrm{L}} + \frac{1}{\sqrt{2}}gW_\mu^+d'_{i\mathrm{L}} \\ \frac{1}{\sqrt{2}}gW_\mu^-u_{i\mathrm{L}} + \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)\right]d'_{i\mathrm{L}} \end{pmatrix},
\end{aligned} \tag{36}
$$

故

$$
\begin{aligned}
\bar{Q}_{i\mathrm{L}}i\not{D}Q_{i\mathrm{L}} \supset{}& \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{u}_{i\mathrm{L}}\gamma^\mu u_{i\mathrm{L}} + \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)\right]\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} \\
&+ \frac{1}{\sqrt{2}}gW_\mu^+\bar{u}_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} + \frac{1}{\sqrt{2}}gW_\mu^-\bar{d}'_{i\mathrm{L}}\gamma^\mu u_{i\mathrm{L}} \\
={}& \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{u}_i\gamma^\mu\frac{1-\gamma_5}{2}u_i + \frac{1}{2}\left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}\right)\bar{d}_i\gamma^\mu\frac{1-\gamma_5}{2}d_i \\
&+ \frac{1}{\sqrt{2}}gW_\mu^+\bar{u}_i\gamma^\mu\frac{1-\gamma_5}{2}V_{ij}d_j + \frac{1}{\sqrt{2}}gW_\mu^-\bar{d}_jV_{ji}^\dagger\gamma^\mu\frac{1-\gamma_5}{2}u_i,
\end{aligned} \tag{37}
$$

其中

$$
g_\mathrm{L} \equiv T^3 - Qs_\mathrm{W}^2. \tag{38}
$$

另一方面,

$$D_\mu d'_{i\mathrm{R}} = (\partial_\mu - ig' B_\mu Y) d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} - ig' Q(c_\mathrm{W} A_\mu - s_\mathrm{W} Z_\mu) d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} - iQeA_\mu d'_{i\mathrm{R}} + i\frac{g}{c_\mathrm{W}} Q s_\mathrm{W}^2 Z_\mu d'_{i\mathrm{R}}, \tag{39}$$

则

$$\begin{aligned}\bar{u}_{i\mathrm{R}} i\not{D} u_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} i\not{D} d'_{i\mathrm{R}} &\supset \left(QeA_\mu - \frac{g}{c_\mathrm{W}} Q s_\mathrm{W}^2 Z_\mu\right) \bar{u}_{i\mathrm{R}} \gamma^\mu u_{i\mathrm{R}} + \left(QeA_\mu - \frac{g}{c_\mathrm{W}} Q s_\mathrm{W}^2 Z_\mu\right) \bar{d}'_{i\mathrm{R}} \gamma^\mu d'_{i\mathrm{R}} \\ &= \left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{R} Z_\mu\right) \bar{u}_i \gamma^\mu \frac{1+\gamma_5}{2} u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{R} Z_\mu\right) \bar{d}_i \gamma^\mu \frac{1+\gamma_5}{2} d_i,\end{aligned} \tag{40}$$

其中

$$g_\mathrm{R} \equiv -Q s_\mathrm{W}^2. \tag{41}$$

定义

$$g_\mathrm{V} \equiv g_\mathrm{L} + g_\mathrm{R} = T^3 - 2Q s_\mathrm{W}^2, \quad g_\mathrm{A} \equiv g_\mathrm{L} - g_\mathrm{R} = T^3, \tag{42}$$

可得

$$\begin{aligned}&\bar{Q}_{i\mathrm{L}} i\not{D} Q_{i\mathrm{L}} + \bar{u}_{i\mathrm{R}} i\not{D} u_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} i\not{D} d'_{i\mathrm{R}} \\ \supset\ & Qe\bar{u}_i \gamma^\mu u_i A_\mu + Qe\bar{d}\gamma^\mu d_i A_\mu + \frac{g}{2c_\mathrm{W}} \bar{u}_i \gamma^\mu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) u_i Z_\mu + \frac{g}{2c_\mathrm{W}} \bar{d}_i \gamma^\mu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) d_i Z_\mu \\ &+ \frac{1}{\sqrt{2}} g W_\mu^+ \bar{u}_i \gamma^\mu P_\mathrm{L} V_{ij} d_j + \frac{1}{\sqrt{2}} g W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_\mathrm{L} u_i.\end{aligned} \tag{43}$$

同理, 有

$$\begin{aligned}\bar{L}_{i\mathrm{L}} i\not{D} L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}} i\not{D} \ell_{i\mathrm{R}} \supset\ & Qe\bar{\ell}_i \gamma^\mu \ell_i A_\mu + \frac{g}{2c_\mathrm{W}} \bar{\ell}_i \gamma^\mu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) \ell_i Z_\mu + \frac{g}{2c_\mathrm{W}} \bar{\nu}_i \gamma^\mu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) \nu_i Z_\mu \\ &+ \frac{1}{\sqrt{2}} g W_\mu^+ \bar{\nu}_i \gamma^\mu P_\mathrm{L} \ell_i + \frac{1}{\sqrt{2}} g W_\mu^- \bar{\ell}_i \gamma^\mu P_\mathrm{L} \nu_i.\end{aligned} \tag{44}$$

总结起来, 可以写成流耦合的形式,

$$\begin{aligned}\mathcal{L}_\mathrm{EWF} &\supset \sum_f \left[Q_f e \bar{f} \gamma^\mu f A_\mu + \frac{g}{2c_\mathrm{W}} \bar{f} \gamma^\mu (g_\mathrm{V}^f - g_\mathrm{A}^f \gamma_5) f Z_\mu\right] + g(W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}) \\ &= eA_\mu J_\mathrm{EM}^\mu + g(Z_\mu J_Z^\mu + W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}),\end{aligned} \tag{45}$$

其中, 流的定义为

$$\begin{aligned}J_\mathrm{EM}^\mu &\equiv \sum_f Q_f \bar{f} \gamma^\mu f, \quad J_Z^\mu \equiv \frac{1}{2c_\mathrm{W}} \sum_f \bar{f} \gamma^\mu (g_\mathrm{V}^f - g_\mathrm{A}^f \gamma_5) f = \frac{1}{c_\mathrm{W}} \sum_f (g_\mathrm{L}^f \bar{f}_\mathrm{L} \gamma^\mu f_\mathrm{L} + g_\mathrm{R}^f \bar{f}_\mathrm{R} \gamma^\mu f_\mathrm{R}), \\ J_W^{+\mu} &\equiv \frac{1}{\sqrt{2}} (\bar{u}_{i\mathrm{L}} \gamma^\mu V_{ij} d_{j\mathrm{L}} + \bar{\nu}_{i\mathrm{L}} \gamma^\mu \ell_{i\mathrm{L}}), \quad J_W^{-\mu} \equiv \frac{1}{\sqrt{2}} (\bar{d}_{j\mathrm{L}} V_{ji}^\dagger \gamma^\mu u_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{L}} \gamma^\mu \nu_{i\mathrm{L}}).\end{aligned} \tag{46}$$

对于各种费米子, 相关系数如下:

$$Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}, \quad Q_{\nu_i} = 0, \quad Q_{\ell_i} = -1; \tag{47}$$

$$g_{\rm V}^{u_i} = \frac{1}{2} - \frac{4}{3}s_{\rm W}^2, \quad g_{\rm A}^{u_i} = \frac{1}{2}; \quad g_{\rm V}^{d_i} = -\frac{1}{2} + \frac{2}{3}s_{\rm W}^2, \quad g_{\rm A}^{d_i} = -\frac{1}{2}; \tag{48}$$

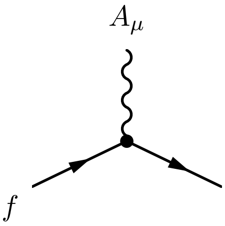
$$g_{\rm V}^{\nu_i} = \frac{1}{2}, \quad g_{\rm A}^{\nu_i} = \frac{1}{2}; \quad g_{\rm V}^{\ell_i} = -\frac{1}{2} + 2s_{\rm W}^2, \quad g_{\rm A}^{\ell_i} = -\frac{1}{2}; \tag{49}$$

$$g_{\rm L}^{u_i} = \frac{1}{2} - \frac{2}{3}s_{\rm W}^2, \quad g_{\rm R}^{u_i} = -\frac{2}{3}s_{\rm W}^2; \quad g_{\rm L}^{d_i} = -\frac{1}{2} + \frac{1}{3}s_{\rm W}^2, \quad g_{\rm R}^{d_i} = \frac{1}{3}s_{\rm W}^2; \tag{50}$$

$$g_{\rm L}^{\nu_i} = \frac{1}{2}, \quad g_{\rm R}^{\nu_i} = 0; \quad g_{\rm L}^{\ell_i} = -\frac{1}{2} + s_{\rm W}^2, \quad g_{\rm R}^{\ell_i} = s_{\rm W}^2. \tag{51}$$

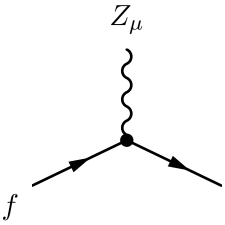
下面给出费米子电弱规范相互作用顶点的费曼规则.

QED 顶点:



$$= iQ_f e\gamma^\mu \quad (\text{对于电子}, Q_f = -1)$$

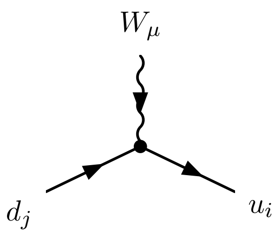
费米子与 Z 玻色子的耦合:



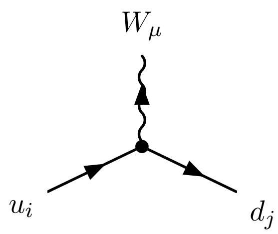
$$= i\frac{g}{2c_{\rm W}}\gamma^\mu(g_{\rm V}^f - g_{\rm A}^f\gamma_5)$$

$$g_{\rm V}^{u_i} = \frac{1}{2} - \frac{4}{3}s_{\rm W}^2, \quad g_{\rm A}^{u_i} = \frac{1}{2}; \quad g_{\rm V}^{d_i} = -\frac{1}{2} + \frac{2}{3}s_{\rm W}^2, \quad g_{\rm A}^{d_i} = -\frac{1}{2};$$
$$g_{\rm V}^{\nu_i} = \frac{1}{2}, \quad g_{\rm A}^{\nu_i} = \frac{1}{2}; \quad g_{\rm V}^{\ell_i} = -\frac{1}{2} + 2s_{\rm W}^2, \quad g_{\rm A}^{\ell_i} = -\frac{1}{2}.$$

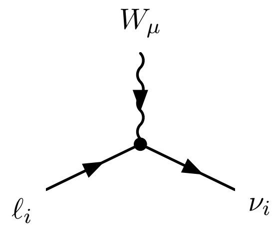
费米子与 $W^\pm$ 玻色子的耦合:



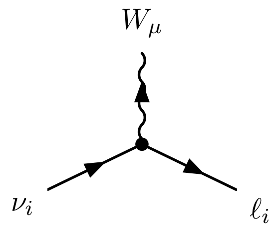
$$= i\frac{g}{\sqrt{2}}V_{ij}\gamma^\mu P_{\rm L}$$



$$= i\frac{g}{\sqrt{2}}V_{ji}^\dagger\gamma^\mu P_{\rm L}$$



$$= i\frac{g}{\sqrt{2}}\gamma^\mu P_{\rm L}$$



$$= i\frac{g}{\sqrt{2}}\gamma^\mu P_{\rm L}$$

5 电弱规范场自相互作用拉氏量和费曼规则

电弱规范场自相互作用拉氏量是

$$\mathcal{L}_{\rm EWG} = -\frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{52}$$

其中

$$W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g\varepsilon^{abc} W^{b\mu} W^{c\nu}, \quad B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu. \tag{53}$$

利用 (32) 式和 (33) 式, 可得

$$\begin{aligned}
& W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 \\
= \; & \frac{i}{\sqrt{2}}[(W_\mu^+ - W_\mu^-)(s_\mathrm{W} A_\nu + c_\mathrm{W} Z_\nu) - (s_\mathrm{W} A_\mu + c_\mathrm{W} Z_\mu)(W_\nu^+ - W_\nu^-)] \\
= \; & \frac{i}{\sqrt{2}}[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) - s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)],
\end{aligned} \tag{54}$$

$$\begin{aligned}
& W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 \\
= \; & \frac{1}{\sqrt{2}}[(s_\mathrm{W} A_\mu + c_\mathrm{W} Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_\mathrm{W} A_\nu + c_\mathrm{W} Z_\nu)] \\
= \; & -\frac{1}{\sqrt{2}}[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) + s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)].
\end{aligned} \tag{55}$$

从而,

$$\begin{aligned}
W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g\varepsilon^{1bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2 \\
&= \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2) \\
&= \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad + \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{1}{\sqrt{2}}(F_{\mu\nu}^+ + F_{\mu\nu}^-),
\end{aligned} \tag{56}$$

其中,

$$F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \tag{57}$$

$$F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ie(W_\mu^- A_\nu - A_\mu W_\nu^-) - igc_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \tag{58}$$

另一方面,

$$\begin{aligned}
W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 + g\varepsilon^{2bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g W_\mu^1 W_\nu^3 + g W_\mu^3 W_\nu^1 \\
&= \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\
&= \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad - \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{i}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-).
\end{aligned} \tag{59}$$

因此,

$$\begin{aligned}
&-\frac{1}{4}W^1_{\mu\nu}W^{1\mu\nu}-\frac{1}{4}W^2_{\mu\nu}W^{2\mu\nu} \qquad (60)\\
=\;& -\frac{1}{8}(F^+_{\mu\nu}+F^-_{\mu\nu})(F^{+\mu\nu}+F^{-\mu\nu})+\frac{1}{8}(F^+_{\mu\nu}-F^-_{\mu\nu})(F^{+\mu\nu}-F^{-\mu\nu})=-\frac{1}{2}F^+_{\mu\nu}F^{-\mu\nu}\\
=\;& -\frac{1}{2}[\partial_\mu W^+_\nu-\partial_\nu W^+_\mu+ie(W^+_\mu A_\nu-A_\mu W^+_\nu)+igc_{\mathrm{W}}(W^+_\mu Z_\nu-Z_\mu W^+_\nu)]\\
&\times[\partial^\mu W^{-\nu}-\partial^\nu W^{-\mu}-ie(W^{-\mu}A^\nu-A^\mu W^{-\nu})-igc_{\mathrm{W}}(W^{-\mu}Z^\nu-Z^\mu W^{-\nu})]\\
=\;& -(\partial_\mu W^+_\nu)(\partial^\mu W^{-\nu})+(\partial_\mu W^+_\nu)(\partial^\nu W^{-\mu})\\
&+ie[(\partial_\mu W^+_\nu)W^{-\mu}A^\nu-(\partial_\mu W^+_\nu)W^{-\nu}A^\mu-W^+_\mu(\partial^\mu W^{-\nu})A_\nu+W^+_\nu(\partial^\mu W^{-\nu})A_\mu]\\
&+igc_{\mathrm{W}}[(\partial_\mu W^+_\nu)W^{-\mu}Z^\nu-(\partial_\mu W^+_\nu)W^{-\nu}Z^\mu-W^+_\mu(\partial^\mu W^{-\nu})Z_\nu+W^+_\nu(\partial^\mu W^{-\nu})Z_\mu]\\
&+e^2(W^+_\mu W^{-\nu}A_\nu A^\mu-W^+_\mu W^{-\mu}A_\nu A^\nu)+g^2c^2_{\mathrm{W}}(W^+_\mu W^{-\nu}Z_\nu Z^\mu-W^+_\mu W^{-\mu}Z_\nu Z^\nu)\\
&+egc_{\mathrm{W}}(W^+_\mu W^{-\nu}A_\nu Z^\mu+W^+_\mu W^{-\nu}A^\mu Z_\nu-2W^+_\mu W^{-\mu}A_\nu Z^\nu). \qquad (61)
\end{aligned}$$

由

$$W^1_\mu W^2_\nu-W^2_\mu W^1_\nu=\frac{i}{2}(W^+_\mu+W^-_\mu)(W^+_\nu-W^-_\nu)-\frac{i}{2}(W^+_\mu-W^-_\mu)(W^+_\nu+W^-_\nu)=-i(W^+_\mu W^-_\nu-W^-_\mu W^+_\nu), \qquad (62)$$

可得

$$\begin{aligned}
W^3_{\mu\nu}\;&=\;\partial_\mu W^3_\nu-\partial_\nu W^3_\mu+g\varepsilon^{3bc}W^b_\mu W^c_\nu=\partial_\mu W^3_\nu-\partial_\nu W^3_\mu+gW^1_\mu W^2_\nu-gW^2_\mu W^1_\nu\\
&=\;s_{\mathrm{W}}\partial_\mu A_\nu+c_{\mathrm{W}}\partial_\mu Z_\nu-s_{\mathrm{W}}\partial_\nu A_\mu+c_{\mathrm{W}}\partial_\nu Z_\mu+g(W^1_\mu W^2_\nu-W^2_\mu W^1_\nu)\\
&=\;s_{\mathrm{W}}(\partial_\mu A_\nu-\partial_\nu A_\mu)+c_{\mathrm{W}}(\partial_\mu Z_\nu-\partial_\nu Z_\mu)-ig(W^+_\mu W^-_\nu-W^-_\mu W^+_\nu), \qquad (63)\\
B_{\mu\nu}\;&=\;\partial_\mu(c_{\mathrm{W}}A_\nu-s_{\mathrm{W}}Z_\nu)-\partial_\nu(c_{\mathrm{W}}A_\mu-s_{\mathrm{W}}Z_\mu)=c_{\mathrm{W}}(\partial_\mu A_\nu-\partial_\nu A_\mu)-s_{\mathrm{W}}(\partial_\mu Z_\nu-\partial_\nu Z_\mu). \qquad (64)
\end{aligned}$$

于是,

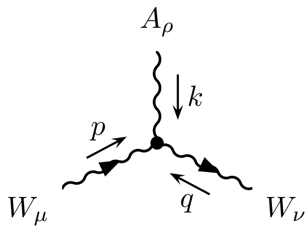
$$\begin{aligned}
&-\frac{1}{4}W^3_{\mu\nu}W^{3\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\\
=\;& -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu)-(\partial_\mu A_\nu)(\partial^\nu A^\mu)]-\frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu)-(\partial_\mu Z_\nu)(\partial^\nu Z^\mu)]\\
&+ie[W^{+\mu}W^{-\nu}(\partial_\mu A_\nu)-W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)]+igc_{\mathrm{W}}[W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu)-W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)]\\
&+\frac{1}{2}g^2(W^+_\mu W^{+\mu}W^-_\nu W^{-\nu}-W^+_\mu W^{+\nu}W^-_\nu W^{-\mu}). \qquad (65)
\end{aligned}$$

综合起来, 有

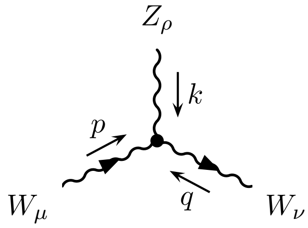
$$\begin{aligned}
\mathcal{L}_{\mathrm{EWG}}\;=\;& \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu)-(\partial_\mu A_\nu)(\partial^\mu A^\nu)]+\frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu)-(\partial_\mu Z_\nu)(\partial^\mu Z^\nu)]\\
&+(\partial_\mu W^+_\nu)(\partial^\nu W^{-\mu})-(\partial_\mu W^+_\nu)(\partial^\mu W^{-\nu})+\frac{1}{2}g^2(W^+_\mu W^{+\mu}W^-_\nu W^{-\nu}-W^+_\mu W^{+\nu}W^-_\nu W^{-\mu})\\
&+ie[(\partial_\mu W^+_\nu)W^{-\mu}A^\nu-(\partial_\mu W^+_\nu)W^{-\nu}A^\mu-W^{+\mu}(\partial_\mu W^-_\nu)A^\nu+W^{+\nu}(\partial_\mu W^-_\nu)A^\mu\\
&\qquad+W^{+\mu}W^{-\nu}(\partial_\mu A_\nu)-W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)]+e^2(W^+_\mu W^{-\nu}A_\nu A^\mu-W^+_\mu W^{-\mu}A_\nu A^\nu)\\
&+igc_{\mathrm{W}}[(\partial_\mu W^+_\nu)W^{-\mu}Z^\nu-(\partial_\mu W^+_\nu)W^{-\nu}Z^\mu-W^{+\mu}(\partial_\mu W^-_\nu)Z^\nu+W^{+\nu}(\partial_\mu W^-_\nu)Z^\mu\\
&\qquad\quad+W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu)-W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)]+g^2c^2_{\mathrm{W}}(W^+_\mu W^{-\nu}Z_\nu Z^\mu-W^+_\mu W^{-\mu}Z_\nu Z^\nu)
\end{aligned}$$

$$+egc_{\mathrm{W}}(W^{+}_{\mu}W^{-\nu}A_{\nu}Z^{\mu}+W^{+}_{\mu}W^{-\nu}A^{\mu}Z_{\nu}-2W^{+}_{\mu}W^{-\mu}A_{\nu}Z^{\nu}). \tag{66}$$

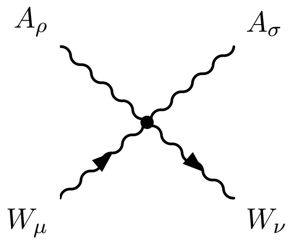
电弱规范玻色子自耦合的费曼规则:



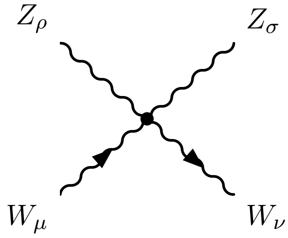
$$= -ie[(g^{\mu\nu}(p-q)^{\rho}+g^{\nu\rho}(q-k)^{\mu}+g^{\rho\mu}(k-p)^{\nu}]$$



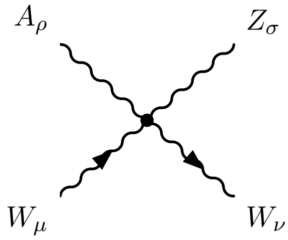
$$= -igc_{\mathrm{W}}[(g^{\mu\nu}(p-q)^{\rho}+g^{\nu\rho}(q-k)^{\mu}+g^{\rho\mu}(k-p)^{\nu}]$$



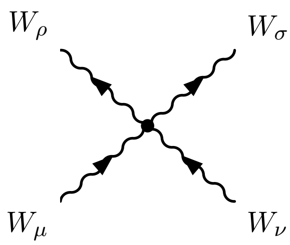
$$= ie^{2}(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma})$$



$$= ig^{2}c_{\mathrm{W}}^{2}(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma})$$



$$= iegc_{\mathrm{W}}(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma})$$



$$= -ig^{2}(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma})$$

6 幺正规范下 Higgs 场相关拉氏量和费曼规则

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_{\mathrm{H}} = (D^\mu\Phi)^\dagger(D_\mu\Phi) - V_{\mathrm{H}}(\Phi), \quad V_{\mathrm{H}}(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \tag{67}$$

其中

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \quad D_\mu\Phi = (\partial_\mu - ig' B_\mu Y_H - igW_\mu^a T^a)\Phi, \quad Y_H = \frac{1}{2}. \tag{68}$$

当 $\lambda > 0$ 且 $\mu^2 > 0$ 时, Higgs 场势能 $V_{\mathrm{H}}(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式, 势能最小值位于方程

$$\Phi^\dagger\Phi = [\mathrm{Re}(\phi^+)]^2 + [\mathrm{Im}(\phi^+)]^2 + [\mathrm{Re}(\phi^0)]^2 + [\mathrm{Im}(\phi^0)]^2 = \frac{v^2}{2} \tag{69}$$

对应的 4 维球面上, 其中 $v \equiv \sqrt{\mu^2/\lambda}$.

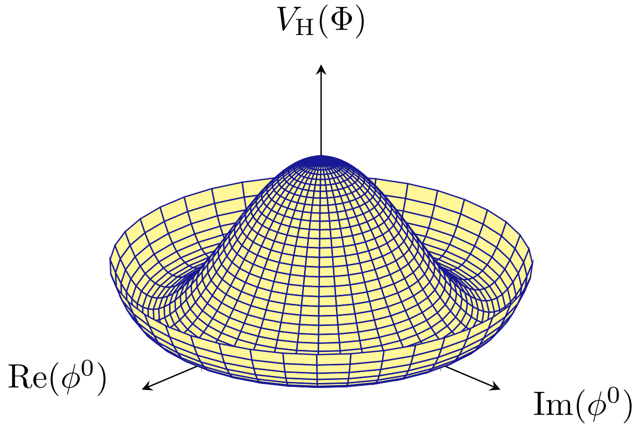


图 1: Higgs 场势能示意图. $\mathrm{Re}(\phi^+)$ 和 $\mathrm{Im}(\phi^+)$ 两个维度已经被压缩掉.

Higgs 场的真空期待值位于这个 4 维球面上的某一点, 不失一般性, 可将它取为

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{70}$$

其它真空期待值可通过整体规范变换

$$\langle\Phi\rangle \to \exp(i\alpha^a T^a)\exp(i\alpha^Y Y_H)\langle\Phi\rangle \tag{71}$$

得到, 因为 $\langle\Phi^\dagger\Phi\rangle$ 在这样的变换下保持不变. 若 $\alpha^1 = \alpha^2 = 0$ 且 $\alpha^3 = \alpha^Y$, 则 $\langle\Phi\rangle$ 在变换下不变. 因此, 有 1 个方向的规范对称性没有受到破坏, 只有 3 个方向的规范对称性发生自发破缺. 根据 Goldstone 定理, 破缺后生成 3 个无质量的 Nambu-Goldstone 玻色子. 最终, 有 3 个规范玻色子自由度通过 Brout–Englert–Higgs 机制获得质量.

以 $\langle\Phi\rangle$ 为基础, 可将 Higgs 场一般地参数化为

$$\Phi(x) = \exp\left[-i\frac{\chi^a(x)}{v}T^a\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{72}$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场. $\exp[-i\chi^a(x)T^a/v]$ 因子能够通过 $\mathrm{SU(2)_L}$ 定域规范变换消去, 因而可将 $\Phi(x)$ 直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger\Phi = \frac{1}{2}(v+H)^2. \tag{73}$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$, 对应于 Higgs 玻色子, 这种取法称为幺正规范.

幺正规范下的势能项化为

$$\begin{aligned}
-V_{\mathrm{H}}(\Phi) &= \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 = \frac{1}{2}\mu^2(v+H)^2 - \frac{1}{4}\lambda(v+H)^4 \\
&= \frac{1}{2}\mu^2(v^2+H^2+2vH) - \frac{1}{4}\lambda(v^4+4v^2H^2+H^4+4v^3H+2v^2H^2+4vH^3) \\
&= \frac{1}{4}\mu^2v^2 + \frac{1}{4}(\mu^2-\lambda v^2)v^2 + (\mu^2-\lambda v^2)vH + \frac{1}{2}(\mu^2-\lambda v^2)H^2 - \lambda v^2H^2 - \lambda vH^3 - \frac{1}{4}\lambda H^4 \\
&= \frac{1}{8}m_H^2v^2 - \frac{1}{2}m_H^2H^2 - \frac{1}{2}\frac{m_H^2}{v}H^3 - \frac{1}{8}\frac{m_H^2}{v^2}H^4,
\end{aligned} \tag{74}$$

其中 Higgs 玻色子的质量为

$$m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \tag{75}$$

利用

$$\begin{aligned}
g'B_\mu + gW_\mu^3 &= g'(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) + g(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu) = 2eA_\mu + \frac{g^2-g'^2}{\sqrt{g^2+g'^2}}Z_\mu \\
&= 2eA_\mu + \frac{g}{c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu,
\end{aligned} \tag{76}$$

有

$$\begin{aligned}
g'B_\mu Y_H + gW_\mu^a T^a &= \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix} \\
&= \begin{pmatrix} eA_\mu + \dfrac{g}{2c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu & \dfrac{1}{\sqrt{2}}gW_\mu^+ \\ \dfrac{1}{\sqrt{2}}gW_\mu^- & -\dfrac{g}{2c_{\mathrm{W}}}Z_\mu \end{pmatrix}.
\end{aligned} \tag{77}$$

于是, 在幺正规范下,

$$\begin{aligned}
&(D^\mu\Phi)^\dagger(D_\mu\Phi) \\
&= \left|\begin{pmatrix} \partial_\mu - ieA_\mu - \dfrac{ig}{2c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu & -\dfrac{i}{\sqrt{2}}gW_\mu^+ \\ -\dfrac{i}{\sqrt{2}}gW_\mu^- & \partial_\mu + \dfrac{ig}{2c_{\mathrm{W}}}Z_\mu \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}\right|^2 \\
&= \frac{1}{2}\left(\frac{i}{\sqrt{2}}gW_\mu^-(v+H),\ \partial_\mu H - \frac{ig}{2c_{\mathrm{W}}}Z_\mu(v+H)\right)\begin{pmatrix} -\dfrac{i}{\sqrt{2}}gW_\mu^+(v+H) \\ \partial_\mu H + \dfrac{ig}{2c_{\mathrm{W}}}Z_\mu(v+H) \end{pmatrix} \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2\left(\frac{g^2}{4}W_\mu^+W^{-\mu} + \frac{g^2}{8c_{\mathrm{W}}^2}Z_\mu Z^\mu\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \\
&\quad + g m_W H W_\mu^+ W^{-\mu} + \frac{g m_Z}{2 c_{\mathrm{W}}} H Z_\mu Z^\mu + \frac{g^2}{4} H^2 W_\mu^+ W^{-\mu} + \frac{g^2}{8 c_{\mathrm{W}}^2} H^2 Z_\mu Z^\mu .
\end{aligned} \tag{78}
$$

故 $W^\pm$ 和 Z 玻色子获得质量, 分别为

$$
m_W \equiv \frac{gv}{2}, \quad m_Z \equiv \frac{gv}{2c_{\mathrm{W}}} = \frac{m_W}{c_{\mathrm{W}}} = \frac{v}{2}\sqrt{g^2 + g'^2}. \tag{79}
$$

$Y = -1/2$ 的 Higgs 场共轭态为

$$
\tilde{\Phi}(x) = i\sigma^2 \Phi^*(x) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}[v + H(x) - i\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \tag{80}
$$

利用它可以写下 Yukawa 耦合项

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Y}} &= -\tilde{y}_d^{ij} \bar{Q}_{i\mathrm{L}} d'_{j\mathrm{R}} \Phi - y_{u_i} \bar{Q}_{i\mathrm{L}} u_{i\mathrm{R}} \tilde{\Phi} - y_{\ell_i} \bar{L}_{i\mathrm{L}} \ell_{i\mathrm{R}} \Phi + h.c. \\
&= -\frac{1}{\sqrt{2}}(v+H) \bar{d}'_{l\mathrm{L}} V_{li}^\dagger \tilde{y}_d^{ij} V_{jk} d'_{k\mathrm{R}} - \frac{y_{u_i}}{\sqrt{2}}(v+H) \bar{u}_{i\mathrm{L}} u_{i\mathrm{R}} - \frac{y_{\ell_i}}{\sqrt{2}}(v+H) \bar{\ell}_{i\mathrm{L}} \ell_{i\mathrm{R}} + h.c. \\
&= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i .
\end{aligned} \tag{81}
$$

这里 CKM 矩阵将 $\tilde{y}_d^{ij}$ 对角化:

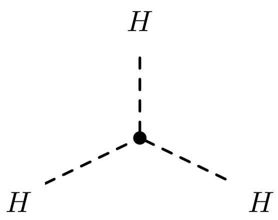
$$
V_{li}^\dagger \tilde{y}_d^{ij} V_{jk} = y_{d_k} \delta_{lk}. \tag{82}
$$

通过 Yukawa 耦合, 费米子获得了质量,

$$
m_{d_i} \equiv \frac{1}{\sqrt{2}} y_{d_i} v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}} y_{u_i} v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}} y_{\ell_i} v. \tag{83}
$$

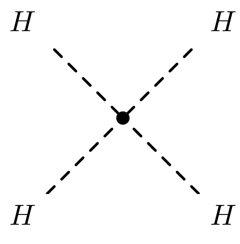
下面给出幺正规范下的顶点费曼规则.

Higgs 玻色子自耦合:



$= -3i\dfrac{m_H^2}{v}$

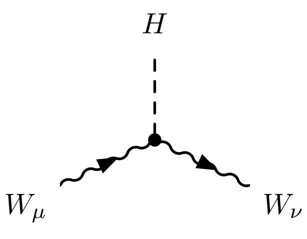
$= -6i\lambda v$



$= -3i\dfrac{m_H^2}{v^2}$

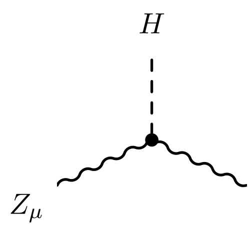
$= -6i\lambda$

Higgs 玻色子与电弱规范玻色子的耦合:



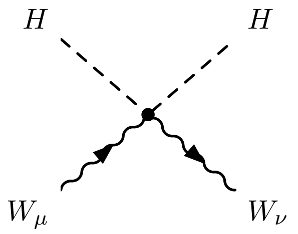
$= i g m_W g^{\mu\nu}$

$= i\dfrac{g^2 v}{2} g^{\mu\nu}$

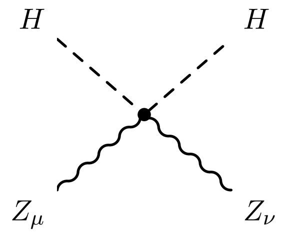


$= i\dfrac{g m_Z}{c_{\mathrm{W}}} g^{\mu\nu}$

$= i\dfrac{g^2 v}{2 c_{\mathrm{W}}^2} g^{\mu\nu}$

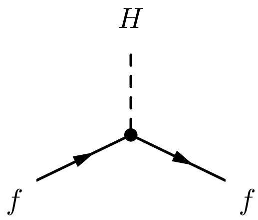


$$= i\frac{g^2}{2}g^{\mu\nu}$$



$$= i\frac{g^2}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$

Higgs 玻色子与费米子的耦合:



$$= -i\frac{m_f}{v}$$
$$= -i\frac{y_f}{\sqrt{2}}$$

7 R_ξ 规范相关拉氏量和费曼规则

将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + i\chi(x)] \end{pmatrix}, \tag{84}$$

其中 ϕ^+ 和 χ 是 Nambu-Goldstone 标量场. 由

$$\begin{aligned} \Phi^\dagger\Phi &= \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \\ (\Phi^\dagger\Phi)^2 &= \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \end{aligned} \tag{85}$$

可得 Higgs 场势能项

$$\begin{aligned} & -V_{\mathrm{H}}(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \\ = & \frac{1}{2}\mu^2(v^2 + H^2 + 2vH + \chi^2) + \mu^2|\phi^+|^2 - \frac{1}{4}\lambda(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda|\phi^+|^4 \\ & -\lambda|\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\ = & \frac{1}{2}\left(\mu^2 - \frac{1}{2}\lambda v^2\right)v^2 + \frac{1}{2}(\mu^2 - 3\lambda v^2)H^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)\chi^2 - \frac{1}{4}\lambda H^4 - \frac{1}{4}\lambda\chi^4 - \lambda vH^3 \\ & -\frac{1}{2}\lambda H^2\chi^2 - \lambda vH\chi^2 + (\mu^2 - \lambda v^2)|\phi^+|^2 - \lambda|\phi^+|^4 - \lambda|\phi^+|^2(H^2 + 2vH + \chi^2) \\ = & \frac{1}{4}\lambda v^4 - \lambda v^2H^2 - \frac{1}{4}\lambda H^4 - \frac{1}{4}\lambda\chi^4 - \lambda vH^3 - \frac{1}{2}\lambda H^2\chi^2 - \lambda vH\chi^2 - \lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2) \\ = & \frac{1}{8}m_H^2v^2 - \frac{1}{2}m_H^2H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4 - \frac{m_H^2}{2v}H\chi^2 - \frac{m_H^2}{4v^2}H^2\chi^2 - \frac{m_H^2}{8v^2}\chi^4 \\ & -\frac{m_H^2}{2v^2}\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2). \end{aligned} \tag{86}$$

由于

$$V_{li}^\dagger\tilde{y}_d^{ij}V_{jk} = y_{d_k}\delta_{lk}, \quad \tilde{y}_d^{ij} = V_{ik}y_{d_k}V_{kj}^\dagger, \tag{87}$$

有

$$
\begin{aligned}
-\tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi &= -\tilde{y}_d^{ij}\left[\bar{u}_{i\mathrm{L}}d'_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}}(v+H+i\chi)\right] \\
&= -\left[\bar{u}_{i\mathrm{L}}V_{ik}y_{d_k}V_{kj}^\dagger V_{jl}d_{l\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\bar{d}_{l\mathrm{L}}V_{li}^\dagger\tilde{y}_d^{ij}V_{jk}d_{k\mathrm{R}}(v+H+i\chi)\right] \\
&= -\left[y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v+H+i\chi)\right], \qquad (88)
\end{aligned}
$$

则 Yukawa 耦合项为

$$
\begin{aligned}
\mathcal{L}_\mathrm{Y} &= -\tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - y_{u_i}\bar{Q}_{i\mathrm{L}}u_{i\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{h.c.} \\
&= -\left[y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v+H+i\chi)\right] - y_{u_i}\left[\frac{1}{\sqrt{2}}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(v+H-i\chi) - \bar{d}_{j\mathrm{L}}V_{ji}^\dagger u_{i\mathrm{R}}\phi^-\right] \\
&\quad -y_{\ell_i}\left[\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(v+H+i\chi)\right] + \mathrm{h.c.} \\
&= -m_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} - m_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} - m_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} - \frac{m_{d_i}}{v}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(H+i\chi) - \frac{m_{u_i}}{v}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(H-i\chi) \\
&\quad -\frac{m_{\ell_i}}{v}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(H+i\chi) - \frac{\sqrt{2}m_{d_j}}{v}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{\sqrt{2}m_{u_i}}{v}\bar{d}_{j\mathrm{L}}V_{ji}^\dagger u_{i\mathrm{R}}\phi^- - \frac{\sqrt{2}m_{\ell_i}}{v}\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ + \mathrm{h.c.} \\
&= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i \\
&\quad -\frac{m_{d_i}}{v}\chi\bar{d}_i i\gamma_5 d_i + \frac{m_{u_i}}{v}\chi\bar{u}_i i\gamma_5 u_i - \frac{m_{\ell_i}}{v}\chi\bar{\ell}_i i\gamma_5\ell_i + \frac{\sqrt{2}V_{ij}}{v}\phi^+\bar{u}_i(m_{u_i}P_\mathrm{L} - m_{d_j}P_\mathrm{R})d_j \\
&\quad -\frac{\sqrt{2}V_{ji}^\dagger}{v}\phi^-\bar{d}_j(m_{d_j}P_\mathrm{L} - m_{u_i}P_\mathrm{R})u_i - \frac{\sqrt{2}m_{\ell_i}}{v}(\phi^+\bar{\nu}_i P_\mathrm{R}\ell_i + \phi^-\bar{\ell}_i P_\mathrm{L}\nu_i). \qquad (89)
\end{aligned}
$$

利用

$$
\begin{aligned}
D_\mu\Phi &= \begin{pmatrix} \partial_\mu - ieA_\mu - \dfrac{ig}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & -\dfrac{i}{\sqrt{2}}gW_\mu^+ \\ -\dfrac{i}{\sqrt{2}}gW_\mu^- & \partial_\mu + \dfrac{ig}{2c_\mathrm{W}}Z_\mu \end{pmatrix}\begin{pmatrix} \phi^+ \\ \dfrac{1}{\sqrt{2}}(v+H+i\chi) \end{pmatrix} \\
&= \begin{pmatrix} \partial_\mu\phi^+ - i\left[eA_\mu + \dfrac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ - \dfrac{ig}{2}W_\mu^+(H+i\chi) - im_W W_\mu^+ \\ \partial_\mu(H+i\chi) - igW_\mu^-\phi^+ + \dfrac{ig}{2c_\mathrm{W}}Z_\mu(H+i\chi) + im_Z Z_\mu \end{pmatrix}, \qquad (90)
\end{aligned}
$$

可将 Higgs 场协变动能项化为

$$
\begin{aligned}
&(D^\mu\Phi)^\dagger D_\mu\Phi \\
&= \left|\partial_\mu\phi^+ - i\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ - \frac{ig}{2}W_\mu^+(H+i\chi) - im_W W_\mu^+\right|^2 \\
&\quad + \frac{1}{2}\left|\partial_\mu(H+i\chi) - igW_\mu^-\phi^+ + \frac{ig}{2c_\mathrm{W}}Z_\mu(H+i\chi) + im_Z Z_\mu\right|^2 \\
&= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi) \\
&\quad + \left(-i\partial^\mu\phi^-\left\{\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H+i\chi) + m_W W_\mu^+\right\} + \mathrm{h.c.}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\left\{-\frac{i}{2}\partial^\mu(H-i\chi)\left[gW_\mu^-\phi^+-\frac{g}{2c_\mathrm{W}}Z_\mu(H+i\chi)-m_ZZ_\mu\right]+\text{h.c.}\right\}\\
&+\left|\left[eA_\mu+\frac{g(c_\mathrm{W}^2-s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^++\frac{g}{2}W_\mu^+(H+i\chi)+m_WW_\mu^+\right|^2\\
&+\frac{1}{2}\left|gW_\mu^-\phi^+-\frac{g}{2c_\mathrm{W}}Z_\mu(H+i\chi)-m_ZZ_\mu\right|^2\\
=\;&(\partial^\mu\phi^+)(\partial_\mu\phi^-)+\frac{1}{2}(\partial^\mu H)(\partial_\mu H)+\frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi)\\
&+m_W^2W^{-\mu}W_\mu^++\frac{1}{2}m_Z^2Z^\mu Z_\mu+gm_WHW_\mu^+W^{-\mu}+\frac{gm_Z}{2c_\mathrm{W}}HZ^\mu Z_\mu\\
&+\frac{g}{2}[W_\mu^+\phi^-i\overleftrightarrow{\partial^\mu}(H+i\chi)+\text{h.c.}]+eA_\mu\phi^-i\overleftrightarrow{\partial^\mu}\phi^++\frac{g}{2c_\mathrm{W}}Z_\mu[i\chi i\overleftrightarrow{\partial^\mu}H+(c_\mathrm{W}^2-s_\mathrm{W}^2)\phi^-i\overleftrightarrow{\partial^\mu}\phi^+]\\
&+\frac{g^2}{4}W_\mu^+W^{-\mu}(2\phi^+\phi^-+H^2+\chi^2)+e^2A_\mu A^\mu\phi^+\phi^-+\frac{g^2}{4c_\mathrm{W}^2}Z_\mu Z^\mu\left[(c_\mathrm{W}^2-s_\mathrm{W}^2)^2\phi^+\phi^-+\frac{1}{2}H^2+\frac{1}{2}\chi^2\right]\\
&+\left[\frac{eg}{2}W_\mu^+A^\mu\phi^-(H+i\chi)-\frac{g^2s_\mathrm{W}^2}{2c_\mathrm{W}}W_\mu^+Z^\mu\phi^-(H+i\chi)+\text{h.c.}\right]+\frac{eg}{c_\mathrm{W}}(c_\mathrm{W}^2-s_\mathrm{W}^2)A_\mu Z^\mu\phi^+\phi^-\\
&+(em_WA^\mu\phi^+W_\mu^--gs_\mathrm{W}^2m_ZZ^\mu\phi^+W_\mu^-+\text{h.c.})+\mathcal{L}_\mathrm{b1},
\end{aligned}
\tag{91}
$$

其中

$$
\mathcal{L}_\mathrm{b1}=-im_W(\partial^\mu\phi^-)W_\mu^++im_W(\partial^\mu\phi^+)W_\mu^-+m_Z(\partial^\mu\chi)Z_\mu. \tag{92}
$$

R_ξ 规范的规范固定函数设为

$$
G^\pm=\frac{1}{\sqrt{\xi}}(\partial^\mu W_\mu^\pm\mp i\xi m_W\phi^\pm),\quad G^Z=\frac{1}{\sqrt{\xi}}(\partial^\mu Z_\mu-\xi m_Z\chi),\quad G^\gamma=\frac{1}{\sqrt{\xi}}\partial^\mu A_\mu, \tag{93}
$$

它们在路径积分量子化中的泛函积分形式为

$$
\begin{aligned}
&\int\mathcal{D}\omega^+\int\mathcal{D}\omega^-\int\mathcal{D}\omega^Z\int\mathcal{D}\omega^\gamma\exp\left[-i\int d^4x\left(\omega^+\omega^-+\frac{1}{2}\omega^Z\omega^Z+\frac{1}{2}\omega^\gamma\omega^\gamma\right)\right]\\
&\qquad\times\delta(G^+-\omega^+)\delta(G^--\omega^-)\delta(G^Z-\omega^Z)\delta(G^\gamma-\omega^\gamma)\\
=\;&\exp\left[-i\int d^4x\left(G^+G^-+\frac{1}{2}G^ZG^Z+\frac{1}{2}G^\gamma G^\gamma\right)\right].
\end{aligned}
\tag{94}
$$

由此可得拉氏量中的规范固定项

$$
\begin{aligned}
\mathcal{L}_\mathrm{EW,GF}=\;&-G^+G^--\frac{1}{2}(G^Z)^2-\frac{1}{2}(G^\gamma)^2\\
=\;&-\frac{1}{\xi}(\partial^\mu W_\mu^+-i\xi m_W\phi^+)(\partial^\nu W_\nu^-+i\xi m_W\phi^-)-\frac{1}{2\xi}(\partial^\mu Z_\mu-\xi m_Z\chi)^2-\frac{1}{2\xi}(\partial^\mu A_\mu)^2\\
=\;&-\frac{1}{\xi}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-)-\frac{1}{2\xi}(\partial^\mu Z_\mu)^2-\frac{1}{2\xi}(\partial^\mu A_\mu)^2-\xi m_W^2\phi^+\phi^--\frac{1}{2}\xi m_Z^2\chi^2+\mathcal{L}_\mathrm{b2}.
\end{aligned}
\tag{95}
$$

可见, Nambu-Goldstone 玻色子在 R_ξ 规范下具有依赖于 ξ 的非物理质量:

$$
m_\phi=\sqrt{\xi}m_W,\quad m_\chi=\sqrt{\xi}m_Z. \tag{96}
$$

这里

$$
\mathcal{L}_\mathrm{b2}=-im_W\phi^-(\partial^\mu W_\mu^+)+im_W\phi^+\partial^\mu W_\mu^-+m_Z\chi\partial^\mu Z_\mu. \tag{97}
$$

由于

$$\mathcal{L}_{\rm b1}+\mathcal{L}_{\rm b2}=-im_W\partial^\mu(\phi^-W^+_\mu)+im_W\partial^\mu(\phi^+W^-_\mu)+m_Z\partial^\mu(\chi Z_\mu), \tag{98}$$

这两项体现为全散度, 不会有物理效应. 可见, 协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 $\mathcal{L}_{\rm b1}$ 被规范固定项中的 $\mathcal{L}_{\rm b2}$ 抵消掉, 这就是如此选取规范固定函数的目的.

这样一来, 电弱规范场传播子相关拉氏量变成

$$\begin{aligned}
\mathcal{L}_{\rm EW,prop} \ =\ & (\partial_\mu W^+_\nu)(\partial^\nu W^{-\mu})-(\partial_\mu W^+_\nu)(\partial^\mu W^{-\nu})-\frac{1}{\xi}(\partial^\mu W^+_\mu)(\partial^\nu W^-_\nu)+m_W^2W^{-\mu}W^+_\mu\\
&+\frac{1}{2}\left[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu)-(\partial_\mu Z_\nu)(\partial^\mu Z^\nu)-\frac{1}{\xi}(\partial^\mu Z_\mu)^2+m_Z^2Z^\mu Z_\mu\right]\\
&+\frac{1}{2}\left[(\partial_\mu A_\nu)(\partial^\nu A^\mu)-(\partial_\mu A_\nu)(\partial^\mu A^\nu)-\frac{1}{\xi}(\partial^\mu A_\mu)^2\right]\\
\to\ & W^+_\mu\left[g^{\mu\nu}(\partial^2+m_W^2)-\left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]W^-_\nu+\frac{1}{2}Z_\mu\left[g^{\mu\nu}(\partial^2+m_Z^2)-\left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]Z_\nu\\
&+\frac{1}{2}A_\mu\left[g^{\mu\nu}\partial-\left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]A_\nu.
\end{aligned} \tag{99}$$

于是, 光子的传播子与胶子形式类似, 为

$$\frac{-i}{p^2+i\varepsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}(1-\xi)\right]. \tag{100}$$

将 $W^\pm$ 传播子相关拉氏量变换到动量空间, 得

$$-g^{\mu\nu}(p^2-m_W^2)+\left(1-\frac{1}{\xi}\right)p^\mu p^\nu=-\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)(p^2-m_W^2)-\frac{p^\mu p^\nu}{p^2}\frac{p^2-\xi m_W^2}{\xi}, \tag{101}$$

它的逆矩阵是

$$-\frac{1}{p^2-m_W^2}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)-\frac{\xi}{p^2-\xi m_W^2}\frac{p_\mu p_\nu}{p^2}=-\frac{1}{p^2-m_W^2}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi m_W^2}(1-\xi)\right], \tag{102}$$

这是因为由

$$\left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)\frac{p^\mu p^\nu}{p^2}=\frac{p_\rho p^\nu}{p^2}-\frac{p_\rho p^\nu}{p^2}=0,\quad \left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)=\delta^\nu_\rho-\frac{p_\rho p^\nu}{p^2}, \tag{103}$$

可得

$$\begin{aligned}
&\left[-\frac{1}{p^2-m_W^2}\left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)-\frac{\xi}{p^2-\xi m_W^2}\frac{p_\rho p_\mu}{p^2}\right]\left[-\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)(p^2-m_W^2)-\frac{p^\mu p^\nu}{p^2}\frac{p^2-\xi m_W^2}{\xi}\right]\\
=\ &\delta^\nu_\rho-\frac{p_\rho p^\nu}{p^2}+\frac{p_\rho p^\nu}{p^2}=\delta^\nu_\rho.
\end{aligned} \tag{104}$$

从而, $W^\pm$ 传播子的形式为

$$\frac{-i}{p^2-m_W^2+i\varepsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi m_W^2}(1-\xi)\right]. \tag{105}$$

同理, Z 传播子的形式为

$$\frac{-i}{p^2-m_Z^2+i\varepsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi m_Z^2}(1-\xi)\right]. \tag{106}$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a=\frac{1}{g}\partial_\mu\alpha^a+\varepsilon^{abc}W_\mu^b\alpha^c, \quad \delta B_\mu=\frac{1}{g'}\partial_\mu\alpha^Y. \tag{107}$$

定义

$$\alpha^\pm\equiv\frac{1}{\sqrt{2}}(\alpha^1\mp i\alpha^2), \quad \alpha^Z\equiv\alpha^3-\alpha^Y, \quad \alpha^\gamma\equiv s_\mathrm{W}^2\alpha^3+c_\mathrm{W}^2\alpha^Y, \tag{108}$$

利用

$$\varepsilon^{1bc}W_\mu^b\alpha^c = W_\mu^2\alpha^3-W_\mu^3\alpha^2, \quad \varepsilon^{2bc}W_\mu^b\alpha^c=-W_\mu^1\alpha^3+W_\mu^3\alpha^1, \tag{109}$$

$$\pm i\sqrt{2}\alpha^\pm = \pm i\alpha^1+\alpha^2, \quad \pm i\sqrt{2}W_\mu^\pm=\pm iW_\mu^1+W_\mu^2, \tag{110}$$

有

$$\begin{aligned}\varepsilon^{1bc}W_\mu^b\alpha^c\mp i\varepsilon^{2bc}W_\mu^b\alpha^c &= (W_\mu^2\alpha^3-W_\mu^3\alpha^2)\mp i(-W_\mu^1\alpha^3+W_\mu^3\alpha^1)=(W_\mu^2\pm iW_\mu^1)\alpha^3-W_\mu^3(\alpha^2\pm i\alpha^1)\\ &= \pm i\sqrt{2}W_\mu^\pm(c_\mathrm{W}^2\alpha^Z+\alpha^\gamma)\mp i\sqrt{2}(s_\mathrm{W}A_\mu+c_\mathrm{W}Z_\mu)\alpha^\pm, \end{aligned}\tag{111}$$

$$\begin{aligned}\varepsilon^{3bc}W_\mu^b\alpha^c &= W_\mu^1\alpha^2-W_\mu^2\alpha^1=\frac{1}{\sqrt{2}}(W_\mu^++W_\mu^-)\frac{i}{\sqrt{2}}(\alpha^+-\alpha^-)-\frac{i}{\sqrt{2}}(W_\mu^+-W_\mu^-)\frac{1}{\sqrt{2}}(\alpha^++\alpha^-)\\ &= -i(W_\mu^+\alpha^--W_\mu^-\alpha^+). \end{aligned}\tag{112}$$

因此,

$$\begin{aligned}\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1-i\delta W_\mu^2)=\frac{1}{\sqrt{2}g}\partial_\mu(\alpha^1-i\alpha^2)+\frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c-i\varepsilon^{2bc}W_\mu^b\alpha^c)\\ &= \frac{1}{g}\partial_\mu\alpha^+-i(s_\mathrm{W}A_\mu+c_\mathrm{W}Z_\mu)\alpha^++iW_\mu^+(c_\mathrm{W}^2\alpha^Z+\alpha^\gamma), \end{aligned}\tag{113}$$

$$\delta W_\mu^- = (\delta W_\mu^+)^\dagger=\frac{1}{g}\partial_\mu\alpha^-+i(s_\mathrm{W}A_\mu+c_\mathrm{W}Z_\mu)\alpha^--iW_\mu^-(c_\mathrm{W}^2\alpha^Z+\alpha^\gamma), \tag{114}$$

$$\delta Z_\mu^a=c_\mathrm{W}\delta W_\mu^3-s_\mathrm{W}\delta B_\mu=\frac{c_\mathrm{W}}{g}\partial_\mu\alpha^3+c_\mathrm{W}\varepsilon^{3bc}W_\mu^b\alpha^c-\frac{s_\mathrm{W}}{g'}\partial_\mu\alpha^Y=\frac{c_\mathrm{W}}{g}\partial_\mu\alpha^Z-ic_\mathrm{W}(W_\mu^+\alpha^--W_\mu^-\alpha^+), \tag{115}$$

$$\delta A_\mu=s_\mathrm{W}\delta W_\mu^3+c_\mathrm{W}\delta B_\mu=\frac{s_\mathrm{W}}{g}\partial_\mu\alpha^3+s_\mathrm{W}\varepsilon^{3bc}W_\mu^b\alpha^c+\frac{c_\mathrm{W}}{g'}\partial_\mu\alpha^Y=\frac{1}{e}\partial_\mu\alpha^\gamma-is_\mathrm{W}(W_\mu^+\alpha^--W_\mu^-\alpha^+). \tag{116}$$

另一方面, 由

$$\alpha^aT^a+\alpha^YY_H=\frac{1}{2}(\alpha^a\sigma^a+\alpha^Y)=\frac{1}{2}\begin{pmatrix}\alpha^3+\alpha^Y & \alpha^1-i\alpha^2\\ \alpha^1+i\alpha^2 & -\alpha^3+\alpha^Y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}2\alpha^\gamma+(c_\mathrm{W}^2-s_\mathrm{W}^2)\alpha^Z & \sqrt{2}\alpha^+\\ \sqrt{2}\alpha^- & -\alpha^Z\end{pmatrix}, \tag{117}$$

可知 Higgs 场的无穷小规范变换形式为

$$\delta\Phi = i(\alpha^aT^a+\alpha^YY_H)\Phi=\frac{i}{2}\begin{pmatrix}2\alpha^\gamma+(c_\mathrm{W}^2-s_\mathrm{W}^2)\alpha^Z & \sqrt{2}\alpha^+\\ \sqrt{2}\alpha^- & -\alpha^Z\end{pmatrix}\begin{pmatrix}\phi^+\\ \frac{1}{\sqrt{2}}(v+H+i\chi)\end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{i}{2}[\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H+i\chi)\alpha^+] \\ \dfrac{1}{\sqrt{2}}\left[i\phi^+\alpha^- - \dfrac{1}{2}(iv + iH - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \dfrac{1}{\sqrt{2}}(\delta H + i\delta\chi) \end{pmatrix}. \tag{118}$$

根据

$$\mathrm{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \mathrm{Im}(\phi^+\alpha^-) = -\frac{i}{2}(\phi^+\alpha^- - \phi^-\alpha^+), \tag{119}$$

可得

$$\delta\phi^+ = \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H+i\chi)\alpha^+\}, \tag{120}$$

$$\delta\phi^- = -\frac{i}{2}\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H-i\chi)\alpha^-\}, \tag{121}$$

$$\delta H = \frac{1}{2}[i(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v+H)\alpha^Z]. \tag{122}$$

于是, 规范固定函数的无穷小规范变换为

$$\begin{aligned}\sqrt{\xi}\delta G^+ &= \partial^\mu \delta W_\mu^+ - i\xi m_W \delta\phi^+ = \partial^\mu \left[\frac{1}{g}\partial_\mu\alpha^+ - i(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^+ + iW_\mu^+(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad + \frac{1}{2}\xi m_W\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H+i\chi)\alpha^+\}, \end{aligned} \tag{123}$$

$$\begin{aligned}\sqrt{\xi}\delta G^- &= \partial^\mu \delta W_\mu^- + i\xi m_W \delta\phi^- = \partial^\mu \left[\frac{1}{g}\partial_\mu\alpha^- + i(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^- - iW_\mu^-(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad + \frac{1}{2}\xi m_W\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H-i\chi)\alpha^-\}, \end{aligned} \tag{124}$$

$$\begin{aligned}\sqrt{\xi}\delta G^Z &= \partial^\mu \delta Z_\mu - \xi m_Z \delta\chi = \partial^\mu \left[\frac{c_\mathrm{W}}{g}\partial_\mu\alpha^Z - ic_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right] \\ &\quad - \frac{1}{2}\xi m_Z[\phi^+\alpha^- + \phi^-\alpha^+ - (v+H)\alpha^Z], \end{aligned} \tag{125}$$

$$\sqrt{\xi}\delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu \left[\frac{1}{e}\partial_\mu\alpha^\gamma - is_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right]. \tag{126}$$

因此,

$$\sqrt{\xi}g\frac{\delta G^+}{\delta\alpha^+} = \partial^2 + \xi m_W^2 - ie\partial^\mu A_\mu - igc_\mathrm{W}\partial^\mu Z_\mu + \frac{1}{2}g\xi m_W(H + i\chi), \tag{127}$$

$$\frac{\sqrt{\xi}g}{c_\mathrm{W}}\frac{\delta G^+}{\delta\alpha^Z} = igc_\mathrm{W}\partial^\mu W_\mu^+ + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi m_W}{2c_\mathrm{W}}\phi^+, \quad \sqrt{\xi}e\frac{\delta G^+}{\delta\alpha^\gamma} = ie\partial^\mu W_\mu^+ + e\xi m_W\phi^+, \tag{128}$$

$$\sqrt{\xi}g\frac{\delta G^-}{\delta\alpha^-} = \partial^2 + \xi m_W^2 + ie\partial^\mu A_\mu + igc_\mathrm{W}\partial^\mu Z_\mu + \frac{1}{2}\xi g m_W(H - i\chi), \tag{129}$$

$$\frac{\sqrt{\xi}g}{c_\mathrm{W}}\frac{\delta G^-}{\delta\alpha^Z} = -igc_\mathrm{W}\partial^\mu W_\mu^- + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi m_W}{2c_\mathrm{W}}\phi^-, \quad \sqrt{\xi}e\frac{\delta G^-}{\delta\alpha^\gamma} = -ie\partial^\mu W_\mu^- + e\xi m_W\phi^-, \tag{130}$$

$$\sqrt{\xi}g\frac{\delta G^Z}{\delta\alpha^+} = igc_\mathrm{W}\partial^\mu W_\mu^- - \frac{1}{2}g\xi m_Z\phi^-, \quad \sqrt{\xi}g\frac{\delta G^Z}{\delta\alpha^-} = -igc_\mathrm{W}\partial^\mu W_\mu^+ - \frac{1}{2}g\xi m_Z\phi^+, \tag{131}$$

$$\frac{\sqrt{\xi}g}{c_\mathrm{W}}\frac{\delta G^Z}{\delta\alpha^Z} = \partial^2 + \xi m_Z^2 + \frac{g\xi m_Z}{2c_\mathrm{W}}H, \tag{132}$$

$$\sqrt{\xi}g\frac{\delta G^\gamma}{\delta\alpha^+} = ie\partial^\mu W_\mu^-, \quad \sqrt{\xi}g\frac{\delta G^\gamma}{\delta\alpha^-} = -ie\partial^\mu W_\mu^+, \quad \sqrt{\xi}e\frac{\delta G^\gamma}{\delta\alpha^\gamma} = \partial^2. \tag{133}$$

最后, 得到以下 Faddeev-Popov 鬼场拉氏量:

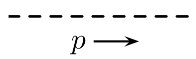
$$
\begin{aligned}
\mathcal{L}_{\mathrm{EWG,FP}} =& -\bar{\eta}^{+}\left(\sqrt{\xi}g\frac{\delta G^{+}}{\delta\alpha^{+}}\right)\eta^{+}-\bar{\eta}^{Z}\left(\sqrt{\xi}g\frac{\delta G^{Z}}{\delta\alpha^{+}}\right)\eta^{+}-\bar{\eta}^{\gamma}\left(\sqrt{\xi}g\frac{\delta G^{\gamma}}{\delta\alpha^{+}}\right)\eta^{+}-\bar{\eta}^{-}\left(\sqrt{\xi}g\frac{\delta G^{-}}{\delta\alpha^{-}}\right)\eta^{-}\\
&-\bar{\eta}^{Z}\left(\sqrt{\xi}g\frac{\delta G^{Z}}{\delta\alpha^{-}}\right)\eta^{-}-\bar{\eta}^{\gamma}\left(\sqrt{\xi}g\frac{\delta G^{\gamma}}{\delta\alpha^{-}}\right)\eta^{-}-\bar{\eta}^{Z}\left(\frac{\sqrt{\xi}g}{c_{\mathrm{W}}}\frac{\delta G^{Z}}{\delta\alpha^{Z}}\right)\eta^{Z}-\bar{\eta}^{+}\left(\frac{\sqrt{\xi}g}{c_{\mathrm{W}}}\frac{\delta G^{+}}{\delta\alpha^{Z}}\right)\eta^{Z}\\
&-\bar{\eta}^{-}\left(\frac{\sqrt{\xi}g}{c_{\mathrm{W}}}\frac{\delta G^{-}}{\delta\alpha^{Z}}\right)\eta^{Z}-\bar{\eta}^{\gamma}\left(\sqrt{\xi}e\frac{\delta G^{\gamma}}{\delta\alpha^{\gamma}}\right)\eta^{\gamma}-\bar{\eta}^{+}\left(\sqrt{\xi}e\frac{\delta G^{+}}{\delta\alpha^{\gamma}}\right)\eta^{\gamma}-\bar{\eta}^{-}\left(\sqrt{\xi}e\frac{\delta G^{-}}{\delta\alpha^{\gamma}}\right)\eta^{\gamma}\\
=& \;\bar{\eta}^{+}\left[-\partial^{2}-\xi m_{W}^{2}-ie\overleftarrow{\partial^{\mu}}A_{\mu}-igc_{\mathrm{W}}\overleftarrow{\partial^{\mu}}Z_{\mu}-\frac{1}{2}g\xi m_{W}(H+i\chi)\right]\eta^{+}\\
&+\bar{\eta}^{Z}\left(igc_{\mathrm{W}}\overleftarrow{\partial^{\mu}}W_{\mu}^{-}+\frac{1}{2}g\xi m_{Z}\phi^{-}\right)\eta^{+}+ie(\partial^{\mu}\bar{\eta}^{\gamma})W_{\mu}^{-}\eta^{+}\\
&+\bar{\eta}^{-}\left[-\partial^{2}-\xi m_{W}^{2}+ie\overleftarrow{\partial^{\mu}}A_{\mu}+igc_{\mathrm{W}}\overleftarrow{\partial^{\mu}}Z_{\mu}-\frac{1}{2}g\xi m_{W}(H-i\chi)\right]\eta^{-}\\
&+\bar{\eta}^{Z}\left(-igc_{\mathrm{W}}\overleftarrow{\partial^{\mu}}W_{\mu}^{+}+\frac{1}{2}g\xi m_{Z}\phi^{+}\right)\eta^{-}-ie(\partial^{\mu}\bar{\eta}^{\gamma})W_{\mu}^{+}\eta^{-}\\
&+\bar{\eta}^{Z}\left(-\partial^{2}-\xi m_{Z}^{2}-\frac{g\xi m_{Z}}{2c_{\mathrm{W}}}H\right)\eta^{Z}+\bar{\eta}^{+}\left(igc_{\mathrm{W}}\overleftarrow{\partial^{\mu}}W_{\mu}^{+}-\frac{g(c_{\mathrm{W}}^{2}-s_{\mathrm{W}}^{2})\xi m_{W}}{2c_{\mathrm{W}}}\phi^{+}\right)\eta^{Z}\\
&+\bar{\eta}^{-}\left(-igc_{\mathrm{W}}\overleftarrow{\partial^{\mu}}W_{\mu}^{-}-\frac{g(c_{\mathrm{W}}^{2}-s_{\mathrm{W}}^{2})\xi m_{W}}{2c_{\mathrm{W}}}\phi^{-}\right)\eta^{Z}\\
&-\bar{\eta}^{\gamma}\partial^{2}\eta^{\gamma}+\bar{\eta}^{+}(ie\overleftarrow{\partial^{\mu}}W_{\mu}^{+}-e\xi m_{W}\phi^{+})\eta^{\gamma}+\bar{\eta}^{-}(-ie\overleftarrow{\partial^{\mu}}W_{\mu}^{-}-e\xi m_{W}\phi^{-})\eta^{\gamma}.
\end{aligned}
\tag{134}
$$

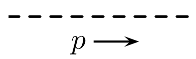
鬼粒子的质量为

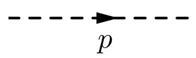
$$
m_{\eta^{+}}=m_{\eta^{-}}=\sqrt{\xi}m_{W},\quad m_{\eta^{Z}}=\sqrt{\xi}m_{Z},\quad m_{\eta^{\gamma}}=0. \tag{135}
$$

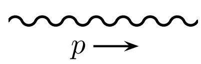
下面给出 R_ξ 规范下的费曼规则. $\xi=1$ 对应 Feynman-'t Hooft 规范, $\xi=0$ 对应 Landau 规范, $\xi\to\infty$ 对应幺正规范.

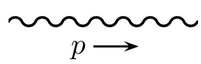
传播子:

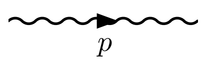
H  $H \quad = \dfrac{i}{p^2-m_H^2+i\varepsilon}$

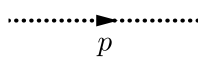
χ  $\chi \quad = \dfrac{i}{p^2-\xi m_Z^2+i\varepsilon}$

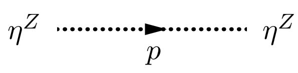
ϕ  $\phi \quad = \dfrac{i}{p^2-\xi m_W^2+i\varepsilon}$

A_μ  $A_\nu \quad = \dfrac{-i}{p^2+i\varepsilon}\left[g_{\mu\nu}-\dfrac{p_\mu p_\nu}{p^2}(1-\xi)\right]$

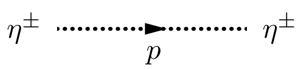
Z_μ  $Z_\nu \quad = \dfrac{-i}{p^2-m_Z^2+i\varepsilon}\left[g_{\mu\nu}-\dfrac{p_\mu p_\nu}{p^2-\xi m_Z^2}(1-\xi)\right]$

W_μ  $W_\nu \quad = \dfrac{-i}{p^2-m_W^2+i\varepsilon}\left[g_{\mu\nu}-\dfrac{p_\mu p_\nu}{p^2-\xi m_W^2}(1-\xi)\right]$

η^γ  $\eta^\gamma \quad = \dfrac{i}{p^2+i\varepsilon}$

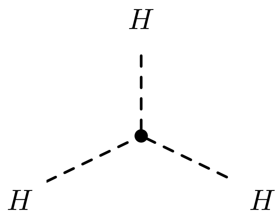


$$= \frac{i}{p^2 - \xi m_Z^2 + i\varepsilon}$$

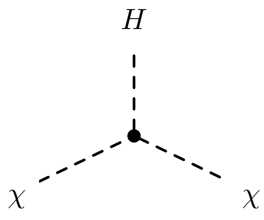


$$= \frac{i}{p^2 - \xi m_W^2 + i\varepsilon}$$

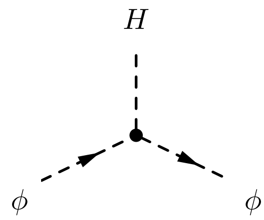
标量玻色子三线性耦合:



$$= -3i\frac{m_H^2}{v}$$
$$= -6i\lambda v$$

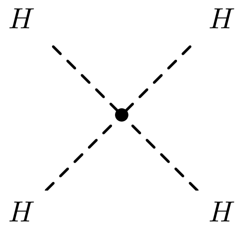


$$= -i\frac{m_H^2}{v}$$
$$= -2i\lambda v$$

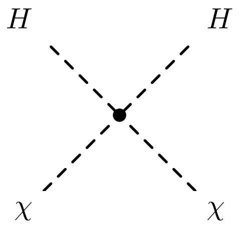


$$= -i\frac{m_H^2}{v}$$
$$= -2i\lambda v$$

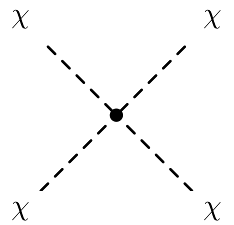
标量玻色子四线性耦合:



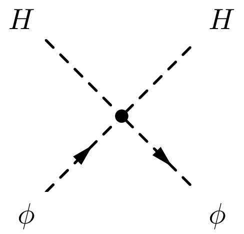
$$= -3i\frac{m_H^2}{v^2}$$
$$= -6i\lambda$$



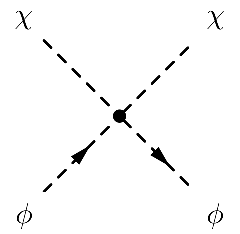
$$= -i\frac{m_H^2}{v^2}$$
$$= -2i\lambda$$



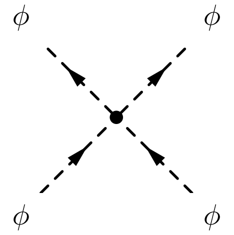
$$= -3i\frac{m_H^2}{v^2}$$
$$= -6i\lambda$$



$$= -i\frac{m_H^2}{v^2}$$
$$= -2i\lambda$$

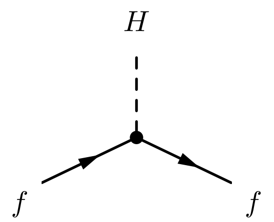


$$= -i\frac{m_H^2}{v^2}$$
$$= -2i\lambda$$

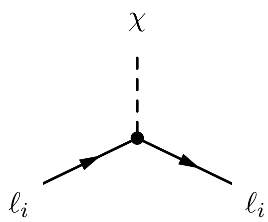


$$= -2i\frac{m_H^2}{v^2}$$
$$= -4i\lambda$$

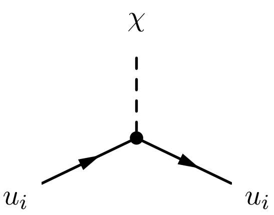
Yukawa 耦合:



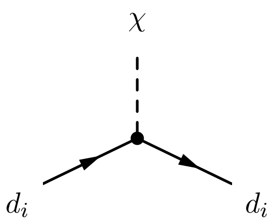
$$= -i\frac{m_f}{v}$$
$$= -i\frac{y_f}{\sqrt{2}}$$



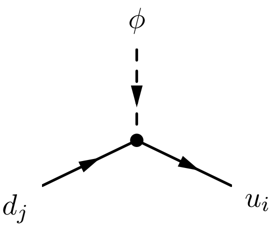
$$= \frac{m_{\ell_i}}{v}\gamma_5$$
$$= \frac{y_{\ell_i}}{\sqrt{2}}\gamma_5$$



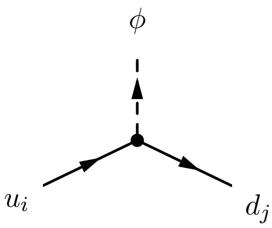
$$= -\frac{m_{u_i}}{v}\gamma_5$$
$$= -\frac{y_{u_i}}{\sqrt{2}}\gamma_5$$



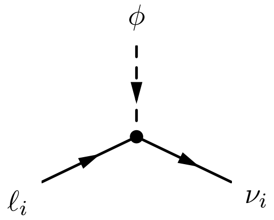
$$= \frac{m_{d_i}}{v}\gamma_5$$
$$= \frac{y_{d_i}}{\sqrt{2}}\gamma_5$$



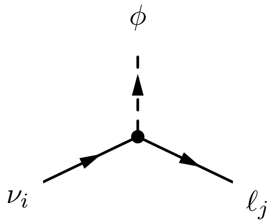
$$= i\frac{\sqrt{2}V_{ij}}{v}(m_{u_i}P_{\mathrm{L}} - m_{d_j}P_{\mathrm{R}})$$
$$= iV_{ij}(y_{u_i}P_{\mathrm{L}} - y_{d_j}P_{\mathrm{R}})$$



$$= -i\frac{\sqrt{2}V_{ji}^\dagger}{v}(m_{d_j}P_{\mathrm{L}} - m_{u_i}P_{\mathrm{R}})$$
$$= -iV_{ji}^\dagger(y_{d_j}P_{\mathrm{L}} - y_{u_i}P_{\mathrm{R}})$$

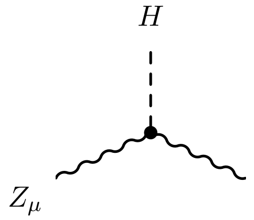


$$= -i\frac{\sqrt{2}m_{\ell_i}}{v}P_{\mathrm{R}}$$
$$= -iy_{\ell_i}P_{\mathrm{R}}$$

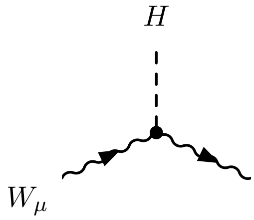


$$= -i\frac{\sqrt{2}m_{\ell_i}}{v}P_{\mathrm{L}}$$
$$= -iy_{\ell_i}P_{\mathrm{L}}$$

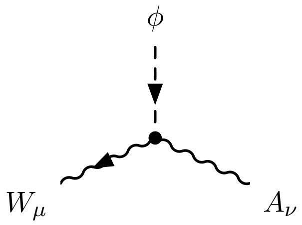
标量玻色子与电弱规范玻色子的三线性耦合:



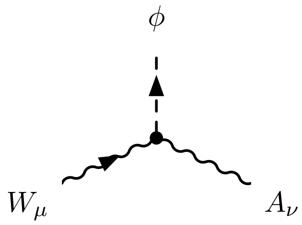
$$= i\frac{gm_Z}{c_{\mathrm{W}}}g^{\mu\nu}$$
$$= i\frac{g^2v}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$



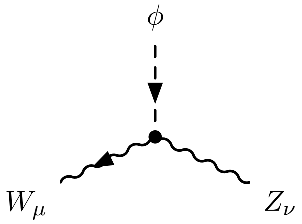
$$= igm_Wg^{\mu\nu}$$
$$= i\frac{g^2v}{2}g^{\mu\nu}$$



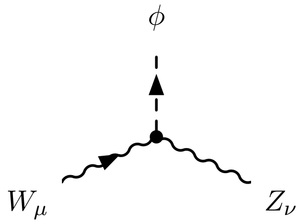
$$= iem_Wg^{\mu\nu}$$
$$= i\frac{egv}{2}g^{\mu\nu}$$



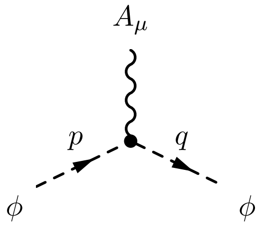
$$= iem_Wg^{\mu\nu}$$
$$= i\frac{egv}{2}g^{\mu\nu}$$



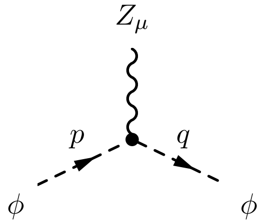
$$= -igs_{\mathrm{W}}^2m_Zg^{\mu\nu}$$
$$= -i\frac{g^2s_{\mathrm{W}}^2v}{2c_{\mathrm{W}}}g^{\mu\nu}$$



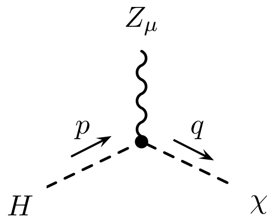
$$= -igs_{\mathrm{W}}^2m_Zg^{\mu\nu}$$
$$= -i\frac{g^2s_{\mathrm{W}}^2v}{2c_{\mathrm{W}}}g^{\mu\nu}$$



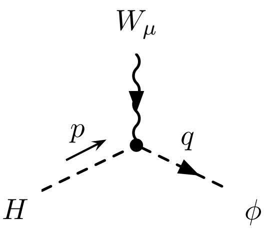
$$= ie(p+q)^\mu$$



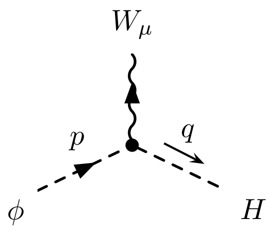
$$= i\frac{g(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}(p+q)^\mu$$



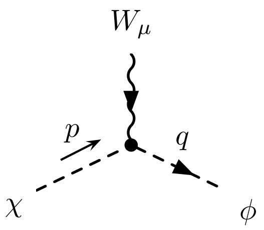
$$= -\frac{g}{2c_{\mathrm{W}}}(p+q)^\mu$$



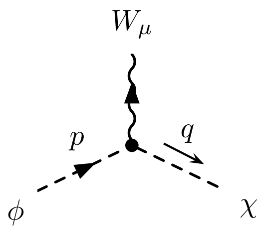
$$= i\frac{g}{2}(p+q)^\mu$$



$$= i\frac{g}{2}(p+q)^\mu$$

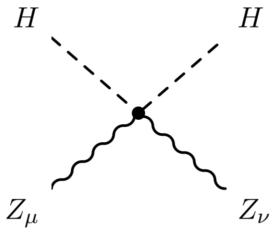


$$= -\frac{g}{2}(p+q)^\mu$$

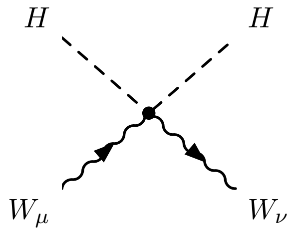


$$= \frac{g}{2}(p+q)^\mu$$

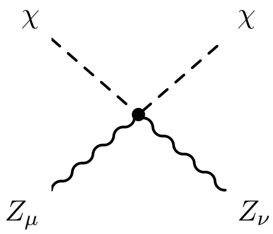
标量玻色子与电弱规范玻色子的四线性耦合:



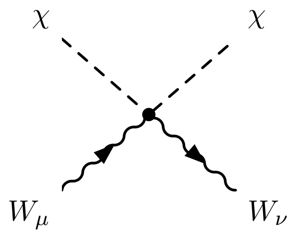
$$= i\frac{g^2}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$



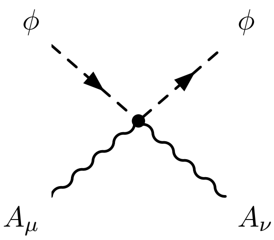
$$= i\frac{g^2}{2}g^{\mu\nu}$$



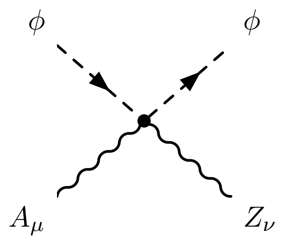
$$= i\frac{g^2}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$



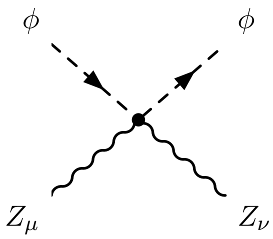
$$= i\frac{g^2}{2}g^{\mu\nu}$$



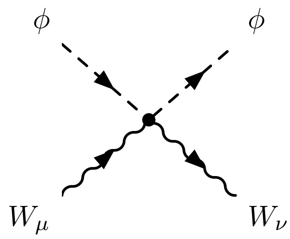
$$= 2ie^2g^{\mu\nu}$$



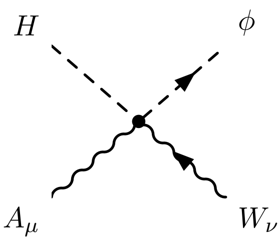
$$= i\frac{eg(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{c_{\mathrm{W}}}g^{\mu\nu}$$



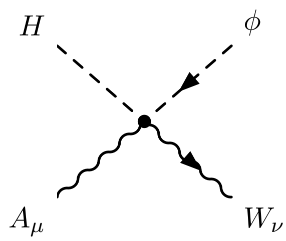
$$= i\frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)^2}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$



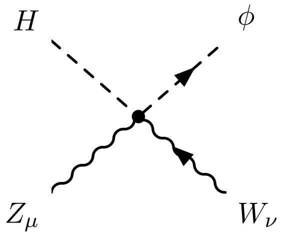
$$= i\frac{g^2}{2}g^{\mu\nu}$$



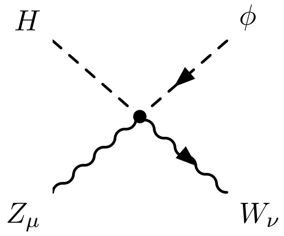
$$= i\frac{eg}{2}g^{\mu\nu}$$



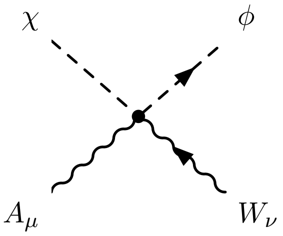
$$= i\frac{eg}{2}g^{\mu\nu}$$



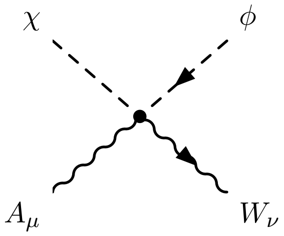
$$= -i\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}} g^{\mu\nu}$$



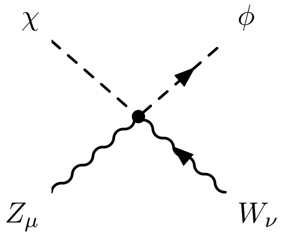
$$= -i\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}} g^{\mu\nu}$$



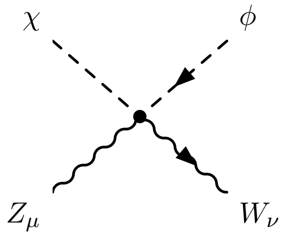
$$= -\frac{eg}{2} g^{\mu\nu}$$



$$= \frac{eg}{2} g^{\mu\nu}$$

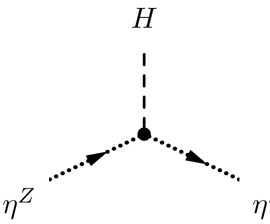


$$= \frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}} g^{\mu\nu}$$

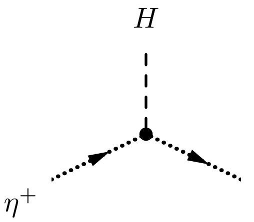


$$= -\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}} g^{\mu\nu}$$

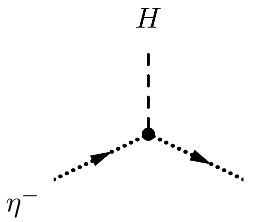
鬼粒子与标量玻色子的耦合:



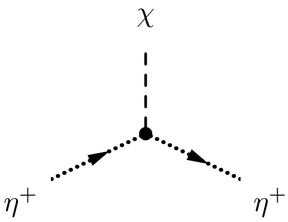
$$= -i\frac{g\xi m_Z}{2c_{\mathrm{W}}}$$
$$= -i\frac{g^2\xi v}{4c_{\mathrm{W}}^2}$$



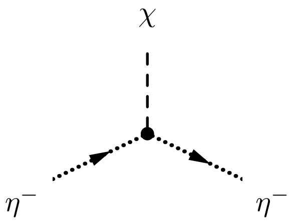
$$= -i\frac{g\xi m_W}{2}$$
$$= -i\frac{g^2\xi v}{4}$$



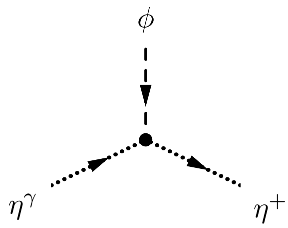
$$= -i\frac{g\xi m_W}{2}$$
$$= -i\frac{g^2\xi v}{4}$$



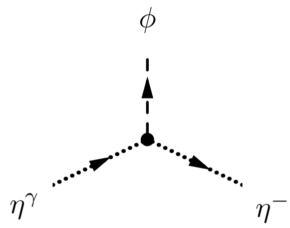
$$= \frac{g\xi m_W}{2}$$
$$= \frac{g^2\xi v}{4}$$



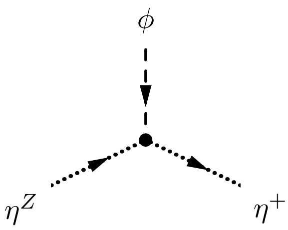
$$= -\frac{g\xi m_W}{2}$$
$$= -\frac{g^2\xi v}{4}$$



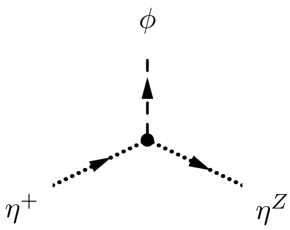
$$= -ie\xi m_W$$
$$= -i\frac{eg\xi v}{2}$$



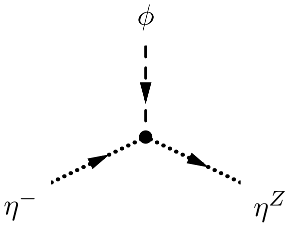
$$= -ie\xi m_W$$
$$= -i\frac{eg\xi v}{2}$$



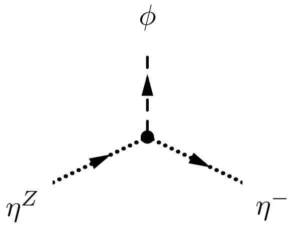
$$= -i\frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi m_W}{2c_{\mathrm{W}}}$$
$$= -i\frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi v}{4c_{\mathrm{W}}}$$



$$= i\frac{g\xi m_Z}{2}$$
$$= i\frac{g^2\xi v}{4c_{\mathrm{W}}}$$

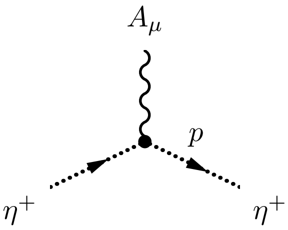


$$= i\frac{g\xi m_Z}{2}$$
$$= i\frac{g^2\xi v}{4c_{\mathrm{W}}}$$

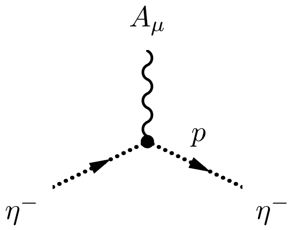


$$= -i\frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi m_W}{2c_{\mathrm{W}}}$$
$$= -i\frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi v}{4c_{\mathrm{W}}}$$

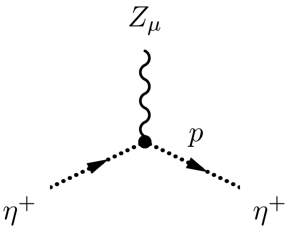
鬼粒子与电弱规范玻色子的耦合:



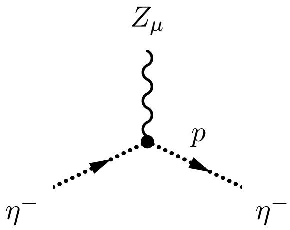
$$= iep^\mu$$



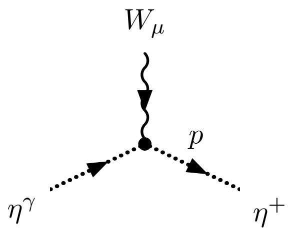
$$= -iep^\mu$$



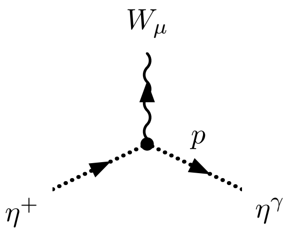
$$= igc_{\mathrm{W}}p^\mu$$



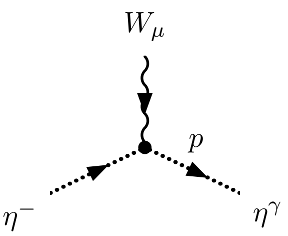
$$= -igc_{\mathrm{W}}p^\mu$$



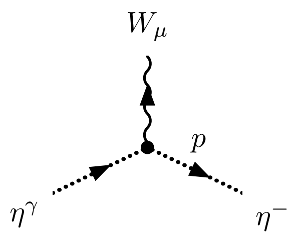
$$= -iep^\mu$$



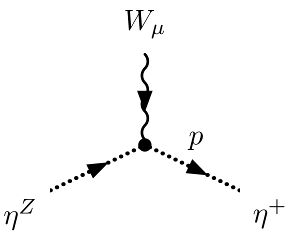
$$= -iep^\mu$$



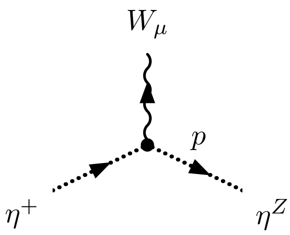
$$= iep^\mu$$



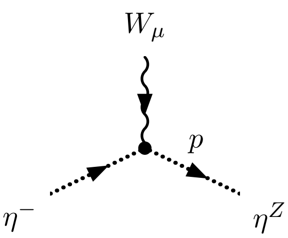
$$= iep^\mu$$



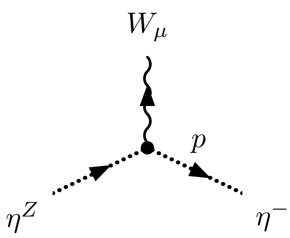
$$= -igc_{\mathrm{W}}p^\mu$$



$$= -igc_{\mathrm{W}}p^\mu$$



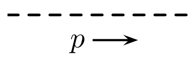
$$= igc_{\mathrm{W}}p^\mu$$



$$= igc_{\mathrm{W}}p^\mu$$

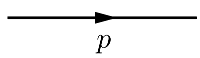
8 内外线一般费曼规则

标量玻色子传播子:



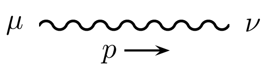
$$= \frac{i}{p^2 - m^2 + i\varepsilon}$$

Dirac 费米子传播子:

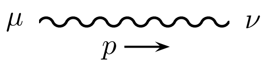


$$= \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$$

无质量规范玻色子 (如光子) 传播子:

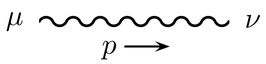


$$= \frac{-ig_{\mu\nu}}{p^2 + i\varepsilon} \quad \text{(费曼规范)}$$

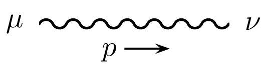


$$= \frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2 + i\varepsilon} \quad \text{(朗道规范)}$$

有质量规范玻色子 (如 $W^\pm$ 和 Z) 传播子:



$$= \frac{-i(g_{\mu\nu} - p_\mu p_\nu / m^2)}{p^2 - m^2 + i\varepsilon} \quad \text{(幺正规范)}$$

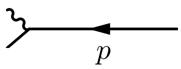


$$= \frac{-ig_{\mu\nu}}{p^2 - m^2 + i\varepsilon} \quad \text{(费曼规范)}$$

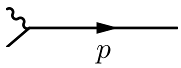
标量玻色子外线:

$$= 1 \quad \text{(初态或末态)}$$

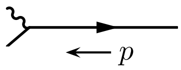
Dirac 费米子外线:



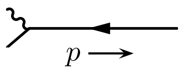
$$= u(p, s) \quad \text{(正粒子初态)}$$



$$= \bar{u}(p, s) \quad \text{(正粒子末态)}$$



$$= \bar{v}(p, s) \quad \text{(反粒子初态)}$$

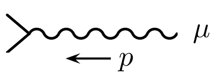


$$= v(p, s) \quad \text{(反粒子末态)}$$

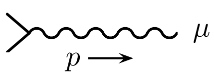
在计算非极化截面时, 可利用自旋求和关系

$$\sum_s u(p, s)\bar{u}(p, s) = \not{p} + m, \quad \sum_s v(p, s)\bar{v}(p, s) = \not{p} - m. \tag{136}$$

矢量玻色子外线:



$$= \varepsilon_\mu(p, \lambda) \quad \text{(初态)}$$



$$= \varepsilon^*_\mu(p, \lambda) \quad \text{(末态)}$$

在计算非极化截面时, 若包含无质量矢量玻色子外线, 可作替换

$$\sum_{\lambda} \varepsilon_{\mu}^{*}(p,\lambda)\varepsilon_{\nu}(p,\lambda) \to -g_{\mu\nu}; \tag{137}$$

若包含有质量矢量玻色子外线, 可作替换

$$\sum_{\lambda} \varepsilon_{\mu}^{*}(p,\lambda)\varepsilon_{\nu}(p,\lambda) \to -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{m^2}. \tag{138}$$

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