

标准模型及其费曼规则

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本文各种约定主要遵从文献 [1].
度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \text{diag}(+1, -1, -1, -1). \quad (1)$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad (2)$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \quad (3)$$

手征表示下的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}. \quad (4)$$

左右手投影算符

$$P_L \equiv \frac{1}{2}(1 - \gamma_5), \quad P_R \equiv \frac{1}{2}(1 + \gamma_5). \quad (5)$$

1 拉氏量

在粒子物理标准模型中, Higgs 场相关拉氏量

$$\mathcal{L}_H = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V_H(\Phi), \quad V_H(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad (6)$$

其中

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + i\chi(x)] \end{pmatrix}, \quad (7)$$

$$D_\mu \Phi = (\partial_\mu - ig' B_\mu Y_H - ig W_\mu^a T^a) \Phi, \quad Y_H = \frac{1}{2}, \quad T^a = \frac{\sigma^a}{2}. \quad (8)$$

由 Higgs 场势能 $V_H(\Phi)$ 的极小值条件可知, 真空期待值 v 应满足

$$\mu^2 = \lambda v^2. \quad (9)$$

在么正规范下,

$$\Phi(x) \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger \Phi \rightarrow \frac{1}{2}(v + H)^2, \quad (10)$$

势能项

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \rightarrow \frac{1}{2} \mu^2 (v + H)^2 - \frac{1}{4} \lambda (v + H)^4 \\ &= \frac{1}{2} \mu^2 (v^2 + H^2 + 2vH) - \frac{1}{4} \lambda (v^4 + 4v^2 H^2 + H^4 + 4v^3 H + 2v^2 H^2 + 4vH^3) \\ &= \frac{1}{4} \mu^2 v^2 + \frac{1}{4} (\mu^2 - \lambda v^2) v^2 + (\mu^2 - \lambda v^2) vH + \frac{1}{2} (\mu^2 - \lambda v^2) H^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{1}{4} \lambda H^4 \\ &= \frac{1}{4} \mu^2 v^2 - \frac{1}{2} m_H^2 H^2 - \frac{1}{2} \frac{m_H^2}{v} H^3 - \frac{1}{8} \frac{m_H^2}{v^2} H^4, \end{aligned} \quad (11)$$

其中 Higgs 粒子的质量为

$$m_H \equiv \sqrt{2} \mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \quad (12)$$

物理的光子场 $A_\mu(x)$, Z^0 玻色子矢量场 $Z_\mu(x)$ 和 W^\pm 玻色子矢量场 $W_\mu^\pm(x)$ 与 $SU(2)_L \times U(1)_Y$ 规范场 $W_\mu^a(x)$ 和 $B_\mu(x)$ 之间的关系为

$$A_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}} (g' W_\mu^3 + g B_\mu) = s_W W_\mu^3 + c_W B_\mu, \quad (13)$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}} (g W_\mu^3 - g' B_\mu) = c_W W_\mu^3 - s_W B_\mu, \quad (14)$$

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2), \quad (15)$$

或

$$B_\mu = c_W A_\mu - s_W Z_\mu, \quad W_\mu^3 = s_W A_\mu + c_W Z_\mu, \quad (16)$$

$$W_\mu^1 = \frac{1}{\sqrt{2}} (W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{i}{\sqrt{2}} (W_\mu^+ - W_\mu^-). \quad (17)$$

参数间有如下关系,

$$s_W \equiv \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_W \equiv \cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = gs_W = g'c_W. \quad (18)$$

从而,

$$\begin{aligned} g'B_\mu + gW_\mu^3 &= g'(c_W A_\mu - s_W Z_\mu) + g(s_W A_\mu + c_W Z_\mu) = 2eA_\mu + \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}} Z_\mu \\ &= 2eA_\mu + \sqrt{g^2 + g'^2}(c_W^2 - s_W^2)Z_\mu, \end{aligned} \quad (19)$$

则

$$\begin{aligned} g'B_\mu Y_H + gW_\mu^a T^a &= \frac{1}{2} \begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 2eA_\mu + \sqrt{g^2 + g'^2}(c_W^2 - s_W^2)Z_\mu & \sqrt{2}gW_\mu^+ \\ \sqrt{2}gW_\mu^- & -\sqrt{g^2 + g'^2}Z_\mu \end{pmatrix}. \end{aligned} \quad (20)$$

于是, 在么正规范下,

$$\begin{aligned} &(D^\mu \Phi)^\dagger (D_\mu \Phi) \\ \rightarrow &\left| \left[\partial_\mu - \frac{i}{2} \begin{pmatrix} 2eA_\mu + \sqrt{g^2 + g'^2}(c_W^2 - s_W^2)Z_\mu & \sqrt{2}gW_\mu^+ \\ \sqrt{2}gW_\mu^- & -\sqrt{g^2 + g'^2}Z_\mu \end{pmatrix} \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \right|^2 \\ = &\frac{1}{2} \begin{pmatrix} \frac{i}{\sqrt{2}}gW_\mu^-(v + H), \partial_\mu H - \frac{i}{2}\sqrt{g^2 + g'^2}Z_\mu(v + H) \end{pmatrix} \begin{pmatrix} -\frac{i}{\sqrt{2}}gW_\mu^+(v + H) \\ \partial_\mu H + \frac{i}{2}\sqrt{g^2 + g'^2}Z_\mu(v + H) \end{pmatrix} \\ = &\frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v + H)^2 \left[\frac{1}{4}g^2 W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2 + g'^2)Z_\mu Z^\mu \right] \\ = &\frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\ &+ 2\frac{m_W^2}{v}HW_\mu^+ W^{-\mu} + \frac{m_Z^2}{v}HZ_\mu Z^\mu + \frac{m_W^2}{v^2}H^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2v^2}H^2 Z_\mu Z^\mu. \end{aligned} \quad (21)$$

故 W^\pm 和 Z^0 玻色子通过 Brout-Englert-Higgs 机制获得质量, 分别为

$$m_W \equiv \frac{1}{2}gv, \quad m_Z \equiv \frac{1}{2}\sqrt{g^2 + g'^2}v. \quad (22)$$

标准模型中的费米子, 包括 3 代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$, 3 代带电轻子 $\ell_i = e, \mu, \tau$, 3 代上型夸克 $u_i = u, c, t$ 和 3 代下型夸克 $d_i = d, s, b$ ($i = 1, 2, 3$). 左手的费米子构成 $SU(2)_L$ 二重态

$$L_{iL} = \begin{pmatrix} \nu_i \\ \ell_i^- \end{pmatrix}_L, \quad Q_{iL} = \begin{pmatrix} u_i \\ d_i' \end{pmatrix}_L. \quad (23)$$

下型夸克的质量态 d_j 与规范态 d_j' 通过 CKM 矩阵 V_{ij} 联系起来,

$$d_i' = V_{ij}d_j. \quad (24)$$

标准模型费米子的量子数见表 1. 电荷数 Q , T^3 和弱超荷 Y 存在如下关系,

$$Q = T^3 + Y. \quad (25)$$

Table 1: 标准模型费米子的量子数.

			Q	T^3	Y	B	L
$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$	0	1/2	-1/2	0	1
$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	2/3	1/2	1/6	1/3	0
e_R^-	μ_R^-	τ_R^-	-1	0	-1	0	1
u_R	c_R	t_R	2/3	0	2/3	1/3	0
d'_R	s'_R	b'_R	-1/3	0	-1/3	1/3	0

$Y = -1/2$ 的 Higgs 场共轭态为

$$\tilde{\Phi}(x) = i\sigma^2 \Phi^*(x) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}[v + H(x) - i\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \quad (26)$$

利用它可以写下 Yukawa 耦合项

$$\begin{aligned} \mathcal{L}_Y &= -\lambda_{d_i} \bar{Q}_{iL} d'_{iR} \Phi - \lambda_{u_i} \bar{Q}_{iL} u_{iR} \tilde{\Phi} - \lambda_{\ell_i} \bar{L}_{iL} \ell_{iR} \Phi + \text{h.c.} \\ &\rightarrow -\frac{\lambda_{d_i}}{\sqrt{2}}(v + H) \bar{d}'_{iL} d'_{iR} - \frac{\lambda_{u_i}}{\sqrt{2}}(v + H) \bar{u}_{iL} u_{iR} - \frac{\lambda_{\ell_i}}{\sqrt{2}}(v + H) \bar{\ell}_{iL} \ell_{iR} + \text{h.c.} \\ &= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i. \end{aligned} \quad (27)$$

通过这种耦合, 费米子获得了质量,

$$m_{d_i} \equiv \frac{1}{\sqrt{2}} \lambda_{d_i} v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}} \lambda_{u_i} v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}} \lambda_{\ell_i} v. \quad (28)$$

费米子与电弱规范玻色子的耦合源于拉氏量

$$\mathcal{L}_{\text{EWF}} = \bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}_{iR} i \not{D} u_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} + \bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR}. \quad (29)$$

利用

$$\begin{aligned} g'Y B_\mu + gT^3 W_\mu^3 &= g'Y(c_W A_\mu - s_W Z_\mu) + gT^3(s_W A_\mu + c_W Z_\mu) \\ &= e(Y + T^3)A_\mu + \left(gc_W T^3 - \frac{gs_W}{c_W} s_W Y \right) Z_\mu = QeA_\mu + \frac{g}{c_W} (T^3 c_W^2 - Y s_W^2) Z_\mu \\ &= QeA_\mu + \frac{g}{c_W} (T^3 - Qs_W^2) Z_\mu, \end{aligned} \quad (30)$$

有

$$D_\mu Q_{iL} = (\partial_\mu - ig' B_\mu Y - ig W_\mu^a T^a) Q_{iL} = \partial_\mu Q_{iL} - i \begin{pmatrix} g'Y B_\mu + gT^3 W_\mu^3 & \frac{1}{2}g(W_\mu^1 - iW_\mu^2) \\ \frac{1}{2}g(W_\mu^1 + iW_\mu^2) & g'Y B_\mu + gT^3 W_\mu^3 \end{pmatrix} Q_{iL}$$

$$\begin{aligned}
&= \partial_\mu Q_{iL} - i \left(\begin{array}{cc} QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu & \frac{1}{\sqrt{2}}gW_\mu^+ \\ \frac{1}{\sqrt{2}}gW_\mu^- & QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu \end{array} \right) Q_{iL} \\
&= \partial_\mu Q_{iL} - i \left(\begin{array}{c} \left[QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu \right] u_{iL} + \frac{1}{\sqrt{2}}gW_\mu^+ d'_{iL} \\ \frac{1}{\sqrt{2}}gW_\mu^- u_{iL} + \left[QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu \right] d'_{iL} \end{array} \right), \tag{31}
\end{aligned}$$

故

$$\begin{aligned}
\bar{Q}_{iL} i \not{D} Q_{iL} &\supset \left[QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu \right] \bar{u}_{iL} \gamma^\mu u_{iL} + \left[QeA_\mu + \frac{g}{c_W}(T^3 - Qs_W^2)Z_\mu \right] \bar{d}'_{iL} \gamma^\mu d'_{iL} \\
&\quad + \frac{1}{\sqrt{2}}gW_\mu^+ \bar{u}_{iL} \gamma^\mu d'_{iL} + \frac{1}{\sqrt{2}}gW_\mu^- \bar{d}'_{iL} \gamma^\mu u_{iL} \\
&= \left(QeA_\mu + \frac{g}{c_W}g_L Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1-\gamma_5}{2} u_i + \frac{1}{2} \left(QeA_\mu + \frac{g}{c_W}g_L \right) \bar{d}_i \gamma^\mu \frac{1-\gamma_5}{2} d_i \\
&\quad + \frac{1}{\sqrt{2}}gW_\mu^+ \bar{u}_i \gamma^\mu \frac{1-\gamma_5}{2} V_{ij} d_j + \frac{1}{\sqrt{2}}gW_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu \frac{1-\gamma_5}{2} u_i, \tag{32}
\end{aligned}$$

其中

$$g_L \equiv T^3 - Qs_W^2. \tag{33}$$

另一方面,

$$D_\mu d'_{iR} = (\partial_\mu - ig' B_\mu Y) d'_{iR} = \partial_\mu d'_{iR} - ig' Q(c_W A_\mu - s_W Z_\mu) d'_{iR} = \partial_\mu d'_{iR} - iQeA_\mu d'_{iR} + i\frac{g}{c_W}Qs_W^2 Z_\mu d'_{iR}, \tag{34}$$

则

$$\begin{aligned}
\bar{u}_{iR} i \not{D} u_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} &\supset \left(QeA_\mu - \frac{g}{c_W}Qs_W^2 Z_\mu \right) \bar{u}_{iR} \gamma^\mu u_{iR} + \left(QeA_\mu - \frac{g}{c_W}Qs_W^2 Z_\mu \right) \bar{d}'_{iR} \gamma^\mu d'_{iR} \\
&= \left(QeA_\mu + \frac{g}{c_W}g_R Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1+\gamma_5}{2} u_i + \left(QeA_\mu + \frac{g}{c_W}g_R Z_\mu \right) \bar{d}_i \gamma^\mu \frac{1+\gamma_5}{2} d_i, \tag{35}
\end{aligned}$$

其中

$$g_R \equiv -Qs_W^2. \tag{36}$$

定义

$$g_V \equiv g_L + g_R = T^3 - 2Qs_W^2, \quad g_A \equiv g_L - g_R = T^3, \tag{37}$$

可得

$$\begin{aligned}
&\bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}_{iR} i \not{D} u_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} \\
&\supset Qe\bar{u}_i \gamma^\mu u_i A_\mu + Qe\bar{d}_i \gamma^\mu d_i A_\mu + \frac{g}{2c_W}\bar{u}_i \gamma^\mu (g_V - g_A \gamma_5) u_i Z_\mu + \frac{g}{2c_W}\bar{d}_i \gamma^\mu (g_V - g_A \gamma_5) d_i Z_\mu \\
&\quad + \frac{1}{\sqrt{2}}gW_\mu^+ \bar{u}_i \gamma^\mu \frac{1-\gamma_5}{2} V_{ij} d_j + \frac{1}{\sqrt{2}}gW_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu \frac{1-\gamma_5}{2} u_i. \tag{38}
\end{aligned}$$

同理, 有

$$\begin{aligned}
\bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR} &\supset Qe\bar{\ell}_i \gamma^\mu \ell_i A_\mu + \frac{g}{2c_W}\bar{\ell}_i \gamma^\mu (g_V - g_A \gamma_5) \ell_i Z_\mu + \frac{g}{2c_W}\bar{\nu}_i \gamma^\mu (g_V - g_A \gamma_5) \nu_i Z_\mu \\
&\quad + \frac{1}{\sqrt{2}}gW_\mu^+ \bar{\nu}_i \gamma^\mu \frac{1-\gamma_5}{2} \ell_i + \frac{1}{\sqrt{2}}gW_\mu^- \bar{\ell}_i \gamma^\mu \frac{1-\gamma_5}{2} \nu_i. \tag{39}
\end{aligned}$$

总结起来, 可以写成流耦合的形式,

$$\begin{aligned}\mathcal{L}_{\text{EWF}} &\supset \sum_f \left[Q_f e \bar{f} \gamma^\mu f A_\mu + \frac{g}{2c_W} \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma_5) f Z_\mu \right] + g(W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}) \\ &= e A_\mu J_{\text{EM}}^\mu + g(Z_\mu J_Z^\mu + W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}),\end{aligned}\quad (40)$$

其中, 流的定义为

$$\begin{aligned}J_{\text{EM}}^\mu &\equiv \sum_f Q_f \bar{f} \gamma^\mu f, \quad J_Z^\mu \equiv \frac{1}{2c_W} \sum_f \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma_5) f = \frac{1}{c_W} \sum_f (g_L^f \bar{f}_L \gamma^\mu f_L + g_R^f \bar{f}_R \gamma^\mu f_R), \\ J_W^{+\mu} &\equiv \frac{1}{\sqrt{2}} (\bar{u}_{iL} \gamma^\mu V_{ij} d_{jL} + \bar{\nu}_{iL} \gamma^\mu \ell_{iL}), \quad J_W^{-\mu} \equiv \frac{1}{\sqrt{2}} (\bar{d}_{jL} V_{ji}^\dagger \gamma^\mu u_{iL} + \bar{\ell}_{iL} \gamma^\mu \nu_{iL}).\end{aligned}\quad (41)$$

对于各种费米子, 相关系数如下:

$$Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}, \quad Q_{\nu_i} = 0, \quad Q_{\ell_i} = -1; \quad (42)$$

$$g_V^{u_i} = \frac{1}{2} - \frac{4}{3} s_W^2, \quad g_A^{u_i} = \frac{1}{2}; \quad g_V^{d_i} = -\frac{1}{2} + \frac{2}{3} s_W^2, \quad g_A^{d_i} = -\frac{1}{2}; \quad (43)$$

$$g_V^{\nu_i} = \frac{1}{2}, \quad g_A^{\nu_i} = \frac{1}{2}; \quad g_V^{\ell_i} = -\frac{1}{2} + 2s_W^2, \quad g_A^{\ell_i} = -\frac{1}{2}; \quad (44)$$

$$g_L^{u_i} = \frac{1}{2} - \frac{2}{3} s_W^2, \quad g_R^{u_i} = -\frac{2}{3} s_W^2; \quad g_L^{d_i} = -\frac{1}{2} + \frac{1}{3} s_W^2, \quad g_R^{d_i} = \frac{1}{3} s_W^2; \quad (45)$$

$$g_L^{\nu_i} = \frac{1}{2}, \quad g_R^{\nu_i} = 0; \quad g_L^{\ell_i} = -\frac{1}{2} + s_W^2, \quad g_R^{\ell_i} = s_W^2. \quad (46)$$

纯电弱规范场相关的拉氏量

$$\mathcal{L}_{\text{EWG}} = -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad (47)$$

其中

$$W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g \varepsilon^{abc} W^{b\mu} W^{c\nu}, \quad B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu. \quad (48)$$

利用(16)式和(17)式, 可得

$$\begin{aligned}&W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 \\ &= \frac{i}{\sqrt{2}} [(W_\mu^+ - W_\mu^-)(s_W A_\nu + c_W Z_\nu) - (s_W A_\mu + c_W Z_\mu)(W_\nu^+ - W_\nu^-)] \\ &= \frac{i}{\sqrt{2}} [s_W (W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W (W_\mu^+ Z_\nu - Z_\mu W_\nu^+) - s_W (W_\mu^- A_\nu - A_\mu W_\nu^-) - c_W (W_\mu^- Z_\nu - Z_\mu W_\nu^-)], \quad (49) \\ &W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 \\ &= \frac{1}{\sqrt{2}} [(s_W A_\mu + c_W Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_W A_\nu + c_W Z_\nu)] \\ &= -\frac{1}{\sqrt{2}} [s_W (W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W (W_\mu^+ Z_\nu - Z_\mu W_\nu^+) + s_W (W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W (W_\mu^- Z_\nu - Z_\mu W_\nu^-)]. \quad (50)\end{aligned}$$

从而,

$$\begin{aligned}W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g \varepsilon^{1bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2 \\ &= \frac{1}{\sqrt{2}} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}} (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g (W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2) \\ &= \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + i g [s_W (W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W (W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \} \\ &\quad + \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - i g [s_W (W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W (W_\mu^- Z_\nu - Z_\mu W_\nu^-)] \}\end{aligned}$$

$$= \frac{1}{\sqrt{2}}(F_{\mu\nu}^+ + F_{\mu\nu}^-), \quad (51)$$

其中,

$$F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \quad (52)$$

$$F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ie(W_\mu^- A_\nu - A_\mu W_\nu^-) - igc_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \quad (53)$$

另一方面,

$$\begin{aligned} W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 + g\varepsilon^{2bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - gW_\mu^1 W_\nu^3 + gW_\mu^3 W_\nu^1 \\ &= \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\ &= \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\ &\quad \times - \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\ &= \frac{i}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-). \end{aligned} \quad (54)$$

因此,

$$\begin{aligned} -\frac{1}{4}W_{\mu\nu}^1 W^{1\mu\nu} - \frac{1}{4}W_{\mu\nu}^2 W^{2\mu\nu} &= -\frac{1}{8}(F_{\mu\nu}^+ + F_{\mu\nu}^-)(F^{+\mu\nu} + F^{-\mu\nu}) + \frac{1}{8}(F_{\mu\nu}^+ - F_{\mu\nu}^-)(F^{+\mu\nu} - F^{-\mu\nu}) = -\frac{1}{2}F_{\mu\nu}^+ F^{-\mu\nu} \\ &= -\frac{1}{2}[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \\ &\quad \times [\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu} - ie(W^{-\mu} A^\nu - A^\mu W^{-\nu}) - igc_W(W^{-\mu} Z^\nu - Z^\mu W^{-\nu})] \\ &= -(\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) \\ &\quad + ie[(\partial_\mu W_\nu^+)W^{-\mu} A^\nu - (\partial_\mu W_\nu^+)W^{-\nu} A^\mu - W_\mu^+(\partial^\mu W^{-\nu})A_\nu + W_\nu^+(\partial^\mu W^{-\nu})A_\mu] \\ &\quad + igc_W[(\partial_\mu W_\nu^+)W^{-\mu} Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu} Z^\mu - W_\mu^+(\partial^\mu W^{-\nu})Z_\nu + W_\nu^+(\partial^\mu W^{-\nu})Z_\mu] \\ &\quad + e^2(W_\mu^+ W^{-\nu} A_\nu A^\mu - W_\mu^+ W^{-\mu} A_\nu A^\nu) + g^2 c_W^2 (W_\mu^+ W^{-\nu} Z_\nu Z^\mu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu) \\ &\quad + egc_W(W_\mu^+ W^{-\nu} A_\nu Z^\mu + W_\mu^+ W^{-\nu} A^\mu Z_\nu - 2W_\mu^+ W^{-\mu} A_\nu Z^\nu). \end{aligned} \quad (55)$$

由

$$W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1 = \frac{i}{2}(W_\mu^+ + W_\mu^-)(W_\nu^+ - W_\nu^-) - \frac{i}{2}(W_\mu^+ - W_\mu^-)(W_\nu^+ + W_\nu^-) = -i(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \quad (56)$$

可得

$$\begin{aligned} W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 + g\varepsilon^{3bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 + gW_\mu^1 W_\nu^2 - gW_\mu^2 W_\nu^1 \\ &= s_W \partial_\mu A_\nu + c_W \partial_\mu Z_\nu - s_W \partial_\nu A_\mu + c_W \partial_\nu Z_\mu + g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\ &= s_W(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_W(\partial_\mu Z_\nu - \partial_\nu Z_\mu) - ig(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \end{aligned} \quad (57)$$

$$B_{\mu\nu} = \partial_\mu(c_W A_\nu - s_W Z_\nu) - \partial_\nu(c_W A_\mu - s_W Z_\mu) = c_W(\partial_\mu A_\nu - \partial_\nu A_\mu) - s_W(\partial_\mu Z_\nu - \partial_\nu Z_\mu). \quad (58)$$

于是,

$$\begin{aligned} &-\frac{1}{4}W_{\mu\nu}^3 W^{3\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} \\ &= -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\ &\quad + ie[W^{+\mu} W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu} W^{-\mu}(\partial_\mu A_\nu)] + igc_W[W^{+\mu} W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu} W^{-\mu}(\partial_\mu Z_\nu)] \end{aligned}$$

$$+\frac{1}{2}g^2(W_\mu^+W^{+\mu}W_\nu^-W^{-\nu}-W_\mu^+W^{+\nu}W_\nu^-W^{-\mu}). \quad (59)$$

综合起来, 有

$$\begin{aligned} \mathcal{L}_{\text{EWG}} = & \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\ & + (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + \frac{1}{2}g^2(W_\mu^+W^{+\mu}W_\nu^-W^{-\nu} - W_\mu^+W^{+\nu}W_\nu^-W^{-\mu}) \\ & + ie[(\partial_\mu W_\nu^+)W^{-\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-\nu}A^\mu - W^{+\mu}(\partial_\mu W_\nu^-)A^\nu + W^{+\nu}(\partial_\mu W_\nu^-)A^\mu \\ & \quad + W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] + e^2(W_\mu^+W^{-\nu}A_\nu A^\mu - W_\mu^+W^{-\mu}A_\nu A^\nu) \\ & + igc_W[(\partial_\mu W_\nu^+)W^{-\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu}Z^\mu - W^{+\mu}(\partial_\mu W_\nu^-)Z^\nu + W^{+\nu}(\partial_\mu W_\nu^-)Z^\mu \\ & \quad + W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)] + g^2c_W^2(W_\mu^+W^{-\nu}Z_\nu Z^\mu - W_\mu^+W^{-\mu}Z_\nu Z^\nu) \\ & + egc_W(W_\mu^+W^{-\nu}A_\nu Z^\mu + W_\mu^+W^{-\nu}A^\mu Z_\nu - 2W_\mu^+W^{-\mu}A_\nu Z^\nu). \end{aligned} \quad (60)$$

QCD 的拉氏量可表达成

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \dots, 8, \quad (61)$$

其中

$$D_\mu = \partial_\mu - ig_s G_\mu^a t^a, \quad G^{a\mu\nu} \equiv \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{abc} G^{b\mu} G^{c\nu}. \quad (62)$$

$SU(3)_C$ 群基础表示生成元 $t^a = \lambda^a/2$, 其中 λ^a 为 Gell-Mann 矩阵. 生成元对易关系为 $[t^a, t^b] = if^{abc}t^c$. 结构常数 f^{abc} 是全反对称的, 其非零分量为

$$f_{123} = 1, \quad f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \quad (63)$$

由

$$\begin{aligned} -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} &= -\frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{ade} G^{d\mu} G^{e\nu}) \\ &= -\frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu}) - (\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu})] - g_s f^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} - \frac{1}{4}g_s^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu}, \end{aligned} \quad (64)$$

可得

$$\begin{aligned} \mathcal{L}_{\text{QCD}} = & \sum_q [\bar{q}(i\gamma^\mu \partial_\mu - m_q)q + g_s G_\mu^a \bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu})] \\ & - g_s f^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} - \frac{1}{4}g_s^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu}. \end{aligned} \quad (65)$$

2 顶点费曼规则

下面列出么正规化下的顶点费曼规则.

Higgs 粒子自耦合:

$$\begin{aligned} \text{Three-point vertex: } & \text{Higgs line (dashed) entering from top, two Higgs lines (dashed) exiting to bottom-left and bottom-right.} \\ & = -3i \frac{m_H^2}{v} \\ \text{Four-point vertex: } & \text{Two Higgs lines (dashed) entering from top-left and top-right, two Higgs lines (dashed) exiting to bottom-left and bottom-right.} \\ & = -3i \frac{m_H^2}{v^2} \end{aligned}$$

Higgs 粒子与电弱规范玻色子的耦合:

$$\begin{aligned}
 & \text{Tree-level } H \rightarrow W^+ W^-: = 2i \frac{m_W^2}{v} g^{\mu\nu} \\
 & \text{Loop } H \rightarrow W^+ W^-: = 2i \frac{m_W^2}{v^2} g^{\mu\nu} \\
 & \text{Tree-level } H \rightarrow Z Z: = 2i \frac{m_Z^2}{v} g^{\mu\nu} \\
 & \text{Loop } H \rightarrow Z Z: = 2i \frac{m_Z^2}{v^2} g^{\mu\nu}
 \end{aligned}$$

Higgs 粒子与费米子的耦合:

$$= -i \frac{m_f}{v}$$

QED 顶点:

$$= i Q_f e \gamma^\mu \quad (\text{对于电子, } Q_f = -1)$$

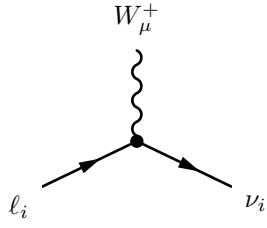
费米子与 Z 玻色子的耦合:

$$= i \frac{g}{2c_W} \gamma^\mu (g_V^f - g_A^f \gamma_5)$$

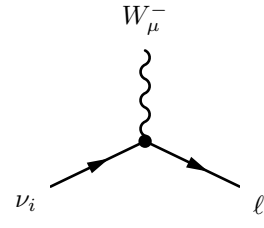
$$\begin{aligned}
 g_V^{u_i} &= \frac{1}{2} - \frac{4}{3} s_W^2, & g_A^{u_i} &= \frac{1}{2}; & g_V^{d_i} &= -\frac{1}{2} + \frac{2}{3} s_W^2, & g_A^{d_i} &= -\frac{1}{2}; \\
 g_V^{\nu_i} &= \frac{1}{2}, & g_A^{\nu_i} &= \frac{1}{2}; & g_V^{\ell_i} &= -\frac{1}{2} + 2s_W^2, & g_A^{\ell_i} &= -\frac{1}{2}.
 \end{aligned}$$

费米子与 W 玻色子的耦合:

$$\begin{aligned}
 & \text{Left: } W^+ \rightarrow d_j u_i: = i \frac{g}{\sqrt{2}} V_{ij} \gamma^\mu \frac{1 - \gamma_5}{2} \\
 & \text{Right: } W^- \rightarrow u_i d_j: = i \frac{g}{\sqrt{2}} V_{ji}^\dagger \gamma^\mu \frac{1 - \gamma_5}{2}
 \end{aligned}$$

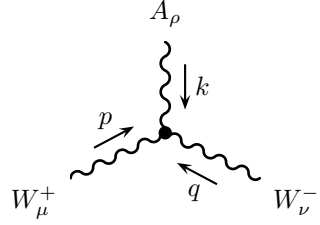


$$= i \frac{g}{\sqrt{2}} \gamma^\mu \frac{1 - \gamma_5}{2}$$



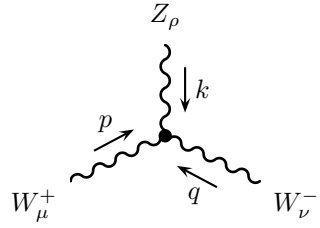
$$= i \frac{g}{\sqrt{2}} \gamma^\mu \frac{1 - \gamma_5}{2}$$

电弱规范玻色子自耦合:

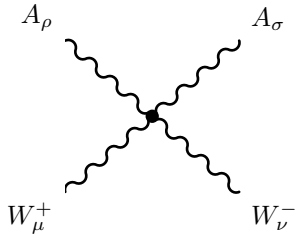


$$= -ie[(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu]$$

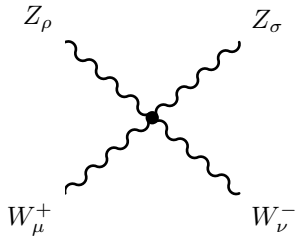
(对于指向顶点的动量 k , 时空导数 ∂_μ 在动量空间中贡献一个 $-ik_\mu$ 因子.)



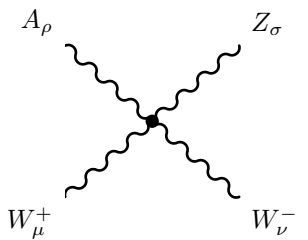
$$= -igc_W[(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu]$$



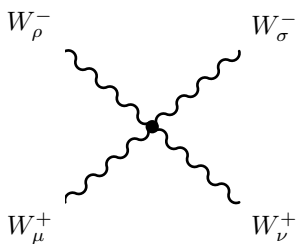
$$= ie^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= ig^2c_W^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= iegc_W(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= -ig^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$

QCD 顶点:

$$= ig_s \gamma^\mu t^a$$

$$= g_s f^{abc} [g^{\mu\nu} (k-p)^\rho + g^{\nu\rho} (p-q)^\mu + g^{\rho\mu} (q-k)^\nu]$$

$$= -ig_s^2 [f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})]$$

3 内外线费曼规则

标量玻色子传播子:

$$= \frac{i}{p^2 - m^2 + i\epsilon}$$

Dirac 费米子传播子:

$$= \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$$

无质量矢量玻色子 (光子) 传播子:

$$= \frac{-ig_{\mu\nu}}{p^2 + i\epsilon} \quad (\text{费曼规范})$$

$$= \frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2 + i\epsilon} \quad (\text{朗道规范})$$

有质量矢量玻色子 (W^\pm, Z^0) 传播子:

$$= \frac{-i(g_{\mu\nu} - p_\mu p_\nu / m^2)}{p^2 - m^2 + i\epsilon} \quad (\text{幺正规范})$$

$$= \frac{-ig_{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad (\text{费曼规范})$$

标量玻色子外线:

$$= 1 \quad (\text{初态或末态})$$

Dirac 费米子外线:

$$\begin{aligned}
 \text{---} \leftarrow \text{---} \quad p &= u^s(p) \quad (\text{正粒子初态}) \\
 \text{---} \rightarrow \text{---} \quad p &= \bar{u}^s(p) \quad (\text{正粒子末态}) \\
 \text{---} \rightarrow \text{---} \quad \leftarrow p &= \bar{v}^s(p) \quad (\text{反粒子初态}) \\
 \text{---} \leftarrow \text{---} \quad p \rightarrow &= v^s(p) \quad (\text{反粒子末态})
 \end{aligned}$$

在计算非极化截面时, 可利用自旋求和关系

$$\sum_s u^s(p) \bar{u}^s(p) = \not{p} + m, \quad \sum_s v^s(p) \bar{v}^s(p) = \not{p} - m. \quad (66)$$

矢量玻色子外线:

$$\begin{aligned}
 \text{---} \leftarrow \text{---} \quad \mu &= \varepsilon_\mu(p) \quad (\text{初态}) \\
 \text{---} \rightarrow \text{---} \quad \mu &= \varepsilon_\mu^*(p) \quad (\text{末态})
 \end{aligned}$$

在计算非极化截面时, 若包含无质量矢量玻色子外线, 可作替换

$$\sum_{\text{polarizations}} \varepsilon_\mu^*(p) \varepsilon_\nu(p) \rightarrow -g_{\mu\nu}; \quad (67)$$

若包含有质量矢量玻色子外线, 可作替换

$$\sum_{\text{polarizations}} \varepsilon_\mu^*(p) \varepsilon_\nu(p) \rightarrow -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}. \quad (68)$$

参考文献

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