β function for the Higgs quartic coupling λ in the standard model (SM)

h^4 vertex $\begin{matrix} h & & h \\ & >< & \\ h & & h \end{matrix} = -6i\lambda$

h self-energy $h-(\mathrm{1PI})-h = i\Pi_h(p^2)$

h-h counter term $h-\otimes-h = i(p^2\delta_h - \delta_{m_h})$, $\quad i(\Pi_h + p^2\delta_h - \delta_{m_h})$ is finite $\;\Rightarrow\; \dfrac{\partial\Pi_h}{\partial p^2} + \delta_h$ is finite

h^4 vertex correction $\begin{matrix} h- & & -h \\ & (\mathrm{1PI}) & \\ h- & & -h \end{matrix} = i\Sigma_\lambda(p_1, p_2, p_3)$

h^4 counter term $\begin{matrix} h- & & -h \\ & \otimes & \\ h- & & -h \end{matrix} = -6i\delta_\lambda, \quad \Sigma_{y_t} - 6\delta_\lambda$ is finite

h^4 Green function $G_{\mathrm{c}}^{(4)}(\{p_i\}) = \left(\displaystyle\prod_{i=1}^{4} \frac{i}{p_i^2}\right)\left\{-6i\lambda - iB\ln\frac{\Lambda^2}{-p^2} - 6i\delta_\lambda - 6i\lambda\left[\sum_{i=1}^{4}\left(A_i \ln\frac{\Lambda^2}{-p_i^2}\right) - 4\delta_h\right]\right\}$

Callan-Symanzik equation $\left[\dfrac{\partial}{\partial\ln\mu_{\mathrm{R}}} + \beta_\lambda\dfrac{\partial}{\partial\lambda} + 2\dfrac{\partial\delta_h}{\partial\ln\mu_{\mathrm{R}}}\right]G_{\mathrm{c}}^{(4)} = 0$

$$\Rightarrow \frac{\partial}{\partial\ln\mu_{\mathrm{R}}}(-6i\delta_\lambda + 24i\lambda\delta_h) - 6i\beta_\lambda - 12i\lambda\frac{\partial\delta_h}{\partial\ln\mu_{\mathrm{R}}} = 0 \quad \text{(lowest order)}$$

$$\Rightarrow \frac{\partial}{\partial\ln\mu_{\mathrm{R}}}(\delta_\lambda - 4\lambda\delta_h) + \beta_\lambda + 2\lambda\frac{\partial\delta_h}{\partial\ln\mu_{\mathrm{R}}} = 0$$

β function for λ: $\quad \beta_\lambda = \dfrac{\partial}{\partial\ln\mu_{\mathrm{R}}}(-\delta_\lambda + 2\lambda\delta_h) = -\dfrac{\partial\delta_\lambda}{\partial\ln\mu_{\mathrm{R}}} + 2\lambda\dfrac{\partial\delta_h}{\partial\ln\mu_{\mathrm{R}}}$

The Feynman-t' Hooft gauge ($\xi = 1$) is adopted in the following calculation

In the SM with this gauge, $\dfrac{\partial\delta_h^{\mathrm{SM}}}{\partial\ln\mu_{\mathrm{R}}} = \dfrac{1}{16\pi^2}(6y_t^2 - 3g^2 - g'^2)$

⊙ Calculation for $\dfrac{\partial\delta_\lambda^{\mathrm{SM}}}{\partial\ln\mu_{\mathrm{R}}}$

For such a calculation, all masses and external momenta can be neglected

There are $3! = 6$ different contractions for the box diagram with a top loop:

$$\begin{matrix} -\!\!\leftarrow\!- \\ \downarrow \quad \uparrow \\ -\!\!\rightarrow\!- \end{matrix} + \begin{matrix} -\!\!\rightarrow\!- \\ {}^{\nwarrow}_{\swarrow}\!\times \\ -\!\!\rightarrow\!- \end{matrix} + \begin{matrix} -\!\!\rightarrow\!- \\ \uparrow \quad \downarrow \\ -\!\!\leftarrow\!- \end{matrix} + \begin{matrix} -- \quad -- \\ \uparrow \times {}^{\swarrow}_{\searrow} \uparrow \\ -- \quad -- \end{matrix} + \begin{matrix} -\!\!\leftarrow\!- \\ \times {}^{\nearrow}_{\searrow} \\ -\!\!\leftarrow\!- \end{matrix} + \begin{matrix} -- \quad -- \\ \downarrow \times {}^{\nearrow}_{\nwarrow} \downarrow \\ -- \quad -- \end{matrix}$$

$$i\Sigma_\lambda^t = 6\times\begin{pmatrix} h & --\bar{t}-- & h \\ & t| \quad\quad |t & \\ h & --t-- & h \end{pmatrix} = 6\cdot(-1)\cdot 3\left(-\frac{iy_t}{\sqrt{2}}\right)^4 \int\frac{d^dq}{(2\pi)^d}\,\mathrm{Tr}\left(\frac{i\not{q}}{q^2}\frac{i\not{q}}{q^2}\frac{i\not{q}}{q^2}\frac{i\not{q}}{q^2}\right)$$

$$= -18y_t^4\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = -18y_t^4\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^t = -\frac{1}{6}18y_t^4\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}3y_t^4\ln\mu_{\mathrm{R}}^2 + \cdots$$

$$\frac{\partial\delta_\lambda^t}{\partial\ln\mu_{\mathrm{R}}} = \frac{1}{16\pi^2}6y_t^4$$

$$\begin{array}{ccc} h & --h-- & h \\ h\big| & & \big|h \\ h & --h-- & h \end{array} \sim \begin{array}{ccc} h & --G^0-- & h \\ G^0\big| & & \big|G^0 \\ h & --G^0-- & h \end{array} \sim \begin{array}{ccc} h & --G^\pm-- & h \\ G^\pm\big| & & \big|G^\pm \\ h & --G^\pm-- & h \end{array} \sim \int \frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^4} = \text{finite}$$

$$i\Sigma_\lambda^h = 3\times\begin{pmatrix} h & h & h \\ & \rangle\langle\overline{\underline{\quad}}\rangle\langle & \\ h & h & h \end{pmatrix} = 3\cdot\frac{1}{2}(-6i\lambda)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{q^2}\frac{i}{q^2} = 54\lambda^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = 54\lambda^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^h = \frac{1}{6}54\lambda^2\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}} + \text{finite} = -\frac{1}{16\pi^2}9\lambda^2\ln\mu_\mathrm{R}^2 + \cdots,\quad \frac{\partial\delta_\lambda^h}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}(-18\lambda^2)$$

$$i\Sigma_\lambda^{G^0} = 3\times\begin{pmatrix} h & G^0 & h \\ & \rangle\langle\overline{\underline{\quad}}\rangle\langle & \\ h & G^0 & h \end{pmatrix} = 3\cdot\frac{1}{2}(-2i\lambda)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{q^2}\frac{i}{q^2} = 6\lambda^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^{G^0} = -\frac{1}{16\pi^2}\lambda^2\ln\mu_\mathrm{R}^2 + \cdots,\quad \frac{\partial\delta_\lambda^{G^0}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}(-2\lambda^2)$$

$$i\Sigma_\lambda^{G^\pm} = 3\times\begin{pmatrix} h & G^+ & h \\ & \rangle\langle\overline{\underline{\quad}}\rangle\langle & \\ h & G^- & h \end{pmatrix} = 3(-2i\lambda)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{q^2}\frac{i}{q^2} = 12\lambda^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^{G^\pm} = -\frac{1}{16\pi^2}2\lambda^2\ln\mu_\mathrm{R}^2 + \cdots,\quad \frac{\partial\delta_\lambda^{G^\pm}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}(-4\lambda^2)$$

$$\begin{array}{ccc} h & --Z-- & h \\ Z\big| & & \big|Z \\ h & --Z-- & h \end{array} \sim \begin{array}{ccc} h & --W^\pm-- & h \\ W^\pm\big| & & \big|W^\pm \\ h & --W^\pm-- & h \end{array} \sim \begin{array}{ccc} h & --\eta^Z-- & h \\ \eta^Z\big| & & \big|\eta^Z \\ h & --\eta^Z-- & h \end{array} \sim \begin{array}{ccc} h & --\eta^\pm-- & h \\ \eta^\pm\big| & & \big|\eta^\pm \\ h & --\eta^\pm-- & h \end{array} \sim \int \frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^4} = \text{finite}$$

$$i\Sigma_\lambda^Z = 3\times\begin{pmatrix} h & Z & h \\ & \rangle\langle\overline{\underline{\quad}}\rangle\langle & \\ h & Z & h \end{pmatrix} = 3\cdot\frac{1}{2}\left(\frac{ig^2}{2c_\mathrm{W}^2}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}}{q^2}\frac{-ig^{\nu\mu}}{q^2} = \frac{3g^4}{8c_\mathrm{W}^4}d\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = \frac{3g^4}{2c_\mathrm{W}^4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^Z = \frac{1}{6}\frac{3g^4}{2c_\mathrm{W}^4}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}} + \text{finite} = -\frac{g^4}{4c_\mathrm{W}^4}\ln\mu_\mathrm{R}^2 + \cdots,\quad \frac{\partial\delta_\lambda^Z}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}\left(-\frac{g^4}{2c_\mathrm{W}^4}\right)$$

$$i\Sigma_\lambda^W = 3\times\begin{pmatrix} h & W^+ & h \\ & \rangle\langle\overline{\underline{\quad}}\rangle\langle & \\ h & W^- & h \end{pmatrix} = 3\left(\frac{ig^2}{2}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}}{q^2}\frac{-ig^{\nu\mu}}{q^2} = \frac{3g^4}{4}d\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = 3g^4\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^W = \frac{1}{6}3g^4\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}} + \text{finite} = -\frac{g^4}{2}\ln\mu_\mathrm{R}^2 + \cdots,\quad \frac{\partial\delta_\lambda^W}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}(-g^4)$$

(2)

$$i\Sigma_\lambda^{G^0Z,1}=3\times\begin{pmatrix} h & --G^0-- & h \\ & Z| \quad\quad |Z & \\ h & --G^0-- & h \end{pmatrix}=3\left(-\frac{g}{2c_\mathrm{W}}\right)^4\int\frac{d^dq}{(2\pi)^d}\,q^\mu\,\frac{i}{q^2}(-q^\nu)\frac{-ig_{\nu\rho}}{q^2}q^\rho\,\frac{i}{q^2}(-q^\sigma)\frac{-ig^{\sigma\mu}}{q^2}$$

$$=\frac{3g^4}{16c_\mathrm{W}^4}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}=\frac{3g^4}{16c_\mathrm{W}^4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma_\lambda^{G^0Z,1}=3\times\begin{pmatrix} h & --G^0-- & h \\ & Z\times Z & \\ h & --G^0-- & h \end{pmatrix}=3\left(-\frac{g}{2c_\mathrm{W}}\right)^4\int\frac{d^dq}{(2\pi)^d}\,q^\mu\,\frac{i}{q^2}(-q^\nu)\frac{-ig_{\nu\sigma}}{q^2}(-q^\sigma)\frac{i}{q^2}q^\rho\,\frac{-ig^{\rho\mu}}{q^2}$$

$$=\frac{3g^4}{16c_\mathrm{W}^4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$\delta_\lambda^{G^0Z}=\frac{1}{6}\cdot 2\cdot\frac{3g^4}{16c_\mathrm{W}^4}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}}+\text{finite}=-\frac{1}{16\pi^2}\frac{g^4}{16c_\mathrm{W}^4}\ln\mu_\mathrm{R}^2+\cdots,\qquad \frac{\partial\delta_\lambda^{G^0Z}}{\partial\ln\mu_\mathrm{R}}=\frac{1}{16\pi^2}\left(-\frac{g^4}{8c_\mathrm{W}^2}\right)$$

$$i\Sigma_\lambda^{G^\pm W,1}=3\times\begin{pmatrix} h & --G^+-- & h \\ & W| \quad\quad |W & \\ h & --G^--- & h \end{pmatrix}=3\left(\frac{ig}{2}\right)^4\int\frac{d^dq}{(2\pi)^d}\,q^\mu\,\frac{i}{q^2}q^\nu\,\frac{-ig_{\nu\rho}}{q^2}q^\rho\,\frac{i}{q^2}q^\sigma\,\frac{-ig^{\sigma\mu}}{q^2}$$

$$=\frac{3g^4}{16}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}=\frac{3g^4}{16}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma_\lambda^{G^\pm W,2}=3\times\begin{pmatrix} h & --G^--- & h \\ & W| \quad\quad |W & \\ h & --G^+-- & h \end{pmatrix}=\frac{3g^4}{16}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma_\lambda^{G^\pm W,3}=3\times\begin{pmatrix} h & --G^+-- & h \\ & W^-\times W^- & \\ h & --G^+-- & h \end{pmatrix}=3\left(\frac{ig}{2}\right)^4\int\frac{d^dq}{(2\pi)^d}\,q^\mu\,\frac{i}{q^2}q^\nu\,\frac{-ig_{\nu\sigma}}{q^2}q^\sigma\,\frac{i}{q^2}q^\rho\,\frac{-ig^{\rho\mu}}{q^2}$$

$$=\frac{3g^4}{16}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma_\lambda^{G^\pm W,4}=3\times\begin{pmatrix} h & --G^--- & h \\ & W^+\times W^+ & \\ h & --G^--- & h \end{pmatrix}=\frac{3g^4}{16}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$\delta_\lambda^{G^\pm W}=\frac{1}{6}\cdot 4\cdot\frac{3g^4}{16}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}}+\text{finite}=-\frac{1}{16\pi^2}\frac{g^4}{8}\ln\mu_\mathrm{R}^2+\cdots,\qquad \frac{\partial\delta_\lambda^{G^\pm W}}{\partial\ln\mu_\mathrm{R}}=\frac{1}{16\pi^2}\left(-\frac{g^4}{4}\right)$$

(3)

$$\begin{matrix} h & --\wedge-- & h \\ & Z / \quad \backslash Z & \\ h & --Z-- & h \end{matrix} \sim \begin{matrix} h & --\wedge-- & h \\ & W / \quad \backslash W & \\ h & --W-- & h \end{matrix} \sim \int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2)^3} = \text{finite}$$

$$i\Sigma_\lambda^{G^0 ZZ} = 2\times 6\times \begin{pmatrix} h & --\wedge-- & h \\ & Z / \quad \backslash Z & \\ h & --G^0-- & h \end{pmatrix} = 12\frac{1}{2}\frac{ig^2}{2c_{\mathrm{W}}^2}\left(-\frac{g}{2c_{\mathrm{W}}}\right)^2 \int \frac{d^d q}{(2\pi)^d}\, g^{\mu\nu}\, \frac{-ig_{\mu\rho}}{q^2}\frac{-ig_{\nu\sigma}}{q^2} q^\rho \frac{i}{q^2}(-q^\sigma)$$

$$= -\frac{3g^4}{4c_{\mathrm{W}}^4}\int \frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^2} = -\frac{3g^4}{4c_{\mathrm{W}}^4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^{G^0 ZZ} = -\frac{1}{6}\frac{3g^4}{4c_{\mathrm{W}}^4}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}\frac{g^4}{8c_{\mathrm{W}}^4}\ln\mu_{\mathrm{R}}^2 + \cdots, \quad \frac{\partial \delta_\lambda^{G^0 ZZ}}{\partial \ln \mu_{\mathrm{R}}} = \frac{1}{16\pi^2}\frac{g^4}{4c_{\mathrm{W}}^4}$$

$$i\Sigma_\lambda^{G^\pm WW,1} = 2\times 3\times \begin{pmatrix} h & --\wedge-- & h \\ & W^- / \quad \backslash W^- & \\ h & --G^+-- & h \end{pmatrix} = 6\frac{ig^2}{2}\left(\frac{ig}{2}\right)^2 \int \frac{d^d q}{(2\pi)^d}\, g^{\mu\nu}\, \frac{-ig_{\mu\rho}}{q^2}\frac{-ig_{\nu\sigma}}{q^2} q^\rho \frac{i}{q^2} q^\sigma$$

$$= -\frac{3g^4}{4}\int \frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^2} = -\frac{3g^4}{4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma_\lambda^{G^\pm WW,2} = 2\times 3\times \begin{pmatrix} h & --\wedge-- & h \\ & W^+ / \quad \backslash W^+ & \\ h & --G^--- & h \end{pmatrix} = 6\frac{ig^2}{2}\left(\frac{ig}{2}\right)^2 \int \frac{d^d q}{(2\pi)^d}\, g^{\mu\nu}\, \frac{-ig_{\mu\rho}}{q^2}\frac{-ig_{\nu\sigma}}{q^2} q^\rho \frac{i}{q^2} q^\sigma$$

$$= -\frac{3g^4}{4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^{G^\pm WW} = -\frac{1}{6}\cdot 2\cdot\frac{3g^4}{4}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}\frac{g^4}{4}\ln\mu_{\mathrm{R}}^2 + \cdots, \quad \frac{\partial \delta_\lambda^{G^\pm WW}}{\partial \ln \mu_{\mathrm{R}}} = \frac{1}{16\pi^2}\frac{g^4}{2}$$

$$\begin{matrix} h & --\wedge-- & h \\ & h / \quad \backslash h & \\ h & --h-- & h \end{matrix} \sim \begin{matrix} h & --\wedge-- & h \\ & G^0 / \quad \backslash G^0 & \\ h & --G^0-- & h \end{matrix} \sim \begin{matrix} h & --\wedge-- & h \\ & G^\pm / \quad \backslash G^\pm & \\ h & --G^\pm-- & h \end{matrix} \sim \int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2)^3} = \text{finite}$$

(4)

$$i\Sigma_\lambda^{G^0G^0Z} = 2\times 6\times \begin{pmatrix} h & --\wedge-- & h \\ & G^0 \,/ \quad \backslash\, G^0 & \\ h & --Z-- & h \end{pmatrix} = 12\cdot\frac{1}{2}(-2i\lambda)\left(-\frac{g}{2c_{\mathrm{W}}}\right)^2 \int \frac{d^dq}{(2\pi)^d}\frac{i}{q^2}\frac{i}{q^2}(-q^\mu)\frac{-ig_{\mu\nu}}{q^2}q^\nu$$

$$= -\frac{3\lambda g^2}{c_{\mathrm{W}}^2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = -\frac{3\lambda g^2}{c_{\mathrm{W}}^2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^{G^0G^0Z} = -\frac{1}{6}\frac{3\lambda g^2}{c_{\mathrm{W}}^2}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}\frac{\lambda g^2}{2c_{\mathrm{W}}^2}\ln\mu_{\mathrm{R}}^2 + \cdots, \quad \frac{\partial\delta_\lambda^{G^0G^0Z}}{\partial\ln\mu_{\mathrm{R}}} = \frac{1}{16\pi^2}\frac{\lambda g^2}{c_{\mathrm{W}}^2}$$

$$i\Sigma_\lambda^{G^\pm G^\pm W,1} = 2\times 3\times \begin{pmatrix} h & --\wedge-- & h \\ & G^- \,/ \quad \backslash\, G^- & \\ h & --W^+-- & h \end{pmatrix} = 6(-2i\lambda)\left(\frac{ig}{2}\right)^2 \int \frac{d^dq}{(2\pi)^d}\frac{i}{q^2}\frac{i}{q^2}q^\mu\frac{-ig_{\mu\nu}}{q^2}q^\nu$$

$$= -3\lambda g^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = -\frac{3\lambda g^2}{2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma_\lambda^{G^\pm G^\pm W,2} = 2\times 3\times \begin{pmatrix} h & --\wedge-- & h \\ & G^+ \,/ \quad \backslash\, G^+ & \\ h & --W^--- & h \end{pmatrix} = 6(-2i\lambda)\left(\frac{ig}{2}\right)^2 \int \frac{d^dq}{(2\pi)^d}\frac{i}{q^2}\frac{i}{q^2}q^\mu\frac{-ig_{\mu\nu}}{q^2}q^\nu = -3\lambda g^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_\lambda^{G^\pm G^\pm W} = -\frac{1}{6}\cdot 2\cdot 3\lambda g^2\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}\lambda g^2\ln\mu_{\mathrm{R}}^2 + \cdots, \quad \frac{\partial\delta_\lambda^{G^\pm G^\pm W}}{\partial\ln\mu_{\mathrm{R}}} = \frac{1}{16\pi^2}2\lambda g^2$$

$$\frac{g^2}{c_{\mathrm{W}}^2} = g^2 + g'^2, \quad \frac{\lambda g^2}{c_{\mathrm{W}}^2} + 2\lambda g^2 = \lambda(g^2 + g'^2 + 2g^2) = \lambda(3g^2 + g'^2)$$

$$-\frac{3g^4}{8c_{\mathrm{W}}^4} - \frac{3}{4}g^4 = -\frac{3}{8}\left(\frac{g^4}{c_{\mathrm{W}}^4} + 2g^4\right) = -\frac{3}{8}[(g^2+g'^2)^2 + 2g^4] = -\frac{3}{8}(3g^4 + 2g^2g'^2 + g'^4)$$

$$\frac{\partial\delta_\lambda^{\mathrm{SM}}}{\partial\ln\mu_{\mathrm{R}}} = \frac{\partial}{\partial\ln\mu_{\mathrm{R}}}\sum_i\delta_\lambda^i = \frac{1}{16\pi^2}\left(6y_t^4 - 18\lambda^2 - 2\lambda^2 - 4\lambda^2 - \frac{g^4}{2c_{\mathrm{W}}^4} - g^4 - \frac{g^4}{8c_{\mathrm{W}}^2} - \frac{g^4}{4} + \frac{g^4}{4c_{\mathrm{W}}^4} + \frac{g^4}{2} + \frac{\lambda g^2}{c_{\mathrm{W}}^2} + 2\lambda g^2\right)$$

$$= \frac{1}{16\pi^2}\left(6y_t^4 - 24\lambda^2 - \frac{3g^4}{8c_{\mathrm{W}}^4} - \frac{3}{4}g^4 + \frac{\lambda g^2}{c_{\mathrm{W}}^2} + 2\lambda g^2\right)$$

$$= \frac{1}{16\pi^2}\left[6y_t^4 - 24\lambda^2 - \frac{3}{8}(3g^4 + 2g^2g'^2 + g'^4) + \lambda(3g^2 + g'^2)\right]$$

⊙ Explicit, gauge-independent expressions for the β functions of λ

$$\beta_\lambda^{\mathrm{SM}} = -\frac{\partial\delta_\lambda^{\mathrm{SM}}}{\partial\ln\mu_{\mathrm{R}}} + 2\lambda\frac{\partial\delta_h^{\mathrm{SM}}}{\partial\ln\mu_{\mathrm{R}}}$$

$$= \frac{1}{16\pi^2}\left[-6y_t^4 + 24\lambda^2 + \frac{3}{8}(3g^4 + 2g^2g'^2 + g'^4) - \lambda(3g^2 + g'^2)\right] + \frac{1}{16\pi^2}2\lambda(6y_t^2 - 3g^2 - g'^2)$$

$$= \frac{1}{16\pi^2}\left[24\lambda^2 + \lambda(12y_t^2 - 9g^2 - 3g'^2) - 6y_t^4 + \frac{3}{8}(3g^4 + 2g^2g'^2 + g'^4)\right]$$

(5)

SU(2) gauge symmetry: matrix notation vs tensor notation

Matrix notation

Consider a SU(2) triplet T as an example

Gauge transformation matrix $U(x) = \exp[i\theta^a(x) t_{\mathrm{T}}^a], \quad U^\dagger = \exp(-i\theta^a t_{\mathrm{T}}^a)$

$D_\mu T \equiv (\partial_\mu - igW_\mu^a t_{\mathrm{T}}^a)T$

$$T \to UT, \quad W_\mu^a t_{\mathrm{T}}^a \to UW_\mu^a t_{\mathrm{T}}^a U^\dagger + \frac{i}{g} U\partial_\mu U^\dagger$$

$$D_\mu T \to \left[\partial_\mu - ig\left(UW_\mu^a t_{\mathrm{T}}^a U^\dagger + \frac{i}{g}U\partial_\mu U^\dagger\right)\right]UT$$
$$= UU^\dagger(\partial_\mu U)T + U\partial_\mu T - igUW_\mu^a t_{\mathrm{T}}^a T + U(\partial_\mu U^\dagger)UT$$
$$= U(\partial_\mu - igW_\mu^a t_{\mathrm{T}}^a)T + U\partial_\mu(U^\dagger U)UT = UD_\mu T$$

$D_\mu T$ transforms as T

Infinitesimal transformation $U \simeq 1 + i\theta^a t_{\mathrm{T}}^a$

$$\delta T = i\theta^a t_{\mathrm{T}}^a T, \quad \delta(W_\mu^a t_{\mathrm{T}}^a) = i\theta^b [t_{\mathrm{T}}^b, W_\mu^a t_{\mathrm{T}}^a] + \frac{1}{g}(\partial_\mu \theta^a) t_{\mathrm{T}}^a$$

$$\delta(D_\mu T) = (\partial_\mu - igW_\mu^a t_{\mathrm{T}}^a)\delta T - ig\delta(W_\mu^a t_{\mathrm{T}}^a)T$$
$$= (\partial_\mu - igW_\mu^a t_{\mathrm{T}}^a) i\theta^b t_{\mathrm{T}}^b T - ig\left\{i\theta^b[t_{\mathrm{T}}^b, W_\mu^a t_{\mathrm{T}}^a] + \frac{1}{g}(\partial_\mu\theta^a)t_{\mathrm{T}}^a\right\}T$$
$$= i(\partial_\mu \theta^a)t_{\mathrm{T}}^a T + i\theta^a t_{\mathrm{T}}^a \partial_\mu T + g\theta^b W_\mu^a t_{\mathrm{T}}^a t_{\mathrm{T}}^b T + g\theta^b [t_{\mathrm{T}}^b, W_\mu^a t_{\mathrm{T}}^a]T - i(\partial_\mu\theta^a)t_{\mathrm{T}}^a T$$
$$= i\theta^a t_{\mathrm{T}}^a \partial_\mu T + g\theta^b t_{\mathrm{T}}^b W_\mu^a t_{\mathrm{T}}^a T = i\theta^a t_{\mathrm{T}}^a(\partial_\mu - igW_\mu^b t_{\mathrm{T}}^b)T = i\theta^a t_{\mathrm{T}}^a D_\mu T$$

$$D_\mu D_\nu T = \partial_\mu \partial_\nu T - ig(\partial_\mu W_\nu^a)t_{\mathrm{T}}^a T - igW_\nu^a t_{\mathrm{T}}^a \partial_\mu T - igW_\mu^a t_{\mathrm{T}}^a(\partial_\nu T - igW_\nu^b t_{\mathrm{T}}^b T)$$

$$[D_\mu, D_\nu]T = -ig(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)t_{\mathrm{T}}^a T - g^2[W_\mu^a t_{\mathrm{T}}^a, W_\nu^b t_{\mathrm{T}}^b]T$$

$$W_{\mu\nu}^a t_{\mathrm{T}}^a T \equiv \frac{i}{g}[D_\mu, D_\nu]T = (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)t_{\mathrm{T}}^a T - ig[W_\mu^a t_{\mathrm{T}}^a, W_\nu^b t_{\mathrm{T}}^b]T = (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\varepsilon^{abc}W_\mu^b W_\nu^c)t_{\mathrm{T}}^a T$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\varepsilon^{abc} W_\mu^b W_\nu^c$$

$$[D_\mu, D_\nu]T \to U[D_\mu, D_\nu]T = U[D_\mu, D_\nu]U^\dagger UT \quad \Rightarrow \quad W_{\mu\nu}^a t_{\mathrm{T}}^a \to UW_{\mu\nu}^a t_{\mathrm{T}}^a U^\dagger$$

$\mathrm{tr}(W_{\mu\nu}^a t_{\mathrm{T}}^a W^{b\mu\nu} t_{\mathrm{T}}^b)$ is a gauge invariant

Tensor notation

$$V^i_j(x) = \exp[i\theta^a(x)\tau^a]^i_j, \quad (V^\dagger)^i_j = \exp(-i\theta^a\tau^a)^i_j, \quad \tau^a \equiv \frac{\sigma^a}{2}$$

$$\underline{(D_\mu T)^i_j \equiv \partial_\mu T^i_j - ig(W_\mu)^i_k T^k_j + igT^i_k(W_\mu)^k_j}$$

[Ref: Ta-Pei Cheng & Ling-Fong Li, *Gauge Theory of Elementary Particle Physics*, Eq.(4.140)]

$$T^i_j \to V^i_k T^k_l (V^\dagger)^l_j, \quad (W_\mu)^i_j \to V^i_k (W_\mu)^k_l (V^\dagger)^l_j + \frac{i}{g} V^i_k \partial_\mu (V^\dagger)^k_j$$

Infinitesimal transformation $V^i_j \simeq \delta^i_j + i\theta^a(\tau^a)^i_j, \quad (V^\dagger)^i_j \simeq \delta^i_j - i\theta^a(\tau^a)^i_j$

$$\delta T^i_j = i\theta^a(\tau^a)^i_k T^k_j - i\theta^a T^i_l (\tau^a)^l_j = i\theta^a[(\tau^a)^i_k T^k_j - T^i_k(\tau^a)^k_j]$$

$$\delta(W_\mu)^i_j = i\theta^a[(\tau^a)^i_k (W_\mu)^k_j - (W_\mu)^i_k (\tau^a)^k_j] + \frac{1}{g}(\partial_\mu\theta^a)(\tau^a)^i_j$$

$$\begin{aligned}
\delta(D_\mu T)^i_j &= \delta(\partial_\mu T^i_j) - ig\delta(W_\mu)^i_l T^l_j + ig\delta(W_\mu)^l_j T^i_l - ig(W_\mu)^i_l \delta T^l_j + ig\delta T^i_l (W_\mu)^l_j \\
&= i(\partial_\mu\theta^a)[(\tau^a)^i_k T^k_j - T^i_k(\tau^a)^k_j] + i\theta^a[(\tau^a)^i_k \partial_\mu T^k_j - (\partial_\mu T^i_k)(\tau^a)^k_j] \\
&\quad + g\theta^a[(\tau^a)^i_k (W_\mu)^k_l - (W_\mu)^i_k (\tau^a)^k_l] T^l_j - i(\partial_\mu\theta^a)(\tau^a)^i_l T^l_j \\
&\quad - g\theta^a T^i_l [(\tau^a)^l_k (W_\mu)^k_j - (W_\mu)^l_k (\tau^a)^k_j] + i(\partial_\mu\theta^a) T^i_l (\tau^a)^l_j \\
&\quad + g(W_\mu)^i_l \theta^a[(\tau^a)^l_k T^k_j - T^l_k(\tau^a)^k_j] - g\theta^a[(\tau^a)^i_k T^k_l - T^i_k(\tau^a)^k_l](W_\mu)^l_j \\
&= i\theta^a(\tau^a)^i_k \partial_\mu T^k_j + g\theta^a(\tau^a)^i_k (W_\mu)^k_l T^l_j - g\theta^a(\tau^a)^i_k T^k_l (W_\mu)^l_j \\
&\quad - i\theta^a(\partial_\mu T^i_k)(\tau^a)^k_j - g\theta^a(W_\mu)^i_l T^l_k(\tau^a)^k_j + g\theta^a T^i_l (W_\mu)^l_k (\tau^a)^k_j \\
&= i\theta^a(\tau^a)^i_k[\partial_\mu T^k_j - ig(W_\mu)^k_l T^l_j + igT^k_l(W_\mu)^l_j] - i\theta^a[\partial_\mu T^i_k - ig(W_\mu)^i_l T^l_k + igT^i_l(W_\mu)^l_k](\tau^a)^k_j \\
&= i\theta^a[(\tau^a)^i_k (D_\mu T)^k_j - (D_\mu T)^i_k (\tau^a)^k_j]
\end{aligned}$$

$\underline{(D_\mu T)^i_j \text{ transforms as } T^i_j}$

$$\begin{aligned}
(D_\mu D_\nu T)^i_j &= \partial_\mu\partial_\nu T^i_j - ig\partial_\mu(W_\nu)^i_k T^k_j - ig(W_\nu)^i_k \partial_\mu T^k_j + ig(\partial_\mu T^i_k)(W_\nu)^k_j + igT^i_k \partial_\mu (W_\nu)^k_j \\
&\quad - ig(W_\mu)^i_l[\partial_\nu T^l_j - ig(W_\nu)^l_k T^k_j + igT^l_k(W_\nu)^k_j] + ig[\partial_\nu T^i_l - ig(W_\nu)^i_k T^k_l + igT^i_k(W_\nu)^k_l](W_\mu)^l_j
\end{aligned}$$

$$\begin{aligned}
([D_\mu, D_\nu]T)^i_j &= -ig[\partial_\mu(W_\nu)^i_k - \partial_\nu(W_\mu)^i_k]T^k_j + igT^i_k[\partial_\mu(W_\nu)^k_j - \partial_\nu(W_\mu)^k_j] \\
&\quad - g^2[(W_\mu)^i_l(W_\nu)^l_k - (W_\nu)^i_l(W_\mu)^l_k]T^k_j + g^2 T^i_k[(W_\mu)^k_l(W_\nu)^l_j - (W_\nu)^k_l(W_\mu)^l_j]
\end{aligned}$$

$$\begin{aligned}
(W_{\mu\nu})^i_k T^k_j - T^i_k (W_{\mu\nu})^k_j \equiv \frac{i}{g}([D_\mu, D_\nu]T)^i_j &= \{\partial_\mu(W_\nu)^i_k - \partial_\nu(W_\mu)^i_k - ig[(W_\mu)^i_l(W_\nu)^l_k - (W_\nu)^i_l(W_\mu)^l_k]\}T^k_j \\
&\quad - T^i_k\{\partial_\mu(W_\nu)^k_j - \partial_\nu(W_\mu)^k_j - ig[(W_\mu)^k_l(W_\nu)^l_j - (W_\nu)^k_l(W_\mu)^l_j]\}
\end{aligned}$$

$$(W_{\mu\nu})^i_k = \partial_\mu(W_\nu)^i_k - \partial_\nu(W_\mu)^i_k - ig[(W_\mu)^i_l(W_\nu)^l_k - (W_\nu)^i_l(W_\mu)^l_k]$$

$$\begin{aligned}
(W_{\mu\nu})^i_k T^k_j - T^i_k(W_{\mu\nu})^k_j &\to V^i_m[(W_{\mu\nu})^m_k T^k_n - T^m_k(W_{\mu\nu})^k_n](V^\dagger)^n_j \\
&= V^i_m(W_{\mu\nu})^m_k(V^\dagger)^k_l V^l_p T^p_n (V^\dagger)^n_j - V^i_m T^m_k (V^\dagger)^k_l V^l_p (W_{\mu\nu})^p_n (V^\dagger)^n_j
\end{aligned}$$

$\Rightarrow \quad (W_{\mu\nu})^i_j$ transforms as T^i_j

$(W_{\mu\nu})^i_j (W^{\mu\nu})^j_i$ is a gauge invariant

SU(2) Gauge interactions

For an SU(2) doublet D, $D_\mu D = (\partial_\mu - igW_\mu^a \tau^a)D$,

or equivalently, $\underline{(D_\mu D)^i = \partial_\mu D^i - ig(W_\mu)^i_j D^j}$

$$\Rightarrow \quad (W_\mu)^i_j = W_\mu^a (\tau^a)^i_j = \frac{1}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix}$$

$$W_\mu^+ = \sqrt{2}(W_\mu)^1_2, \quad W_\mu^- = \sqrt{2}(W_\mu)^2_1, \quad W_\mu^3 = 2(W_\mu)^1_1 = -2(W_\mu)^2_2$$

For an SU(2) triplet T, $\underline{(D_\mu T)^i_j = \partial_\mu T^i_j - ig(W_\mu)^i_k T^k_j + igT^i_k (W_\mu)^k_j}$

$$T^+ = T^1_2, \quad T^- = T^2_1, \quad T^0 = \sqrt{2}T^1_1 = -\sqrt{2}T^2_2$$

$$(T^+)^\dagger = (T^\dagger)^2_1, \quad (T^-)^\dagger = (T^\dagger)^1_2, \quad (T^0)^\dagger = \sqrt{2}(T^\dagger)^1_1 = -\sqrt{2}(T^\dagger)^2_2$$

$$\begin{aligned}
(T^\dagger)^i_j \bar\sigma^\mu (W_\mu)^j_k T^k_i &= (T^\dagger)^1_1 \bar\sigma^\mu (W_\mu)^1_1 T^1_1 + (T^\dagger)^1_1 \bar\sigma^\mu (W_\mu)^1_2 T^2_1 + (T^\dagger)^1_2 \bar\sigma^\mu (W_\mu)^2_1 T^1_1 + (T^\dagger)^1_2 \bar\sigma^\mu (W_\mu)^2_2 T^2_1 \\
&\quad + (T^\dagger)^2_1 \bar\sigma^\mu (W_\mu)^1_1 T^1_2 + (T^\dagger)^2_1 \bar\sigma^\mu (W_\mu)^1_2 T^2_2 + (T^\dagger)^2_2 \bar\sigma^\mu (W_\mu)^2_1 T^1_2 + (T^\dagger)^2_2 \bar\sigma^\mu (W_\mu)^2_2 T^2_2 \\
&= \frac{1}{4}(T^0)^\dagger \bar\sigma^\mu W_\mu^3 T^0 + \frac{1}{2}(T^0)^\dagger \bar\sigma^\mu W_\mu^+ T^- + \frac{1}{2}(T^-)^\dagger \bar\sigma^\mu W_\mu^- T^0 - \frac{1}{2}(T^-)^\dagger \bar\sigma^\mu W_\mu^3 T^- \\
&\quad + \frac{1}{2}(T^+)^\dagger \bar\sigma^\mu W_\mu^3 T^+ - \frac{1}{2}(T^+)^\dagger \bar\sigma^\mu W_\mu^+ T^0 - \frac{1}{2}(T^0)^\dagger \bar\sigma^\mu W_\mu^- T^+ - \frac{1}{4}(T^0)^\dagger \bar\sigma^\mu W_\mu^3 T^0 \\
&= \frac{1}{2}[W_\mu^3 (T^+)^\dagger \bar\sigma^\mu T^+ - W_\mu^+ (T^+)^\dagger \bar\sigma^\mu T^0 - W_\mu^- (T^0)^\dagger \bar\sigma^\mu T^+ \\
&\qquad + W_\mu^+ (T^0)^\dagger \bar\sigma^\mu T^- + W_\mu^- (T^-)^\dagger \bar\sigma^\mu T^0 - W_\mu^3 (T^-)^\dagger \bar\sigma^\mu T^-]
\end{aligned}$$

$$\begin{aligned}
(T^\dagger)^i_j \bar\sigma^\mu T^j_k (W_\mu)^k_i &= (T^\dagger)^1_1 \bar\sigma^\mu T^1_1 (W_\mu)^1_1 + (T^\dagger)^1_1 \bar\sigma^\mu T^1_2 (W_\mu)^2_1 + (T^\dagger)^1_2 \bar\sigma^\mu T^2_1 (W_\mu)^1_1 + (T^\dagger)^1_2 \bar\sigma^\mu T^2_2 (W_\mu)^2_1 \\
&\quad + (T^\dagger)^2_1 \bar\sigma^\mu T^1_1 (W_\mu)^1_2 + (T^\dagger)^2_1 \bar\sigma^\mu T^1_2 (W_\mu)^2_2 + (T^\dagger)^2_2 \bar\sigma^\mu T^2_1 (W_\mu)^1_2 + (T^\dagger)^2_2 \bar\sigma^\mu T^2_2 (W_\mu)^2_2 \\
&= \frac{1}{4}(T^0)^\dagger \bar\sigma^\mu T^0 W_\mu^3 - \frac{1}{2}(T^0)^\dagger \bar\sigma^\mu T^+ W_\mu^- - \frac{1}{2}(T^-)^\dagger \bar\sigma^\mu T^- W_\mu^3 - \frac{1}{2}(T^-)^\dagger \bar\sigma^\mu T^0 W_\mu^- \\
&\quad + \frac{1}{2}(T^+)^\dagger \bar\sigma^\mu T^0 W_\mu^+ + \frac{1}{2}(T^+)^\dagger \bar\sigma^\mu T^+ W_\mu^3 + \frac{1}{2}(T^0)^\dagger \bar\sigma^\mu T^- W_\mu^+ - \frac{1}{4}(T^0)^\dagger \bar\sigma^\mu T^0 W_\mu^3 \\
&= \frac{1}{2}[-W_\mu^3 (T^+)^\dagger \bar\sigma^\mu T^+ + W_\mu^+ (T^+)^\dagger \bar\sigma^\mu T^0 + W_\mu^- (T^0)^\dagger \bar\sigma^\mu T^+ \\
&\qquad - W_\mu^+ (T^0)^\dagger \bar\sigma^\mu T^- - W_\mu^- (T^-)^\dagger \bar\sigma^\mu T^0 + W_\mu^3 (T^-)^\dagger \bar\sigma^\mu T^-]
\end{aligned}$$

$$\begin{aligned}
\mathcal{L} \supset\ & g[(T^\dagger)^i_j \bar\sigma^\mu (W_\mu)^j_k T^k_i - (T^\dagger)^i_j \bar\sigma^\mu T^j_k (W_\mu)^k_i] \\
&= g[W_\mu^3 (T^+)^\dagger \bar\sigma^\mu T^+ - W_\mu^+ (T^+)^\dagger \bar\sigma^\mu T^0 \\
&\qquad - W_\mu^- (T^0)^\dagger \bar\sigma^\mu T^+ + W_\mu^+ (T^0)^\dagger \bar\sigma^\mu T^- \\
&\qquad + W_\mu^- (T^-)^\dagger \bar\sigma^\mu T^0 - W_\mu^3 (T^-)^\dagger \bar\sigma^\mu T^-]
\end{aligned}$$

This is equivalent to a choice of SU(2) generators in **3** as

$$t^1_{\mathrm{T}} = \frac{1}{\sqrt{2}}\begin{pmatrix} & -1 & \\ -1 & & 1 \\ & 1 & \end{pmatrix}, \quad t^2_{\mathrm{T}} = \frac{1}{\sqrt{2}}\begin{pmatrix} & i & \\ -i & & -i \\ & i & \end{pmatrix}, \quad t^3_{\mathrm{T}} = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$$

[Note: t^1_{T} and t^2_{T} differ from those given in 1601.01354 by a minus sign!]

For a generic SU(2) multiplet $\psi^{i_1\cdots i_p}_{j_1\cdots j_q}$, we have

$$(D_\mu\psi)^{i_1\cdots i_p}_{j_1\cdots j_q} = \partial_\mu\psi^{i_1\cdots i_p}_{j_1\cdots j_q} - ig\left[\sum_{m=1}^{p}(W_\mu)^{i_m}_{k_m}\psi^{i_1\cdots i_{m-1}k_m i_{m+1}\cdots i_p}_{j_1\cdots j_q} - \sum_{n=1}^{q}\psi^{i_1\cdots i_p}_{j_1\cdots j_{n-1}k_n j_{n+1}\cdots j_q}(W_\mu)^{k_n}_{j_n}\right]$$

Note: in differential geometry, the gauge field $(W_\mu)^i_j$ is a connection form.
The gauge connection defines a principal bundle whose base space is the spacetime and structure group is the gauge group.

The upper (lower) indices of $\psi^{i_1\cdots i_p}_{j_1\cdots j_q}$ and $(D_\mu\psi)^{i_1\cdots i_p}_{j_1\cdots j_q}$ are symmetric

$$\sum_{\{i_i,j_i,k_i\}}\sum_{m=1}^{p}(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}(W_\mu)^{i_m}_{k_m}\psi^{i_1\cdots i_{m-1}k_m i_{m+1}\cdots i_p}_{j_1\cdots j_q} = \sum_{\{i_i,j_i,k_i\}}\sum_{m=1}^{p}(\psi^\dagger)^{j_1\cdots j_q}_{i_m i_1\cdots i_{m-1}i_{m+1}\cdots i_p}(W_\mu)^{i_m}_{k_m}\psi^{k_m i_1\cdots i_{m-1}i_{m+1}\cdots i_p}_{j_1\cdots j_q}$$

$$= p\sum_{\{i_i,j_i\},k}(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}(W_\mu)^{i_1}_{k}\psi^{ki_2\cdots i_p}_{j_1\cdots j_q}$$

$$\varepsilon^{ij} = -\varepsilon^{ji}, \quad \varepsilon_{ij} = -\varepsilon_{ji}, \quad \varepsilon^{ik}\varepsilon_{kj} = \delta^i_j$$

$$(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}\psi^{i_1\cdots i_p}_{kj_2\cdots j_q}(W_\mu)^k_{j_1} = (\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}\psi^{i_1\cdots i_p}_{kj_2\cdots j_q}\varepsilon^{km}\varepsilon_{j_1 n}(W_\mu)^n_m = (\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}\varepsilon_{j_1 n}(W_\mu)^n_m\varepsilon^{km}\psi^{i_1\cdots i_p}_{kj_2\cdots j_q}$$

$$= \varepsilon^{j_1 s}(\psi^\dagger)^{j_2\cdots j_q}_{si_1\cdots i_p}\varepsilon_{j_1 n}(W_\mu)^n_m\varepsilon^{km}\varepsilon^{i_1 r}\psi^{i_2\cdots i_p}_{rkj_2\cdots j_q} = (-\varepsilon^{ri_1})(\psi^\dagger)^{j_2\cdots j_q}_{si_1\cdots i_p}(-\varepsilon^{sj_1})\varepsilon_{j_1 n}(W_\mu)^n_m(-\varepsilon^{mk})\psi^{i_2\cdots i_p}_{rkj_2\cdots j_q}$$

$$= -(\psi^\dagger)^{rj_2\cdots j_q}_{si_2\cdots i_p}\delta^s_n(W_\mu)^n_m\psi^{mi_2\cdots i_p}_{rj_2\cdots j_q} = -(\psi^\dagger)^{rj_2\cdots j_q}_{ni_2\cdots i_p}(W_\mu)^n_m\psi^{mi_2\cdots i_p}_{rj_2\cdots j_q} = -(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}(W_\mu)^{i_1}_k\psi^{ki_2\cdots i_p}_{j_1\cdots j_q}$$

$$-\sum_{\{i_i,j_i,k_i\}}\sum_{n=1}^{q}(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}\psi^{i_1\cdots i_p}_{j_1\cdots j_{n-1}k_n j_{n+1}\cdots j_q}(W_\mu)^{k_n}_{j_n} = -q\sum_{\{i_i,j_i\},k}(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}\psi^{i_1\cdots i_p}_{kj_2\cdots j_q}(W_\mu)^k_{j_1}$$

$$= q\sum_{\{i_i,j_i\},k}(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}(W_\mu)^{i_1}_k\psi^{ki_2\cdots i_p}_{j_1\cdots j_q}$$

Simplified form:

$$\underline{i(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}\bar\sigma^\mu(D_\mu\psi)^{i_1\cdots i_p}_{j_1\cdots j_q} = i(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}\bar\sigma^\mu\partial_\mu\psi^{i_1\cdots i_p}_{j_1\cdots j_q} + g(p+q)(\psi^\dagger)^{j_1\cdots j_q}_{i_1\cdots i_p}\bar\sigma^\mu(W_\mu)^{i_1}_k\psi^{ki_2\cdots i_p}_{j_1\cdots j_q}}$$

Gauge interactions for a SU(2) quadruplet

Weyl spinor quadruplet $Q = \begin{pmatrix} Q_{3/2} \\ Q_{1/2} \\ Q_{-1/2} \\ Q_{-3/2} \end{pmatrix}$

$$\begin{cases} Q_{3/2} = Q^{11}_2 \\ Q_{1/2} = \sqrt{3}Q^{11}_1 = -\sqrt{3}Q^{12}_2 = -\sqrt{3}Q^{21}_2 \\ Q_{-1/2} = \sqrt{3}Q^{22}_2 = -\sqrt{3}Q^{12}_1 = -\sqrt{3}Q^{21}_1 \\ Q_{-3/2} = Q^{22}_1 \end{cases} \qquad \begin{cases} Q^\dagger_{3/2} = (Q^\dagger)^2_{11} \\ Q^\dagger_{1/2} = \sqrt{3}(Q^\dagger)^1_{11} = -\sqrt{3}(Q^\dagger)^2_{12} = -\sqrt{3}(Q^\dagger)^2_{21} \\ Q^\dagger_{-1/2} = \sqrt{3}(Q^\dagger)^2_{22} = -\sqrt{3}(Q^\dagger)^1_{12} = -\sqrt{3}(Q^\dagger)^1_{21} \\ Q^\dagger_{-3/2} = (Q^\dagger)^1_{22} \end{cases}$$

$$
\begin{aligned}
&(Q^\dagger)^k_{ij}\bar{\sigma}^\mu (W_\mu)^i_l Q^{lj}_k \\
&= (Q^\dagger)^1_{11}\bar{\sigma}^\mu (W_\mu)^1_1 Q^{11}_1 + (Q^\dagger)^1_{12}\bar{\sigma}^\mu (W_\mu)^1_2 Q^{22}_1 + (Q^\dagger)^1_{21}\bar{\sigma}^\mu (W_\mu)^2_1 Q^{11}_1 + (Q^\dagger)^1_{22}\bar{\sigma}^\mu (W_\mu)^2_2 Q^{22}_1 \\
&\quad + (Q^\dagger)^1_{11}\bar{\sigma}^\mu (W_\mu)^1_2 Q^{21}_1 + (Q^\dagger)^1_{12}\bar{\sigma}^\mu (W_\mu)^1_1 Q^{12}_1 + (Q^\dagger)^1_{21}\bar{\sigma}^\mu (W_\mu)^2_2 Q^{21}_1 + (Q^\dagger)^1_{22}\bar{\sigma}^\mu (W_\mu)^2_1 Q^{12}_1 \\
&\quad + (Q^\dagger)^2_{11}\bar{\sigma}^\mu (W_\mu)^1_1 Q^{11}_2 + (Q^\dagger)^2_{12}\bar{\sigma}^\mu (W_\mu)^1_2 Q^{22}_2 + (Q^\dagger)^2_{21}\bar{\sigma}^\mu (W_\mu)^2_1 Q^{11}_2 + (Q^\dagger)^2_{22}\bar{\sigma}^\mu (W_\mu)^2_2 Q^{22}_2 \\
&\quad + (Q^\dagger)^2_{11}\bar{\sigma}^\mu (W_\mu)^1_2 Q^{21}_2 + (Q^\dagger)^2_{12}\bar{\sigma}^\mu (W_\mu)^1_1 Q^{12}_2 + (Q^\dagger)^2_{21}\bar{\sigma}^\mu (W_\mu)^2_2 Q^{21}_2 + (Q^\dagger)^2_{22}\bar{\sigma}^\mu (W_\mu)^2_1 Q^{12}_2 \\
&= \frac{1}{\sqrt{3}}Q^\dagger_{1/2}\bar{\sigma}^\mu \frac{1}{2}W^3_\mu \frac{1}{\sqrt{3}}Q_{1/2} + \left(-\frac{1}{\sqrt{3}}Q^\dagger_{-1/2}\right)\bar{\sigma}^\mu \frac{1}{\sqrt{2}}W^+_\mu Q_{-3/2} \\
&\qquad\qquad + \left(-\frac{1}{\sqrt{3}}Q^\dagger_{-1/2}\right)\bar{\sigma}^\mu \frac{1}{\sqrt{2}}W^-_\mu \frac{1}{\sqrt{3}}Q_{1/2} + Q^\dagger_{-3/2}\bar{\sigma}^\mu\left(-\frac{1}{2}W^3_\mu\right)Q_{-3/2} \\
&\quad + \frac{1}{\sqrt{3}}Q^\dagger_{1/2}\bar{\sigma}^\mu \frac{1}{\sqrt{2}}W^+_\mu\left(-\frac{1}{\sqrt{3}}Q_{-1/2}\right) + \left(-\frac{1}{\sqrt{3}}Q^\dagger_{-1/2}\right)\bar{\sigma}^\mu \frac{1}{2}W^3_\mu\left(-\frac{1}{\sqrt{3}}Q_{-1/2}\right) \\
&\qquad\qquad + \left(-\frac{1}{\sqrt{3}}Q^\dagger_{-1/2}\right)\bar{\sigma}^\mu\left(-\frac{1}{2}W^3_\mu\right)\left(-\frac{1}{\sqrt{3}}Q_{-1/2}\right) + Q^\dagger_{-3/2}\bar{\sigma}^\mu \frac{1}{\sqrt{2}}W^-_\mu\left(-\frac{1}{\sqrt{3}}Q_{-1/2}\right) \\
&\quad + Q^\dagger_{3/2}\bar{\sigma}^\mu \frac{1}{2}W^3_\mu Q_{3/2} + \left(-\frac{1}{\sqrt{3}}Q^\dagger_{1/2}\right)\bar{\sigma}^\mu \frac{1}{\sqrt{2}}W^+_\mu \frac{1}{\sqrt{3}}Q_{-1/2} \\
&\qquad\qquad + \left(-\frac{1}{\sqrt{3}}Q^\dagger_{1/2}\right)\bar{\sigma}^\mu \frac{1}{\sqrt{2}}W^-_\mu Q_{3/2} + \frac{1}{\sqrt{3}}Q^\dagger_{-1/2}\bar{\sigma}^\mu\left(-\frac{1}{2}W^3_\mu\right)\frac{1}{\sqrt{3}}Q_{-1/2} \\
&\quad + Q^\dagger_{3/2}\bar{\sigma}^\mu \frac{1}{\sqrt{2}}W^+_\mu\left(-\frac{1}{\sqrt{3}}Q_{1/2}\right) + \left(-\frac{1}{\sqrt{3}}Q^\dagger_{1/2}\right)\bar{\sigma}^\mu \frac{1}{2}W^3_\mu\left(-\frac{1}{\sqrt{3}}Q_{1/2}\right) \\
&\qquad\qquad + \left(-\frac{1}{\sqrt{3}}Q^\dagger_{1/2}\right)\bar{\sigma}^\mu\left(-\frac{1}{2}W^3_\mu\right)\left(-\frac{1}{\sqrt{3}}Q_{1/2}\right) + \frac{1}{\sqrt{3}}Q^\dagger_{-1/2}\bar{\sigma}^\mu \frac{1}{\sqrt{2}}W^-_\mu\left(-\frac{1}{\sqrt{3}}Q_{1/2}\right) \\
&= \frac{1}{6}Q^\dagger_{1/2}\bar{\sigma}^\mu W^3_\mu Q_{1/2} - \frac{1}{\sqrt{6}}Q^\dagger_{-1/2}\bar{\sigma}^\mu W^+_\mu Q_{-3/2} - \frac{1}{3\sqrt{2}}Q^\dagger_{-1/2}\bar{\sigma}^\mu W^-_\mu Q_{1/2} - \frac{1}{2}Q^\dagger_{-3/2}\bar{\sigma}^\mu W^3_\mu Q_{-3/2} \\
&\quad - \frac{1}{3\sqrt{2}}Q^\dagger_{1/2}\bar{\sigma}^\mu W^+_\mu Q_{-1/2} + \frac{1}{6}Q^\dagger_{-1/2}\bar{\sigma}^\mu W^3_\mu Q_{-1/2} - \frac{1}{6}Q^\dagger_{-1/2}\bar{\sigma}^\mu W^3_\mu Q_{-1/2} - \frac{1}{\sqrt{6}}Q^\dagger_{-3/2}\bar{\sigma}^\mu W^-_\mu Q_{-1/2} \\
&\quad + \frac{1}{2}Q^\dagger_{3/2}\bar{\sigma}^\mu W^3_\mu Q_{3/2} - \frac{1}{3\sqrt{2}}Q^\dagger_{1/2}\bar{\sigma}^\mu W^+_\mu Q_{-1/2} - \frac{1}{\sqrt{6}}Q^\dagger_{1/2}\bar{\sigma}^\mu W^-_\mu Q_{3/2} - \frac{1}{6}Q^\dagger_{-1/2}\bar{\sigma}^\mu W^3_\mu Q_{-1/2} \\
&\quad - \frac{1}{\sqrt{6}}Q^\dagger_{3/2}\bar{\sigma}^\mu W^+_\mu Q_{1/2} + \frac{1}{6}Q^\dagger_{1/2}\bar{\sigma}^\mu W^3_\mu Q_{1/2} - \frac{1}{6}Q^\dagger_{1/2}\bar{\sigma}^\mu W^3_\mu Q_{1/2} - \frac{1}{3\sqrt{2}}Q^\dagger_{-1/2}\bar{\sigma}^\mu W^-_\mu Q_{1/2} \\
&= \frac{1}{2}W^3_\mu Q^\dagger_{3/2}\bar{\sigma}^\mu Q_{3/2} - \frac{1}{\sqrt{6}}W^-_\mu Q^\dagger_{1/2}\bar{\sigma}^\mu Q_{3/2} \\
&\quad - \frac{1}{\sqrt{6}}W^+_\mu Q^\dagger_{3/2}\bar{\sigma}^\mu Q_{1/2} + \frac{1}{6}W^3_\mu Q^\dagger_{1/2}\bar{\sigma}^\mu Q_{1/2} - \frac{2}{3\sqrt{2}}W^-_\mu Q^\dagger_{-1/2}\bar{\sigma}^\mu Q_{1/2} \\
&\quad - \frac{2}{3\sqrt{2}}W^+_\mu Q^\dagger_{1/2}\bar{\sigma}^\mu Q_{-1/2} - \frac{1}{6}W^3_\mu Q^\dagger_{-1/2}\bar{\sigma}^\mu Q_{-1/2} - \frac{1}{\sqrt{6}}W^-_\mu Q^\dagger_{-3/2}\bar{\sigma}^\mu Q_{-1/2} \\
&\quad - \frac{1}{\sqrt{6}}W^+_\mu Q^\dagger_{-1/2}\bar{\sigma}^\mu Q_{-3/2} - \frac{1}{2}W^3_\mu Q^\dagger_{-3/2}\bar{\sigma}^\mu Q_{-3/2}
\end{aligned}
$$

(10)

$$(D_\mu Q)^{ij}_k = \partial_\mu \psi^{ij}_k - ig(W_\mu)^i_l Q^{lj}_k - ig(W_\mu)^j_l Q^{il}_k + igQ^{ij}_l (W_\mu)^l_k$$
$$\mathcal{L} \supset i(Q^\dagger)^k_{ij} \bar{\sigma}^\mu (D_\mu Q)^{ij}_k = i(Q^\dagger)^k_{ij} \partial_\mu Q^{ij}_k + 3g(Q^\dagger)^k_{ij} \bar{\sigma}^\mu (W_\mu)^i_l Q^{lj}_k$$

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} &= 3g(Q^\dagger)^k_{ij} \bar{\sigma}^\mu (W_\mu)^i_l Q^{lj}_k \\
&= g\Bigg[\frac{3}{2} W^3_\mu Q^\dagger_{3/2} \bar{\sigma}^\mu Q_{3/2} - \frac{\sqrt{6}}{2} W^-_\mu Q^\dagger_{1/2} \bar{\sigma}^\mu Q_{3/2} \\
&\quad - \frac{\sqrt{6}}{2} W^+_\mu Q^\dagger_{3/2} \bar{\sigma}^\mu Q_{1/2} + \frac{1}{2} W^3_\mu Q^\dagger_{1/2} \bar{\sigma}^\mu Q_{1/2} - \sqrt{2} W^-_\mu Q^\dagger_{-1/2} \bar{\sigma}^\mu Q_{1/2} \\
&\quad - \sqrt{2} W^+_\mu Q^\dagger_{1/2} \bar{\sigma}^\mu Q_{-1/2} - \frac{1}{2} W^3_\mu Q^\dagger_{-1/2} \bar{\sigma}^\mu Q_{-1/2} - \frac{\sqrt{6}}{2} W^-_\mu Q^\dagger_{-3/2} \bar{\sigma}^\mu Q_{-1/2} \\
&\quad - \frac{\sqrt{6}}{2} W^+_\mu Q^\dagger_{-1/2} \bar{\sigma}^\mu Q_{-3/2} - \frac{3}{2} W^3_\mu Q^\dagger_{-3/2} \bar{\sigma}^\mu Q_{-3/2} \Bigg]
\end{aligned}$$

This is equivalent to a choice of SU(2) generators in **4** as

$$t^1_{\text{Q}} = \begin{pmatrix} & -\sqrt{3}/2 & & \\ -\sqrt{3}/2 & & -1 & \\ & -1 & & -\sqrt{3}/2 \\ & & -\sqrt{3}/2 & \end{pmatrix}$$
$$t^2_{\text{Q}} = \begin{pmatrix} & \sqrt{3}i/2 & & \\ -\sqrt{3}i/2 & & i & \\ & -i & & \sqrt{3}i/2 \\ & & -\sqrt{3}i/2 & \end{pmatrix}$$
$$t^3_{\text{Q}} = \begin{pmatrix} 3/2 & & & \\ & 1/2 & & \\ & & -1/2 & \\ & & & -3/2 \end{pmatrix}$$

$\Big[$Note: t^1_{Q} and t^2_{Q} differ from those given in 1601.01354 by a minus sign!$\Big]$

1) TQFDM model

Left-handed Weyl spinors: $T=\begin{pmatrix}T^+\\T^0\\T^-\end{pmatrix}\in(\mathbf{3},0),\quad Q_1=\begin{pmatrix}Q_1^+\\Q_1^0\\Q_1^-\\Q_1^{--}\end{pmatrix}\in\left(\mathbf{4},-\frac{1}{2}\right), Q_2=\begin{pmatrix}Q_2^{++}\\Q_2^+\\Q_2^0\\Q_2^-\end{pmatrix}\in\left(\mathbf{4},\frac{1}{2}\right)$

$$\mathcal{L}_{\mathrm{T}}=iT^\dagger\bar{\sigma}^\mu D_\mu T-\frac{1}{2}(m_T TT+\mathrm{h.c.}),\quad \mathcal{L}_{\mathrm{Q}}=iQ_1^\dagger\bar{\sigma}^\mu D_\mu Q_1+iQ_2^\dagger\bar{\sigma}^\mu D_\mu Q_2-(m_Q Q_1Q_2+\mathrm{h.c.})$$

$$\mathcal{L}_{\mathrm{HTQ}}=y_1Q_1TH-y_2Q_2TH^\dagger+\mathrm{h.c.}$$

4-component spinors:

$$\mathcal{T}^0=\begin{pmatrix}T^0\\(T^0)^\dagger\end{pmatrix},\quad \mathcal{T}^+=\begin{pmatrix}T^+\\(T^-)^\dagger\end{pmatrix},\quad \mathcal{Q}^-=\begin{pmatrix}-Q_2^-\\(Q_1^+)^\dagger\end{pmatrix},\quad \mathcal{Q}^0=\begin{pmatrix}Q_2^0\\(Q_1^0)^\dagger\end{pmatrix},\quad \mathcal{Q}^+=\begin{pmatrix}-Q_2^+\\(Q_1^-)^\dagger\end{pmatrix},\quad \mathcal{Q}^{++}=\begin{pmatrix}Q_2^{++}\\(Q_1^{--})^\dagger\end{pmatrix}$$

$\mathcal{T}^0=(\mathcal{T}^0)^{\mathrm{c}}$ is a Majorana spinor, and the others are Dirac spinors

$$\bar{\mathcal{T}}^+\mathcal{T}^+=\begin{pmatrix}(T^+)^\dagger & T^-\end{pmatrix}\begin{pmatrix} & 1\\ 1 & \end{pmatrix}\begin{pmatrix}T^+\\(T^-)^\dagger\end{pmatrix}=\begin{pmatrix}T^- & (T^+)^\dagger\end{pmatrix}\begin{pmatrix}T^+\\(T^-)^\dagger\end{pmatrix}=T^-T^++\mathrm{h.c.},\quad \bar{\mathcal{T}}^0\mathcal{T}^0=T^0T^0+\mathrm{h.c.}$$

$$\bar{\mathcal{Q}}^-\mathcal{Q}^-=-Q_1^+Q_2^-+\mathrm{h.c.},\quad \bar{\mathcal{Q}}^0\mathcal{Q}^0=Q_1^0Q_2^0+\mathrm{h.c.},\quad \bar{\mathcal{Q}}^+\mathcal{Q}^+=-Q_1^-Q_2^++\mathrm{h.c.},\quad \bar{\mathcal{Q}}^{++}\mathcal{Q}^{++}=Q_1^{--}Q_2^{++}+\mathrm{h.c.}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{TQ,mass}}&=-\frac{1}{2}m_T TT-m_Q Q_1Q_2+\mathrm{h.c.}\\&=-m_T T^-T^+-\frac{1}{2}m_T T^0T^0-m_Q(Q_1^{--}Q_2^{++}-Q_1^-Q_2^+-Q_1^+Q_2^-)-m_Q Q_1^0Q_2^0+\mathrm{h.c.}\\&=-m_T\left(\frac{1}{2}\bar{\mathcal{T}}^0\mathcal{T}^0+\bar{\mathcal{T}}^+\mathcal{T}^+\right)-m_Q(\bar{\mathcal{Q}}^-\mathcal{Q}^-+\bar{\mathcal{Q}}^0\mathcal{Q}^0+\bar{\mathcal{Q}}^+\mathcal{Q}^++\bar{\mathcal{Q}}^{++}\mathcal{Q}^{++})\end{aligned}$$

$$\gamma^0\gamma^\mu=\begin{pmatrix} & 1\\ 1 & \end{pmatrix}\begin{pmatrix} & \sigma^\mu\\ \bar{\sigma}^\mu & \end{pmatrix}=\begin{pmatrix}\bar{\sigma}^\mu & \\ & \sigma^\mu\end{pmatrix},\quad T^-\sigma^\mu(T^0)^\dagger=-(T^0)^\dagger\bar{\sigma}^\mu T^-$$

$$\bar{\mathcal{T}}^+\gamma^\mu\mathcal{T}^0=\begin{pmatrix}(T^+)^\dagger & T^-\end{pmatrix}\begin{pmatrix}\bar{\sigma}^\mu & \\ & \sigma^\mu\end{pmatrix}\begin{pmatrix}T^0\\(T^0)^\dagger\end{pmatrix}=(T^+)^\dagger\bar{\sigma}^\mu T^0+T^-\sigma^\mu(T^0)^\dagger=(T^+)^\dagger\bar{\sigma}^\mu T^0-(T^0)^\dagger\bar{\sigma}^\mu T^-$$

$$\begin{aligned}\mathcal{L}_{\mathrm{T,gauge}}&=T^\dagger\bar{\sigma}^\mu gW_\mu^a t_{\mathrm{T}}^a T\\&=(eA_\mu+gc_{\mathrm{W}}Z_\mu)(T^+)^\dagger\bar{\sigma}^\mu T^+-gW_\mu^+(T^+)^\dagger\bar{\sigma}^\mu T^0\\&\quad-gW_\mu^-(T^0)^\dagger\bar{\sigma}^\mu T^++gW_\mu^+(T^0)^\dagger\bar{\sigma}^\mu T^-\\&\quad+gW_\mu^-(T^-)^\dagger\bar{\sigma}^\mu T^0-(eA_\mu+gc_{\mathrm{W}}Z_\mu)(T^-)^\dagger\bar{\sigma}^\mu T^-\\&=(eA_\mu+gc_{\mathrm{W}}Z_\mu)\bar{\mathcal{T}}^+\gamma^\mu\mathcal{T}^+-gW_\mu^+\bar{\mathcal{T}}^+\gamma^\mu\mathcal{T}^0-gW_\mu^-\bar{\mathcal{T}}^0\gamma^\mu\mathcal{T}^+\end{aligned}$$

[Note: the couplings to W differ from those given in 1601.01354 by a minus sign!]

$$\bar{\mathcal{Q}}^{+}\gamma^{\mu}\mathcal{Q}^{+}=\begin{pmatrix}-(Q_2^{+})^{\dagger} & Q_1^{-}\end{pmatrix}\begin{pmatrix}\bar{\sigma}^{\mu} & \\ & \sigma^{\mu}\end{pmatrix}\begin{pmatrix}-Q_2^{+}\\ (Q_1^{-})^{\dagger}\end{pmatrix}=(Q_2^{+})^{\dagger}\bar{\sigma}^{\mu}Q_2^{+}-(Q_1^{-})^{\dagger}\bar{\sigma}^{\mu}Q_1^{-}$$

$$\bar{\mathcal{Q}}^{-}\gamma^{\mu}\mathcal{Q}^{-}=(Q_2^{-})^{\dagger}\bar{\sigma}^{\mu}Q_2^{-}-(Q_1^{+})^{\dagger}\bar{\sigma}^{\mu}Q_1^{+},\quad \bar{\mathcal{Q}}^{0}\gamma^{\mu}\mathcal{Q}^{0}=(Q_2^{0})^{\dagger}\bar{\sigma}^{\mu}Q_2^{0}-(Q_1^{0})^{\dagger}\bar{\sigma}^{\mu}Q_1^{0}$$

$$\bar{\mathcal{Q}}^{++}\gamma^{\mu}\mathcal{Q}^{++}=(Q_2^{++})^{\dagger}\bar{\sigma}^{\mu}Q_2^{++}-(Q_1^{--})^{\dagger}\bar{\sigma}^{\mu}Q_1^{--}$$

$$\bar{\mathcal{Q}}^{+}\gamma^{\mu}\mathcal{Q}^{0}=\begin{pmatrix}-(Q_2^{+})^{\dagger} & Q_1^{-}\end{pmatrix}\begin{pmatrix}\bar{\sigma}^{\mu} & \\ & \sigma^{\mu}\end{pmatrix}\begin{pmatrix}Q_2^{0}\\ (Q_1^{0})^{\dagger}\end{pmatrix}=-(Q_2^{+})^{\dagger}\bar{\sigma}^{\mu}Q_2^{0}-(Q_1^{0})^{\dagger}\bar{\sigma}^{\mu}Q_1^{-}$$

$$\bar{\mathcal{Q}}^{0}\gamma^{\mu}\mathcal{Q}^{-}=-(Q_2^{0})^{\dagger}\bar{\sigma}^{\mu}Q_2^{-}-(Q_1^{+})^{\dagger}\bar{\sigma}^{\mu}Q_1^{0},\quad \bar{\mathcal{Q}}^{++}\gamma^{\mu}\mathcal{Q}^{+}=-(Q_2^{++})^{\dagger}\bar{\sigma}^{\mu}Q_2^{+}-(Q_1^{-})^{\dagger}\bar{\sigma}^{\mu}Q_1^{--}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{Q,gauge}}&=Q_1^{\dagger}\bar{\sigma}^{\mu}(g'B_{\mu}Y_{Q_1}+gW_{\mu}^{a}t_{\mathrm{Q}}^{a})Q_1+Q_2^{\dagger}\bar{\sigma}^{\mu}(g'B_{\mu}Y_{Q_2}+gW_{\mu}^{a}t_{\mathrm{Q}}^{a})Q_2\\
&=\frac{g}{2c_{\mathrm{W}}}Z_{\mu}(Q_1^{0})^{\dagger}\bar{\sigma}^{\mu}Q_1^{0}-\frac{g}{2c_{\mathrm{W}}}Z_{\mu}(Q_2^{0})^{\dagger}\bar{\sigma}^{\mu}Q_2^{0}\\
&\quad+\left[eA_{\mu}+\frac{g(3c_{\mathrm{W}}^2+s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_{\mu}\right](Q_1^{+})^{\dagger}\bar{\sigma}^{\mu}Q_1^{+}+\left[eA_{\mu}+\frac{g(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_{\mu}\right](Q_2^{+})^{\dagger}\bar{\sigma}^{\mu}Q_2^{+}\\
&\quad+\left[-eA_{\mu}-\frac{g(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_{\mu}\right](Q_1^{-})^{\dagger}\bar{\sigma}^{\mu}Q_1^{-}+\left[-eA_{\mu}-\frac{g(3c_{\mathrm{W}}^2+s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_{\mu}\right](Q_2^{-})^{\dagger}\bar{\sigma}^{\mu}Q_2^{-}\\
&\quad+\left[2eA_{\mu}+\frac{g(3c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_{\mu}\right](Q_2^{++})^{\dagger}\bar{\sigma}^{\mu}Q_2^{++}+\left[-2eA_{\mu}-\frac{g(3c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_{\mu}\right](Q_1^{--})^{\dagger}\bar{\sigma}^{\mu}Q_1^{--}\\
&\quad+\Bigg(-gW_{\mu}^{+}\Bigg\{\sqrt{2}[(Q_1^{0})^{\dagger}\bar{\sigma}^{\mu}Q_1^{-}+(Q_2^{+})^{\dagger}\bar{\sigma}^{\mu}Q_2^{0}]+\frac{\sqrt{6}}{2}[(Q_1^{+})^{\dagger}\bar{\sigma}^{\mu}Q_1^{0}+(Q_2^{0})^{\dagger}\bar{\sigma}^{\mu}Q_2^{-}]\\
&\qquad\qquad+\frac{\sqrt{6}}{2}[(Q_2^{++})^{\dagger}\bar{\sigma}^{\mu}Q_2^{+}+(Q_1^{-})^{\dagger}\bar{\sigma}^{\mu}Q_1^{--}]\Bigg\}+\mathrm{h.c.}\Bigg)\\
&=eA_{\mu}(-\bar{\mathcal{Q}}^{-}\gamma^{\mu}\mathcal{Q}^{-}+\bar{\mathcal{Q}}^{+}\gamma^{\mu}\mathcal{Q}^{+}+2\bar{\mathcal{Q}}^{++}\gamma^{\mu}\mathcal{Q}^{++})\\
&\quad+\frac{g}{2c_{\mathrm{W}}}Z_{\mu}[-(3c_{\mathrm{W}}^2+s_{\mathrm{W}}^2)\bar{\mathcal{Q}}^{-}\gamma^{\mu}\mathcal{Q}^{-}-\bar{\mathcal{Q}}^{0}\gamma^{\mu}\mathcal{Q}^{0}+(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)\bar{\mathcal{Q}}^{+}\gamma^{\mu}\mathcal{Q}^{+}+(3c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)\bar{\mathcal{Q}}^{++}\gamma^{\mu}\mathcal{Q}^{++}]\\
&\quad+\left[gW_{\mu}^{+}\left(\sqrt{2}\bar{\mathcal{Q}}^{+}\gamma^{\mu}\mathcal{Q}^{0}+\frac{\sqrt{6}}{2}\bar{\mathcal{Q}}^{0}\gamma^{\mu}\mathcal{Q}^{-}+\frac{\sqrt{6}}{2}\bar{\mathcal{Q}}^{++}\gamma^{\mu}\mathcal{Q}^{+}\right)+\mathrm{h.c.}\right]
\end{aligned}$$

[Note: the couplings to W differ from those given in 1601.01354 by a minus sign!]

$$P_{\mathrm{L}}=\begin{pmatrix}1 & \\ & 0\end{pmatrix},\quad P_{\mathrm{R}}=\begin{pmatrix}0 & \\ & 1\end{pmatrix},\quad (P_{\mathrm{L}}\psi)^\dagger\gamma^0=\psi^\dagger P_{\mathrm{L}}\gamma^0=\bar{\psi}P_{\mathrm{R}},\quad (P_{\mathrm{R}}\psi)^\dagger\gamma^0=\bar{\psi}P_{\mathrm{L}},\quad (\bar{\psi}_1P_{\mathrm{L}}\psi_2)^\dagger=\psi_2^\dagger P_{\mathrm{L}}\gamma^0\psi_1=\bar{\psi}_2P_{\mathrm{R}}\psi_1$$

$$\bar{\mathcal{Q}}^+\mathcal{T}^0=\begin{pmatrix}-(\mathcal{Q}_2^+)^\dagger & \mathcal{Q}_1^-\end{pmatrix}\begin{pmatrix} & 1\\ 1 & \end{pmatrix}\begin{pmatrix}T^0\\ (T^0)^\dagger\end{pmatrix}=\mathcal{Q}_1^-T^0-(\mathcal{Q}_2^+)^\dagger(T^0)^\dagger$$

$$\bar{\mathcal{Q}}^+P_{\mathrm{L}}\mathcal{T}^0=\mathcal{Q}_1^-T^0,\quad \bar{\mathcal{Q}}^+P_{\mathrm{R}}\mathcal{T}^0=-(\mathcal{Q}_2^+)^\dagger(T^0)^\dagger$$

$$(\mathcal{T}^+)^{\mathrm{c}}=\mathcal{C}(\bar{\mathcal{T}}^+)^{\mathrm{T}}=\begin{pmatrix}T^-\\ (T^+)^\dagger\end{pmatrix},\quad \overline{(\mathcal{T}^+)^{\mathrm{c}}}=(\mathcal{T}^+)^{\mathrm{T}}\mathcal{C}$$

$$\bar{\mathcal{Q}}^0(\mathcal{T}^+)^{\mathrm{c}}=\begin{pmatrix}(\mathcal{Q}_2^0)^\dagger & \mathcal{Q}_1^0\end{pmatrix}\begin{pmatrix} & 1\\ 1 & \end{pmatrix}\begin{pmatrix}T^-\\ (T^+)^\dagger\end{pmatrix}=\mathcal{Q}_1^0T^-+(\mathcal{Q}_2^0)^\dagger(T^+)^\dagger$$

$$\bar{\mathcal{Q}}^0P_{\mathrm{L}}(\mathcal{T}^+)^{\mathrm{c}}=\mathcal{Q}_1^0T^-,\quad \bar{\mathcal{Q}}^0P_{\mathrm{R}}(\mathcal{T}^+)^{\mathrm{c}}=(\mathcal{Q}_2^0)^\dagger(T^+)^\dagger$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{HTQ}}&=y_1G^+\left(\mathcal{Q}_1^{--}T^+-\frac{2}{\sqrt{6}}\mathcal{Q}_1^-T^0-\frac{1}{\sqrt{3}}\mathcal{Q}_1^0T^-\right)+y_1(v+h+iG^0)\left(-\frac{1}{\sqrt{3}}\mathcal{Q}_1^0T^0+\frac{1}{\sqrt{6}}\mathcal{Q}_1^-T^+-\frac{1}{\sqrt{2}}\mathcal{Q}_1^+T^-\right)\\
&\quad+y_2G^-\left(-\mathcal{Q}_2^{++}T^--\frac{2}{\sqrt{6}}\mathcal{Q}_2^+T^0+\frac{1}{\sqrt{3}}\mathcal{Q}_2^0T^+\right)+y_2(v+h-iG^0)\left(\frac{1}{\sqrt{3}}\mathcal{Q}_2^0T^0+\frac{1}{\sqrt{6}}\mathcal{Q}_2^+T^--\frac{1}{\sqrt{2}}\mathcal{Q}_2^-T^+\right)+\text{h.c.}\\
&=y_1G^+\left[\bar{\mathcal{Q}}^{++}P_{\mathrm{L}}\mathcal{T}^+-\frac{2}{\sqrt{6}}\bar{\mathcal{Q}}^+P_{\mathrm{L}}\mathcal{T}^0-\frac{1}{\sqrt{3}}\bar{\mathcal{Q}}^0P_{\mathrm{L}}(\mathcal{T}^+)^{\mathrm{c}}\right]+y_1(v+h+iG^0)\left(-\frac{1}{\sqrt{3}}\bar{\mathcal{Q}}^0P_{\mathrm{L}}\mathcal{T}^0+\frac{1}{\sqrt{6}}\bar{\mathcal{Q}}^+P_{\mathrm{L}}\mathcal{T}^+-\frac{1}{\sqrt{2}}\bar{\mathcal{Q}}^-P_{\mathrm{L}}(\mathcal{T}^+)^{\mathrm{c}}\right)\\
&\quad+y_1G^-\left[\bar{\mathcal{T}}^+P_{\mathrm{R}}\mathcal{Q}^{++}-\frac{2}{\sqrt{6}}\bar{\mathcal{T}}^0P_{\mathrm{R}}\mathcal{Q}^+-\frac{1}{\sqrt{3}}\overline{(\mathcal{T}^+)^{\mathrm{c}}}P_{\mathrm{R}}\mathcal{Q}^0\right]+y_1(v+h-iG^0)\left(-\frac{1}{\sqrt{3}}\bar{\mathcal{T}}^0P_{\mathrm{R}}\mathcal{Q}^0+\frac{1}{\sqrt{6}}\bar{\mathcal{T}}^+P_{\mathrm{R}}\mathcal{Q}^+-\frac{1}{\sqrt{2}}\overline{(\mathcal{T}^+)^{\mathrm{c}}}P_{\mathrm{R}}\mathcal{Q}^-\right)\\
&\quad+y_2G^-\left[-\bar{\mathcal{T}}^+P_{\mathrm{L}}\mathcal{Q}^{++}+\frac{2}{\sqrt{6}}\bar{\mathcal{T}}^0P_{\mathrm{L}}\mathcal{Q}^++\frac{1}{\sqrt{3}}\overline{(\mathcal{T}^+)^{\mathrm{c}}}P_{\mathrm{L}}\mathcal{Q}^0\right]+y_2(v+h-iG^0)\left(\frac{1}{\sqrt{3}}\bar{\mathcal{T}}^0P_{\mathrm{L}}\mathcal{Q}^0-\frac{1}{\sqrt{6}}\bar{\mathcal{T}}^+P_{\mathrm{L}}\mathcal{Q}^++\frac{1}{\sqrt{2}}\overline{(\mathcal{T}^+)^{\mathrm{c}}}P_{\mathrm{L}}\mathcal{Q}^-\right)\\
&\quad+y_2G^+\left[-\bar{\mathcal{Q}}^{++}P_{\mathrm{R}}\mathcal{T}^++\frac{2}{\sqrt{6}}\bar{\mathcal{Q}}^+P_{\mathrm{R}}\mathcal{T}^0+\frac{1}{\sqrt{3}}\bar{\mathcal{Q}}^0P_{\mathrm{R}}(\mathcal{T}^+)^{\mathrm{c}}\right]+y_2(v+h+iG^0)\left(\frac{1}{\sqrt{3}}\bar{\mathcal{Q}}^0P_{\mathrm{R}}\mathcal{T}^0-\frac{1}{\sqrt{6}}\bar{\mathcal{Q}}^+P_{\mathrm{R}}\mathcal{T}^++\frac{1}{\sqrt{2}}\bar{\mathcal{Q}}^-P_{\mathrm{R}}(\mathcal{T}^+)^{\mathrm{c}}\right)\\
&=G^+\left[\bar{\mathcal{Q}}^{++}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\mathcal{T}^++\frac{2}{\sqrt{6}}\bar{\mathcal{Q}}^+(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\mathcal{T}^0+\frac{1}{\sqrt{3}}\bar{\mathcal{Q}}^0(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\mathcal{T}^+)^{\mathrm{c}}\right]\\
&\quad+G^-\left[\bar{\mathcal{T}}^+(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\mathcal{Q}^{++}+\frac{2}{\sqrt{6}}\bar{\mathcal{T}}^0(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^++\frac{1}{\sqrt{3}}\overline{(\mathcal{T}^+)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^0\right]\\
&\quad+(v+h+iG^0)\left[\frac{1}{\sqrt{3}}\bar{\mathcal{Q}}^0(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\mathcal{T}^0+\frac{1}{\sqrt{6}}\bar{\mathcal{Q}}^+(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\mathcal{T}^++\frac{1}{\sqrt{2}}\bar{\mathcal{Q}}^-(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\mathcal{T}^+)^{\mathrm{c}}\right]\\
&\quad+(v+h-iG^0)\left[\frac{1}{\sqrt{3}}\bar{\mathcal{T}}^0(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^0+\frac{1}{\sqrt{6}}\bar{\mathcal{T}}^+(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\mathcal{Q}^++\frac{1}{\sqrt{2}}\overline{(\mathcal{T}^+)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^-\right]
\end{aligned}$$

⊙ Examples for Feynman rules

$$\mathcal{T}^+=\int\frac{d^3p}{(2\pi)^3 2p^0}\sum_s[a_{\mathcal{T}^+}(p,s)u(p,s)e^{-ip\cdot x}+b_{\mathcal{T}^+}^\dagger(p,s)v(p,s)e^{ip\cdot x}],\quad \bar{\mathcal{T}}^+\sim b_{\mathcal{T}^+}\bar{v}e^{-ip\cdot x}+a_{\mathcal{T}^+}^\dagger\bar{u}e^{ip\cdot x}$$

$$G^+\sim a_{G^+}e^{-ip\cdot x}+b_{G^+}^\dagger e^{ip\cdot x},\quad G^-=(G^+)^\dagger\sim b_{G^+}e^{-ip\cdot x}+a_{G^+}^\dagger e^{ip\cdot x}$$

$$\left.\begin{array}{l} i\langle 0|a_{\mathcal{Q}^{++}}G^+\bar{\mathcal{Q}}^{++}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\mathcal{T}^+a_{\mathcal{T}^+}^\dagger a_{G^+}^\dagger|0\rangle\\ i\langle 0|b_{G^+}b_{\mathcal{T}^+}G^+\bar{\mathcal{Q}}^{++}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\mathcal{T}^+b_{\mathcal{Q}^{++}}^\dagger|0\rangle\end{array}\right\}\rightarrow\quad \begin{array}{ccc} & G^+ & \\ & \downarrow & \\ \mathcal{T}^+ & \rightarrow\times\rightarrow & \mathcal{Q}^{++}\end{array}\quad =i(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})$$

$$\left.\begin{array}{l} i\langle 0|a_{G^+}a_{\mathcal{T}^+}G^-\bar{\mathcal{T}}^+(y_1P_{\rm R} - y_2P_{\rm L})\mathcal{Q}^{++}a^\dagger_{\mathcal{Q}^{++}}|0\rangle \\ i\langle 0|b_{\mathcal{Q}^{++}}G^-\bar{\mathcal{T}}^+(y_1P_{\rm R}-y_2P_{\rm L})\mathcal{Q}^{++}b^\dagger_{\mathcal{T}^+}b^\dagger_{G^+}|0\rangle\end{array}\right\}\to \begin{array}{ccc} & G^+ & \\ & \uparrow & \\ \mathcal{Q}^{++} & \to\times\to & \mathcal{T}^+\end{array} = i(y_1P_{\rm R}-y_2P_{\rm L})$$

$$(\mathcal{T}^+)^{\rm c} = b_{\mathcal{T}^+}ue^{-ip\cdot x} + a^\dagger_{\mathcal{T}^+}ve^{ip\cdot x}, \quad \overline{(\mathcal{T}^+)^{\rm c}} \sim a_{\mathcal{T}^+}\bar{v}e^{-ip\cdot x} + b^\dagger_{\mathcal{T}^+}\bar{u}e^{ip\cdot x}$$

[Denner, Eck, Hahn & Küblbeck, Nucl. Phys. B387, 467-481 (1992)]

$$\left.\begin{array}{l} \dfrac{i}{\sqrt{3}}y_1\langle 0|a_{\mathcal{Q}^0}G^+\bar{\mathcal{Q}}^0(-y_1P_{\rm L}+y_2P_{\rm R})(\mathcal{T}^+)^{\rm c}b^\dagger_{\mathcal{T}^+}a^\dagger_{G^+}|0\rangle \\ \dfrac{i}{\sqrt{3}}y_1\langle 0|b_{G^+}a_{\mathcal{T}^+}G^+\bar{\mathcal{Q}}^0(-y_1P_{\rm L}+y_2P_{\rm R})(\mathcal{T}^+)^{\rm c}b^\dagger_{\mathcal{Q}^0}|0\rangle\end{array}\right\}\to \begin{array}{ccc} & G^+ & \\ & \downarrow & \\ (\mathcal{T}^+)^{\rm c} & \to\times\to & \mathcal{Q}^0\end{array} = \frac{i}{\sqrt{3}}(-y_1P_{\rm L}+y_2P_{\rm R})$$

$$\left.\begin{array}{l} \dfrac{i}{\sqrt{3}}y_1\langle 0|a_{G^+}b_{\mathcal{T}^+}G^-\overline{(\mathcal{T}^+)^{\rm c}}(-y_1P_{\rm R}+y_2P_{\rm L})\mathcal{Q}^0a^\dagger_{\mathcal{Q}^0}|0\rangle \\ \dfrac{i}{\sqrt{3}}y_1\langle 0|b_{\mathcal{Q}^0}G^-\overline{(\mathcal{T}^+)^{\rm c}}(-y_1P_{\rm R}+y_2P_{\rm L})\mathcal{Q}^0a^\dagger_{\mathcal{T}^+}b^\dagger_{G^+}|0\rangle\end{array}\right\}\to \begin{array}{ccc} & G^+ & \\ & \uparrow & \\ \mathcal{Q}^0 & \to\times\to & (\mathcal{T}^+)^{\rm c}\end{array} = \frac{i}{\sqrt{3}}(-y_1P_{\rm R}+y_2P_{\rm L})$$

2) Contribution to the β function of the Higgs self-coupling

⊙ Contribution to $\dfrac{\partial\delta_h}{\partial\ln\mu_{\rm R}}$

$$\mathrm{Tr}[(\not{p}+\not{q})(-y_1P_{\rm L}+y_2P_{\rm R})\not{q}(-y_1P_{\rm R}+y_2P_{\rm L})] = \mathrm{Tr}[(\not{p}+\not{q})\not{q}(y_1^2P_{\rm R}+y_2^2P_{\rm L})] = \frac{1}{2}(y_1^2+y_2^2)\mathrm{Tr}[(\not{p}+\not{q})\not{q}]$$

$$x(p+q)^2+(1-x)q^2 = xp^2+2xp\cdot q+q^2 = (q+xp)^2+x(1-x)p^2 = \ell^2-K_0$$

$$\ell = q+xp, \quad K_0 = -x(1-x)p^2$$

$$\mathrm{Tr}[(\not{p}+\not{q})\not{q}] = (p+q)_\mu q_\nu\mathrm{Tr}(\gamma^\mu\gamma^\nu) = (p+q)_\mu q_\nu 4g^{\mu\nu} = 4(p+q)\cdot q = 4[\ell+(1-x)p]\cdot(\ell-xp)$$

$$\to 4[\ell^2-x(1-x)p^2] = 4(\ell^2+K_0)$$

$$\frac{1}{(p+q)^2}\frac{1}{q^2} = \int_0^1 dx\frac{1}{x(p+q)^2+(1-x)q^2} = \int_0^1 dx\frac{1}{(\ell^2-K_0)^2}$$

$$i\Pi_h^{\rm TQ,1} = h-\left(\begin{array}{c}\mathcal{T}^0\\ \\ \mathcal{Q}^0\end{array}\right)-h = (-1)\int\frac{d^dq}{(2\pi)^d}\mathrm{Tr}\left[\frac{i(\not{p}+\not{q})}{(p+q)^2}\frac{i}{\sqrt{3}}(-y_1P_{\rm L}+y_2P_{\rm R})\frac{i\not{q}}{q^2}\frac{i}{\sqrt{3}}(-y_1P_{\rm R}+y_2P_{\rm L})\right]$$

$$= -\frac{1}{3}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2q^2}\mathrm{Tr}[(\not{p}+\not{q})(-y_1P_{\rm L}+y_2P_{\rm R})\not{q}(-y_1P_{\rm R}+y_2P_{\rm L})] = -\frac{2}{3}(y_1^2+y_2^2)\int_0^1dx\int\frac{d^d\ell}{(2\pi)^d}\frac{\ell^2+K_0}{(\ell^2-K_0)^2}$$

$$= -\frac{2}{3}(y_1^2+y_2^2)\int_0^1dx\left[-\frac{d}{2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K_0^{1-d/2}} - x(1-x)p^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}\right]$$

$$\frac{\partial}{\partial p^2}\left[\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K_0^{1-d/2}}\right] = -\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}\frac{\partial K_0}{\partial p^2} = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}x(1-x), \quad \int_0^1dx\,x(1-x) = \frac{1}{6}$$

$$\frac{\partial(i\Pi_h^{\rm TQ,1})}{\partial p^2} = -\frac{2}{3}(y_1^2+y_2^2)\int_0^1dx\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_t^{2-d/2}}\left[-\frac{d}{2}x(1-x)-x(1-x)\right]$$

$$= -\frac{2}{3}(y_1^2+y_2^2)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left(-\frac{d}{2}\frac{1}{6}-\frac{1}{6}\right)+\text{finite} = \frac{1}{3}(y_1^2+y_2^2)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$\mathrm{Tr}[\not{q}(-y_1P_\mathrm{R}+y_2P_\mathrm{L})(\not{p}+\not{q})(-y_1P_\mathrm{L}+y_2P_\mathrm{R})]=\mathrm{Tr}[(\not{p}+\not{q})(-y_1P_\mathrm{L}+y_2P_\mathrm{R})\not{q}(-y_1P_\mathrm{R}+y_2P_\mathrm{L})]$$

$$i\Pi_h^{\mathrm{TQ},2}=h-\overset{\mathcal{T}^0}{\underset{\bar{\mathcal{Q}}^0}{(\quad)}}-h=(-1)\int\frac{d^dq}{(2\pi)^d}\mathrm{Tr}\left[\frac{-i\not{q}}{q^2}\frac{i}{\sqrt{3}}(-y_1P_\mathrm{R}+y_2P_\mathrm{L})\frac{-i(\not{p}+\not{q})}{(p+q)^2}\frac{i}{\sqrt{3}}(-y_1P_\mathrm{L}+y_2P_\mathrm{R})\right]$$

$$=-\frac{1}{3}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2q^2}\mathrm{Tr}[\not{q}(-y_1P_\mathrm{R}+y_2P_\mathrm{L})(\not{p}+\not{q})(-y_1P_\mathrm{L}+y_2P_\mathrm{R})]=i\Pi_h^{\mathrm{TQ},1}$$

$$\frac{\partial(i\Pi_h^{\mathrm{TQ},2})}{\partial p^2}=\frac{\partial(i\Pi_h^{\mathrm{TQ},1})}{\partial p^2}$$

$$i\Pi_h^{\mathrm{TQ},3}=h-\overset{\bar{\mathcal{T}}^+}{\underset{\mathcal{Q}^+}{(\quad)}}-h=(-1)\int\frac{d^dq}{(2\pi)^d}\mathrm{Tr}\left[\frac{i(\not{p}+\not{q})}{(p+q)^2}\frac{i}{\sqrt{6}}(y_1P_\mathrm{L}-y_2P_\mathrm{R})\frac{i\not{q}}{q^2}\frac{i}{\sqrt{6}}(y_1P_\mathrm{R}-y_2P_\mathrm{L})\right]=\frac{1}{2}i\Pi_h^{\mathrm{TQ},1}$$

$$\frac{\partial(i\Pi_h^{\mathrm{TQ},3})}{\partial p^2}=\frac{1}{2}\frac{\partial(i\Pi_h^{\mathrm{TQ},1})}{\partial p^2}$$

$$i\Pi_h^{\mathrm{TQ},4}=h-\overset{\mathcal{T}^+}{\underset{\bar{\mathcal{Q}}^+}{(\quad)}}-h=(-1)\int\frac{d^dq}{(2\pi)^d}\mathrm{Tr}\left[\frac{-i\not{q}}{q^2}\frac{i}{\sqrt{6}}(y_1P_\mathrm{R}-y_2P_\mathrm{L})\frac{-i(\not{p}+\not{q})}{(p+q)^2}\frac{i}{\sqrt{6}}(y_1P_\mathrm{L}-y_2P_\mathrm{R})\right]=i\Pi_h^{\mathrm{TQ},3}$$

$$\frac{\partial(i\Pi_h^{\mathrm{TQ},4})}{\partial p^2}=\frac{\partial(i\Pi_h^{\mathrm{TQ},3})}{\partial p^2}$$

$$i\Pi_h^{\mathrm{TQ},5}=h-\overset{\overline{(\mathcal{T}^+)^\mathrm{c}}}{\underset{\mathcal{Q}^-}{(\quad)}}-h=(-1)\int\frac{d^dq}{(2\pi)^d}\mathrm{Tr}\left[\frac{i(\not{p}+\not{q})}{(p+q)^2}\frac{i}{\sqrt{2}}(-y_1P_\mathrm{L}+y_2P_\mathrm{R})\frac{i\not{q}}{q^2}\frac{i}{\sqrt{2}}(-y_1P_\mathrm{R}+y_2P_\mathrm{L})\right]=\frac{3}{2}i\Pi_h^{\mathrm{TQ},1}$$

$$\frac{\partial(i\Pi_h^{\mathrm{TQ},5})}{\partial p^2}=\frac{3}{2}\frac{\partial(i\Pi_h^{\mathrm{TQ},1})}{\partial p^2}$$

$$i\Pi_h^{\mathrm{TQ},6}=h-\overset{(\mathcal{T}^+)^\mathrm{c}}{\underset{\bar{\mathcal{Q}}^-}{(\quad)}}-h=(-1)\int\frac{d^dq}{(2\pi)^d}\mathrm{Tr}\left[\frac{-i\not{q}}{q^2}\frac{i}{\sqrt{2}}(-y_1P_\mathrm{R}+y_2P_\mathrm{L})\frac{-i(\not{p}+\not{q})}{(p+q)^2}\frac{i}{\sqrt{2}}(-y_1P_\mathrm{L}+y_2P_\mathrm{R})\right]=i\Pi_h^{\mathrm{TQ},5}$$

$$\frac{\partial(i\Pi_h^{\mathrm{TQ},6})}{\partial p^2}=\frac{\partial(i\Pi_h^{\mathrm{TQ},5})}{\partial p^2}$$

$$\frac{\partial(i\Pi_h^{\mathrm{TQ}})}{\partial p^2}=\frac{\partial}{\partial p^2}\sum_i(i\Pi_h^{\mathrm{TQ},i})=2\cdot\left(1+\frac{1}{2}+\frac{3}{2}\right)\cdot\frac{1}{3}(y_1^2+y_2^2)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}=2(y_1^2+y_2^2)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}$$

$$\delta_h^{\mathrm{TQ}}=-2(y_1^2+y_2^2)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}}+\mathrm{finite}=\frac{1}{16\pi^2}2(y_1^2+y_2^2)\ln\mu_\mathrm{R}^2+\cdots$$

$$\frac{\partial\delta_h^{\mathrm{TQ}}}{\partial\ln\mu_\mathrm{R}}=\frac{1}{16\pi^2}4(y_1^2+y_2^2)$$

(16)

$\odot$ Contribution to $\dfrac{\partial \delta_\lambda}{\partial \ln \mu_{\mathrm{R}}}$

$$\mathrm{Tr}\left\{[\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})]^2\right\}=\mathrm{Tr}\left\{[\not{q}\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})]^2\right\}$$
$$=\mathrm{Tr}\left\{[\not{q}\not{q}(y_1^2P_{\mathrm{R}}+y_2^2P_{\mathrm{L}})]^2\right\}=(q^2)^2\,\mathrm{Tr}[(y_1^2P_{\mathrm{R}}+y_2^2P_{\mathrm{L}})(y_1^2P_{\mathrm{R}}+y_2^2P_{\mathrm{L}})]$$
$$=(q^2)^2\,\mathrm{Tr}(y_1^4P_{\mathrm{R}}+y_2^4P_{\mathrm{L}})=\frac{1}{2}(y_1^4+y_2^4)(q^2)^2\,\mathrm{Tr}(1)=2(y_1^4+y_2^4)(q^2)^2$$

$$i\Sigma_\lambda^{\mathrm{TQ},1}=6\times\begin{pmatrix} h & --\xleftarrow{\mathcal{Q}^0}-- & h \\ & \mathcal{T}^0\big| \qquad \big|\mathcal{T}^0 & \\ h & --\xrightarrow{\mathcal{Q}^0}-- & h \end{pmatrix}+6\times\begin{pmatrix} h & --\xrightarrow{\mathcal{Q}^0}-- & h \\ & \mathcal{T}^0\big| \qquad \big|\mathcal{T}^0 & \\ h & --\xleftarrow{\mathcal{Q}^0}-- & h \end{pmatrix}=2\times 6\times\begin{pmatrix} h & --\xleftarrow{\mathcal{Q}^0}-- & h \\ & \mathcal{T}^0\big| \qquad \big|\mathcal{T}^0 & \\ h & --\xrightarrow{\mathcal{Q}^0}-- & h \end{pmatrix}$$
$$=12\cdot(-1)\left(\frac{i}{\sqrt{3}}\right)^4\int\frac{d^dq}{(2\pi)^d}\,\mathrm{Tr}\left[\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\right]$$
$$=-\frac{4}{3}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^4}\,\mathrm{Tr}\left\{[\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})]^2\right\}$$
$$=-\frac{8}{3}(y_1^4+y_2^4)\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}=-\frac{8}{3}(y_1^4+y_2^4)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$
$$\delta_\lambda^{\mathrm{TQ},1}=-\frac{1}{6}\frac{8}{3}(y_1^4+y_2^4)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}+\text{finite}=\frac{1}{16\pi^2}\frac{4}{9}(y_1^4+y_2^4)\ln\mu_{\mathrm{R}}^2+\cdots$$
$$\frac{\partial\delta_\lambda^{\mathrm{TQ},1}}{\partial\ln\mu_{\mathrm{R}}}=\frac{1}{16\pi^2}\frac{8}{9}(y_1^4+y_2^4)$$

$$\mathcal{C}(P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}=P_{\mathrm{L}},\quad \mathcal{C}(P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}=P_{\mathrm{R}}$$
$$\mathrm{Tr}[\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\not{q}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})]$$
$$=\mathrm{Tr}[\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})]$$
$$=\mathrm{Tr}[\not{q}\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})]$$
$$=(q^2)^2\mathrm{Tr}[(-y_1y_2P_{\mathrm{R}}-y_1y_2P_{\mathrm{L}})(-y_1y_2P_{\mathrm{L}}-y_1y_2P_{\mathrm{R}})]$$
$$=y_1^2y_2^2(q^2)^2\mathrm{Tr}(1)=4y_1^2y_2^2(q^2)^2$$

$$i\Sigma_\lambda^{\mathrm{TQ},2}=6\times\begin{pmatrix} h & --\xrightarrow{\mathcal{Q}^0}-- & h \\ & \mathcal{T}^0\big| \qquad \big|\mathcal{T}^0 & \\ h & --\xrightarrow{\mathcal{Q}^0}-- & h \end{pmatrix}+6\times\begin{pmatrix} h & --\xleftarrow{\mathcal{Q}^0}-- & h \\ & \mathcal{T}^0\big| \qquad \big|\mathcal{T}^0 & \\ h & --\xleftarrow{\mathcal{Q}^0}-- & h \end{pmatrix}=2\times 6\times\begin{pmatrix} h & --\xrightarrow{\mathcal{Q}^0}-- & h \\ & \mathcal{T}^0\big| \qquad \big|\mathcal{T}^0 & \\ h & --\xrightarrow{\mathcal{Q}^0}-- & h \end{pmatrix}$$
$$=12\cdot(-1)\left(\frac{i}{\sqrt{3}}\right)^4\int\frac{d^dq}{(2\pi)^d}\,\mathrm{Tr}\left[\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i\not{q}}{q^2}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\right]$$
$$=-\frac{4}{3}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^4}\mathrm{Tr}[\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\not{q}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})]$$
$$=-\frac{16}{3}y_1^2y_2^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}=-\frac{16}{3}y_1^2y_2^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$
$$\delta_\lambda^{\mathrm{TQ},2}=-\frac{1}{6}\frac{16}{3}y_1^2y_2^2\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}+\text{finite}=\frac{1}{16\pi^2}\frac{8}{9}y_1^2y_2^2\ln\mu_{\mathrm{R}}^2+\cdots$$
$$\frac{\partial\delta_\lambda^{\mathrm{TQ},2}}{\partial\ln\mu_{\mathrm{R}}}=\frac{1}{16\pi^2}\frac{16}{9}y_1^2y_2^2$$

$$\mathrm{Tr}\left\{[\not{q}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\not{q}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})]^2\right\}=\mathrm{Tr}\left\{[\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})]^2\right\}=2(y_1^4+y_2^4)(q^2)^2$$

$$i\Sigma_\lambda^{\mathrm{TQ},3}=6\times\begin{pmatrix} h & --\xleftarrow{\mathcal{Q}^+}-- & h \\ & \mathcal{T}^+\downarrow \qquad \uparrow\mathcal{T}^+ & \\ h & --\xrightarrow{\mathcal{Q}^+}-- & h \end{pmatrix}+6\times\begin{pmatrix} h & --\xleftarrow{\mathcal{T}^+}-- & h \\ & \mathcal{Q}^+\downarrow \qquad \uparrow\mathcal{Q}^+ & \\ h & --\xrightarrow{\mathcal{T}^+}-- & h \end{pmatrix}$$

$$=2\times6\times\begin{pmatrix} h & --\xleftarrow{\mathcal{Q}^+}-- & h \\ & \mathcal{T}^+\downarrow \qquad \uparrow\mathcal{T}^+ & \\ h & --\xrightarrow{\mathcal{Q}^+}-- & h \end{pmatrix}$$

$$=12\cdot(-1)\left(\frac{i}{\sqrt{6}}\right)^4\int\frac{d^dq}{(2\pi)^d}\,\mathrm{Tr}\left[\frac{i\not{q}}{q^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\frac{i\not{q}}{q^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\right]$$

$$=-\frac{1}{3}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^4}\mathrm{Tr}\left\{[\not{q}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\not{q}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})]^2\right\}$$

$$=-\frac{2}{3}(y_1^4+y_2^4)\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}=-\frac{2}{3}(y_1^4+y_2^4)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\mathrm{finite}$$

$$\delta_\lambda^{\mathrm{TQ},3}=-\frac{1}{6}\frac{2}{3}(y_1^4+y_2^4)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}+\mathrm{finite}=\frac{1}{16\pi^2}\frac{1}{9}(y_1^4+y_2^4)\ln\mu_{\mathrm{R}}^2+\cdots$$

$$\frac{\partial\delta_\lambda^{\mathrm{TQ},3}}{\partial\ln\mu_{\mathrm{R}}}=\frac{1}{16\pi^2}\frac{2}{9}(y_1^4+y_2^4)$$

$$i\Sigma_\lambda^{\mathrm{TQ},4}=6\times\begin{pmatrix} h & ---\xleftarrow{\mathcal{Q}^-}--- & h \\ & (\mathcal{T}^+)^{\mathrm{c}}\downarrow \qquad \uparrow(\mathcal{T}^+)^{\mathrm{c}} & \\ h & ---\xrightarrow{\mathcal{Q}^-}--- & h \end{pmatrix}+6\times\begin{pmatrix} h & --\xleftarrow{(\mathcal{T}^+)^{\mathrm{c}}}-- & h \\ & \mathcal{Q}^-\downarrow \qquad \uparrow\mathcal{Q}^- & \\ h & --\xrightarrow{(\mathcal{T}^+)^{\mathrm{c}}}-- & h \end{pmatrix}$$

$$=2\times6\times\begin{pmatrix} h & ---\xleftarrow{\mathcal{Q}^-}--- & h \\ & (\mathcal{T}^+)^{\mathrm{c}}\downarrow \qquad \uparrow(\mathcal{T}^+)^{\mathrm{c}} & \\ h & ---\xrightarrow{\mathcal{Q}^-}--- & h \end{pmatrix}$$

$$=12\cdot(-1)\left(\frac{i}{\sqrt{2}}\right)^4\int\frac{d^dq}{(2\pi)^d}\,\mathrm{Tr}\left[\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\right]$$

$$=-3\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^4}\mathrm{Tr}\left\{[\not{q}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\not{q}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})]^2\right\}$$

$$=-6(y_1^4+y_2^4)\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}=-6(y_1^4+y_2^4)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\mathrm{finite}$$

$$\delta_\lambda^{\mathrm{TQ},4}=-\frac{1}{6}6(y_1^4+y_2^4)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}+\mathrm{finite}=\frac{1}{16\pi^2}(y_1^4+y_2^4)\ln\mu_{\mathrm{R}}^2+\cdots$$

$$\frac{\partial\delta_\lambda^{\mathrm{TQ},4}}{\partial\ln\mu_{\mathrm{R}}}=\frac{1}{16\pi^2}2(y_1^4+y_2^4)$$

(18)

$$\frac{1}{\sqrt{2}}h\bar{\mathcal{Q}}^{-}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\mathcal{T}^{+})^{\mathrm{c}}=\frac{1}{\sqrt{2}}h\left[\bar{\mathcal{Q}}^{-}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\mathcal{C}(\bar{\mathcal{T}}^{+})^{\mathrm{T}}\right]^{\mathrm{T}}=-\frac{1}{\sqrt{2}}h\bar{\mathcal{T}}^{+}\mathcal{C}^{\mathrm{T}}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}(\bar{\mathcal{Q}}^{-})^{\mathrm{T}}$$

$$=\frac{1}{\sqrt{2}}h\bar{\mathcal{T}}^{+}\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\mathcal{C}(\bar{\mathcal{Q}}^{-})^{\mathrm{T}}=\frac{1}{\sqrt{2}}h\bar{\mathcal{T}}^{+}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\bar{\mathcal{Q}}^{-})^{\mathrm{c}}$$

$$\frac{1}{\sqrt{2}}h\overline{(\mathcal{T}^{+})^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^{-}=\frac{1}{\sqrt{2}}h\left[(\mathcal{T}^{+})^{\mathrm{T}}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^{-}\right]^{\mathrm{T}}=-\frac{1}{\sqrt{2}}h(\mathcal{Q}^{-})^{\mathrm{T}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{\mathrm{T}}\mathcal{T}^{+}$$

$$=\frac{1}{\sqrt{2}}h(\mathcal{Q}^{-})^{\mathrm{T}}\mathcal{C}\mathcal{C}^{-1}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}\mathcal{T}^{+}=\frac{1}{\sqrt{2}}h\overline{(\mathcal{Q}^{-})^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{T}^{+}$$

$$\mathrm{Tr}\left[\not{q}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\not{q}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\right]$$

$$=(q^2)^2\mathrm{Tr}\left[(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\right]$$

$$=(q^2)^2\mathrm{Tr}\left[(y_1y_2P_{\mathrm{L}}+y_1y_2P_{\mathrm{R}})(y_1y_2P_{\mathrm{L}}+y_1y_2P_{\mathrm{R}})\right]=y_1^2y_2^2(q^2)^2\mathrm{Tr}(1)=4y_1^2y_2^2(q^2)^2$$

$$i\Sigma_{\lambda}^{\mathrm{TQ},5}=6\times\begin{pmatrix} h & \xleftarrow{(\mathcal{Q}^{-})^{\mathrm{c}}} & h \\ \mathcal{T}^{+}\downarrow & & \uparrow\mathcal{T}^{+} \\ h & \xrightarrow{\mathcal{Q}^{+}} & h \end{pmatrix}+6\times\begin{pmatrix} h & \xleftarrow{\mathcal{T}^{+}} & h \\ \mathcal{Q}^{+}\downarrow & & \uparrow(\mathcal{Q}^{-})^{\mathrm{c}} \\ h & \xrightarrow{\mathcal{T}^{+}} & h \end{pmatrix}$$

$$+6\times\begin{pmatrix} h & \xleftarrow{\mathcal{Q}^{+}} & h \\ \mathcal{T}^{+}\downarrow & & \uparrow\mathcal{T}^{+} \\ h & \xrightarrow{(\mathcal{Q}^{-})^{\mathrm{c}}} & h \end{pmatrix}+6\times\begin{pmatrix} h & \xleftarrow{\mathcal{T}^{+}} & h \\ (\mathcal{Q}^{-})^{\mathrm{c}}\downarrow & & \uparrow\mathcal{Q}^{+} \\ h & \xrightarrow{\mathcal{T}^{+}} & h \end{pmatrix}$$

$$=4\times6\times\begin{pmatrix} h & \xleftarrow{(\mathcal{Q}^{-})^{\mathrm{c}}} & h \\ \mathcal{T}^{+}\downarrow & & \uparrow\mathcal{T}^{+} \\ h & \xrightarrow{\mathcal{Q}^{+}} & h \end{pmatrix}$$

$$=24\cdot(-1)\left(\frac{i}{\sqrt{6}}\right)^2\left(\frac{i}{\sqrt{2}}\right)^2\int\frac{d^dq}{(2\pi)^d}\mathrm{Tr}\left[\frac{i\not{q}}{q^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\frac{i\not{q}}{q^2}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\right]$$

$$=-2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^4}\mathrm{Tr}\left[\not{q}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\not{q}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\not{q}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\right]$$

$$=-8y_1^2y_2^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}=-8y_1^2y_2^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$\delta_{\lambda}^{\mathrm{TQ},5}=-\frac{1}{6}8y_1^2y_2^2\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}+\text{finite}=\frac{1}{16\pi^2}\frac{4}{3}(y_1^4+y_2^4)\ln\mu_{\mathrm{R}}^2+\cdots$$

$$\frac{\partial\delta_{\lambda}^{\mathrm{TQ},5}}{\partial\ln\mu_{\mathrm{R}}}=\frac{1}{16\pi^2}\frac{8}{3}(y_1^4+y_2^4)$$

$$\frac{\partial\delta_{\lambda}^{\mathrm{TQ}}}{\partial\ln\mu_{\mathrm{R}}}=\frac{\partial}{\partial\ln\mu_{\mathrm{R}}}\sum_i\delta_{\lambda}^{\mathrm{TQ},i}=\frac{1}{16\pi^2}\left[\left(\frac{8}{9}+\frac{2}{9}+2\right)(y_1^4+y_2^4)+\left(\frac{16}{9}+\frac{8}{3}\right)y_1^2y_2^2\right]=\frac{1}{16\pi^2}\left[\frac{28}{9}(y_1^4+y_2^4)+\frac{40}{9}y_1^2y_2^2\right]$$

⊙ Contribution to β_{λ}

$$\beta_{\lambda}^{\mathrm{TQ}}=-\frac{\partial\delta_{\lambda}^{\mathrm{TQ}}}{\partial\ln\mu_{\mathrm{R}}}+2\lambda\frac{\partial\delta_{h}^{\mathrm{TQ}}}{\partial\ln\mu_{\mathrm{R}}}=-\frac{1}{16\pi^2}\left[\frac{28}{9}(y_1^4+y_2^4)+\frac{40}{9}y_1^2y_2^2\right]+2\lambda\frac{1}{16\pi^2}4(y_1^2+y_2^2)$$

$$=\frac{1}{16\pi^2}\left[8\lambda(y_1^2+y_2^2)-\frac{28}{9}(y_1^4+y_2^4)-\frac{40}{9}y_1^2y_2^2\right]$$

3) Calculation for β functions of y_1 and y_2

h-$\mathcal{Q}^+$-$\mathcal{T}^+$ vertex $h - \begin{smallmatrix} \nearrow \mathcal{Q}^+ \\ \nwarrow \mathcal{T}^+ \end{smallmatrix} = \dfrac{i}{\sqrt{6}}(y_1 P_\mathrm{L} - y_2 P_\mathrm{R})$

For $\mathcal{Q}^+$, self-energy $\mathcal{Q}^+ - (1\mathrm{PI}) - \mathcal{Q}^+ = i\Pi_{\mathcal{Q}^+}(p)$

$$\frac{\partial \Pi_{\mathcal{Q}^+}}{\partial \not{p}} = \frac{\partial \Pi_{\mathcal{Q}^+,\mathrm{L}}}{\partial \not{p}} P_\mathrm{L} + \frac{\partial \Pi_{\mathcal{Q}^+,\mathrm{R}}}{\partial \not{p}} P_\mathrm{R}, \quad \Pi_{\mathcal{Q}^+} = \not{p} \frac{\partial \Pi_{\mathcal{Q}^+}}{\partial \not{p}} + \cdots = \not{p} \frac{\partial \Pi_{\mathcal{Q}^+,\mathrm{L}}}{\partial \not{p}} P_\mathrm{L} + \not{p} \frac{\partial \Pi_{\mathcal{Q}^+,\mathrm{R}}}{\partial \not{p}} P_\mathrm{R} + \cdots$$

$$\mathcal{Q}^+ = \begin{pmatrix} -Q_2^+ \\ (Q_1^-)^\dagger \end{pmatrix}, \quad (Q_2^+)_0 = \sqrt{Z_{\mathcal{Q}^+}^\mathrm{L}} Q_2^+ = \left(1 + \frac{1}{2}\delta Z_{\mathcal{Q}^+}^\mathrm{L}\right) Q_2^+, \quad Q_1^- = \sqrt{Z_{\mathcal{Q}^+}^\mathrm{R}} Q_1^- = \left(1 + \frac{1}{2}\delta Z_{\mathcal{Q}^+}^\mathrm{R}\right) Q_1^-$$

$$(\mathcal{Q}^+)_0 = \mathcal{Q}^+ + \frac{1}{2}(\delta Z_{\mathcal{Q}^+}^\mathrm{L} P_\mathrm{L} + \delta Z_{\mathcal{Q}^+}^{\mathrm{R}*} P_\mathrm{R}) \mathcal{Q}^+$$

$$\overline{(\mathcal{Q}^+)}_0 i\gamma^\mu \partial_\mu (\mathcal{Q}^+)_0 = \bar{\mathcal{Q}}^+ \left(1 + \frac{1}{2}\delta Z_{\mathcal{Q}^+}^{\mathrm{L}*} P_\mathrm{R} + \frac{1}{2}\delta Z_{\mathcal{Q}^+}^\mathrm{R} P_\mathrm{L}\right) i\gamma^\mu \partial_\mu \left(1 + \frac{1}{2}\delta Z_{\mathcal{Q}^+}^\mathrm{L} P_\mathrm{L} + \frac{1}{2}\delta Z_{\mathcal{Q}^+}^{\mathrm{R}*} P_\mathrm{R}\right) \mathcal{Q}^+$$

$$= \overline{(\mathcal{Q}^+)} i\gamma^\mu \partial_\mu (\mathcal{Q}^+) + \frac{1}{2}\bar{\mathcal{Q}}^+ [(\delta Z_{\mathcal{Q}^+}^\mathrm{L} + \delta Z_{\mathcal{Q}^+}^{\mathrm{L}*}) P_\mathrm{R} + (\delta Z_{\mathcal{Q}^+}^\mathrm{R} + \delta Z_{\mathcal{Q}^+}^{\mathrm{R}*}) P_\mathrm{L}] i\gamma^\mu \partial_\mu \mathcal{Q}^+$$

$$\mathcal{Q}^+ - \otimes - \mathcal{Q}^+ \supset \frac{1}{2}[(\delta Z_{\mathcal{Q}^+}^\mathrm{L} + \delta Z_{\mathcal{Q}^+}^{\mathrm{L}*}) P_\mathrm{R} + (\delta Z_{\mathcal{Q}^+}^\mathrm{R} + \delta Z_{\mathcal{Q}^+}^{\mathrm{R}*}) P_\mathrm{L}] i\not{p}$$

$$= \frac{i}{2}[(\delta Z_{\mathcal{Q}^+}^\mathrm{L} + \delta Z_{\mathcal{Q}^+}^{\mathrm{L}*}) \not{p} P_\mathrm{L} + (\delta Z_{\mathcal{Q}^+}^\mathrm{R} + \delta Z_{\mathcal{Q}^+}^{\mathrm{R}*}) \not{p} P_\mathrm{R}]$$

$$\delta_{\mathcal{Q}^+,\mathrm{L}} \equiv \frac{1}{2}(\delta Z_{\mathcal{Q}^+}^\mathrm{L} + \delta Z_{\mathcal{Q}^+}^{\mathrm{L}*}), \quad \delta_{\mathcal{Q}^+,\mathrm{R}} \equiv \frac{1}{2}(\delta Z_{\mathcal{Q}^+}^\mathrm{R} + \delta Z_{\mathcal{Q}^+}^{\mathrm{R}*})$$

$\mathcal{Q}^+$-$\mathcal{Q}^+$ counter term $\mathcal{Q}^+ - \otimes - \mathcal{Q}^+ \supset i\not{p}(\delta_{\mathcal{Q}^+,\mathrm{L}} P_\mathrm{L} + \delta_{\mathcal{Q}^+,\mathrm{R}} P_\mathrm{R})$

$\dfrac{\partial \Pi_{\mathcal{Q}^+,\mathrm{L}}}{\partial \not{p}} + \delta_{\mathcal{Q}^+,\mathrm{L}}$ and $\dfrac{\partial \Pi_{\mathcal{Q}^+,\mathrm{R}}}{\partial \not{p}} + \delta_{\mathcal{Q}^+,\mathrm{R}}$ are finite

For $\mathcal{T}^+$, self-energy $\mathcal{T}^+ - (1\mathrm{PI}) - \mathcal{T}^+ = i\Pi_{\mathcal{T}^+}(p), \quad \Pi_{\mathcal{T}^+} = \not{p} \dfrac{\partial \Pi_{\mathcal{T}^+}}{\partial \not{p}} + \cdots = \not{p} \dfrac{\partial \Pi_{\mathcal{T}^+,\mathrm{L}}}{\partial \not{p}} P_\mathrm{L} + \not{p} \dfrac{\partial \Pi_{\mathcal{T}^+,\mathrm{R}}}{\partial \not{p}} P_\mathrm{R} + \cdots$

$\mathcal{T}^+$-$\mathcal{T}^+$ counter term $\mathcal{T}^+ - \otimes - \mathcal{T}^+ \supset i\not{p}(\delta_{\mathcal{T}^+,\mathrm{L}} P_\mathrm{L} + \delta_{\mathcal{T}^+,\mathrm{R}} P_\mathrm{R})$

$\dfrac{\partial \Pi_{\mathcal{T}^+,\mathrm{L}}}{\partial \not{p}} + \delta_{\mathcal{T}^+,\mathrm{L}}$ and $\dfrac{\partial \Pi_{\mathcal{T}^+,\mathrm{R}}}{\partial \not{p}} + \delta_{\mathcal{T}^+,\mathrm{R}}$ are finite

h-$\mathcal{Q}^+$-$\mathcal{T}^+$ vertex correction $h - \langle 1\mathrm{PI} \big| \begin{smallmatrix} \nearrow \mathcal{Q}^+ \\ \nwarrow \mathcal{T}^+ \end{smallmatrix} = i\Sigma_{y_{1,2}}(p_1, p_2, p_3), \quad \Sigma_{y_{1,2}} = \Sigma_{y_{1,2},\mathrm{L}} P_\mathrm{L} + \Sigma_{y_{1,2},\mathrm{R}} P_\mathrm{R}$

h-$\mathcal{Q}^+$-$\mathcal{T}^+$ counter term $h - \otimes \begin{smallmatrix} \nearrow \mathcal{Q}^+ \\ \nwarrow \mathcal{T}^+ \end{smallmatrix} = \dfrac{i}{\sqrt{6}}(\delta_{y_1} P_\mathrm{L} - \delta_{y_2} P_\mathrm{R})$

$\Sigma_{y_{1,2},\mathrm{L}} + \dfrac{\delta_{y_1}}{\sqrt{6}}$ and $\Sigma_{y_{1,2},\mathrm{R}} - \dfrac{\delta_{y_2}}{\sqrt{6}}$ are finite

$h(p_1)$-$\mathcal{Q}^+(p_2)$-$\mathcal{T}^+(p_3)$ Green function:

$$
\begin{aligned}
G_{\mathrm{c}}^{(3)}(\{p_i\}) &= \frac{i}{p_1^2}\frac{i}{\not p_2}\left[\tilde{G}_{\mathrm{c,L}}^{(3)}(\{p_i\})P_{\mathrm{L}} + \tilde{G}_{\mathrm{c,R}}^{(3)}(\{p_i\})P_{\mathrm{R}}\right]\frac{i}{\not p_3} \\
&= \begin{pmatrix}\text{Tree-level}\\ \text{propagator}\end{pmatrix} + \begin{pmatrix}\text{1PI loop}\\ \text{diagrams}\end{pmatrix} + \begin{pmatrix}\text{Vertex}\\ \text{counterterm}\end{pmatrix} + \begin{pmatrix}\text{External leg}\\ \text{corrections}\end{pmatrix} \\
&= \frac{i}{p_1^2}\frac{i}{\not p_2}\left\{\frac{i}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}}) - i(B_{\mathrm{L}}P_{\mathrm{L}} + B_{\mathrm{R}}P_{\mathrm{R}})\ln\frac{\Lambda^2}{-p^2} + \frac{i}{\sqrt{6}}(\delta_{y_1}P_{\mathrm{L}} - \delta_{y_2}P_{\mathrm{R}})\right. \\
&\qquad + \frac{i}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\left[(A_{1,\mathrm{L}}P_{\mathrm{L}} + A_{1,\mathrm{R}}P_{\mathrm{R}})\ln\frac{\Lambda^2}{-p_1^2} - \delta_h\right] \\
&\qquad + \left[(A_{2,\mathrm{L}}P_{\mathrm{L}} + A_{2,\mathrm{R}}P_{\mathrm{R}})\ln\frac{\Lambda^2}{-p_2^2} + i\not p_2(\delta_{\mathcal{Q}^+,\mathrm{L}}P_{\mathrm{L}} + \delta_{\mathcal{Q}^+,\mathrm{R}}P_{\mathrm{R}})\frac{i\not p_2}{p_2^2}\right]\frac{i}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}}) \\
&\qquad \left. + \frac{i}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\left[(A_{3,\mathrm{L}}P_{\mathrm{L}} + A_{3,\mathrm{R}}P_{\mathrm{R}})\ln\frac{\Lambda^2}{-p_3^2} + \frac{i\not p_3}{p_3^2}i\not p_3(\delta_{\mathcal{T}^+,\mathrm{L}}P_{\mathrm{L}} + \delta_{\mathcal{T}^+,\mathrm{R}}P_{\mathrm{R}})\right]\right\}\frac{i}{\not p_3} \\
&= \frac{i}{p_1^2}\frac{i}{\not p_2}\left\{\frac{i}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}}) - i(B_{\mathrm{L}}P_{\mathrm{L}} + B_{\mathrm{R}}P_{\mathrm{R}})\ln\frac{\Lambda^2}{-p^2} + \frac{i}{\sqrt{6}}(\delta_{y_1}P_{\mathrm{L}} - \delta_{y_2}P_{\mathrm{R}})\right. \\
&\qquad + \frac{i}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\left[(A_{1,\mathrm{L}}P_{\mathrm{L}} + A_{1,\mathrm{R}}P_{\mathrm{R}})\ln\frac{\Lambda^2}{-p_1^2} - \delta_h\right] \\
&\qquad + \left[(A_{2,\mathrm{L}}P_{\mathrm{L}} + A_{2,\mathrm{R}}P_{\mathrm{R}})\ln\frac{\Lambda^2}{-p_2^2} - (\delta_{\mathcal{Q}^+,\mathrm{L}}P_{\mathrm{R}} + \delta_{\mathcal{Q}^+,\mathrm{R}}P_{\mathrm{L}})\right]\frac{i}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}}) \\
&\qquad \left. + \frac{i}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\left[(A_{3,\mathrm{L}}P_{\mathrm{L}} + A_{3,\mathrm{R}}P_{\mathrm{R}})\ln\frac{\Lambda^2}{-p_3^2} - (\delta_{\mathcal{T}^+,\mathrm{L}}P_{\mathrm{L}} + \delta_{\mathcal{T}^+,\mathrm{R}}P_{\mathrm{R}})\right]\right\}\frac{i}{\not p_3}
\end{aligned}
$$

$$
\tilde{G}_{\mathrm{c,L}}^{(3)}(\{p_i\}) = \frac{i}{\sqrt{6}}y_1 - iB_{\mathrm{L}}\ln\frac{\Lambda^2}{-p^2} + \frac{i}{\sqrt{6}}\delta_{y_1} + \frac{i}{\sqrt{6}}y_1\left[\sum_{i=1}^{3}\left(A_{i,\mathrm{L}}\ln\frac{\Lambda^2}{-p_i^2}\right) - \delta_h - \delta_{\mathcal{Q}^+,\mathrm{R}} - \delta_{\mathcal{T}^+,\mathrm{L}}\right]
$$

$$
\tilde{G}_{\mathrm{c,R}}^{(3)}(\{p_i\}) = -\frac{i}{\sqrt{6}}y_2 - iB_{\mathrm{R}}\ln\frac{\Lambda^2}{-p^2} - \frac{i}{\sqrt{6}}\delta_{y_2} - \frac{i}{\sqrt{6}}y_2\left[\sum_{i=1}^{3}\left(A_{i,\mathrm{R}}\ln\frac{\Lambda^2}{-p_i^2}\right) - \delta_h - \delta_{\mathcal{Q}^+,\mathrm{L}} - \delta_{\mathcal{T}^+,\mathrm{R}}\right]
$$

Callan-Symanzik equations
$$
\begin{cases}
\left[\dfrac{\partial}{\partial\ln\mu_{\mathrm{R}}} + \beta_{y_1}\dfrac{\partial}{\partial y_1} + \dfrac{1}{2}\dfrac{\partial(\delta_h + \delta_{\mathcal{Q}^+,\mathrm{R}} + \delta_{\mathcal{T}^+,\mathrm{L}})}{\partial\ln\mu_{\mathrm{R}}}\right]\tilde{G}_{\mathrm{c,L}}^{(3)} = 0 \\
\left[\dfrac{\partial}{\partial\ln\mu_{\mathrm{R}}} + \beta_{y_2}\dfrac{\partial}{\partial y_2} + \dfrac{1}{2}\dfrac{\partial(\delta_h + \delta_{\mathcal{Q}^+,\mathrm{L}} + \delta_{\mathcal{T}^+,\mathrm{R}})}{\partial\ln\mu_{\mathrm{R}}}\right]\tilde{G}_{\mathrm{c,R}}^{(3)} = 0
\end{cases}
$$

Lowest order $\Rightarrow$
$$
\begin{cases}
\dfrac{\partial}{\partial\ln\mu_{\mathrm{R}}}\left[\dfrac{i}{\sqrt{6}}\delta_{y_1} + \dfrac{i}{\sqrt{6}}y_1(-\delta_h - \delta_{\mathcal{Q}^+,\mathrm{R}} - \delta_{\mathcal{T}^+,\mathrm{L}})\right] + \dfrac{i}{\sqrt{6}}\beta_{y_1} + \dfrac{i}{\sqrt{6}}y_1\dfrac{1}{2}\dfrac{\partial(\delta_h + \delta_{\mathcal{Q}^+,\mathrm{R}} + \delta_{\mathcal{T}^+,\mathrm{L}})}{\partial\ln\mu_{\mathrm{R}}} = 0 \\
\dfrac{\partial}{\partial\ln\mu_{\mathrm{R}}}\left[-\dfrac{i}{\sqrt{6}}\delta_{y_2} - \dfrac{i}{\sqrt{6}}y_2(-\delta_h - \delta_{\mathcal{Q}^+,\mathrm{L}} - \delta_{\mathcal{T}^+,\mathrm{R}})\right] - \dfrac{i}{\sqrt{6}}\beta_{y_2} - \dfrac{i}{\sqrt{6}}y_2\dfrac{1}{2}\dfrac{\partial(\delta_h + \delta_{\mathcal{Q}^+,\mathrm{L}} + \delta_{\mathcal{T}^+,\mathrm{R}})}{\partial\ln\mu_{\mathrm{R}}} = 0
\end{cases}
$$

$$
\Rightarrow \begin{cases}
\dfrac{\partial\delta_{y_1}}{\partial\ln\mu_{\mathrm{R}}} + \beta_{y_1} - \dfrac{1}{2}y_1\dfrac{\partial(\delta_h + \delta_{\mathcal{Q}^+,\mathrm{R}} + \delta_{\mathcal{T}^+,\mathrm{L}})}{\partial\ln\mu_{\mathrm{R}}} = 0 \\
\dfrac{\partial\delta_{y_2}}{\partial\ln\mu_{\mathrm{R}}} + \beta_{y_2} - \dfrac{1}{2}y_2\dfrac{\partial(\delta_h + \delta_{\mathcal{Q}^+,\mathrm{L}} + \delta_{\mathcal{T}^+,\mathrm{R}})}{\partial\ln\mu_{\mathrm{R}}} = 0
\end{cases}
\Rightarrow \begin{cases}
\beta_{y_1} = -\dfrac{\partial\delta_{y_1}}{\partial\ln\mu_{\mathrm{R}}} + \dfrac{1}{2}y_1\left(\dfrac{\partial\delta_h}{\partial\ln\mu_{\mathrm{R}}} + \dfrac{\partial\delta_{\mathcal{Q}^+,\mathrm{R}}}{\partial\ln\mu_{\mathrm{R}}} + \dfrac{\partial\delta_{\mathcal{T}^+,\mathrm{L}}}{\partial\ln\mu_{\mathrm{R}}}\right) \\
\beta_{y_2} = -\dfrac{\partial\delta_{y_2}}{\partial\ln\mu_{\mathrm{R}}} + \dfrac{1}{2}y_2\left(\dfrac{\partial\delta_h}{\partial\ln\mu_{\mathrm{R}}} + \dfrac{\partial\delta_{\mathcal{Q}^+,\mathrm{L}}}{\partial\ln\mu_{\mathrm{R}}} + \dfrac{\partial\delta_{\mathcal{T}^+,\mathrm{R}}}{\partial\ln\mu_{\mathrm{R}}}\right)
\end{cases}
$$

(21)

$\odot$ Calculation for $\dfrac{\partial \delta_{\mathcal{T}^+,\mathrm{L}}}{\partial \ln \mu_{\mathrm{R}}}$ and $\dfrac{\partial \delta_{\mathcal{T}^+,\mathrm{R}}}{\partial \ln \mu_{\mathrm{R}}}$

$$\gamma^\mu \gamma^\nu \gamma_\mu = -(d-2)\gamma^\nu, \quad \gamma^\mu (\not{p}+\not{q})\gamma_\mu = (p+q)_\nu \gamma^\mu \gamma^\nu \gamma_\mu = -(d-2)(\not{p}+\not{q})$$
$$x(p+q)^2 + (1-x)q^2 = xp^2 + 2xp\cdot q + q^2 = (q+xp)^2 + x(1-x)p^2 = \ell^2 - K_0$$
$$\ell = q + xp, \quad K_0 = -x(1-x)p^2, \quad \not{p}+\not{q} = \not{\ell} + (1-x)\not{p} \to (1-x)\not{p}$$
$$\frac{1}{(p+q)^2}\frac{1}{q^2} = \int_0^1 dx \frac{1}{[x(p+q)^2 + (1-x)q^2]^2} = \int_0^1 dx \frac{1}{(\ell^2 - K_0)^2}$$

$$i\Pi^{\gamma}_{\mathcal{T}^+} = \begin{array}{c} \gamma \\ (\qquad) \\ \mathcal{T}^+ \quad --\mathcal{T}^+-- \quad \mathcal{T}^+ \end{array} = (ie)^2 \int \frac{d^d q}{(2\pi)^d} \gamma^\mu \frac{i(\not{p}+\not{q})}{(p+q)^2} \gamma^\nu \frac{-ig_{\mu\nu}}{q^2} = -e^2 \int \frac{d^d q}{(2\pi)^d} \frac{\gamma^\mu(\not{p}+\not{q})\gamma_\mu}{(p+q)^2 q^2}$$
$$= e^2(d-2)\int \frac{d^d q}{(2\pi)^d}\frac{\not{p}+\not{q}}{(p+q)^2 q^2} = g^2 s_{\mathrm{W}}^2 (d-2)\int_0^1 dx \int \frac{d^d \ell}{(2\pi)^d}\frac{(1-x)\not{p}}{(\ell^2-K_0)^2} = g^2 s_{\mathrm{W}}^2(d-2)\int_0^1 dx(1-x)\not{p}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}$$
$$\int_0^1 dx(1-x) = \frac{1}{2}$$
$$\frac{\partial(i\Pi^{\gamma}_{\mathcal{T}^+})}{\partial \not{p}} = g^2 s_{\mathrm{W}}^2 (d-2)\int_0^1 dx(1-x)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}} + \text{finite} = g^2 s_{\mathrm{W}}^2 \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{Z}_{\mathcal{T}^+} = \begin{array}{c} Z \\ (\qquad) \\ \mathcal{T}^+ \quad --\mathcal{T}^+-- \quad \mathcal{T}^+ \end{array} = (igc_{\mathrm{W}})^2 \int \frac{d^d q}{(2\pi)^d}\gamma^\mu \frac{i(\not{p}+\not{q})}{(p+q)^2}\gamma^\nu \frac{-ig_{\mu\nu}}{q^2} = g^2 c_{\mathrm{W}}^2 (d-2)\int_0^1 dx(1-x)\not{p}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}$$
$$\frac{\partial(i\Pi^{Z}_{\mathcal{T}^+})}{\partial \not{p}} = g^2 c_{\mathrm{W}}^2 \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{W}_{\mathcal{T}^+} = \begin{array}{c} W^+ \\ (\qquad) \\ \mathcal{T}^+ \quad --\mathcal{T}^0-- \quad \mathcal{T}^+ \end{array} = (-ig)^2 \int \frac{d^d q}{(2\pi)^d}\gamma^\mu \frac{i(\not{p}+\not{q})}{(p+q)^2}\gamma^\nu \frac{-ig_{\mu\nu}}{q^2} = g^2 (d-2)\int_0^1 dx(1-x)\not{p}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}$$
$$\frac{\partial(i\Pi^{W}_{\mathcal{T}^+})}{\partial \not{p}} = g^2 \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\left(\frac{i}{\sqrt{6}}\right)^2 \langle 0| a_{\mathcal{T}^+} \overline{h h}\, \bar{\mathcal{T}}^+ (y_1 P_{\mathrm{R}} - y_2 P_{\mathrm{L}}) \overline{\mathcal{Q}^+ \bar{\mathcal{Q}}^+} (y_1 P_{\mathrm{L}} - y_2 P_{\mathrm{R}}) \mathcal{T}^+ a^\dagger_{\mathcal{T}^+} |0\rangle$$
$$(y_1 P_{\mathrm{R}} - y_2 P_{\mathrm{L}})(\not{p}+\not{q})(y_1 P_{\mathrm{L}} - y_2 P_{\mathrm{R}}) = (\not{p}+\not{q})(y_1 P_{\mathrm{L}} - y_2 P_{\mathrm{R}})(y_1 P_{\mathrm{L}} - y_2 P_{\mathrm{R}}) = (\not{p}+\not{q})(y_1^2 P_{\mathrm{L}} + y_2^2 P_{\mathrm{R}})$$
$$i\Pi^{h,1}_{\mathcal{T}^+} = \begin{array}{c} h \\ (\qquad) \\ \mathcal{T}^+ \quad \to - \ \mathcal{Q}^+ \to - \quad \mathcal{T}^+ \end{array} = \left(\frac{i}{\sqrt{6}}\right)^2 \int \frac{d^d q}{(2\pi)^d}(y_1 P_{\mathrm{R}} - y_2 P_{\mathrm{L}})\frac{i(\not{p}+\not{q})}{(p+q)^2}(y_1 P_{\mathrm{L}} - y_2 P_{\mathrm{R}})\frac{i}{q^2}$$
$$= \frac{1}{6}\int \frac{d^d q}{(2\pi)^d}\frac{(y_1 P_{\mathrm{R}} - y_2 P_{\mathrm{L}})(\not{p}+\not{q})(y_1 P_{\mathrm{L}} - y_2 P_{\mathrm{R}})}{(p+q)^2 q^2} = \frac{1}{6}\int \frac{d^d q}{(2\pi)^d}\frac{\not{p}+\not{q}}{(p+q)^2 q^2}(y_1^2 P_{\mathrm{L}} + y_2^2 P_{\mathrm{R}})$$
$$= \frac{1}{6}\int_0^1 dx \int \frac{d^d \ell}{(2\pi)^d}\frac{(1-x)\not{p}}{(\ell^2 - K_0)^2}(y_1^2 P_{\mathrm{L}} + y_2^2 P_{\mathrm{R}}) = \frac{1}{6}\int_0^1 dx(1-x)\not{p}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}(y_1^2 P_{\mathrm{L}} + y_2^2 P_{\mathrm{R}})$$
$$\frac{\partial(i\Pi^{h,1}_{\mathcal{T}^+})}{\partial \not{p}} = \frac{1}{6}\int_0^1 dx(1-x)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}(y_1^2 P_{\mathrm{L}} + y_2^2 P_{\mathrm{R}}) + \text{finite} = \frac{1}{12}(y_1^2 P_{\mathrm{L}} + y_2^2 P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$(\mathcal{T}^+)^{\mathrm{c}} = \mathcal{C}(\bar{\mathcal{T}}^+)^{\mathrm{T}}, \quad \overline{(\mathcal{T}^+)^{\mathrm{c}}} = (\mathcal{T}^+)^{\mathrm{T}}\mathcal{C}$$

$$\overline{(\mathcal{T}^+)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^-\bar{\mathcal{Q}}^-(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\mathcal{T}^+)^{\mathrm{c}} = \left[\overline{(\mathcal{T}^+)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^-\bar{\mathcal{Q}}^-(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\mathcal{T}^+)^{\mathrm{c}}\right]^{\mathrm{T}}$$

$$= \bar{\mathcal{T}}^+\mathcal{C}^{\mathrm{T}}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}(\bar{\mathcal{Q}}^-)^{\mathrm{T}}(\mathcal{Q}^-)^{\mathrm{T}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{\mathrm{T}}\mathcal{T}^+$$

$$= \bar{\mathcal{T}}^+\mathcal{C}^{\mathrm{T}}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\mathcal{C}(\bar{\mathcal{Q}}^-)^{\mathrm{T}}(\mathcal{Q}^-)^{\mathrm{T}}\mathcal{C}\mathcal{C}^{-1}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{\mathrm{T}}\mathcal{T}^+$$

$$= \bar{\mathcal{T}}^+\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}(\mathcal{Q}^-)^{\mathrm{c}}\overline{(\mathcal{Q}^-)^{\mathrm{c}}}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\mathcal{T}^+$$

$$\left(\frac{i}{\sqrt{2}}\right)^2\langle 0|a_{\mathcal{T}^+}\, hh\,\overline{(\mathcal{T}^+)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^-\bar{\mathcal{Q}}^-(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\mathcal{T}^+)^{\mathrm{c}}a^\dagger_{\mathcal{T}^+}|0\rangle$$

$$= \left(\frac{i}{\sqrt{2}}\right)^2\langle 0|a_{\mathcal{T}^+}\, hh\,\bar{\mathcal{T}}^+\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}(\mathcal{Q}^-)^{\mathrm{c}}\overline{(\mathcal{Q}^-)^{\mathrm{c}}}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\mathcal{T}^+a^\dagger_{\mathcal{T}^+}|0\rangle$$

$$(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\not{p}+\not{q})(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}}) = (\not{p}+\not{q})(y_2^2P_{\mathrm{L}}+y_1^2P_{\mathrm{R}})$$

$$i\Pi^{h,2}_{\mathcal{T}^+} = \underset{(\mathcal{T}^+)^{\mathrm{c}} \quad -\leftarrow \mathcal{Q}^- -\leftarrow \quad (\mathcal{T}^+)^{\mathrm{c}}}{\overset{h}{(\qquad)}} = \left(\frac{i}{\sqrt{2}}\right)^2\int\frac{d^dq}{(2\pi)^d}\,\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i(\not{p}+\not{q})}{(p+q)^2}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i}{q^2}$$

$$= \frac{1}{2}\int\frac{d^dq}{(2\pi)^d}\frac{(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\not{p}+\not{q})(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})}{(p+q)^2q^2} = \frac{1}{2}\int\frac{d^dq}{(2\pi)^d}\frac{\not{p}+\not{q}}{(p+q)^2q^2}(y_2^2P_{\mathrm{L}}+y_1^2P_{\mathrm{R}})$$

$$\frac{\partial(i\Pi^{h,1}_{\mathcal{T}^+})}{\partial\not{p}} = \frac{1}{4}(y_2^2P_{\mathrm{L}}+y_1^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{G^0,1}_{\mathcal{T}^+} = \underset{\mathcal{T}^+ \quad \to - \mathcal{Q}^+ \to - \quad \mathcal{T}^+}{\overset{G^0}{(\qquad)}} = \left(-\frac{1}{\sqrt{6}}\right)\frac{1}{\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\frac{i(\not{p}+\not{q})}{(p+q)^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i}{q^2} = i\Pi^{h,1}_{\mathcal{T}^+}$$

$$\frac{\partial(i\Pi^{G^0,1}_{\mathcal{T}^+})}{\partial\not{p}} = \frac{1}{12}(y_1^2P_{\mathrm{L}}+y_2^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{G^0,2}_{\mathcal{T}^+} = \underset{(\mathcal{T}^+)^{\mathrm{c}} \quad -\leftarrow \mathcal{Q}^- -\leftarrow \quad (\mathcal{T}^+)^{\mathrm{c}}}{\overset{G^0}{(\qquad)}} = \left(-\frac{1}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}\int\frac{d^dq}{(2\pi)^d}\,\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i(\not{p}+\not{q})}{(p+q)^2}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i}{q^2} = i\Pi^{h,2}_{\mathcal{T}^+}$$

$$\frac{\partial(i\Pi^{G^0,2}_{\mathcal{T}^+})}{\partial\not{p}} = \frac{1}{4}(y_2^2P_{\mathrm{L}}+y_1^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{G^\pm,1}_{\mathcal{T}^+} = \underset{\mathcal{T}^+ \quad \to - \mathcal{Q}^{++} \to - \quad \mathcal{T}^+}{\overset{G^-}{(\qquad)}} = i^2\int\frac{d^dq}{(2\pi)^d}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\frac{i(\not{p}+\not{q})}{(p+q)^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i}{q^2}$$

$$\frac{\partial(i\Pi^{G^\pm,1}_{\mathcal{T}^+})}{\partial\not{p}} = \frac{1}{2}(y_1^2P_{\mathrm{L}}+y_2^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{G^\pm,2}_{\mathcal{T}^+} = \underset{(\mathcal{T}^+)^{\mathrm{c}} \quad -\leftarrow \mathcal{Q}^0 -\leftarrow \quad (\mathcal{T}^+)^{\mathrm{c}}}{\overset{G^+}{(\qquad)}} = \left(\frac{i}{\sqrt{3}}\right)^2\int\frac{d^dq}{(2\pi)^d}\,\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i(\not{p}+\not{q})}{(p+q)^2}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i}{q^2}$$

$$\frac{\partial(i\Pi^{G^\pm,2}_{\mathcal{T}^+})}{\partial\not{p}} = \frac{1}{6}(y_2^2P_{\mathrm{L}}+y_1^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\frac{\partial(i\Pi_{\mathcal{T}^+,\mathrm{L}})}{\partial \boldsymbol{p}} = \sum_j \frac{\partial(i\Pi^j_{\mathcal{T}^+,\mathrm{L}})}{\partial \boldsymbol{p}} = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left[g^2 s_\mathrm{W}^2 + g^2 c_\mathrm{W}^2 + g^2 + 2\cdot\left(\frac{1}{12}y_1^2 + \frac{1}{4}y_2^2\right) + \frac{1}{2}y_1^2 + \frac{1}{6}y_2^2\right] + \text{finite}$$

$$= \left[2g^2 + \frac{2}{3}(y_1^2 + y_2^2)\right]\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_{\mathcal{T}^+,\mathrm{L}} = -\left[2g^2 + \frac{2}{3}(y_1^2 + y_2^2)\right]\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}\left[2g^2 + \frac{2}{3}(y_1^2 + y_2^2)\right]\ln\mu_\mathrm{R}^2 + \cdots$$

$$\frac{\partial\delta_{\mathcal{T}^+,\mathrm{L}}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}\left[4g^2 + \frac{4}{3}(y_1^2 + y_2^2)\right]$$

$$\frac{\partial(i\Pi_{\mathcal{T}^+,\mathrm{R}})}{\partial \boldsymbol{p}} = \sum_j \frac{\partial(i\Pi^j_{\mathcal{T}^+,\mathrm{R}})}{\partial \boldsymbol{p}} = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left[g^2 s_\mathrm{W}^2 + g^2 c_\mathrm{W}^2 + g^2 + 2\cdot\left(\frac{1}{12}y_2^2 + \frac{1}{4}y_1^2\right) + \frac{1}{2}y_2^2 + \frac{1}{6}y_1^2\right] + \text{finite}$$

$$= \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left[2g^2 + \frac{2}{3}(y_1^2 + y_2^2)\right] + \text{finite}$$

$$\frac{\partial\delta_{\mathcal{T}^+,\mathrm{R}}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}\left[4g^2 + \frac{4}{3}(y_1^2 + y_2^2)\right]$$

⊙ Calculation for $\frac{\partial\delta_{\mathcal{Q}^+,\mathrm{L}}}{\partial\ln\mu_\mathrm{R}}$ and $\frac{\partial\delta_{\mathcal{Q}^+,\mathrm{R}}}{\partial\ln\mu_\mathrm{R}}$

$$i\Pi^{\gamma}_{\mathcal{Q}^+} = \begin{array}{c}\gamma \\ (\qquad) \\ \mathcal{Q}^+ \quad --\mathcal{Q}^+-- \quad \mathcal{Q}^+\end{array} = (ie)^2\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i(\boldsymbol{p}+\boldsymbol{q})}{(p+q)^2}\gamma^\nu\frac{-ig_{\mu\nu}}{q^2}$$

$$\frac{\partial(i\Pi^{\gamma}_{\mathcal{Q}^+})}{\partial \boldsymbol{p}} = g^2 s_\mathrm{W}^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{Z}_{\mathcal{Q}^+} = \begin{array}{c}Z \\ (\qquad) \\ \mathcal{Q}^+ \quad --\mathcal{Q}^+-- \quad \mathcal{Q}^+\end{array} = \left[\frac{ig(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}\right]^2\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i(\boldsymbol{p}+\boldsymbol{q})}{(p+q)^2}\gamma^\nu\frac{-ig_{\mu\nu}}{q^2}$$

$$\frac{\partial(i\Pi^{Z}_{\mathcal{Q}^+})}{\partial \boldsymbol{p}} = \frac{g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)^2}{4c_\mathrm{W}^2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{W,1}_{\mathcal{Q}^+} = \begin{array}{c}W^+ \\ (\qquad) \\ \mathcal{Q}^+ \quad --\mathcal{Q}^0-- \quad \mathcal{Q}^+\end{array} = \left(i\sqrt{2}g\right)^2\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i(\boldsymbol{p}+\boldsymbol{q})}{(p+q)^2}\gamma^\nu\frac{-ig_{\mu\nu}}{q^2}, \quad \frac{\partial(i\Pi^{W,1}_{\mathcal{Q}^+})}{\partial \boldsymbol{p}} = 2g^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{W,2}_{\mathcal{Q}^+} = \begin{array}{c}W^- \\ (\qquad) \\ \mathcal{Q}^+ \quad --\mathcal{Q}^{++}-- \quad \mathcal{Q}^+\end{array} = \left(i\frac{\sqrt{6}}{2}g\right)^2\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i(\boldsymbol{p}+\boldsymbol{q})}{(p+q)^2}\gamma^\nu\frac{-ig_{\mu\nu}}{q^2}, \quad \frac{\partial(i\Pi^{W,2}_{\mathcal{Q}^+})}{\partial \boldsymbol{p}} = \frac{3}{2}g^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{h}_{\mathcal{Q}^+} = \begin{array}{c}h \\ (\qquad) \\ \mathcal{Q}^+ \quad \to -\ \mathcal{T}^+ \to - \quad \mathcal{Q}^+\end{array} = \left(\frac{i}{\sqrt{6}}\right)^2\int\frac{d^dq}{(2\pi)^d}(y_1P_\mathrm{L} - y_2P_\mathrm{R})\frac{i(\boldsymbol{p}+\boldsymbol{q})}{(p+q)^2}(y_1P_\mathrm{R} - y_2P_\mathrm{L})\frac{i}{q^2}$$

$$\frac{\partial(i\Pi^{h}_{\mathcal{Q}^+})}{\partial \boldsymbol{p}} = \frac{1}{12}(y_2^2P_\mathrm{L} + y_1^2P_\mathrm{R})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{G^0}_{\mathcal{Q}^+} = \begin{array}{ccc} & G^0 & \\ & (\qquad) & \\ \mathcal{Q}^+ \ \to - & \mathcal{T}^+ & \to - \ \mathcal{Q}^+ \end{array} = \left(-\frac{1}{\sqrt{6}}\right)\frac{1}{\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\frac{i(\not{p}+\not{q})}{(p+q)^2}(y_1P_{\mathrm{R}} - y_2P_{\mathrm{L}})\frac{i}{q^2} = i\Pi^h_{\mathcal{Q}^+}$$

$$\frac{\partial(i\Pi^{G^0}_{\mathcal{Q}^+})}{\partial\not{p}} = \frac{1}{12}(y_2^2P_{\mathrm{L}} + y_1^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi^{G^\pm}_{\mathcal{Q}^+} = \begin{array}{ccc} & G^+ & \\ & (\qquad) & \\ \mathcal{Q}^+ \ \to - & \mathcal{T}^0 & \to - \ \mathcal{Q}^+ \end{array} = \left(\frac{2i}{\sqrt{6}}\right)^2\int\frac{d^dq}{(2\pi)^d}(-y_1P_{\mathrm{L}} + y_2P_{\mathrm{R}})\frac{i(\not{p}+\not{q})}{(p+q)^2}(-y_1P_{\mathrm{R}} + y_2P_{\mathrm{L}})\frac{i}{q^2} = i\Pi^h_{\mathcal{Q}^+}$$

$$\frac{\partial(i\Pi^{G^0}_{\mathcal{Q}^+})}{\partial\not{p}} = \frac{1}{3}(y_2^2P_{\mathrm{L}} + y_1^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$g^2s_{\mathrm{W}}^2 + \frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)^2}{4c_{\mathrm{W}}^2} = g^2s_{\mathrm{W}}^2 + \frac{g^2(2c_{\mathrm{W}}^2 - 1)^2}{4c_{\mathrm{W}}^2} = g^2s_{\mathrm{W}}^2 + g^2c_{\mathrm{W}}^2 - g^2 + \frac{g^2}{4c_{\mathrm{W}}^2} = \frac{1}{4}g^2 + \frac{1}{4}g'^2$$

$$\frac{\partial(i\Pi_{\mathcal{Q}^+,\mathrm{L}})}{\partial\not{p}} = \sum_j\frac{\partial(i\Pi^j_{\mathcal{Q}^+,\mathrm{L}})}{\partial\not{p}} = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left[g^2s_{\mathrm{W}}^2 + \frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)^2}{4c_{\mathrm{W}}^2} + 2g^2 + \frac{3}{2}g^2 + 2\cdot\frac{1}{12}y_2^2 + \frac{1}{3}y_2^2\right] + \text{finite}$$

$$= \left(\frac{15}{4}g^2 + \frac{1}{4}g'^2 + \frac{1}{2}y_2^2\right)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_{\mathcal{Q}^+,\mathrm{L}} = -\left(\frac{15}{4}g^2 + \frac{1}{4}g'^2 + \frac{1}{2}y_2^2\right)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}\left(\frac{15}{4}g^2 + \frac{1}{4}g'^2 + \frac{1}{2}y_2^2\right)\ln\mu_{\mathrm{R}}^2 + \cdots$$

$$\frac{\partial\delta_{\mathcal{Q}^+,\mathrm{L}}}{\partial\ln\mu_{\mathrm{R}}} = \frac{1}{16\pi^2}\left(\frac{15}{2}g^2 + \frac{1}{2}g'^2 + y_2^2\right)$$

$$\frac{\partial(i\Pi_{\mathcal{Q}^+,\mathrm{R}})}{\partial\not{p}} = \sum_j\frac{\partial(i\Pi^j_{\mathcal{Q}^+,\mathrm{R}})}{\partial\not{p}} = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left[g^2s_{\mathrm{W}}^2 + \frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)^2}{4c_{\mathrm{W}}^4} + 2g^2 + \frac{3}{2}g^2 + 2\cdot\frac{1}{12}y_1^2 + \frac{1}{3}y_1^2\right] + \text{finite}$$

$$= \left(\frac{15}{4}g^2 + \frac{1}{4}g'^2 + \frac{1}{2}y_1^2\right)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\frac{\partial\delta_{\mathcal{Q}^+,\mathrm{R}}}{\partial\ln\mu_{\mathrm{R}}} = \frac{1}{16\pi^2}\left(\frac{15}{2}g^2 + \frac{1}{2}g'^2 + y_1^2\right)$$

⊙ Calculation for $\frac{\partial\delta_{y_1}}{\partial\ln\mu_{\mathrm{R}}}$ and $\frac{\partial\delta_{y_2}}{\partial\ln\mu_{\mathrm{R}}}$

$$\gamma^\mu\not{q}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\not{q}\gamma_\mu = q^2\gamma^\mu\gamma_\mu(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}}) = dq^2(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})$$

$$i\Sigma^{\mathcal{Q}^+\mathcal{T}^+\gamma}_{y_{1,2}} = \begin{array}{ccc} & h & \\ \mathcal{T}^+ \ / & & \backslash \ \mathcal{Q}^+ \\ \mathcal{T}^+ & --\gamma-- & \mathcal{Q}^+ \end{array} = (ie)^2\frac{i}{\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i\not{q}}{q^2}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}\gamma_\mu\frac{-i}{q^2} = \frac{1}{\sqrt{6}}g^2s_{\mathrm{W}}^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^3}\gamma^\mu\not{q}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\not{q}\gamma_\mu$$

$$= \frac{1}{\sqrt{6}}g^2s_{\mathrm{W}}^2d\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}}) = \frac{4g^2s_{\mathrm{W}}^2}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma^{\mathcal{Q}^+\mathcal{T}^+Z}_{y_{1,2}} = \begin{array}{ccc} & h & \\ \mathcal{T}^+ \ / & & \backslash \ \mathcal{Q}^+ \\ \mathcal{T}^+ & --Z-- & \mathcal{Q}^+ \end{array} = \frac{ig(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}igc_{\mathrm{W}}\frac{i}{\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i\not{q}}{q^2}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}\gamma_\mu\frac{-i}{q^2}$$

$$= \frac{2g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{\sqrt{6}}(y_1P_{\mathrm{L}} - y_2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma^{\mathcal{Q}^0\mathcal{T}^0W}_{y_{1,2}} = \begin{array}{ccccc} & & h & & \\ & \mathcal{T}^0 / & & \searrow \mathcal{Q}^0 & \\ \mathcal{T}^+ & \rightarrow -W^+ - \rightarrow - & & & \mathcal{Q}^+ \end{array} = \left(i\sqrt{2}g\right)(-ig)\frac{i}{\sqrt{3}}\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}\gamma_\mu\frac{-i}{q^2}$$

$$=\frac{\sqrt{6}}{3}g^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^3}\gamma^\mu\not{q}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\not{q}\gamma_\mu=\frac{4\sqrt{6}}{3}g^2(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma^{\mathcal{T}^+\mathcal{Q}^+h}_{y_{1,2}} = \begin{array}{ccccc} & & h & & \\ & \mathcal{Q}^+ / & & \backslash\, \mathcal{T}^+ & \\ \mathcal{T}^+ & & --h-- & & \mathcal{Q}^+ \end{array} = \left(\frac{i}{\sqrt{6}}\right)^3\int\frac{d^dq}{(2\pi)^d}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(y_1P_{\mathrm{R}}-y_2P_{\mathrm{L}})\frac{i\not{q}}{q^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i}{q^2}$$

$$=-\frac{1}{6\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(y_1^3P_{\mathrm{L}}-y_2^3P_{\mathrm{R}})=-\frac{1}{6\sqrt{6}}(y_1^3P_{\mathrm{L}}-y_2^3P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma^{\mathcal{T}^+\mathcal{Q}^+G^0}_{y_{1,2}} = \begin{array}{ccccc} & & h & & \\ & \mathcal{Q}^+ / & & \backslash\, \mathcal{T}^+ & \\ \mathcal{T}^+ & & --G^0-- & & \mathcal{Q}^+ \end{array} = i^2\cdot\left(\begin{array}{ccccc} & & h & & \\ & \mathcal{Q}^+ / & & \backslash\, \mathcal{T}^+ & \\ \mathcal{T}^+ & & --h-- & & \mathcal{Q}^+ \end{array}\right) = -i\Sigma^{\mathcal{T}^+\mathcal{Q}^+h}_{y_{1,2}}$$

$$\frac{1}{\sqrt{2}}h\bar{\mathcal{Q}}^-(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\mathcal{T}^+)^{\mathrm{c}}=\frac{1}{\sqrt{2}}h\left[\bar{\mathcal{Q}}^-(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\mathcal{C}(\bar{\mathcal{T}}^+)^{\mathrm{T}}\right]^{\mathrm{T}}=-\frac{1}{\sqrt{2}}h\bar{\mathcal{T}}^+\mathcal{C}^{\mathrm{T}}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}(\bar{\mathcal{Q}}^-)^{\mathrm{T}}$$

$$=\frac{1}{\sqrt{2}}h\bar{\mathcal{T}}^+\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\mathcal{C}(\bar{\mathcal{Q}}^-)^{\mathrm{T}}=\frac{1}{\sqrt{2}}h\bar{\mathcal{T}}^+(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})(\bar{\mathcal{Q}}^-)^{\mathrm{c}}$$

$$\frac{1}{\sqrt{2}}h\overline{(\mathcal{T}^+)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^-=\frac{1}{\sqrt{2}}h\left[(\mathcal{T}^+)^{\mathrm{T}}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^-\right]^{\mathrm{T}}=-\frac{1}{\sqrt{2}}h(\mathcal{Q}^-)^{\mathrm{T}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{\mathrm{T}}\mathcal{T}^+$$

$$=\frac{1}{\sqrt{2}}h(\mathcal{Q}^-)^{\mathrm{T}}\mathcal{C}\mathcal{C}^{-1}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}\mathcal{T}^+=\frac{1}{\sqrt{2}}h\overline{(\mathcal{Q}^-)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{T}^+$$

$$i\Sigma^{\mathcal{T}^+(\mathcal{Q}^-)^{\mathrm{c}}h}_{y_{1,2}} = \begin{array}{ccccc} & & h & & \\ & (\mathcal{Q}^-)^{\mathrm{c}} / & & \backslash\, \mathcal{T}^+ & \\ \mathcal{T}^+ & & --h-- & & \mathcal{Q}^+ \end{array} = \left(\frac{i}{\sqrt{6}}\right)\left(\frac{i}{\sqrt{2}}\right)^2\int\frac{d^dq}{(2\pi)^d}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\frac{i}{q^2}$$

$$=-\frac{1}{2\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(y_1y_2^2P_{\mathrm{L}}-y_2y_1^2P_{\mathrm{R}})=-\frac{1}{2\sqrt{6}}(y_1y_2^2P_{\mathrm{L}}-y_2y_1^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$-\frac{i}{\sqrt{2}}G^0\overline{(\mathcal{T}^+)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{Q}^-=-\frac{i}{\sqrt{2}}G^0\overline{(\mathcal{Q}^-)^{\mathrm{c}}}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\mathcal{T}^+$$

$$i\Sigma^{\mathcal{T}^+(\mathcal{Q}^-)^{\mathrm{c}}G^0}_{y_{1,2}} = \begin{array}{ccccc} & & h & & \\ & (\mathcal{Q}^-)^{\mathrm{c}} / & & \backslash\, \mathcal{T}^+ & \\ \mathcal{T}^+ & & --G^0-- & & \mathcal{Q}^+ \end{array} = \left(-\frac{1}{\sqrt{6}}\right)\left(\frac{i}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)\int\frac{d^dq}{(2\pi)^d}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})\frac{i}{q^2}$$

$$=-\frac{1}{2\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(y_1y_2^2P_{\mathrm{L}}-y_2y_1^2P_{\mathrm{R}})=-\frac{1}{2\sqrt{6}}(y_1y_2^2P_{\mathrm{L}}-y_2y_1^2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma^{\mathcal{T}^0\mathcal{Q}^0G^+}_{y_{1,2}} = \begin{array}{ccccc} & & h & & \\ & \mathcal{Q}^0 \swarrow & & \backslash\, \mathcal{T}^0 & \\ (\mathcal{T}^+)^{\mathrm{c}} & & -\leftarrow -G^+ - \rightarrow - & & \mathcal{Q}^+ \end{array}$$

$$=\left(i\frac{2}{\sqrt{6}}\right)\left(\frac{i}{\sqrt{3}}\right)\left(\frac{i}{\sqrt{3}}\right)\int\frac{d^dq}{(2\pi)^d}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}\mathcal{C}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i\not{q}}{q^2}\mathcal{C}(-y_1P_{\mathrm{R}}+y_2P_{\mathrm{L}})^{\mathrm{T}}\mathcal{C}^{-1}\frac{i}{q^2}$$

$$=-\frac{2}{3\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(-y_1y_2^2P_{\mathrm{L}}+y_1^2y_2P_{\mathrm{R}})=\frac{2}{3\sqrt{6}}(y_1y_2^2P_{\mathrm{L}}-y_1^2y_2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

(26)

$$i\Sigma^{ZG^0\mathcal{Q}^+}_{y_{1,2}} = \begin{array}{c} h \\ G^0 \;/ \quad \backslash\; Z \\ \mathcal{T}^+ \quad --\mathcal{Q}^+-- \quad \mathcal{Q}^+ \end{array} = \frac{ig(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}\left(-\frac{1}{\sqrt{6}}\right)\left(-\frac{g}{2c_{\mathrm{W}}}\right)\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i\not{q}}{q^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{-ig_{\mu\nu}}{q^2}q^\nu\frac{i}{q^2}$$

$$=-\frac{g^2(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{4\sqrt{6}c_{\mathrm{W}}^2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}}) = -\frac{g^2(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{4\sqrt{6}c_{\mathrm{W}}^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma^{G^0Z\mathcal{T}^+}_{y_{1,2}} = \begin{array}{c} h \\ Z \;/ \quad \backslash\; G^0 \\ \mathcal{T}^+ \quad --\mathcal{T}^+-- \quad \mathcal{Q}^+ \end{array} = \left(-\frac{1}{\sqrt{6}}\right)igc_{\mathrm{W}}\frac{g}{2c_{\mathrm{W}}}\int\frac{d^dq}{(2\pi)^d}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}\gamma^\mu\frac{i}{q^2}q^\nu\frac{-ig_{\mu\nu}}{q^2}$$

$$=\frac{g^2}{2\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}}) = \frac{g^2}{2\sqrt{6}}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma^{W^-G^-\mathcal{Q}^{++}}_{y_{1,2}} = \begin{array}{c} h \\ G^- \swarrow\!\nwarrow W^- \\ \mathcal{T}^+ \quad \to-\mathcal{Q}^{++}\to- \quad \mathcal{Q}^+ \end{array} = \left(i\frac{\sqrt{6}g}{2}\right)i\frac{ig}{2}\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i\not{q}}{q^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{-ig_{\mu\nu}}{q^2}q^\nu\frac{i}{q^2}$$

$$=\frac{\sqrt{6}g^2}{4}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}}) = \frac{\sqrt{6}g^2}{4}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma^{G^+W^+\mathcal{T}^0}_{y_{1,2}} = \begin{array}{c} h \\ W^+ \nearrow\!\searrow G^+ \\ \mathcal{T}^+ \quad \to-\mathcal{T}^0\to- \quad \mathcal{Q}^+ \end{array} = \left(\frac{2i}{\sqrt{6}}\right)(-ig)\left(-\frac{ig}{2}\right)\int\frac{d^dq}{(2\pi)^d}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}})\frac{i\not{q}}{q^2}\gamma^\mu\frac{i}{q^2}q^\nu\frac{-ig_{\mu\nu}}{q^2}$$

$$=\frac{g^2}{\sqrt{6}}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}(-y_1P_{\mathrm{L}}+y_2P_{\mathrm{R}}) = -\frac{g^2}{\sqrt{6}}(y_1P_{\mathrm{L}}-y_2P_{\mathrm{R}})\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Sigma_{y_{1,2},\mathrm{L}} = \sum_j(i\Sigma^j_{y_{1,2},\mathrm{L}})$$

$$=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}y_1\left[\frac{4g^2s_{\mathrm{W}}^2}{\sqrt{6}}+\frac{2g^2(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{\sqrt{6}}+\frac{4\sqrt{6}}{3}g^2-2\cdot\frac{y_2^2}{2\sqrt{6}}+\frac{2y_2^2}{3\sqrt{6}}-\frac{g^2(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)}{4\sqrt{6}c_{\mathrm{W}}^2}+\frac{g^2}{2\sqrt{6}}+\frac{\sqrt{6}g^2}{4}-\frac{g^2}{\sqrt{6}}\right]+\text{finite}$$

$$=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}y_1\left(\frac{2}{\sqrt{6}}g^2+\frac{4\sqrt{6}}{3}g^2-\frac{3\sqrt{6}y_2^2}{18}+\frac{2\sqrt{6}y_2^2}{18}-\frac{1}{4\sqrt{6}}g^2+\frac{1}{4\sqrt{6}}g'^2+\frac{1}{2\sqrt{6}}g^2+\frac{\sqrt{6}}{4}g^2-\frac{1}{\sqrt{6}}g^2\right)+\text{finite}$$

$$=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}y_1\left(\frac{8\sqrt{6}}{24}g^2+\frac{32\sqrt{6}}{24}g^2-\frac{\sqrt{6}y_2^2}{18}-\frac{\sqrt{6}}{24}g^2+\frac{\sqrt{6}}{24}g'^2+\frac{2\sqrt{6}}{24}g^2+\frac{6\sqrt{6}}{24}g^2-\frac{4\sqrt{6}}{24}g^2\right)+\text{finite}$$

$$=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\frac{\sqrt{6}}{24}y_1\left(-\frac{4}{3}y_2^2+43g^2+g'^2\right)+\text{finite}$$

$$\delta_{y_1}=-\sqrt{6}\frac{\sqrt{6}}{24}y_1\left(-\frac{4}{3}y_2^2+43g^2+g'^2\right)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}+\text{finite}=\frac{1}{16\pi^2}y_1\left(-\frac{1}{3}y_2^2+\frac{43}{4}g^2+\frac{1}{4}g'^2\right)\ln\mu_{\mathrm{R}}^2+\cdots$$

$$\frac{\partial\delta_{y_1}}{\partial\ln\mu_{\mathrm{R}}}=\frac{1}{16\pi^2}y_1\left(-\frac{2}{3}y_2^2+\frac{43}{2}g^2+\frac{1}{2}g'^2\right)$$

$$
\begin{aligned}
i\Sigma_{y_{1,2},\mathrm{R}} &= \sum_j (i\Sigma^j_{y_{1,2},\mathrm{R}}) \\
&= \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}(-y_2)\left[\frac{4g^2 s_\mathrm{W}^2}{\sqrt{6}} + \frac{2g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{\sqrt{6}} + \frac{4\sqrt{6}}{3}g^2 - 2\cdot\frac{y_1^2}{2\sqrt{6}} + \frac{2y_1^2}{3\sqrt{6}} - \frac{g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{4\sqrt{6}c_\mathrm{W}^2} + \frac{g^2}{2\sqrt{6}} + \frac{\sqrt{6}g^2}{4} - \frac{g^2}{\sqrt{6}}\right] + \text{finite} \\
&= \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left(-\frac{\sqrt{6}}{24}y_2\right)\left(-\frac{4}{3}y_1^2 + 43g^2 + g'^2\right) + \text{finite}
\end{aligned}
$$

$$
\delta_{y_2} = \sqrt{6}\left(-\frac{\sqrt{6}}{24}y_2\right)\left(-\frac{4}{3}y_1^2 + 43g^2 + g'^2\right)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}y_2\left(-\frac{1}{3}y_1^2 + \frac{43}{4}g^2 + \frac{1}{4}g'^2\right)\ln\mu_\mathrm{R}^2 + \cdots
$$

$$
\frac{\partial\delta_{y_2}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}y_2\left(-\frac{2}{3}y_1^2 + \frac{43}{2}g^2 + \frac{1}{2}g'^2\right)
$$

⊙ Expressions for β_{y_1} and β_{y_2}

$$
\frac{\partial\delta_h}{\partial\ln\mu_\mathrm{R}} = \frac{\partial\delta_h^\mathrm{SM}}{\partial\ln\mu_\mathrm{R}} + \frac{\partial\delta_h^\mathrm{TQ}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}[6y_t^2 - 3g^2 - g'^2 + 4(y_1^2 + y_2^2)]
$$

$$
\begin{aligned}
\frac{16\pi^2}{y_1}\beta_{y_1} &= 16\pi^2\left(-\frac{1}{y_1}\frac{\partial\delta_{y_1}}{\partial\ln\mu_\mathrm{R}} + \frac{1}{2}\frac{\partial\delta_h}{\partial\ln\mu_\mathrm{R}} + \frac{1}{2}\frac{\partial\delta_{\mathcal{Q}^+,\mathrm{R}}}{\partial\ln\mu_\mathrm{R}} + \frac{1}{2}\frac{\partial\delta_{\mathcal{T}^+,\mathrm{L}}}{\partial\ln\mu_\mathrm{R}}\right) \\
&= -\left(-\frac{2}{3}y_2^2 + \frac{43}{2}g^2 + \frac{1}{2}g'^2\right) + \frac{1}{2}[6y_t^2 - 3g^2 - g'^2 + 4(y_1^2 + y_2^2)] + \frac{1}{2}\left(\frac{15}{2}g^2 + \frac{1}{2}g'^2 + y_1^2\right) + \frac{1}{2}\left[4g^2 + \frac{4}{3}(y_1^2 + y_2^2)\right] \\
&= \left(\frac{12}{6} + \frac{3}{6} + \frac{4}{6}\right)y_1^2 + \left(\frac{2}{3} + \frac{6}{3} + \frac{2}{3}\right)y_2^2 + \left(-\frac{86}{4} - \frac{6}{4} + \frac{15}{4} + \frac{8}{4}\right)g^2 + \left(-\frac{2}{4} - \frac{2}{4} + \frac{1}{4}\right)g'^2 + 3y_t^2 \\
&= \frac{19}{6}y_1^2 + \frac{10}{3}y_2^2 - \frac{69}{4}g^2 - \frac{3}{4}g'^2 + 3y_t^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{16\pi^2}{y_2}\beta_{y_2} &= 16\pi^2\left(-\frac{1}{y_2}\frac{\partial\delta_{y_2}}{\partial\ln\mu_\mathrm{R}} + \frac{1}{2}\frac{\partial\delta_h}{\partial\ln\mu_\mathrm{R}} + \frac{1}{2}\frac{\partial\delta_{\mathcal{Q}^+,\mathrm{L}}}{\partial\ln\mu_\mathrm{R}} + \frac{1}{2}\frac{\partial\delta_{\mathcal{T}^+,\mathrm{R}}}{\partial\ln\mu_\mathrm{R}}\right) \\
&= -\left(-\frac{2}{3}y_1^2 + \frac{39}{2}g^2 + \frac{1}{2}g'^2\right) + \frac{1}{2}[6y_t^2 - 3g^2 - g'^2 + 4(y_1^2 + y_2^2)] + \frac{1}{2}\left(\frac{15}{2}g^2 + \frac{1}{2}g'^2 + y_2^2\right) + \frac{1}{2}\left[4g^2 + \frac{4}{3}(y_1^2 + y_2^2)\right] \\
&= \frac{19}{6}y_2^2 + \frac{10}{3}y_1^2 - \frac{69}{4}g^2 - \frac{3}{4}g'^2 + 3y_t^2
\end{aligned}
$$

4) Contribution to the β function of the top Yukawa coupling

$$
\beta_{y_t} = -\frac{\partial\delta_{y_t}}{\partial\ln\mu_\mathrm{R}} + y_t\frac{\partial\delta_{t,\mathrm{V}}}{\partial\ln\mu_\mathrm{R}} + \frac{1}{2}y_t\frac{\partial\delta_h}{\partial\ln\mu_\mathrm{R}}
$$

$$
\delta_{y_t}^\mathrm{TQ} = 0, \quad \delta_{t,\mathrm{V}}^\mathrm{TQ} = 0, \quad \frac{\partial\delta_h^\mathrm{TQ}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}4(y_1^2 + y_2^2)
$$

$$
\beta_{y_t}^\mathrm{TQ} = \frac{1}{2}y_t\frac{\partial\delta_h^\mathrm{TQ}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}2y_t(y_1^2 + y_2^2)
$$

5) Contributions to β functions of gauge couplings

$$\mathrm{SU}(2): \quad C(\mathbf{3}) = \mathrm{Tr}(t_{\mathrm{T}}^3 t_{\mathrm{T}}^3) = 1^2 + 0^2 + (-1)^2 = 2$$

$$C(\mathbf{4}) = \mathrm{Tr}(t_{\mathrm{Q}}^3 t_{\mathrm{Q}}^3) = \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{3}{2}\right)^2 = 5$$

$$\beta_g^{\mathrm{T}} = -\frac{g^3}{16\pi^2}\frac{1}{3}[-2C(\mathbf{3})] = \frac{1}{16\pi^2}\frac{4}{3}g^3, \quad \beta_g^{\mathrm{Q}} = -\frac{g^3}{16\pi^2}\frac{1}{3}[-2\cdot 2C(\mathbf{4})] = \frac{1}{16\pi^2}\frac{20}{3}g^3$$

$$\beta_{g'}^{\mathrm{Q}} = \frac{g'^3}{16\pi^2}\frac{1}{3}\left[2\cdot 4\left(\frac{1}{2}\right)^2 + 2\cdot 4\left(-\frac{1}{2}\right)^2\right] = \frac{1}{16\pi^2}\frac{4}{3}g'^3$$

6) Summary

$$\beta_{g_{\mathrm{s}}} = \beta_{g_{\mathrm{s}}}^{\mathrm{SM}}, \quad \beta_g = \beta_g^{\mathrm{SM}} + \beta_g^{\mathrm{T}} + \beta_g^{\mathrm{Q}}, \quad \beta_{g'} = \beta_{g'}^{\mathrm{SM}} + \beta_{g'}^{\mathrm{Q}}, \quad \beta_\lambda = \beta_\lambda^{\mathrm{SM}} + \beta_\lambda^{\mathrm{TQ}}, \quad \beta_{y_t} = \beta_{y_t}^{\mathrm{SM}} + \beta_{y_t}^{\mathrm{TQ}}$$

$$16\pi^2\beta_{g_{\mathrm{s}}}^{\mathrm{SM}} = -7g_{\mathrm{s}}^3, \quad 16\pi^2\beta_g^{\mathrm{SM}} = -\frac{19}{6}g^3, \quad 16\pi^2\beta_{g'}^{\mathrm{SM}} = \frac{41}{6}g'^3$$

$$16\pi^2\beta_\lambda^{\mathrm{SM}} = 24\lambda^2 + \lambda(12y_t^2 - 9g^2 - 3g'^2) - 6y_t^4 + \frac{3}{8}(3g^4 + 2g^2g'^2 + g'^4)$$

$$16\pi^2\beta_{y_t}^{\mathrm{SM}} = y_t\left(\frac{9}{2}y_t^2 - 8g_{\mathrm{s}}^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right)$$

$$16\pi^2\beta_g^{\mathrm{T}} = \frac{4}{3}g^3, \quad 16\pi^2\beta_g^{\mathrm{Q}} = \frac{20}{3}g^3, \quad 16\pi^2\beta_{g'}^{\mathrm{Q}} = \frac{4}{3}g'^3$$

$$16\pi^2\beta_\lambda^{\mathrm{TQ}} = 8\lambda(y_1^2 + y_2^2) - \frac{28}{9}(y_1^4 + y_2^4) - \frac{40}{9}y_1^2y_2^2, \quad 16\pi^2\beta_{y_t}^{\mathrm{TQ}} = 2y_t(y_1^2 + y_2^2)$$

$$16\pi^2\beta_{y_1} = y_1\left(\frac{19}{6}y_1^2 + \frac{10}{3}y_2^2 - \frac{69}{4}g^2 - \frac{3}{4}g'^2 + 3y_t^2\right)$$

$$16\pi^2\beta_{y_2} = y_2\left(\frac{19}{6}y_2^2 + \frac{10}{3}y_1^2 - \frac{69}{4}g^2 - \frac{3}{4}g'^2 + 3y_t^2\right)$$

This result is consistent with the PyR@TE result