粒子物理标准模型拉氏量和费曼规则

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目　录

1 约定

本文采用自然单位制，各种约定主要遵从文献 [1]，推导和计算参考文献 [1, 2, 3, 4]。

Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{1}$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \tag{2}$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \tag{3}$$

手征表示中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}. \tag{4}$$

左右手投影算符

$$P_{\mathrm{L}} \equiv \frac{1}{2}(1-\gamma_5) = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \quad P_{\mathrm{R}} \equiv \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}. \tag{5}$$

Levi-Civita 张量约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \tag{6}$$

费曼规则约定:

- 对于指向相互作用顶点的动量 p，时空偏导数 ∂_μ 在动量空间费曼规则里贡献一个 $-ip_\mu$ 因子。
- 实线表示费米子，实线上的箭头表示费米子数流动的方向。
- 虚线表示标量玻色子，虚线上的箭头表示电荷数流动的方向。
- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示电荷数流动的方向。
- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。
- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

2 标准模型概述

粒子物理标准模型是一个 $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 规范理论。模型中有三代费米子，包括三代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$，三代带电轻子 $\ell_i = e, \mu, \tau$，三代上型夸克 $u_i = u, c, t$ 和三代下型夸克 $d_i = d, s, b$ $(i = 1, 2, 3)$。规范玻色子传递费米子间相互作用。

$\mathrm{SU}(3)_\mathrm{C}$ 部分描述夸克的强相互作用，称为量子色动力学 (Quantum Chromodynamics, QCD), 相应的规范玻色子是胶子。$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 部分统一描述夸克和轻子的电磁和弱相互作用，称为电弱统一理论。理论中有一个 Higgs 二重态，通过 Brout–Englert–Higgs 机制引发规范群的自发对称性破缺，使 $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 群破缺为 $\mathrm{U}(1)_\mathrm{EM}$ 群。$\mathrm{U}(1)_\mathrm{EM}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前。理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度；左手费米子和右手费米子都没有质量。具有不同量子数。

破缺后，3 个规范玻色子与 3 个 Higgs 自由度结合，从而获得质量，成为 $W^{\pm}$ 和 Z^0 玻色子，传递弱相互作用；剩下的 1 个无质量规范玻色子是光子，即是 $\mathrm{U}(1)_{\mathrm{EM}}$ 群的规范玻色子，传递电磁相互作用；剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子；与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量，组合成 Dirac 费米子。

理论中的中微子没有右手分量，因而没有获得质量。1998 年实验发现中微子振荡，证明中微子具有质量，所以需要扩充标准模型才能正确描述中微子物理。

3 QCD 拉氏量和费曼规则

QCD 的拉氏量可表达成

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \cdots, 8, \tag{7}$$

其中

$$D_\mu = \partial_\mu - ig_{\mathrm{s}}G^a_\mu t^a, \quad G^{a\mu\nu} \equiv \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_{\mathrm{s}}f^{abc}G^{b\mu}G^{c\nu}. \tag{8}$$

$\mathrm{SU}(3)_{\mathrm{C}}$ 群基础表示生成元 $t^a = \lambda^a/2$，其中 λ^a 为 Gell-Mann 矩阵。生成元对易关系为 $[t^a, t^b] = if^{abc}t^c$。结构常数 f^{abc} 是全反对称的，其非零分量为

$$f_{123} = 1, \quad f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \tag{9}$$

由

$$\begin{aligned}
-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} &= -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_{\mathrm{s}}f^{abc}G^b_\mu G^c_\nu)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_{\mathrm{s}}f^{ade}G^{d\mu}G^{e\nu}) \\
&= -\frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu}) - (\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu})] - g_{\mathrm{s}}f^{abc}(\partial_\mu G^a_\nu)G^{b\mu}G^{c\nu} \\
&\quad -\frac{1}{4}g_{\mathrm{s}}^2 f^{abc}f^{ade}G^b_\mu G^c_\nu G^{d\mu}G^{e\nu},
\end{aligned} \tag{10}$$

可得

$$\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \sum_q [\bar{q}(i\gamma^\mu\partial_\mu - m_q)q + g_{\mathrm{s}}G^a_\mu\bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu}) - (\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu})] \\
&\quad -g_{\mathrm{s}}f^{abc}(\partial_\mu G^a_\nu)G^{b\mu}G^{c\nu} - \frac{1}{4}g_{\mathrm{s}}^2 f^{abc}f^{ade}G^b_\mu G^c_\nu G^{d\mu}G^{e\nu}.
\end{aligned} \tag{11}$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G^a_\mu(x) - \omega^a(x)$，其中 $\omega^a(x)$ 是某个任意函数，规范固定条件是 $G^a(x) = 0$。这是 Lorenz 规范的推广，$\omega^a(x) = 0$ 对应于 Lorenz 规范。在路径积分量子化中，以中心为 $\omega^a(x) = 0$ 的 Gauss 权重对 $\omega^a(x)$ 作泛函积分，有

$$\int \mathcal{D}\omega^a \exp\left[-i\int d^4x \frac{1}{2\xi}(\omega^a)^2\right]\delta(G^a) = \exp\left[-i\int d^4x \frac{1}{2\xi}(\partial^\mu G^a_\mu)^2\right]. \tag{12}$$

可见，拉氏量中的规范固定项为

$$\mathcal{L}_{\text{QCD,GF}} = -\frac{1}{2\xi}(\partial^\mu G^a_\mu)^2. \tag{13}$$

ξ 的任何一个取值对应于一种规范。$\xi = 1$ 称为 Feynman-'t Hooft 规范，$\xi = 0$ 称为 Landau 规范。于是，胶子传播子相关拉氏量为

$$\begin{aligned}\mathcal{L}_{\text{QCD,prop}} &= \frac{1}{2}\left[(\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu}) - (\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu}) - \frac{1}{\xi}(\partial^\mu G^a_\mu)^2\right] \\ &\to \frac{1}{2}G^a_\mu\left[g^{\mu\nu}\partial^2 - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]G^a_\nu. \end{aligned}\tag{14}$$

变换到动量空间，得

$$-g^{\mu\nu}p^2 + \left(1-\frac{1}{\xi}\right)p^\mu p^\nu, \tag{15}$$

它的逆矩阵是

$$-\frac{1}{p^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right], \tag{16}$$

这是因为

$$\begin{aligned}&-\frac{1}{p^2}\left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}(1-\xi)\right]\left[-g^{\mu\nu}p^2 + \left(1-\frac{1}{\xi}\right)p^\mu p^\nu\right] \\ &= \delta^\nu_\rho - \frac{p_\rho p^\nu}{p^2}\left(1-\frac{1}{\xi}\right) - \frac{p_\rho p^\nu}{p^2}(1-\xi) + \frac{p_\rho p^\nu}{p^2}(1-\xi)\left(1-\frac{1}{\xi}\right) = \delta^\nu_\rho. \end{aligned}\tag{17}$$

从而，胶子传播子的形式为

$$\frac{-i\delta^{ab}}{p^2+i\varepsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right]. \tag{18}$$

$\text{SU}(3)_\text{C}$ 定域规范变换为

$$q \to Uq, \quad G^a_\mu t^a \to UG^a_\mu t^a U^\dagger + \frac{i}{g_\text{s}}U\partial_\mu U^\dagger, \tag{19}$$

其中 $U(x) = \exp[i\alpha^a(x)t^a]$。胶子场的无穷小规范变换形式是

$$\begin{aligned}G^a_\mu t^a &\to (1+i\alpha^a t^a)G^b_\mu t^b(1-i\alpha^c t^c) + \frac{i}{g_\text{s}}(1+i\alpha^a t^a)\partial_\mu(1-i\alpha^c t^c) \\ &= G^b_\mu t^b + i\alpha^a G^b_\mu[t^a, t^b] + \frac{1}{g_\text{s}}(\partial_\mu\alpha^c)t^c + \mathcal{O}(\alpha^2) = G^a_\mu t^a - f^{abc}\alpha^a G^b_\mu t^c + \frac{1}{g_\text{s}}(\partial_\mu\alpha^a)t^a + \mathcal{O}(\alpha^2) \\ &= \left(G^a_\mu + f^{abc}G^b_\mu\alpha^c + \frac{1}{g_\text{s}}\partial_\mu\alpha^a\right)t^a + \mathcal{O}(\alpha^2), \end{aligned}\tag{20}$$

即

$$\delta G^a_\mu = \frac{1}{g_\text{s}}\partial_\mu\alpha^a + f^{abc}G^b_\mu\alpha^c = \left(\frac{1}{g_\text{s}}\delta^{ac}\partial_\mu + f^{abc}G^b_\mu\right)\alpha^c, \tag{21}$$

因而规范固定函数 G^a 的无穷小规范变换为

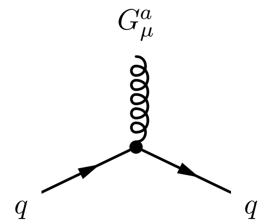
$$\delta G^a = \partial^\mu\delta G^a_\mu = \frac{1}{g_\text{s}}\delta^{ac}\partial^2\alpha^c + f^{abc}\partial^\mu G^b_\mu\alpha^c, \quad g_\text{s}\frac{\delta G^a}{\delta\alpha^c} = \delta^{ab}\partial^2 + g_\text{s}f^{abc}\partial^\mu G^b_\mu. \tag{22}$$

Faddeev-Popov 鬼场的拉氏量是

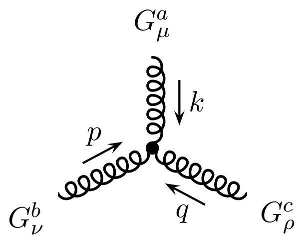
$$\mathcal{L}_{\text{QCD,FP}} = -\bar{\eta}_g^a \left(g_{\text{s}} \frac{\delta G^a}{\delta \alpha^c} \right) \eta_g^c = -\bar{\eta}_g^a (\delta^{ac}\partial^2 + g_{\text{s}} f^{abc} \partial^\mu G_\mu^b) \eta_g^c \to -\bar{\eta}_g^a \delta^{ab} \partial^2 \eta_g^b + g_{\text{s}} f^{abc} (\partial^\mu \bar{\eta}_g^a) G_\mu^b \eta_g^c. \quad (23)$$

下面列出 QCD 费曼规则。

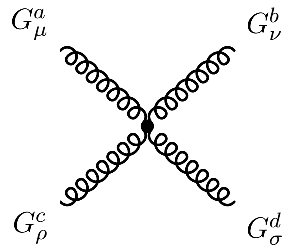
QCD 顶点:



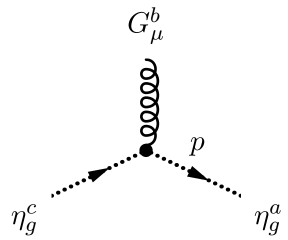
$$= i g_{\text{s}} \gamma^\mu t^a$$



$$= g_{\text{s}} f^{abc} [g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu]$$

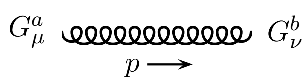


$$= -i g_{\text{s}}^2 [f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})]$$



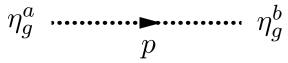
$$= -g_{\text{s}} f^{abc} p^\mu$$

胶子传播子:



$$G_\mu^a \quad G_\nu^b = \frac{-i\delta^{ab}}{p^2 + i\varepsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1-\xi) \right]$$

鬼粒子传播子:



$$\eta_g^a \quad \eta_g^b = \frac{i\delta^{ab}}{p^2 + i\varepsilon}$$

4 费米子电弱规范相互作用拉氏量和费曼规则

表 1 列出标准模型费米子场的量子数电荷数 Q、弱同位旋第 3 分量 T^3、弱超荷 Y、重子数 B 和轻子数 $L_e/L_\mu/L_\tau$。每代左手费米子场构成 2 个 $\mathrm{SU}(2)_\mathrm{L}$ 二重态

$$L_{i\mathrm{L}} = \begin{pmatrix} P_\mathrm{L}\nu_i \\ P_\mathrm{L}\ell_i \end{pmatrix} = \begin{pmatrix} \nu_{i\mathrm{L}} \\ \ell_{i\mathrm{L}} \end{pmatrix}, \quad Q_{i\mathrm{L}} = \begin{pmatrix} P_\mathrm{L}u_i \\ P_\mathrm{L}d'_i \end{pmatrix} = \begin{pmatrix} u_{i\mathrm{L}} \\ d'_{i\mathrm{L}} \end{pmatrix}, \quad i = 1,2,3. \tag{24}$$

下型夸克的质量本征态 d_j 与规范本征态 d'_j 通过 CKM 矩阵 V_{ij} 联系起来：

$$d'_i = V_{ij}d_j. \tag{25}$$

右手费米子场 $\ell_{i\mathrm{R}} = P_\mathrm{R}\ell_i$、$u_{i\mathrm{R}} = P_\mathrm{R}u_i$ 和 $d'_{i\mathrm{R}} = P_\mathrm{R}d'_i$ 是 $\mathrm{SU}(2)_\mathrm{L}$ 单态。它们的电荷数 Q、弱同位旋第 3 分量 T^3 和弱超荷 Y 满足关系

$$Q = T^3 + Y. \tag{26}$$

表 1: 标准模型费米子场的量子数。

统一记号	第一代	第二代	第三代	Q	T^3	Y	B	$L_e/L_\mu/L_\tau$
$L_{i\mathrm{L}} = \begin{pmatrix} \nu_{i\mathrm{L}} \\ \ell_{i\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} \nu_{e\mathrm{L}} \\ e_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$	0 -1	$1/2$ $-1/2$	$-1/2$ $-1/2$	0 0	1 1
$Q_{i\mathrm{L}} = \begin{pmatrix} u_{i\mathrm{L}} \\ d'_{i\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} u_\mathrm{L} \\ d'_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} c_\mathrm{L} \\ s'_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} t_\mathrm{L} \\ b'_\mathrm{L} \end{pmatrix}$	$2/3$ $-1/3$	$1/2$ $-1/2$	$1/6$ $1/6$	$1/3$ $1/3$	0 0
$\ell_{i\mathrm{R}}$	e_R	μ_R	τ_R	-1	0	-1	0	1
$u_{i\mathrm{R}}$	u_R	c_R	t_R	$2/3$	0	$2/3$	$1/3$	0
$d'_{i\mathrm{R}}$	d'_R	s'_R	b'_R	$-1/3$	0	$-1/3$	$1/3$	0

$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 规范不变的费米子协变动能项为

$$\mathcal{L}_\mathrm{EWF} = \bar{Q}_{i\mathrm{L}} i \not{D} Q_{i\mathrm{L}} + \bar{u}_{i\mathrm{R}} i \not{D} u_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} i \not{D} d'_{i\mathrm{R}} + \bar{L}_{i\mathrm{L}} i \not{D} L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}} i \not{D} \ell_{i\mathrm{R}}. \tag{27}$$

对于 $\mathrm{SU}(2)_\mathrm{L}$ 二重态 $Q_{i\mathrm{L}}$ 和 $L_{i\mathrm{L}}$，协变导数为

$$D_\mu = \partial_\mu - ig' B_\mu Y - igW^a_\mu \tau^a, \quad \tau^a = \frac{\sigma^a}{2}. \tag{28}$$

弱同位旋第 3 分量 T^3 是生成元 τ^3 的本征值。对于 $\mathrm{SU}(2)_\mathrm{L}$ 单态 $u_{i\mathrm{R}}$、$d'_{i\mathrm{R}}$ 和 $\ell_{i\mathrm{R}}$，协变导数为

$$D_\mu = \partial_\mu - ig' B_\mu Y. \tag{29}$$

规范场 $W^a_\mu(x)$ 和 $B_\mu(x)$ 跟左手费米子场的相互作用与右手费米子场不同，而在 QED 中，电磁场 $A_\mu(x)$ 跟左手费米子场的相互作用却与右手费米子场相同。为了回到 QED 的情况，需要把 $W^3_\mu(x)$ 和 $B_\mu(x)$ 混

合起来，得到电磁场 $A_\mu(x)$ 和另一个中性规范场 $Z_\mu(x)$，即定义

$$
\begin{aligned}
A_\mu &\equiv \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3+gB_\mu)=s_{\mathrm{W}}W_\mu^3+c_{\mathrm{W}}B_\mu, &(30)\\
Z_\mu &\equiv \frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3-g'B_\mu)=c_{\mathrm{W}}W_\mu^3-s_{\mathrm{W}}B_\mu, &(31)\\
W_\mu^\pm &\equiv \frac{1}{\sqrt{2}}(W_\mu^1\mp iW_\mu^2), &(32)
\end{aligned}
$$

或

$$
\begin{aligned}
B_\mu &= c_{\mathrm{W}}A_\mu-s_{\mathrm{W}}Z_\mu,\quad W_\mu^3=s_{\mathrm{W}}A_\mu+c_{\mathrm{W}}Z_\mu, &(33)\\
W_\mu^1 &= \frac{1}{\sqrt{2}}(W_\mu^++W_\mu^-),\quad W_\mu^2=\frac{i}{\sqrt{2}}(W_\mu^+-W_\mu^-). &(34)
\end{aligned}
$$

参数间有如下关系，

$$
s_{\mathrm{W}}\equiv\sin\theta_{\mathrm{W}}=\frac{g'}{\sqrt{g^2+g'^2}},\quad c_{\mathrm{W}}\equiv\cos\theta_{\mathrm{W}}=\frac{g}{\sqrt{g^2+g'^2}},\quad e\equiv\frac{gg'}{\sqrt{g^2+g'^2}}=gs_{\mathrm{W}}=g'c_{\mathrm{W}}. \tag{35}
$$

这里 θ_{W} 称为 Weinberg 角。

利用

$$
\begin{aligned}
g'YB_\mu+gT^3W_\mu^3 &= g'Y(c_{\mathrm{W}}A_\mu-s_{\mathrm{W}}Z_\mu)+gT^3(s_{\mathrm{W}}A_\mu+c_{\mathrm{W}}Z_\mu)\\
&= e(Y+T^3)A_\mu+\left(gc_{\mathrm{W}}T^3-\frac{gs_{\mathrm{W}}}{c_{\mathrm{W}}}s_{\mathrm{W}}Y\right)Z_\mu=QeA_\mu+\frac{g}{c_{\mathrm{W}}}(T^3c_{\mathrm{W}}^2-Ys_{\mathrm{W}}^2)Z_\mu\\
&= QeA_\mu+\frac{g}{c_{\mathrm{W}}}(T^3-Qs_{\mathrm{W}}^2)Z_\mu,
\end{aligned}\tag{36}
$$

有

$$
\begin{aligned}
D_\mu Q_{i\mathrm{L}} &= (\partial_\mu-ig'B_\mu Y-igW_\mu^a\tau^a)Q_{i\mathrm{L}}=\partial_\mu Q_{i\mathrm{L}}-i\begin{pmatrix} g'YB_\mu+gT^3W_\mu^3 & \frac{1}{2}g(W_\mu^1-iW_\mu^2)\\ \frac{1}{2}g(W_\mu^1+iW_\mu^2) & g'YB_\mu+gT^3W_\mu^3\end{pmatrix}Q_{i\mathrm{L}}\\
&= \partial_\mu Q_{i\mathrm{L}}-i\begin{pmatrix} QeA_\mu+\frac{g}{c_{\mathrm{W}}}(T^3-Qs_{\mathrm{W}}^2)Z_\mu & \frac{1}{\sqrt{2}}gW_\mu^+\\ \frac{1}{\sqrt{2}}gW_\mu^- & QeA_\mu+\frac{g}{c_{\mathrm{W}}}(T^3-Qs_{\mathrm{W}}^2)Z_\mu\end{pmatrix}Q_{i\mathrm{L}}\\
&= \partial_\mu Q_{i\mathrm{L}}-i\begin{pmatrix} \left[QeA_\mu+\frac{g}{c_{\mathrm{W}}}(T^3-Qs_{\mathrm{W}}^2)Z_\mu\right]u_{i\mathrm{L}}+\frac{1}{\sqrt{2}}gW_\mu^+d'_{i\mathrm{L}}\\ \frac{1}{\sqrt{2}}gW_\mu^-u_{i\mathrm{L}}+\left[QeA_\mu+\frac{g}{c_{\mathrm{W}}}(T^3-Qs_{\mathrm{W}}^2)\right]d'_{i\mathrm{L}}\end{pmatrix},
\end{aligned}\tag{37}
$$

故

$$
\begin{aligned}
\bar{Q}_{i\mathrm{L}}i\not{D}Q_{i\mathrm{L}} \supset{}& \left[QeA_\mu+\frac{g}{c_{\mathrm{W}}}(T^3-Qs_{\mathrm{W}}^2)Z_\mu\right]\bar{u}_{i\mathrm{L}}\gamma^\mu u_{i\mathrm{L}}+\left[QeA_\mu+\frac{g}{c_{\mathrm{W}}}(T^3-Qs_{\mathrm{W}}^2)Z_\mu\right]\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}}\\
&+\frac{1}{\sqrt{2}}gW_\mu^+\bar{u}_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}}+\frac{1}{\sqrt{2}}gW_\mu^-\bar{d}'_{i\mathrm{L}}\gamma^\mu u_{i\mathrm{L}}
\end{aligned}
$$

$$
\begin{aligned}
&= \left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{L} Z_\mu\right) \bar{u}_i \gamma^\mu \frac{1-\gamma_5}{2} u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{L} Z_\mu\right) \bar{d}_i \gamma^\mu \frac{1-\gamma_5}{2} d_i \\
&\quad + \frac{1}{\sqrt{2}} g W_\mu^+ \bar{u}_i \gamma^\mu \frac{1-\gamma_5}{2} V_{ij} d_j + \frac{1}{\sqrt{2}} g W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu \frac{1-\gamma_5}{2} u_i, \qquad (38)
\end{aligned}
$$

其中

$$
g_\mathrm{L} \equiv T^3 - Q s_\mathrm{W}^2. \qquad (39)
$$

另一方面,

$$
D_\mu d'_{i\mathrm{R}} = (\partial_\mu - i g' B_\mu Y) d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} - i g' Q (c_\mathrm{W} A_\mu - s_\mathrm{W} Z_\mu) d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} - iQeA_\mu d'_{i\mathrm{R}} + i\frac{g}{c_\mathrm{W}} Q s_\mathrm{W}^2 Z_\mu d'_{i\mathrm{R}}, \qquad (40)
$$

则

$$
\begin{aligned}
\bar{u}_{i\mathrm{R}} i \not{D} u_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} i \not{D} d'_{i\mathrm{R}} &\supset \left(QeA_\mu - \frac{g}{c_\mathrm{W}} Q s_\mathrm{W}^2 Z_\mu\right) \bar{u}_{i\mathrm{R}} \gamma^\mu u_{i\mathrm{R}} + \left(QeA_\mu - \frac{g}{c_\mathrm{W}} Q s_\mathrm{W}^2 Z_\mu\right) \bar{d}'_{i\mathrm{R}} \gamma^\mu d'_{i\mathrm{R}} \\
&= \left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{R} Z_\mu\right) \bar{u}_i \gamma^\mu \frac{1+\gamma_5}{2} u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}} g_\mathrm{R} Z_\mu\right) \bar{d}_i \gamma^\mu \frac{1+\gamma_5}{2} d_i, \\
&\qquad (41)
\end{aligned}
$$

其中

$$
g_\mathrm{R} \equiv -Q s_\mathrm{W}^2. \qquad (42)
$$

定义

$$
g_\mathrm{V} \equiv g_\mathrm{L} + g_\mathrm{R} = T^3 - 2Q s_\mathrm{W}^2, \quad g_\mathrm{A} \equiv g_\mathrm{L} - g_\mathrm{R} = T^3, \qquad (43)
$$

可得

$$
\begin{aligned}
&\bar{Q}_{i\mathrm{L}} i \not{D} Q_{i\mathrm{L}} + \bar{u}_{i\mathrm{R}} i \not{D} u_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} i \not{D} d'_{i\mathrm{R}} \\
&\supset Qe\bar{u}_i \gamma^\mu u_i A_\mu + Qe\bar{d}\gamma^\mu d_i A_\mu + \frac{g}{2c_\mathrm{W}} \bar{u}_i \gamma^\mu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) u_i Z_\mu + \frac{g}{2c_\mathrm{W}} \bar{d}_i \gamma^\mu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) d_i Z_\mu \\
&\quad + \frac{1}{\sqrt{2}} g W_\mu^+ \bar{u}_i \gamma^\mu P_\mathrm{L} V_{ij} d_j + \frac{1}{\sqrt{2}} g W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_\mathrm{L} u_i. \qquad (44)
\end{aligned}
$$

同理，有

$$
\begin{aligned}
\bar{L}_{i\mathrm{L}} i \not{D} L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}} i \not{D} \ell_{i\mathrm{R}} &\supset Qe\bar{\ell}_i \gamma^\mu \ell_i A_\mu + \frac{g}{2c_\mathrm{W}} \bar{\ell}_i \gamma^\mu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) \ell_i Z_\mu + \frac{g}{2c_\mathrm{W}} \bar{\nu}_i \gamma^\mu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) \nu_i Z_\mu \\
&\quad + \frac{1}{\sqrt{2}} g W_\mu^+ \bar{\nu}_i \gamma^\mu P_\mathrm{L} \ell_i + \frac{1}{\sqrt{2}} g W_\mu^- \bar{\ell}_i \gamma^\mu P_\mathrm{L} \nu_i. \qquad (45)
\end{aligned}
$$

总结起来，可以写成流耦合的形式,

$$
\begin{aligned}
\mathcal{L}_\mathrm{EWF} &\supset \sum_f \left[Q_f e \bar{f} \gamma^\mu f A_\mu + \frac{g}{2c_\mathrm{W}} \bar{f} \gamma^\mu (g_\mathrm{V}^f - g_\mathrm{A}^f \gamma_5) f Z_\mu\right] + g(W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}) \\
&= eA_\mu J_\mathrm{EM}^\mu + g(Z_\mu J_Z^\mu + W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}), \qquad (46)
\end{aligned}
$$

其中，流的定义为

$$J^\mu_{\rm EM} \equiv \sum_f Q_f \bar f \gamma^\mu f, \quad J^\mu_Z \equiv \frac{1}{2c_{\rm W}} \sum_f \bar f \gamma^\mu (g^f_{\rm V} - g^f_{\rm A}\gamma_5) f = \frac{1}{c_{\rm W}} \sum_f (g^f_{\rm L} \bar f_{\rm L} \gamma^\mu f_{\rm L} + g^f_{\rm R} \bar f_{\rm R} \gamma^\mu f_{\rm R}),$$

$$J^{+\mu}_W \equiv \frac{1}{\sqrt 2}(\bar u_{i\rm L}\gamma^\mu V_{ij} d_{j\rm L} + \bar\nu_{i\rm L}\gamma^\mu \ell_{i\rm L}), \quad J^{-\mu}_W \equiv \frac{1}{\sqrt 2}(\bar d_{j\rm L} V^\dagger_{ji} \gamma^\mu u_{i\rm L} + \bar\ell_{i\rm L}\gamma^\mu \nu_{i\rm L}). \tag{47}$$

对于各种费米子，相关系数如下，

$$Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}, \quad Q_{\nu_i} = 0, \quad Q_{\ell_i} = -1; \tag{48}$$

$$g^{u_i}_{\rm V} = \frac12 - \frac43 s^2_{\rm W}, \quad g^{u_i}_{\rm A} = \frac12; \quad g^{d_i}_{\rm V} = -\frac12 + \frac23 s^2_{\rm W}, \quad g^{d_i}_{\rm A} = -\frac12; \tag{49}$$

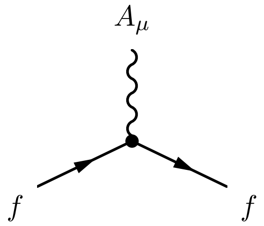
$$g^{\nu_i}_{\rm V} = \frac12, \quad g^{\nu_i}_{\rm A} = \frac12; \quad g^{\ell_i}_{\rm V} = -\frac12 + 2s^2_{\rm W}, \quad g^{\ell_i}_{\rm A} = -\frac12; \tag{50}$$

$$g^{u_i}_{\rm L} = \frac12 - \frac23 s^2_{\rm W}, \quad g^{u_i}_{\rm R} = -\frac23 s^2_{\rm W}; \quad g^{d_i}_{\rm L} = -\frac12 + \frac13 s^2_{\rm W}, \quad g^{d_i}_{\rm R} = \frac13 s^2_{\rm W}; \tag{51}$$

$$g^{\nu_i}_{\rm L} = \frac12, \quad g^{\nu_i}_{\rm R} = 0; \quad g^{\ell_i}_{\rm L} = -\frac12 + s^2_{\rm W}, \quad g^{\ell_i}_{\rm R} = s^2_{\rm W}. \tag{52}$$

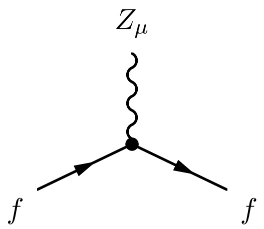
下面给出费米子电弱规范相互作用顶点的费曼规则。

QED 顶点:



$= iQ_f e\gamma^\mu$ （对于电子，$Q_e = -1$）

费米子与 Z 玻色子的耦合:

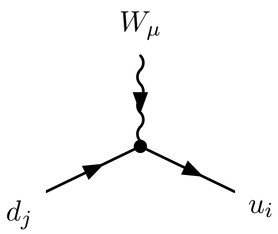


$$= i\frac{g}{2c_{\rm W}}\gamma^\mu(g^f_{\rm V} - g^f_{\rm A}\gamma_5)$$

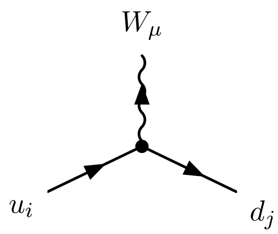
$$g^{u_i}_{\rm V} = \frac12 - \frac43 s^2_{\rm W}, \quad g^{u_i}_{\rm A} = \frac12; \quad g^{d_i}_{\rm V} = -\frac12 + \frac23 s^2_{\rm W}, \quad g^{d_i}_{\rm A} = -\frac12;$$

$$g^{\nu_i}_{\rm V} = \frac12, \quad g^{\nu_i}_{\rm A} = \frac12; \quad g^{\ell_i}_{\rm V} = -\frac12 + 2s^2_{\rm W}, \quad g^{\ell_i}_{\rm A} = -\frac12.$$

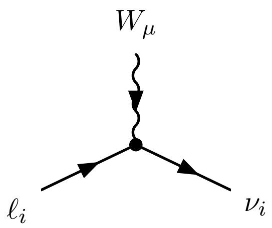
费米子与 $W^\pm$ 玻色子的耦合:



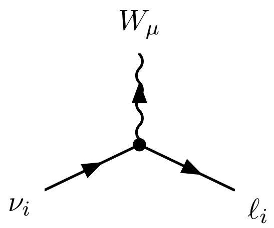
$$= i\frac{g}{\sqrt2}V_{ij}\gamma^\mu P_{\rm L}$$



$$= i\frac{g}{\sqrt2}V^\dagger_{ji}\gamma^\mu P_{\rm L}$$



$$= i\frac{g}{\sqrt{2}}\gamma^\mu P_\mathrm{L}$$



$$= i\frac{g}{\sqrt{2}}\gamma^\mu P_\mathrm{L}$$

5 电弱规范场自相互作用拉氏量和费曼规则

电弱规范场自相互作用拉氏量是

$$\mathcal{L}_\mathrm{EWG} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{53}$$

其中

$$W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g\varepsilon^{abc}W^{b\mu}W^{c\nu}, \quad B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu. \tag{54}$$

利用 (33) 式和 (34) 式，可得

$$\begin{aligned}
& W^2_\mu W^3_\nu - W^3_\mu W^2_\nu \\
= \ & \frac{i}{\sqrt{2}}[(W^+_\mu - W^-_\mu)(s_\mathrm{W}A_\nu + c_\mathrm{W}Z_\nu) - (s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)(W^+_\nu - W^-_\nu)] \\
= \ & \frac{i}{\sqrt{2}}[s_\mathrm{W}(W^+_\mu A_\nu - A_\mu W^+_\nu) + c_\mathrm{W}(W^+_\mu Z_\nu - Z_\mu W^+_\nu) - s_\mathrm{W}(W^-_\mu A_\nu - A_\mu W^-_\nu) - c_\mathrm{W}(W^-_\mu Z_\nu - Z_\mu W^-_\nu)],
\end{aligned} \tag{55}$$

$$\begin{aligned}
& W^3_\mu W^1_\nu - W^1_\mu W^3_\nu \\
= \ & \frac{1}{\sqrt{2}}[(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)(W^+_\nu + W^-_\nu) - (W^+_\mu + W^-_\mu)(s_\mathrm{W}A_\nu + c_\mathrm{W}Z_\nu)] \\
= \ & -\frac{1}{\sqrt{2}}[s_\mathrm{W}(W^+_\mu A_\nu - A_\mu W^+_\nu) + c_\mathrm{W}(W^+_\mu Z_\nu - Z_\mu W^+_\nu) + s_\mathrm{W}(W^-_\mu A_\nu - A_\mu W^-_\nu) + c_\mathrm{W}(W^-_\mu Z_\nu - Z_\mu W^-_\nu)].
\end{aligned} \tag{56}$$

从而，

$$\begin{aligned}
W^1_{\mu\nu} &= \partial_\mu W^1_\nu - \partial_\nu W^1_\mu + g\varepsilon^{1bc}W^b_\mu W^c_\nu = \partial_\mu W^1_\nu - \partial_\nu W^1_\mu + gW^2_\mu W^3_\nu - gW^3_\mu W^2_\nu \\
&= \frac{1}{\sqrt{2}}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) + \frac{1}{\sqrt{2}}(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) + g(W^2_\mu W^3_\nu - gW^3_\mu W^2_\nu) \\
&= \frac{1}{\sqrt{2}}\{\partial_\mu W^+_\nu - \partial_\nu W^+_\mu + ig[s_\mathrm{W}(W^+_\mu A_\nu - A_\mu W^+_\nu) + c_\mathrm{W}(W^+_\mu Z_\nu - Z_\mu W^+_\nu)]\} \\
&\quad + \frac{1}{\sqrt{2}}\{\partial_\mu W^-_\nu - \partial_\nu W^-_\mu - ig[s_\mathrm{W}(W^-_\mu A_\nu - A_\mu W^-_\nu) + c_\mathrm{W}(W^-_\mu Z_\nu - Z_\mu W^-_\nu)]\} \\
&= \frac{1}{\sqrt{2}}(F^+_{\mu\nu} + F^-_{\mu\nu}),
\end{aligned} \tag{57}$$

其中，

$$F^+_{\mu\nu} \equiv \partial_\mu W^+_\nu - \partial_\nu W^+_\mu + ie(W^+_\mu A_\nu - A_\mu W^+_\nu) + igc_\mathrm{W}(W^+_\mu Z_\nu - Z_\mu W^+_\nu), \tag{58}$$

$$F^-_{\mu\nu} \equiv \partial_\mu W^-_\nu - \partial_\nu W^-_\mu - ie(W^-_\mu A_\nu - A_\mu W^-_\nu) - igc_\mathrm{W}(W^-_\mu Z_\nu - Z_\mu W^-_\nu). \tag{59}$$

另一方面,

$$\begin{aligned}
W^2_{\mu\nu} &= \partial_\mu W^2_\nu - \partial_\nu W^2_\mu + g\varepsilon^{2bc}W^b_\mu W^c_\nu = \partial_\mu W^2_\nu - \partial_\nu W^2_\mu - gW^1_\mu W^3_\nu + gW^3_\mu W^1_\nu \\
&= \frac{i}{\sqrt{2}}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) - \frac{i}{\sqrt{2}}(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) + g(W^3_\mu W^1_\nu - W^1_\mu W^3_\nu) \\
&= \frac{i}{\sqrt{2}}\{\partial_\mu W^+_\nu - \partial_\nu W^+_\mu + ig[s_{\mathrm{W}}(W^+_\mu A_\nu - A_\mu W^+_\nu) + c_{\mathrm{W}}(W^+_\mu Z_\nu - Z_\mu W^+_\nu)]\} \\
&\quad -\frac{i}{\sqrt{2}}\{\partial_\mu W^-_\nu - \partial_\nu W^-_\mu - ig[s_{\mathrm{W}}(W^-_\mu A_\nu - A_\mu W^-_\nu) + c_{\mathrm{W}}(W^-_\mu Z_\nu - Z_\mu W^-_\nu)]\} \\
&= \frac{i}{\sqrt{2}}(F^+_{\mu\nu} - F^-_{\mu\nu}).
\end{aligned} \tag{60}$$

因此,

$$-\frac{1}{4}W^1_{\mu\nu}W^{1\mu\nu} - \frac{1}{4}W^2_{\mu\nu}W^{2\mu\nu} \tag{61}$$

$$\begin{aligned}
&= -\frac{1}{8}(F^+_{\mu\nu} + F^-_{\mu\nu})(F^{+\mu\nu} + F^{-\mu\nu}) + \frac{1}{8}(F^+_{\mu\nu} - F^-_{\mu\nu})(F^{+\mu\nu} - F^{-\mu\nu}) = -\frac{1}{2}F^+_{\mu\nu}F^{-\mu\nu} \\
&= -\frac{1}{2}[\partial_\mu W^+_\nu - \partial_\nu W^+_\mu + ie(W^+_\mu A_\nu - A_\mu W^+_\nu) + igc_{\mathrm{W}}(W^+_\mu Z_\nu - Z_\mu W^+_\nu)] \\
&\quad \times[\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu} - ie(W^{-\mu}A^\nu - A^\mu W^{-\nu}) - igc_{\mathrm{W}}(W^{-\mu}Z^\nu - Z^\mu W^{-\nu})] \\
&= -(\partial_\mu W^+_\nu)(\partial^\mu W^{-\nu}) + (\partial_\mu W^+_\nu)(\partial^\nu W^{-\mu}) \\
&\quad +ie[(\partial_\mu W^+_\nu)W^{-\mu}A^\nu - (\partial_\mu W^+_\nu)W^{-\nu}A^\mu - W^+_\mu(\partial^\mu W^{-\nu})A_\nu + W^+_\nu(\partial^\mu W^{-\nu})A_\mu] \\
&\quad +igc_{\mathrm{W}}[(\partial_\mu W^+_\nu)W^{-\mu}Z^\nu - (\partial_\mu W^+_\nu)W^{-\nu}Z^\mu - W^+_\mu(\partial^\mu W^{-\nu})Z_\nu + W^+_\nu(\partial^\mu W^{-\nu})Z_\mu] \\
&\quad +e^2(W^+_\mu W^{-\nu}A_\nu A^\mu - W^+_\mu W^{-\mu}A_\nu A^\nu) + g^2c^2_{\mathrm{W}}(W^+_\mu W^{-\nu}Z_\nu Z^\mu - W^+_\mu W^{-\mu}Z_\nu Z^\nu) \\
&\quad +egc_{\mathrm{W}}(W^+_\mu W^{-\nu}A_\nu Z^\mu + W^+_\mu W^{-\nu}A^\mu Z_\nu - 2W^+_\mu W^{-\mu}A_\nu Z^\nu).
\end{aligned} \tag{62}$$

由

$$W^1_\mu W^2_\nu - W^2_\mu W^1_\nu = \frac{i}{2}(W^+_\mu + W^-_\mu)(W^+_\nu - W^-_\nu) - \frac{i}{2}(W^+_\mu - W^-_\mu)(W^+_\nu + W^-_\nu) = -i(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu), \tag{63}$$

可得

$$\begin{aligned}
W^3_{\mu\nu} &= \partial_\mu W^3_\nu - \partial_\nu W^3_\mu + g\varepsilon^{3bc}W^b_\mu W^c_\nu = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu + gW^1_\mu W^2_\nu - gW^2_\mu W^1_\nu \\
&= s_{\mathrm{W}}\partial_\mu A_\nu + c_{\mathrm{W}}\partial_\mu Z_\nu - s_{\mathrm{W}}\partial_\nu A_\mu + c_{\mathrm{W}}\partial_\nu Z_\mu + g(W^1_\mu W^2_\nu - W^2_\mu W^1_\nu) \\
&= s_{\mathrm{W}}(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_{\mathrm{W}}(\partial_\mu Z_\nu - \partial_\nu Z_\mu) - ig(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu),
\end{aligned} \tag{64}$$

$$B_{\mu\nu} = \partial_\mu(c_{\mathrm{W}}A_\nu - s_{\mathrm{W}}Z_\nu) - \partial_\nu(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) = c_{\mathrm{W}}(\partial_\mu A_\nu - \partial_\nu A_\mu) - s_{\mathrm{W}}(\partial_\mu Z_\nu - \partial_\nu Z_\mu). \tag{65}$$

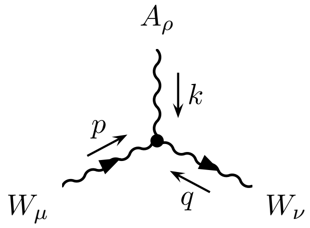
于是,

$$\begin{aligned}
&-\frac{1}{4}W^3_{\mu\nu}W^{3\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&= -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
&\quad +ie[W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] + igc_{\mathrm{W}}[W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)] \\
&\quad +\frac{1}{2}g^2(W^+_\mu W^{+\mu}W^-_\nu W^{-\nu} - W^+_\mu W^{+\nu}W^-_\nu W^{-\mu}).
\end{aligned} \tag{66}$$

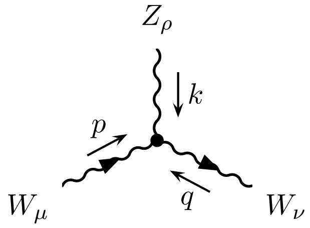
综合起来，有

$$\begin{aligned}
\mathcal{L}_{\mathrm{EWG}} = \ & \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\
& + (\partial_\mu W^+_\nu)(\partial^\nu W^{-\mu}) - (\partial_\mu W^+_\nu)(\partial^\mu W^{-\nu}) + \frac{1}{2} g^2 (W^+_\mu W^{+\mu} W^-_\nu W^{-\nu} - W^+_\mu W^{+\nu} W^-_\nu W^{-\mu}) \\
& + ie[(\partial_\mu W^+_\nu) W^{-\mu} A^\nu - (\partial_\mu W^+_\nu) W^{-\nu} A^\mu - W^{+\mu}(\partial_\mu W^-_\nu) A^\nu + W^{+\nu}(\partial_\mu W^-_\nu) A^\mu \\
& \qquad + W^{+\mu} W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu} W^{-\mu}(\partial_\mu A_\nu)] + e^2 (W^+_\mu W^{-\nu} A_\nu A^\mu - W^+_\mu W^{-\mu} A_\nu A^\nu) \\
& + igc_{\mathrm{W}}[(\partial_\mu W^+_\nu) W^{-\mu} Z^\nu - (\partial_\mu W^+_\nu) W^{-\nu} Z^\mu - W^{+\mu}(\partial_\mu W^-_\nu) Z^\nu + W^{+\nu}(\partial_\mu W^-_\nu) Z^\mu \\
& \qquad\quad + W^{+\mu} W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu} W^{-\mu}(\partial_\mu Z_\nu)] + g^2 c^2_{\mathrm{W}} (W^+_\mu W^{-\nu} Z_\nu Z^\mu - W^+_\mu W^{-\mu} Z_\nu Z^\nu) \\
& + egc_{\mathrm{W}} (W^+_\mu W^{-\nu} A_\nu Z^\mu + W^+_\mu W^{-\nu} A^\mu Z_\nu - 2 W^+_\mu W^{-\mu} A_\nu Z^\nu).
\end{aligned} \tag{67}$$

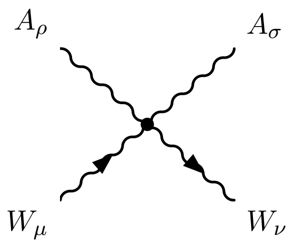
下面是电弱规范玻色子自耦合的费曼规则：



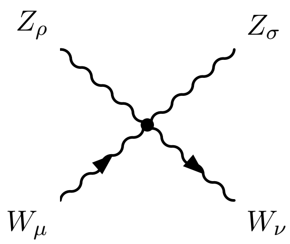
$$= -ie[(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu]$$



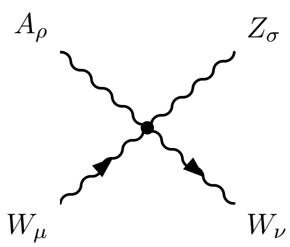
$$= -igc_{\mathrm{W}}[(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu]$$



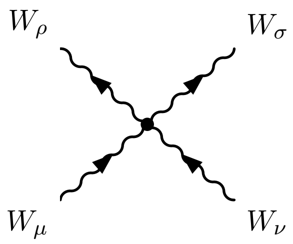
$$= ie^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= ig^2c^2_{\mathrm{W}}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= iegc_{\mathrm{W}}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= -ig^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$

6 幺正规范下 Higgs 场相关拉氏量和费曼规则

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_{\mathrm{H}} = (D^\mu\Phi)^\dagger(D_\mu\Phi) - V_{\mathrm{H}}(\Phi), \quad V_{\mathrm{H}}(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \tag{68}$$

其中

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \quad D_\mu\Phi = (\partial_\mu - ig'B_\mu Y_H - igW_\mu^a\tau^a)\Phi, \quad Y_H = \frac{1}{2}. \tag{69}$$

当 $\lambda > 0$ 且 $\mu^2 > 0$ 时，Higgs 场势能 $V_{\mathrm{H}}(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式，势能最小值位于方程

$$\Phi^\dagger\Phi = [\mathrm{Re}(\phi^+)]^2 + [\mathrm{Im}(\phi^+)]^2 + [\mathrm{Re}(\phi^0)]^2 + [\mathrm{Im}(\phi^0)]^2 = \frac{v^2}{2} \tag{70}$$

对应的 4 维球面上，其中 $v \equiv \sqrt{\mu^2/\lambda}$。

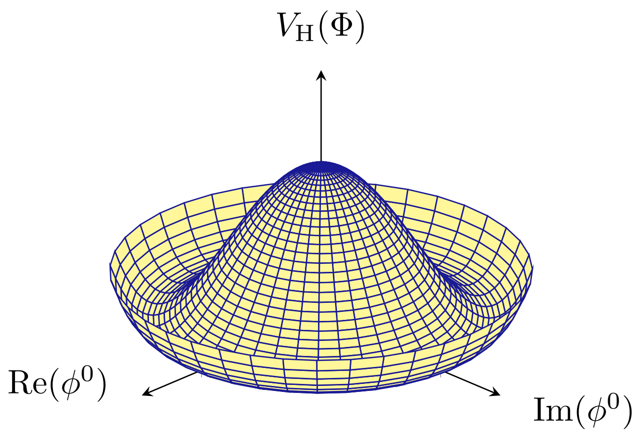


图 1: Higgs 场势能示意图。这里压缩掉 $\mathrm{Re}(\phi^+)$ 和 $\mathrm{Im}(\phi^+)$ 两个维度。

Higgs 场的真空期待值位于这个 4 维球面上的某一点，不失一般性，可将它取为

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{71}$$

其它真空期待值可通过整体规范变换

$$\langle\Phi\rangle \to \exp(i\alpha^a\tau^a)\exp(i\alpha^Y Y_H)\langle\Phi\rangle \tag{72}$$

得到，因为 $\langle\Phi^\dagger\Phi\rangle$ 在这样的变换下保持不变。若 $\alpha^1 = \alpha^2 = 0$ 且 $\alpha^3 = \alpha^Y$，则 $\langle\Phi\rangle$ 在变换下不变。因此，有 1 个方向的规范对称性没有受到破坏，只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定

理，破缺后生成 3 个无质量的 Nambu-Goldstone 玻色子。最终，有 3 个规范玻色子自由度通过 Brout–Englert–Higgs 机制获得质量。

以 $\langle\Phi\rangle$ 为基础，将 Higgs 场一般地参数化为

$$\Phi(x) = \exp\left[-i\frac{\chi^a(x)}{v}\tau^a\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix}, \tag{73}$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场。$\exp[-i\chi^a(x)\tau^a/v]$ 因子能够通过 $\mathrm{SU}(2)_\mathrm{L}$ 定域规范变换消去，因而可将 $\Phi(x)$ 直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix}, \quad \Phi^\dagger\Phi = \frac{1}{2}(v+H)^2. \tag{74}$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$，对应于 Higgs 玻色子，这种取法称为幺正规范。

在幺正规范下，势能项化为

$$\begin{aligned} -V_\mathrm{H}(\Phi) &= \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 = \frac{1}{2}\mu^2(v+H)^2 - \frac{1}{4}\lambda(v+H)^4 \\ &= \frac{1}{2}\mu^2(v^2+H^2+2vH) - \frac{1}{4}\lambda(v^4+4v^2H^2+H^4+4v^3H+2v^2H^2+4vH^3) \\ &= \frac{1}{4}\mu^2v^2 + \frac{1}{4}(\mu^2-\lambda v^2)v^2 + (\mu^2-\lambda v^2)vH + \frac{1}{2}(\mu^2-\lambda v^2)H^2 - \lambda v^2H^2 - \lambda vH^3 - \frac{1}{4}\lambda H^4 \\ &= \frac{1}{8}m_H^2v^2 - \frac{1}{2}m_H^2H^2 - \frac{1}{2}\frac{m_H^2}{v}H^3 - \frac{1}{8}\frac{m_H^2}{v^2}H^4, \end{aligned} \tag{75}$$

其中 Higgs 玻色子的质量为

$$m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \tag{76}$$

利用

$$\begin{aligned} g'B_\mu + gW_\mu^3 &= g'(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) + g(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu) = 2eA_\mu + \frac{g^2-g'^2}{\sqrt{g^2+g'^2}}Z_\mu \\ &= 2eA_\mu + \frac{g}{c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu, \end{aligned} \tag{77}$$

有

$$\begin{aligned} g'B_\mu Y_H + gW_\mu^a\tau^a &= \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix} \\ &= \begin{pmatrix} eA_\mu + \dfrac{g}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & \dfrac{1}{\sqrt{2}}gW_\mu^+ \\ \dfrac{1}{\sqrt{2}}gW_\mu^- & -\dfrac{g}{2c_\mathrm{W}}Z_\mu \end{pmatrix}. \end{aligned} \tag{78}$$

于是，在幺正规范下，

$$\begin{aligned} &(D^\mu\Phi)^\dagger(D_\mu\Phi) \\ &= \left|\begin{pmatrix} \partial_\mu - ieA_\mu - \dfrac{ig}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & -\dfrac{i}{\sqrt{2}}gW_\mu^+ \\ -\dfrac{i}{\sqrt{2}}gW_\mu^- & \partial_\mu + \dfrac{ig}{2c_\mathrm{W}}Z_\mu \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}\right|^2 \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}\left(\frac{i}{\sqrt{2}}gW_\mu^-(v+H),\ \partial_\mu H - \frac{ig}{2c_\mathrm{W}}Z_\mu(v+H)\right)\begin{pmatrix} -\dfrac{i}{\sqrt{2}}gW_\mu^+(v+H) \\ \partial_\mu H + \dfrac{ig}{2c_\mathrm{W}}Z_\mu(v+H)\end{pmatrix} \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2\left(\frac{g^2}{4}W_\mu^+W^{-\mu} + \frac{g^2}{8c_\mathrm{W}^2}Z_\mu Z^\mu\right) \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\
&\quad + g m_W H W_\mu^+ W^{-\mu} + \frac{g m_Z}{2c_\mathrm{W}} H Z_\mu Z^\mu + \frac{g^2}{4}H^2 W_\mu^+ W^{-\mu} + \frac{g^2}{8c_\mathrm{W}^2}H^2 Z_\mu Z^\mu.
\end{aligned} \tag{79}
$$

故 $W^\pm$ 和 Z 玻色子获得质量，分别为

$$
m_W \equiv \frac{gv}{2},\quad m_Z \equiv \frac{gv}{2c_\mathrm{W}} = \frac{m_W}{c_\mathrm{W}} = \frac{v}{2}\sqrt{g^2+g'^2}. \tag{80}
$$

$Y=-1/2$ 的 Higgs 场共轭态为

$$
\tilde{\Phi}(x) = i\sigma^2\Phi^*(x) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix}\phi^-(x) \\ \phi^{0*}(x)\end{pmatrix} = \begin{pmatrix}\phi^{0*}(x) \\ -\phi^-(x)\end{pmatrix} = \begin{pmatrix}\dfrac{1}{\sqrt{2}}[v+H(x)-i\chi(x)] \\ -\phi^-(x)\end{pmatrix}. \tag{81}
$$

利用它可以写下 Yukawa 耦合项

$$
\begin{aligned}
\mathcal{L}_\mathrm{Y} &= -\tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - y_{u_i}\bar{Q}_{i\mathrm{L}}u_{i\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + h.c. \\
&= -\frac{1}{\sqrt{2}}(v+H)\bar{d}'_{l\mathrm{L}}V_{li}^\dagger\tilde{y}_d^{ij}V_{jk}d'_{k\mathrm{R}} - \frac{y_{u_i}}{\sqrt{2}}(v+H)\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} - \frac{y_{\ell_i}}{\sqrt{2}}(v+H)\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + h.c. \\
&= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i.
\end{aligned} \tag{82}
$$

这里 CKM 矩阵将 $\tilde{y}_d^{ij}$ 对角化：

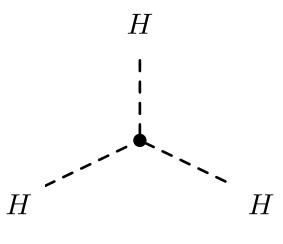
$$
V_{li}^\dagger\tilde{y}_d^{ij}V_{jk} = y_{d_k}\delta_{lk}. \tag{83}
$$

通过 Yukawa 耦合，费米子获得了质量，

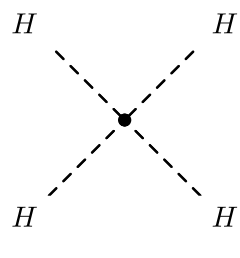
$$
m_{d_i} \equiv \frac{1}{\sqrt{2}}y_{d_i}v,\quad m_{u_i} \equiv \frac{1}{\sqrt{2}}y_{u_i}v,\quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}}y_{\ell_i}v. \tag{84}
$$

下面给出幺正规范下的顶点费曼规则。

Higgs 玻色子自耦合：

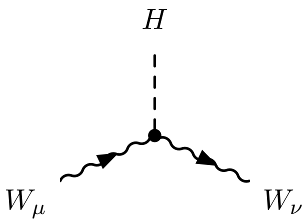


$$
\begin{aligned}
&= -3i\frac{m_H^2}{v} \\
&= -6i\lambda v
\end{aligned}
$$



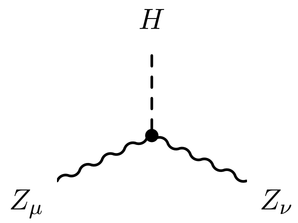
$$
\begin{aligned}
&= -3i\frac{m_H^2}{v^2} \\
&= -6i\lambda
\end{aligned}
$$

Higgs 玻色子与电弱规范玻色子的耦合：



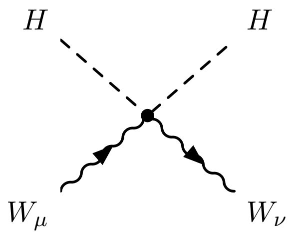
$$= igm_W g^{\mu\nu}$$

$$= i\frac{g^2 v}{2} g^{\mu\nu}$$

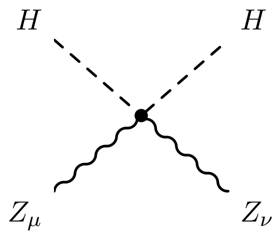


$$= i\frac{gm_Z}{c_{\mathrm{W}}} g^{\mu\nu}$$

$$= i\frac{g^2 v}{2c_{\mathrm{W}}^2} g^{\mu\nu}$$

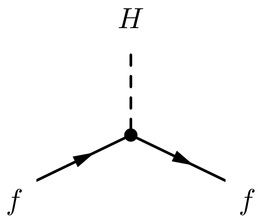


$$= i\frac{g^2}{2} g^{\mu\nu}$$



$$= i\frac{g^2}{2c_{\mathrm{W}}^2} g^{\mu\nu}$$

Higgs 玻色子与费米子的耦合:



$$= -i\frac{m_f}{v}$$

$$= -i\frac{y_f}{\sqrt{2}}$$

7 R_ξ 规范相关拉氏量和费曼规则

将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + i\chi(x)] \end{pmatrix}, \tag{85}$$

其中 ϕ^+ 和 χ 是 Nambu-Goldstone 标量场。由

$$\begin{aligned} \Phi^\dagger\Phi &= \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \\ (\Phi^\dagger\Phi)^2 &= \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \end{aligned} \tag{86}$$

可得 Higgs 场势能项

$$\begin{aligned} & -V_{\mathrm{H}}(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \\ = & \frac{1}{2}\mu^2(v^2 + H^2 + 2vH + \chi^2) + \mu^2|\phi^+|^2 - \frac{1}{4}\lambda(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda|\phi^+|^4 \\ & -\lambda|\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\ = & \frac{1}{2}\left(\mu^2 - \frac{1}{2}\lambda v^2\right)v^2 + \frac{1}{2}(\mu^2 - 3\lambda v^2)H^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)\chi^2 - \frac{1}{4}\lambda H^4 - \frac{1}{4}\lambda\chi^4 - \lambda v H^3 \\ & -\frac{1}{2}\lambda H^2\chi^2 - \lambda v H\chi^2 + (\mu^2 - \lambda v^2)|\phi^+|^2 - \lambda|\phi^+|^4 - \lambda|\phi^+|^2(H^2 + 2vH + \chi^2) \\ = & \frac{1}{4}\lambda v^4 - \lambda v^2 H^2 - \frac{1}{4}\lambda H^4 - \frac{1}{4}\lambda\chi^4 - \lambda v H^3 - \frac{1}{2}\lambda H^2\chi^2 - \lambda v H\chi^2 - \lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2) \\ = & \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4 - \frac{m_H^2}{2v}H\chi^2 - \frac{m_H^2}{4v^2}H^2\chi^2 - \frac{m_H^2}{8v^2}\chi^4 \\ & -\frac{m_H^2}{2v^2}\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2). \end{aligned} \tag{87}$$

由于

$$V_{li}^{\dagger}\tilde{y}_d^{ij}V_{jk} = y_{d_k}\delta_{lk}, \quad \tilde{y}_d^{ij} = V_{ik}y_{d_k}V_{kj}^{\dagger}, \tag{88}$$

有

$$\begin{aligned}
-\tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi &= -\tilde{y}_d^{ij}\left[\bar{u}_{i\mathrm{L}}d'_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}}(v+H+i\chi)\right] \\
&= -\left[\bar{u}_{i\mathrm{L}}V_{ik}y_{d_k}V_{kj}^{\dagger}V_{jl}d_{l\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\bar{d}_{l\mathrm{L}}V_{li}^{\dagger}\tilde{y}_d^{ij}V_{jk}d_{k\mathrm{R}}(v+H+i\chi)\right] \\
&= -\left[y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v+H+i\chi)\right],
\end{aligned} \tag{89}$$

则 Yukawa 耦合项为

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}} &= -\tilde{y}_d^{ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - y_{u_i}\bar{Q}_{i\mathrm{L}}u_{i\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{h.c.} \\
&= -\left[y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v+H+i\chi)\right] - y_{u_i}\left[\frac{1}{\sqrt{2}}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(v+H-i\chi) - \bar{d}_{j\mathrm{L}}V_{ji}^{\dagger}u_{i\mathrm{R}}\phi^-\right] \\
&\quad -y_{\ell_i}\left[\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(v+H+i\chi)\right] + \mathrm{h.c.} \\
&= -m_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} - m_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} - m_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} - \frac{m_{d_i}}{v}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(H+i\chi) - \frac{m_{u_i}}{v}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(H-i\chi) \\
&\quad -\frac{m_{\ell_i}}{v}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(H+i\chi) - \frac{\sqrt{2}m_{d_j}}{v}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{\sqrt{2}m_{u_i}}{v}\bar{d}_{j\mathrm{L}}V_{ji}^{\dagger}u_{i\mathrm{R}}\phi^- - \frac{\sqrt{2}m_{\ell_i}}{v}\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ + \mathrm{h.c.} \\
&= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i \\
&\quad -\frac{m_{d_i}}{v}\chi\bar{d}_i i\gamma_5 d_i + \frac{m_{u_i}}{v}\chi\bar{u}_i i\gamma_5 u_i - \frac{m_{\ell_i}}{v}\chi\bar{\ell}_i i\gamma_5\ell_i + \frac{\sqrt{2}V_{ij}}{v}\phi^+\bar{u}_i(m_{u_i}P_{\mathrm{L}} - m_{d_j}P_{\mathrm{R}})d_j \\
&\quad -\frac{\sqrt{2}V_{ji}^{\dagger}}{v}\phi^-\bar{d}_j(m_{d_j}P_{\mathrm{L}} - m_{u_i}P_{\mathrm{R}})u_i - \frac{\sqrt{2}m_{\ell_i}}{v}(\phi^+\bar{\nu}_i P_{\mathrm{R}}\ell_i + \phi^-\bar{\ell}_i P_{\mathrm{L}}\nu_i).
\end{aligned} \tag{90}$$

利用

$$\begin{aligned}
D_\mu\Phi &= \begin{pmatrix} \partial_\mu - ieA_\mu - \dfrac{ig}{2c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu & -\dfrac{i}{\sqrt{2}}gW_\mu^+ \\ -\dfrac{i}{\sqrt{2}}gW_\mu^- & \partial_\mu + \dfrac{ig}{2c_{\mathrm{W}}}Z_\mu \end{pmatrix} \begin{pmatrix} \phi^+ \\ \dfrac{1}{\sqrt{2}}(v+H+i\chi) \end{pmatrix} \\
&= \begin{pmatrix} \partial_\mu\phi^+ - i\left[eA_\mu + \dfrac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_\mu\right]\phi^+ - \dfrac{ig}{2}W_\mu^+(H+i\chi) - im_W W_\mu^+ \\ \dfrac{1}{\sqrt{2}}\left[\partial_\mu(H+i\chi) - igW_\mu^-\phi^+ + \dfrac{ig}{2c_{\mathrm{W}}}Z_\mu(H+i\chi) + im_Z Z_\mu\right] \end{pmatrix},
\end{aligned} \tag{91}$$

可将 Higgs 场协变动能项化为

$$\begin{aligned}
&(D^\mu\Phi)^\dagger D_\mu\Phi \\
={}& \left|\partial_\mu\phi^+ - i\left[eA_\mu + \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}Z_\mu\right]\phi^+ - \frac{ig}{2}W_\mu^+(H+i\chi) - im_W W_\mu^+\right|^2 \\
&+\frac{1}{2}\left|\partial_\mu(H+i\chi) - igW_\mu^-\phi^+ + \frac{ig}{2c_{\mathrm{W}}}Z_\mu(H+i\chi) + im_Z Z_\mu\right|^2
\end{aligned}$$

$$
\begin{aligned}
&= (\partial^\mu \phi^+)(\partial_\mu \phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu \chi)(\partial_\mu \chi) \\
&\quad + \left(-i\partial^\mu \phi^- \left\{\left[eA_\mu + \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}} Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H+i\chi) + m_W W_\mu^+\right\} + \text{h.c.}\right) \\
&\quad + \left\{-\frac{i}{2}\partial^\mu (H - i\chi)\left[gW_\mu^- \phi^+ - \frac{g}{2c_{\mathrm{W}}} Z_\mu (H+i\chi) - m_Z Z_\mu\right] + \text{h.c.}\right\} \\
&\quad + \left|\left[eA_\mu + \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}} Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H+i\chi) + m_W W_\mu^+\right|^2 \\
&\quad + \frac{1}{2}\left|gW_\mu^- \phi^+ - \frac{g}{2c_{\mathrm{W}}} Z_\mu (H+i\chi) - m_Z Z_\mu\right|^2 \\
&= (\partial^\mu \phi^+)(\partial_\mu \phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu \chi)(\partial_\mu \chi) \\
&\quad + m_W^2 W^{-\mu} W_\mu^+ + \frac{1}{2} m_Z^2 Z^\mu Z_\mu + g m_W H W_\mu^+ W^{-\mu} + \frac{g m_Z}{2c_{\mathrm{W}}} H Z^\mu Z_\mu \\
&\quad + \frac{g}{2}[W_\mu^+ \phi^- i\overleftrightarrow{\partial^\mu}(H+i\chi) + \text{h.c.}] + eA_\mu \phi^- i\overleftrightarrow{\partial^\mu}\phi^+ + \frac{g}{2c_{\mathrm{W}}} Z_\mu [i\chi i\overleftrightarrow{\partial^\mu} H + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\phi^- i\overleftrightarrow{\partial^\mu}\phi^+] \\
&\quad + \frac{g^2}{4} W_\mu^+ W^{-\mu}(2\phi^+\phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu \phi^+\phi^- + \frac{g^2}{4c_{\mathrm{W}}^2} Z_\mu Z^\mu \left[(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)^2 \phi^+\phi^- + \frac{1}{2}H^2 + \frac{1}{2}\chi^2\right] \\
&\quad + \left[\frac{eg}{2} W_\mu^+ A^\mu \phi^- (H+i\chi) - \frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}} W_\mu^+ Z^\mu \phi^- (H+i\chi) + \text{h.c.}\right] + \frac{eg}{c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2) A_\mu Z^\mu \phi^+\phi^- \\
&\quad + (e m_W A^\mu \phi^+ W_\mu^- - g s_{\mathrm{W}}^2 m_Z Z^\mu \phi^+ W_\mu^- + \text{h.c.}) + \mathcal{L}_{\mathrm{b1}},
\end{aligned} \tag{92}
$$

其中

$$
\mathcal{L}_{\mathrm{b1}} = -i m_W (\partial^\mu \phi^-) W_\mu^+ + i m_W (\partial^\mu \phi^+) W_\mu^- + m_Z (\partial^\mu \chi) Z_\mu. \tag{93}
$$

R_ξ 规范的规范固定函数设为

$$
G^\pm = \frac{1}{\sqrt{\xi}}(\partial^\mu W_\mu^\pm \mp i\xi m_W \phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi}}(\partial^\mu Z_\mu - \xi m_Z \chi), \quad G^\gamma = \frac{1}{\sqrt{\xi}}\partial^\mu A_\mu, \tag{94}
$$

它们在路径积分量子化中的泛函积分形式为

$$
\begin{aligned}
&\int \mathcal{D}\omega^+ \int \mathcal{D}\omega^- \int \mathcal{D}\omega^Z \int \mathcal{D}\omega^\gamma \exp\left[-i\int d^4x \left(\omega^+\omega^- + \frac{1}{2}\omega^Z\omega^Z + \frac{1}{2}\omega^\gamma\omega^\gamma\right)\right] \\
&\qquad \times \delta(G^+ - \omega^+)\delta(G^- - \omega^-)\delta(G^Z - \omega^Z)\delta(G^\gamma - \omega^\gamma) \\
&= \exp\left[-i\int d^4x \left(G^+G^- + \frac{1}{2}G^Z G^Z + \frac{1}{2}G^\gamma G^\gamma\right)\right].
\end{aligned} \tag{95}
$$

由此可得拉氏量中的规范固定项

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EW,GF}} &= -G^+G^- - \frac{1}{2}(G^Z)^2 - \frac{1}{2}(G^\gamma)^2 \\
&= -\frac{1}{\xi}(\partial^\mu W_\mu^+ - i\xi m_W \phi^+)(\partial^\nu W_\nu^- + i\xi m_W \phi^-) - \frac{1}{2\xi}(\partial^\mu Z_\mu - \xi m_Z \chi)^2 - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 \\
&= -\frac{1}{\xi}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) - \frac{1}{2\xi}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 - \xi m_W^2 \phi^+\phi^- - \frac{1}{2}\xi m_Z^2 \chi^2 + \mathcal{L}_{\mathrm{b2}}.
\end{aligned} \tag{96}
$$

可见，Nambu-Goldstone 玻色子在 R_ξ 规范下具有依赖于 ξ 的非物理质量，

$$m_\phi = \sqrt{\xi} m_W, \quad m_\chi = \sqrt{\xi} m_Z. \tag{97}$$

这里，

$$\mathcal{L}_{\mathrm{b2}} = -im_W \phi^- (\partial^\mu W_\mu^+) + im_W \phi^+ \partial^\mu W_\mu^- + m_Z \chi \partial^\mu Z_\mu. \tag{98}$$

由于

$$\mathcal{L}_{\mathrm{b1}} + \mathcal{L}_{\mathrm{b2}} = -im_W \partial^\mu (\phi^- W_\mu^+) + im_W \partial^\mu (\phi^+ W_\mu^-) + m_Z \partial^\mu (\chi Z_\mu), \tag{99}$$

这两项体现为全散度，不会有物理效应。可见，协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 $\mathcal{L}_{\mathrm{b1}}$ 被规范固定项中的 $\mathcal{L}_{\mathrm{b2}}$ 抵消掉，这就是如此选取规范固定函数的目的。

这样一来，电弱规范场传播子相关拉氏量变成

$$\begin{aligned}
\mathcal{L}_{\mathrm{EW,prop}} \;=\;& (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) - \frac{1}{\xi}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) + m_W^2 W^{-\mu} W_\mu^+ \\
&+\frac{1}{2}\left[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - \frac{1}{\xi}(\partial^\mu Z_\mu)^2 + m_Z^2 Z^\mu Z_\mu\right] \\
&+\frac{1}{2}\left[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu) - \frac{1}{\xi}(\partial^\mu A_\mu)^2\right] \\
\to\;& W_\mu^+ \left[g^{\mu\nu}(\partial^2 + m_W^2) - \left(1 - \frac{1}{\xi}\right)\partial^\mu \partial^\nu\right] W_\nu^- + \frac{1}{2} Z_\mu \left[g^{\mu\nu}(\partial^2 + m_Z^2) - \left(1 - \frac{1}{\xi}\right)\partial^\mu \partial^\nu\right] Z_\nu \\
&+\frac{1}{2} A_\mu \left[g^{\mu\nu}\partial - \left(1 - \frac{1}{\xi}\right)\partial^\mu \partial^\nu\right] A_\nu.
\end{aligned} \tag{100}$$

于是，光子的传播子与胶子形式类似，为

$$\frac{-i}{p^2 + i\varepsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi)\right]. \tag{101}$$

将 $W^\pm$ 传播子相关拉氏量变换到动量空间，得

$$-g^{\mu\nu}(p^2 - m_W^2) + \left(1 - \frac{1}{\xi}\right) p^\mu p^\nu = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2}\frac{p^2 - \xi m_W^2}{\xi}, \tag{102}$$

它的逆矩阵是

$$-\frac{1}{p^2 - m_W^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - \frac{\xi}{p^2 - \xi m_W^2}\frac{p_\mu p_\nu}{p^2} = -\frac{1}{p^2 - m_W^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi m_W^2}(1 - \xi)\right], \tag{103}$$

这是因为由

$$\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right)\frac{p^\mu p^\nu}{p^2} = \frac{p_\rho p^\nu}{p^2} - \frac{p_\rho p^\nu}{p^2} = 0, \quad \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) = \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \tag{104}$$

可得

$$\begin{aligned}
&\left[-\frac{1}{p^2 - m_W^2}\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right) - \frac{\xi}{p^2 - \xi m_W^2}\frac{p_\rho p_\mu}{p^2}\right]\left[-\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2}\frac{p^2 - \xi m_W^2}{\xi}\right] \\
&= \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} + \frac{p_\rho p^\nu}{p^2} = \delta_\rho^\nu.
\end{aligned} \tag{105}$$

从而，$W^{\pm}$ 传播子的形式为

$$\frac{-i}{p^2-m_W^2+i\varepsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi m_W^2}(1-\xi)\right]. \tag{106}$$

同理，Z 传播子的形式为

$$\frac{-i}{p^2-m_Z^2+i\varepsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi m_Z^2}(1-\xi)\right]. \tag{107}$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a=\frac{1}{g}\partial_\mu\alpha^a+\varepsilon^{abc}W_\mu^b\alpha^c, \quad \delta B_\mu=\frac{1}{g'}\partial_\mu\alpha^Y. \tag{108}$$

定义

$$\alpha^{\pm}\equiv\frac{1}{\sqrt{2}}(\alpha^1\mp i\alpha^2), \quad \alpha^Z\equiv\alpha^3-\alpha^Y, \quad \alpha^\gamma\equiv s_{\mathrm{W}}^2\alpha^3+c_{\mathrm{W}}^2\alpha^Y, \tag{109}$$

利用

$$\begin{aligned}\varepsilon^{1bc}W_\mu^b\alpha^c &= W_\mu^2\alpha^3-W_\mu^3\alpha^2, \quad \varepsilon^{2bc}W_\mu^b\alpha^c=-W_\mu^1\alpha^3+W_\mu^3\alpha^1, &(110)\\ \pm i\sqrt{2}\alpha^{\pm} &= \pm i\alpha^1+\alpha^2, \quad \pm i\sqrt{2}W_\mu^{\pm}=\pm iW_\mu^1+W_\mu^2, &(111)\end{aligned}$$

有

$$\begin{aligned}\varepsilon^{1bc}W_\mu^b\alpha^c\mp i\varepsilon^{2bc}W_\mu^b\alpha^c &= (W_\mu^2\alpha^3-W_\mu^3\alpha^2)\mp i(-W_\mu^1\alpha^3+W_\mu^3\alpha^1)=(W_\mu^2\pm iW_\mu^1)\alpha^3-W_\mu^3(\alpha^2\pm i\alpha^1)\\ &= \pm i\sqrt{2}W_\mu^{\pm}(c_{\mathrm{W}}^2\alpha^Z+\alpha^\gamma)\mp i\sqrt{2}(s_{\mathrm{W}}A_\mu+c_{\mathrm{W}}Z_\mu)\alpha^{\pm}, &(112)\\ \varepsilon^{3bc}W_\mu^b\alpha^c &= W_\mu^1\alpha^2-W_\mu^2\alpha^1=\frac{1}{\sqrt{2}}(W_\mu^++W_\mu^-)\frac{i}{\sqrt{2}}(\alpha^+-\alpha^-)-\frac{i}{\sqrt{2}}(W_\mu^+-W_\mu^-)\frac{1}{\sqrt{2}}(\alpha^++\alpha^-)\\ &= -i(W_\mu^+\alpha^--W_\mu^-\alpha^+). &(113)\end{aligned}$$

因此，

$$\begin{aligned}\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1-i\delta W_\mu^2)=\frac{1}{\sqrt{2}g}\partial_\mu(\alpha^1-i\alpha^2)+\frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c-i\varepsilon^{2bc}W_\mu^b\alpha^c)\\ &= \frac{1}{g}\partial_\mu\alpha^+-i(s_{\mathrm{W}}A_\mu+c_{\mathrm{W}}Z_\mu)\alpha^++iW_\mu^+(c_{\mathrm{W}}^2\alpha^Z+\alpha^\gamma), &(114)\\ \delta W_\mu^- &= (\delta W_\mu^+)^\dagger=\frac{1}{g}\partial_\mu\alpha^-+i(s_{\mathrm{W}}A_\mu+c_{\mathrm{W}}Z_\mu)\alpha^--iW_\mu^-(c_{\mathrm{W}}^2\alpha^Z+\alpha^\gamma), &(115)\end{aligned}$$

$$\delta Z_\mu^a=c_{\mathrm{W}}\delta W_\mu^3-s_{\mathrm{W}}\delta B_\mu=\frac{c_{\mathrm{W}}}{g}\partial_\mu\alpha^3+c_{\mathrm{W}}\varepsilon^{3bc}W_\mu^b\alpha^c-\frac{s_{\mathrm{W}}}{g'}\partial_\mu\alpha^Y=\frac{c_{\mathrm{W}}}{g}\partial_\mu\alpha^Z-ic_{\mathrm{W}}(W_\mu^+\alpha^--W_\mu^-\alpha^+), \tag{116}$$

$$\delta A_\mu=s_{\mathrm{W}}\delta W_\mu^3+c_{\mathrm{W}}\delta B_\mu=\frac{s_{\mathrm{W}}}{g}\partial_\mu\alpha^3+s_{\mathrm{W}}\varepsilon^{3bc}W_\mu^b\alpha^c+\frac{c_{\mathrm{W}}}{g'}\partial_\mu\alpha^Y=\frac{1}{e}\partial_\mu\alpha^\gamma-is_{\mathrm{W}}(W_\mu^+\alpha^--W_\mu^-\alpha^+). \tag{117}$$

另一方面，根据

$$\alpha^aT^a+\alpha^YY_H=\frac{1}{2}(\alpha^a\sigma^a+\alpha^Y)=\frac{1}{2}\begin{pmatrix}\alpha^3+\alpha^Y & \alpha^1-i\alpha^2\\ \alpha^1+i\alpha^2 & -\alpha^3+\alpha^Y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}2\alpha^\gamma+(c_{\mathrm{W}}^2-s_{\mathrm{W}}^2)\alpha^Z & \sqrt{2}\alpha^+\\ \sqrt{2}\alpha^- & -\alpha^Z\end{pmatrix}, \tag{118}$$

可知 Higgs 场的无穷小规范变换形式为

$$\begin{aligned}\delta\Phi &= i(\alpha^a T^a + \alpha^Y Y_H)\Phi = \frac{i}{2}\begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}\begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\ &= \begin{pmatrix} \frac{i}{2}[\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+] \\ \frac{1}{\sqrt{2}}\left[i\phi^+\alpha^- - \frac{1}{2}(iv + iH - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \frac{1}{\sqrt{2}}(\delta H + i\delta\chi) \end{pmatrix}. \end{aligned} \tag{119}$$

利用

$$\mathrm{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \mathrm{Im}(\phi^+\alpha^-) = -\frac{i}{2}(\phi^+\alpha^- - \phi^-\alpha^+), \tag{120}$$

可得

$$\delta\phi^+ = \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\}, \tag{121}$$

$$\delta\phi^- = -\frac{i}{2}\{\phi^-[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H - i\chi)\alpha^-\}, \tag{122}$$

$$\delta H = \frac{1}{2}[i(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z]. \tag{123}$$

于是，规范固定函数的无穷小规范变换为

$$\begin{aligned}\sqrt{\xi}\delta G^+ &= \partial^\mu \delta W_\mu^+ - i\xi m_W \delta\phi^+ = \partial^\mu\left[\frac{1}{g}\partial_\mu\alpha^+ - i(s_W A_\mu + c_W Z_\mu)\alpha^+ + iW_\mu^+(c_W^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad + \frac{1}{2}\xi m_W\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\},\end{aligned} \tag{124}$$

$$\begin{aligned}\sqrt{\xi}\delta G^- &= \partial^\mu \delta W_\mu^- + i\xi m_W \delta\phi^- = \partial^\mu\left[\frac{1}{g}\partial_\mu\alpha^- + i(s_W A_\mu + c_W Z_\mu)\alpha^- - iW_\mu^-(c_W^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad + \frac{1}{2}\xi m_W\{\phi^-[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H - i\chi)\alpha^-\},\end{aligned} \tag{125}$$

$$\begin{aligned}\sqrt{\xi}\delta G^Z &= \partial^\mu \delta Z_\mu - \xi m_Z \delta\chi = \partial^\mu\left[\frac{c_W}{g}\partial_\mu\alpha^Z - ic_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right] \\ &\quad - \frac{1}{2}\xi m_Z[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z],\end{aligned} \tag{126}$$

$$\sqrt{\xi}\delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu\left[\frac{1}{e}\partial_\mu\alpha^\gamma - is_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right]. \tag{127}$$

因此,

$$\sqrt{\xi}g\frac{\delta G^+}{\delta\alpha^+} = \partial^2 + \xi m_W^2 - ie\partial^\mu A_\mu - igc_W\partial^\mu Z_\mu + \frac{1}{2}g\xi m_W(H + i\chi), \tag{128}$$

$$\frac{\sqrt{\xi}g}{c_W}\frac{\delta G^+}{\delta\alpha^Z} = igc_W\partial^\mu W_\mu^+ + \frac{g(c_W^2 - s_W^2)\xi m_W}{2c_W}\phi^+, \quad \sqrt{\xi}e\frac{\delta G^+}{\delta\alpha^\gamma} = ie\partial^\mu W_\mu^+ + e\xi m_W\phi^+, \tag{129}$$

$$\sqrt{\xi}g\frac{\delta G^-}{\delta\alpha^-} = \partial^2 + \xi m_W^2 + ie\partial^\mu A_\mu + igc_W\partial^\mu Z_\mu + \frac{1}{2}\xi g m_W(H - i\chi), \tag{130}$$

$$\frac{\sqrt{\xi}g}{c_W}\frac{\delta G^-}{\delta\alpha^Z} = -igc_W\partial^\mu W_\mu^- + \frac{g(c_W^2 - s_W^2)\xi m_W}{2c_W}\phi^-, \quad \sqrt{\xi}e\frac{\delta G^-}{\delta\alpha^\gamma} = -ie\partial^\mu W_\mu^- + e\xi m_W\phi^-, \tag{131}$$

$$\sqrt{\xi}g\frac{\delta G^Z}{\delta\alpha^+} = igc_W\partial^\mu W_\mu^- - \frac{1}{2}g\xi m_Z\phi^-, \quad \sqrt{\xi}g\frac{\delta G^Z}{\delta\alpha^-} = -igc_W\partial^\mu W_\mu^+ - \frac{1}{2}g\xi m_Z\phi^+, \tag{132}$$

$$\frac{\sqrt{\xi}g}{c_{\rm W}}\frac{\delta G^Z}{\delta\alpha^Z} = \partial^2+\xi m_Z^2+\frac{g\xi m_Z}{2c_{\rm W}}H, \tag{133}$$

$$\sqrt{\xi}g\frac{\delta G^\gamma}{\delta\alpha^+} = ie\partial^\mu W_\mu^-, \quad \sqrt{\xi}g\frac{\delta G^\gamma}{\delta\alpha^-} = -ie\partial^\mu W_\mu^+, \quad \sqrt{\xi}e\frac{\delta G^\gamma}{\delta\alpha^\gamma} = \partial^2. \tag{134}$$

最后，得到以下 Faddeev-Popov 鬼场拉氏量，

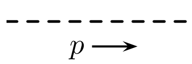
$$\begin{aligned}
\mathcal{L}_{\rm EWG,FP} =& -\bar\eta^+\left(\sqrt{\xi}g\frac{\delta G^+}{\delta\alpha^+}\right)\eta^+ - \bar\eta^Z\left(\sqrt{\xi}g\frac{\delta G^Z}{\delta\alpha^+}\right)\eta^+ - \bar\eta^\gamma\left(\sqrt{\xi}g\frac{\delta G^\gamma}{\delta\alpha^+}\right)\eta^+ - \bar\eta^-\left(\sqrt{\xi}g\frac{\delta G^-}{\delta\alpha^-}\right)\eta^- \\
& -\bar\eta^Z\left(\sqrt{\xi}g\frac{\delta G^Z}{\delta\alpha^-}\right)\eta^- - \bar\eta^\gamma\left(\sqrt{\xi}g\frac{\delta G^\gamma}{\delta\alpha^-}\right)\eta^- - \bar\eta^Z\left(\frac{\sqrt{\xi}g}{c_{\rm W}}\frac{\delta G^Z}{\delta\alpha^Z}\right)\eta^Z - \bar\eta^+\left(\frac{\sqrt{\xi}g}{c_{\rm W}}\frac{\delta G^+}{\delta\alpha^Z}\right)\eta^Z \\
& -\bar\eta^-\left(\frac{\sqrt{\xi}g}{c_{\rm W}}\frac{\delta G^-}{\delta\alpha^Z}\right)\eta^Z - \bar\eta^\gamma\left(\sqrt{\xi}e\frac{\delta G^\gamma}{\delta\alpha^\gamma}\right)\eta^\gamma - \bar\eta^+\left(\sqrt{\xi}e\frac{\delta G^+}{\delta\alpha^\gamma}\right)\eta^\gamma - \bar\eta^-\left(\sqrt{\xi}e\frac{\delta G^-}{\delta\alpha^\gamma}\right)\eta^\gamma \\
=& \;\bar\eta^+\left[-\partial^2-\xi m_W^2 - ie\overleftarrow{\partial^\mu}A_\mu - igc_{\rm W}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi m_W(H+i\chi)\right]\eta^+ \\
& +\bar\eta^Z\left(igc_{\rm W}\overleftarrow{\partial^\mu}W_\mu^- + \frac{1}{2}g\xi m_Z\phi^-\right)\eta^+ + ie(\partial^\mu\bar\eta^\gamma)W_\mu^-\eta^+ \\
& +\bar\eta^-\left[-\partial^2-\xi m_W^2 + ie\overleftarrow{\partial^\mu}A_\mu + igc_{\rm W}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi m_W(H-i\chi)\right]\eta^- \\
& +\bar\eta^Z\left(-igc_{\rm W}\overleftarrow{\partial^\mu}W_\mu^+ + \frac{1}{2}g\xi m_Z\phi^+\right)\eta^- - ie(\partial^\mu\bar\eta^\gamma)W_\mu^+\eta^- \\
& +\bar\eta^Z\left(-\partial^2-\xi m_Z^2 - \frac{g\xi m_Z}{2c_{\rm W}}H\right)\eta^Z + \bar\eta^+\left(igc_{\rm W}\overleftarrow{\partial^\mu}W_\mu^+ - \frac{g(c_{\rm W}^2-s_{\rm W}^2)\xi m_W}{2c_{\rm W}}\phi^+\right)\eta^Z \\
& +\bar\eta^-\left(-igc_{\rm W}\overleftarrow{\partial^\mu}W_\mu^- - \frac{g(c_{\rm W}^2-s_{\rm W}^2)\xi m_W}{2c_{\rm W}}\phi^-\right)\eta^Z \\
& -\bar\eta^\gamma\partial^2\eta^\gamma + \bar\eta^+(ie\overleftarrow{\partial^\mu}W_\mu^+ - e\xi m_W\phi^+)\eta^\gamma + \bar\eta^-(-ie\overleftarrow{\partial^\mu}W_\mu^- - e\xi m_W\phi^-)\eta^\gamma.
\end{aligned} \tag{135}$$

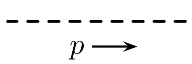
鬼粒子的质量为

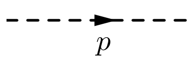
$$m_{\eta^+} = m_{\eta^-} = \sqrt{\xi}m_W, \quad m_{\eta^Z} = \sqrt{\xi}m_Z, \quad m_{\eta^\gamma} = 0. \tag{136}$$

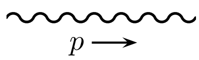
下面给出 R_ξ 规范下的费曼规则。$\xi = 1$ 对应 Feynman-'t Hooft 规范，$\xi = 0$ 对应 Landau 规范，$\xi \to \infty$ 对应幺正规范。

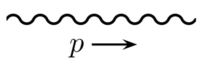
传播子：

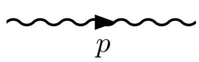
H  H $= \dfrac{i}{p^2-m_H^2+i\varepsilon}$

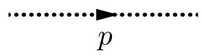
χ  χ $= \dfrac{i}{p^2-\xi m_Z^2+i\varepsilon}$

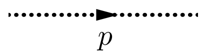
ϕ  ϕ $= \dfrac{i}{p^2-\xi m_W^2+i\varepsilon}$

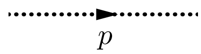
A_μ  A_ν $= \dfrac{-i}{p^2+i\varepsilon}\left[g_{\mu\nu}-\dfrac{p_\mu p_\nu}{p^2}(1-\xi)\right]$

Z_μ  Z_ν $= \dfrac{-i}{p^2-m_Z^2+i\varepsilon}\left[g_{\mu\nu}-\dfrac{p_\mu p_\nu}{p^2-\xi m_Z^2}(1-\xi)\right]$

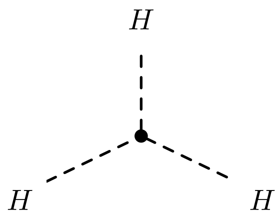
W_μ  W_ν $= \dfrac{-i}{p^2-m_W^2+i\varepsilon}\left[g_{\mu\nu}-\dfrac{p_\mu p_\nu}{p^2-\xi m_W^2}(1-\xi)\right]$

η^γ  $\eta^\gamma \quad = \dfrac{i}{p^2 + i\varepsilon}$

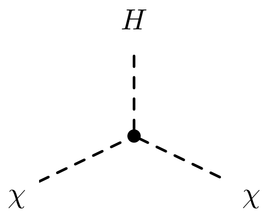
η^Z  $\eta^Z \quad = \dfrac{i}{p^2 - \xi m_Z^2 + i\varepsilon}$

$\eta^\pm$  $\eta^\pm \quad = \dfrac{i}{p^2 - \xi m_W^2 + i\varepsilon}$

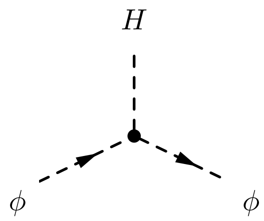
标量玻色子三线性耦合:



$$= -3i\frac{m_H^2}{v} \qquad = -6i\lambda v$$

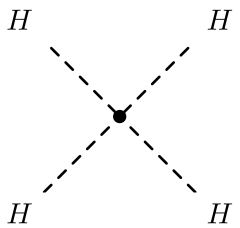


$$= -i\frac{m_H^2}{v} \qquad = -2i\lambda v$$

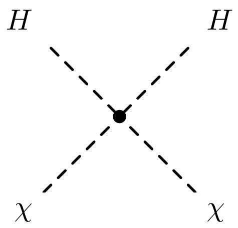


$$= -i\frac{m_H^2}{v} \qquad = -2i\lambda v$$

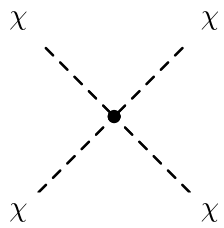
标量玻色子四线性耦合:



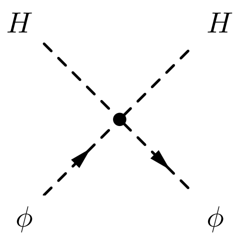
$$= -3i\frac{m_H^2}{v^2} \qquad = -6i\lambda$$



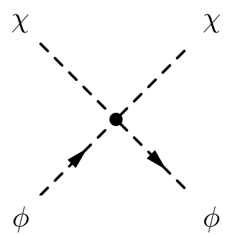
$$= -i\frac{m_H^2}{v^2} \qquad = -2i\lambda$$



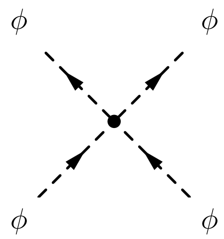
$$= -3i\frac{m_H^2}{v^2} \qquad = -6i\lambda$$



$$= -i\frac{m_H^2}{v^2} \qquad = -2i\lambda$$

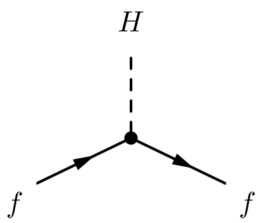


$$= -i\frac{m_H^2}{v^2} \qquad = -2i\lambda$$

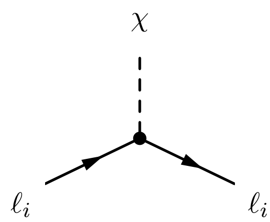


$$= -2i\frac{m_H^2}{v^2} \qquad = -4i\lambda$$

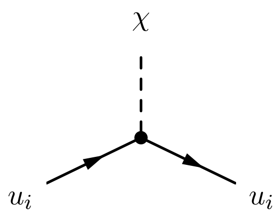
Yukawa 耦合:



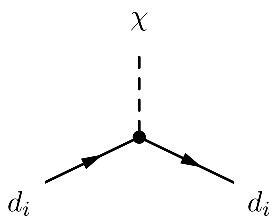
$$= -i\frac{m_f}{v} \qquad = -i\frac{y_f}{\sqrt{2}}$$



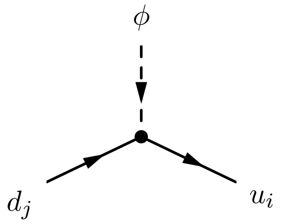
$$= \frac{m_{\ell_i}}{v}\gamma_5 \qquad = \frac{y_{\ell_i}}{\sqrt{2}}\gamma_5$$



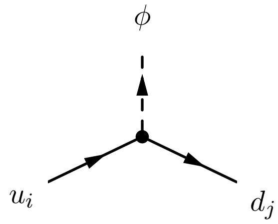
$$= -\frac{m_{u_i}}{v}\gamma_5 \qquad = -\frac{y_{u_i}}{\sqrt{2}}\gamma_5$$



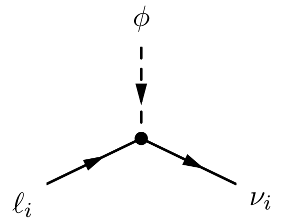
$$= \frac{m_{d_i}}{v}\gamma_5 \qquad = \frac{y_{d_i}}{\sqrt{2}}\gamma_5$$



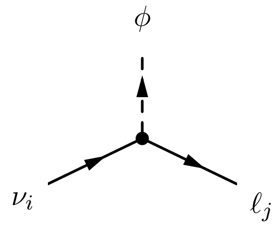
$$= i\frac{\sqrt{2}V_{ij}}{v}(m_{u_i}P_{\mathrm{L}} - m_{d_j}P_{\mathrm{R}})$$
$$= iV_{ij}(y_{u_i}P_{\mathrm{L}} - y_{d_j}P_{\mathrm{R}})$$



$$= -i\frac{\sqrt{2}V^{\dagger}_{ji}}{v}(m_{d_j}P_{\mathrm{L}} - m_{u_i}P_{\mathrm{R}})$$
$$= -iV^{\dagger}_{ji}(y_{d_j}P_{\mathrm{L}} - y_{u_i}P_{\mathrm{R}})$$

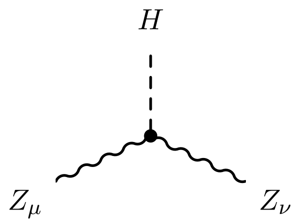


$$= -i\frac{\sqrt{2}m_{\ell_i}}{v}P_{\mathrm{R}}$$
$$= -iy_{\ell_i}P_{\mathrm{R}}$$

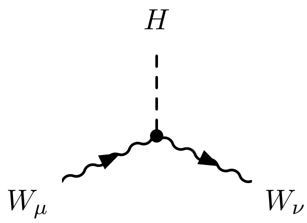


$$= -i\frac{\sqrt{2}m_{\ell_i}}{v}P_{\mathrm{L}}$$
$$= -iy_{\ell_i}P_{\mathrm{L}}$$

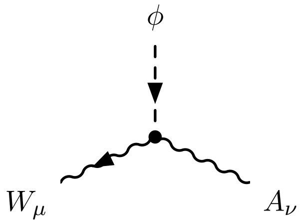
标量玻色子与电弱规范玻色子的三线性耦合:



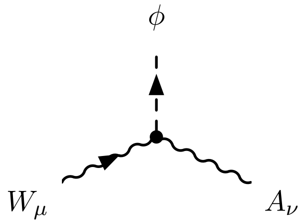
$$= i\frac{gm_Z}{c_{\mathrm{W}}}g^{\mu\nu}$$
$$= i\frac{g^2v}{2c^2_{\mathrm{W}}}g^{\mu\nu}$$



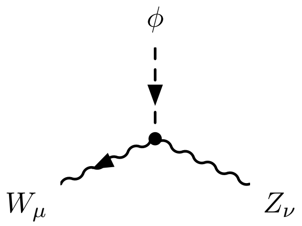
$$= igm_Wg^{\mu\nu}$$
$$= i\frac{g^2v}{2}g^{\mu\nu}$$



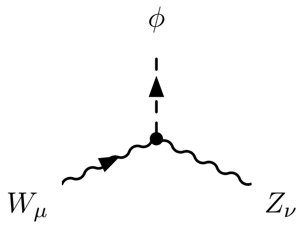
$$= iem_Wg^{\mu\nu}$$
$$= i\frac{egv}{2}g^{\mu\nu}$$



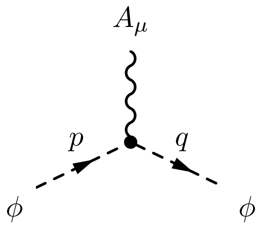
$$= iem_Wg^{\mu\nu}$$
$$= i\frac{egv}{2}g^{\mu\nu}$$



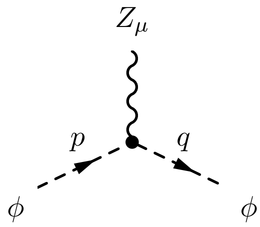
$$= -igs^2_{\mathrm{W}}m_Zg^{\mu\nu}$$
$$= -i\frac{g^2s^2_{\mathrm{W}}v}{2c_{\mathrm{W}}}g^{\mu\nu}$$



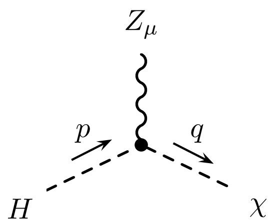
$$= -igs^2_{\mathrm{W}}m_Zg^{\mu\nu}$$
$$= -i\frac{g^2s^2_{\mathrm{W}}v}{2c_{\mathrm{W}}}g^{\mu\nu}$$



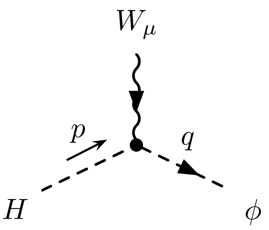
$$= ie(p+q)^\mu$$



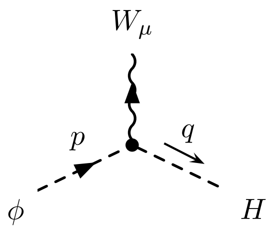
$$= i\frac{g(c^2_{\mathrm{W}} - s^2_{\mathrm{W}})}{2c_{\mathrm{W}}}(p+q)^\mu$$



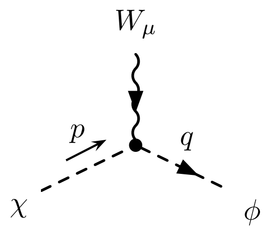
$$= -\frac{g}{2c_{\mathrm{W}}}(p+q)^\mu$$



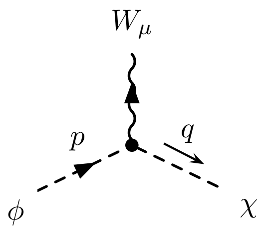
$$= i\frac{g}{2}(p+q)^\mu$$



$$= i\frac{g}{2}(p+q)^\mu$$

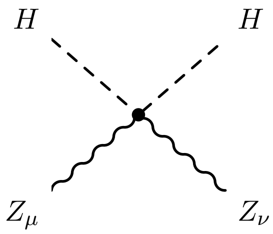


$$= -\frac{g}{2}(p+q)^\mu$$

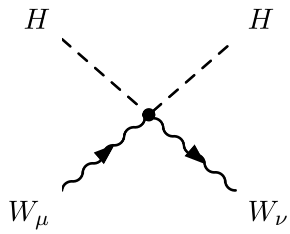


$$= \frac{g}{2}(p+q)^\mu$$

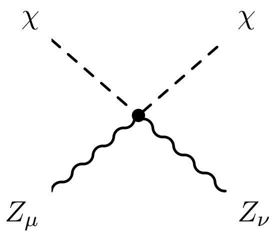
标量玻色子与电弱规范玻色子的四线性耦合:



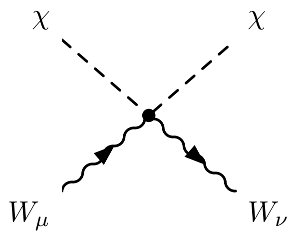
$$= i\frac{g^2}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$



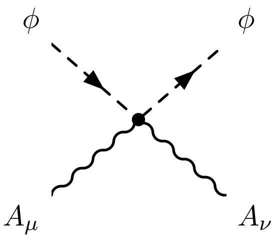
$$= i\frac{g^2}{2}g^{\mu\nu}$$



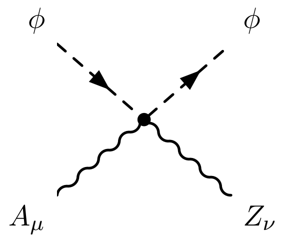
$$= i\frac{g^2}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$



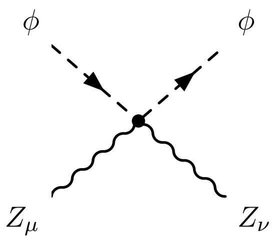
$$= i\frac{g^2}{2}g^{\mu\nu}$$



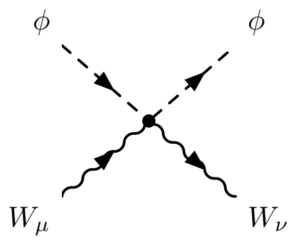
$$= 2ie^2 g^{\mu\nu}$$



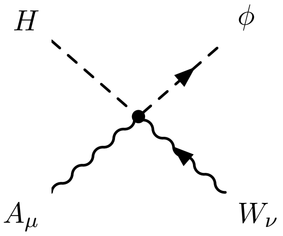
$$= i\frac{eg(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{c_{\mathrm{W}}}g^{\mu\nu}$$



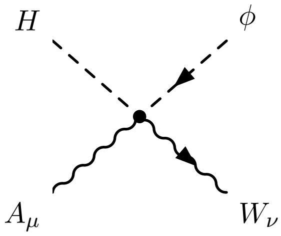
$$= i\frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)^2}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$



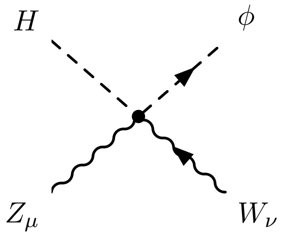
$$= i\frac{g^2}{2}g^{\mu\nu}$$



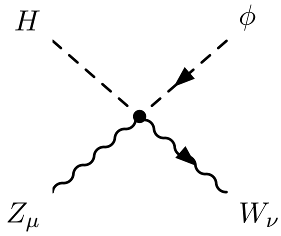
$$= i\frac{eg}{2}g^{\mu\nu}$$



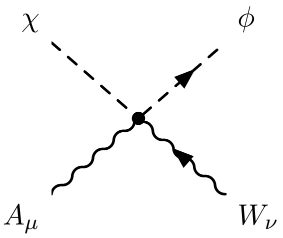
$$= i\frac{eg}{2}g^{\mu\nu}$$



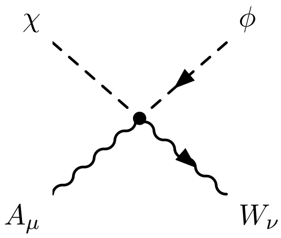
$$= -i\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}g^{\mu\nu}$$



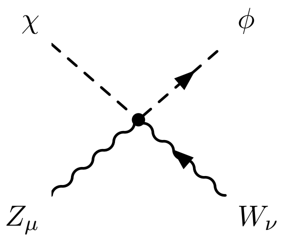
$$= -i\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}g^{\mu\nu}$$



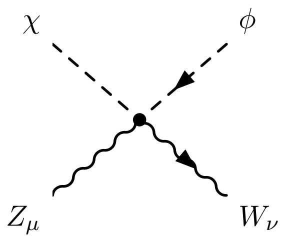
$$= -\frac{eg}{2}g^{\mu\nu}$$



$$= \frac{eg}{2}g^{\mu\nu}$$

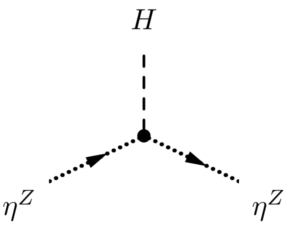


$$= \frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}g^{\mu\nu}$$

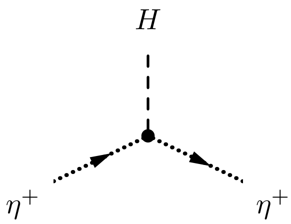


$$= -\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}g^{\mu\nu}$$

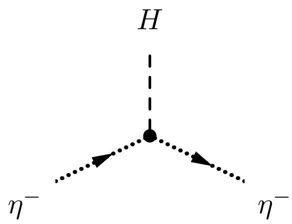
鬼粒子与标量玻色子的耦合:



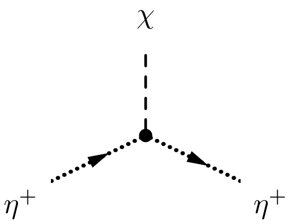
$$= -i\frac{g\xi m_Z}{2c_{\mathrm{W}}} = -i\frac{g^2\xi v}{4c_{\mathrm{W}}^2}$$



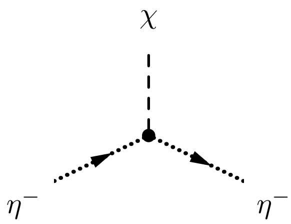
$$= -i\frac{g\xi m_W}{2} = -i\frac{g^2\xi v}{4}$$



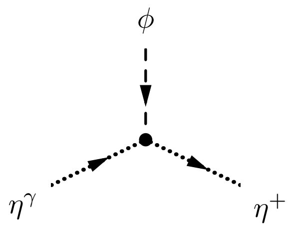
$$= -i\frac{g\xi m_W}{2} = -i\frac{g^2\xi v}{4}$$



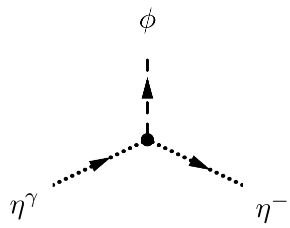
$$= \frac{g\xi m_W}{2} = \frac{g^2\xi v}{4}$$



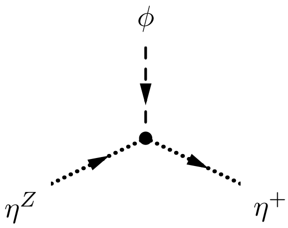
$$= -\frac{g\xi m_W}{2} = -\frac{g^2\xi v}{4}$$



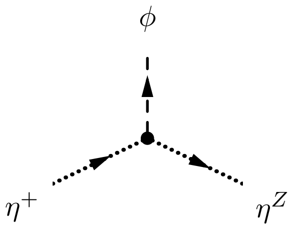
$$= -ie\xi m_W$$
$$= -i\frac{eg\xi v}{2}$$



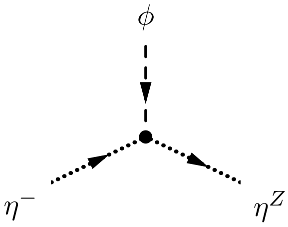
$$= -ie\xi m_W$$
$$= -i\frac{eg\xi v}{2}$$



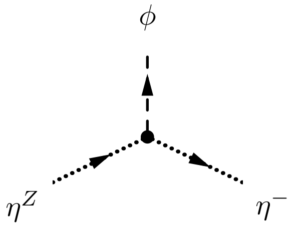
$$= -i\frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi m_W}{2c_{\mathrm{W}}}$$
$$= -i\frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi v}{4c_{\mathrm{W}}}$$



$$= i\frac{g\xi m_Z}{2}$$
$$= i\frac{g^2\xi v}{4c_{\mathrm{W}}}$$

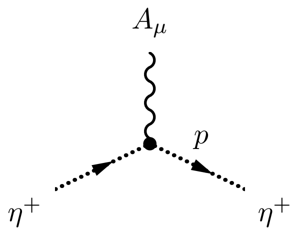


$$= i\frac{g\xi m_Z}{2}$$
$$= i\frac{g^2\xi v}{4c_{\mathrm{W}}}$$

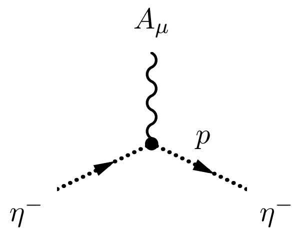


$$= -i\frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi m_W}{2c_{\mathrm{W}}}$$
$$= -i\frac{g^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi v}{4c_{\mathrm{W}}}$$

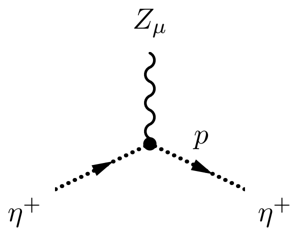
鬼粒子与电弱规范玻色子的耦合:



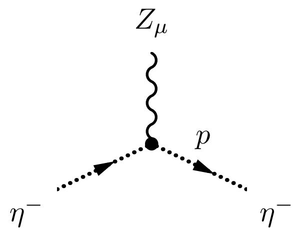
$$= iep^\mu$$



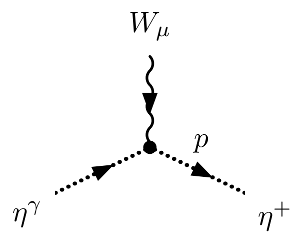
$$= -iep^\mu$$



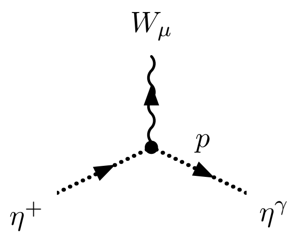
$$= igc_{\mathrm{W}}p^\mu$$



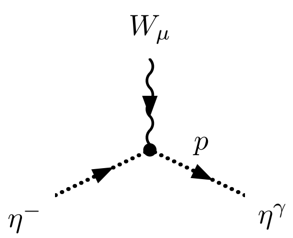
$$= -igc_{\mathrm{W}}p^\mu$$



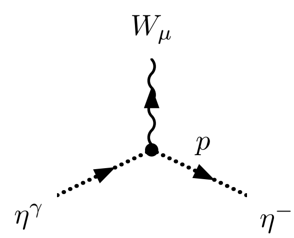
$$= -iep^\mu$$



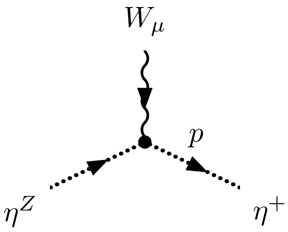
$$= -iep^\mu$$



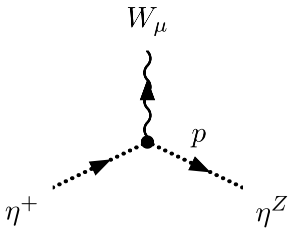
$$= iep^\mu$$



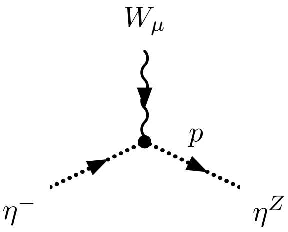
$$= iep^\mu$$



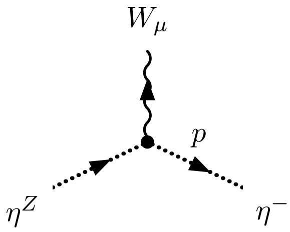
$= -igc_\text{W}p^\mu$



$= -igc_\text{W}p^\mu$



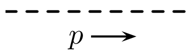
$= igc_\text{W}p^\mu$



$= igc_\text{W}p^\mu$

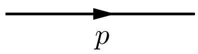
8　内外线一般费曼规则

标量玻色子传播子:



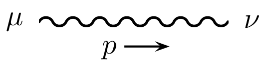
$$= \frac{i}{p^2 - m^2 + i\varepsilon}$$

Dirac 费米子传播子:

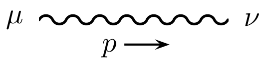


$$= \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$$

无质量规范玻色子（如光子）传播子:

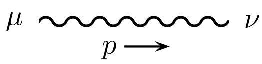


$$= \frac{-ig_{\mu\nu}}{p^2 + i\varepsilon} \quad \text{(Feynman 规范)}$$

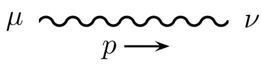


$$= \frac{-i(g_{\mu\nu} - p_\mu p_\nu/p^2)}{p^2 + i\varepsilon} \quad \text{(Landau 规范)}$$

有质量规范玻色子 (如 $W^\pm$ 和 Z) 传播子:



$$= \frac{-i(g_{\mu\nu} - p_\mu p_\nu/m^2)}{p^2 - m^2 + i\varepsilon} \quad \text{(幺正规范)}$$

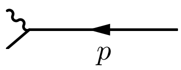


$$= \frac{-ig_{\mu\nu}}{p^2 - m^2 + i\varepsilon} \quad \text{(Feynman 规范)}$$

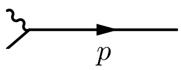
标量玻色子外线:

$= 1$ (初态或末态)

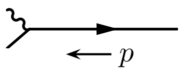
Dirac 费米子外线:



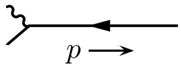
$= u(p,s)$ (正粒子初态)



$= \bar{u}(p,s)$ (正粒子末态)



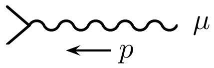
$= \bar{v}(p,s)$ (反粒子初态)

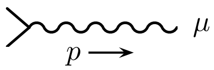
 $= v(p,s)$ （反粒子末态）

在计算非极化截面时，可利用自旋求和关系

$$\sum_s u(p,s)\bar{u}(p,s) = \not{p} + m, \quad \sum_s v(p,s)\bar{v}(p,s) = \not{p} - m. \tag{137}$$

矢量玻色子外线:

 $= \varepsilon_\mu(p,\lambda)$ （初态）

 $= \varepsilon^*_\mu(p,\lambda)$ （末态）

在计算非极化截面时，若包含无质量矢量玻色子外线，可作替换

$$\sum_\lambda \varepsilon^*_\mu(p,\lambda)\varepsilon_\nu(p,\lambda) \to -g_{\mu\nu}\,; \tag{138}$$

若包含有质量矢量玻色子外线，可作替换

$$\sum_\lambda \varepsilon^*_\mu(p,\lambda)\varepsilon_\nu(p,\lambda) \to -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}. \tag{139}$$

9 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2018 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中，光速、约化 Planck 常数和真空介电常数均取为 1，即 $c = \hbar = \varepsilon_0 = 1$。从而，速度没有量纲 (dimension)；长度量纲与时间量纲相同，是能量量纲的倒数；能量、质量和动量具有相同的量纲；精细结构常数表达为 $\alpha = e^2/(4\pi)$，而单位电荷量 $e = \sqrt{4\pi\alpha}$ 是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系取为

$$1 = c = 2.997\,924\,58 \times 10^{10}\ \text{cm}\cdot\text{s}^{-1}, \tag{140}$$
$$1 = \hbar = 6.582\,119\,514(40) \times 10^{-25}\ \text{GeV}\cdot\text{s}, \tag{141}$$
$$1 = \hbar c = 1.973\,269\,788(12) \times 10^{-14}\ \text{GeV}\cdot\text{cm}, \tag{142}$$

括号内数字代表测量值的 1σ 不确定度，由此可得

$$1\ \text{s} = 2.997\,925 \times 10^{10}\ \text{cm}, \qquad 1\ \text{cm} = 3.335\,641 \times 10^{-11}\ \text{s}, \tag{143}$$
$$1\ \text{s} = 1.519\,267 \times 10^{24}\ \text{GeV}^{-1}, \quad 1\ \text{GeV}^{-1} = 6.582\,120 \times 10^{-25}\ \text{s}, \tag{144}$$
$$1\ \text{cm} = 5.067\,731 \times 10^{13}\ \text{GeV}^{-1}, \quad 1\ \text{GeV}^{-1} = 1.973\,270 \times 10^{-14}\ \text{cm}, \tag{145}$$
$$1\ \text{cm}^2 = 2.568\,190 \times 10^{27}\ \text{GeV}^{-2}, \quad 1\ \text{GeV}^{-2} = 3.893\,794 \times 10^{-28}\ \text{cm}^2, \tag{146}$$
$$1\ \text{cm}^3\cdot\text{s}^{-1} = 8.566\,558 \times 10^{16}\ \text{GeV}^{-2}, \quad 1\ \text{GeV}^{-2} = 1.167\,330 \times 10^{-17}\ \text{cm}^3\cdot\text{s}^{-1}. \tag{147}$$

靶 (barn) 是散射截面的常用单位，记作 b，满足

$$1\ \mathrm{b} = 10^{-24}\ \mathrm{cm}^2 = 10^9\ \mathrm{nb} = 10^{12}\ \mathrm{pb} = 10^{15}\ \mathrm{fb} = 10^{18}\ \mathrm{ab}, \tag{148}$$

$$1\ \mathrm{pb} = 10^{-36}\ \mathrm{cm}^2 = 2.568\,190 \times 10^{-9}\ \mathrm{GeV}^{-2}, \quad 1\ \mathrm{GeV}^{-2} = 3.893\,794 \times 10^{8}\ \mathrm{pb}. \tag{149}$$

Fermi 耦合常数是

$$G_{\mathrm{F}} = 1.166\,378\,7(6) \times 10^{-5}\ \mathrm{GeV}^{-2}. \tag{150}$$

由树图阶关系式

$$\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_{\mathrm{W}}^2}, \tag{151}$$

可得 Higgs 场真空期待值为

$$v = (\sqrt{2}G_{\mathrm{F}})^{-1/2} = 246.2197\ \mathrm{GeV}. \tag{152}$$

在低能标 (Thomson 极限) 处，精细结构常数为

$$\alpha = 7.297\,352\,5664(17) \times 10^{-3} = \frac{1}{137.035\,999\,139(31)}; \tag{153}$$

在 $\overline{\mathrm{MS}}$ 重整化方案（以 $\widehat{\ }$ 为标志）中，α^{-1} 跑动到 $\mu = m_Z$ 能标处的数值是

$$\widehat{\alpha}^{-1}(m_Z) = 127.955 \pm 0.010\,. \tag{154}$$

在 $\overline{\mathrm{MS}}$ 方案中，$\mu = m_Z$ 能标处强耦合常数 $\alpha_{\mathrm{s}} = g_{\mathrm{s}}^2/(4\pi)$ 的数值为

$$\widehat{\alpha}_{\mathrm{s}}(m_Z) = 0.1181 \pm 0.0011, \tag{155}$$

Weinberg 角 θ_{W} 的数值对应于

$$\widehat{s}_{\mathrm{W}}^2 = \sin^2\widehat{\theta}_{\mathrm{W}}(m_Z) = 0.23122 \pm 0.00004\,. \tag{156}$$

在标准模型中，光子、胶子和中微子没有质量，其它基本粒子的质量为

$$m_W = 80.379 \pm 0.012\ \mathrm{GeV}, \quad m_Z = 91.1876 \pm 0.0021\ \mathrm{GeV}, \quad m_H = 125.18 \pm 0.16\ \mathrm{GeV}, \tag{157}$$

$$m_e = 0.510\,998\,9461(31)\ \mathrm{MeV}, \quad m_\mu = 105.658\,3745(24)\ \mathrm{MeV}, \quad m_\tau = 1776.86 \pm 0.12\ \mathrm{MeV}, \tag{158}$$

$$m_u = 2.2^{+0.5}_{-0.4}\ \mathrm{MeV}, \quad m_d = 4.7^{+0.5}_{-0.3}\ \mathrm{MeV}, \quad m_s = 95^{+9}_{-3}\ \mathrm{MeV}, \tag{159}$$

$$m_c = 1.275^{+0.025}_{-0.035}\ \mathrm{GeV}, \quad m_b = 4.18^{+0.04}_{-0.03}\ \mathrm{GeV}, \quad m_t = 173.0 \pm 0.4\ \mathrm{GeV}. \tag{160}$$

这里，u、d、s 夸克的质量是 $\mu \simeq 2$ GeV 能标处的流夸克质量 (current-quark mass)，c、b 夸克的质量是 $\overline{\mathrm{MS}}$ 方案中的跑动质量 (running mass)，其余粒子的质量均为极点质量 (pole mass)。质子和中子的质量为

$$m_p = 938.272\,0813(58)\ \mathrm{MeV}, \quad m_n = 939.565\,413(6)\ \mathrm{MeV}. \tag{161}$$

在电弱能标附近作领头阶计算时，可将单位电荷量 e 取为

$$e = \sqrt{4\pi\widehat{\alpha}(m_Z)} = 0.313\,383\,6\,, \tag{162}$$

将强耦合常数 g_s 取为

$$g_\mathrm{s}=\sqrt{4\pi\widehat{\alpha}_\mathrm{s}(m_Z)}=1.218\,232\,. \tag{163}$$

从树图阶关系计算 Higgs 场四线性耦合常数 λ 和 Yukawa 耦合常数 y_t、y_b、y_τ、y_c，得

$$\lambda=\frac{m_H^2}{2v^2}=0.129\,239\,3,\quad y_t=\frac{\sqrt{2}m_t}{v}=0.993\,661\,3,\quad y_b=\frac{\sqrt{2}m_b}{v}=2.400\,870\times10^{-2}, \tag{164}$$

$$y_\tau=\frac{\sqrt{2}m_\tau}{v}=1.020\,576\times10^{-2},\quad y_c=\frac{\sqrt{2}m_c}{v}=7.323\,227\times10^{-3}. \tag{165}$$

耦合常数 g 和 g' 有以下两种取值方式。

1. 根据树图阶关系 $\sin^2\theta_\mathrm{W}=1-m_W^2/m_Z^2$ 计算 Weinberg 角，得

$$s_\mathrm{W}^2=1-\frac{m_W^2}{m_Z^2}=0.223\,013\,2,\quad c_\mathrm{W}^2=1-s_\mathrm{W}^2=0.776\,986\,8, \tag{166}$$

$$s_\mathrm{W}=\sqrt{s_\mathrm{W}^2}=0.472\,242\,8,\quad c_\mathrm{W}=\sqrt{c_\mathrm{W}^2}=0.881\,468\,5, \tag{167}$$

故

$$g=\frac{e}{s_\mathrm{W}}=0.663\,607\,1,\quad g'=\frac{e}{c_\mathrm{W}}=0.355\,524\,5\,. \tag{168}$$

2. 根据 $\overline{\mathrm{MS}}$ 方案中 Weinberg 角的数值 (156) 计算 g 和 g'，得

$$c_\mathrm{W}^2=1-\widehat{s}_\mathrm{W}^2=0.76878,\quad s_\mathrm{W}=\sqrt{\widehat{s}_\mathrm{W}^2}=0.480\,853\,4,\quad c_\mathrm{W}=\sqrt{c_\mathrm{W}^2}=0.876\,801\,0, \tag{169}$$

$$g=\frac{e}{s_\mathrm{W}}=0.651\,723\,8,\quad g'=\frac{e}{c_\mathrm{W}}=0.357\,417\,0\,. \tag{170}$$

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[2] T. P. Cheng and L. F. Li, “Gauge Theory of Elementary Particle Physics,” Oxford, UK: Clarendon (1984), 536 pages.

[3] A. Denner, “Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200,” Fortsch. Phys. **41**, 307 (1993) [arXiv:0709.1075 [hep-ph]].

[4] 杜东生，杨茂志，《粒子物理导论》，中国北京：科学出版社 (2014)，402 页。

[5] M. Tanabashi *et al.* [Particle Data Group], “Review of Particle Physics,” Phys. Rev. D **98**, 030001 (2018).