

# 粒子物理标准模型拉氏量和费曼规则

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## 1 约定

本文采用自然单位制，各种约定主要遵从文献 [1]，推导和计算参考文献 [1, 2, 3, 4]。  
Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \quad (1)$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad (2)$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \quad (3)$$

手征表示中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}. \quad (4)$$

左右手投影算符

$$P_L \equiv \frac{1}{2}(1 - \gamma_5) = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \quad P_R \equiv \frac{1}{2}(1 + \gamma_5) = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}. \quad (5)$$

Levi-Civita 张量约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \quad (6)$$

费曼规则约定：

- 对于指向相互作用顶点的动量  $p$ ，时空偏导数  $\partial_\mu$  在动量空间费曼规则里贡献一个  $-ip_\mu$  因子。
- 实线表示费米子，实线上的箭头表示费米子数流动的方向。
- 虚线表示标量玻色子，虚线上的箭头表示电荷数流动的方向。
- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示电荷数流动的方向。
- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。
- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

## 2 标准模型概述

粒子物理标准模型是一个  $SU(3)_C \times SU(2)_L \times U(1)_Y$  规范理论。模型中有三代费米子，包括三代中微子  $\nu_i = \nu_e, \nu_\mu, \nu_\tau$ ，三代带电轻子  $\ell_i = e, \mu, \tau$ ，三代上型夸克  $u_i = u, c, t$  和三代下型夸克  $d_i = d, s, b$  ( $i = 1, 2, 3$ )。规范玻色子传递费米子间相互作用。

$SU(3)_C$  部分描述夸克的强相互作用，称为量子色动力学 (Quantum Chromodynamics, QCD)，相应的规范玻色子是胶子。 $SU(2)_L \times U(1)_Y$  部分统一描述夸克和轻子的电磁和弱相互作用，称为电弱统一理论。理论中有一个 Higgs 二重态，通过 Brout-Englert-Higgs 机制引发规范群的自发对称性破缺，使  $SU(2)_L \times U(1)_Y$  群破缺为  $U(1)_{EM}$  群。 $U(1)_{EM}$  规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前。理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度；左手费米子和右手费米子都没有质量。具有不同量子数。

破缺后，3 个规范玻色子与 3 个 Higgs 自由度结合，从而获得质量，成为  $W^\pm$  和  $Z^0$  玻色子，传递弱相互作用；剩下的 1 个无质量规范玻色子是光子，即是  $U(1)_{\text{EM}}$  群的规范玻色子，传递电磁相互作用；剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子；与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量，组合成 Dirac 费米子。

理论中的中微子没有右手分量，因而没有获得质量。1998 年实验发现中微子振荡，证明中微子具有质量，所以需要扩充标准模型才能正确描述中微子物理。

### 3 QCD 拉氏量和费曼规则

QCD 的拉氏量可表达成

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \dots, 8, \quad (7)$$

其中

$$D_\mu = \partial_\mu - ig_s G_\mu^a t^a, \quad G^{a\mu\nu} \equiv \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{abc} G^{b\mu} G^{c\nu}. \quad (8)$$

$SU(3)_C$  群基础表示生成元  $t^a = \lambda^a/2$ ，其中  $\lambda^a$  为 Gell-Mann 矩阵。生成元对易关系为  $[t^a, t^b] = if^{abc}t^c$ 。结构常数  $f^{abc}$  是全反对称的，其非零分量为

$$f_{123} = 1, \quad f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \quad (9)$$

由

$$\begin{aligned} -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} &= -\frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{ade} G^{d\mu} G^{e\nu}) \\ &= -\frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu}) - (\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu})] - g_s f^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} \\ &\quad - \frac{1}{4}g_s^2 f^{abc} f^{ade} G_\mu^b G_\nu^c G^{d\mu} G^{e\nu}, \end{aligned} \quad (10)$$

可得

$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= \sum_q [\bar{q}(i\gamma^\mu \partial_\mu - m_q)q + g_s G_\mu^a \bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu})] \\ &\quad - g_s f^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} - \frac{1}{4}g_s^2 f^{abc} f^{ade} G_\mu^b G_\nu^c G^{d\mu} G^{e\nu}. \end{aligned} \quad (11)$$

设用于固定胶子场规范的函数  $G^a(x) = \partial^\mu G_\mu^a(x) - \omega^a(x)$ ，其中  $\omega^a(x)$  是某个任意函数，规范固定条件是  $G^a(x) = 0$ 。这是 Lorenz 规范的推广， $\omega^a(x) = 0$  对应于 Lorenz 规范。在路径积分量子化中，以中心为  $\omega^a(x) = 0$  的 Gauss 权重对  $\omega^a(x)$  作泛函积分，有

$$\int \mathcal{D}\omega^a \exp \left[ -i \int d^4x \frac{1}{2\xi} (\omega^a)^2 \right] \delta(G^a) = \exp \left[ -i \int d^4x \frac{1}{2\xi} (\partial^\mu G_\mu^a)^2 \right]. \quad (12)$$

可见，拉氏量中的规范固定项为

$$\mathcal{L}_{\text{QCD,GF}} = -\frac{1}{2\xi}(\partial^\mu G_\mu^a)^2. \quad (13)$$

$\xi$  的任何一个取值对应于一种规范。 $\xi = 1$  称为 Feynman-'t Hooft 规范， $\xi = 0$  称为 Landau 规范。于是，胶子传播子相关拉氏量为

$$\begin{aligned} \mathcal{L}_{\text{QCD,prop}} &= \frac{1}{2} \left[ (\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu}) - \frac{1}{\xi}(\partial^\mu G_\mu^a)^2 \right] \\ &\rightarrow \frac{1}{2} G_\mu^a \left[ g^{\mu\nu} \partial^2 - \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial^\nu \right] G_\nu^a. \end{aligned} \quad (14)$$

变换到动量空间，得

$$-g^{\mu\nu} p^2 + \left(1 - \frac{1}{\xi}\right) p^\mu p^\nu, \quad (15)$$

它的逆矩阵是

$$-\frac{1}{p^2} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right], \quad (16)$$

这是因为

$$\begin{aligned} &-\frac{1}{p^2} \left[ g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} (1 - \xi) \right] \left[ -g^{\mu\nu} p^2 + \left(1 - \frac{1}{\xi}\right) p^\mu p^\nu \right] \\ &= \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \left(1 - \frac{1}{\xi}\right) - \frac{p_\rho p^\nu}{p^2} (1 - \xi) + \frac{p_\rho p^\nu}{p^2} (1 - \xi) \left(1 - \frac{1}{\xi}\right) = \delta_\rho^\nu. \end{aligned} \quad (17)$$

从而，胶子传播子的形式为

$$\frac{-i\delta^{ab}}{p^2 + i\varepsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right]. \quad (18)$$

$\text{SU}(3)_C$  规范变换为

$$q \rightarrow Uq, \quad G_\mu^a t^a \rightarrow U G_\mu^a t^a U^\dagger + \frac{i}{g_s} U \partial_\mu U^\dagger, \quad (19)$$

其中  $U(x) = \exp[i\alpha^a(x)t^a]$ 。胶子场的无穷小规范变换形式是

$$\begin{aligned} G_\mu^a t^a &\rightarrow (1 + i\alpha^a t^a) G_\mu^b t^b (1 - i\alpha^c t^c) + \frac{i}{g_s} (1 + i\alpha^a t^a) \partial_\mu (1 - i\alpha^c t^c) \\ &= G_\mu^b t^b + i\alpha^a G_\mu^b [t^a, t^b] + \frac{1}{g_s} (\partial_\mu \alpha^c) t^c + \mathcal{O}(\alpha^2) = G_\mu^a t^a - f^{abc} \alpha^a G_\mu^b t^c + \frac{1}{g_s} (\partial_\mu \alpha^a) t^a + \mathcal{O}(\alpha^2) \\ &= \left( G_\mu^a + f^{abc} G_\mu^b \alpha^c + \frac{1}{g_s} \partial_\mu \alpha^a \right) t^a + \mathcal{O}(\alpha^2), \end{aligned} \quad (20)$$

即

$$\delta G_\mu^a = \frac{1}{g_s} \partial_\mu \alpha^a + f^{abc} G_\mu^b \alpha^c = \left( \frac{1}{g_s} \delta^{ac} \partial_\mu + f^{abc} G_\mu^b \right) \alpha^c, \quad (21)$$

因而规范固定函数  $G^a$  的无穷小规范变换为

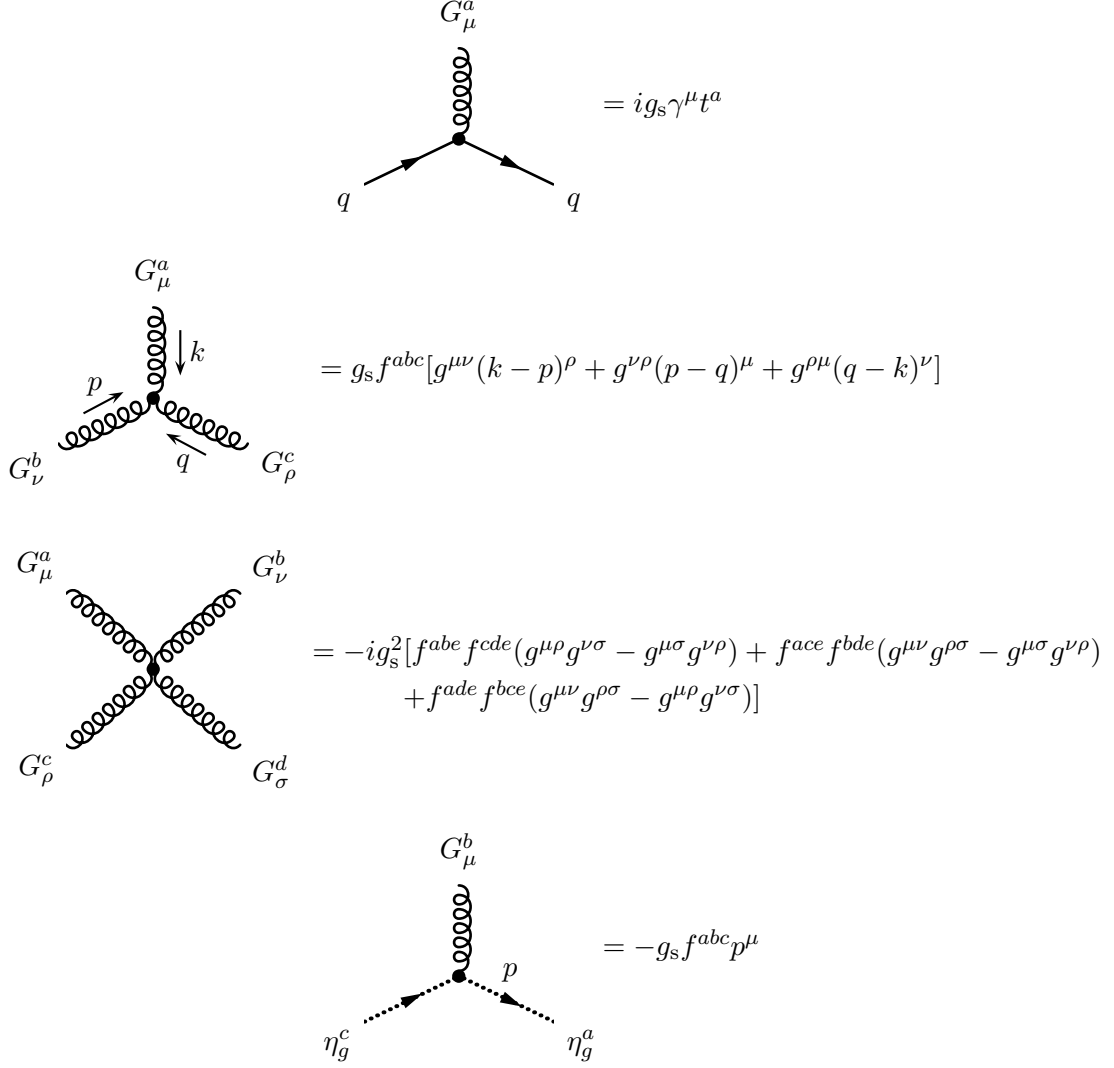
$$\delta G^a = \partial^\mu \delta G_\mu^a = \frac{1}{g_s} \delta^{ac} \partial^2 \alpha^c + f^{abc} \partial^\mu G_\mu^b \alpha^c, \quad g_s \frac{\delta G^a}{\delta \alpha^c} = \delta^{ab} \partial^2 + g_s f^{abc} \partial^\mu G_\mu^b. \quad (22)$$

Faddeev-Popov 鬼场的拉氏量是

$$\mathcal{L}_{\text{QCD,FP}} = -\bar{\eta}_g^a \left( g_s \frac{\delta G^a}{\delta \alpha^c} \right) \eta_g^c = -\bar{\eta}_g^a (\delta^{ac} \partial^2 + g_s f^{abc} \partial^\mu G_\mu^b) \eta_g^c \rightarrow -\bar{\eta}_g^a \delta^{ab} \partial^2 \eta_g^b + g_s f^{abc} (\partial^\mu \bar{\eta}_g^a) G_\mu^b \eta_g^c. \quad (23)$$

下面列出 QCD 费曼规则。

QCD 顶点：



$$= ig_s \gamma^\mu t^a$$

$$= g_s f^{abc} [g^{\mu\nu} (k-p)^\rho + g^{\nu\rho} (p-q)^\mu + g^{\rho\mu} (q-k)^\nu]$$

$$= -ig_s^2 [f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})]$$

$$= -g_s f^{abc} p^\mu$$

胶子传播子：

$$G_\mu^a \xrightarrow{p} G_\nu^b = \frac{-i\delta^{ab}}{p^2 + i\varepsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right]$$

鬼粒子传播子：

$$\eta_g^a \xrightarrow{p} \eta_g^b = \frac{i\delta^{ab}}{p^2 + i\varepsilon}$$

## 4 费米子电弱规范相互作用拉氏量和费曼规则

表 1 列出标准模型费米子场的量子数电荷数  $Q$ 、弱同位旋第 3 分量  $T^3$ 、弱超荷  $Y$ 、重子数  $B$  和轻子数  $L_e/L_\mu/L_\tau$ 。每代左手费米子场构成 2 个  $SU(2)_L$  二重态

$$L_{iL} = \begin{pmatrix} P_L \nu_i \\ P_L \ell_i \end{pmatrix} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}, \quad Q_{iL} = \begin{pmatrix} P_L u_i \\ P_L d'_i \end{pmatrix} = \begin{pmatrix} u_{iL} \\ d'_{iL} \end{pmatrix}, \quad i = 1, 2, 3. \quad (24)$$

下型夸克的质量本征态  $d_j$  与规范本征态  $d'_j$  通过 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵  $V_{ij}$  联系起来：

$$d'_i = V_{ij} d_j. \quad (25)$$

右手费米子场  $\ell_{iR} = P_R \ell_i$ 、 $u_{iR} = P_R u_i$  和  $d'_{iR} = P_R d'_i$  是  $SU(2)_L$  单态。它们的电荷数  $Q$ 、弱同位旋第 3 分量  $T^3$  和弱超荷  $Y$  满足关系

$$Q = T^3 + Y. \quad (26)$$

表 1: 标准模型费米子场的量子数。

统一记号	第一代	第二代	第三代	$Q$	$T^3$	$Y$	$B$	$L_e/L_\mu/L_\tau$
$L_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix}$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	0	1/2	-1/2	0	1
				-1	-1/2	-1/2	0	1
$Q_{iL} = \begin{pmatrix} u_{iL} \\ d'_{iL} \end{pmatrix}$	$\begin{pmatrix} u_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b'_L \end{pmatrix}$	2/3	1/2	1/6	1/3	0
				-1/3	-1/2	1/6	1/3	0
$\ell_{iR}$	$e_R$	$\mu_R$	$\tau_R$	-1	0	-1	0	1
$u_{iR}$	$u_R$	$c_R$	$t_R$	2/3	0	2/3	1/3	0
$d'_{iR}$	$d'_R$	$s'_R$	$b'_R$	-1/3	0	-1/3	1/3	0

$SU(2)_L \times U(1)_Y$  规范不变的费米子协变动能项为

$$\mathcal{L}_{\text{EWF}} = \bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}_{iR} i \not{D} u_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} + \bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR}. \quad (27)$$

对于  $SU(2)_L$  二重态  $Q_{iL}$  和  $L_{iL}$ ，协变导数为

$$D_\mu = \partial_\mu - ig' B_\mu Y - ig W_\mu^a \tau^a, \quad \tau^a = \frac{\sigma^a}{2}. \quad (28)$$

弱同位旋第 3 分量  $T^3$  是生成元  $\tau^3$  的本征值。对于  $SU(2)_L$  单态  $u_{iR}$ 、 $d'_{iR}$  和  $\ell_{iR}$ ，协变导数为

$$D_\mu = \partial_\mu - ig' B_\mu Y. \quad (29)$$

规范场  $W_\mu^a(x)$  和  $B_\mu(x)$  跟左手费米子场的相互作用与右手费米子场不同，而在 QED 中，电磁场  $A_\mu(x)$  跟左手费米子场的相互作用却与右手费米子场相同。为了回到 QED 的情况，需要把  $W_\mu^3(x)$  和  $B_\mu(x)$  混

合起来, 得到电磁场  $A_\mu(x)$  和另一个中性规范场  $Z_\mu(x)$ , 即定义

$$A_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}}(g'W_\mu^3 + gB_\mu) = s_W W_\mu^3 + c_W B_\mu, \quad (30)$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}}(gW_\mu^3 - g'B_\mu) = c_W W_\mu^3 - s_W B_\mu, \quad (31)$$

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad (32)$$

或

$$B_\mu = c_W A_\mu - s_W Z_\mu, \quad W_\mu^3 = s_W A_\mu + c_W Z_\mu, \quad (33)$$

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \quad (34)$$

参数间有如下关系,

$$s_W \equiv \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_W \equiv \cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = g s_W = g' c_W. \quad (35)$$

这里  $\theta_W$  称为 Weinberg 角。

利用

$$\begin{aligned} g'Y B_\mu + gT^3 W_\mu^3 &= g'Y (c_W A_\mu - s_W Z_\mu) + gT^3 (s_W A_\mu + c_W Z_\mu) \\ &= e(Y + T^3)A_\mu + \left( g c_W T^3 - \frac{g s_W}{c_W} s_W Y \right) Z_\mu = Q e A_\mu + \frac{g}{c_W} (T^3 c_W^2 - Y s_W^2) Z_\mu \\ &= Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu, \end{aligned} \quad (36)$$

有

$$\begin{aligned} D_\mu Q_{iL} &= (\partial_\mu - i g' B_\mu Y - i g W_\mu^a \tau^a) Q_{iL} = \partial_\mu Q_{iL} - i \begin{pmatrix} g'Y B_\mu + gT^3 W_\mu^3 & \frac{1}{2}g(W_\mu^1 - iW_\mu^2) \\ \frac{1}{2}g(W_\mu^1 + iW_\mu^2) & g'Y B_\mu + gT^3 W_\mu^3 \end{pmatrix} Q_{iL} \\ &= \partial_\mu Q_{iL} - i \begin{pmatrix} Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu & \frac{1}{\sqrt{2}} g W_\mu^+ \\ \frac{1}{\sqrt{2}} g W_\mu^- & Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \end{pmatrix} Q_{iL} \\ &= \partial_\mu Q_{iL} - i \begin{pmatrix} \left[ Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \right] u_{iL} + \frac{1}{\sqrt{2}} g W_\mu^+ d'_{iL} \\ \frac{1}{\sqrt{2}} g W_\mu^- u_{iL} + \left[ Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \right] d'_{iL} \end{pmatrix}, \end{aligned} \quad (37)$$

故

$$\begin{aligned} \bar{Q}_{iL} i \not{D} Q_{iL} &\supset \left[ Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \right] \bar{u}_{iL} \gamma^\mu u_{iL} + \left[ Q e A_\mu + \frac{g}{c_W} (T^3 - Q s_W^2) Z_\mu \right] \bar{d}'_{iL} \gamma^\mu d'_{iL} \\ &\quad + \frac{1}{\sqrt{2}} g W_\mu^+ \bar{u}_{iL} \gamma^\mu d'_{iL} + \frac{1}{\sqrt{2}} g W_\mu^- \bar{d}'_{iL} \gamma^\mu u_{iL} \end{aligned}$$

$$\begin{aligned}
&= \left( QeA_\mu + \frac{g}{c_W} g_L Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1-\gamma_5}{2} u_i + \left( QeA_\mu + \frac{g}{c_W} g_L Z_\mu \right) \bar{d}_i \gamma^\mu \frac{1-\gamma_5}{2} d_i \\
&\quad + \frac{1}{\sqrt{2}} g W_\mu^+ \bar{u}_i \gamma^\mu \frac{1-\gamma_5}{2} V_{ij} d_j + \frac{1}{\sqrt{2}} g W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu \frac{1-\gamma_5}{2} u_i,
\end{aligned} \tag{38}$$

其中

$$g_L \equiv T^3 - Qs_W^2. \tag{39}$$

另一方面,

$$D_\mu d'_{iR} = (\partial_\mu - ig' B_\mu Y) d'_{iR} = \partial_\mu d'_{iR} - ig' Q(c_W A_\mu - s_W Z_\mu) d'_{iR} = \partial_\mu d'_{iR} - iQeA_\mu d'_{iR} + i\frac{g}{c_W} Qs_W^2 Z_\mu d'_{iR}, \tag{40}$$

则

$$\begin{aligned}
\bar{u}_{iR} i \not{D} u_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} &\supset \left( QeA_\mu - \frac{g}{c_W} Qs_W^2 Z_\mu \right) \bar{u}_{iR} \gamma^\mu u_{iR} + \left( QeA_\mu - \frac{g}{c_W} Qs_W^2 Z_\mu \right) \bar{d}'_{iR} \gamma^\mu d'_{iR} \\
&= \left( QeA_\mu + \frac{g}{c_W} g_R Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1+\gamma_5}{2} u_i + \left( QeA_\mu + \frac{g}{c_W} g_R Z_\mu \right) \bar{d}_i \gamma^\mu \frac{1+\gamma_5}{2} d_i,
\end{aligned} \tag{41}$$

其中

$$g_R \equiv -Qs_W^2. \tag{42}$$

定义

$$g_V \equiv g_L + g_R = T^3 - 2Qs_W^2, \quad g_A \equiv g_L - g_R = T^3, \tag{43}$$

可得

$$\begin{aligned}
&\bar{Q}_{iL} i \not{D} Q_{iL} + \bar{u}_{iR} i \not{D} u_{iR} + \bar{d}'_{iR} i \not{D} d'_{iR} \\
&\supset Qe\bar{u}_i \gamma^\mu u_i A_\mu + Qe\bar{d}_i \gamma^\mu d_i A_\mu + \frac{g}{2c_W} \bar{u}_i \gamma^\mu (g_V - g_A \gamma_5) u_i Z_\mu + \frac{g}{2c_W} \bar{d}_i \gamma^\mu (g_V - g_A \gamma_5) d_i Z_\mu \\
&\quad + \frac{1}{\sqrt{2}} g W_\mu^+ \bar{u}_i \gamma^\mu P_L V_{ij} d_j + \frac{1}{\sqrt{2}} g W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_L u_i.
\end{aligned} \tag{44}$$

同理, 有

$$\begin{aligned}
\bar{L}_{iL} i \not{D} L_{iL} + \bar{\ell}_{iR} i \not{D} \ell_{iR} &\supset Qe\bar{\ell}_i \gamma^\mu \ell_i A_\mu + \frac{g}{2c_W} \bar{\ell}_i \gamma^\mu (g_V - g_A \gamma_5) \ell_i Z_\mu + \frac{g}{2c_W} \bar{\nu}_i \gamma^\mu (g_V - g_A \gamma_5) \nu_i Z_\mu \\
&\quad + \frac{1}{\sqrt{2}} g W_\mu^+ \bar{\nu}_i \gamma^\mu P_L \ell_i + \frac{1}{\sqrt{2}} g W_\mu^- \bar{\ell}_i \gamma^\mu P_L \nu_i.
\end{aligned} \tag{45}$$

总结起来, 可以写成流耦合的形式,

$$\begin{aligned}
\mathcal{L}_{\text{EWF}} &\supset \sum_f \left[ Q_f e \bar{f} \gamma^\mu f A_\mu + \frac{g}{2c_W} \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma_5) f Z_\mu \right] + g(W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}) \\
&= eA_\mu J_{\text{EM}}^\mu + g(Z_\mu J_Z^\mu + W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}),
\end{aligned} \tag{46}$$



其中，流的定义为

$$J_{\text{EM}}^\mu \equiv \sum_f Q_f \bar{f} \gamma^\mu f, \quad J_Z^\mu \equiv \frac{1}{2c_W} \sum_f \bar{f} \gamma^\mu (g_V^f - g_A^f \gamma_5) f = \frac{1}{c_W} \sum_f (g_L^f \bar{f}_L \gamma^\mu f_L + g_R^f \bar{f}_R \gamma^\mu f_R),$$

$$J_W^{+\mu} \equiv \frac{1}{\sqrt{2}} (\bar{u}_{iL} \gamma^\mu V_{ij} d_{jL} + \bar{\nu}_{iL} \gamma^\mu \ell_{iL}), \quad J_W^{-\mu} \equiv \frac{1}{\sqrt{2}} (\bar{d}_{jL} V_{ji}^\dagger \gamma^\mu u_{iL} + \bar{\ell}_{iL} \gamma^\mu \nu_{iL}). \quad (47)$$

对于各种费米子，相关系数如下，

$$Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}, \quad Q_{\nu_i} = 0, \quad Q_{\ell_i} = -1; \quad (48)$$

$$g_V^{u_i} = \frac{1}{2} - \frac{4}{3}s_W^2, \quad g_A^{u_i} = \frac{1}{2}; \quad g_V^{d_i} = -\frac{1}{2} + \frac{2}{3}s_W^2, \quad g_A^{d_i} = -\frac{1}{2}; \quad (49)$$

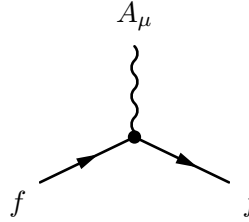
$$g_V^{\nu_i} = \frac{1}{2}, \quad g_A^{\nu_i} = \frac{1}{2}; \quad g_V^{\ell_i} = -\frac{1}{2} + 2s_W^2, \quad g_A^{\ell_i} = -\frac{1}{2}; \quad (50)$$

$$g_L^{u_i} = \frac{1}{2} - \frac{2}{3}s_W^2, \quad g_R^{u_i} = -\frac{2}{3}s_W^2; \quad g_L^{d_i} = -\frac{1}{2} + \frac{1}{3}s_W^2, \quad g_R^{d_i} = \frac{1}{3}s_W^2; \quad (51)$$

$$g_L^{\nu_i} = \frac{1}{2}, \quad g_R^{\nu_i} = 0; \quad g_L^{\ell_i} = -\frac{1}{2} + s_W^2, \quad g_R^{\ell_i} = s_W^2. \quad (52)$$

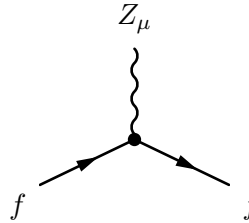
下面给出费米子电弱规范相互作用顶点的费曼规则。

QED 顶点：



$$= iQ_f e \gamma^\mu \quad (\text{对于电子, } Q_e = -1)$$

费米子与  $Z$  玻色子的耦合：

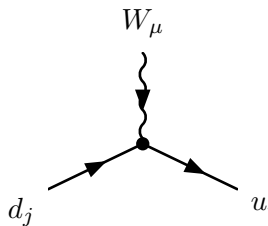


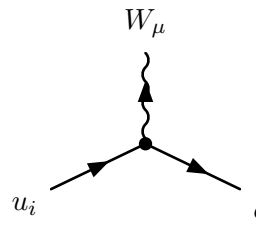
$$= i \frac{g}{2c_W} \gamma^\mu (g_V^f - g_A^f \gamma_5)$$

$$g_V^{u_i} = \frac{1}{2} - \frac{4}{3}s_W^2, \quad g_A^{u_i} = \frac{1}{2}; \quad g_V^{d_i} = -\frac{1}{2} + \frac{2}{3}s_W^2, \quad g_A^{d_i} = -\frac{1}{2};$$

$$g_V^{\nu_i} = \frac{1}{2}, \quad g_A^{\nu_i} = \frac{1}{2}; \quad g_V^{\ell_i} = -\frac{1}{2} + 2s_W^2, \quad g_A^{\ell_i} = -\frac{1}{2}.$$

费米子与  $W^\pm$  玻色子的耦合：



$$= i \frac{g}{\sqrt{2}} V_{ij} \gamma^\mu P_L$$


$$= i \frac{g}{\sqrt{2}} V_{ji}^\dagger \gamma^\mu P_L$$

$$\begin{aligned}
& \text{Diagram 1: } l_i \text{ and } \nu_i \text{ meet at a vertex, with } W_\mu \text{ exiting.} = i \frac{g}{\sqrt{2}} \gamma^\mu P_L \\
& \text{Diagram 2: } \nu_i \text{ and } l_i \text{ meet at a vertex, with } W_\mu \text{ exiting.} = i \frac{g}{\sqrt{2}} \gamma^\mu P_L
\end{aligned}$$

## 5 电弱规范场自相互作用拉氏量和费曼规则

电弱规范场自相互作用拉氏量是

$$\mathcal{L}_{\text{EWG}} = -\frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \quad (53)$$

其中

$$W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g\varepsilon^{abc}W^{b\mu}W^{c\nu}, \quad B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu. \quad (54)$$

利用 (33) 式和 (34) 式, 可得

$$\begin{aligned}
& W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 \\
&= \frac{i}{\sqrt{2}}[(W_\mu^+ - W_\mu^-)(s_W A_\nu + c_W Z_\nu) - (s_W A_\mu + c_W Z_\mu)(W_\nu^+ - W_\nu^-)] \\
&= \frac{i}{\sqrt{2}}[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) - s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)],
\end{aligned} \quad (55)$$

$$\begin{aligned}
& W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 \\
&= \frac{1}{\sqrt{2}}[(s_W A_\mu + c_W Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_W A_\nu + c_W Z_\nu)] \\
&= -\frac{1}{\sqrt{2}}[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) + s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)].
\end{aligned} \quad (56)$$

从而,

$$\begin{aligned}
W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g\varepsilon^{1bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g(W_\mu^2 W_\nu^3 - gW_\mu^3 W_\nu^2) \\
&= \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^2 W_\nu^3 - gW_\mu^3 W_\nu^2) \\
&= \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad + \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{1}{\sqrt{2}}(F_{\mu\nu}^+ + F_{\mu\nu}^-),
\end{aligned} \quad (57)$$

其中,

$$F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \quad (58)$$

$$F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ie(W_\mu^- A_\nu - A_\mu W_\nu^-) - igc_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \quad (59)$$

另一方面,

$$\begin{aligned}
W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 + g\varepsilon^{2bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - gW_\mu^1 W_\nu^3 + gW_\mu^3 W_\nu^1 \\
&= \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\
&= \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_W(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad - \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_W(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_W(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{i}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-).
\end{aligned} \tag{60}$$

因此,

$$\begin{aligned}
&-\frac{1}{4}W_{\mu\nu}^1 W^{1\mu\nu} - \frac{1}{4}W_{\mu\nu}^2 W^{2\mu\nu} \\
&= -\frac{1}{8}(F_{\mu\nu}^+ + F_{\mu\nu}^-)(F^{+\mu\nu} + F^{-\mu\nu}) + \frac{1}{8}(F_{\mu\nu}^+ - F_{\mu\nu}^-)(F^{+\mu\nu} - F^{-\mu\nu}) = -\frac{1}{2}F_{\mu\nu}^+ F^{-\mu\nu} \\
&= -\frac{1}{2}[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_W(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \\
&\quad \times [\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu} - ie(W^{-\mu} A^\nu - A^\mu W^{-\nu}) - igc_W(W^{-\mu} Z^\nu - Z^\mu W^{-\nu})] \\
&= -(\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) \\
&\quad + ie[(\partial_\mu W_\nu^+)W^{-\mu} A^\nu - (\partial_\mu W_\nu^+)W^{-\nu} A^\mu - W_\mu^+(\partial^\mu W^{-\nu})A_\nu + W_\nu^+(\partial^\mu W^{-\nu})A_\mu] \\
&\quad + igc_W[(\partial_\mu W_\nu^+)W^{-\mu} Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu} Z^\mu - W_\mu^+(\partial^\mu W^{-\nu})Z_\nu + W_\nu^+(\partial^\mu W^{-\nu})Z_\mu] \\
&\quad + e^2(W_\mu^+ W^{-\nu} A_\nu A^\mu - W_\mu^+ W^{-\mu} A_\nu A^\nu) + g^2 c_W^2 (W_\mu^+ W^{-\nu} Z_\nu Z^\mu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu) \\
&\quad + egc_W(W_\mu^+ W^{-\nu} A_\nu Z^\mu + W_\mu^+ W^{-\nu} A^\mu Z_\nu - 2W_\mu^+ W^{-\mu} A_\nu Z^\nu).
\end{aligned} \tag{61}$$

由

$$W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1 = \frac{i}{2}(W_\mu^+ + W_\mu^-)(W_\nu^+ - W_\nu^-) - \frac{i}{2}(W_\mu^+ - W_\mu^-)(W_\nu^+ + W_\nu^-) = -i(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \tag{62}$$

可得

$$\begin{aligned}
W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 + g\varepsilon^{3bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 + gW_\mu^1 W_\nu^2 - gW_\mu^2 W_\nu^1 \\
&= s_W \partial_\mu A_\nu + c_W \partial_\mu Z_\nu - s_W \partial_\nu A_\mu + c_W \partial_\nu Z_\mu + g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
&= s_W(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_W(\partial_\mu Z_\nu - \partial_\nu Z_\mu) - ig(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+),
\end{aligned} \tag{63}$$

$$B_{\mu\nu} = \partial_\mu(c_W A_\nu - s_W Z_\nu) - \partial_\nu(c_W A_\mu - s_W Z_\mu) = c_W(\partial_\mu A_\nu - \partial_\nu A_\mu) - s_W(\partial_\mu Z_\nu - \partial_\nu Z_\mu). \tag{64}$$

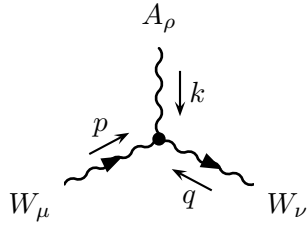
于是,

$$\begin{aligned}
&-\frac{1}{4}W_{\mu\nu}^3 W^{3\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} \\
&= -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
&\quad + ie[W^{+\mu} W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu} W^{-\mu}(\partial_\mu A_\nu)] + igc_W[W^{+\mu} W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu} W^{-\mu}(\partial_\mu Z_\nu)] \\
&\quad + \frac{1}{2}g^2(W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu} - W_\mu^+ W^{+\nu} W_\nu^- W^{-\mu}).
\end{aligned} \tag{65}$$

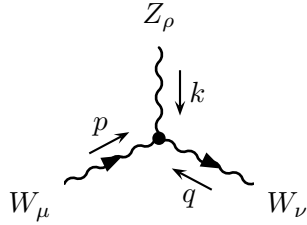
综合起来，有

$$\begin{aligned}
\mathcal{L}_{\text{EWG}} = & \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\
& + (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + \frac{1}{2}g^2(W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu} - W_\mu^+ W^{+\nu} W_\nu^- W^{-\mu}) \\
& + ie[(\partial_\mu W_\nu^+)W^{-\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-\nu}A^\mu - W^{+\mu}(\partial_\mu W_\nu^-)A^\nu + W^{+\nu}(\partial_\mu W_\nu^-)A^\mu \\
& \quad + W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] + e^2(W_\mu^+ W^{-\nu} A_\nu A^\mu - W_\mu^+ W^{-\mu} A_\nu A^\nu) \\
& + igc_W[(\partial_\mu W_\nu^+)W^{-\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu}Z^\mu - W^{+\mu}(\partial_\mu W_\nu^-)Z^\nu + W^{+\nu}(\partial_\mu W_\nu^-)Z^\mu \\
& \quad + W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)] + g^2 c_W^2(W_\mu^+ W^{-\nu} Z_\nu Z^\mu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu) \\
& + egc_W(W_\mu^+ W^{-\nu} A_\nu Z^\mu + W_\mu^+ W^{-\nu} A^\mu Z_\nu - 2W_\mu^+ W^{-\mu} A_\nu Z^\nu). \tag{66}
\end{aligned}$$

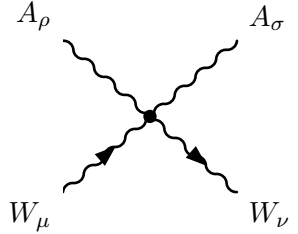
下面是电弱规范玻色子自耦合的费曼规则：



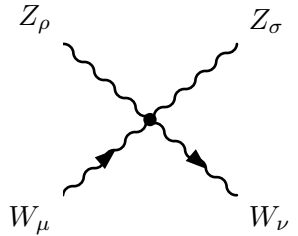
$$= -ie[(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu)]$$



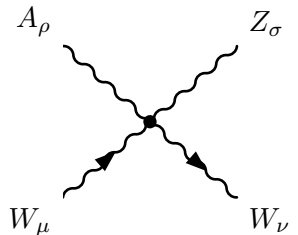
$$= -igc_W[(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu)]$$



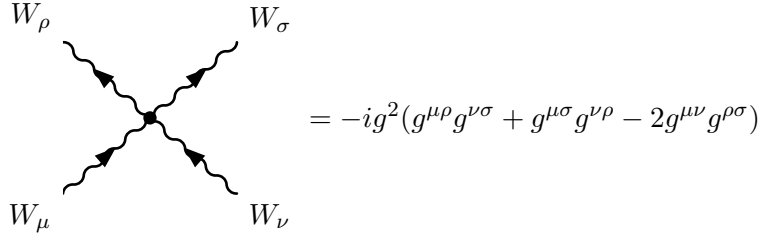
$$= ie^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= ig^2 c_W^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= iegc_W(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$



$$= -ig^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma})$$

## 6 么正规范下 Higgs 场相关拉氏量和费曼规则

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_H = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V_H(\Phi), \quad V_H(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad (67)$$

其中

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \quad D_\mu \Phi = (\partial_\mu - ig' B_\mu Y_H - ig W_\mu^a \tau^a) \Phi, \quad Y_H = \frac{1}{2}. \quad (68)$$

当  $\lambda > 0$  且  $\mu^2 > 0$  时, Higgs 场势能  $V_H(\Phi)$  呈现出图 1 所示墨西哥草帽状的形式, 势能最小值位于方程

$$\Phi^\dagger \Phi = [\text{Re}(\phi^+)]^2 + [\text{Im}(\phi^+)]^2 + [\text{Re}(\phi^0)]^2 + [\text{Im}(\phi^0)]^2 = \frac{v^2}{2} \quad (69)$$

对应的 4 维球面上, 其中  $v \equiv \sqrt{\mu^2/\lambda}$ , 满足

$$\mu^2 = \lambda v^2. \quad (70)$$

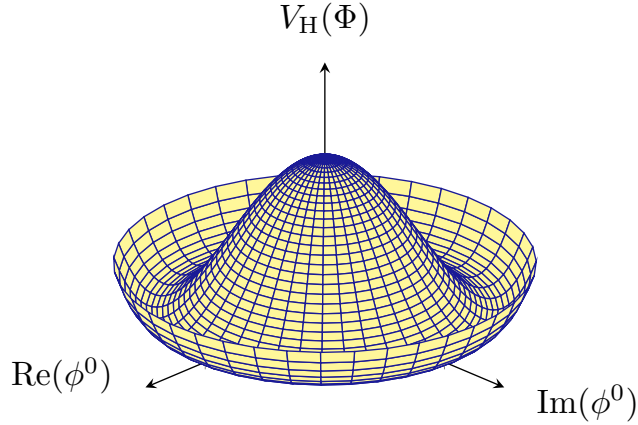


图 1: Higgs 场势能示意图。这里压缩掉  $\text{Re}(\phi^+)$  和  $\text{Im}(\phi^+)$  两个维度。

Higgs 场的真空期待值位于这个 4 维球面上的某一点, 不失一般性, 可将它取为

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (71)$$

其它真空期待值可通过整体变换

$$\langle \Phi \rangle \rightarrow \exp(i\alpha^a \tau^a) \exp(i\alpha^Y Y_H) \langle \Phi \rangle \quad (72)$$

得到, 因为  $\langle \Phi^\dagger \Phi \rangle$  在这样的变换下保持不变。若  $\alpha^1 = \alpha^2 = 0$  且  $\alpha^3 = \alpha^Y$ , 则  $\langle \Phi \rangle$  在变换下不变。因此, 有 1 个方向的规范对称性没有受到破坏, 只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理, 破缺后生成 3 个无质量的 Nambu-Goldstone 玻色子。最终, 有 3 个规范玻色子自由度通过 Brout-Englert-Higgs 机制获得质量。

以  $\langle \Phi \rangle$  为基础, 将 Higgs 场一般地参数化为

$$\Phi(x) = \exp \left[ -i \frac{\chi^a(x)}{v} \tau^a \right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad (73)$$

其中  $\chi^a(x)$  和  $H(x)$  都是实标量场。  $\exp[-i\chi^a(x)\tau^a/v]$  因子能够通过  $SU(2)_L$  规范变换消去, 因而可将  $\Phi(x)$  直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger \Phi = \frac{1}{2}(v + H)^2. \quad (74)$$

此时 Higgs 场只剩下一个物理自由度  $H(x)$ , 对应于 Higgs 玻色子, 这种取法称为么正规范。

在么正规范下, 势能项化为

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 = \frac{1}{2} \mu^2 (v + H)^2 - \frac{1}{4} \lambda (v + H)^4 \\ &= \frac{1}{2} \mu^2 (v^2 + H^2 + 2vH) - \frac{1}{4} \lambda (v^4 + 4v^2 H^2 + H^4 + 4v^3 H + 2v^2 H^2 + 4vH^3) \\ &= \frac{1}{4} \mu^2 v^2 + \frac{1}{4} (\mu^2 - \lambda v^2) v^2 + (\mu^2 - \lambda v^2) vH + \frac{1}{2} (\mu^2 - \lambda v^2) H^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{1}{4} \lambda H^4 \\ &= \frac{1}{8} m_H^2 v^2 - \frac{1}{2} m_H^2 H^2 - \frac{1}{2} \frac{m_H^2}{v} H^3 - \frac{1}{8} \frac{m_H^2}{v^2} H^4, \end{aligned} \quad (75)$$

其中 Higgs 玻色子的质量为

$$m_H \equiv \sqrt{2} \mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \quad (76)$$

利用

$$\begin{aligned} g' B_\mu + g W_\mu^3 &= g' (c_W A_\mu - s_W Z_\mu) + g (s_W A_\mu + c_W Z_\mu) = 2e A_\mu + \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}} Z_\mu \\ &= 2e A_\mu + \frac{g}{c_W} (c_W^2 - s_W^2) Z_\mu, \end{aligned} \quad (77)$$

有

$$\begin{aligned} g' B_\mu Y_H + g W_\mu^a \tau^a &= \frac{1}{2} \begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix} \\ &= \begin{pmatrix} e A_\mu + \frac{g}{2c_W} (c_W^2 - s_W^2) Z_\mu & \frac{1}{\sqrt{2}} g W_\mu^+ \\ \frac{1}{\sqrt{2}} g W_\mu^- & -\frac{g}{2c_W} Z_\mu \end{pmatrix}. \end{aligned} \quad (78)$$

于是, 在么正规范下,

$$(D^\mu \Phi)^\dagger (D_\mu \Phi)$$

$$\begin{aligned}
&= \left| \begin{pmatrix} \partial_\mu - ieA_\mu - \frac{ig}{2c_W}(c_W^2 - s_W^2)Z_\mu & -\frac{i}{\sqrt{2}}gW_\mu^+ \\ -\frac{i}{\sqrt{2}}gW_\mu^- & \partial_\mu + \frac{ig}{2c_W}Z_\mu \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix} \right|^2 \\
&= \frac{1}{2} \left( \frac{i}{\sqrt{2}}gW_\mu^-(v+H), \partial_\mu H - \frac{ig}{2c_W}Z_\mu(v+H) \right) \begin{pmatrix} -\frac{i}{\sqrt{2}}gW_\mu^+(v+H) \\ \partial_\mu H + \frac{ig}{2c_W}Z_\mu(v+H) \end{pmatrix} \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2 \left( \frac{g^2}{4}W_\mu^+W^{-\mu} + \frac{g^2}{8c_W^2}Z_\mu Z^\mu \right) \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\
&\quad + gm_W H W_\mu^+ W^{-\mu} + \frac{gm_Z}{2c_W} H Z_\mu Z^\mu + \frac{g^2}{4} H^2 W_\mu^+ W^{-\mu} + \frac{g^2}{8c_W^2} H^2 Z_\mu Z^\mu. \tag{79}
\end{aligned}$$

故  $W^\pm$  和  $Z$  玻色子获得质量，分别为

$$m_W \equiv \frac{gv}{2}, \quad m_Z \equiv \frac{gv}{2c_W} = \frac{m_W}{c_W} = \frac{v}{2}\sqrt{g^2 + g'^2}. \tag{80}$$

$Y = -1/2$  的 Higgs 场共轭态为

$$\tilde{\Phi}(x) = i\sigma^2 \Phi^*(x) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}, \tag{81}$$

其中  $\phi^- \equiv (\phi^+)^*$ 。在么正规范下， $\tilde{\Phi}(x)$  化为

$$\tilde{\Phi}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ v+H(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v+H(x) \\ 0 \end{pmatrix}. \tag{82}$$

Yukawa 耦合项是

$$\begin{aligned}
\mathcal{L}_Y &= -\tilde{y}_d^{ij} \bar{Q}_{iL} d'_{jR} \Phi - y_{u_i} \bar{Q}_{iL} u_{iR} \tilde{\Phi} - y_{\ell_i} \bar{L}_{iL} \ell_{iR} \Phi + h.c. \\
&= -\frac{1}{\sqrt{2}}(v+H) \bar{d}_{iL} V_{li}^\dagger \tilde{y}_d^{ij} V_{jk} d_{kR} - \frac{y_{u_i}}{\sqrt{2}}(v+H) \bar{u}_{iL} u_{iR} - \frac{y_{\ell_i}}{\sqrt{2}}(v+H) \bar{\ell}_{iL} \ell_{iR} + h.c. \\
&= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i. \tag{83}
\end{aligned}$$

这里 CKM 矩阵将  $\tilde{y}_d^{ij}$  对角化：

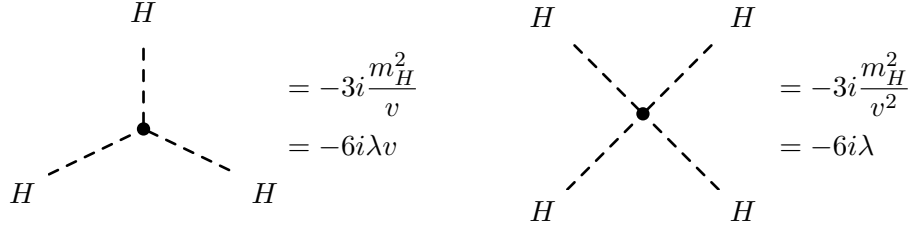
$$V_{li}^\dagger \tilde{y}_d^{ij} V_{jk} = y_{d_k} \delta_{lk}. \tag{84}$$

通过 Yukawa 耦合，费米子获得了质量，

$$m_{d_i} \equiv \frac{1}{\sqrt{2}} y_{d_i} v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}} y_{u_i} v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}} y_{\ell_i} v. \tag{85}$$

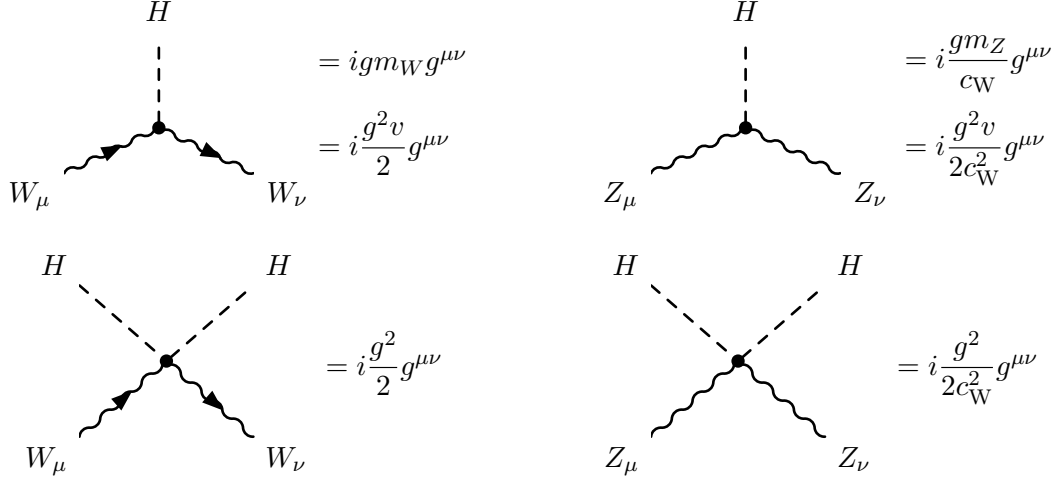
下面给出么正规范下的顶点费曼规则。

Higgs 玻色子自耦合:



$$\begin{aligned}
 &= -3i \frac{m_H^2}{v} \\
 &= -6i\lambda
 \end{aligned}
 \qquad
 \begin{aligned}
 &= -3i \frac{m_H^2}{v^2} \\
 &= -6i\lambda
 \end{aligned}$$

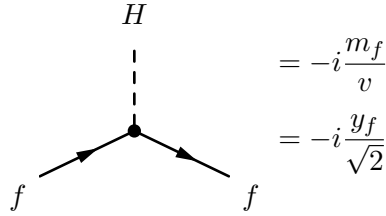
Higgs 玻色子与电弱规范玻色子的耦合:



$$\begin{aligned}
 &= igm_W g^{\mu\nu} \\
 &= i \frac{g^2 v}{2} g^{\mu\nu}
 \end{aligned}
 \qquad
 \begin{aligned}
 &= i \frac{gm_Z}{c_W} g^{\mu\nu} \\
 &= i \frac{g^2 v}{2c_W^2} g^{\mu\nu}
 \end{aligned}$$

$$\begin{aligned}
 &= i \frac{g^2}{2} g^{\mu\nu} \\
 &= i \frac{g^2}{2c_W^2} g^{\mu\nu}
 \end{aligned}$$

Higgs 玻色子与费米子的耦合:



$$\begin{aligned}
 &= -i \frac{m_f}{v} \\
 &= -i \frac{y_f}{\sqrt{2}}
 \end{aligned}$$

## 7 $R_\xi$ 规范相关拉氏量和费曼规则

将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + i\chi(x)] \end{pmatrix}, \quad (86)$$

其中  $\phi^+$  和  $\chi$  是 Nambu-Goldstone 标量场。那么,  $\tilde{\Phi}(x)$  的形式是

$$\tilde{\Phi}(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}[v + H(x) - i\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \quad (87)$$

由

$$\Phi^\dagger \Phi = \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2,$$



$$(\Phi^\dagger \Phi)^2 = \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \quad (88)$$

可得 Higgs 场势能项

$$\begin{aligned} -V_H(\Phi) &= \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2 \\ &= \frac{1}{2}\mu^2(v^2 + H^2 + 2vH + \chi^2) + \mu^2|\phi^+|^2 - \frac{1}{4}\lambda(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda|\phi^+|^4 \\ &\quad - \lambda|\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\ &= \frac{1}{2}\left(\mu^2 - \frac{1}{2}\lambda v^2\right)v^2 + \frac{1}{2}(\mu^2 - 3\lambda v^2)H^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)\chi^2 - \frac{1}{4}\lambda H^4 - \frac{1}{4}\lambda\chi^4 - \lambda vH^3 \\ &\quad - \frac{1}{2}\lambda H^2\chi^2 - \lambda vH\chi^2 + (\mu^2 - \lambda v^2)|\phi^+|^2 - \lambda|\phi^+|^4 - \lambda|\phi^+|^2(H^2 + 2vH + \chi^2) \\ &= \frac{1}{4}\lambda v^4 - \lambda v^2 H^2 - \frac{1}{4}\lambda H^4 - \frac{1}{4}\lambda\chi^4 - \lambda vH^3 - \frac{1}{2}\lambda H^2\chi^2 - \lambda vH\chi^2 - \lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2) \\ &= \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4 - \frac{m_H^2}{2v}H\chi^2 - \frac{m_H^2}{4v^2}H^2\chi^2 - \frac{m_H^2}{8v^2}\chi^4 \\ &\quad - \frac{m_H^2}{2v^2}\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2). \end{aligned} \quad (89)$$

由于

$$V_{li}^\dagger \tilde{y}_d^{ij} V_{jk} = y_{dk} \delta_{lk}, \quad \tilde{y}_d^{ij} = V_{ik} y_{dk} V_{kj}^\dagger, \quad (90)$$

有

$$\begin{aligned} -\tilde{y}_d^{ij} \bar{Q}_{iL} d'_{jR} \Phi &= -\tilde{y}_d^{ij} \left[ \bar{u}_{iL} d'_{jR} \phi^+ + \frac{1}{\sqrt{2}} \bar{d}_{iL} d'_{jR} (v + H + i\chi) \right] \\ &= - \left[ \bar{u}_{iL} V_{ik} y_{dk} V_{kj}^\dagger V_{jl} d_{lR} \phi^+ + \frac{1}{\sqrt{2}} \bar{d}_{iL} V_{li}^\dagger \tilde{y}_d^{ij} V_{jk} d_{kR} (v + H + i\chi) \right] \\ &= - \left[ y_{dj} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{1}{\sqrt{2}} y_{di} \bar{d}_{iL} d_{iR} (v + H + i\chi) \right], \end{aligned} \quad (91)$$

则 Yukawa 耦合项为

$$\begin{aligned} \mathcal{L}_Y &= -\tilde{y}_d^{ij} \bar{Q}_{iL} d'_{jR} \Phi - y_{ui} \bar{Q}_{iL} u_{iR} \tilde{\Phi} - y_{\ell i} \bar{L}_{iL} \ell_{iR} \Phi + \text{h.c.} \\ &= - \left[ y_{dj} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{1}{\sqrt{2}} y_{di} \bar{d}_{iL} d_{iR} (v + H + i\chi) \right] - y_{ui} \left[ \frac{1}{\sqrt{2}} \bar{u}_{iL} u_{iR} (v + H - i\chi) - \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^- \right] \\ &\quad - y_{\ell i} \left[ \bar{\nu}_{iL} \ell_{iR} \phi^+ + \frac{1}{\sqrt{2}} \bar{\ell}_{iL} \ell_{iR} (v + H + i\chi) \right] + \text{h.c.} \\ &= -m_{d_i} \bar{d}_{iL} d_{iR} - m_{u_i} \bar{u}_{iL} u_{iR} - m_{\ell_i} \bar{\ell}_{iL} \ell_{iR} - \frac{m_{d_i}}{v} \bar{d}_{iL} d_{iR} (H + i\chi) - \frac{m_{u_i}}{v} \bar{u}_{iL} u_{iR} (H - i\chi) \\ &\quad - \frac{m_{\ell_i}}{v} \bar{\ell}_{iL} \ell_{iR} (H + i\chi) - \frac{\sqrt{2}m_{d_j}}{v} \bar{u}_{iL} V_{ij} d_{jR} \phi^+ + \frac{\sqrt{2}m_{u_i}}{v} \bar{d}_{jL} V_{ji}^\dagger u_{iR} \phi^- - \frac{\sqrt{2}m_{\ell_i}}{v} \bar{\nu}_{iL} \ell_{iR} \phi^+ + \text{h.c.} \\ &= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i \\ &\quad - \frac{m_{d_i}}{v} \chi \bar{d}_i i\gamma_5 d_i + \frac{m_{u_i}}{v} \chi \bar{u}_i i\gamma_5 u_i - \frac{m_{\ell_i}}{v} \chi \bar{\ell}_i i\gamma_5 \ell_i + \frac{\sqrt{2}V_{ij}}{v} \phi^+ \bar{u}_i (m_{u_i} P_L - m_{d_j} P_R) d_j \\ &\quad - \frac{\sqrt{2}V_{ji}^\dagger}{v} \phi^- \bar{d}_j (m_{d_j} P_L - m_{u_i} P_R) u_i - \frac{\sqrt{2}m_{\ell_i}}{v} (\phi^+ \bar{\nu}_i P_R \ell_i + \phi^- \bar{\ell}_i P_L \nu_i). \end{aligned} \quad (92)$$

利用

$$\begin{aligned}
D_\mu \Phi &= \begin{pmatrix} \partial_\mu - ieA_\mu - \frac{ig}{2c_W}(c_W^2 - s_W^2)Z_\mu & -\frac{i}{\sqrt{2}}gW_\mu^+ \\ -\frac{i}{\sqrt{2}}gW_\mu^- & \partial_\mu + \frac{ig}{2c_W}Z_\mu \end{pmatrix} \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\
&= \begin{pmatrix} \partial_\mu \phi^+ - i \left[ eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu \right] \phi^+ - \frac{ig}{2}W_\mu^+(H + i\chi) - im_W W_\mu^+ \\ \frac{1}{\sqrt{2}} \left[ \partial_\mu(H + i\chi) - igW_\mu^- \phi^+ + \frac{ig}{2c_W}Z_\mu(H + i\chi) + im_Z Z_\mu \right] \end{pmatrix}, \quad (93)
\end{aligned}$$

可将 Higgs 场协变动能项化为

$$\begin{aligned}
&(D^\mu \Phi)^\dagger D_\mu \Phi \\
&= \left| \partial_\mu \phi^+ - i \left[ eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu \right] \phi^+ - \frac{ig}{2}W_\mu^+(H + i\chi) - im_W W_\mu^+ \right|^2 \\
&\quad + \frac{1}{2} \left| \partial_\mu(H + i\chi) - igW_\mu^- \phi^+ + \frac{ig}{2c_W}Z_\mu(H + i\chi) + im_Z Z_\mu \right|^2 \\
&= (\partial^\mu \phi^+)(\partial_\mu \phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu \chi)(\partial_\mu \chi) \\
&\quad + \left( -i\partial^\mu \phi^- \left\{ \left[ eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu \right] \phi^+ + \frac{g}{2}W_\mu^+(H + i\chi) + m_W W_\mu^+ \right\} + \text{h.c.} \right) \\
&\quad + \left\{ -\frac{i}{2}\partial^\mu(H - i\chi) \left[ gW_\mu^- \phi^+ - \frac{g}{2c_W}Z_\mu(H + i\chi) - m_Z Z_\mu \right] + \text{h.c.} \right\} \\
&\quad + \left| \left[ eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu \right] \phi^+ + \frac{g}{2}W_\mu^+(H + i\chi) + m_W W_\mu^+ \right|^2 \\
&\quad + \frac{1}{2} \left| gW_\mu^- \phi^+ - \frac{g}{2c_W}Z_\mu(H + i\chi) - m_Z Z_\mu \right|^2 \\
&= (\partial^\mu \phi^+)(\partial_\mu \phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu \chi)(\partial_\mu \chi) \\
&\quad + m_W^2 W^{-\mu} W_\mu^+ + \frac{1}{2}m_Z^2 Z^\mu Z_\mu + gm_W H W_\mu^+ W^{-\mu} + \frac{gm_Z}{2c_W} H Z^\mu Z_\mu \\
&\quad + \frac{g}{2} [W_\mu^+ \phi^- i \overleftrightarrow{\partial}^\mu (H + i\chi) + \text{h.c.}] + eA_\mu \phi^- i \overleftrightarrow{\partial}^\mu \phi^+ + \frac{g}{2c_W} Z_\mu [i\chi i \overleftrightarrow{\partial}^\mu H + (c_W^2 - s_W^2) \phi^- i \overleftrightarrow{\partial}^\mu \phi^+] \\
&\quad + \frac{g^2}{4} W_\mu^+ W^{-\mu} (2\phi^+ \phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu \phi^+ \phi^- + \frac{g^2}{4c_W^2} Z_\mu Z^\mu \left[ (c_W^2 - s_W^2)^2 \phi^+ \phi^- + \frac{1}{2}H^2 + \frac{1}{2}\chi^2 \right] \\
&\quad + \left[ \frac{eg}{2} W_\mu^+ A^\mu \phi^- (H + i\chi) - \frac{g^2 s_W^2}{2c_W} W_\mu^+ Z^\mu \phi^- (H + i\chi) + \text{h.c.} \right] + \frac{eg}{c_W} (c_W^2 - s_W^2) A_\mu Z^\mu \phi^+ \phi^- \\
&\quad + (em_W A^\mu \phi^+ W_\mu^- - gs_W^2 m_Z Z^\mu \phi^+ W_\mu^- + \text{h.c.}) + \mathcal{L}_{b1}, \quad (94)
\end{aligned}$$

其中

$$\mathcal{L}_{b1} = -im_W(\partial^\mu \phi^-)W_\mu^+ + im_W(\partial^\mu \phi^+)W_\mu^- + m_Z(\partial^\mu \chi)Z_\mu. \quad (95)$$

$R_\xi$  规范规范固定函数设为

$$G^\pm = \frac{1}{\sqrt{\xi}}(\partial^\mu W_\mu^\pm \mp i\xi m_W \phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi}}(\partial^\mu Z_\mu - \xi m_Z \chi), \quad G^\gamma = \frac{1}{\sqrt{\xi}}\partial^\mu A_\mu, \quad (96)$$

它们在路径积分量子化中的泛函积分形式为

$$\begin{aligned}
& \int \mathcal{D}\omega^+ \int \mathcal{D}\omega^- \int \mathcal{D}\omega^Z \int \mathcal{D}\omega^\gamma \exp \left[ -i \int d^4x \left( \omega^+ \omega^- + \frac{1}{2} \omega^Z \omega^Z + \frac{1}{2} \omega^\gamma \omega^\gamma \right) \right] \\
& \quad \times \delta(G^+ - \omega^+) \delta(G^- - \omega^-) \delta(G^Z - \omega^Z) \delta(G^\gamma - \omega^\gamma) \\
& = \exp \left[ -i \int d^4x \left( G^+ G^- + \frac{1}{2} G^Z G^Z + \frac{1}{2} G^\gamma G^\gamma \right) \right].
\end{aligned} \tag{97}$$

由此可得拉氏量中的规范固定项

$$\begin{aligned}
\mathcal{L}_{\text{EW,GF}} &= -G^+ G^- - \frac{1}{2} (G^Z)^2 - \frac{1}{2} (G^\gamma)^2 \\
&= -\frac{1}{\xi} (\partial^\mu W_\mu^+ - i\xi m_W \phi^+) (\partial^\nu W_\nu^- + i\xi m_W \phi^-) - \frac{1}{2\xi} (\partial^\mu Z_\mu - \xi m_Z \chi)^2 - \frac{1}{2\xi} (\partial^\mu A_\mu)^2 \\
&= -\frac{1}{\xi} (\partial^\mu W_\mu^+) (\partial^\nu W_\nu^-) - \frac{1}{2\xi} (\partial^\mu Z_\mu)^2 - \frac{1}{2\xi} (\partial^\mu A_\mu)^2 - \xi m_W^2 \phi^+ \phi^- - \frac{1}{2} \xi m_Z^2 \chi^2 + \mathcal{L}_{\text{b2}}.
\end{aligned} \tag{98}$$

可见, Nambu-Goldstone 玻色子在  $R_\xi$  规范下具有依赖于  $\xi$  的非物理质量,

$$m_\phi = \sqrt{\xi} m_W, \quad m_\chi = \sqrt{\xi} m_Z. \tag{99}$$

这里,

$$\mathcal{L}_{\text{b2}} = -im_W \phi^- (\partial^\mu W_\mu^+) + im_W \phi^+ \partial^\mu W_\mu^- + m_Z \chi \partial^\mu Z_\mu. \tag{100}$$

由于

$$\mathcal{L}_{\text{b1}} + \mathcal{L}_{\text{b2}} = -im_W \partial^\mu (\phi^- W_\mu^+) + im_W \partial^\mu (\phi^+ W_\mu^-) + m_Z \partial^\mu (\chi Z_\mu), \tag{101}$$

这两项体现为全散度, 不会有物理效应。可见, 协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项  $\mathcal{L}_{\text{b1}}$  被规范固定项中的  $\mathcal{L}_{\text{b2}}$  抵消掉, 这就是如此选取规范固定函数的目的。

这样一来, 电弱规范场传播子相关拉氏量变成

$$\begin{aligned}
\mathcal{L}_{\text{EW,prop}} &= (\partial_\mu W_\nu^+) (\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+) (\partial^\mu W^{-\nu}) - \frac{1}{\xi} (\partial^\mu W_\mu^+) (\partial^\nu W_\nu^-) + m_W^2 W^{-\mu} W_\mu^+ \\
&\quad + \frac{1}{2} \left[ (\partial_\mu Z_\nu) (\partial^\nu Z^\mu) - (\partial_\mu Z_\nu) (\partial^\mu Z^\nu) - \frac{1}{\xi} (\partial^\mu Z_\mu)^2 + m_Z^2 Z^\mu Z_\mu \right] \\
&\quad + \frac{1}{2} \left[ (\partial_\mu A_\nu) (\partial^\nu A^\mu) - (\partial_\mu A_\nu) (\partial^\mu A^\nu) - \frac{1}{\xi} (\partial^\mu A_\mu)^2 \right] \\
&\rightarrow W_\mu^+ \left[ g^{\mu\nu} (\partial^2 + m_W^2) - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] W_\nu^- + \frac{1}{2} Z_\mu \left[ g^{\mu\nu} (\partial^2 + m_Z^2) - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] Z_\nu \\
&\quad + \frac{1}{2} A_\mu \left[ g^{\mu\nu} \partial - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] A_\nu.
\end{aligned} \tag{102}$$

于是, 光子的传播子与胶子形式类似, 为

$$\frac{-i}{p^2 + i\varepsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right]. \tag{103}$$

将  $W^\pm$  传播子相关拉氏量变换到动量空间, 得

$$-g^{\mu\nu}(p^2 - m_W^2) + \left(1 - \frac{1}{\xi}\right)p^\mu p^\nu = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2} \frac{p^2 - \xi m_W^2}{\xi}, \quad (104)$$

它的逆矩阵是

$$-\frac{1}{p^2 - m_W^2} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - \frac{\xi}{p^2 - \xi m_W^2} \frac{p_\mu p_\nu}{p^2} = -\frac{1}{p^2 - m_W^2} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi m_W^2} (1 - \xi)\right], \quad (105)$$

这是因为由

$$\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right) \frac{p^\mu p^\nu}{p^2} = \frac{p_\rho p^\nu}{p^2} - \frac{p_\rho p^\nu}{p^2} = 0, \quad \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) = \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} \quad (106)$$

可得

$$\begin{aligned} & \left[-\frac{1}{p^2 - m_W^2} \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right) - \frac{\xi}{p^2 - \xi m_W^2} \frac{p_\rho p_\mu}{p^2}\right] \left[-\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2} \frac{p^2 - \xi m_W^2}{\xi}\right] \\ &= \delta_\rho^\nu - \frac{p_\rho p^\nu}{p^2} + \frac{p_\rho p^\nu}{p^2} = \delta_\rho^\nu. \end{aligned} \quad (107)$$

从而,  $W^\pm$  传播子的形式为

$$\frac{-i}{p^2 - m_W^2 + i\varepsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi m_W^2} (1 - \xi)\right]. \quad (108)$$

同理,  $Z$  传播子的形式为

$$\frac{-i}{p^2 - m_Z^2 + i\varepsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi m_Z^2} (1 - \xi)\right]. \quad (109)$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a = \frac{1}{g} \partial_\mu \alpha^a + \varepsilon^{abc} W_\mu^b \alpha^c, \quad \delta B_\mu = \frac{1}{g'} \partial_\mu \alpha^Y. \quad (110)$$

定义

$$\alpha^\pm \equiv \frac{1}{\sqrt{2}}(\alpha^1 \mp i\alpha^2), \quad \alpha^Z \equiv \alpha^3 - \alpha^Y, \quad \alpha^\gamma \equiv s_W^2 \alpha^3 + c_W^2 \alpha^Y, \quad (111)$$

利用

$$\varepsilon^{1bc} W_\mu^b \alpha^c = W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2, \quad \varepsilon^{2bc} W_\mu^b \alpha^c = -W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1, \quad (112)$$

$$\pm i\sqrt{2}\alpha^\pm = \pm i\alpha^1 + \alpha^2, \quad \pm i\sqrt{2}W_\mu^\pm = \pm iW_\mu^1 + W_\mu^2, \quad (113)$$

有

$$\begin{aligned} \varepsilon^{1bc} W_\mu^b \alpha^c \mp i\varepsilon^{2bc} W_\mu^b \alpha^c &= (W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2) \mp i(-W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1) = (W_\mu^2 \pm iW_\mu^1) \alpha^3 - W_\mu^3 (\alpha^2 \pm i\alpha^1) \\ &= \pm i\sqrt{2}W_\mu^\pm (c_W^2 \alpha^Z + \alpha^\gamma) \mp i\sqrt{2}(s_W A_\mu + c_W Z_\mu) \alpha^\pm, \end{aligned} \quad (114)$$

$$\begin{aligned} \varepsilon^{3bc} W_\mu^b \alpha^c &= W_\mu^1 \alpha^2 - W_\mu^2 \alpha^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-) \frac{i}{\sqrt{2}}(\alpha^+ - \alpha^-) - \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-) \frac{1}{\sqrt{2}}(\alpha^+ + \alpha^-) \\ &= -i(W_\mu^+ \alpha^- - W_\mu^- \alpha^+). \end{aligned} \quad (115)$$

因此,

$$\begin{aligned}\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1 - i\delta W_\mu^2) = \frac{1}{\sqrt{2}g}\partial_\mu(\alpha^1 - i\alpha^2) + \frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c - i\varepsilon^{2bc}W_\mu^b\alpha^c) \\ &= \frac{1}{g}\partial_\mu\alpha^+ - i(s_W A_\mu + c_W Z_\mu)\alpha^+ + iW_\mu^+(c_W^2\alpha^Z + \alpha^\gamma),\end{aligned}\quad (116)$$

$$\delta W_\mu^- = (\delta W_\mu^+)^\dagger = \frac{1}{g}\partial_\mu\alpha^- + i(s_W A_\mu + c_W Z_\mu)\alpha^- - iW_\mu^-(c_W^2\alpha^Z + \alpha^\gamma),\quad (117)$$

$$\delta Z_\mu^a = c_W\delta W_\mu^3 - s_W\delta B_\mu = \frac{c_W}{g}\partial_\mu\alpha^3 + c_W\varepsilon^{3bc}W_\mu^b\alpha^c - \frac{s_W}{g'}\partial_\mu\alpha^Y = \frac{c_W}{g}\partial_\mu\alpha^Z - ic_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+),\quad (118)$$

$$\delta A_\mu = s_W\delta W_\mu^3 + c_W\delta B_\mu = \frac{s_W}{g}\partial_\mu\alpha^3 + s_W\varepsilon^{3bc}W_\mu^b\alpha^c + \frac{c_W}{g'}\partial_\mu\alpha^Y = \frac{1}{e}\partial_\mu\alpha^\gamma - is_W(W_\mu^+\alpha^- - W_\mu^-\alpha^+).\quad (119)$$

另一方面, 根据

$$\alpha^a T^a + \alpha^Y Y_H = \frac{1}{2}(\alpha^a \sigma^a + \alpha^Y) = \frac{1}{2}\begin{pmatrix} \alpha^3 + \alpha^Y & \alpha^1 - i\alpha^2 \\ \alpha^1 + i\alpha^2 & -\alpha^3 + \alpha^Y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix},\quad (120)$$

可知 Higgs 场的无穷小规范变换形式为

$$\begin{aligned}\delta\Phi &= i(\alpha^a T^a + \alpha^Y Y_H)\Phi = \frac{i}{2}\begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}\begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\ &= \begin{pmatrix} \frac{i}{2}[\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+] \\ \frac{1}{\sqrt{2}}[i\phi^+\alpha^- - \frac{1}{2}(iv + iH - \chi)\alpha^Z] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \frac{1}{\sqrt{2}}(\delta H + i\delta\chi) \end{pmatrix}.\end{aligned}\quad (121)$$

利用

$$\text{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \text{Im}(\phi^+\alpha^-) = -\frac{i}{2}(\phi^+\alpha^- - \phi^-\alpha^+),\quad (122)$$

可得

$$\delta\phi^+ = \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\},\quad (123)$$

$$\delta\phi^- = -\frac{i}{2}\{\phi^-[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H - i\chi)\alpha^-\},\quad (124)$$

$$\delta H = \frac{1}{2}[i(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z].\quad (125)$$

于是, 规范固定函数的无穷小规范变换为

$$\begin{aligned}\sqrt{\xi}\delta G^+ &= \partial^\mu\delta W_\mu^+ - i\xi m_W\delta\phi^+ = \partial^\mu\left[\frac{1}{g}\partial_\mu\alpha^+ - i(s_W A_\mu + c_W Z_\mu)\alpha^+ + iW_\mu^+(c_W^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad + \frac{1}{2}\xi m_W\{\phi^+[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H + i\chi)\alpha^+\},\end{aligned}\quad (126)$$

$$\begin{aligned}\sqrt{\xi}\delta G^- &= \partial^\mu\delta W_\mu^- + i\xi m_W\delta\phi^- = \partial^\mu\left[\frac{1}{g}\partial_\mu\alpha^- + i(s_W A_\mu + c_W Z_\mu)\alpha^- - iW_\mu^-(c_W^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad + \frac{1}{2}\xi m_W\{\phi^-[2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z] + (v + H - i\chi)\alpha^-\},\end{aligned}\quad (127)$$

$$\begin{aligned}\sqrt{\xi}\delta G^Z &= \partial^\mu \delta Z_\mu - \xi m_Z \delta \chi = \partial^\mu \left[ \frac{c_W}{g} \partial_\mu \alpha^Z - i c_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+) \right] \\ &\quad - \frac{1}{2} \xi m_Z [\phi^+ \alpha^- + \phi^- \alpha^+ - (v + H) \alpha^Z],\end{aligned}\quad (128)$$

$$\sqrt{\xi}\delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu \left[ \frac{1}{e} \partial_\mu \alpha^\gamma - i s_W (W_\mu^+ \alpha^- - W_\mu^- \alpha^+) \right]. \quad (129)$$

因此,

$$\sqrt{\xi}g \frac{\delta G^+}{\delta \alpha^+} = \partial^2 + \xi m_W^2 - ie \partial^\mu A_\mu - i g c_W \partial^\mu Z_\mu + \frac{1}{2} g \xi m_W (H + i\chi), \quad (130)$$

$$\frac{\sqrt{\xi}g}{c_W} \frac{\delta G^+}{\delta \alpha^Z} = i g c_W \partial^\mu W_\mu^+ + \frac{g(c_W^2 - s_W^2)\xi m_W}{2c_W} \phi^+, \quad \sqrt{\xi}e \frac{\delta G^+}{\delta \alpha^\gamma} = ie \partial^\mu W_\mu^+ + e \xi m_W \phi^+, \quad (131)$$

$$\sqrt{\xi}g \frac{\delta G^-}{\delta \alpha^-} = \partial^2 + \xi m_W^2 + ie \partial^\mu A_\mu + i g c_W \partial^\mu Z_\mu + \frac{1}{2} g \xi m_W (H - i\chi), \quad (132)$$

$$\frac{\sqrt{\xi}g}{c_W} \frac{\delta G^-}{\delta \alpha^Z} = -i g c_W \partial^\mu W_\mu^- + \frac{g(c_W^2 - s_W^2)\xi m_W}{2c_W} \phi^-, \quad \sqrt{\xi}e \frac{\delta G^-}{\delta \alpha^\gamma} = -ie \partial^\mu W_\mu^- + e \xi m_W \phi^-, \quad (133)$$

$$\sqrt{\xi}g \frac{\delta G^Z}{\delta \alpha^+} = i g c_W \partial^\mu W_\mu^- - \frac{1}{2} g \xi m_Z \phi^-, \quad \sqrt{\xi}g \frac{\delta G^Z}{\delta \alpha^-} = -i g c_W \partial^\mu W_\mu^+ - \frac{1}{2} g \xi m_Z \phi^+, \quad (134)$$

$$\frac{\sqrt{\xi}g}{c_W} \frac{\delta G^Z}{\delta \alpha^Z} = \partial^2 + \xi m_Z^2 + \frac{g \xi m_Z}{2c_W} H, \quad (135)$$

$$\sqrt{\xi}g \frac{\delta G^\gamma}{\delta \alpha^+} = ie \partial^\mu W_\mu^-, \quad \sqrt{\xi}g \frac{\delta G^\gamma}{\delta \alpha^-} = -ie \partial^\mu W_\mu^+, \quad \sqrt{\xi}e \frac{\delta G^\gamma}{\delta \alpha^\gamma} = \partial^2. \quad (136)$$

最后, 得到以下 Faddeev-Popov 鬼场拉氏量,

$$\begin{aligned}\mathcal{L}_{\text{EWG,FP}} &= -\bar{\eta}^+ \left( \sqrt{\xi}g \frac{\delta G^+}{\delta \alpha^+} \right) \eta^+ - \bar{\eta}^Z \left( \sqrt{\xi}g \frac{\delta G^Z}{\delta \alpha^+} \right) \eta^+ - \bar{\eta}^\gamma \left( \sqrt{\xi}g \frac{\delta G^\gamma}{\delta \alpha^+} \right) \eta^+ - \bar{\eta}^- \left( \sqrt{\xi}g \frac{\delta G^-}{\delta \alpha^+} \right) \eta^- \\ &\quad - \bar{\eta}^Z \left( \sqrt{\xi}g \frac{\delta G^Z}{\delta \alpha^-} \right) \eta^- - \bar{\eta}^\gamma \left( \sqrt{\xi}g \frac{\delta G^\gamma}{\delta \alpha^-} \right) \eta^- - \bar{\eta}^Z \left( \frac{\sqrt{\xi}g}{c_W} \frac{\delta G^Z}{\delta \alpha^Z} \right) \eta^Z - \bar{\eta}^+ \left( \frac{\sqrt{\xi}g}{c_W} \frac{\delta G^+}{\delta \alpha^Z} \right) \eta^Z \\ &\quad - \bar{\eta}^- \left( \frac{\sqrt{\xi}g}{c_W} \frac{\delta G^-}{\delta \alpha^Z} \right) \eta^Z - \bar{\eta}^\gamma \left( \sqrt{\xi}e \frac{\delta G^\gamma}{\delta \alpha^\gamma} \right) \eta^\gamma - \bar{\eta}^+ \left( \sqrt{\xi}e \frac{\delta G^+}{\delta \alpha^\gamma} \right) \eta^\gamma - \bar{\eta}^- \left( \sqrt{\xi}e \frac{\delta G^-}{\delta \alpha^\gamma} \right) \eta^\gamma \\ &= \bar{\eta}^+ \left[ -\partial^2 - \xi m_W^2 - ie \overleftarrow{\partial}^\mu A_\mu - i g c_W \overleftarrow{\partial}^\mu Z_\mu - \frac{1}{2} g \xi m_W (H + i\chi) \right] \eta^+ \\ &\quad + \bar{\eta}^Z \left( i g c_W \overleftarrow{\partial}^\mu W_\mu^- + \frac{1}{2} g \xi m_Z \phi^- \right) \eta^+ + ie (\partial^\mu \bar{\eta}^\gamma) W_\mu^- \eta^+ \\ &\quad + \bar{\eta}^- \left[ -\partial^2 - \xi m_W^2 + ie \overleftarrow{\partial}^\mu A_\mu + i g c_W \overleftarrow{\partial}^\mu Z_\mu - \frac{1}{2} g \xi m_W (H - i\chi) \right] \eta^- \\ &\quad + \bar{\eta}^Z \left( -i g c_W \overleftarrow{\partial}^\mu W_\mu^+ + \frac{1}{2} g \xi m_Z \phi^+ \right) \eta^- - ie (\partial^\mu \bar{\eta}^\gamma) W_\mu^+ \eta^- \\ &\quad + \bar{\eta}^Z \left( -\partial^2 - \xi m_Z^2 - \frac{g \xi m_Z}{2c_W} H \right) \eta^Z + \bar{\eta}^+ \left( i g c_W \overleftarrow{\partial}^\mu W_\mu^+ - \frac{g(c_W^2 - s_W^2)\xi m_W}{2c_W} \phi^+ \right) \eta^Z \\ &\quad + \bar{\eta}^- \left( -i g c_W \overleftarrow{\partial}^\mu W_\mu^- - \frac{g(c_W^2 - s_W^2)\xi m_W}{2c_W} \phi^- \right) \eta^Z \\ &\quad - \bar{\eta}^\gamma \partial^2 \eta^\gamma + \bar{\eta}^+ (ie \overleftarrow{\partial}^\mu W_\mu^+ - e \xi m_W \phi^+) \eta^\gamma + \bar{\eta}^- (-ie \overleftarrow{\partial}^\mu W_\mu^- - e \xi m_W \phi^-) \eta^\gamma.\end{aligned}\quad (137)$$

鬼粒子的质量为

$$m_{\eta^+} = m_{\eta^-} = \sqrt{\xi} m_W, \quad m_{\eta^Z} = \sqrt{\xi} m_Z, \quad m_{\eta^\gamma} = 0. \quad (138)$$

下面给出  $R_\xi$  规范下的费曼规则。 $\xi = 1$  对应 Feynman-'t Hooft 规范,  $\xi = 0$  对应 Landau 规范,  $\xi \rightarrow \infty$  对应么正规范。

传播子:

$$H \text{ --- } p \text{ --- } H = \frac{i}{p^2 - m_H^2 + i\varepsilon}$$

$$\chi \text{ --- } p \text{ --- } \chi = \frac{i}{p^2 - \xi m_Z^2 + i\varepsilon}$$

$$\phi \text{ --- } p \text{ --- } \phi = \frac{i}{p^2 - \xi m_W^2 + i\varepsilon}$$

$$A_\mu \text{ --- } p \text{ --- } A_\nu = \frac{-i}{p^2 + i\varepsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi) \right]$$

$$Z_\mu \text{ --- } p \text{ --- } Z_\nu = \frac{-i}{p^2 - m_Z^2 + i\varepsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi m_Z^2} (1 - \xi) \right]$$

$$W_\mu \text{ --- } p \text{ --- } W_\nu = \frac{-i}{p^2 - m_W^2 + i\varepsilon} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi m_W^2} (1 - \xi) \right]$$

$$\eta^\gamma \text{ --- } p \text{ --- } \eta^\gamma = \frac{i}{p^2 + i\varepsilon}$$

$$\eta^Z \text{ --- } p \text{ --- } \eta^Z = \frac{i}{p^2 - \xi m_Z^2 + i\varepsilon}$$

$$\eta^\pm \text{ --- } p \text{ --- } \eta^\pm = \frac{i}{p^2 - \xi m_W^2 + i\varepsilon}$$

标量玻色子三线性耦合:

$$\begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} = -3i \frac{m_H^2}{v} = -6i\lambda v$$

$$\begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ \chi \quad \chi \end{array} = -i \frac{m_H^2}{v} = -2i\lambda v$$

$$\begin{array}{c} H \\ | \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} = -i \frac{m_H^2}{v} = -2i\lambda v$$

标量玻色子四线性耦合:

$$\begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ H \quad H \end{array} = -3i \frac{m_H^2}{v^2} = -6i\lambda$$

$$\begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \chi \quad \chi \end{array} = -i \frac{m_H^2}{v^2} = -2i\lambda$$

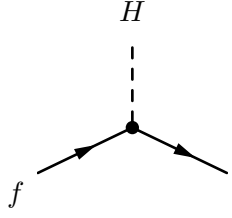
$$\begin{array}{c} \chi \quad \chi \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \chi \quad \chi \end{array} = -3i \frac{m_H^2}{v^2} = -6i\lambda$$

$$\begin{array}{c} H \quad H \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} = -i \frac{m_H^2}{v^2} = -2i\lambda$$

$$\begin{array}{c} \chi \quad \chi \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} = -i \frac{m_H^2}{v^2} = -2i\lambda$$

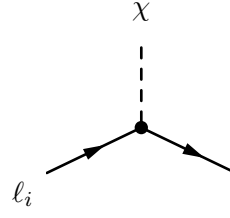
$$\begin{array}{c} \phi \quad \phi \\ \backslash \quad / \\ \bullet \\ / \quad \backslash \\ \phi \quad \phi \end{array} = -2i \frac{m_H^2}{v^2} = -4i\lambda$$

Yukawa 耦合:



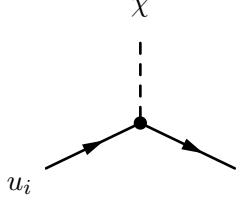
$$= -i \frac{m_f}{v}$$

$$= -i \frac{y_f}{\sqrt{2}}$$



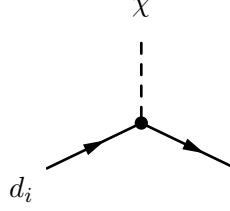
$$= \frac{m_{l_i}}{v} \gamma_5$$

$$= \frac{y_{l_i}}{\sqrt{2}} \gamma_5$$



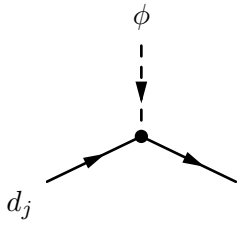
$$= -\frac{m_{u_i}}{v} \gamma_5$$

$$= -\frac{y_{u_i}}{\sqrt{2}} \gamma_5$$



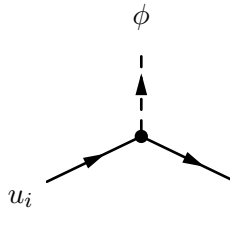
$$= \frac{m_{d_i}}{v} \gamma_5$$

$$= \frac{y_{d_i}}{\sqrt{2}} \gamma_5$$



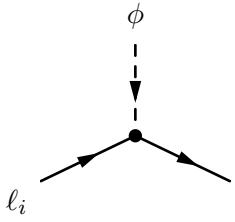
$$= i \frac{\sqrt{2} V_{ij}}{v} (m_{u_i} P_L - m_{d_j} P_R)$$

$$= i V_{ij} (y_{u_i} P_L - y_{d_j} P_R)$$



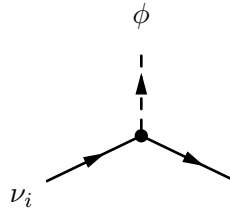
$$= -i \frac{\sqrt{2} V_{ji}^\dagger}{v} (m_{d_j} P_L - m_{u_i} P_R)$$

$$= -i V_{ji}^\dagger (y_{d_j} P_L - y_{u_i} P_R)$$



$$= -i \frac{\sqrt{2} m_{l_i}}{v} P_R$$

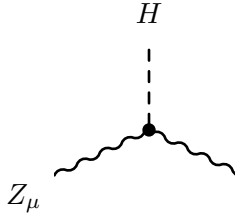
$$= -i y_{l_i} P_R$$



$$= -i \frac{\sqrt{2} m_{l_i}}{v} P_L$$

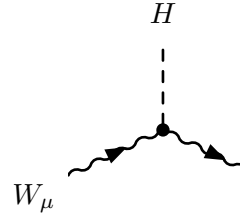
$$= -i y_{l_i} P_L$$

标量玻色子与电弱规范玻色子的三线耦合:



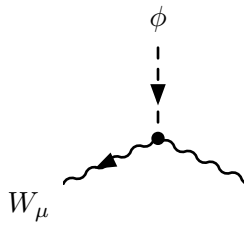
$$= i \frac{g m_Z}{c_W} g^{\mu\nu}$$

$$= i \frac{g^2 v}{2 c_W^2} g^{\mu\nu}$$



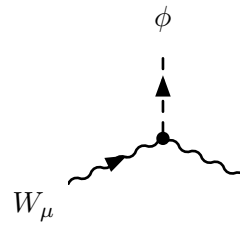
$$= i g m_W g^{\mu\nu}$$

$$= i \frac{g^2 v}{2} g^{\mu\nu}$$



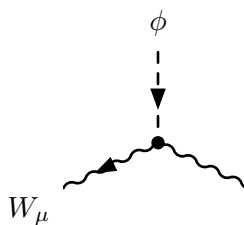
$$= i e m_W g^{\mu\nu}$$

$$= i \frac{e g v}{2} g^{\mu\nu}$$



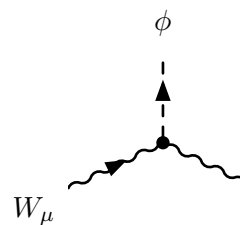
$$= i e m_W g^{\mu\nu}$$

$$= i \frac{e g v}{2} g^{\mu\nu}$$



$$= -i g s_W^2 m_Z g^{\mu\nu}$$

$$= -i \frac{g^2 s_W^2 v}{2 c_W} g^{\mu\nu}$$



$$= -i g s_W^2 m_Z g^{\mu\nu}$$

$$= -i \frac{g^2 s_W^2 v}{2 c_W} g^{\mu\nu}$$



$$= ie(p+q)^\mu$$

$$= i \frac{g(c_W^2 - s_W^2)}{2c_W} (p+q)^\mu$$

$$= -\frac{g}{2c_W} (p+q)^\mu$$

$$= i \frac{g}{2} (p+q)^\mu$$

$$= i \frac{g}{2} (p+q)^\mu$$

$$= -\frac{g}{2} (p+q)^\mu$$

$$= \frac{g}{2} (p+q)^\mu$$

标量玻色子与电弱规范玻色子的四线性耦合：

$$= i \frac{g^2}{2c_W^2} g^{\mu\nu}$$

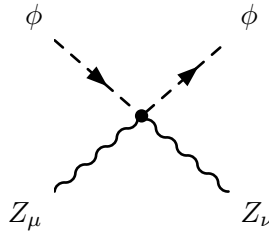
$$= i \frac{g^2}{2} g^{\mu\nu}$$

$$= i \frac{g^2}{2c_W^2} g^{\mu\nu}$$

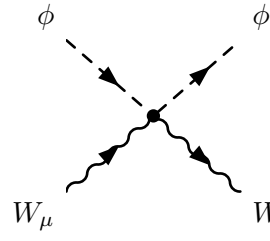
$$= i \frac{g^2}{2} g^{\mu\nu}$$

$$= 2ie^2 g^{\mu\nu}$$

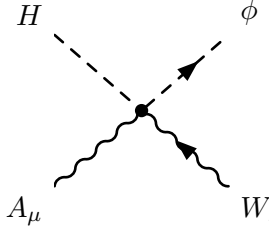
$$= i \frac{eg(c_W^2 - s_W^2)}{c_W} g^{\mu\nu}$$



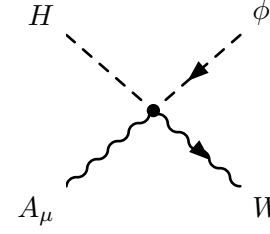
$$= i \frac{g^2 (c_W^2 - s_W^2)^2}{2c_W^2} g^{\mu\nu}$$



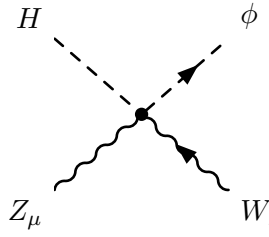
$$= i \frac{g^2}{2} g^{\mu\nu}$$



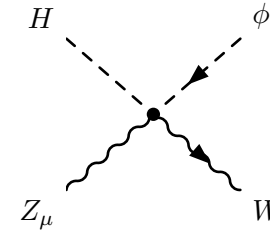
$$= i \frac{eg}{2} g^{\mu\nu}$$



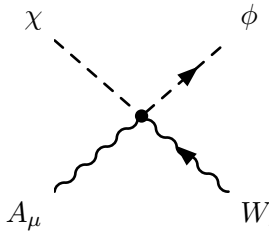
$$= i \frac{eg}{2} g^{\mu\nu}$$



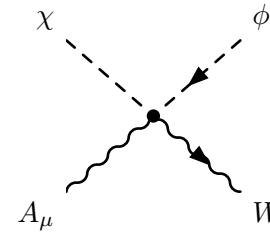
$$= -i \frac{g^2 s_W^2}{2c_W} g^{\mu\nu}$$



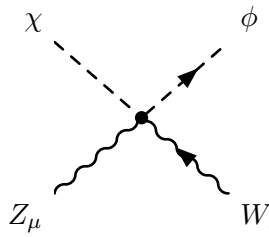
$$= -i \frac{g^2 s_W^2}{2c_W} g^{\mu\nu}$$



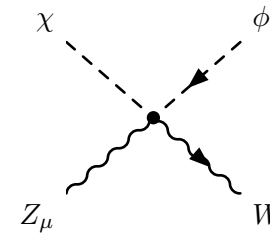
$$= -\frac{eg}{2} g^{\mu\nu}$$



$$= \frac{eg}{2} g^{\mu\nu}$$

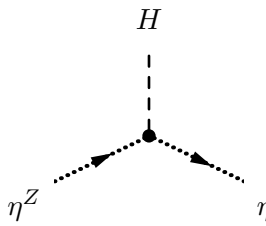


$$= \frac{g^2 s_W^2}{2c_W} g^{\mu\nu}$$



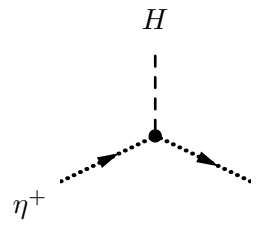
$$= -\frac{g^2 s_W^2}{2c_W} g^{\mu\nu}$$

鬼粒子与标量玻色子的耦合:



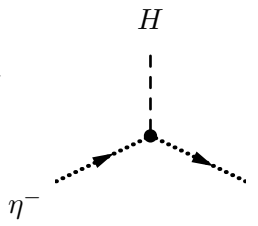
$$= -i \frac{g\xi m_Z}{2c_W}$$

$$= -i \frac{g^2 \xi v}{4c_W^2}$$



$$= -i \frac{g\xi m_W}{2}$$

$$= -i \frac{g^2 \xi v}{4}$$



$$= -i \frac{g\xi m_W}{2}$$

$$= -i \frac{g^2 \xi v}{4}$$

$$= \frac{g\xi m_W}{2}$$

$$= \frac{g^2\xi v}{4}$$

$$= -\frac{g\xi m_W}{2}$$

$$= -\frac{g^2\xi v}{4}$$

$$= -ie\xi m_W$$

$$= -i\frac{eg\xi v}{2}$$

$$= -ie\xi m_W$$

$$= -i\frac{eg\xi v}{2}$$

$$= -i\frac{g(c_W^2 - s_W^2)\xi m_W}{2c_W}$$

$$= -i\frac{g^2(c_W^2 - s_W^2)\xi v}{4c_W}$$

$$= i\frac{g\xi m_Z}{2}$$

$$= i\frac{g^2\xi v}{4c_W}$$

$$= i\frac{g\xi m_Z}{2}$$

$$= i\frac{g^2\xi v}{4c_W}$$

$$= -i\frac{g(c_W^2 - s_W^2)\xi m_W}{2c_W}$$

$$= -i\frac{g^2(c_W^2 - s_W^2)\xi v}{4c_W}$$

鬼粒子与电弱规范玻色子的耦合：

$$= iep^\mu$$

$$= -iep^\mu$$

$$= igc_W p^\mu$$

$$= -igc_W p^\mu$$

$$= -iep^\mu$$

$$= -iep^\mu$$

$$\begin{array}{cc}
\begin{array}{c} W_\mu \\ \text{---} \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^\gamma \end{array} & = i e p^\mu \\
\begin{array}{c} W_\mu \\ \text{---} \bullet \\ \swarrow \quad \searrow \\ \eta^\gamma \quad \eta^- \end{array} & = i e p^\mu \\
\begin{array}{c} W_\mu \\ \text{---} \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^+ \end{array} & = -i g c_W p^\mu \\
\begin{array}{c} W_\mu \\ \text{---} \bullet \\ \swarrow \quad \searrow \\ \eta^+ \quad \eta^Z \end{array} & = -i g c_W p^\mu \\
\begin{array}{c} W_\mu \\ \text{---} \bullet \\ \swarrow \quad \searrow \\ \eta^- \quad \eta^Z \end{array} & = i g c_W p^\mu \\
\begin{array}{c} W_\mu \\ \text{---} \bullet \\ \swarrow \quad \searrow \\ \eta^Z \quad \eta^- \end{array} & = i g c_W p^\mu
\end{array}$$

## 8 内外线一般费曼规则

标量玻色子传播子:

$$\text{---} \xrightarrow{p} \text{---} = \frac{i}{p^2 - m^2 + i\varepsilon}$$

Dirac 费米子传播子:

$$\text{---} \xrightarrow{p} \text{---} = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$$

无质量规范玻色子 (如光子) 传播子:

$$\mu \text{---} \xrightarrow{p} \text{---} \nu = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon} \quad (\text{Feynman 规范})$$

$$\mu \text{---} \xrightarrow{p} \text{---} \nu = \frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2 + i\varepsilon} \quad (\text{Landau 规范})$$

有质量规范玻色子 (如  $W^\pm$  和  $Z$ ) 传播子:

$$\mu \text{---} \xrightarrow{p} \text{---} \nu = \frac{-i(g_{\mu\nu} - p_\mu p_\nu / m^2)}{p^2 - m^2 + i\varepsilon} \quad (\text{么正规规范})$$

$$\mu \text{---} \xrightarrow{p} \text{---} \nu = \frac{-i g_{\mu\nu}}{p^2 - m^2 + i\varepsilon} \quad (\text{Feynman 规范})$$

标量玻色子外线:

$$\text{---} \text{---} \text{---} = 1 \quad (\text{初态或末态})$$

Dirac 费米子外线:

$$\begin{aligned}
\text{---} \leftarrow p &= u(p, s) \quad (\text{正粒子初态}) \\
\text{---} \rightarrow p &= \bar{u}(p, s) \quad (\text{正粒子末态}) \\
\text{---} \leftarrow p &= \bar{v}(p, s) \quad (\text{反粒子初态}) \\
\text{---} \rightarrow p &= v(p, s) \quad (\text{反粒子末态})
\end{aligned}$$

在计算非极化截面时，可利用自旋求和关系

$$\sum_s u(p, s) \bar{u}(p, s) = \not{p} + m, \quad \sum_s v(p, s) \bar{v}(p, s) = \not{p} - m. \quad (139)$$

矢量玻色子外线：

$$\begin{aligned}
\text{---} \leftarrow p &= \varepsilon_\mu(p, \lambda) \quad (\text{初态}) \\
\text{---} \rightarrow p &= \varepsilon_\mu^*(p, \lambda) \quad (\text{末态})
\end{aligned}$$

在计算非极化截面时，若包含无质量矢量玻色子外线，可作替换

$$\sum_\lambda \varepsilon_\mu^*(p, \lambda) \varepsilon_\nu(p, \lambda) \rightarrow -g_{\mu\nu}; \quad (140)$$

若包含有质量矢量玻色子外线，可作替换

$$\sum_\lambda \varepsilon_\mu^*(p, \lambda) \varepsilon_\nu(p, \lambda) \rightarrow -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}. \quad (141)$$

## 9 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2018 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中，光速、约化 Planck 常数和真空介电常数均取为 1，即  $c = \hbar = \varepsilon_0 = 1$ 。从而，速度没有量纲 (dimension)；长度量纲与时间量纲相同，是能量量纲的倒数；能量、质量和动量具有相同的量纲；精细结构常数表达为  $\alpha = e^2/(4\pi)$ ，而单位电荷量  $e = \sqrt{4\pi\alpha}$  是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系取为

$$1 = c = 2.99792458 \times 10^{10} \text{ cm} \cdot \text{s}^{-1}, \quad (142)$$

$$1 = \hbar = 6.582119514(40) \times 10^{-25} \text{ GeV} \cdot \text{s}, \quad (143)$$

$$1 = \hbar c = 1.973269788(12) \times 10^{-14} \text{ GeV} \cdot \text{cm}, \quad (144)$$

括号内数字代表测量值的  $1\sigma$  不确定度，由此可得

$$1 \text{ s} = 2.997925 \times 10^{10} \text{ cm}, \quad 1 \text{ cm} = 3.335641 \times 10^{-11} \text{ s}, \quad (145)$$

$$1 \text{ s} = 1.519267 \times 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 6.582120 \times 10^{-25} \text{ s}, \quad (146)$$

$$1 \text{ cm} = 5.067731 \times 10^{13} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 1.973270 \times 10^{-14} \text{ cm}, \quad (147)$$

$$1 \text{ cm}^2 = 2.568190 \times 10^{27} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^{-28} \text{ cm}^2, \quad (148)$$

$$1 \text{ cm}^3 \cdot \text{s}^{-1} = 8.566558 \times 10^{16} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 1.167330 \times 10^{-17} \text{ cm}^3 \cdot \text{s}^{-1}. \quad (149)$$

靶 (barn) 是散射截面的常用单位, 记作 b, 满足

$$1 \text{ b} = 10^{-24} \text{ cm}^2 = 10^9 \text{ nb} = 10^{12} \text{ pb} = 10^{15} \text{ fb} = 10^{18} \text{ ab}, \quad (150)$$

$$1 \text{ pb} = 10^{-36} \text{ cm}^2 = 2.568190 \times 10^{-9} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^8 \text{ pb}. \quad (151)$$

Fermi 耦合常数是

$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}. \quad (152)$$

由树图阶关系式

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_W^2}, \quad (153)$$

可得 Higgs 场真空期待值为

$$v = (\sqrt{2}G_F)^{-1/2} = 246.2197 \text{ GeV}. \quad (154)$$

在低能标 (Thomson 极限) 处, 精细结构常数为

$$\alpha = 7.2973525664(17) \times 10^{-3} = \frac{1}{137.035999139(31)}; \quad (155)$$

在  $\overline{\text{MS}}$  重整化方案 (以  $\wedge$  为标志) 中,  $\alpha^{-1}$  跑动到  $\mu = m_Z$  能标处的数值是

$$\hat{\alpha}^{-1}(m_Z) = 127.955 \pm 0.010. \quad (156)$$

在  $\overline{\text{MS}}$  方案中,  $\mu = m_Z$  能标处强耦合常数  $\alpha_s = g_s^2/(4\pi)$  的数值为

$$\hat{\alpha}_s(m_Z) = 0.1181 \pm 0.0011, \quad (157)$$

Weinberg 角  $\theta_W$  的数值对应于

$$\hat{s}_W^2 = \sin^2 \hat{\theta}_W(m_Z) = 0.23122 \pm 0.00004. \quad (158)$$

在标准模型中, 光子、胶子和中微子没有质量, 其它基本粒子的质量为

$$m_W = 80.379 \pm 0.012 \text{ GeV}, \quad m_Z = 91.1876 \pm 0.0021 \text{ GeV}, \quad m_H = 125.18 \pm 0.16 \text{ GeV}, \quad (159)$$

$$m_e = 0.5109989461(31) \text{ MeV}, \quad m_\mu = 105.6583745(24) \text{ MeV}, \quad m_\tau = 1776.86 \pm 0.12 \text{ MeV}, \quad (160)$$

$$m_u = 2.2_{-0.4}^{+0.5} \text{ MeV}, \quad m_d = 4.7_{-0.3}^{+0.5} \text{ MeV}, \quad m_s = 95_{-3}^{+9} \text{ MeV}, \quad (161)$$

$$m_c = 1.67 \pm 0.07 \text{ GeV}, \quad m_b = 4.78 \pm 0.06 \text{ GeV}, \quad m_t = 173.0 \pm 0.4 \text{ GeV}. \quad (162)$$

这里,  $u$ 、 $d$ 、 $s$  夸克的质量是  $\mu \simeq 2 \text{ GeV}$  能标处的流夸克质量 (current-quark mass), 其余粒子的质量均为极点质量 (pole mass)。  $c$ 、 $b$  夸克在  $\overline{\text{MS}}$  方案中的跑动质量 (running mass) 为

$$\bar{m}_c(\bar{m}_c) = 1.275_{-0.035}^{+0.025} \text{ GeV}, \quad \bar{m}_b(\bar{m}_b) = 4.18_{-0.03}^{+0.04} \text{ GeV}. \quad (163)$$

质子和中子的质量为

$$m_p = 938.2720813(58) \text{ MeV}, \quad m_n = 939.565413(6) \text{ MeV}. \quad (164)$$

在电弱能标附近作领头阶计算时, 可将单位电荷量  $e$  取为

$$e = \sqrt{4\pi\hat{\alpha}(m_Z)} = 0.3133836, \quad (165)$$

将强耦合常数  $g_s$  取为

$$g_s = \sqrt{4\pi\hat{\alpha}_s(m_Z)} = 1.218232. \quad (166)$$

从树图阶关系计算 Higgs 场四线性耦合常数  $\lambda$  和 Yukawa 耦合常数  $y_t$ 、 $y_b$ 、 $y_\tau$ 、 $y_c$ , 得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1292393, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9936613, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.400870 \times 10^{-2}, \quad (167)$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020576 \times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 7.323227 \times 10^{-3}. \quad (168)$$

耦合常数  $g$  和  $g'$  有以下两种取值方式。

1. 根据树图阶关系  $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$  计算 Weinberg 角, 得

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2230132, \quad c_W^2 = 1 - s_W^2 = 0.7769868, \quad (169)$$

$$s_W = \sqrt{s_W^2} = 0.4722428, \quad c_W = \sqrt{c_W^2} = 0.8814685, \quad (170)$$

故

$$g = \frac{e}{s_W} = 0.6636071, \quad g' = \frac{e}{c_W} = 0.3555245. \quad (171)$$

2. 根据  $\overline{\text{MS}}$  方案中 Weinberg 角的数值 (158) 计算  $g$  和  $g'$ , 得

$$c_W^2 = 1 - \hat{s}_W^2 = 0.76878, \quad s_W = \sqrt{\hat{s}_W^2} = 0.4808534, \quad c_W = \sqrt{c_W^2} = 0.8768010, \quad (172)$$

$$g = \frac{e}{s_W} = 0.6517238, \quad g' = \frac{e}{c_W} = 0.3574170. \quad (173)$$

CKM 矩阵分量  $V_{ij}$  之模的测量值为

$$|V_{ij}| = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359_{-0.00011}^{+0.00010} & 0.04214 \pm 0.00076 \\ 0.00896_{-0.00023}^{+0.00024} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}. \quad (174)$$

如果仅仅近似地考虑 Cabibbo 转动角  $\theta_C$ , 有

$$V_{ij} \simeq \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \\ & & 1 \end{pmatrix}, \quad \sin \theta_C = |V_{12}| = 0.22452. \quad (175)$$

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