标准模型拉氏量和费曼规则

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目 录

1 约定

本文采用有理化的自然单位制，约定主要遵从文献 [1]，推导和计算参考文献 [1, 2, 3, 4]。Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{1}$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \tag{2}$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \tag{3}$$

Weyl 表象中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -\mathbf{1} & \\ & \mathbf{1} \end{pmatrix}. \tag{4}$$

左右手投影算符

$$P_{\mathrm{L}} \equiv \frac{1}{2}(1 - \gamma_5) = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{0} \end{pmatrix}, \quad P_{\mathrm{R}} \equiv \frac{1}{2}(1 + \gamma_5) = \begin{pmatrix} \mathbf{0} & \\ & \mathbf{1} \end{pmatrix} \tag{5}$$

用于定义左手旋量场 $\psi_{\mathrm{L}} \equiv P_{\mathrm{L}}\psi$ 和右手旋量场 $\psi_{\mathrm{R}} \equiv P_{\mathrm{R}}\psi$。Levi-Civita 符号的约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \tag{6}$$

费曼规则约定如下。

- 对于指向相互作用顶点的动量 p，时空导数 ∂_μ 在动量空间费曼规则里贡献一个 $-ip_\mu$ 因子。

- 实线表示费米子，实线上的箭头表示费米子数流动的方向。

- 虚线表示标量玻色子，虚线上的箭头表示玻色子数流动的方向。

- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示玻色子数流动的方向。

- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。

- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

2 标准模型概述

粒子物理标准模型是一个 $SU(3)_C \times SU(2)_L \times U(1)_Y$ 规范理论。模型中有三代费米子，包括三代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$，三代带电轻子 $\ell_i = e, \mu, \tau$，三代上型夸克 $u_i = u, c, t$ 和三代下型夸克 $d_i = d, s, b$ $(i = 1, 2, 3)$。规范玻色子传递费米子之间的规范相互作用。

$SU(3)_C$ 部分描述夸克的强相互作用，称为量子色动力学 (Quantum Chromodynamics, QCD)，相应的规范玻色子是胶子。$SU(2)_L \times U(1)_Y$ 部分统一描述夸克和轻子的电磁和弱相互作用，称为电弱规范理论。理论中有一个 Higgs 二重态，通过 Brout–Englert–Higgs (BEH) 机制引发规范群的自发对称性破缺，使 $SU(2)_L \times U(1)_Y$ 群破缺为 $U(1)_{EM}$ 群。$U(1)_{EM}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前。理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度；左手费米子和右手费米子都没有质量，具有不同的量子数。

破缺后，3 个规范玻色子与 3 个 Higgs 自由度结合，从而获得质量，成为 $W^\pm$ 和 Z^0 玻色子，传递弱相互作用。剩下的 1 个无质量规范玻色子是光子，即是 $U(1)_{EM}$ 群的规范玻色子，传递电磁相互作用。剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子。费米子与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量，组合成 Dirac 费米子。

理论中的中微子没有右手分量，因而没有获得质量。1998 年实验发现中微子振荡，证明中微子具有质量，所以需要扩充标准模型才能正确描述中微子物理。

3 量子色动力学

QCD 的拉氏量表达为

$$\mathcal{L}_{QCD} = \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \cdots, 8, \tag{7}$$

其中

$$D_\mu = \partial_\mu - ig_s G^a_\mu t^a, \quad G^{a\mu\nu} \equiv \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{abc} G^{b\mu} G^{c\nu}. \tag{8}$$

q 为夸克旋量场 $SU(3)_C$ 三重态，$SU(3)_C$ 规范场 G^a_μ 对应于胶子 g，g_s 是 $SU(3)_C$ 规范耦合常数。$t^a = \lambda^a/2$ 是 $SU(3)_C$ 群基础表示的生成元，其中 λ^a 为 Gell-Mann 矩阵。$SU(3)_C$ 生成元对易关系为 $[t^a, t^b] = if^{abc}t^c$，结构常数 f^{abc} 是全反对称的，非零分量为

$$f_{123} = 1, \quad f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \tag{9}$$

由

$$-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} = -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_s f^{ade} G^{d\mu} G^{e\nu})$$

$$= -\frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu}) - (\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu})] - g_s f^{abc}(\partial_\mu G^a_\nu)G^{b\mu}G^{c\nu}$$

$$-\frac{1}{4}g_{\mathrm{s}}^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu}, \tag{10}$$

推出

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q [\bar{q}(i\gamma^\mu\partial_\mu - m_q)q + g_{\mathrm{s}}G_\mu^a\bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu})]$$

$$-g_{\mathrm{s}}f^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} - \frac{1}{4}g_{\mathrm{s}}^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu}. \tag{11}$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G_\mu^a(x) - \omega^a(x)$，其中 $\omega^a(x)$ 是某个任意函数，规范固定条件是 $G^a(x) = 0$。这是 Lorenz 规范的推广，$\omega^a(x) = 0$ 对应于 Lorenz 规范。在路径积分量子化中，以中心为 $\omega^a(x) = 0$ 的 Gauss 权重对 $\omega^a(x)$ 作泛函积分，有

$$\int \mathcal{D}\omega^a \exp\left[-i\int d^4x \frac{1}{2\xi}(\omega^a)^2\right]\delta(G^a) = \exp\left[-i\int d^4x \frac{1}{2\xi}(\partial^\mu G_\mu^a)^2\right]. \tag{12}$$

可见，拉氏量中的规范固定项为

$$\mathcal{L}_{\mathrm{QCD,GF}} = -\frac{1}{2\xi}(\partial^\mu G_\mu^a)^2. \tag{13}$$

ξ 的任何一个取值对应于一种规范。$\xi = 1$ 称为 Feynman-'t Hooft 规范，$\xi = 0$ 称为 Landau 规范。于是，胶子传播子相关拉氏量为

$$\mathcal{L}_{\mathrm{QCD,prop}} = \frac{1}{2}\left[(\partial_\mu G_\nu^a)(\partial^\nu G^{a\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a\nu}) - \frac{1}{\xi}(\partial^\mu G_\mu^a)^2\right]$$

$$\to \frac{1}{2}G_\mu^a\left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]G_\nu^a. \tag{14}$$

变换到动量空间，得

$$-g^{\mu\nu}p^2 + \left(1 - \frac{1}{\xi}\right)p^\mu p^\nu, \tag{15}$$

它的逆矩阵是

$$-\frac{1}{p^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi)\right], \tag{16}$$

这是因为

$$-\frac{1}{p^2}\left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}(1 - \xi)\right]\left[-g^{\mu\nu}p^2 + \left(1 - \frac{1}{\xi}\right)p^\mu p^\nu\right]$$

$$= \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2}\left(1 - \frac{1}{\xi}\right) - \frac{p_\rho p^\nu}{p^2}(1 - \xi) + \frac{p_\rho p^\nu}{p^2}(1 - \xi)\left(1 - \frac{1}{\xi}\right) = \delta_\rho{}^\nu. \tag{17}$$

从而胶子传播子的形式为

$$\frac{-i\delta^{ab}}{p^2 + i\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi)\right]. \tag{18}$$

SU(3)$_{\mathrm{C}}$ 规范变换为

$$q \to Uq, \quad G_\mu^a t^a \to U G_\mu^a t^a U^\dagger + \frac{i}{g_{\mathrm{s}}} U \partial_\mu U^\dagger, \tag{19}$$

其中 $U(x) = \exp[i\alpha^a(x)t^a]$。胶子场的无穷小规范变换形式是

$$
\begin{aligned}
G_\mu^a t^a &\to (1 + i\alpha^a t^a) G_\mu^b t^b (1 - i\alpha^c t^c) + \frac{i}{g_{\mathrm{s}}}(1 + i\alpha^a t^a) \partial_\mu (1 - i\alpha^c t^c) \\
&= G_\mu^b t^b + i\alpha^a G_\mu^b [t^a, t^b] + \frac{1}{g_{\mathrm{s}}}(\partial_\mu \alpha^c) t^c + \mathcal{O}(\alpha^2) = G_\mu^a t^a - f^{abc} \alpha^a G_\mu^b t^c + \frac{1}{g_{\mathrm{s}}}(\partial_\mu \alpha^a) t^a + \mathcal{O}(\alpha^2) \\
&= \left(G_\mu^a + f^{abc} G_\mu^b \alpha^c + \frac{1}{g_{\mathrm{s}}} \partial_\mu \alpha^a \right) t^a + \mathcal{O}(\alpha^2),
\end{aligned}
\tag{20}
$$

即

$$\delta G_\mu^a = \frac{1}{g_{\mathrm{s}}} \partial_\mu \alpha^a + f^{abc} G_\mu^b \alpha^c = \left(\frac{1}{g_{\mathrm{s}}} \delta^{ac} \partial_\mu + f^{abc} G_\mu^b \right) \alpha^c, \tag{21}$$
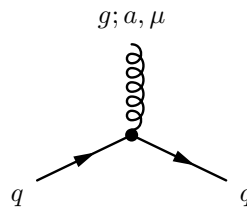
因而规范固定函数 G^a 的无穷小规范变换为

$$\delta G^a = \partial^\mu \delta G_\mu^a = \frac{1}{g_{\mathrm{s}}} \delta^{ac} \partial^2 \alpha^c + f^{abc} \partial^\mu G_\mu^b \alpha^c, \quad g_{\mathrm{s}} \frac{\delta G^a}{\delta \alpha^c} = \delta^{ab} \partial^2 + g_{\mathrm{s}} f^{abc} \partial^\mu G_\mu^b. \tag{22}$$
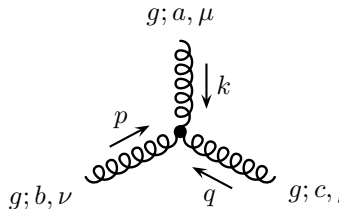
Faddeev-Popov 鬼场的拉氏量是

$$\mathcal{L}_{\mathrm{QCD,FP}} = -\bar\eta_g^a \left(g_{\mathrm{s}} \frac{\delta G^a}{\delta \alpha^c} \right) \eta_g^c = -\bar\eta_g^a (\delta^{ac} \partial^2 + g_{\mathrm{s}} f^{abc} \partial^\mu G_\mu^b) \eta_g^c \to -\bar\eta_g^a \delta^{ab} \partial^2 \eta_g^b + g_{\mathrm{s}} f^{abc} (\partial^\mu \bar\eta_g^a) G_\mu^b \eta_g^c. \tag{23}$$
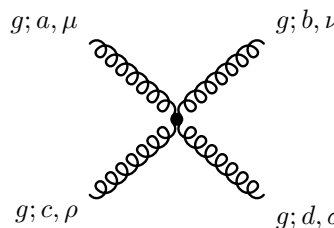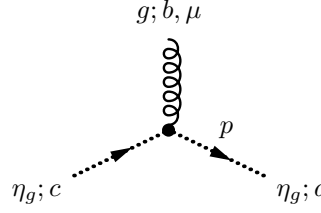
下面列出 QCD 费曼规则。

QCD 耦合顶点：

$$= ig_{\mathrm{s}} \gamma^\mu t^a \tag{24}$$

$$= g_{\mathrm{s}} f^{abc} [g^{\mu\nu}(k - p)^\rho + g^{\nu\rho}(p - q)^\mu + g^{\rho\mu}(q - k)^\nu] \tag{25}$$
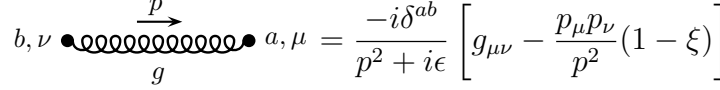

$$
\begin{aligned}
= -ig_{\mathrm{s}}^2 [&f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\
&+ f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\
&+ f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})]
\end{aligned}
\tag{26}
$$

$$= -g_{\mathrm{s}} f^{abc} p^{\mu} \tag{27}$$

胶子传播子：

$$= \frac{-i\delta^{ab}}{p^2 + i\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi) \right] \tag{28}$$

鬼粒子传播子：

$$= \frac{i\delta^{ab}}{p^2 + i\epsilon} \tag{29}$$

4 电弱规范理论

电弱规范理论的规范群是 $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$，每一代左手旋量场构成 2 个 $\mathrm{SU}(2)_{\mathrm{L}}$ 二重态

$$L_{i\mathrm{L}} = \begin{pmatrix} P_{\mathrm{L}}\nu_i \\ P_{\mathrm{L}}\ell_i \end{pmatrix} = \begin{pmatrix} \nu_{i\mathrm{L}} \\ \ell_{i\mathrm{L}} \end{pmatrix}, \quad Q_{i\mathrm{L}} = \begin{pmatrix} P_{\mathrm{L}}u'_i \\ P_{\mathrm{L}}d'_i \end{pmatrix} = \begin{pmatrix} u'_{i\mathrm{L}} \\ d'_{i\mathrm{L}} \end{pmatrix}, \quad i = 1, 2, 3. \tag{30}$$

它们的协变导数是

$$D_\mu = \partial_\mu - ig'B_\mu Y - igW^a_\mu \tau^a, \tag{31}$$

其中 $W^a_\mu(x)$ $(a = 1, 2, 3)$ 是 $\mathrm{SU}(2)_{\mathrm{L}}$ 规范场，$B_\mu(x)$ 是 $\mathrm{U}(1)_{\mathrm{Y}}$ 规范场，g 和 g' 分别是 $\mathrm{SU}(2)_{\mathrm{L}}$ 和 $\mathrm{U}(1)_{\mathrm{Y}}$ 的规范耦合常数。

$$\tau^a = \frac{\sigma^a}{2} \tag{32}$$

是 $\mathrm{SU}(2)_{\mathrm{L}}$ 群 2 维表示的生成元，对应于弱同位旋。生成元 τ^3 的本征值是弱同位旋第 3 分量，记为 T^3。右手旋量场 $\ell_{i\mathrm{R}} = P_{\mathrm{R}}\ell_i$、$u'_{i\mathrm{R}} = P_{\mathrm{R}}u'_i$ 和 $d'_{i\mathrm{R}} = P_{\mathrm{R}}d'_i$ 是 $\mathrm{SU}(2)_{\mathrm{L}}$ 单态，协变导数为

$$D_\mu = \partial_\mu - ig'B_\mu Y. \tag{33}$$

表 1 列出费米子场的电荷 Q、弱同位旋第 3 分量 T^3、弱超荷 Y、重子数 B 和轻子数 $L_e/L_\mu/L_\tau$，其中电荷 Q 由 T^3 和 Y 定义，

$$Q \equiv T^3 + Y. \tag{34}$$

4.1 Brout–Englert–Higgs 机制

由于左手费米子和右手费米子参与不同的 $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$ 规范相互作用，耦合左右手费米子场的质量项会破坏规范对称性。另一方面，规范对称性也禁止规范玻色子具有质量。为了让费米子和弱规范玻色子获得质量，需要引入 BEH 机制，使 $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$ 规范对称性自发破

表 1: 标准模型费米子场的量子数。

统一记号	第一代	第二代	第三代	Q	T^3	Y	B	$L_e/L_\mu/L_\tau$
$L_{i\mathrm{L}} = \begin{pmatrix} \nu_{i\mathrm{L}} \\ \ell_{i\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} \nu_{e\mathrm{L}} \\ e_{\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu\mathrm{L}} \\ \mu_{\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau\mathrm{L}} \\ \tau_{\mathrm{L}} \end{pmatrix}$	0 -1	$1/2$ $-1/2$	$-1/2$ $-1/2$	0 0	1 1
$Q_{i\mathrm{L}} = \begin{pmatrix} u'_{i\mathrm{L}} \\ d'_{i\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} u'_{\mathrm{L}} \\ d'_{\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} c'_{\mathrm{L}} \\ s'_{\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} t'_{\mathrm{L}} \\ b'_{\mathrm{L}} \end{pmatrix}$	$2/3$ $-1/3$	$1/2$ $-1/2$	$1/6$ $1/6$	$1/3$ $1/3$	0 0
$\ell_{i\mathrm{R}}$	e_{R}	μ_{R}	τ_{R}	-1	0	-1	0	1
$u'_{i\mathrm{R}}$	u'_{R}	c'_{R}	t'_{R}	$2/3$	0	$2/3$	$1/3$	0
$d'_{i\mathrm{R}}$	d'_{R}	s'_{R}	b'_{R}	$-1/3$	0	$-1/3$	$1/3$	0

缺。因此，引入 Higgs 标量场

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \tag{35}$$

其中 ϕ^+ 和 ϕ^0 都是复标量场。Φ 是 SU(2)$_\mathrm{L}$ 二重态，弱超荷是

$$Y_H = \frac{1}{2}. \tag{36}$$

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_\mathrm{H} = (D^\mu\Phi)^\dagger(D_\mu\Phi) - V_\mathrm{H}(\Phi), \quad V_\mathrm{H}(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \tag{37}$$

其中协变导数为

$$D_\mu\Phi = (\partial_\mu - ig'B_\mu Y_H - igW_\mu^a\tau^a)\Phi. \tag{38}$$

当 $\lambda > 0$ 且 $\mu^2 > 0$ 时，Higgs 场势能 $V_\mathrm{H}(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式，势能最小值位于方程

$$\Phi^\dagger\Phi = [\mathrm{Re}(\phi^+)]^2 + [\mathrm{Im}(\phi^+)]^2 + [\mathrm{Re}(\phi^0)]^2 + [\mathrm{Im}(\phi^0)]^2 = \frac{v^2}{2} \tag{39}$$

对应的 3 维球面上，其中 $v \equiv \sqrt{\mu^2/\lambda}$，满足
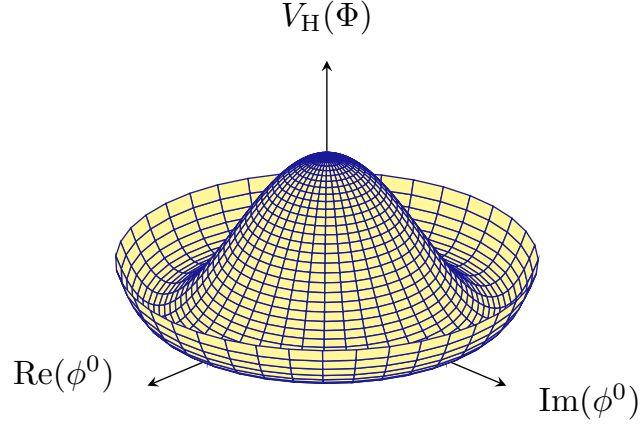
$$\mu^2 = \lambda v^2. \tag{40}$$

图 1: Higgs 场势能示意图。这里压缩掉 $\mathrm{Re}(\phi^+)$ 和 $\mathrm{Im}(\phi^+)$ 两个维度。

Higgs 场的真空期待值位于这个 3 维球面上的某一点，不失一般性，可将它取为

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{41}$$

其它真空期待值可通过 $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 整体变换

$$\langle \Phi \rangle \to \exp(i\alpha^a \tau^a) \exp(i\alpha^Y Y_H) \langle \Phi \rangle \tag{42}$$

得到，因为 $\langle \Phi^\dagger \Phi \rangle$ 在这样的变换下保持不变。若 $\alpha^1 = \alpha^2 = 0$ 且 $\alpha^3 = \alpha^Y$，则 $\langle \Phi \rangle$ 在变换下不变。因此，有 1 个方向的规范对称性没有受到破坏，只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理，破缺后生成 3 个无质量的 Nambu-Goldstone 玻色子。最终，有 3 个规范玻色子结合 Nambu-Goldstone 玻色子，通过 BEH 机制获得质量。

以 $\langle \Phi \rangle$ 为基础，将 Higgs 场参数化为

$$\Phi(x) = \exp\left[-i\frac{\chi^a(x)}{v}\tau^a\right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{43}$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场。$\exp[-i\chi^a(x)\tau^a/v]$ 因子能够通过 $\mathrm{SU}(2)_\mathrm{L}$ 规范变换消去，因而可将 $\Phi(x)$ 直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger \Phi = \frac{1}{2}(v+H)^2. \tag{44}$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$，对应于 Higgs 玻色子，这种取法称为**幺正规范**。

在幺正规范下，势能项化为

$$-V_\mathrm{H}(\Phi) = \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2 = \frac{1}{2}\mu^2(v+H)^2 - \frac{1}{4}\lambda(v+H)^4$$

$$= \frac{1}{2}\mu^2(v^2 + H^2 + 2vH) - \frac{1}{4}\lambda(v^4 + 4v^2H^2 + H^4 + 4v^3H + 2v^2H^2 + 4vH^3)$$

$$= \frac{1}{4}\mu^2v^2 + \frac{1}{4}(\mu^2 - \lambda v^2)v^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)H^2 - \lambda v^2H^2 - \lambda vH^3 - \frac{1}{4}\lambda H^4$$

$$= \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{1}{2}\frac{m_H^2}{v}H^3 - \frac{1}{8}\frac{m_H^2}{v^2}H^4, \tag{45}$$

其中 Higgs 玻色子的质量为

$$m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \tag{46}$$

由于

$$g'B_\mu Y_H + gW_\mu^a \tau^a = \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}, \tag{47}$$

Higgs 场真空期待值 v 对协变导数 $D_\mu\Phi$ 的贡献为

$$D_\mu\Phi \supset (g'B_\mu Y_H + gW_\mu^a \tau^a)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{v}{2\sqrt{2}}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \supset \frac{v}{2\sqrt{2}}\begin{pmatrix} g(W_\mu^1 - iW_\mu^2) \\ (g'B_\mu - gW_\mu^3) \end{pmatrix}, \tag{48}$$

故 v 对协变动能项的贡献是

$$(D^\mu\Phi)^\dagger(D_\mu\Phi) \supset \frac{v^2}{8}[g^2|W_\mu^1 - iW_\mu^2|^2 + (g'B_\mu - gW_\mu^3)^2]$$

$$= \frac{v^2}{8}(g^2 W^{a\mu}W_\mu^a + g'^2 B^\mu B_\mu - 2gg'B^\mu W_\mu^3)$$

$$= \frac{v^2}{8}\begin{pmatrix} W^{1\mu} & W^{2\mu} & W^{3\mu} & B^\mu \end{pmatrix}\begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \\ B_\mu \end{pmatrix}. \tag{49}$$

这些项是规范玻色子的质量项，重新表达为

$$\mathcal{L}_{\text{GBM}} = \frac{1}{2}m_W^2(W^{1\mu}W_\mu^1 + W^{2\mu}W_\mu^2) + \frac{1}{2}\begin{pmatrix} W^{3\mu} & B^\mu \end{pmatrix}M_{W^3B}^2\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{50}$$

其中

$$m_W \equiv \frac{1}{2}gv \tag{51}$$

是 W_μ^1 和 W_μ^2 获得的质量，而 $W^{3\mu}$ 和 B^μ 的质量平方矩阵为

$$M_{W^3B}^2 \equiv \frac{v^2}{4} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \tag{52}$$

为了使 $M_{W^3B}^2$ 矩阵对角化，定义

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{53}$$

其中

$$s_W \equiv \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_W \equiv \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \tag{54}$$

θ_W 称为 Weinberg 角，也称为弱混合角。从后面的讨论可以看出 $A_\mu(x)$ 就是电磁场，对应于光子。$Z_\mu(x)$ 对应于矢量玻色子 Z。反过来，有

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \tag{55}$$

由

$$\begin{aligned} M_{W^3B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W^2 & -s_W c_W \\ -s_W c_W & s_W^2 \end{pmatrix} \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W & 0 \\ -s_W & 0 \end{pmatrix} \end{aligned} \tag{56}$$

得

$$\begin{aligned} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} c_W & 0 \\ -s_W & 0 \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \end{aligned} \tag{57}$$

因此

$$\begin{aligned} \frac{1}{2} \begin{pmatrix} W^{3\mu} & B^\mu \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\ &= \frac{(g^2 + g'^2)v^2}{8} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} m_Z^2 Z^\mu Z_\mu, \quad (58) \end{aligned}$$

其中

$$m_Z \equiv \frac{1}{2}\sqrt{g^2 + g'^2}\, v = \frac{gv}{2c_{\mathrm{W}}} = \frac{m_W}{c_{\mathrm{W}}} \tag{59}$$

是 Z 玻色子的质量，而光子没有质量。另一方面，用质量相同的实矢量场 W_μ^1 和 W_μ^2 线性组合出复矢量场

$$W_\mu^+ \equiv \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2), \tag{60}$$

它的厄米共轭为

$$W_\mu^- \equiv (W_\mu^+)^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2), \tag{61}$$

则

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \tag{62}$$

从而

$$\frac{1}{2}(W^{1\mu}W_\mu^1 + W^{2\mu}W_\mu^2) = \frac{1}{4}[(W^{+\mu} + W^{-\mu})(W_\mu^+ + W_\mu^-) - (W^{+\mu} - W^{-\mu})(W_\mu^+ - W_\mu^-)] = W^{+\mu}W_\mu^-, \tag{63}$$

[(50)](#) 式化为

$$\mathcal{L}_{\mathrm{GBM}} = m_W^2 W^{+\mu}W_\mu^- + \frac{1}{2}m_Z^2 Z^\mu Z_\mu. \tag{64}$$

复矢量场 $W_\mu^\pm$ 描述一对正反矢量玻色子 $W^\pm$，质量为 m_W。可见，BEH 机制使传递弱相互作用的规范玻色子 $W^\pm$ 和 Z 获得了质量。

接下来用质量本征态 $W_\mu^\pm$ 和 Z_μ 表达协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$。注意到

$$A_\mu = s_{\mathrm{W}}W_\mu^3 + c_{\mathrm{W}}B_\mu, \quad Z_\mu = c_{\mathrm{W}}W_\mu^3 - s_{\mathrm{W}}B_\mu, \tag{65}$$

$$B_\mu = c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu, \quad W_\mu^3 = s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu, \tag{66}$$

有

$$g'B_\mu + gW_\mu^3 = g'(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) + g(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu) = \frac{2gg'}{\sqrt{g^2 + g'^2}}A_\mu + \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}}Z_\mu$$

$$= 2eA_\mu + \frac{g}{c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu, \tag{67}$$

其中

$$e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = gs_{\mathrm{W}} = g'c_{\mathrm{W}}. \tag{68}$$

后面的讨论将表明 e 就是单位电荷量。于是

$$g'B_\mu Y_H + gW_\mu^a \tau^a = \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}$$

$$= \begin{pmatrix} eA_\mu + \dfrac{g}{2c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu & \dfrac{1}{\sqrt{2}}gW_\mu^+ \\[2ex] \dfrac{1}{\sqrt{2}}gW_\mu^- & -\dfrac{g}{2c_{\mathrm{W}}}Z_\mu \end{pmatrix}. \tag{69}$$

在幺正规范下，

$$\begin{aligned}
(D^\mu\Phi)^\dagger(D_\mu\Phi) &= \left| \begin{pmatrix} \partial_\mu - ieA_\mu - \dfrac{ig}{2c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu & -\dfrac{i}{\sqrt{2}}gW_\mu^+ \\[2ex] -\dfrac{i}{\sqrt{2}}gW_\mu^- & \partial_\mu + \dfrac{ig}{2c_{\mathrm{W}}}Z_\mu \end{pmatrix} \dfrac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix} \right|^2 \\[3ex]
&= \dfrac{1}{2}\left(\dfrac{i}{\sqrt{2}}gW_\mu^-(v+H) \quad \partial_\mu H - \dfrac{ig}{2c_{\mathrm{W}}}Z_\mu(v+H) \right) \begin{pmatrix} -\dfrac{i}{\sqrt{2}}gW_\mu^+(v+H) \\[2ex] \partial_\mu H + \dfrac{ig}{2c_{\mathrm{W}}}Z_\mu(v+H) \end{pmatrix} \\[3ex]
&= \dfrac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2\left(\dfrac{g^2}{4}W_\mu^+ W^{-\mu} + \dfrac{g^2}{8c_{\mathrm{W}}^2}Z_\mu Z^\mu \right) \\[2ex]
&= \dfrac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-\mu} + \dfrac{1}{2}m_Z^2 Z_\mu Z^\mu \\[2ex]
&\quad + gm_W H W_\mu^+ W^{-\mu} + \dfrac{gm_Z}{2c_{\mathrm{W}}}H Z_\mu Z^\mu + \dfrac{g^2}{4}H^2 W_\mu^+ W^{-\mu} + \dfrac{g^2}{8c_{\mathrm{W}}^2}H^2 Z_\mu Z^\mu.
\end{aligned} \tag{70}$$

除了 $W^\pm$ 和 Z 玻色子的质量项之外，还出现了 Higgs 玻色子 H 与 $W^\pm$、Z 的三线性和四线性耦合项。

Higgs 场 $\Phi(x)$ 的弱超荷为 $+1/2$。引入 $\Phi(x)$ 的共轭态

$$\tilde{\Phi}(x) = i\sigma^2\Phi^*(x) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}, \tag{71}$$

其中 $\phi^- \equiv (\phi^+)^*$，则 $\tilde{\Phi}(x)$ 是弱超荷为 $-1/2$ 的 $\mathrm{SU(2)_L}$ 二重态。在幺正规范下，$\tilde{\Phi}(x)$ 化为

$$\tilde{\Phi}(x) = \dfrac{1}{\sqrt{2}}\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} v+H(x) \\ 0 \end{pmatrix}. \tag{72}$$

用 $\Phi(x)$、$\tilde{\Phi}(x)$ 和费米子场组成满足 $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ 的 Yukawa 相互作用拉氏量

$$\mathcal{L}_{\mathrm{Y}} = -\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - \tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{H.c.}, \tag{73}$$

其中 H.c. 表示厄米共轭，Yukawa 耦合常数 $\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 联系着不同代的夸克场，而 y_{ℓ_i} 只联系同一代的轻子场。在幺正规范下，利用

$$\bar{Q}_{i\mathrm{L}}\Phi = \dfrac{1}{\sqrt{2}}\begin{pmatrix} \bar{u}'_{i\mathrm{L}} & \bar{d}'_{i\mathrm{L}} \end{pmatrix}\begin{pmatrix} 0 \\ v+H \end{pmatrix} = \dfrac{1}{\sqrt{2}}(v+H)\bar{d}'_{i\mathrm{L}}, \tag{74}$$

$$\bar{Q}_{i\mathrm{L}}\tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{u}'_{i\mathrm{L}} & \bar{d}'_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}v+H \\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{u}'_{i\mathrm{L}}, \tag{75}$$

$$\bar{L}_{i\mathrm{L}}\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{\nu}_{i\mathrm{L}} & \bar{\ell}_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}0 \\ v+H\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{\ell}_{i\mathrm{L}}, \tag{76}$$

推出

$$\mathcal{L}_{\mathrm{Y}} = -\frac{1}{\sqrt{2}}(v+H)(\tilde{y}_{d,ij}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}} + \tilde{y}_{u,ij}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}} + y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + \mathrm{H.c.}). \tag{77}$$

$\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 可看作 3×3 矩阵 $\tilde{y}_d$ 和 $\tilde{y}_u$ 的元素。$\tilde{y}_d\tilde{y}_d^\dagger$ 和 $\tilde{y}_u\tilde{y}_u^\dagger$ 是厄米矩阵，必定可以分别通过幺正矩阵 U_d 和 U_u 对角化成 y_{D}^2 和 y_{U}^2 两个对角元为实数的对角矩阵，满足 $U_d^\dagger\tilde{y}_d\tilde{y}_d^\dagger U_d = y_{\mathrm{D}}^2$ 和 $U_u^\dagger\tilde{y}_u\tilde{y}_u^\dagger U_u = y_{\mathrm{U}}^2$，即

$$\tilde{y}_d\tilde{y}_d^\dagger = U_d y_{\mathrm{D}}^2 U_d^\dagger, \quad \tilde{y}_u\tilde{y}_u^\dagger = U_u y_{\mathrm{U}}^2 U_u^\dagger. \tag{78}$$

符合这两条式子的 $\tilde{y}_d$ 和 $\tilde{y}_u$ 可以表达为

$$\tilde{y}_d = U_d y_{\mathrm{D}} K_d^\dagger, \quad \tilde{y}_u = U_u y_{\mathrm{U}} K_u^\dagger, \tag{79}$$

其中对角矩阵 y_{D} 和 y_{U} 满足 $y_{\mathrm{D}}y_{\mathrm{D}} = y_{\mathrm{D}}^2$ 和 $y_{\mathrm{U}}y_{\mathrm{U}} = y_{\mathrm{U}}^2$，而 $K_d^\dagger$ 和 $K_u^\dagger$ 是两个幺正矩阵。

将 y_{D} 和 y_{U} 表示成

$$y_{\mathrm{D}} = \begin{pmatrix}y_{d_1} & & \\ & y_{d_2} & \\ & & y_{d_3}\end{pmatrix} = \begin{pmatrix}y_d & & \\ & y_s & \\ & & y_b\end{pmatrix}, \quad y_{\mathrm{U}} = \begin{pmatrix}y_{u_1} & & \\ & y_{d_2} & \\ & & y_{d_3}\end{pmatrix} = \begin{pmatrix}y_u & & \\ & y_c & \\ & & y_t\end{pmatrix}. \tag{80}$$

通过幺正变换定义

$$d_{i\mathrm{L}} \equiv (U_d^\dagger)_{ij}d'_{j\mathrm{L}}, \quad d_{i\mathrm{R}} \equiv (K_d^\dagger)_{ij}d'_{j\mathrm{R}}, \quad u_{i\mathrm{L}} \equiv (U_u^\dagger)_{ij}u'_{j\mathrm{L}}, \quad u_{i\mathrm{R}} \equiv (K_u^\dagger)_{ij}u'_{j\mathrm{R}}, \tag{81}$$

则 $\bar{d}_{i\mathrm{L}} = \bar{d}'_{j\mathrm{L}}U_{d,ji}$，$\bar{u}_{i\mathrm{L}} = \bar{u}'_{j\mathrm{L}}U_{u,ji}$，从而

$$\tilde{y}_{d,ij}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}(U_d y_d K_d^\dagger)_{ij}d'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}U_{d,ik}y_{d_k}(K_d^\dagger)_{kj}d'_{j\mathrm{R}} = y_{d_k}\bar{d}_{k\mathrm{L}}d_{k\mathrm{R}} = y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}, \tag{82}$$

$$\tilde{y}_{u,ij}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}}(U_u y_u K_u^\dagger)_{ij}u'_{j\mathrm{R}} = y_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}, \tag{83}$$

故

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}} &= -\frac{1}{\sqrt{2}}(v+H)(y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} + y_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} + y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + \mathrm{H.c.}) \\
&= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i, \tag{84}
\end{aligned}$$

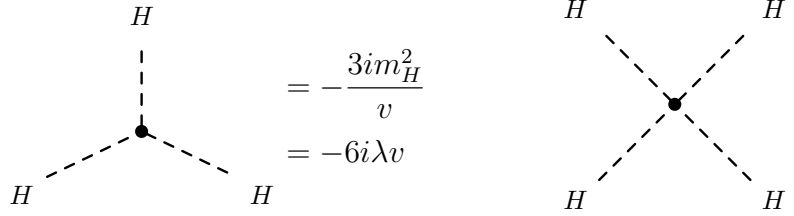其中前三项是费米子质量项，后三项是 Higgs 玻色子与费米子的 Yukawa 耦合项。于是，三代

夸克和带电轻子获得了质量

$$m_{d_i} \equiv \frac{1}{\sqrt{2}} y_{d_i} v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}} y_{u_i} v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}} y_{\ell_i} v. \tag{85}$$

$d'_{i\mathrm{L}}$、$d'_{i\mathrm{R}}$、$u'_{i\mathrm{L}}$ 和 $u'_{i\mathrm{R}}$ 称为规范本征态，$d_{i\mathrm{L}}$、$d_{i\mathrm{R}}$、$u_{i\mathrm{L}}$ 和 $u_{i\mathrm{R}}$ 称为质量本征态。
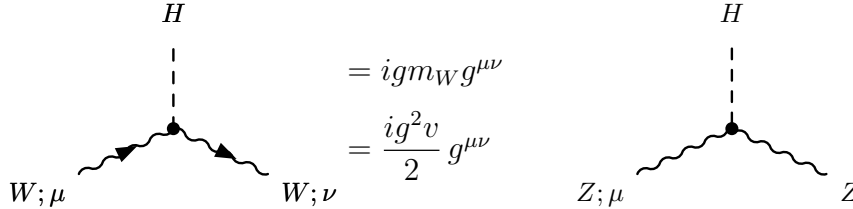
下面给出幺正规范下 Higgs 场的顶点费曼规则。

Higgs 玻色子自耦合：
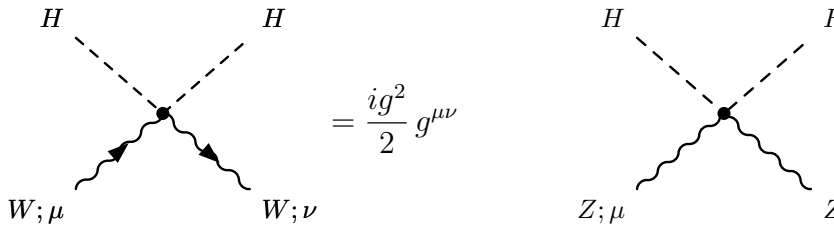
$$\tag{86}$$

Higgs 玻色子与电弱规范玻色子的耦合：
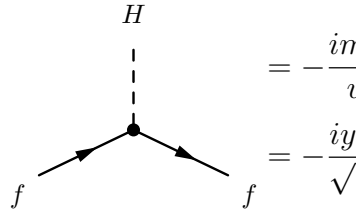
$$\tag{87}$$

$$\tag{88}$$

Higgs 玻色子与费米子 $f = d_i, u_i, \ell_i$ 的耦合：

$$\tag{89}$$

4.2 费米子电弱规范相互作用

(81) 式意味着

$$d'_{i\mathrm{L}} = U_{d,ij} d_{j\mathrm{L}}, \quad d'_{i\mathrm{R}} = K_{d,ij} d_{j\mathrm{R}}, \quad u'_{i\mathrm{L}} = U_{u,ij} u_{j\mathrm{L}}, \quad u'_{i\mathrm{R}} = K_{u,ij} u_{j\mathrm{R}}, \tag{90}$$

从而

$$\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} = \bar{d}_{j\mathrm{L}}(U_d^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}}\delta_{jk}\gamma^\mu d_{k\mathrm{L}} = \bar{d}_{i\mathrm{L}}\gamma^\mu d_{i\mathrm{L}} \tag{91}$$

同理有

$$\bar{u}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} = \bar{u}_{i\mathrm{L}}\gamma^\mu u_{i\mathrm{L}}, \quad \bar{d}'_{i\mathrm{R}}\gamma^\mu d'_{i\mathrm{R}} = \bar{d}_{i\mathrm{R}}\gamma^\mu d_{i\mathrm{R}}, \quad \bar{u}'_{i\mathrm{R}}\gamma^\mu u'_{i\mathrm{R}} = \bar{u}_{i\mathrm{R}}\gamma^\mu u_{i\mathrm{R}}. \tag{92}$$

另一方面,

$$\bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} = \bar{u}_{j\mathrm{L}}(U_u^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{k\mathrm{L}} = \bar{u}_{i\mathrm{L}}\gamma^\mu V_{ij}d_{j\mathrm{L}} \tag{93}$$

$$\bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} = \bar{d}_{j\mathrm{L}}(U_d^\dagger)_{ji}\gamma^\mu U_{u,ik}u_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}}V_{ji}^\dagger\gamma^\mu u_{i\mathrm{L}} \tag{94}$$

其中

$$V \equiv U_u^\dagger U_d \tag{95}$$

称为 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵, 其厄米共轭矩阵为 $V^\dagger = U_d^\dagger U_u$。

$\mathrm{SU(2)_L \times U(1)_Y}$ 规范不变的费米子协变动能项为

$$\mathcal{L}_{\mathrm{EWF}} = \bar{Q}_{i\mathrm{L}}i\slashed{D}Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}}i\slashed{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}i\slashed{D}d'_{i\mathrm{R}} + \bar{L}_{i\mathrm{L}}i\slashed{D}L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}}i\slashed{D}\ell_{i\mathrm{R}}. \tag{96}$$

根据 Q 的定义 (34), 有

$$\begin{aligned} g'YB_\mu + gT^3W_\mu^3 &= g'Y(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) + gT^3(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu) \\ &= e(Y+T^3)A_\mu + \left(gc_\mathrm{W}T^3 - \frac{gs_\mathrm{W}}{c_\mathrm{W}}s_\mathrm{W}Y\right)Z_\mu = QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3c_\mathrm{W}^2 - Ys_\mathrm{W}^2)Z_\mu \\ &= QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu, \end{aligned} \tag{97}$$

故

$$\begin{aligned} D_\mu Q_{i\mathrm{L}} &= (\partial_\mu - ig'B_\mu Y - igW_\mu^a\tau^a)Q_{i\mathrm{L}} \\ &= \partial_\mu Q_{i\mathrm{L}} - i\begin{pmatrix} g'YB_\mu + gT^3W_\mu^3 & \dfrac{1}{2}g(W_\mu^1 - iW_\mu^2) \\ \dfrac{1}{2}g(W_\mu^1 + iW_\mu^2) & g'YB_\mu + gT^3W_\mu^3 \end{pmatrix}Q_{i\mathrm{L}} \\ &= \partial_\mu Q_{i\mathrm{L}} - i\begin{pmatrix} QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu & \dfrac{1}{\sqrt{2}}gW_\mu^+ \\ \dfrac{1}{\sqrt{2}}gW_\mu^- & QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu \end{pmatrix}Q_{i\mathrm{L}} \\ &= \partial_\mu Q_{i\mathrm{L}} - i\begin{pmatrix} \left[QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]u'_{i\mathrm{L}} + \dfrac{1}{\sqrt{2}}gW_\mu^+ d'_{i\mathrm{L}} \\ \dfrac{1}{\sqrt{2}}gW_\mu^- u'_{i\mathrm{L}} + \left[QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)\right]d'_{i\mathrm{L}} \end{pmatrix}. \end{aligned} \tag{98}$$

于是

$$\bar{Q}_{i\mathrm{L}}i\not{D}Q_{i\mathrm{L}} \supset \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{u}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} + \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}}$$

$$+ \frac{1}{\sqrt{2}}gW_\mu^+\bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} + \frac{1}{\sqrt{2}}gW_\mu^-\bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}}$$

$$= \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{u}_i\gamma^\mu\frac{1-\gamma_5}{2}u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{d}_i\gamma^\mu\frac{1-\gamma_5}{2}d_i$$

$$+ \frac{1}{\sqrt{2}}gW_\mu^+\bar{u}_i\gamma^\mu\frac{1-\gamma_5}{2}V_{ij}d_j + \frac{1}{\sqrt{2}}gW_\mu^-\bar{d}_jV_{ji}^\dagger\gamma^\mu\frac{1-\gamma_5}{2}u_i, \tag{99}$$

其中左手耦合系数

$$g_\mathrm{L} \equiv T^3 - Qs_\mathrm{W}^2. \tag{100}$$

另一方面,

$$D_\mu d'_{i\mathrm{R}} = (\partial_\mu - ig'B_\mu Y)d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} - ig'Q(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu)d'_{i\mathrm{R}}$$

$$= \partial_\mu d'_{i\mathrm{R}} - iQeA_\mu d'_{i\mathrm{R}} + i\frac{g}{c_\mathrm{W}}Qs_\mathrm{W}^2 Z_\mu d'_{i\mathrm{R}}, \tag{101}$$

则

$$\bar{u}'_{i\mathrm{R}}i\not{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}i\not{D}d'_{i\mathrm{R}}$$

$$\supset \left(QeA_\mu - \frac{g}{c_\mathrm{W}}Qs_\mathrm{W}^2 Z_\mu\right)\bar{u}'_{i\mathrm{R}}\gamma^\mu u'_{i\mathrm{R}} + \left(QeA_\mu - \frac{g}{c_\mathrm{W}}Qs_\mathrm{W}^2 Z_\mu\right)\bar{d}'_{i\mathrm{R}}\gamma^\mu d'_{i\mathrm{R}}$$

$$= \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{R}Z_\mu\right)\bar{u}_i\gamma^\mu\frac{1+\gamma_5}{2}u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{R}Z_\mu\right)\bar{d}_i\gamma^\mu\frac{1+\gamma_5}{2}d_i, \tag{102}$$

其中右手耦合系数

$$g_\mathrm{R} \equiv -Qs_\mathrm{W}^2. \tag{103}$$

引入矢量流和轴矢量流耦合系数

$$g_\mathrm{V} \equiv g_\mathrm{L} + g_\mathrm{R} = T^3 - 2Qs_\mathrm{W}^2, \quad g_\mathrm{A} \equiv g_\mathrm{L} - g_\mathrm{R} = T^3, \tag{104}$$

得

$$\bar{Q}_{i\mathrm{L}}i\not{D}Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}}i\not{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}i\not{D}d'_{i\mathrm{R}}$$

$$\supset Qe\bar{u}_i\gamma^\mu u_i A_\mu + Qe\bar{d}_i\gamma^\mu d_i A_\mu + \frac{g}{2c_\mathrm{W}}\bar{u}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma_5)u_i Z_\mu + \frac{g}{2c_\mathrm{W}}\bar{d}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma_5)d_i Z_\mu$$

$$+ \frac{1}{\sqrt{2}}gW_\mu^+\bar{u}_i\gamma^\mu P_\mathrm{L}V_{ij}d_j + \frac{1}{\sqrt{2}}gW_\mu^-\bar{d}_jV_{ji}^\dagger\gamma^\mu P_\mathrm{L}u_i. \tag{105}$$

同理，有

$$\bar{L}_{i\mathrm{L}}i\not{D}L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}}i\not{D}\ell_{i\mathrm{R}} \supset Qe\bar{\ell}_i\gamma^\mu\ell_i A_\mu + \frac{g}{2c_\mathrm{W}}\bar{\ell}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma_5)\ell_i Z_\mu + \frac{g}{2c_\mathrm{W}}\bar{\nu}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma_5)\nu_i Z_\mu$$

$$+ \frac{1}{\sqrt{2}}gW_\mu^+ \bar{\nu}_i\gamma^\mu P_\mathrm{L}\ell_i + \frac{1}{\sqrt{2}}gW_\mu^- \bar{\ell}_i\gamma^\mu P_\mathrm{L}\nu_i. \tag{106}$$

总结起来，将费米子电弱规范相互作用写成流耦合的形式，

$$\mathcal{L}_\mathrm{EWF} \supset \sum_f \left[Q_f e\bar{f}\gamma^\mu f A_\mu + \frac{g}{2c_\mathrm{W}}\bar{f}\gamma^\mu(g_\mathrm{V}^f - g_\mathrm{A}^f\gamma_5)f Z_\mu \right] + g(W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu})$$

$$= eA_\mu J_\mathrm{EM}^\mu + g(Z_\mu J_Z^\mu + W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu}), \tag{107}$$

其中 f 代表任意费米子，电磁流

$$J_\mathrm{EM}^\mu \equiv \sum_f Q_f \bar{f}\gamma^\mu f, \tag{108}$$

弱中性流

$$J_Z^\mu \equiv \frac{1}{2c_\mathrm{W}}\sum_f \bar{f}\gamma^\mu(g_\mathrm{V}^f - g_\mathrm{A}^f\gamma_5)f = \frac{1}{c_\mathrm{W}}\sum_f (g_\mathrm{L}^f \bar{f}_\mathrm{L}\gamma^\mu f_\mathrm{L} + g_\mathrm{R}^f \bar{f}_\mathrm{R}\gamma^\mu f_\mathrm{R}), \tag{109}$$

弱带电流

$$J_W^{+\mu} \equiv \frac{1}{\sqrt{2}}(\bar{u}_{i\mathrm{L}}\gamma^\mu V_{ij}d_{j\mathrm{L}} + \bar{\nu}_{i\mathrm{L}}\gamma^\mu \ell_{i\mathrm{L}}), \quad J_W^{-\mu} \equiv (J_W^{+\mu})^\dagger = \frac{1}{\sqrt{2}}(\bar{d}_{j\mathrm{L}}V_{ji}^\dagger\gamma^\mu u_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{L}}\gamma^\mu \nu_{i\mathrm{L}}). \tag{110}$$

对于各种费米子，相关的系数如下，

$$Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}, \quad Q_{\nu_i} = 0, \quad Q_{\ell_i} = -1; \tag{111}$$

$$g_\mathrm{V}^{u_i} = \frac{1}{2} - \frac{4}{3}s_\mathrm{W}^2, \quad g_\mathrm{A}^{u_i} = \frac{1}{2}; \quad g_\mathrm{V}^{d_i} = -\frac{1}{2} + \frac{2}{3}s_\mathrm{W}^2, \quad g_\mathrm{A}^{d_i} = -\frac{1}{2}; \tag{112}$$

$$g_\mathrm{V}^{\nu_i} = \frac{1}{2}, \quad g_\mathrm{A}^{\nu_i} = \frac{1}{2}; \quad g_\mathrm{V}^{\ell_i} = -\frac{1}{2} + 2s_\mathrm{W}^2, \quad g_\mathrm{A}^{\ell_i} = -\frac{1}{2}; \tag{113}$$
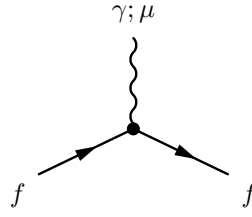
$$g_\mathrm{L}^{u_i} = \frac{1}{2} - \frac{2}{3}s_\mathrm{W}^2, \quad g_\mathrm{R}^{u_i} = -\frac{2}{3}s_\mathrm{W}^2; \quad g_\mathrm{L}^{d_i} = -\frac{1}{2} + \frac{1}{3}s_\mathrm{W}^2, \quad g_\mathrm{R}^{d_i} = \frac{1}{3}s_\mathrm{W}^2; \tag{114}$$

$$g_\mathrm{L}^{\nu_i} = \frac{1}{2}, \quad g_\mathrm{R}^{\nu_i} = 0; \quad g_\mathrm{L}^{\ell_i} = -\frac{1}{2} + s_\mathrm{W}^2, \quad g_\mathrm{R}^{\ell_i} = s_\mathrm{W}^2. \tag{115}$$

可以看到，电磁流耦合与 QED 耦合完全相同，由此辩认出 A_μ 是电磁场，e 是单位电荷量，由 $Q = T^3 + Y$ 定义的 Q 确实是电荷。不同代夸克之间的相互作用只发生在弱带电流耦合中，源自 CKM 矩阵 V 的非对角元。
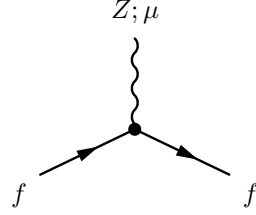
下面给出费米子电弱规范相互作用顶点的费曼规则。

QED 耦合：

$$= iQ_f e\gamma^\mu \quad (\text{对于电子，} Q_e = -1) \tag{116}$$

费米子与 Z 玻色子的耦合：
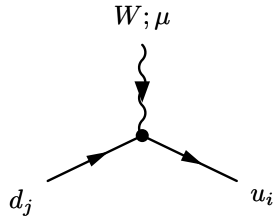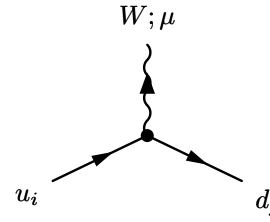
$$= \frac{ig}{2c_{\mathrm{W}}}\gamma^\mu(g_{\mathrm{V}}^f - g_{\mathrm{A}}^f\gamma_5) \tag{117}$$

$$g_{\mathrm{V}}^{u_i} = \frac{1}{2} - \frac{4}{3}s_{\mathrm{W}}^2, \quad g_{\mathrm{A}}^{u_i} = \frac{1}{2}; \quad g_{\mathrm{V}}^{d_i} = -\frac{1}{2} + \frac{2}{3}s_{\mathrm{W}}^2, \quad g_{\mathrm{A}}^{d_i} = -\frac{1}{2}; \tag{118}$$
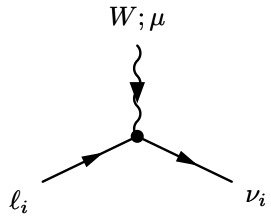
$$g_{\mathrm{V}}^{\nu_i} = \frac{1}{2}, \quad g_{\mathrm{A}}^{\nu_i} = \frac{1}{2}; \quad g_{\mathrm{V}}^{\ell_i} = -\frac{1}{2} + 2s_{\mathrm{W}}^2, \quad g_{\mathrm{A}}^{\ell_i} = -\frac{1}{2}. \tag{119}$$
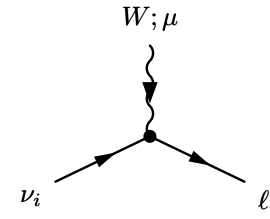
费米子与 $W^\pm$ 玻色子的耦合：

$$= \frac{ig}{\sqrt{2}}V_{ij}\gamma^\mu P_{\mathrm{L}} \qquad = \frac{ig}{\sqrt{2}}V_{ji}^\dagger\gamma^\mu P_{\mathrm{L}} \tag{120}$$

$$= \frac{ig}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}} \qquad = \frac{ig}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}} \tag{121}$$

4.3 电弱规范场的自相互作用

电弱规范场自相互作用的拉氏量是

$$\mathcal{L}_{\mathrm{EWG}} = -\frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{122}$$

其中

$$W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g\varepsilon^{abc}W^{b\mu}W^{c\nu}, \quad B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu. \tag{123}$$

利用 (62) 和 (66) 式，推出

$$
\begin{aligned}
W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 &= \frac{i}{\sqrt{2}}[(W_\mu^+ - W_\mu^-)(s_{\mathrm{W}}A_\nu + c_{\mathrm{W}}Z_\nu) - (s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu)(W_\nu^+ - W_\nu^-)] \\
&= \frac{i}{\sqrt{2}}[s_{\mathrm{W}}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_{\mathrm{W}}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
&\qquad - s_{\mathrm{W}}(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_{\mathrm{W}}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)], \qquad (124)
\end{aligned}
$$

$$
\begin{aligned}
W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 &= \frac{1}{\sqrt{2}}[(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_{\mathrm{W}}A_\nu + c_{\mathrm{W}}Z_\nu)] \\
&= -\frac{1}{\sqrt{2}}[s_{\mathrm{W}}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_{\mathrm{W}}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
&\qquad + s_{\mathrm{W}}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_{\mathrm{W}}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]. \qquad (125)
\end{aligned}
$$

从而

$$
\begin{aligned}
W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g\varepsilon^{1bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + gW_\mu^2 W_\nu^3 - gW_\mu^3 W_\nu^2 \\
&= \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^2 W_\nu^3 - gW_\mu^3 W_\nu^2) \\
&= \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_{\mathrm{W}}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_{\mathrm{W}}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad + \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_{\mathrm{W}}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_{\mathrm{W}}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{1}{\sqrt{2}}(F_{\mu\nu}^+ + F_{\mu\nu}^-), \qquad (126)
\end{aligned}
$$

其中，

$$
F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - A_\mu W_\nu^+) + igc_{\mathrm{W}}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \qquad (127)
$$

$$
F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ie(W_\mu^- A_\nu - A_\mu W_\nu^-) - igc_{\mathrm{W}}(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \qquad (128)
$$

另一方面，

$$
\begin{aligned}
W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 + g\varepsilon^{2bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - gW_\mu^1 W_\nu^3 + gW_\mu^3 W_\nu^1 \\
&= \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{i}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\
&= \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig[s_{\mathrm{W}}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_{\mathrm{W}}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad - \frac{i}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - ig[s_{\mathrm{W}}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_{\mathrm{W}}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{i}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-). \qquad (129)
\end{aligned}
$$

因此

$$-\frac{1}{4}W^1_{\mu\nu}W^{1\mu\nu} - \frac{1}{4}W^2_{\mu\nu}W^{2\mu\nu}$$

$$= -\frac{1}{8}(F^+_{\mu\nu} + F^-_{\mu\nu})(F^{+\mu\nu} + F^{-\mu\nu}) + \frac{1}{8}(F^+_{\mu\nu} - F^-_{\mu\nu})(F^{+\mu\nu} - F^{-\mu\nu}) = -\frac{1}{2}F^+_{\mu\nu}F^{-\mu\nu}$$

$$= -\frac{1}{2}[\partial_\mu W^+_\nu - \partial_\nu W^+_\mu + ie(W^+_\mu A_\nu - A_\mu W^+_\nu) + igc_{\mathrm{W}}(W^+_\mu Z_\nu - Z_\mu W^+_\nu)]$$

$$\times[\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu} - ie(W^{-\mu}A^\nu - A^\mu W^{-\nu}) - igc_{\mathrm{W}}(W^{-\mu}Z^\nu - Z^\mu W^{-\nu})]$$

$$= -(\partial_\mu W^+_\nu)(\partial^\mu W^{-\nu}) + (\partial_\mu W^+_\nu)(\partial^\nu W^{-\mu})$$

$$+ie[(\partial_\mu W^+_\nu)W^{-\mu}A^\nu - (\partial_\mu W^+_\nu)W^{-\nu}A^\mu - W^+_\mu(\partial^\mu W^{-\nu})A_\nu + W^+_\nu(\partial^\mu W^{-\nu})A_\mu]$$

$$+igc_{\mathrm{W}}[(\partial_\mu W^+_\nu)W^{-\mu}Z^\nu - (\partial_\mu W^+_\nu)W^{-\nu}Z^\mu - W^+_\mu(\partial^\mu W^{-\nu})Z_\nu + W^+_\nu(\partial^\mu W^{-\nu})Z_\mu]$$

$$+e^2(W^+_\mu W^{-\nu}A_\nu A^\mu - W^+_\mu W^{-\mu}A_\nu A^\nu) + g^2 c^2_{\mathrm{W}}(W^+_\mu W^{-\nu}Z_\nu Z^\mu - W^+_\mu W^{-\mu}Z_\nu Z^\nu)$$

$$+egc_{\mathrm{W}}(W^+_\mu W^{-\nu}A_\nu Z^\mu + W^+_\mu W^{-\nu}A^\mu Z_\nu - 2W^+_\mu W^{-\mu}A_\nu Z^\nu). \tag{130}$$

由

$$W^1_\mu W^2_\nu - W^2_\mu W^1_\nu = \frac{i}{2}(W^+_\mu + W^-_\mu)(W^+_\nu - W^-_\nu) - \frac{i}{2}(W^+_\mu - W^-_\mu)(W^+_\nu + W^-_\nu)$$

$$= -i(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu), \tag{131}$$

得到

$$W^3_{\mu\nu} = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu + g\varepsilon^{3bc}W^b_\mu W^c_\nu = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu + gW^1_\mu W^2_\nu - gW^2_\mu W^1_\nu$$

$$= s_{\mathrm{W}}\partial_\mu A_\nu + c_{\mathrm{W}}\partial_\mu Z_\nu - s_{\mathrm{W}}\partial_\nu A_\mu + c_{\mathrm{W}}\partial_\nu Z_\mu + g(W^1_\mu W^2_\nu - W^2_\mu W^1_\nu)$$

$$= s_{\mathrm{W}}(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_{\mathrm{W}}(\partial_\mu Z_\nu - \partial_\nu Z_\mu) - ig(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu), \tag{132}$$

$$B_{\mu\nu} = \partial_\mu(c_{\mathrm{W}}A_\nu - s_{\mathrm{W}}Z_\nu) - \partial_\nu(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu)$$

$$= c_{\mathrm{W}}(\partial_\mu A_\nu - \partial_\nu A_\mu) - s_{\mathrm{W}}(\partial_\mu Z_\nu - \partial_\nu Z_\mu). \tag{133}$$

于是
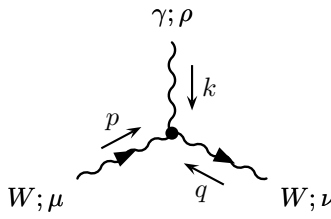
$$-\frac{1}{4}W^3_{\mu\nu}W^{3\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$= -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)]$$

$$+ie[W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] + igc_{\mathrm{W}}[W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)]$$

$$+\frac{1}{2}g^2(W^+_\mu W^{+\mu}W^-_\nu W^{-\nu} - W^+_\mu W^{+\nu}W^-_\nu W^{-\mu}). \tag{134}$$

综合起来，有
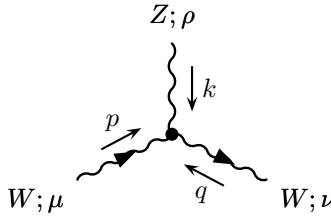
$$\mathcal{L}_{\mathrm{EWG}} = \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)]$$

$$+(\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) + \frac{1}{2}g^2(W_\mu^+ W^{+\mu}W_\nu^- W^{-\nu} - W_\mu^+ W^{+\nu}W_\nu^- W^{-\mu})$$

$$+ie[(\partial_\mu W_\nu^+)W^{-\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-\nu}A^\mu - W^{+\mu}(\partial_\mu W_\nu^-)A^\nu + W^{+\nu}(\partial_\mu W_\nu^-)A^\mu$$

$$+W^{+\mu}W^{-\nu}(\partial_\mu A_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)] + e^2(W_\mu^+ W^{-\nu}A_\nu A^\mu - W_\mu^+ W^{-\mu}A_\nu A^\nu)$$

$$+igc_{\mathrm{W}}[(\partial_\mu W_\nu^+)W^{-\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-\nu}Z^\mu - W^{+\mu}(\partial_\mu W_\nu^-)Z^\nu + W^{+\nu}(\partial_\mu W_\nu^-)Z^\mu$$

$$+W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu) - W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)]$$

$$+g^2 c_{\mathrm{W}}^2(W_\mu^+ W^{-\nu}Z_\nu Z^\mu - W_\mu^+ W^{-\mu}Z_\nu Z^\nu)$$

$$+egc_{\mathrm{W}}(W_\mu^+ W^{-\nu}A_\nu Z^\mu + W_\mu^+ W^{-\nu}A^\mu Z_\nu - 2W_\mu^+ W^{-\mu}A_\nu Z^\nu). \tag{135}$$
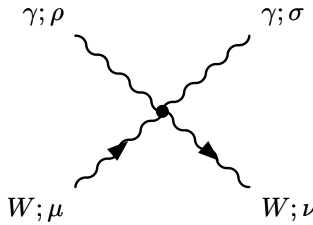
下面是电弱规范玻色子自耦合的费曼规则:
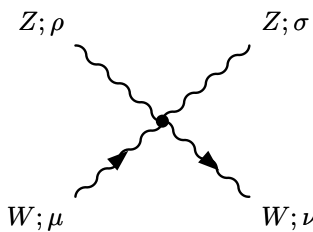
$$= -ie[(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{136}$$
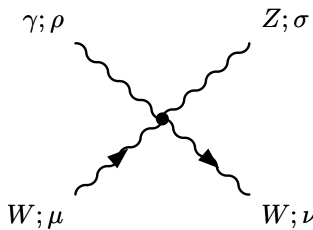
$$= -igc_{\mathrm{W}}[(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{137}$$
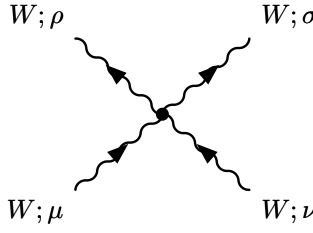
$$= ie^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{138}$$

$$= ig^2 c_{\mathrm{W}}^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{139}$$

$$= iegc_{\mathrm{W}}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{140}$$

$$= -ig^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \qquad (141)$$

5 R$_\xi$ 规范下电弱拉氏量和费曼规则

在 **R$_\xi$** 规范下，将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \dfrac{1}{\sqrt{2}}[v + H(x) + i\chi(x)] \end{pmatrix}, \qquad (142)$$

其中 ϕ^+ 和 χ 是 Nambu-Goldstone 标量场。那么，$\tilde{\Phi}(x)$ 的形式是

$$\tilde{\Phi}(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\sqrt{2}}[v + H(x) - i\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \qquad (143)$$

利用

$$\Phi^\dagger\Phi = \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \qquad (144)$$

$$(\Phi^\dagger\Phi)^2 = \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \qquad (145)$$

推出 Higgs 场势能项

$$
\begin{aligned}
&-V_{\mathrm{H}}(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \\
&= \frac{1}{2}\mu^2(v^2 + H^2 + 2vH + \chi^2) + \mu^2|\phi^+|^2 - \frac{1}{4}\lambda(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda|\phi^+|^4 \\
&\quad - \lambda|\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\
&= \frac{1}{2}\left(\mu^2 - \frac{1}{2}\lambda v^2\right)v^2 + \frac{1}{2}(\mu^2 - 3\lambda v^2)H^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)\chi^2 - \frac{1}{4}\lambda H^4 - \frac{1}{4}\lambda\chi^4 \\
&\quad - \lambda vH^3 - \frac{1}{2}\lambda H^2\chi^2 - \lambda vH\chi^2 + (\mu^2 - \lambda v^2)|\phi^+|^2 - \lambda|\phi^+|^4 - \lambda|\phi^+|^2(H^2 + 2vH + \chi^2) \\
&= \frac{1}{4}\lambda v^4 - \lambda v^2 H^2 - \frac{1}{4}\lambda H^4 - \frac{1}{4}\lambda\chi^4 - \lambda vH^3 - \frac{1}{2}\lambda H^2\chi^2 - \lambda vH\chi^2 \\
&\quad - \lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2) \\
&= \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4 - \frac{m_H^2}{2v}H\chi^2 - \frac{m_H^2}{4v^2}H^2\chi^2 - \frac{m_H^2}{8v^2}\chi^4 \\
&\quad - \frac{m_H^2}{2v^2}\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2).
\end{aligned}
\qquad (146)
$$

由 $V = U_u^\dagger U_d$ 得到 $V^\dagger = U_d^\dagger U_u$ 和 $U_d = U_u V$, $U_u = U_d V^\dagger$，则

$$\tilde{y}_d = U_d y_{\mathrm{D}} K_d^\dagger = U_u V y_{\mathrm{D}} K_d^\dagger, \quad \tilde{y}_u = U_u y_{\mathrm{U}} K_u^\dagger = U_d V^\dagger y_{\mathrm{U}} K_u^\dagger, \tag{147}$$

故

$$\tilde{y}_{d,ij} \bar{u}'_{i\mathrm{L}} d'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}} (U_u V y_{\mathrm{D}} K_d^\dagger)_{ij} d'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}} U_{u,ik} V_{kl} y_{d_l} (K_d^\dagger)_{lj} d'_{m\mathrm{R}} = y_{d_j} \bar{u}_{i\mathrm{L}} V_{ij} d_{j\mathrm{R}}, \tag{148}$$

$$\tilde{y}_{u,ij} \bar{d}'_{i\mathrm{L}} u'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}} (U_d V^\dagger y_{\mathrm{U}} K_u^\dagger)_{ij} u'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}} U_{d,ik} V_{kl}^\dagger y_{u_l} (K_u^\dagger)_{lj} u'_{j\mathrm{R}} = y_{u_i} \bar{d}_{j\mathrm{L}} V_{ji}^\dagger u_{i\mathrm{R}}. \tag{149}$$

结合 (82) 和 (83) 式，得

$$\begin{aligned}
\tilde{y}_{d,ij} \bar{Q}_{i\mathrm{L}} d'_{j\mathrm{R}} \Phi &= \tilde{y}_{d,ij} \left[\bar{u}'_{i\mathrm{L}} d'_{j\mathrm{R}} \phi^+ + \frac{1}{\sqrt{2}} \bar{d}'_{i\mathrm{L}} d'_{j\mathrm{R}} (v + H + i\chi) \right] \\
&= y_{d_j} \bar{u}_{i\mathrm{L}} V_{ij} d_{j\mathrm{R}} \phi^+ + \frac{1}{\sqrt{2}} y_{d_i} \bar{d}_{i\mathrm{L}} d_{i\mathrm{R}} (v + H + i\chi),
\end{aligned} \tag{150}$$

$$\begin{aligned}
\tilde{y}_{u,ij} \bar{Q}_{i\mathrm{L}} u'_{j\mathrm{R}} \tilde{\Phi} &= \tilde{y}_{u,ij} \left[\frac{1}{\sqrt{2}} \bar{u}'_{i\mathrm{L}} u'_{j\mathrm{R}} (v + H - i\chi) - \bar{d}'_{i\mathrm{L}} u'_{j\mathrm{R}} \phi^- \right] \\
&= \frac{1}{\sqrt{2}} y_{u_i} \bar{u}_{i\mathrm{L}} u_{j\mathrm{R}} (v + H - i\chi) - y_{u_i} \bar{d}_{j\mathrm{L}} V_{ji}^\dagger u_{i\mathrm{R}} \phi^-.
\end{aligned} \tag{151}$$

从而，Yukawa 相互作用拉氏量化为

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}} &= -\tilde{y}_d^{ij} \bar{Q}_{i\mathrm{L}} d'_{j\mathrm{R}} \Phi - y_{u_i} \bar{Q}_{i\mathrm{L}} u_{i\mathrm{R}} \tilde{\Phi} - y_{\ell_i} \bar{L}_{i\mathrm{L}} \ell_{i\mathrm{R}} \Phi + \mathrm{H.c.} \\
&= - \left[y_{d_j} \bar{u}_{i\mathrm{L}} V_{ij} d_{j\mathrm{R}} \phi^+ + \frac{1}{\sqrt{2}} y_{d_i} \bar{d}_{i\mathrm{L}} d_{i\mathrm{R}} (v + H + i\chi) \right] \\
&\quad - \left[\frac{1}{\sqrt{2}} y_{u_i} \bar{u}_{i\mathrm{L}} u_{i\mathrm{R}} (v + H - i\chi) - y_{u_i} \bar{d}_{j\mathrm{L}} V_{ji}^\dagger u_{i\mathrm{R}} \phi^- \right] \\
&\quad - y_{\ell_i} \left[\bar{\nu}_{i\mathrm{L}} \ell_{i\mathrm{R}} \phi^+ + \frac{1}{\sqrt{2}} \bar{\ell}_{i\mathrm{L}} \ell_{i\mathrm{R}} (v + H + i\chi) \right] + \mathrm{H.c.} \\
&= -m_{d_i} \bar{d}_{i\mathrm{L}} d_{i\mathrm{R}} - m_{u_i} \bar{u}_{i\mathrm{L}} u_{i\mathrm{R}} - m_{\ell_i} \bar{\ell}_{i\mathrm{L}} \ell_{i\mathrm{R}} - \frac{m_{d_i}}{v} \bar{d}_{i\mathrm{L}} d_{i\mathrm{R}} (H + i\chi) - \frac{m_{u_i}}{v} \bar{u}_{i\mathrm{L}} u_{i\mathrm{R}} (H - i\chi) \\
&\quad - \frac{m_{\ell_i}}{v} \bar{\ell}_{i\mathrm{L}} \ell_{i\mathrm{R}} (H + i\chi) - \frac{\sqrt{2} m_{d_j}}{v} \bar{u}_{i\mathrm{L}} V_{ij} d_{j\mathrm{R}} \phi^+ + \frac{\sqrt{2} m_{u_i}}{v} \bar{d}_{j\mathrm{L}} V_{ji}^\dagger u_{i\mathrm{R}} \phi^- - \frac{\sqrt{2} m_{\ell_i}}{v} \bar{\nu}_{i\mathrm{L}} \ell_{i\mathrm{R}} \phi^+ + \mathrm{H.c.} \\
&= -m_{d_i} \bar{d}_i d_i - m_{u_i} \bar{u}_i u_i - m_{\ell_i} \bar{\ell}_i \ell_i - \frac{m_{d_i}}{v} H \bar{d}_i d_i - \frac{m_{u_i}}{v} H \bar{u}_i u_i - \frac{m_{\ell_i}}{v} H \bar{\ell}_i \ell_i \\
&\quad - \frac{m_{d_i}}{v} \chi \bar{d}_i i\gamma_5 d_i + \frac{m_{u_i}}{v} \chi \bar{u}_i i\gamma_5 u_i - \frac{m_{\ell_i}}{v} \chi \bar{\ell}_i i\gamma_5 \ell_i + \frac{\sqrt{2} V_{ij}}{v} \phi^+ \bar{u}_i (m_{u_i} P_{\mathrm{L}} - m_{d_j} P_{\mathrm{R}}) d_j \\
&\quad - \frac{\sqrt{2} V_{ji}^\dagger}{v} \phi^- \bar{d}_j (m_{d_j} P_{\mathrm{L}} - m_{u_i} P_{\mathrm{R}}) u_i - \frac{\sqrt{2} m_{\ell_i}}{v} (\phi^+ \bar{\nu}_i P_{\mathrm{R}} \ell_i + \phi^- \bar{\ell}_i P_{\mathrm{L}} \nu_i).
\end{aligned} \tag{152}$$

利用

$$D_\mu\Phi = \begin{pmatrix} \partial_\mu - ieA_\mu - \dfrac{ig}{2c_W}(c_W^2 - s_W^2)Z_\mu & -\dfrac{i}{\sqrt{2}}gW_\mu^+ \\[2mm] -\dfrac{i}{\sqrt{2}}gW_\mu^- & \partial_\mu + \dfrac{ig}{2c_W}Z_\mu \end{pmatrix} \begin{pmatrix} \phi^+ \\[2mm] \dfrac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix}$$

$$= \begin{pmatrix} \partial_\mu\phi^+ - i\left[eA_\mu + \dfrac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu\right]\phi^+ - \dfrac{ig}{2}W_\mu^+(H + i\chi) - im_W W_\mu^+ \\[3mm] \dfrac{1}{\sqrt{2}}\left[\partial_\mu(H + i\chi) - igW_\mu^-\phi^+ + \dfrac{ig}{2c_W}Z_\mu(H + i\chi) + im_Z Z_\mu\right] \end{pmatrix}, \quad (153)$$

将 Higgs 场协变动能项化为

$$(D^\mu\Phi)^\dagger D_\mu\Phi$$

$$= \left|\partial_\mu\phi^+ - i\left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu\right]\phi^+ - \frac{ig}{2}W_\mu^+(H + i\chi) - im_W W_\mu^+\right|^2$$

$$+ \frac{1}{2}\left|\partial_\mu(H + i\chi) - igW_\mu^-\phi^+ + \frac{ig}{2c_W}Z_\mu(H + i\chi) + im_Z Z_\mu\right|^2$$

$$= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi)$$

$$+ \left(-i\partial^\mu\phi^-\left\{\left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H + i\chi) + m_W W_\mu^+\right\} + \text{H.c.}\right)$$

$$+ \left\{-\frac{i}{2}\partial^\mu(H - i\chi)\left[gW_\mu^-\phi^+ - \frac{g}{2c_W}Z_\mu(H + i\chi) - m_Z Z_\mu\right] + \text{H.c.}\right\}$$

$$+ \left|\left[eA_\mu + \frac{g(c_W^2 - s_W^2)}{2c_W}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H + i\chi) + m_W W_\mu^+\right|^2$$

$$+ \frac{1}{2}\left|gW_\mu^-\phi^+ - \frac{g}{2c_W}Z_\mu(H + i\chi) - m_Z Z_\mu\right|^2$$

$$= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi)$$

$$+ m_W^2 W^{-\mu}W_\mu^+ + \frac{1}{2}m_Z^2 Z^\mu Z_\mu + gm_W HW_\mu^+ W^{-\mu} + \frac{gm_Z}{2c_W}HZ^\mu Z_\mu$$

$$+ \frac{g}{2}[W_\mu^+\phi^- i\overleftrightarrow{\partial^\mu}(H + i\chi) + \text{H.c.}] + eA_\mu\phi^- i\overleftrightarrow{\partial^\mu}\phi^+ + \frac{g}{2c_W}Z_\mu[i\chi i\overleftrightarrow{\partial^\mu}H + (c_W^2 - s_W^2)\phi^- i\overleftrightarrow{\partial^\mu}\phi^+]$$

$$+ \frac{g^2}{4}W_\mu^+ W^{-\mu}(2\phi^+\phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu\phi^+\phi^-$$

$$+ \frac{g^2}{4c_W^2}Z_\mu Z^\mu\left[(c_W^2 - s_W^2)^2\phi^+\phi^- + \frac{1}{2}H^2 + \frac{1}{2}\chi^2\right]$$

$$+ \left[\frac{eg}{2}W_\mu^+ A^\mu\phi^-(H + i\chi) - \frac{g^2 s_W^2}{2c_W}W_\mu^+ Z^\mu\phi^-(H + i\chi) + \text{H.c.}\right] + \frac{eg}{c_W}(c_W^2 - s_W^2)A_\mu Z^\mu\phi^+\phi^-$$

$$+ (em_W A^\mu\phi^+ W_\mu^- - gs_W^2 m_Z Z^\mu\phi^+ W_\mu^- + \text{H.c.}) + \mathcal{L}_{b1}, \quad (154)$$

其中

$$\mathcal{L}_{\text{b1}} = -im_W(\partial^\mu\phi^-)W_\mu^+ + im_W(\partial^\mu\phi^+)W_\mu^- + m_Z(\partial^\mu\chi)Z_\mu. \tag{155}$$

将 R$_\xi$ 规范的规范固定函数设为

$$G^\pm = \frac{1}{\sqrt{\xi_W}}(\partial^\mu W_\mu^\pm \mp i\xi_W m_W\phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi_Z}}(\partial^\mu Z_\mu - \xi_Z m_Z\chi), \quad G^\gamma = \frac{1}{\sqrt{\xi_\gamma}}\partial^\mu A_\mu, \tag{156}$$

它们在路径积分量子化中的泛函积分形式为

$$\int \mathcal{D}\omega^+ \int \mathcal{D}\omega^- \int \mathcal{D}\omega^Z \int \mathcal{D}\omega^\gamma \exp\left[-i\int d^4x \left(\omega^+\omega^- + \frac{1}{2}\omega^Z\omega^Z + \frac{1}{2}\omega^\gamma\omega^\gamma\right)\right]$$
$$\times \delta(G^+ - \omega^+)\delta(G^- - \omega^-)\delta(G^Z - \omega^Z)\delta(G^\gamma - \omega^\gamma)$$
$$= \exp\left[-i\int d^4x \left(G^+G^- + \frac{1}{2}G^ZG^Z + \frac{1}{2}G^\gamma G^\gamma\right)\right]. \tag{157}$$

由此得到拉氏量中的规范固定项

$$\mathcal{L}_{\text{EW,GF}} = -G^+G^- - \frac{1}{2}(G^Z)^2 - \frac{1}{2}(G^\gamma)^2$$
$$= -\frac{1}{\xi_W}(\partial^\mu W_\mu^+ - i\xi_W m_W\phi^+)(\partial^\nu W_\nu^- + i\xi_W m_W\phi^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu - \xi_Z m_Z\chi)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2$$
$$= -\frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2 - \xi_W m_W^2\phi^+\phi^- - \frac{1}{2}\xi_Z m_Z^2\chi^2 + \mathcal{L}_{\text{b2}}. \tag{158}$$

可见，Nambu-Goldstone 玻色子 $\phi^\pm$ 和 χ 在 R$_\xi$ 规范下具有依赖于 ξ_W 和 ξ_Z 的非物理质量，

$$m_\phi = \sqrt{\xi_W}\,m_W, \quad m_\chi = \sqrt{\xi_Z}\,m_Z. \tag{159}$$

这里

$$\mathcal{L}_{\text{b2}} = -im_W\phi^-(\partial^\mu W_\mu^+) + im_W\phi^+\partial^\mu W_\mu^- + m_Z\chi\partial^\mu Z_\mu. \tag{160}$$

由于

$$\mathcal{L}_{\text{b1}} + \mathcal{L}_{\text{b2}} = -im_W\partial^\mu(\phi^- W_\mu^+) + im_W\partial^\mu(\phi^+ W_\mu^-) + m_Z\partial^\mu(\chi Z_\mu), \tag{161}$$

这两项体现为全散度，不会有物理效应。可见，协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 $\mathcal{L}_{\text{b1}}$ 被规范固定项中的 $\mathcal{L}_{\text{b2}}$ 抵消掉，这就是如此选取规范固定函数的目的。

这样一来，电弱规范场传播子相关拉氏量变成

$$\mathcal{L}_{\text{EW,prop}} = (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) - \frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) + m_W^2 W^{-\mu}W_\mu^+$$
$$+ \frac{1}{2}\left[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - \frac{1}{\xi_Z}(\partial^\mu Z_\mu)^2 + m_Z^2 Z^\mu Z_\mu\right]$$

$$+\frac{1}{2}\left[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu) - \frac{1}{\xi_\gamma}(\partial^\mu A_\mu)^2\right]$$

$$\to W_\mu^+\left[g^{\mu\nu}(\partial^2 + m_W^2) - \left(1 - \frac{1}{\xi_W}\right)\partial^\mu\partial^\nu\right]W_\nu^-$$

$$+\frac{1}{2}Z_\mu\left[g^{\mu\nu}(\partial^2 + m_Z^2) - \left(1 - \frac{1}{\xi_Z}\right)\partial^\mu\partial^\nu\right]Z_\nu$$

$$+\frac{1}{2}A_\mu\left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi_\gamma}\right)\partial^\mu\partial^\nu\right]A_\nu. \tag{162}$$

于是，光子的传播子与胶子形式类似，为

$$\frac{-i}{p^2 + i\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi_\gamma)\right]. \tag{163}$$

将 $W^\pm$ 传播子相关拉氏量变换到动量空间，得

$$-g^{\mu\nu}(p^2 - m_W^2) + \left(1 - \frac{1}{\xi_W}\right)p^\mu p^\nu = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2}\frac{p^2 - \xi_W m_W^2}{\xi_W}, \tag{164}$$

它的逆矩阵是

$$-\frac{1}{p^2 - m_W^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - \frac{\xi_W}{p^2 - \xi_W m_W^2}\frac{p_\mu p_\nu}{p^2} = -\frac{1}{p^2 - m_W^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W)\right], \tag{165}$$

这是因为由

$$\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right)\frac{p^\mu p^\nu}{p^2} = \frac{p_\rho p^\nu}{p^2} - \frac{p_\rho p^\nu}{p^2} = 0, \quad \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) = \delta_\rho^{\ \nu} - \frac{p_\rho p^\nu}{p^2} \tag{166}$$

可得

$$\left[-\frac{1}{p^2 - m_W^2}\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right) - \frac{\xi_W}{p^2 - \xi_W m_W^2}\frac{p_\rho p_\mu}{p^2}\right]$$

$$\times\left[-\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2}\frac{p^2 - \xi_W m_W^2}{\xi_W}\right]$$

$$= \delta_\rho^{\ \nu} - \frac{p_\rho p^\nu}{p^2} + \frac{p_\rho p^\nu}{p^2} = \delta_\rho^{\ \nu}. \tag{167}$$

从而，$W^\pm$ 传播子的形式为

$$\frac{-i}{p^2 - m_W^2 + i\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W)\right]. \tag{168}$$

同理，Z 传播子的形式为

$$\frac{-i}{p^2 - m_Z^2 + i\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2}(1 - \xi_Z)\right]. \tag{169}$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a = \frac{1}{g}\partial_\mu \alpha^a + \varepsilon^{abc} W_\mu^b \alpha^c, \quad \delta B_\mu = \frac{1}{g'}\partial_\mu \alpha^Y. \tag{170}$$

定义

$$\alpha^\pm \equiv \frac{1}{\sqrt{2}}(\alpha^1 \mp i\alpha^2), \quad \alpha^Z \equiv \alpha^3 - \alpha^Y, \quad \alpha^\gamma \equiv s_W^2 \alpha^3 + c_W^2 \alpha^Y, \tag{171}$$

利用

$$\varepsilon^{1bc} W_\mu^b \alpha^c = W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2, \quad \varepsilon^{2bc} W_\mu^b \alpha^c = -W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1, \tag{172}$$

$$\pm i\sqrt{2}\alpha^\pm = \pm i\alpha^1 + \alpha^2, \qquad \pm i\sqrt{2}W_\mu^\pm = \pm iW_\mu^1 + W_\mu^2, \tag{173}$$

有

$$\begin{aligned}
\varepsilon^{1bc} W_\mu^b \alpha^c \mp i\varepsilon^{2bc} W_\mu^b \alpha^c &= (W_\mu^2 \alpha^3 - W_\mu^3 \alpha^2) \mp i(-W_\mu^1 \alpha^3 + W_\mu^3 \alpha^1) \\
&= (W_\mu^2 \pm iW_\mu^1)\alpha^3 - W_\mu^3(\alpha^2 \pm i\alpha^1) \\
&= \pm i\sqrt{2}W_\mu^\pm(c_W^2 \alpha^Z + \alpha^\gamma) \mp i\sqrt{2}(s_W A_\mu + c_W Z_\mu)\alpha^\pm, \tag{174}
\end{aligned}$$

$$\begin{aligned}
\varepsilon^{3bc} W_\mu^b \alpha^c = W_\mu^1 \alpha^2 - W_\mu^2 \alpha^1 &= \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-)\frac{i}{\sqrt{2}}(\alpha^+ - \alpha^-) - \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-)\frac{1}{\sqrt{2}}(\alpha^+ + \alpha^-) \\
&= -i(W_\mu^+ \alpha^- - W_\mu^- \alpha^+). \tag{175}
\end{aligned}$$

因此,

$$\begin{aligned}
\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1 - i\delta W_\mu^2) = \frac{1}{\sqrt{2}g}\partial_\mu(\alpha^1 - i\alpha^2) + \frac{1}{\sqrt{2}}(\varepsilon^{1bc} W_\mu^b \alpha^c - i\varepsilon^{2bc} W_\mu^b \alpha^c) \\
&= \frac{1}{g}\partial_\mu \alpha^+ - i(s_W A_\mu + c_W Z_\mu)\alpha^+ + iW_\mu^+(c_W^2 \alpha^Z + \alpha^\gamma), \tag{176}
\end{aligned}$$

$$\delta W_\mu^- = (\delta W_\mu^+)^\dagger = \frac{1}{g}\partial_\mu \alpha^- + i(s_W A_\mu + c_W Z_\mu)\alpha^- - iW_\mu^-(c_W^2 \alpha^Z + \alpha^\gamma), \tag{177}$$

$$\begin{aligned}
\delta Z_\mu^a &= c_W \delta W_\mu^3 - s_W \delta B_\mu = \frac{c_W}{g}\partial_\mu \alpha^3 + c_W \varepsilon^{3bc} W_\mu^b \alpha^c - \frac{s_W}{g'}\partial_\mu \alpha^Y \\
&= \frac{c_W}{g}\partial_\mu \alpha^Z - ic_W(W_\mu^+ \alpha^- - W_\mu^- \alpha^+), \tag{178}
\end{aligned}$$

$$\begin{aligned}
\delta A_\mu &= s_W \delta W_\mu^3 + c_W \delta B_\mu = \frac{s_W}{g}\partial_\mu \alpha^3 + s_W \varepsilon^{3bc} W_\mu^b \alpha^c + \frac{c_W}{g'}\partial_\mu \alpha^Y \\
&= \frac{1}{e}\partial_\mu \alpha^\gamma - is_W(W_\mu^+ \alpha^- - W_\mu^- \alpha^+). \tag{179}
\end{aligned}$$

另一方面,根据

$$\alpha^a T^a + \alpha^Y Y_H = \frac{1}{2}(\alpha^a \sigma^a + \alpha^Y) = \frac{1}{2}\begin{pmatrix} \alpha^3 + \alpha^Y & \alpha^1 - i\alpha^2 \\ \alpha^1 + i\alpha^2 & -\alpha^3 + \alpha^Y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2\alpha^\gamma + (c_W^2 - s_W^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}, \tag{180}$$

可知 Higgs 场的无穷小规范变换形式为

$$
\begin{aligned}
\delta\Phi = i(\alpha^a T^a + \alpha^Y Y_H)\Phi &= \frac{i}{2}\begin{pmatrix} 2\alpha^\gamma + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}\begin{pmatrix} \phi^+ \\ \dfrac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix} \\
&= \begin{pmatrix} \dfrac{i}{2}[\phi^+[2\alpha^\gamma + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\alpha^Z] + (v + H + i\chi)\alpha^+] \\ \dfrac{1}{\sqrt{2}}\left[i\phi^+\alpha^- - \dfrac{1}{2}(iv + iH - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \dfrac{1}{\sqrt{2}}(\delta H + i\delta\chi) \end{pmatrix}.
\end{aligned}
\tag{181}
$$

利用

$$
\mathrm{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \mathrm{Im}(\phi^+\alpha^-) = -\frac{i}{2}(\phi^+\alpha^- - \phi^-\alpha^+),
\tag{182}
$$

推出

$$
\delta\phi^+ = \frac{i}{2}\{\phi^+[2\alpha^\gamma + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\alpha^Z] + (v + H + i\chi)\alpha^+\},
\tag{183}
$$

$$
\delta\phi^- = -\frac{i}{2}\{\phi^-[2\alpha^\gamma + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\alpha^Z] + (v + H - i\chi)\alpha^-\},
\tag{184}
$$

$$
\delta H = \frac{1}{2}[i(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z].
\tag{185}
$$

于是，规范固定函数的无穷小规范变换为

$$
\begin{aligned}
\sqrt{\xi_W}\,\delta G^+ &= \partial^\mu \delta W_\mu^+ - i\xi_W m_W \delta\phi^+ \\
&= \partial^\mu\left[\frac{1}{g}\partial_\mu\alpha^+ - i(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu)\alpha^+ + iW_\mu^+(c_{\mathrm{W}}^2\alpha^Z + \alpha^\gamma)\right] \\
&\quad + \frac{1}{2}\xi_W m_W\left\{\phi^+[2\alpha^\gamma + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\alpha^Z] + (v + H + i\chi)\alpha^+\right\},
\end{aligned}
\tag{186}
$$

$$
\begin{aligned}
\sqrt{\xi_W}\,\delta G^- &= \partial^\mu \delta W_\mu^- + i\xi_W m_W \delta\phi^- \\
&= \partial^\mu\left[\frac{1}{g}\partial_\mu\alpha^- + i(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu)\alpha^- - iW_\mu^-(c_{\mathrm{W}}^2\alpha^Z + \alpha^\gamma)\right] \\
&\quad + \frac{1}{2}\xi_W m_W\left\{\phi^-[2\alpha^\gamma + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\alpha^Z] + (v + H - i\chi)\alpha^-\right\},
\end{aligned}
\tag{187}
$$

$$
\begin{aligned}
\sqrt{\xi_Z}\,\delta G^Z &= \partial^\mu \delta Z_\mu - \xi_Z m_Z \delta\chi \\
&= \partial^\mu\left[\frac{c_{\mathrm{W}}}{g}\partial_\mu\alpha^Z - ic_{\mathrm{W}}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right] \\
&\quad - \frac{1}{2}\xi_Z m_Z\left[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z\right],
\end{aligned}
\tag{188}
$$

$$
\sqrt{\xi_\gamma}\,\delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu\left[\frac{1}{e}\partial_\mu\alpha^\gamma - is_{\mathrm{W}}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right].
\tag{189}
$$

因此，

$$
\sqrt{\xi_W}\,g\,\frac{\delta G^+}{\delta\alpha^+} = \partial^2 + \xi_W m_W^2 - ie\partial^\mu A_\mu - igc_{\mathrm{W}}\partial^\mu Z_\mu + \frac{1}{2}g\xi_W m_W(H + i\chi),
\tag{190}
$$

$$\frac{\sqrt{\xi_W}\,g}{c_{\mathrm{W}}}\frac{\delta G^+}{\delta \alpha^Z} = igc_{\mathrm{W}}\partial^\mu W_\mu^+ + \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}\phi^+, \tag{191}$$

$$\sqrt{\xi_W}\,e\,\frac{\delta G^+}{\delta \alpha^\gamma} = ie\partial^\mu W_\mu^+ + e\xi_W m_W \phi^+, \tag{192}$$

$$\sqrt{\xi_W}\,g\,\frac{\delta G^-}{\delta \alpha^-} = \partial^2 + \xi_W m_W^2 + ie\partial^\mu A_\mu + igc_{\mathrm{W}}\partial^\mu Z_\mu + \frac{1}{2}\xi_W g m_W (H - i\chi), \tag{193}$$

$$\frac{\sqrt{\xi_W}\,g}{c_{\mathrm{W}}}\frac{\delta G^-}{\delta \alpha^Z} = -igc_{\mathrm{W}}\partial^\mu W_\mu^- + \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}\phi^-, \tag{194}$$

$$\sqrt{\xi_W}\,e\,\frac{\delta G^-}{\delta \alpha^\gamma} = -ie\partial^\mu W_\mu^- + e\xi_W m_W \phi^-, \tag{195}$$

$$\sqrt{\xi_Z}\,g\,\frac{\delta G^Z}{\delta \alpha^+} = igc_{\mathrm{W}}\partial^\mu W_\mu^- - \frac{1}{2}g\xi_Z m_Z \phi^-, \tag{196}$$

$$\sqrt{\xi_Z}\,g\,\frac{\delta G^Z}{\delta \alpha^-} = -igc_{\mathrm{W}}\partial^\mu W_\mu^+ - \frac{1}{2}g\xi_Z m_Z \phi^+, \tag{197}$$

$$\frac{\sqrt{\xi_Z}\,g}{c_{\mathrm{W}}}\frac{\delta G^Z}{\delta \alpha^Z} = \partial^2 + \xi_Z m_Z^2 + \frac{g\xi_Z m_Z}{2c_{\mathrm{W}}}H, \tag{198}$$

$$\sqrt{\xi_\gamma}\,g\,\frac{\delta G^\gamma}{\delta \alpha^+} = ie\partial^\mu W_\mu^-, \quad \sqrt{\xi_\gamma}\,g\,\frac{\delta G^\gamma}{\delta \alpha^-} = -ie\partial^\mu W_\mu^+, \quad \sqrt{\xi_\gamma}\,e\,\frac{\delta G^\gamma}{\delta \alpha^\gamma} = \partial^2. \tag{199}$$

最后，得到以下 Faddeev-Popov 鬼场拉氏量，

$$\begin{aligned}
\mathcal{L}_{\mathrm{EWG,FP}} = &-\bar\eta^+\left(\sqrt{\xi_W}\,g\,\frac{\delta G^+}{\delta \alpha^+}\right)\eta^+ - \bar\eta^Z\left(\sqrt{\xi_Z}\,g\,\frac{\delta G^Z}{\delta \alpha^+}\right)\eta^+ - \bar\eta^\gamma\left(\sqrt{\xi_\gamma}\,g\,\frac{\delta G^\gamma}{\delta \alpha^+}\right)\eta^+ \\
&-\bar\eta^-\left(\sqrt{\xi_W}\,g\,\frac{\delta G^-}{\delta \alpha^-}\right)\eta^- - \bar\eta^Z\left(\sqrt{\xi_Z}\,g\,\frac{\delta G^Z}{\delta \alpha^-}\right)\eta^- - \bar\eta^\gamma\left(\sqrt{\xi_\gamma}\,g\,\frac{\delta G^\gamma}{\delta \alpha^-}\right)\eta^- \\
&-\bar\eta^Z\left(\frac{\sqrt{\xi_Z}\,g}{c_{\mathrm{W}}}\frac{\delta G^Z}{\delta \alpha^Z}\right)\eta^Z - \bar\eta^+\left(\frac{\sqrt{\xi_W}\,g}{c_{\mathrm{W}}}\frac{\delta G^+}{\delta \alpha^Z}\right)\eta^Z - \bar\eta^-\left(\frac{\sqrt{\xi_W}\,g}{c_{\mathrm{W}}}\frac{\delta G^-}{\delta \alpha^Z}\right)\eta^Z \\
&-\bar\eta^\gamma\left(\sqrt{\xi_\gamma}\,e\,\frac{\delta G^\gamma}{\delta \alpha^\gamma}\right)\eta^\gamma - \bar\eta^+\left(\sqrt{\xi_W}\,e\,\frac{\delta G^+}{\delta \alpha^\gamma}\right)\eta^\gamma - \bar\eta^-\left(\sqrt{\xi_W}\,e\,\frac{\delta G^-}{\delta \alpha^\gamma}\right)\eta^\gamma \\
= &\;\bar\eta^+\left[-\partial^2 - \xi_W m_W^2 - ie\overleftarrow{\partial^\mu}A_\mu - igc_{\mathrm{W}}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi_W m_W(H + i\chi)\right]\eta^+ \\
&+\bar\eta^Z\left(igc_{\mathrm{W}}\overleftarrow{\partial^\mu}W_\mu^- + \frac{1}{2}g\xi_Z m_Z \phi^-\right)\eta^+ + ie(\partial^\mu \bar\eta^\gamma)W_\mu^-\eta^+ \\
&+\bar\eta^-\left[-\partial^2 - \xi_W m_W^2 + ie\overleftarrow{\partial^\mu}A_\mu + igc_{\mathrm{W}}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi_W m_W(H - i\chi)\right]\eta^- \\
&+\bar\eta^Z\left(-igc_{\mathrm{W}}\overleftarrow{\partial^\mu}W_\mu^+ + \frac{1}{2}g\xi_Z m_Z \phi^+\right)\eta^- - ie(\partial^\mu \bar\eta^\gamma)W_\mu^+\eta^- \\
&+\bar\eta^Z\left(-\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_{\mathrm{W}}}H\right)\eta^Z \\
&+\bar\eta^+\left(igc_{\mathrm{W}}\overleftarrow{\partial^\mu}W_\mu^+ - \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}\phi^+\right)\eta^Z \\
&+\bar\eta^-\left(-igc_{\mathrm{W}}\overleftarrow{\partial^\mu}W_\mu^- - \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}\phi^-\right)\eta^Z \\
&-\bar\eta^\gamma \partial^2 \eta^\gamma + \bar\eta^+(ie\overleftarrow{\partial^\mu}W_\mu^+ - e\xi_W m_W \phi^+)\eta^\gamma + \bar\eta^-(-ie\overleftarrow{\partial^\mu}W_\mu^- - e\xi_W m_W \phi^-)\eta^\gamma.
\end{aligned} \tag{200}$$

鬼粒子的质量为

$$m_{\eta^+} = m_{\eta^-} = \sqrt{\xi_W}\, m_W, \quad m_{\eta^Z} = \sqrt{\xi_Z}\, m_Z, \quad m_{\eta^\gamma} = 0. \tag{201}$$

下面给出 R_ξ 规范下的费曼规则。$\xi_i = 1$ 对应 Feynman-'t Hooft 规范，$\xi_i = 0$ 对应 Landau 规范，$\xi_W, \xi_Z \to \infty$ 对应幺正规范。在树图计算中，常取 $\xi_\gamma = 1$ 和 $\xi_W, \xi_Z \to \infty$。在圈图计算中，常取 $\xi_\gamma = \xi_W = \xi_Z = 1$。

传播子：

$$\bullet\text{---}\!\!\xrightarrow{\ p\ }\!\!\text{---}\bullet \atop H \quad = \quad \frac{i}{p^2 - m_H^2 + i\epsilon} \tag{202}$$

$$\bullet\text{---}\!\!\xrightarrow{\ p\ }\!\!\text{---}\bullet \atop \chi \quad = \quad \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon} \tag{203}$$

$$\bullet\text{---}\!\!\xrightarrow{\ p\ }\!\!\text{---}\bullet \atop \phi \quad = \quad \frac{i}{p^2 - \xi_W m_W^2 + i\epsilon} \tag{204}$$

$$\nu\,\bullet\!\!\sim\!\!\xrightarrow{\ p\ }\!\!\sim\!\!\bullet\,\mu \atop \gamma \quad = \quad \frac{-i}{p^2 + i\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi_\gamma)\right] \tag{205}$$

$$\nu\,\bullet\!\!\sim\!\!\xrightarrow{\ p\ }\!\!\sim\!\!\bullet\,\mu \atop Z \quad = \quad \frac{-i}{p^2 - m_Z^2 + i\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2}(1 - \xi_Z)\right] \tag{206}$$

$$\nu\,\bullet\!\!\sim\!\!\xrightarrow{\ p\ }\!\!\sim\!\!\bullet\,\mu \atop W \quad = \quad \frac{-i}{p^2 - m_W^2 + i\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W)\right] \tag{207}$$

$$\bullet\cdots\!\!\xrightarrow{\ p\ }\!\!\cdots\bullet \atop \eta^\gamma \quad = \quad \frac{i}{p^2 + i\epsilon} \tag{208}$$

$$\bullet\cdots\!\!\xrightarrow{\ p\ }\!\!\cdots\bullet \atop \eta^Z \quad = \quad \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon} \tag{209}$$

$$\bullet\cdots\!\!\xrightarrow{\ p\ }\!\!\cdots\bullet \atop \eta^\pm \quad = \quad \frac{i}{p^2 - \xi_W m_W^2 + i\epsilon} \tag{210}$$

标量玻色子三线性耦合：

$$
\begin{array}{ccc}
\begin{array}{c} H \\ \mid \\ \bullet \\ H \diagup \ \diagdown H \end{array}
\begin{array}{c} = -\dfrac{3im_H^2}{v} \\[2mm] = -6i\lambda v \end{array}
&
\begin{array}{c} H \\ \mid \\ \bullet \\ \chi \diagup \ \diagdown \chi \end{array}
\begin{array}{c} = -\dfrac{im_H^2}{v} \\[2mm] = -2i\lambda v \end{array}
&
\begin{array}{c} H \\ \mid \\ \bullet \\ \phi \diagup \ \diagdown \phi \end{array}
\begin{array}{c} = -\dfrac{im_H^2}{v} \\[2mm] = -2i\lambda v \end{array}
\end{array}
\tag{211}
$$

标量玻色子四线性耦合：

$$
\begin{aligned}
&= -\frac{3im_H^2}{v^2} \\
&= -6i\lambda
\end{aligned}
\qquad
\begin{aligned}
&= -\frac{im_H^2}{v^2} \\
&= -2i\lambda
\end{aligned}
\qquad
\begin{aligned}
&= -\frac{3im_H^2}{v^2} \\
&= -6i\lambda
\end{aligned}
\tag{212}
$$

$$
\begin{aligned}
&= -\frac{im_H^2}{v^2} \\
&= -2i\lambda
\end{aligned}
\qquad
\begin{aligned}
&= -\frac{im_H^2}{v^2} \\
&= -2i\lambda
\end{aligned}
\qquad
\begin{aligned}
&= -\frac{2im_H^2}{v^2} \\
&= -4i\lambda
\end{aligned}
\tag{213}
$$

Yukawa 耦合：

$$
\begin{aligned}
&= -\frac{im_f}{v} \\
&= -\frac{iy_f}{\sqrt{2}}
\end{aligned}
\qquad
\begin{aligned}
&= \frac{m_{\ell_i}}{v}\,\gamma_5 \\
&= \frac{y_{\ell_i}}{\sqrt{2}}\,\gamma_5
\end{aligned}
\tag{214}
$$

$$
\begin{aligned}
&= -\frac{m_{u_i}}{v}\,\gamma_5 \\
&= -\frac{y_{u_i}}{\sqrt{2}}\,\gamma_5
\end{aligned}
\qquad
\begin{aligned}
&= \frac{m_{d_i}}{v}\,\gamma_5 \\
&= \frac{y_{d_i}}{\sqrt{2}}\,\gamma_5
\end{aligned}
\tag{215}
$$

$$
\begin{aligned}
&= -\frac{i\sqrt{2}m_{\ell_i}}{v}\,P_{\mathrm{R}} \\
&= -iy_{\ell_i}P_{\mathrm{R}}
\end{aligned}
\qquad
\begin{aligned}
&= -\frac{i\sqrt{2}m_{\ell_i}}{v}\,P_{\mathrm{L}} \\
&= -iy_{\ell_i}P_{\mathrm{L}}
\end{aligned}
\tag{216}
$$

$$
\begin{aligned}
&= \frac{i\sqrt{2}V_{ij}}{v}(m_{u_i}P_{\mathrm{L}} - m_{d_j}P_{\mathrm{R}}) \\
&= iV_{ij}(y_{u_i}P_{\mathrm{L}} - y_{d_j}P_{\mathrm{R}})
\end{aligned}
\qquad
\begin{aligned}
&= -\frac{i\sqrt{2}V_{ji}^\dagger}{v}(m_{d_j}P_{\mathrm{L}} - m_{u_i}P_{\mathrm{R}}) \\
&= -iV_{ji}^\dagger(y_{d_j}P_{\mathrm{L}} - y_{u_i}P_{\mathrm{R}})
\end{aligned}
\tag{217}
$$

标量玻色子与电弱规范玻色子的三线性耦合：

$$= \frac{igm_Z}{c_{\mathrm{W}}}\, g^{\mu\nu}$$
$$= \frac{ig^2 v}{2c_{\mathrm{W}}^2}\, g^{\mu\nu}$$

$$= igm_W g^{\mu\nu}$$
$$= \frac{ig^2 v}{2}\, g^{\mu\nu} \tag{218}$$

$$= iem_W g^{\mu\nu}$$
$$= \frac{iegv}{2}\, g^{\mu\nu}$$

$$= iem_W g^{\mu\nu}$$
$$= \frac{iegv}{2}\, g^{\mu\nu} \tag{219}$$

$$= -igs_{\mathrm{W}}^2 m_Z g^{\mu\nu}$$
$$= -\frac{ig^2 s_{\mathrm{W}}^2 v}{2c_{\mathrm{W}}}\, g^{\mu\nu}$$

$$= -igs_{\mathrm{W}}^2 m_Z g^{\mu\nu}$$
$$= -\frac{ig^2 s_{\mathrm{W}}^2 v}{2c_{\mathrm{W}}}\, g^{\mu\nu} \tag{220}$$

$$= ie(p+q)^{\mu} \tag{221}$$

$$= \frac{ig(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}\,(p+q)^{\mu}$$

$$= -\frac{g}{2c_{\mathrm{W}}}\,(p+q)^{\mu} \tag{222}$$

$$= \frac{ig}{2}\,(p+q)^{\mu}$$

$$= \frac{ig}{2}\,(p+q)^{\mu} \tag{223}$$

$$= -\frac{g}{2}\,(p+q)^{\mu}$$

$$= \frac{g}{2}\,(p+q)^{\mu} \tag{224}$$

标量玻色子与电弱规范玻色子的四线性耦合:

$$= \frac{ig^2}{2c_W^2} g^{\mu\nu} \qquad (225)$$

$$= \frac{ig^2}{2} g^{\mu\nu}$$

$$= \frac{ig^2}{2c_W^2} g^{\mu\nu} \qquad (226)$$

$$= \frac{ig^2}{2} g^{\mu\nu}$$

$$= 2ie^2 g^{\mu\nu} \qquad (227)$$

$$= \frac{ieg(c_W^2 - s_W^2)}{c_W} g^{\mu\nu}$$

$$= \frac{ig^2(c_W^2 - s_W^2)^2}{2c_W^2} g^{\mu\nu} \qquad (228)$$

$$= \frac{ig^2}{2} g^{\mu\nu}$$

$$= \frac{ieg}{2} g^{\mu\nu} \qquad (229)$$

$$= \frac{ieg}{2} g^{\mu\nu}$$

$$= -\frac{ig^2 s_W^2}{2c_W} g^{\mu\nu} \qquad (230)$$

$$= -\frac{ig^2 s_W^2}{2c_W} g^{\mu\nu}$$

$$= -\frac{eg}{2} g^{\mu\nu} \qquad (231)$$

$$= \frac{eg}{2} g^{\mu\nu}$$

$$= \frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}} g^{\mu\nu}$$

$$= -\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}} g^{\mu\nu} \tag{232}$$
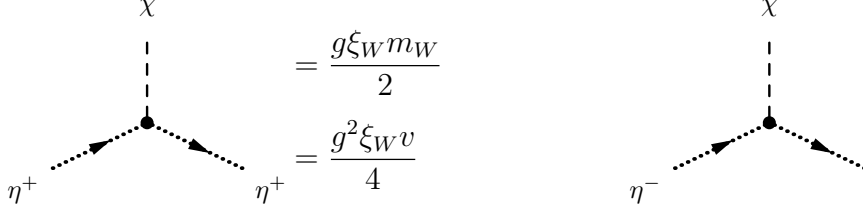
鬼粒子与标量玻色子的耦合:
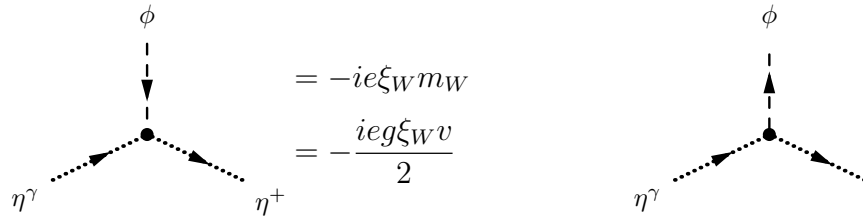
$$= -\frac{ig\xi_Z m_Z}{2c_{\mathrm{W}}}$$

$$= -\frac{ig^2\xi_Z v}{4c_{\mathrm{W}}^2} \tag{233}$$
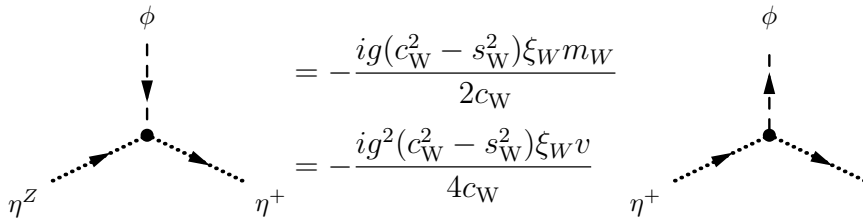

$$= -\frac{ig\xi_W m_W}{2}$$

$$= -\frac{ig^2\xi_W v}{4}$$

$$= -\frac{ig\xi_W m_W}{2}$$

$$= -\frac{ig^2\xi_W v}{4} \tag{234}$$

$$= \frac{g\xi_W m_W}{2}$$

$$= \frac{g^2\xi_W v}{4}$$

$$= -\frac{g\xi_W m_W}{2}$$

$$= -\frac{g^2\xi_W v}{4} \tag{235}$$

$$= -ie\xi_W m_W$$

$$= -\frac{ieg\xi_W v}{2}$$

$$= -ie\xi_W m_W$$

$$= -\frac{ieg\xi_W v}{2} \tag{236}$$

$$= -\frac{ig(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}$$

$$= -\frac{ig^2(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W v}{4c_{\mathrm{W}}}$$

$$= \frac{ig\xi_Z m_Z}{2}$$

$$= \frac{ig^2\xi_Z v}{4c_{\mathrm{W}}} \tag{237}$$

$$= \frac{ig\xi_Z m_Z}{2}$$

$$= \frac{ig^2\xi_Z v}{4c_W}$$

$$= -\frac{ig(c_W^2 - s_W^2)\xi_W m_W}{2c_W}$$

$$= -\frac{ig^2(c_W^2 - s_W^2)\xi_W v}{4c_W}$$

$$(238)$$

鬼粒子与电弱规范玻色子的耦合：

$$= iep^\mu \qquad = -iep^\mu \qquad (239)$$

$$= igc_W p^\mu \qquad = -igc_W p^\mu \qquad (240)$$

$$= -iep^\mu \qquad = -iep^\mu \qquad (241)$$

$$= iep^\mu \qquad = iep^\mu \qquad (242)$$

$$= -igc_W p^\mu \qquad = -igc_W p^\mu \qquad (243)$$

$$= igc_W p^\mu \qquad = igc_W p^\mu \qquad (244)$$

6 内外线一般费曼规则

本节列出一些通用的内外线费曼规则。

内线费曼规则如下。

- 实标量玻色子传播子: $\quad\bullet\text{-}\xrightarrow{p}\text{-}\bullet = \dfrac{i}{p^2 - m^2 + i\epsilon}$

- 复标量玻色子传播子: $\quad\bullet\text{-}\xrightarrow{p}\text{-}\bullet = \dfrac{i}{p^2 - m^2 + i\epsilon}$

- Dirac 费米子传播子: $\quad\bullet\xrightarrow{p}\bullet = \dfrac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$

- 无质量实矢量玻色子传播子:

$$\nu\bullet\!\!\underset{}{\overset{p}{\leadsto}}\!\!\bullet\mu = \frac{-ig^{\mu\nu}}{p^2 + i\epsilon} \quad （\text{Feynman 规范}）$$

$$\nu\bullet\!\!\underset{}{\overset{p}{\leadsto}}\!\!\bullet\mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu/p^2)}{p^2 + i\epsilon} \quad （\text{Landau 规范}）$$

- 有质量实矢量玻色子传播子:

$$\nu\bullet\!\!\underset{}{\overset{p}{\leadsto}}\!\!\bullet\mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu/m^2)}{p^2 - m^2 + i\epsilon} \quad （\text{幺正规范}）$$

$$\nu\bullet\!\!\underset{}{\overset{p}{\leadsto}}\!\!\bullet\mu = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad （\text{Feynman 规范}）$$

- 有质量复矢量玻色子传播子:

$$\nu\bullet\!\!\underset{}{\overset{p}{\leadsto}}\!\!\bullet\mu = \frac{-i(g^{\mu\nu} - p^\mu p^\nu/m^2)}{p^2 - m^2 + i\epsilon} \quad （\text{幺正规范}）$$

$$\nu\bullet\!\!\underset{}{\overset{p}{\leadsto}}\!\!\bullet\mu = \frac{-ig^{\mu\nu}}{p^2 - m^2 + i\epsilon} \quad （\text{Feynman 规范}）$$

实标量场外线费曼规则如下。

- 实标量玻色子入射外线: $\quad\text{-}\text{-}\xrightarrow{p}\text{-}\bullet = 1$

- 实标量玻色子出射外线: $\quad\bullet\text{-}\text{-}\xrightarrow{p}\text{-}\text{-} = 1$

复标量场外线费曼规则如下。

- 正标量玻色子入射外线: $\quad\text{-}\text{-}\text{-}\xrightarrow{p}\text{-}\text{-}\bullet = 1$

- 反标量玻色子入射外线: $\quad\text{-}\text{-}\xleftarrow{p}\text{-}\text{-}\bullet = 1$

- 正标量玻色子出射外线:　$\bullet\, {-}{-}\!\!\overset{p}{\rightarrow}{-}{-}{-} \;= 1$

- 反标量玻色子出射外线:　$\bullet\, {-}{-}\!\!\overset{p}{\underset{}{\leftarrow}}{-}{-}{-} \;= 1$

以 λ 代表自旋极化指标（如螺旋度），**Dirac 旋量场**外线费曼规则如下。

- Dirac 正费米子入射外线:　$\lambda \,\overset{p}{\longrightarrow}\!\!\!\bullet\; = u(\mathbf{p}, \lambda)$

- Dirac 反费米子入射外线:　$\lambda \,\overset{p}{\underset{\leftarrow}{\longrightarrow}}\!\!\bullet\; = \bar{v}(\mathbf{p}, \lambda)$

- Dirac 正费米子出射外线:　$\bullet\!\!\overset{p}{\longrightarrow}\; \lambda\; = \bar{u}(\mathbf{p}, \lambda)$

- Dirac 反费米子出射外线:　$\bullet\!\!\overset{p}{\underset{\leftarrow}{\longrightarrow}}\; \lambda\; = v(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，可利用自旋求和关系

$$\sum_\lambda u(\mathbf{p}, \lambda)\bar{u}(\mathbf{p}, \lambda) = \not{p} + m, \quad \sum_\lambda v(\mathbf{p}, \lambda)\bar{v}(\mathbf{p}, \lambda) = \not{p} - m. \tag{245}$$

以 λ 代表自旋极化指标，**实矢量场**外线费曼规则如下。

- 实矢量玻色子入射外线:　$\lambda;\mu \,\overset{p}{\sim\!\!\sim\!\!\rightarrow\!\!\sim}\!\!\bullet\; = \varepsilon^\mu(\mathbf{p}, \lambda)$

- 实矢量玻色子出射外线:　$\bullet\!\!\overset{p}{\sim\!\!\rightarrow\!\!\sim\!\!\sim}\; \lambda;\mu\; = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

复矢量场外线费曼规则如下。

- 正矢量玻色子入射外线:　$\lambda;\mu \,\overset{p}{\sim\!\!\sim\!\!\blacktriangleright\!\!\sim}\!\!\bullet\; = \varepsilon^\mu(\mathbf{p}, \lambda)$

- 反矢量玻色子入射外线:　$\lambda;\mu \,\overset{p}{\underset{\blacktriangleleft}{\sim\!\!\sim\!\!\sim}}\!\!\bullet\; = \varepsilon^\mu(\mathbf{p}, \lambda)$

- 正矢量玻色子出射外线:　$\bullet\!\!\overset{p}{\sim\!\!\blacktriangleright\!\!\sim}\; \lambda;\mu\; = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

- 反矢量玻色子出射外线:　$\bullet\!\!\overset{p}{\underset{\blacktriangleleft}{\sim\!\!\sim\!\!\sim}}\; \lambda;\mu\; = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，若包含无质量矢量玻色子外线，可利用极化求和替换关系

$$\sum_\lambda \varepsilon_\mu^*(\mathbf{p}, \lambda)\varepsilon_\nu(\mathbf{p}, \lambda) \to -g_{\mu\nu}\,; \tag{246}$$

若包含有质量矢量玻色子外线，可利用极化求和关系

$$\sum_\lambda \varepsilon_\mu^*(\mathbf{p}, \lambda)\varepsilon_\nu(\mathbf{p}, \lambda) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}. \tag{247}$$

7 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2020 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中，光速、约化 Planck 常数和真空介电常数均取为 1，即 $c = \hbar = \varepsilon_0 = 1$。从而，速度没有量纲 (dimension)；长度量纲与时间量纲相同，是能量量纲的倒数；能量、质量和动量具有相同的量纲；精细结构常数表达为

$$\alpha = \frac{e^2}{4\pi}, \tag{248}$$

而单位电荷量 $e = \sqrt{4\pi\alpha}$ 是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系为

$$1 = c = 2.99792458 \times 10^{10} \text{ cm} \cdot \text{s}^{-1}, \tag{249}$$

$$1 = \hbar = 6.582119569 \times 10^{-25} \text{ GeV} \cdot \text{s}, \tag{250}$$

$$1 = \hbar c = 1.973269804 \times 10^{-14} \text{ GeV} \cdot \text{cm}, \tag{251}$$

$$1 = (\hbar c)^2 = 3.893793721 \times 10^8 \text{ GeV}^2 \cdot \text{pb}, \tag{252}$$

由此得到

$$1 \text{ s} = 2.997925 \times 10^{10} \text{ cm}, \qquad 1 \text{ cm} = 3.335641 \times 10^{-11} \text{ s}, \tag{253}$$

$$1 \text{ s} = 1.519267 \times 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 6.582120 \times 10^{-25} \text{ s}, \tag{254}$$

$$1 \text{ cm} = 5.067731 \times 10^{13} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 1.973270 \times 10^{-14} \text{ cm}, \tag{255}$$

$$1 \text{ cm}^2 = 2.568189 \times 10^{27} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^{-28} \text{ cm}^2, \tag{256}$$

$$1 \text{ cm}^3 \cdot \text{s}^{-1} = 8.566558 \times 10^{16} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 1.167330 \times 10^{-17} \text{ cm}^3 \cdot \text{s}^{-1}. \tag{257}$$

靶 (barn) 是散射截面的常用单位，记作 b，满足

$$1 \text{ b} = 10^{-24} \text{ cm}^2 = 10^9 \text{ nb} = 10^{12} \text{ pb} = 10^{15} \text{ fb} = 10^{18} \text{ ab}, \tag{258}$$

$$1 \text{ pb} = 10^{-36} \text{ cm}^2 = 2.568189 \times 10^{-9} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^{8} \text{ pb}. \tag{259}$$

Fermi 耦合常数是

$$G_{\text{F}} = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}, \tag{260}$$

括号内数字代表测量值的 1σ 不确定度，由树图阶关系式

$$\frac{G_{\text{F}}}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_{\text{W}}^2}, \tag{261}$$

得到 Higgs 场真空期待值为

$$v = (\sqrt{2}G_{\text{F}})^{-1/2} = 246.2197 \text{ GeV}. \tag{262}$$

在低能标（Thomson 极限）处，精细结构常数为

$$\alpha = 7.2973525693(11) \times 10^{-3} = \frac{1}{137.035999084(21)}; \tag{263}$$

在 $\overline{\text{MS}}$ 重整化方案（以 ^ 为标志）中，α^{-1} 跑动到 $\mu = m_Z$ 能标处的数值是

$$\widehat{\alpha}^{-1}(m_Z) = 127.952 \pm 0.009. \tag{264}$$

在 $\overline{\text{MS}}$ 方案中，$\mu = m_Z$ 能标处强耦合常数 $\alpha_{\text{s}} = g_{\text{s}}^2/(4\pi)$ 的数值为

$$\widehat{\alpha}_{\text{s}}(m_Z) = 0.1179 \pm 0.0010, \tag{265}$$

Weinberg 角 θ_{W} 的数值对应于

$$\widehat{s}_{\text{W}}^2 = \sin^2\widehat{\theta}_{\text{W}}(m_Z) = 0.23121 \pm 0.00004. \tag{266}$$

在标准模型中，光子、胶子和中微子没有质量，其它基本粒子的质量为

$$m_W = 80.379 \pm 0.012 \text{ GeV}, \qquad m_Z = 91.1876 \pm 0.0021 \text{ GeV}, \tag{267}$$

$$m_H = 125.10 \pm 0.14 \text{ GeV}, \qquad m_e = 0.51099895000(15) \text{ MeV}, \tag{268}$$

$$m_\mu = 105.6583745(24) \text{ MeV}, \qquad m_\tau = 1776.86 \pm 0.12 \text{ MeV}, \tag{269}$$

$$m_u = 2.16^{+0.49}_{-0.26} \text{ MeV}, \qquad m_d = 4.67^{+0.48}_{-0.17} \text{ MeV}, \tag{270}$$

$$m_s = 93^{+11}_{-5} \text{ MeV}, \qquad m_c = 1.67 \pm 0.07 \text{ GeV}, \tag{271}$$

$$m_b = 4.78 \pm 0.06 \text{ GeV}, \qquad m_t = 172.76 \pm 0.30 \text{ GeV}. \tag{272}$$

这里，u、d、s 夸克的质量是 $\mu \simeq 2 \text{ GeV}$ 能标处的流夸克质量 (current-quark mass)，其余粒

子的质量均为极点质量 (pole mass)。c、b 夸克在 $\overline{\text{MS}}$ 方案中的跑动质量 (running mass) 为

$$\bar{m}_c(\bar{m}_c) = 1.27 \pm 0.02 \text{ GeV}, \quad \bar{m}_b(\bar{m}_b) = 4.18^{+0.03}_{-0.02} \text{ GeV}. \tag{273}$$

质子和中子的质量为

$$m_p = 938.27208816(29) \text{ MeV}, \quad m_n = 939.56542052(54) \text{ MeV}. \tag{274}$$

在电弱能标附近作领头阶计算时，可将单位电荷量取为

$$e = \sqrt{4\pi\widehat{\alpha}(m_Z)} = 0.3133873, \tag{275}$$

将强耦合常数取为

$$g_{\text{s}} = \sqrt{4\pi\widehat{\alpha}_{\text{s}}(m_Z)} = 1.217200\,. \tag{276}$$

从树图阶关系计算 Higgs 场四线性耦合常数 λ 和 Yukawa 耦合常数 y_t、y_b、y_τ、y_c，得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1290741, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9922828, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.745492 \times 10^{-2}, \tag{277}$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020576 \times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 9.591991 \times 10^{-3}. \tag{278}$$

耦合常数 g 和 g' 有以下两种取值方式。

1. 根据树图阶关系 $\sin^2\theta_{\text{W}} = 1 - m_W^2/m_Z^2$ 计算 Weinberg 角，得

$$s_{\text{W}}^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2230132, \quad c_{\text{W}}^2 = 1 - s_{\text{W}}^2 = 0.7769868, \tag{279}$$

$$s_{\text{W}} = \sqrt{s_{\text{W}}^2} = 0.4722428, \qquad c_{\text{W}} = \sqrt{c_{\text{W}}^2} = 0.8814685, \tag{280}$$

故

$$g = \frac{e}{s_{\text{W}}} = 0.6636148, \quad g' = \frac{e}{c_{\text{W}}} = 0.3555286\,. \tag{281}$$

2. 根据 $\overline{\text{MS}}$ 方案中 Weinberg 角的数值 (266) 计算 g 和 g'，得

$$c_{\text{W}}^2 = 1 - \widehat{s}_{\text{W}}^2 = 0.76879, \quad s_{\text{W}} = \sqrt{\widehat{s}_{\text{W}}^2} = 0.4808430, \quad c_{\text{W}} = \sqrt{c_{\text{W}}^2} = 0.8768067, \tag{282}$$

$$g = \frac{e}{s_{\text{W}}} = 0.6517456, \quad g' = \frac{e}{c_{\text{W}}} = 0.3574189\,. \tag{283}$$

CKM 矩阵元 V_{ij} 之模的测量值为

$$|V_{ij}| = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.00024}_{-0.00035} \end{pmatrix}. \tag{284}$$

如果只近似地讨论第一、二代夸克的混合，可利用 Cabibbo 转动角 θ_{C} 将 CKM 矩阵表达为

$$V \simeq \begin{pmatrix} \cos\theta_{\mathrm{C}} & \sin\theta_{\mathrm{C}} & \\ -\sin\theta_{\mathrm{C}} & \cos\theta_{\mathrm{C}} & \\ & & 1 \end{pmatrix}, \quad \sin\theta_{\mathrm{C}} = |V_{12}| = 0.22650. \tag{285}$$

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