# Triplet-Quadruplet Fermionic Dark Matter

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Based on **Tim Tait, ZHY, arXiv:1601.01354, JHEP**

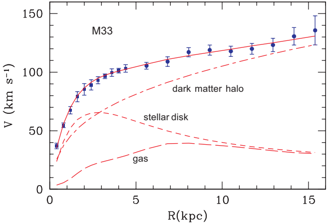
Sun Yat-Sen University

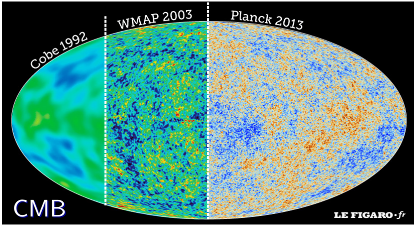
Guangzhou, August 20, 2016

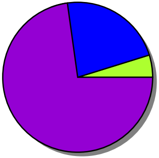
# Dark Matter in the Universe

**Dark matter (DM)** makes up most of the matter component in the Universe, as suggested by astrophysical and cosmological observations





**Planck 2015**
[1502.01589]

**Cold DM** (25.8%)
$\Omega_c h^2 = 0.1186 \pm 0.0020$

**Baryons** (4.8%)
$\Omega_b h^2 = 0.02226 \pm 0.00023$

**Dark energy** (69.3%)
$\Omega_\Lambda = 0.692 \pm 0.012$

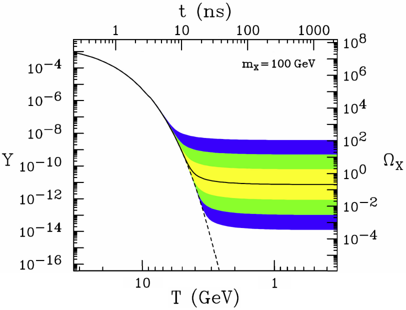
# DM Relic Abundance

If DM particles ($\chi$) were thermally produced in the early Universe, their **relic abundance** would be determined by the annihilation cross section $\langle \sigma_{\text{ann}} v \rangle$:

$$\Omega_\chi h^2 \simeq \frac{3 \times 10^{-27}\ \text{cm}^3\ \text{s}^{-1}}{\langle \sigma_{\text{ann}} v \rangle}$$

Observation value $\Omega_\chi h^2 \simeq 0.1$

$$\Rightarrow \quad \langle \sigma_{\text{ann}} v \rangle \simeq 3 \times 10^{-26}\ \text{cm}^3\ \text{s}^{-1}$$




[Feng, arXiv:1003.0904]

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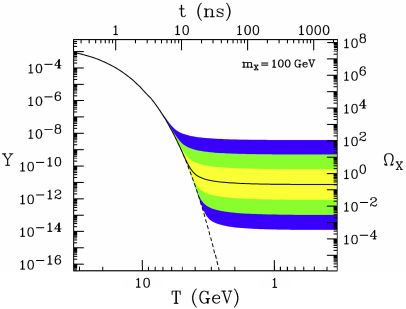


[Feng, arXiv:1003.0904]

Assuming the annihilation process consists of two weak interaction vertices with the $SU(2)_L$ gauge coupling $g \simeq 0.64$, for $m_\chi \sim \mathcal{O}(\mathrm{TeV})$ we have

$$\langle\sigma_{\mathrm{ann}}v\rangle \sim \frac{g^4}{16\pi^2 m_\chi^2} \sim \mathcal{O}(10^{-26})\ \mathrm{cm}^3\ \mathrm{s}^{-1}$$

$\Rightarrow$ A very attractive class of DM candidates:

**Weakly interacting massive particles (WIMPs)**

# WIMP Models

WIMPs are typically introduced in the extensions of the Standard Model (SM) aiming at solving the **gauge hierarchy problem**

- **Supersymmetry (SUSY):** the lightest neutralino ($\tilde{\chi}_1^0$)
- **Universal extra dimensions:** the lightest KK particle ($B^{(1)}$, $W^{3(1)}$, or $\nu^{(1)}$)

# WIMP Models

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- **Universal extra dimensions:** the lightest KK particle ($B^{(1)}$, $W^{3(1)}$, or $\nu^{(1)}$)

For DM phenomenology, it is quite natural to construct WIMP models by extending the SM with a dark sector consisting of $SU(2)_L$ **multiplets**, whose neutral components could provide a viable DM candidate

- 1 multiplet in a high-dimensional representation:
  **minimal DM model** [Cirelli *et al.*, hep-ph/0512090]
  (DM stability is explained by an accidental symmetry)
- 2 types of multiplets: an artificial $Z_2$ symmetry is usually needed
  - **Singlet-doublet DM model** [Mahbubani & Senatore, hep-ph/0510064; D'Eramo, 0705.4493; Cohen *et al.*, 1109.2604]
  - **Doublet-triplet DM model** [Dedes & Karamitros, 1403.7744]
  - ... ...

# Connection to SUSY models

The above models with $SU(2)_L$ multiplets can be understood as **simplifications** of more complete models, but the model parameters are much more **free**

**Singlet-doublet** fermionic DM model:

- **Bino-higgsino** sector in the MSSM

$$\mathcal{L}_{\text{mass}} \supset -\frac{1}{2}M_1\tilde{B}\tilde{B} - \mu(\tilde{H}_u^+\tilde{H}_d^- - \tilde{H}_u^0\tilde{H}_d^0) + \frac{g'v_d}{\sqrt{2}}\tilde{B}\tilde{H}_d^0 - \frac{g'v_u}{\sqrt{2}}\tilde{B}\tilde{H}_u^0 + \text{h.c.}$$

- **Singlino-higgsino** sector in the NMSSM

$$\mathcal{L}_{\text{mass}} \supset -\kappa v_s\tilde{S}\tilde{S} - \lambda v_s(\tilde{H}_u^+\tilde{H}_d^- - \tilde{H}_u^0\tilde{H}_d^0) + \lambda v_u\tilde{S}\tilde{H}_d^0 + \lambda v_d\tilde{S}\tilde{H}_u^0 + \text{h.c.}$$

**Doublet-triplet** fermionic DM model: **higgsino-wino** sector in the MSSM

$$\mathcal{L}_{\text{mass}} \supset -\frac{1}{2}M_2\tilde{W}^0\tilde{W}^0 - M_2\tilde{W}^+\tilde{W}^- - \mu(\tilde{H}_u^+\tilde{H}_d^- - \tilde{H}_u^0\tilde{H}_d^0) - \frac{gv_d}{\sqrt{2}}\tilde{W}^0\tilde{H}_d^0$$
$$+\frac{gv_u}{\sqrt{2}}\tilde{W}^0\tilde{H}_u^0 - gv_u\tilde{H}_u^+\tilde{W}^- - gv_d\tilde{W}^+\tilde{H}_d^- + \text{h.c.}$$

**Triplet-quadruplet** fermionic DM model: **no analogue** in usual SUSY models

# Triplet-Quadruplet DM Model

Introduce left-handed Weyl fermions in the dark sector:

$$T = \begin{pmatrix} T^+ \\ T^0 \\ T^- \end{pmatrix} : (\mathbf{3}, 0), \quad Q_1 = \begin{pmatrix} Q_1^+ \\ Q_1^0 \\ Q_1^- \\ Q_1^{--} \end{pmatrix} : \left(\mathbf{4}, -\frac{1}{2}\right), \quad Q_2 = \begin{pmatrix} Q_2^{++} \\ Q_2^+ \\ Q_2^0 \\ Q_2^- \end{pmatrix} : \left(\mathbf{4}, +\frac{1}{2}\right)$$

Covariant kinetic and mass terms:

$$\mathcal{L}_\mathrm{T} = iT^\dagger \bar{\sigma}^\mu D_\mu T - \frac{1}{2}(m_T T T + \mathrm{h.c.})$$

$$\mathcal{L}_\mathrm{Q} = iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 - (m_Q Q_1 Q_2 + \mathrm{h.c.})$$

Yukawa couplings: $\mathcal{L}_\mathrm{HTQ} = y_1 \varepsilon_{jl} (Q_1)_i^{jk} T_k^i H^l - y_2 (Q_2)_i^{jk} T_k^i H_j^\dagger + \mathrm{h.c.}$

$Z_2$ symmetry: odd for dark sector fermions, even for SM particles

$\Rightarrow$ forbids operators like $TLH$, $Te^c H^\dagger H^\dagger$, $Q_1 L^\dagger H H^\dagger$, $Q_2 L H H^\dagger$, ...

# State Mixing

$$\mathcal{L}_{\text{mass}} = -m_Q Q_1^{--} Q_2^{++} - \frac{1}{2}(T^0, Q_1^0, Q_2^0)\mathcal{M}_N \begin{pmatrix} T^0 \\ Q_1^0 \\ Q_2^0 \end{pmatrix} - (T^-, Q_1^-, Q_2^-)\mathcal{M}_C \begin{pmatrix} T^+ \\ Q_1^+ \\ Q_2^+ \end{pmatrix} + \text{h.c.}$$

$$= -m_Q \chi^{--}\chi^{++} - \frac{1}{2}\sum_{i=1}^{3} m_{\chi_i^0}\chi_i^0\chi_i^0 - \sum_{i=1}^{3} m_{\chi_i^\pm}\chi_i^-\chi_i^+ + \text{h.c.}$$

$$\mathcal{M}_N = \begin{pmatrix} m_T & \frac{1}{\sqrt{3}}y_1 v & -\frac{1}{\sqrt{3}}y_2 v \\ \frac{1}{\sqrt{3}}y_1 v & 0 & m_Q \\ -\frac{1}{\sqrt{3}}y_2 v & m_Q & 0 \end{pmatrix}, \quad \mathcal{M}_C = \begin{pmatrix} m_T & \frac{1}{\sqrt{2}}y_1 v & -\frac{1}{\sqrt{6}}y_2 v \\ -\frac{1}{\sqrt{6}}y_1 v & 0 & -m_Q \\ \frac{1}{\sqrt{2}}y_2 v & -m_Q & 0 \end{pmatrix}$$

$$\begin{pmatrix} T^0 \\ Q_1^0 \\ Q_2^0 \end{pmatrix} = \mathcal{N}\begin{pmatrix} \chi_1^0 \\ \chi_2^0 \\ \chi_3^0 \end{pmatrix}, \quad \begin{pmatrix} T^+ \\ Q_1^+ \\ Q_2^+ \end{pmatrix} = \mathcal{C}_L\begin{pmatrix} \chi_1^+ \\ \chi_2^+ \\ \chi_3^+ \end{pmatrix}, \quad \begin{pmatrix} T^- \\ Q_1^- \\ Q_2^- \end{pmatrix} = \mathcal{C}_R\begin{pmatrix} \chi_1^- \\ \chi_2^- \\ \chi_3^- \end{pmatrix}$$

$$\chi^{--} \equiv Q_1^{--}, \quad \chi^{++} \equiv Q_2^{++}$$

3 Majorana fermions, 3 singly charged fermions, 1 doubly charged fermion

$\chi_1^0$ can be an excellent DM candidate if it is the lightest dark sector fermion

# $y_1 = y_2$: a Custodial Global Symmetry

When the two Yukawa couplings are equal ($y = y_1 = y_2$), the Lagrangians have an $SU(2)_L \times SU(2)_R$ **invariant form:**

$$\mathcal{L}_{\mathrm{Q}} + \mathcal{L}_{\mathrm{HTQ}} = i(\mathbf{Q}^{\dagger A})^k_{ij}\bar{\sigma}^\mu D_\mu (\mathbf{Q}_A)^{ij}_k - \frac{1}{2}[m_Q \varepsilon^{AB}\varepsilon_{il}(\mathbf{Q}_A)^{ij}_k(\mathbf{Q}_B)^{lk}_j + \mathrm{h.c.}]$$
$$+[y\,\varepsilon^{AB}(\mathbf{Q}_A)^{jk}_i T^i_k (\mathbf{H}_B)_j + \mathrm{h.c.}]$$

$$SU(2)_R \text{ doublets: } (\mathbf{Q}_A)^{ij}_k = \begin{pmatrix} (Q_1)^{ij}_k \\ (Q_2)^{ij}_k \end{pmatrix}, \quad (\mathbf{H}_A)_i = \begin{pmatrix} H^\dagger_i \\ H_i \end{pmatrix}$$

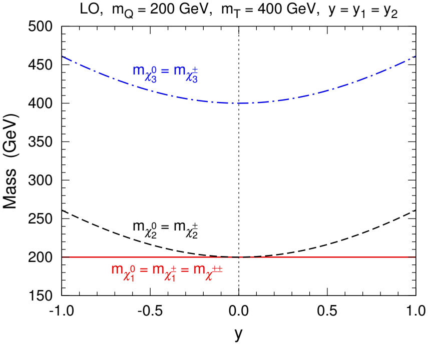
This symmetry is explicitly broken by the $U(1)_Y$ gauge symmetry

There are still some important properties under this approximate symmetry

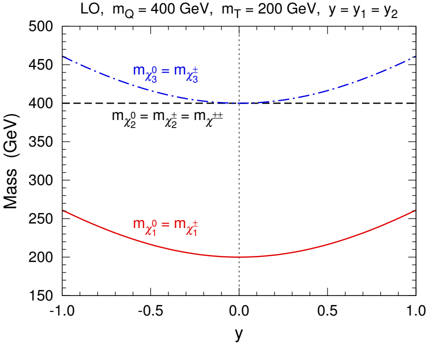
# $y_1 = y_2$: a Custodial Global Symmetry

In the custodial symmetry limit, each of the dark sector neutral fermions is **exactly degenerate in mass** with a singly charged fermion at the LO.

Mass corrections at the NLO are required to check if $m_{\chi_1^0} < m_{\chi_1^\pm}, m_{\chi^{\pm\pm}}$.



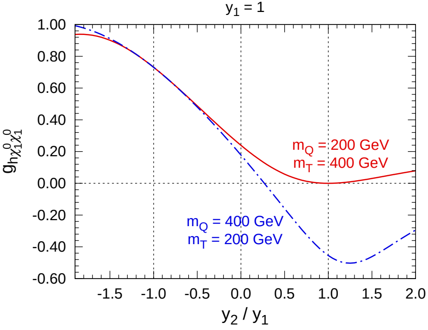

$m_Q < m_T$ case



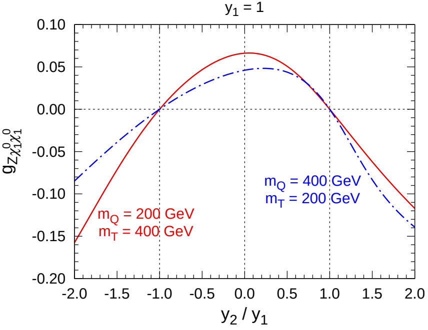

$m_T < m_Q$ case

# $y_1 = y_2$: a Custodial Global Symmetry

In the custodial symmetry limit, when $m_Q < m_T$, we have $\chi_1^0 = (Q_1^0 + Q_2^0)/\sqrt{2}$, which leads to vanishing $\chi_1^0$ couplings to $h$ and $Z$ at the tree level. As a result, $\chi_1^0$ cannot interacts with nuclei at the LO and could easily escape from current DM direct detection bounds.

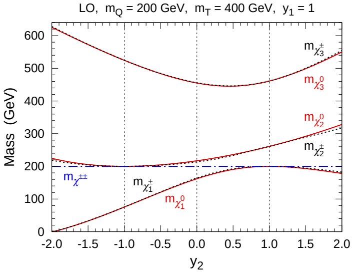


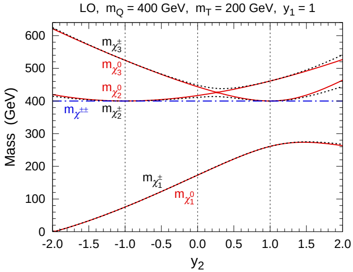

$h\chi_1^0\chi_1^0$ coupling

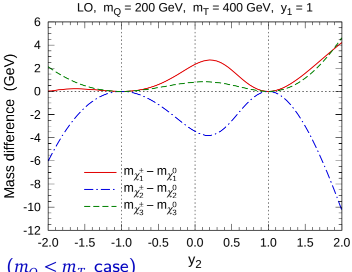


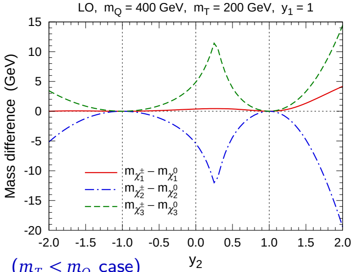

$Z\chi_1^0\chi_1^0$ coupling

# LO Mass Spectrum: $m_{\chi_i^0} \simeq m_{\chi_i^\pm}$ in Any Cases



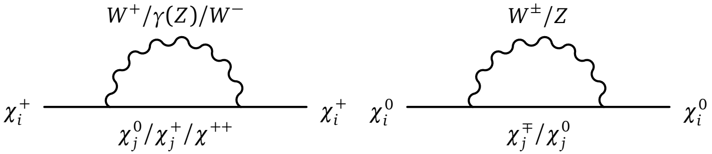




($m_Q < m_T$ case) ($m_T < m_Q$ case)

# Mass Corrections at the NLO

One-loop corrections to an $SU(2)_L$ multiplet from **electroweak gauge boson loops** drive a charged component **heavier** than the neutral component (by $\sim Q^2 \cdot 170$ MeV for a multiplet much heavier than $Z$ with $Y = 0$).

[Feng *et al.*, hep-ph/9904250; Cirelli *et al.*, hep-ph/0512090; Hill & Solon, 1111.0016]




$\Rightarrow\ m_{\chi_1^0} < m_{\chi_1^\pm}$

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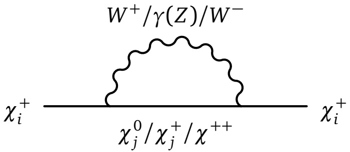
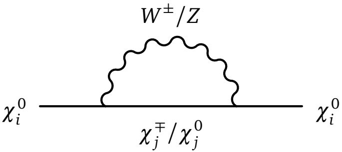
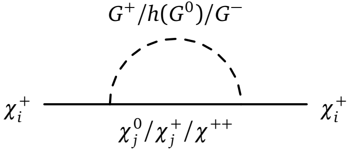
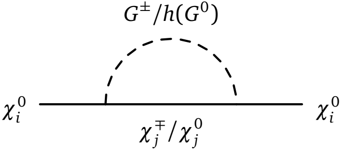



$\Rightarrow \; m_{\chi_1^0} < m_{\chi_1^\pm}$

There are **mixings** among $T$, $Q_1$, and $Q_2$, and corrections from the Higgs sector due to the $HTQ$ **Yukawa couplings**. The situation is more complicated.

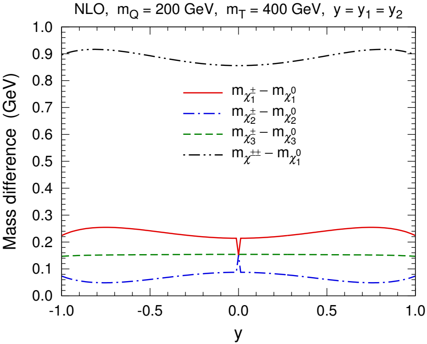


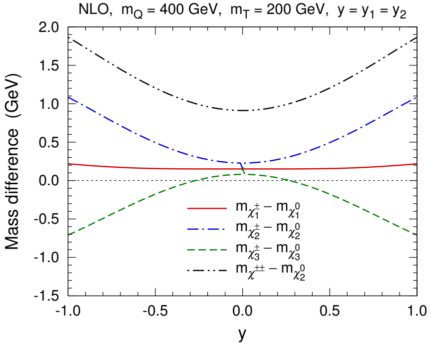

**?**

# Mass Corrections at the NLO

In the **custodial symmetry limit**, we always have $m_{\chi_1^0} < m_{\chi_1^\pm}$ at the NLO and hence $\chi_1^0$ is stable as required for a DM candidate.



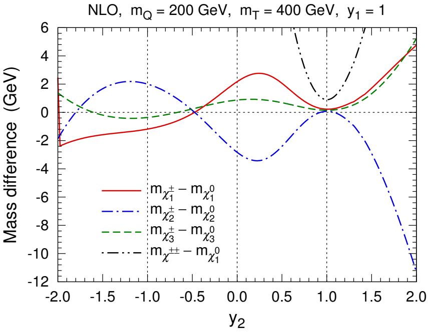

$m_Q < m_T$ case



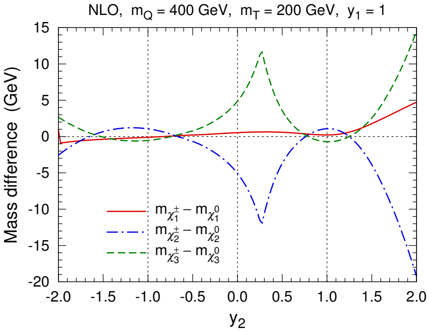

$m_T < m_Q$ case

# Mass Corrections at the NLO

Beyond the custodial symmetry limit:




$m_Q < m_T$ case




$m_T < m_Q$ case

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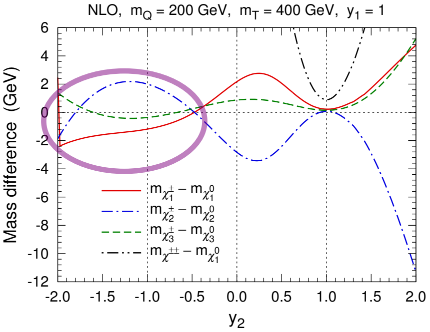
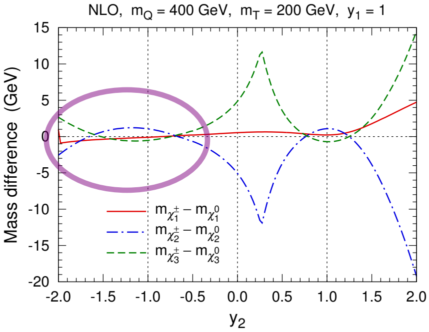



$m_Q < m_T$ case




$m_T < m_Q$ case

When $y_1$ and $y_2$ have opposite signs, we may have $m_{\chi_1^\pm} < m_{\chi_1^0}$ at the NLO and $\chi_1^0$ is no longer a viable DM candidate because it can decay.

## Relic Abundance

In this model, we always have the mass degeneracy $m_{\chi_1^\pm} \simeq m_{\chi_1^0}$. Besides,

$$m_Q < m_T \quad \Rightarrow \quad \text{maybe } m_{\chi^{\pm\pm}} \simeq m_{\chi_1^0}$$

$$|y_{1,2}v| \ll m_Q < m_T \quad \Rightarrow \quad m_{\chi_2^0} \simeq m_{\chi_2^\pm} \simeq m_{\chi_1^0}$$

These dark sector fermions, with close masses and comparable interaction strengths, basically decoupled at the same time in the early Universe.

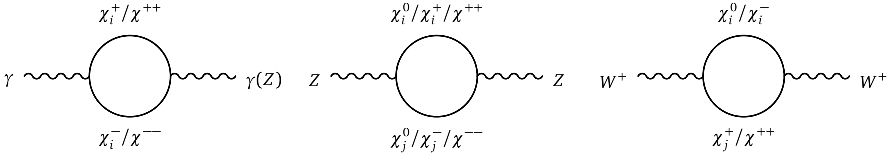
**Coannihilation processes** among them significantly affected their abundances.

After freeze-out, $\chi_1^\pm$, $\chi^{\pm\pm}$, $\chi_2^0$, and $\chi_2^\pm$ decayed into $\chi_1^0$ and contributed to the DM relic abundance.

`FeynRules` $\rightarrow$ `MadGraph` $\rightarrow$ `MadDM`:
includes all annihilation and coannihilation channels

Observed DM abundance $\Omega h^2 = 0.1186 \quad \Longleftrightarrow \quad m_{\chi_1^0} \sim 2.4$ TeV

# Electroweak Oblique Parameters




Gauge interactions of the triplet and quadruplets would affect the **electroweak oblique parameters** [Peskin & Takeuchi, '90, '92]

$$S = \frac{16\pi c_W^2 s_W^2}{e^2}\left[\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{c_W s_W}\Pi'_{ZA}(0) - \Pi'_{AA}(0)\right]$$

$$T = \frac{4\pi}{e^2}\left[\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}\right]$$

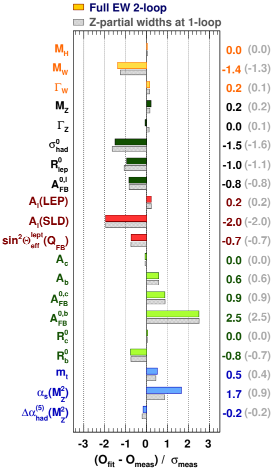
$$U = \frac{16\pi s_W^2}{e^2}\left[\Pi'_{WW}(0) - c_W^2\Pi'_{ZZ}(0) - 2c_W s_W \Pi'_{ZA}(0) - s_W^2 \Pi'_{AA}(0)\right]$$

**The Standard Model predicts $S = T = U = 0$.**

# Electroweak Oblique Parameters

A global fit based on the measurements of **electroweak precision observables**:

[Gfitter Group, 1407.3792]



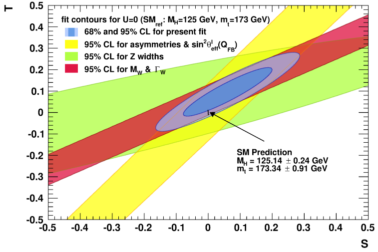

Fixed $U = 0$ $\rightarrow$

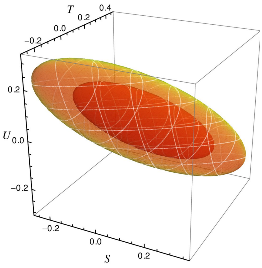
$$S = 0.06 \pm 0.09,\ \ T = 0.10 \pm 0.07,\ \ \rho_{ST} = +0.91$$

Free $U$ $\rightarrow$

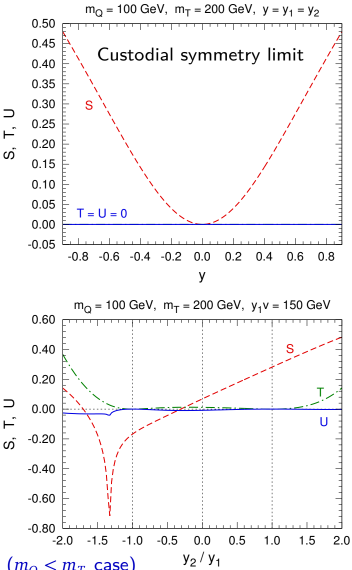
$$S = 0.05 \pm 0.11,\ \ T = 0.09 \pm 0.13,\ \ U = 0.01 \pm 0.11$$

$$\rho_{ST} = +0.90,\ \ \rho_{SU} = -0.59,\ \ \rho_{TU} = -0.83$$

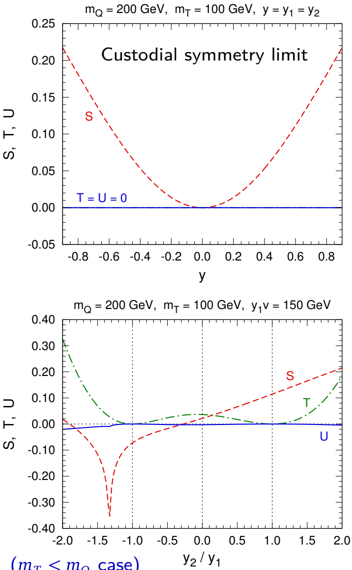


# Electroweak Oblique Parameters




($m_Q < m_T$ case)




($m_T < m_Q$ case)

# Direct Detection

$$\mathcal{L} \supset \frac{1}{2} g_{h\chi_1^0\chi_1^0} h \bar{\chi}_1^0 \chi_1^0 + \frac{1}{2} g_{Z\chi_1^0\chi_1^0} Z_\mu \bar{\chi}_1^0 \gamma^\mu \gamma_5 \chi_1^0$$

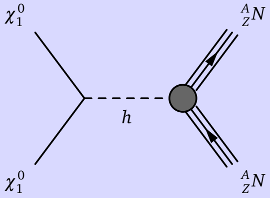
$$g_{h\chi_1^0\chi_1^0} = -\frac{2}{\sqrt{3}} (y_1 \mathcal{N}_{21} - y_2 \mathcal{N}_{31}) \mathcal{N}_{11}$$

$$g_{Z\chi_1^0\chi_1^0} = \frac{g}{2c_W} (|\mathcal{N}_{31}|^2 - |\mathcal{N}_{21}|^2)$$

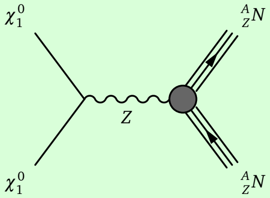
For $m_Q < m_T$ in the custodial symmetry limit, we have $\mathcal{N}_{11} = 0$ and $|\mathcal{N}_{31}| = |\mathcal{N}_{21}|$, and both $g_{h\chi_1^0\chi_1^0}$ and $g_{Z\chi_1^0\chi_1^0}$ vanish

Current direct detection experiments are much more sensitive to the SI DM-nucleus scatterings than the SD scatterings

The exclusion limit on the SI cross section from the **LUX experiment** [1310.8214] is used to constrain the model




Spin independent (SI)




Spin dependent (SD)

## Indirect Detection

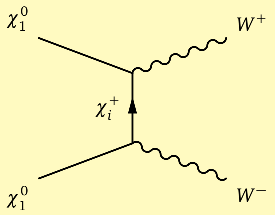
Indirect detection searches for products from **nonrelativistic DM annihilations**

Suppressions on $\chi_1^0\chi_1^0$ annihilations into SM particles

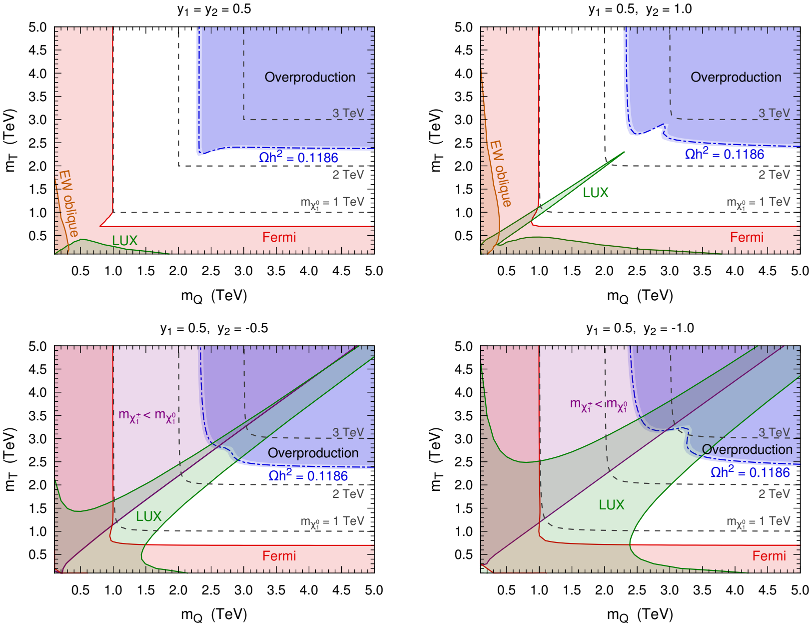
- $\chi_1^0\chi_1^0 \to Z^* \to f\bar{f}$: helicity suppression in $s$ wave ($\langle\sigma v\rangle \propto m_f^2/m_{\chi_1^0}^2$)
- $\chi_1^0\chi_1^0 \to h^* \to f\bar{f}$: $p$-wave suppression ($\langle\sigma v\rangle \propto v^2$)
- $\chi_1^0\chi_1^0 \to hh$: $p$-wave suppression ($\langle\sigma v\rangle \propto v^2$)

The cross section of $\chi_1^0\chi_1^0 \to W^+W^-$ is typically larger than those of $\chi_1^0\chi_1^0 \to ZZ$, $Zh$, $t\bar{t}$ by at least 1 to 2 orders of magnitude

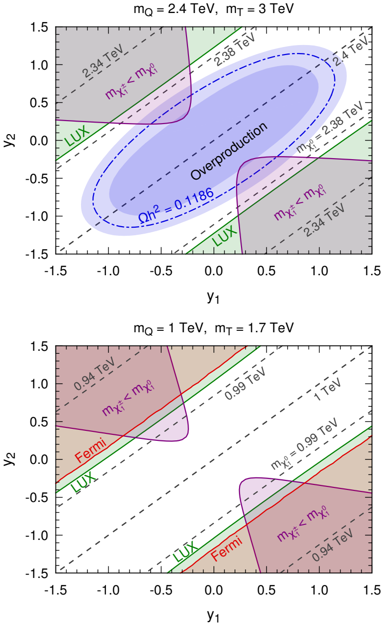
The upper limit on the annihilation cross section into $W^+W^-$ given by **Fermi-LAT** 6-year $\gamma$-ray observations of dwarf galaxies [1503.02641] is used to constrain the model



Dominant channel



$y_1 = y_2 = 0.5$
$y_1 = 0.5, \ y_2 = 1.0$
$y_1 = 0.5, \ y_2 = -0.5$
$y_1 = 0.5, \ y_2 = -1.0$
Overproduction
3 TeV
2 TeV
$m_{\chi_1^0} = 1$ TeV
$\Omega h^2 = 0.1186$
EW oblique
LUX
Fermi
$m_{\chi_1^\pm} < m_{\chi_1^0}$
$m_T$ (TeV)
$m_Q$ (TeV)



$m_Q = 2.4$ TeV, $m_T = 3$ TeV
2.34 TeV
2.38 TeV
2.4 TeV
$m_{\chi_1^\pm} < m_{\chi_1^0}$
Overproduction
$\Omega h^2 = 0.1186$
$m_{\chi_1^0} = 2.38$ TeV
LUX
$m_Q = 3$ TeV, $m_T = 2.4$ TeV
2.41 TeV
2.4 TeV
Overp.
$m_{\chi_1^0} = 2.38$ TeV
$m_Q = 1$ TeV, $m_T = 1.7$ TeV
0.94 TeV
0.99 TeV
1 TeV
$m_{\chi_1^0} = 0.99$ TeV
Fermi
$m_Q = 1.5$ TeV, $m_T = 0.7$ TeV
630 GeV
670 GeV
700 GeV
710 GeV
$m_{\chi_1^0} = 700$ GeV
$y_1$
$y_2$

# Conclusion

1. We investigate a **triplet-quadruplet WIMP model**, whose dark sector involves 3 Majorana fermions, 3 singly charged fermions, and 1 doubly charged fermion.
2. The triplet and quadruplets can interact with the SM Higgs doublet through two Yukawa couplings, whose equality leads to **an approximate custodial symmetry** that would make the DM candidate $\chi_1^0$ easily escaping from direct searches.
3. There are mass degeneracies among dark sector fermions. **One-loop mass corrections** are calculated to check if $\chi_1^0$ can be stable.
4. The observed relic abundance suggests $m_{\chi_1^0} \sim 2.4$ TeV. Phenomenological constraints from EW oblique parameters and direct and indirect detection experiments are also considered.

## Conclusion

5. There may be extra constraints from 8 TeV LHC results, such as $h \to \gamma\gamma$ **measurements, monojet searches, and direct searches for exotic charged particles**. But it is unlikely that they could give more stringent constraints than the Fermi-LAT bound.

## Conclusion

5. There may be extra constraints from 8 TeV LHC results, such as $h \to \gamma\gamma$ **measurements, monojet searches, and direct searches for exotic charged particles**. But it is unlikely that they could give more stringent constraints than the Fermi-LAT bound.

# Thanks for your attention!