标准模型拉氏量和费曼规则

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2022 年 9 月 2 日

目　录

1 约定

本文采用有理化的自然单位制，推导过程参考文献 [1, 2, 3, 4]。

Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{1}$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -\mathrm{i} \\ \mathrm{i} & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \tag{2}$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \tag{3}$$

Weyl 表象中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbf{1} & \\ & \mathbf{1} \end{pmatrix}. \tag{4}$$

左右手投影算符

$$P_{\mathrm{L}} \equiv \frac{1}{2}(1-\gamma^5) = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{0} \end{pmatrix}, \quad P_{\mathrm{R}} \equiv \frac{1}{2}(1+\gamma^5) = \begin{pmatrix} \mathbf{0} & \\ & \mathbf{1} \end{pmatrix} \tag{5}$$

用于定义左手旋量场 $\psi_{\mathrm{L}} \equiv P_{\mathrm{L}}\psi$ 和右手旋量场 $\psi_{\mathrm{R}} \equiv P_{\mathrm{R}}\psi$。Levi-Civita 符号的约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \tag{6}$$

费曼规则约定如下。

- 对于指向相互作用顶点的动量 p，时空导数 ∂_μ 在动量空间费曼规则里贡献一个 $-\mathrm{i}p_\mu$ 因子。
- 实线表示费米子，实线上的箭头表示费米子数流动的方向。
- 虚线表示标量玻色子，虚线上的箭头表示玻色子数流动的方向。
- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示玻色子数流动的方向。
- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。
- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

2 标准模型概述

粒子物理标准模型是一个 $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$ 规范理论。模型中有三代费米子，包括三代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$，三代带电轻子 $\ell_i = e, \mu, \tau$，三代上型夸克 $u_i = u, c, t$ 和三代下型夸克 $d_i = d, s, b$ $(i = 1, 2, 3)$。规范玻色子传递费米子之间的规范相互作用。

$\mathrm{SU(3)_C}$ 部分描述夸克的强相互作用,称为量子色动力学 (Quantum Chromodynamics, QCD), 相应的规范玻色子是胶子。$\mathrm{SU(2)_L \times U(1)_Y}$ 部分统一描述夸克和轻子的电磁和弱相互作用，称为电弱规范理论。理论中有一个 Higgs 二重态，通过 Brout–Englert–Higgs (BEH) 机制引发规范群的自发对称性破缺，使 $\mathrm{SU(2)_L \times U(1)_Y}$ 群破缺为 $\mathrm{U(1)_{EM}}$ 群。$\mathrm{U(1)_{EM}}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前。理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度；左手费米子和右手费米子都没有质量，具有不同的量子数。

破缺后，3 个规范玻色子与 3 个 Higgs 自由度结合，从而获得质量，成为 $W^\pm$ 和 Z^0 玻色子，传递弱相互作用。剩下的 1 个无质量规范玻色子是光子，即是 $\mathrm{U(1)_{EM}}$ 群的规范玻色子，传递电磁相互作用。剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子。费米子与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量，组合成 Dirac 费米子。

理论中的中微子没有右手分量，因而没有获得质量。1998 年实验发现中微子振荡，证明中微子具有质量，所以需要扩充标准模型才能正确描述中微子物理。

3 量子色动力学

QCD 的拉氏量表达为

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{q}(\mathrm{i}\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \cdots, 8, \tag{7}$$

其中

$$D_\mu = \partial_\mu - \mathrm{i}g_{\mathrm{s}}G^a_\mu t^a, \quad G^{a\mu\nu} \equiv \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_{\mathrm{s}} f^{abc} G^{b\mu} G^{c\nu}. \tag{8}$$

q 为夸克旋量场 $\mathrm{SU(3)_C}$ 三重态，$\mathrm{SU(3)_C}$ 规范场 G^a_μ 对应于胶子 g，g_{s} 是 $\mathrm{SU(3)_C}$ 规范耦合常数。$t^a = \lambda^a/2$ 是 $\mathrm{SU(3)_C}$ 群基础表示的生成元，其中 λ^a 为 Gell-Mann 矩阵。$\mathrm{SU(3)_C}$ 生成元对易关系为 $[t^a, t^b] = \mathrm{i}f^{abc}t^c$，结构常数 f^{abc} 是全反对称的，非零分量为

$$f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}. \tag{9}$$

由

$$\begin{aligned}
-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} &= -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_{\mathrm{s}} f^{abc} G^b_\mu G^c_\nu)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + g_{\mathrm{s}} f^{ade} G^{d\mu} G^{e\nu}) \\
&= -\frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu}) - (\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu})] - g_{\mathrm{s}} f^{abc}(\partial_\mu G^a_\nu) G^{b\mu} G^{c\nu}
\end{aligned}$$

$$-\frac{1}{4}g_{\rm s}^2 f^{abc} f^{ade} G^b_\mu G^c_\nu G^{d\mu} G^{e\nu}, \tag{10}$$

推出

$$\begin{aligned}\mathcal{L}_{\rm QCD} &= \sum_q [\bar{q}(\mathrm{i}\gamma^\mu\partial_\mu - m_q)q + g_{\rm s} G^a_\mu \bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu}) - (\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu})] \\ &\quad - g_{\rm s} f^{abc}(\partial_\mu G^a_\nu) G^{b\mu} G^{c\nu} - \frac{1}{4} g_{\rm s}^2 f^{abc} f^{ade} G^b_\mu G^c_\nu G^{d\mu} G^{e\nu}.\end{aligned} \tag{11}$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G^a_\mu(x) - \omega^a(x)$，其中 $\omega^a(x)$ 是某个任意函数，规范固定条件是 $G^a(x) = 0$。这是 Lorenz 规范的推广，$\omega^a(x) = 0$ 对应于 Lorenz 规范。在路径积分量子化中，以中心为 $\omega^a(x) = 0$ 的 Gauss 权重对 $\omega^a(x)$ 作泛函积分，有

$$\int \mathcal{D}\omega^a \exp\left[-\mathrm{i}\int \mathrm{d}^4x\, \frac{1}{2\xi}(\omega^a)^2\right] \delta(G^a) = \exp\left[-\mathrm{i}\int \mathrm{d}^4x\, \frac{1}{2\xi}(\partial^\mu G^a_\mu)^2\right]. \tag{12}$$

可见，拉氏量中的规范固定项为

$$\mathcal{L}_{\rm QCD,GF} = -\frac{1}{2\xi}(\partial^\mu G^a_\mu)^2. \tag{13}$$

ξ 的任何一个取值对应于一种规范。$\xi = 1$ 称为 Feynman-'t Hooft 规范，$\xi = 0$ 称为 Landau 规范。于是，胶子传播子相关拉氏量为

$$\begin{aligned}\mathcal{L}_{\rm QCD,prop} &= \frac{1}{2}\left[(\partial_\mu G^a_\nu)(\partial^\nu G^{a\mu}) - (\partial_\mu G^a_\nu)(\partial^\mu G^{a\nu}) - \frac{1}{\xi}(\partial^\mu G^a_\mu)^2\right] \\ &\to \frac{1}{2} G^a_\mu \left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu\right] G^a_\nu.\end{aligned} \tag{14}$$

变换到动量空间，得

$$-g^{\mu\nu}p^2 + \left(1 - \frac{1}{\xi}\right) p^\mu p^\nu, \tag{15}$$

它的逆矩阵是

$$-\frac{1}{p^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right], \tag{16}$$

这是因为

$$\begin{aligned}&-\frac{1}{p^2}\left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}(1-\xi)\right]\left[-g^{\mu\nu}p^2 + \left(1 - \frac{1}{\xi}\right)p^\mu p^\nu\right] \\ &= \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2}\left(1 - \frac{1}{\xi}\right) - \frac{p_\rho p^\nu}{p^2}(1-\xi) + \frac{p_\rho p^\nu}{p^2}(1-\xi)\left(1 - \frac{1}{\xi}\right) = \delta_\rho{}^\nu.\end{aligned} \tag{17}$$

从而胶子传播子的形式为

$$\frac{-\mathrm{i}\delta^{ab}}{p^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right]. \tag{18}$$

$\mathrm{SU(3)_C}$ 规范变换为

$$q \to Uq, \quad G_\mu^a t^a \to U G_\mu^a t^a U^\dagger + \frac{\mathrm{i}}{g_\mathrm{s}} U \partial_\mu U^\dagger, \tag{19}$$

其中 $U(x) = \exp[\mathrm{i}\alpha^a(x)t^a]$。胶子场的无穷小规范变换形式是

$$\begin{aligned} G_\mu^a t^a &\to (1+\mathrm{i}\alpha^a t^a) G_\mu^b t^b (1-\mathrm{i}\alpha^c t^c) + \frac{\mathrm{i}}{g_\mathrm{s}}(1+\mathrm{i}\alpha^a t^a)\partial_\mu(1-\mathrm{i}\alpha^c t^c) \\ &= G_\mu^b t^b + \mathrm{i}\alpha^a G_\mu^b [t^a, t^b] + \frac{1}{g_\mathrm{s}}(\partial_\mu \alpha^c) t^c + \mathcal{O}(\alpha^2) = G_\mu^a t^a - f^{abc}\alpha^a G_\mu^b t^c + \frac{1}{g_\mathrm{s}}(\partial_\mu\alpha^a)t^a + \mathcal{O}(\alpha^2) \\ &= \left(G_\mu^a + f^{abc} G_\mu^b \alpha^c + \frac{1}{g_\mathrm{s}}\,\partial_\mu \alpha^a\right) t^a + \mathcal{O}(\alpha^2), \end{aligned} \tag{20}$$

即

$$\delta G_\mu^a = \frac{1}{g_\mathrm{s}}\,\partial_\mu \alpha^a + f^{abc} G_\mu^b \alpha^c = \left(\frac{1}{g_\mathrm{s}}\,\delta^{ac}\partial_\mu + f^{abc} G_\mu^b\right)\alpha^c, \tag{21}$$

因而规范固定函数 G^a 的无穷小规范变换为

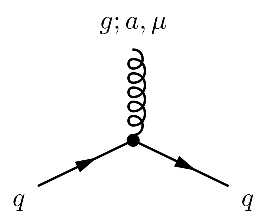
$$\delta G^a = \partial^\mu \delta G_\mu^a = \frac{1}{g_\mathrm{s}}\,\delta^{ac}\partial^2\alpha^c + f^{abc}\partial^\mu G_\mu^b \alpha^c, \quad g_\mathrm{s}\,\frac{\delta G^a}{\delta\alpha^c} = \delta^{ab}\partial^2 + g_\mathrm{s} f^{abc}\partial^\mu G_\mu^b. \tag{22}$$

Faddeev-Popov 鬼场的拉氏量是

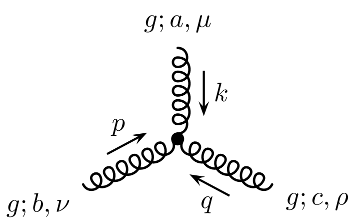
$$\mathcal{L}_{\mathrm{QCD,FP}} = -\bar\eta_g^a \left(g_\mathrm{s}\,\frac{\delta G^a}{\delta\alpha^c}\right)\eta_g^c = -\bar\eta_g^a(\delta^{ac}\partial^2 + g_\mathrm{s} f^{abc}\partial^\mu G_\mu^b)\eta_g^c \to -\bar\eta_g^a\delta^{ab}\partial^2\eta_g^b + g_\mathrm{s} f^{abc}(\partial^\mu\bar\eta_g^a) G_\mu^b \eta_g^c. \tag{23}$$

下面列出 QCD 费曼规则。

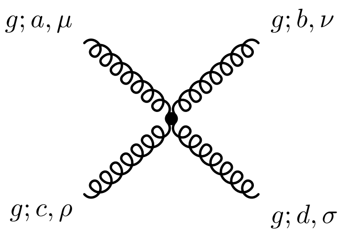
QCD 耦合顶点:



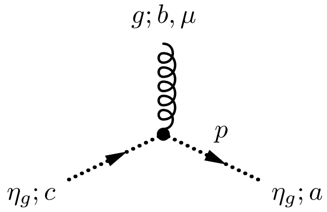
$$= \mathrm{i}g_\mathrm{s}\gamma^\mu t^a \tag{24}$$



$$= g_\mathrm{s} f^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu] \tag{25}$$

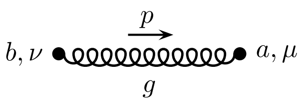


$$\begin{aligned} = -\mathrm{i}g_\mathrm{s}^2[&f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})] \end{aligned} \tag{26}$$



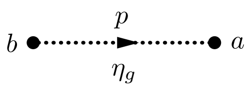
$$= -g_\mathrm{s} f^{abc} p^\mu \tag{27}$$

胶子传播子:



$$= \frac{-\mathrm{i}\delta^{ab}}{p^2 + \mathrm{i}\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi) \right] \tag{28}$$

鬼粒子传播子:



$$= \frac{\mathrm{i}\delta^{ab}}{p^2 + \mathrm{i}\epsilon} \tag{29}$$

4　电弱规范理论

电弱规范理论的规范群是 $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$，每一代左手旋量场构成 2 个 $\mathrm{SU}(2)_\mathrm{L}$ 二重态

$$L_{i\mathrm{L}} = \begin{pmatrix} P_\mathrm{L}\nu_i \\ P_\mathrm{L}\ell_i \end{pmatrix} = \begin{pmatrix} \nu_{i\mathrm{L}} \\ \ell_{i\mathrm{L}} \end{pmatrix}, \quad Q_{i\mathrm{L}} = \begin{pmatrix} P_\mathrm{L}u'_i \\ P_\mathrm{L}d'_i \end{pmatrix} = \begin{pmatrix} u'_{i\mathrm{L}} \\ d'_{i\mathrm{L}} \end{pmatrix}, \quad i = 1, 2, 3. \tag{30}$$

它们的协变导数是

$$D_\mu = \partial_\mu - \mathrm{i}gW_\mu^a \tau^a - \mathrm{i}g' B_\mu Y, \tag{31}$$

其中 $W_\mu^a(x)$ $(a = 1, 2, 3)$ 是 $\mathrm{SU}(2)_\mathrm{L}$ 规范场，$B_\mu(x)$ 是 $\mathrm{U}(1)_\mathrm{Y}$ 规范场，g 和 g' 分别是 $\mathrm{SU}(2)_\mathrm{L}$ 和 $\mathrm{U}(1)_\mathrm{Y}$ 的规范耦合常数。

$$\tau^a = \frac{\sigma^a}{2} \tag{32}$$

是 $\mathrm{SU}(2)_\mathrm{L}$ 群 2 维表示的生成元，对应于弱同位旋。生成元 τ^3 的本征值是弱同位旋第 3 分量，记为 T^3 。Y 是弱超荷。各代右手旋量场 $\ell_{i\mathrm{R}} = P_\mathrm{R}\ell_i$、$u'_{i\mathrm{R}} = P_\mathrm{R}u'_i$ 和 $d'_{i\mathrm{R}} = P_\mathrm{R}d'_i$ 是 $\mathrm{SU}(2)_\mathrm{L}$ 单态，协变导数为

$$D_\mu = \partial_\mu - \mathrm{i}g' B_\mu Y. \tag{33}$$

表 1 列出费米子场的电荷 Q、弱同位旋第 3 分量 T^3、弱超荷 Y、重子数 B 和轻子数 $L_e/L_\mu/L_\tau$，其中电荷 Q 由 T^3 和 Y 定义，

$$Q \equiv T^3 + Y. \tag{34}$$

4.1　Brout–Englert–Higgs 机制

由于左手费米子和右手费米子参与不同的 $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 规范相互作用，耦合左右手费米子场的质量项会破坏规范对称性。另一方面，规范对称性也禁止规范玻色子具有质量。为了让费米子和弱规范玻色子获得质量，需要引入 BEH 机制，使 $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 规范对称性自发破

表 1: 标准模型费米子场的量子数。

统一记号	第一代	第二代	第三代	Q	T^3	Y	B	$L_e/L_\mu/L_\tau$
$L_{i\mathrm{L}}=\begin{pmatrix}\nu_{i\mathrm{L}}\\ \ell_{i\mathrm{L}}\end{pmatrix}$	$\begin{pmatrix}\nu_{e\mathrm{L}}\\ e_{\mathrm{L}}\end{pmatrix}$	$\begin{pmatrix}\nu_{\mu\mathrm{L}}\\ \mu_{\mathrm{L}}\end{pmatrix}$	$\begin{pmatrix}\nu_{\tau\mathrm{L}}\\ \tau_{\mathrm{L}}\end{pmatrix}$	$\begin{matrix}0\\ -1\end{matrix}$	$\begin{matrix}1/2\\ -1/2\end{matrix}$	$\begin{matrix}-1/2\\ -1/2\end{matrix}$	$\begin{matrix}0\\ 0\end{matrix}$	$\begin{matrix}1\\ 1\end{matrix}$
$Q_{i\mathrm{L}}=\begin{pmatrix}u'_{i\mathrm{L}}\\ d'_{i\mathrm{L}}\end{pmatrix}$	$\begin{pmatrix}u'_{\mathrm{L}}\\ d'_{\mathrm{L}}\end{pmatrix}$	$\begin{pmatrix}c'_{\mathrm{L}}\\ s'_{\mathrm{L}}\end{pmatrix}$	$\begin{pmatrix}t'_{\mathrm{L}}\\ b'_{\mathrm{L}}\end{pmatrix}$	$\begin{matrix}2/3\\ -1/3\end{matrix}$	$\begin{matrix}1/2\\ -1/2\end{matrix}$	$\begin{matrix}1/6\\ 1/6\end{matrix}$	$\begin{matrix}1/3\\ 1/3\end{matrix}$	$\begin{matrix}0\\ 0\end{matrix}$
$\ell_{i\mathrm{R}}$	e_{R}	μ_{R}	τ_{R}	-1	0	-1	0	1
$u'_{i\mathrm{R}}$	u'_{R}	c'_{R}	t'_{R}	$2/3$	0	$2/3$	$1/3$	0
$d'_{i\mathrm{R}}$	d'_{R}	s'_{R}	b'_{R}	$-1/3$	0	$-1/3$	$1/3$	0

缺。因此，引入 Higgs 标量场

$$\Phi(x)=\begin{pmatrix}\phi^+(x)\\ \phi^0(x)\end{pmatrix},\tag{35}$$

其中 ϕ^+ 和 ϕ^0 都是复标量场。Φ 是 $\mathrm{SU}(2)_{\mathrm{L}}$ 二重态，弱超荷是

$$Y_H=\frac{1}{2}.\tag{36}$$

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_{\mathrm{H}}=(D^\mu\Phi)^\dagger(D_\mu\Phi)-V_{\mathrm{H}}(\Phi),\quad V_{\mathrm{H}}(\Phi)=-\mu^2\Phi^\dagger\Phi+\lambda(\Phi^\dagger\Phi)^2,\tag{37}$$

其中协变导数为

$$D_\mu\Phi=(\partial_\mu-\mathrm{i}g'B_\mu Y_H-\mathrm{i}gW_\mu^a\tau^a)\Phi.\tag{38}$$

当 $\lambda>0$ 且 $\mu^2>0$ 时，Higgs 场势能 $V_{\mathrm{H}}(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式，势能最小值位于方程

$$\Phi^\dagger\Phi=[\mathrm{Re}(\phi^+)]^2+[\mathrm{Im}(\phi^+)]^2+[\mathrm{Re}(\phi^0)]^2+[\mathrm{Im}(\phi^0)]^2=\frac{v^2}{2}\tag{39}$$

对应的 3 维球面上，其中 $v\equiv\sqrt{\mu^2/\lambda}$，满足

$$\mu^2=\lambda v^2.\tag{40}$$

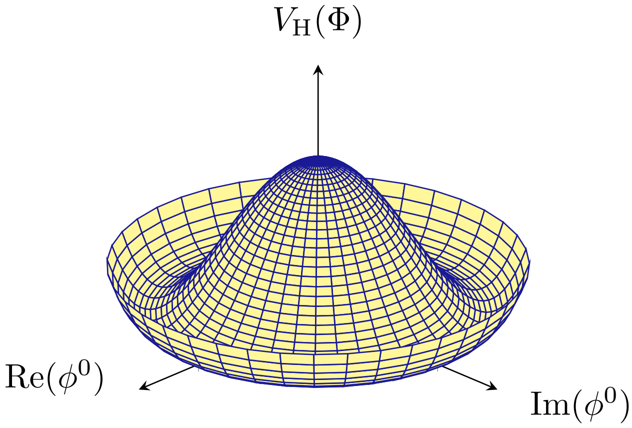


图 1: Higgs 场势能示意图。这里压缩掉 $\mathrm{Re}(\phi^+)$ 和 $\mathrm{Im}(\phi^+)$ 两个维度。

Higgs 场的真空期待值位于这个 3 维球面上的某一点，不失一般性，可将它取为

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}. \tag{41}$$

其它真空期待值可通过 $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_{\mathrm{Y}}$ 整体变换

$$\langle\Phi\rangle \to \exp(\mathrm{i}\alpha^a\tau^a)\exp(\mathrm{i}\alpha^Y Y_H)\langle\Phi\rangle \tag{42}$$

得到，因为 $\langle\Phi^\dagger\Phi\rangle$ 在这样的变换下保持不变。若 $\alpha^1=\alpha^2=0$ 且 $\alpha^3=\alpha^Y$，则

$$\exp(\mathrm{i}\alpha^a\tau^a)\exp(\mathrm{i}\alpha^Y Y_H) = \exp[\mathrm{i}\alpha^3(\sigma^3+\mathbf{1})/2] = \exp\left[\mathrm{i}\alpha^3\begin{pmatrix}1 & \\ & 0\end{pmatrix}\right] = \begin{pmatrix}\mathrm{e}^{\mathrm{i}\alpha^3} & \\ & 1\end{pmatrix}, \tag{43}$$

而 $\langle\Phi\rangle$ 在此变换下不变。因此，有 1 个方向的规范对称性没有受到破坏，只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理，破缺后存在 3 个无质量的 Nambu-Goldstone 玻色子。最终，有 3 个规范玻色子结合 Nambu-Goldstone 玻色子，通过 BEH 机制获得质量。

以 $\langle\Phi\rangle$ 为基础，将 Higgs 场参数化为

$$\Phi(x) = \exp\left[-\mathrm{i}\,\frac{\chi^a(x)}{v}\,\tau^a\right]\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}, \tag{44}$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场。$\exp[-\mathrm{i}\chi^a(x)\tau^a/v]$ 因子能够通过 $\mathrm{SU}(2)_{\mathrm{L}}$ 规范变换消去，因而可将 $\Phi(x)$ 直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}, \quad \Phi^\dagger\Phi = \frac{1}{2}(v+H)^2. \tag{45}$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$，对应于 Higgs 玻色子，这种取法称为**幺正规范**。

在幺正规范下，势能项化为

$$
\begin{aligned}
&-V_{\rm H}(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 = \frac{1}{2}\mu^2(v+H)^2 - \frac{1}{4}\lambda(v+H)^4 \\
&= \frac{1}{2}\mu^2(v^2+H^2+2vH) - \frac{1}{4}\lambda(v^4+4v^2H^2+H^4+4v^3H+2v^2H^2+4vH^3) \\
&= \frac{1}{4}\mu^2v^2 + \frac{1}{4}(\mu^2-\lambda v^2)v^2 + (\mu^2-\lambda v^2)vH + \frac{1}{2}(\mu^2-\lambda v^2)H^2 - \lambda v^2H^2 - \lambda vH^3 - \frac{1}{4}\lambda H^4 \\
&= \frac{1}{8}m_H^2v^2 - \frac{1}{2}m_H^2H^2 - \frac{1}{2}\frac{m_H^2}{v}H^3 - \frac{1}{8}\frac{m_H^2}{v^2}H^4,
\end{aligned} \tag{46}
$$

其中 Higgs 玻色子的质量为

$$
m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \tag{47}
$$

由于

$$
g'B_\mu Y_H + gW_\mu^a\tau^a = \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - {\rm i}W_\mu^2) \\ g(W_\mu^1 + {\rm i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}, \tag{48}
$$

Higgs 场真空期待值 v 对协变导数 $D_\mu\Phi$ 的贡献为

$$
\begin{aligned}
D_\mu\Phi &\supset -{\rm i}(g'B_\mu Y_H + gW_\mu^a\tau^a)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= -\frac{{\rm i}v}{2\sqrt{2}}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - {\rm i}W_\mu^2) \\ g(W_\mu^1 + {\rm i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \supset -\frac{{\rm i}v}{2\sqrt{2}}\begin{pmatrix} g(W_\mu^1 - {\rm i}W_\mu^2) \\ g'B_\mu - gW_\mu^3 \end{pmatrix},
\end{aligned} \tag{49}
$$

故协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$ 中正比于 v^2 的项是

$$
(D^\mu\Phi)^\dagger(D_\mu\Phi) \supset \frac{v^2}{8}[g^2|W_\mu^1 - {\rm i}W_\mu^2|^2 + (g'B_\mu - gW_\mu^3)^2] = \frac{v^2}{8}(g^2W^{a\mu}W_\mu^a + g'^2B^\mu B_\mu - 2gg'B^\mu W_\mu^3). \tag{50}
$$

这些项是规范玻色子的质量项，重新表达为

$$
\mathcal{L}_{\rm GBM} = \frac{1}{2}m_W^2(W^{1\mu}W_\mu^1 + W^{2\mu}W_\mu^2) + \frac{1}{2}\begin{pmatrix} W^{3\mu} & B^\mu \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{51}
$$

其中

$$
m_W \equiv \frac{1}{2}gv \tag{52}
$$

是 W_μ^1 和 W_μ^2 获得的质量，而 $W^{3\mu}$ 和 B^μ 的质量平方矩阵为

$$
M_{W^3B}^2 \equiv \frac{v^2}{4}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \tag{53}
$$

为了使 $M^2_{W^3B}$ 矩阵对角化，定义

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} c_\mathrm{W} & -s_\mathrm{W} \\ s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}, \tag{54}$$

其中

$$s_\mathrm{W} \equiv \sin\theta_\mathrm{W} = \frac{g'}{\sqrt{g^2+g'^2}}, \quad c_\mathrm{W} \equiv \cos\theta_\mathrm{W} = \frac{g}{\sqrt{g^2+g'^2}}, \tag{55}$$

θ_W 称为 Weinberg 角，也称为弱混合角。从后面的讨论可以看出 $A_\mu(x)$ 就是电磁场，对应于光子。$Z_\mu(x)$ 对应于矢量玻色子 Z。反过来，有

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_\mathrm{W} & s_\mathrm{W} \\ -s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \tag{56}$$

由

$$\begin{aligned} M^2_{W^3B} \begin{pmatrix} c_\mathrm{W} & s_\mathrm{W} \\ -s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} &= \frac{(g^2+g'^2)v^2}{4} \begin{pmatrix} c^2_\mathrm{W} & -s_\mathrm{W}c_\mathrm{W} \\ -s_\mathrm{W}c_\mathrm{W} & s^2_\mathrm{W} \end{pmatrix} \begin{pmatrix} c_\mathrm{W} & s_\mathrm{W} \\ -s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} \\ &= \frac{(g^2+g'^2)v^2}{4} \begin{pmatrix} c_\mathrm{W} & 0 \\ -s_\mathrm{W} & 0 \end{pmatrix} \end{aligned} \tag{57}$$

得

$$\begin{aligned} \begin{pmatrix} c_\mathrm{W} & -s_\mathrm{W} \\ s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} M^2_{W^3B} \begin{pmatrix} c_\mathrm{W} & s_\mathrm{W} \\ -s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} &= \frac{(g^2+g'^2)v^2}{4} \begin{pmatrix} c_\mathrm{W} & -s_\mathrm{W} \\ s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} \begin{pmatrix} c_\mathrm{W} & 0 \\ -s_\mathrm{W} & 0 \end{pmatrix} \\ &= \frac{(g^2+g'^2)v^2}{4} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \end{aligned} \tag{58}$$

因此

$$\begin{aligned} \frac{1}{2} \begin{pmatrix} W^{3\mu} & B^\mu \end{pmatrix} M^2_{W^3B} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} c_\mathrm{W} & -s_\mathrm{W} \\ s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} M^2_{W^3B} \begin{pmatrix} c_\mathrm{W} & s_\mathrm{W} \\ -s_\mathrm{W} & c_\mathrm{W} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \\ &= \frac{(g^2+g'^2)v^2}{8} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} m^2_Z Z^\mu Z_\mu, \end{aligned} \tag{59}$$

其中

$$m_Z \equiv \frac{1}{2}\sqrt{g^2+g'^2}\,v = \frac{gv}{2c_\mathrm{W}} = \frac{m_W}{c_\mathrm{W}} \tag{60}$$

是 Z 玻色子的质量，而光子没有质量。另一方面，用质量相同的实矢量场 W^1_μ 和 W^2_μ 线性组合

出复矢量场

$$W_\mu^+ \equiv \frac{1}{\sqrt{2}}(W_\mu^1 - \mathrm{i}W_\mu^2), \tag{61}$$

它的厄米共轭为

$$W_\mu^- \equiv (W_\mu^+)^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 + \mathrm{i}W_\mu^2), \tag{62}$$

则

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{\mathrm{i}}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \tag{63}$$

从而

$$\begin{aligned}\frac{1}{2}(W^{1\mu}W_\mu^1 + W^{2\mu}W_\mu^2) &= \frac{1}{4}[(W^{+\mu} + W^{-\mu})(W_\mu^+ + W_\mu^-) - (W^{+\mu} - W^{-\mu})(W_\mu^+ - W_\mu^-)] \\ &= W^{+\mu}W_\mu^-,\end{aligned} \tag{64}$$

(51) 式化为

$$\mathcal{L}_{\mathrm{GBM}} = m_W^2 W^{+\mu}W_\mu^- + \frac{1}{2}m_Z^2 Z^\mu Z_\mu. \tag{65}$$

复矢量场 $W_\mu^\pm$ 描述一对正反矢量玻色子 $W^\pm$，质量为 m_W。可见，BEH 机制使传递弱相互作用的规范玻色子 $W^\pm$ 和 Z 获得了质量。

接下来用质量本征态 $W_\mu^\pm$ 和 Z_μ 表达协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$。注意到

$$A_\mu = s_{\mathrm{W}}W_\mu^3 + c_{\mathrm{W}}B_\mu, \quad Z_\mu = c_{\mathrm{W}}W_\mu^3 - s_{\mathrm{W}}B_\mu, \tag{66}$$

$$B_\mu = c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu, \quad W_\mu^3 = s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu, \tag{67}$$

有

$$\begin{aligned}g'B_\mu + gW_\mu^3 &= g'(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) + g(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu) = \frac{2gg'}{\sqrt{g^2+g'^2}}A_\mu + \frac{g^2-g'^2}{\sqrt{g^2+g'^2}}Z_\mu \\ &= 2eA_\mu + \frac{g}{c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)Z_\mu,\end{aligned} \tag{68}$$

其中

$$e \equiv \frac{gg'}{\sqrt{g^2+g'^2}} = gs_{\mathrm{W}} = g'c_{\mathrm{W}}. \tag{69}$$

后面的讨论将表明 e 就是单位电荷量。再利用

$$g'B_\mu - gW_\mu^3 = g'(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) - g(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu) = -\left(\frac{gs_{\mathrm{W}}^2}{c_{\mathrm{W}}} + gc_{\mathrm{W}}\right)Z_\mu = -\frac{g}{c_{\mathrm{W}}}Z_\mu, \tag{70}$$

得

$$g'B_\mu Y_H + gW_\mu^a\tau^a = \frac{1}{2}\begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g(W_\mu^1 + \mathrm{i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}$$

$$= \begin{pmatrix} eA_\mu + \dfrac{g}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & \dfrac{g}{\sqrt{2}}\,W_\mu^+ \\ \dfrac{g}{\sqrt{2}}\,W_\mu^- & -\dfrac{g}{2c_\mathrm{W}}\,Z_\mu \end{pmatrix}. \tag{71}$$

在幺正规范下，

$$\begin{aligned}
(D^\mu\Phi)^\dagger(D_\mu\Phi) &= \left|\begin{pmatrix} \partial_\mu - \mathrm{i}eA_\mu - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & -\dfrac{\mathrm{i}g}{\sqrt{2}}\,W_\mu^+ \\ -\dfrac{\mathrm{i}g}{\sqrt{2}}\,W_\mu^- & \partial_\mu + \dfrac{\mathrm{i}g}{2c_\mathrm{W}}\,Z_\mu \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}\right|^2 \\
&= \frac{1}{2}\begin{pmatrix} \dfrac{\mathrm{i}g}{\sqrt{2}}\,W_\mu^-(v+H) & \partial_\mu H - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu(v+H) \end{pmatrix} \begin{pmatrix} -\dfrac{\mathrm{i}g}{\sqrt{2}}\,W_\mu^+(v+H) \\ \partial_\mu H + \dfrac{\mathrm{i}g}{2c_\mathrm{W}}\,Z_\mu(v+H) \end{pmatrix} \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2\left(\frac{g^2}{4}\,W_\mu^+W^{-\mu} + \frac{g^2}{8c_\mathrm{W}^2}\,Z_\mu Z^\mu\right) \\
&= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\
&\quad + g m_W H W_\mu^+ W^{-\mu} + \frac{g m_Z}{2c_\mathrm{W}}\,H Z_\mu Z^\mu + \frac{g^2}{4}\,H^2 W_\mu^+ W^{-\mu} + \frac{g^2}{8c_\mathrm{W}^2}\,H^2 Z_\mu Z^\mu.
\end{aligned} \tag{72}$$

除了 $W^\pm$ 和 Z 玻色子的质量项之外，还出现了 Higgs 玻色子 H 与 $W^\pm$、Z 的三线性和四线性耦合项。

Higgs 场 $\Phi(x)$ 的弱超荷为 $+1/2$。引入 $\Phi(x)$ 的共轭态

$$\tilde{\Phi}(x) = \mathrm{i}\sigma^2\Phi^*(x) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix} \phi^-(x) \\ \phi^{0*}(x) \end{pmatrix} = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}, \tag{73}$$

其中 $\phi^- \equiv (\phi^+)^*$，则 $\tilde{\Phi}(x)$ 是弱超荷为 $-1/2$ 的 $\mathrm{SU}(2)_\mathrm{L}$ 二重态。在幺正规范下，$\tilde{\Phi}(x)$ 化为

$$\tilde{\Phi}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v+H(x) \\ 0 \end{pmatrix}. \tag{74}$$

用 $\Phi(x)$、$\tilde{\Phi}(x)$ 和费米子场组成满足 $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$ 规范对称性的 Yukawa 相互作用拉氏量

$$\mathcal{L}_\mathrm{Y} = -\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - \tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{H.c.}, \tag{75}$$

其中 H.c. 表示厄米共轭，Yukawa 耦合常数 $\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 联系着不同代的夸克场，而 y_{ℓ_i} 只联系同一代的轻子场。在幺正规范下，利用

$$\bar{Q}_{i\mathrm{L}}\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{u}'_{i\mathrm{L}} & \bar{d}'_{i\mathrm{L}} \end{pmatrix}\begin{pmatrix} 0 \\ v+H \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{d}'_{i\mathrm{L}}, \tag{76}$$

$$\begin{aligned}\bar{Q}_{i\mathrm{L}}\tilde{\Phi} &= \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{u}'_{i\mathrm{L}} & \bar{d}'_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}v+H\\0\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{u}'_{i\mathrm{L}}, &(77)\\ \bar{L}_{i\mathrm{L}}\Phi &= \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{\nu}_{i\mathrm{L}} & \bar{\ell}_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}0\\v+H\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{\ell}_{i\mathrm{L}}, &(78)\end{aligned}$$

推出

$$\mathcal{L}_{\mathrm{Y}} = -\frac{1}{\sqrt{2}}(v+H)(\tilde{y}_{d,ij}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}} + \tilde{y}_{u,ij}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}} + y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + \mathrm{H.c.}). \tag{79}$$

$\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 可看作 3×3 矩阵 $\tilde{y}_d$ 和 $\tilde{y}_u$ 的元素。$\tilde{y}_d\tilde{y}_d^\dagger$ 和 $\tilde{y}_u\tilde{y}_u^\dagger$ 是厄米矩阵，必定可以分别通过幺正矩阵 U_d 和 U_u 对角化成 y_{D}^2 和 y_{U}^2 两个对角元为实数的对角矩阵，满足 $U_d^\dagger\tilde{y}_d\tilde{y}_d^\dagger U_d = y_{\mathrm{D}}^2$ 和 $U_u^\dagger\tilde{y}_u\tilde{y}_u^\dagger U_u = y_{\mathrm{U}}^2$，即

$$\tilde{y}_d\tilde{y}_d^\dagger = U_d y_{\mathrm{D}}^2 U_d^\dagger, \quad \tilde{y}_u\tilde{y}_u^\dagger = U_u y_{\mathrm{U}}^2 U_u^\dagger. \tag{80}$$

符合这两条式子的 $\tilde{y}_d$ 和 $\tilde{y}_u$ 可以表达为

$$\tilde{y}_d = U_d y_{\mathrm{D}} K_d^\dagger, \quad \tilde{y}_u = U_u y_{\mathrm{U}} K_u^\dagger, \tag{81}$$

其中对角矩阵 y_{D} 和 y_{U} 满足 $y_{\mathrm{D}}y_{\mathrm{D}} = y_{\mathrm{D}}^2$ 和 $y_{\mathrm{U}}y_{\mathrm{U}} = y_{\mathrm{U}}^2$，而 $K_d^\dagger$ 和 $K_u^\dagger$ 是两个幺正矩阵。

将 y_{D} 和 y_{U} 表示成

$$y_{\mathrm{D}} = \begin{pmatrix} y_{d_1} & & \\ & y_{d_2} & \\ & & y_{d_3}\end{pmatrix} = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b\end{pmatrix}, \quad y_{\mathrm{U}} = \begin{pmatrix} y_{u_1} & & \\ & y_{u_2} & \\ & & y_{u_3}\end{pmatrix} = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t\end{pmatrix}. \tag{82}$$

通过幺正变换定义

$$d_{i\mathrm{L}} \equiv (U_d^\dagger)_{ij}d'_{j\mathrm{L}}, \quad d_{i\mathrm{R}} \equiv (K_d^\dagger)_{ij}d'_{j\mathrm{R}}, \quad u_{i\mathrm{L}} \equiv (U_u^\dagger)_{ij}u'_{j\mathrm{L}}, \quad u_{i\mathrm{R}} \equiv (K_u^\dagger)_{ij}u'_{j\mathrm{R}}, \tag{83}$$

则 $\bar{d}_{i\mathrm{L}} = \bar{d}'_{j\mathrm{L}}U_{d,ji}$，$\bar{u}_{i\mathrm{L}} = \bar{u}'_{j\mathrm{L}}U_{u,ji}$，从而

$$\begin{aligned}\tilde{y}_{d,ij}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}} &= \bar{d}'_{i\mathrm{L}}(U_d y_d K_d^\dagger)_{ij}d'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}U_{d,ik}y_{d_k}(K_d^\dagger)_{kj}d'_{j\mathrm{R}} = y_{d_k}\bar{d}_{k\mathrm{L}}d_{k\mathrm{R}} = y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}, &(84)\\ \tilde{y}_{u,ij}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}} &= \bar{u}'_{i\mathrm{L}}(U_u y_u K_u^\dagger)_{ij}u'_{j\mathrm{R}} = y_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}, &(85)\end{aligned}$$

故

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}} &= -\frac{1}{\sqrt{2}}(v+H)(y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} + y_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} + y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + \mathrm{H.c.})\\ &= -m_{d_i}\bar{d}_id_i - m_{u_i}\bar{u}_iu_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_id_i - \frac{m_{u_i}}{v}H\bar{u}_iu_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i, &(86)\end{aligned}$$

其中前三项是费米子质量项，后三项是 Higgs 玻色子与费米子的 Yukawa 耦合项。于是，三代

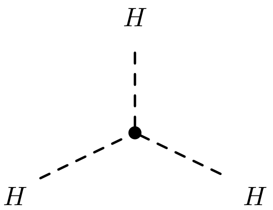
夸克和带电轻子获得了质量

$$m_{d_i} \equiv \frac{1}{\sqrt{2}}\, y_{d_i} v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}}\, y_{u_i} v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}}\, y_{\ell_i} v. \tag{87}$$

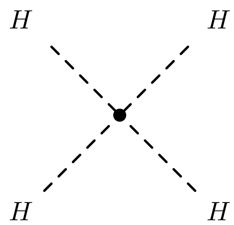
$d'_{i\mathrm{L}}$、$d'_{i\mathrm{R}}$、$u'_{i\mathrm{L}}$ 和 $u'_{i\mathrm{R}}$ 称为规范本征态，$d_{i\mathrm{L}}$、$d_{i\mathrm{R}}$、$u_{i\mathrm{L}}$ 和 $u_{i\mathrm{R}}$ 称为质量本征态。

下面给出幺正规范下 Higgs 场的顶点费曼规则。

Higgs 玻色子自耦合：

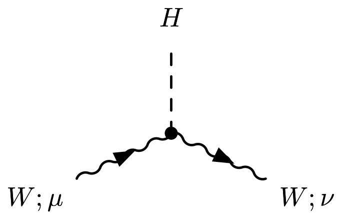


$$= -\frac{3\mathrm{i}m_H^2}{v} = -6\mathrm{i}\lambda v$$

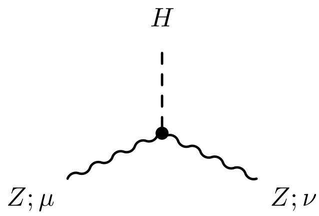


$$= -\frac{3\mathrm{i}m_H^2}{v^2} = -6\mathrm{i}\lambda \tag{88}$$

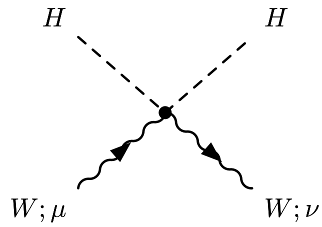
Higgs 玻色子与电弱规范玻色子的耦合：



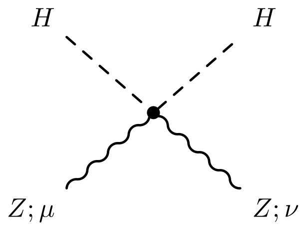
$$= \mathrm{i}g m_W g^{\mu\nu} = \frac{\mathrm{i}g^2 v}{2}\, g^{\mu\nu}$$



$$= \frac{\mathrm{i}g m_Z}{c_\mathrm{W}}\, g^{\mu\nu} = \frac{\mathrm{i}g^2 v}{2c_\mathrm{W}^2}\, g^{\mu\nu} \tag{89}$$

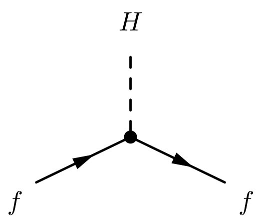


$$= \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu}$$



$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}\, g^{\mu\nu} \tag{90}$$

Higgs 玻色子与费米子 $f = d_i, u_i, \ell_i$ 的耦合：



$$= -\frac{\mathrm{i}m_f}{v} = -\frac{\mathrm{i}y_f}{\sqrt{2}} \tag{91}$$

4.2 费米子电弱规范相互作用

(83) 式意味着

$$d'_{i\mathrm{L}} = U_{d,ij} d_{j\mathrm{L}}, \quad d'_{i\mathrm{R}} = K_{d,ij} d_{j\mathrm{R}}, \quad u'_{i\mathrm{L}} = U_{u,ij} u_{j\mathrm{L}}, \quad u'_{i\mathrm{R}} = K_{u,ij} u_{j\mathrm{R}}, \tag{92}$$

从而

$$\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} = \bar{d}_{j\mathrm{L}}(U_d^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}}\delta_{jk}\gamma^\mu d_{k\mathrm{L}} = \bar{d}_{i\mathrm{L}}\gamma^\mu d_{i\mathrm{L}} \tag{93}$$

同理有

$$\bar{u}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} = \bar{u}_{i\mathrm{L}}\gamma^\mu u_{i\mathrm{L}},\quad \bar{d}'_{i\mathrm{R}}\gamma^\mu d'_{i\mathrm{R}} = \bar{d}_{i\mathrm{R}}\gamma^\mu d_{i\mathrm{R}},\quad \bar{u}'_{i\mathrm{R}}\gamma^\mu u'_{i\mathrm{R}} = \bar{u}_{i\mathrm{R}}\gamma^\mu u_{i\mathrm{R}}. \tag{94}$$

另一方面，

$$\begin{aligned}\bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} &= \bar{u}_{j\mathrm{L}}(U_u^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{k\mathrm{L}} = \bar{u}_{i\mathrm{L}}\gamma^\mu V_{ij}d_{j\mathrm{L}} &(95)\\ \bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} &= \bar{d}_{j\mathrm{L}}(U_d^\dagger)_{ji}\gamma^\mu U_{u,ik}u_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}}V_{ji}^\dagger\gamma^\mu u_{i\mathrm{L}} &(96)\end{aligned}$$

其中

$$V \equiv U_u^\dagger U_d \tag{97}$$

称为 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵，其厄米共轭矩阵为 $V^\dagger = U_d^\dagger U_u$。

$\mathrm{SU(2)_L\times U(1)_Y}$ 规范不变的费米子协变动能项为

$$\mathcal{L}_{\mathrm{EWF}} = \bar{Q}_{i\mathrm{L}}\mathrm{i}\not{D}Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}}\mathrm{i}\not{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}\mathrm{i}\not{D}d'_{i\mathrm{R}} + \bar{L}_{i\mathrm{L}}\mathrm{i}\not{D}L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}}\mathrm{i}\not{D}\ell_{i\mathrm{R}}. \tag{98}$$

根据 Q 的定义 (34)，有

$$\begin{aligned}g'YB_\mu + gT^3W_\mu^3 &= g'Y(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) + gT^3(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\\ &= e(Y+T^3)A_\mu + \left(gc_\mathrm{W}T^3 - \frac{gs_\mathrm{W}}{c_\mathrm{W}}s_\mathrm{W}Y\right)Z_\mu = QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3c_\mathrm{W}^2 - Ys_\mathrm{W}^2)Z_\mu\\ &= QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu, \end{aligned}\tag{99}$$

故

$$\begin{aligned}D_\mu Q_{i\mathrm{L}} &= (\partial_\mu - \mathrm{i}g'B_\mu Y - \mathrm{i}gW_\mu^a\tau^a)Q_{i\mathrm{L}}\\ &= \partial_\mu Q_{i\mathrm{L}} - \mathrm{i}\begin{pmatrix} g'YB_\mu + gT^3W_\mu^3 & \dfrac{g}{2}(W_\mu^1 - \mathrm{i}W_\mu^2)\\ \dfrac{g}{2}(W_\mu^1 + \mathrm{i}W_\mu^2) & g'YB_\mu + gT^3W_\mu^3\end{pmatrix}Q_{i\mathrm{L}}\\ &= \partial_\mu Q_{i\mathrm{L}} - \mathrm{i}\begin{pmatrix} QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu & \dfrac{g}{\sqrt{2}}W_\mu^+\\ \dfrac{g}{\sqrt{2}}W_\mu^- & QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\end{pmatrix}Q_{i\mathrm{L}}\\ &= \partial_\mu Q_{i\mathrm{L}} - \mathrm{i}\begin{pmatrix}\left[QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]u'_{i\mathrm{L}} + \dfrac{g}{\sqrt{2}}W_\mu^+d'_{i\mathrm{L}}\\ \dfrac{g}{\sqrt{2}}W_\mu^-u'_{i\mathrm{L}} + \left[QeA_\mu + \dfrac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)\right]d'_{i\mathrm{L}}\end{pmatrix}.\end{aligned}\tag{100}$$

于是

$$
\begin{aligned}
\bar{Q}_{i\mathrm{L}}\mathrm{i}\not{D}Q_{i\mathrm{L}} \supset & \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{u}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} + \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} \\
& + \frac{g}{\sqrt{2}}W_\mu^+\bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} + \frac{g}{\sqrt{2}}W_\mu^-\bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} \\
= & \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{u}_i\gamma^\mu\frac{1-\gamma^5}{2}u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{d}_i\gamma^\mu\frac{1-\gamma^5}{2}d_i \\
& + \frac{g}{\sqrt{2}}W_\mu^+\bar{u}_i\gamma^\mu P_\mathrm{L}V_{ij}d_j + \frac{g}{\sqrt{2}}W_\mu^-\bar{d}_jV_{ji}^\dagger\gamma^\mu P_\mathrm{L}u_i, \qquad (101)
\end{aligned}
$$

其中左手耦合系数

$$g_\mathrm{L} \equiv T^3 - Qs_\mathrm{W}^2. \qquad (102)$$

另一方面,

$$
\begin{aligned}
D_\mu d'_{i\mathrm{R}} &= (\partial_\mu - \mathrm{i}g'B_\mu Y)d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} - \mathrm{i}g'Q(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu)d'_{i\mathrm{R}} \\
&= \partial_\mu d'_{i\mathrm{R}} - \mathrm{i}QeA_\mu d'_{i\mathrm{R}} + \frac{\mathrm{i}g}{c_\mathrm{W}}Qs_\mathrm{W}^2Z_\mu d'_{i\mathrm{R}}, \qquad (103)
\end{aligned}
$$

则

$$
\begin{aligned}
& \bar{u}'_{i\mathrm{R}}\mathrm{i}\not{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}\mathrm{i}\not{D}d'_{i\mathrm{R}} \\
\supset & \left(QeA_\mu - \frac{g}{c_\mathrm{W}}Qs_\mathrm{W}^2Z_\mu\right)\bar{u}'_{i\mathrm{R}}\gamma^\mu u'_{i\mathrm{R}} + \left(QeA_\mu - \frac{g}{c_\mathrm{W}}Qs_\mathrm{W}^2Z_\mu\right)\bar{d}'_{i\mathrm{R}}\gamma^\mu d'_{i\mathrm{R}} \\
= & \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{R}Z_\mu\right)\bar{u}_i\gamma^\mu\frac{1+\gamma^5}{2}u_i + \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{R}Z_\mu\right)\bar{d}_i\gamma^\mu\frac{1+\gamma^5}{2}d_i, \qquad (104)
\end{aligned}
$$

其中右手耦合系数

$$g_\mathrm{R} \equiv -Qs_\mathrm{W}^2. \qquad (105)$$

引入矢量流和轴矢量流耦合系数

$$g_\mathrm{V} \equiv g_\mathrm{L} + g_\mathrm{R} = T^3 - 2Qs_\mathrm{W}^2, \quad g_\mathrm{A} \equiv g_\mathrm{L} - g_\mathrm{R} = T^3, \qquad (106)$$

得

$$
\begin{aligned}
& \bar{Q}_{i\mathrm{L}}\mathrm{i}\not{D}Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}}\mathrm{i}\not{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}\mathrm{i}\not{D}d'_{i\mathrm{R}} \\
\supset & Qe\bar{u}_i\gamma^\mu u_iA_\mu + Qe\bar{d}_i\gamma^\mu d_iA_\mu + \frac{g}{2c_\mathrm{W}}\bar{u}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma^5)u_iZ_\mu + \frac{g}{2c_\mathrm{W}}\bar{d}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma^5)d_iZ_\mu \\
& + \frac{g}{\sqrt{2}}W_\mu^+\bar{u}_i\gamma^\mu P_\mathrm{L}V_{ij}d_j + \frac{g}{\sqrt{2}}W_\mu^-\bar{d}_jV_{ji}^\dagger\gamma^\mu P_\mathrm{L}u_i. \qquad (107)
\end{aligned}
$$

同理，有

$$\bar{L}_{i\mathrm{L}}\mathrm{i}\not{D}L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}}\mathrm{i}\not{D}\ell_{i\mathrm{R}} \supset Qe\bar{\ell}_i\gamma^\mu\ell_i A_\mu + \frac{g}{2c_\mathrm{W}}\,\bar{\ell}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma^5)\ell_i Z_\mu + \frac{g}{2c_\mathrm{W}}\,\bar{\nu}_i\gamma^\mu(g_\mathrm{V} - g_\mathrm{A}\gamma^5)\nu_i Z_\mu$$
$$+\frac{g}{\sqrt{2}}\,W_\mu^+\bar{\nu}_i\gamma^\mu P_\mathrm{L}\ell_i + \frac{g}{\sqrt{2}}\,W_\mu^-\bar{\ell}_i\gamma^\mu P_\mathrm{L}\nu_i. \tag{108}$$

总结起来，将费米子电弱规范相互作用写成流耦合的形式，

$$\begin{aligned}\mathcal{L}_\mathrm{EWF} &\supset \sum_f \left[Q_f e\bar{f}\gamma^\mu f A_\mu + \frac{g}{2c_\mathrm{W}}\,\bar{f}\gamma^\mu(g_\mathrm{V}^f - g_\mathrm{A}^f\gamma^5)fZ_\mu\right] + g(W_\mu^+ J_W^{+\mu} + W_\mu^- J_W^{-\mu})\\ &= eA_\mu J_\mathrm{EM}^\mu + g(Z_\mu J_Z^\mu + W_\mu^+ J_W^{+,\mu} + W_\mu^- J_W^{-,\mu}),\end{aligned} \tag{109}$$

其中 f 代表任意费米子场，电磁流

$$J_\mathrm{EM}^\mu \equiv \sum_f Q_f\bar{f}\gamma^\mu f, \tag{110}$$

弱中性流

$$J_Z^\mu \equiv \frac{1}{2c_\mathrm{W}}\sum_f \bar{f}\gamma^\mu(g_\mathrm{V}^f - g_\mathrm{A}^f\gamma^5)f = \frac{1}{c_\mathrm{W}}\sum_f(g_\mathrm{L}^f\bar{f}_\mathrm{L}\gamma^\mu f_\mathrm{L} + g_\mathrm{R}^f\bar{f}_\mathrm{R}\gamma^\mu f_\mathrm{R}), \tag{111}$$

弱带电流

$$J_W^{+,\mu} \equiv \frac{1}{\sqrt{2}}(\bar{u}_{i\mathrm{L}}\gamma^\mu V_{ij}d_{j\mathrm{L}} + \bar{\nu}_{i\mathrm{L}}\gamma^\mu\ell_{i\mathrm{L}}), \quad J_W^{-,\mu} \equiv (J_W^{+\mu})^\dagger = \frac{1}{\sqrt{2}}(\bar{d}_{j\mathrm{L}}V_{ji}^\dagger\gamma^\mu u_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{L}}\gamma^\mu\nu_{i\mathrm{L}}). \tag{112}$$

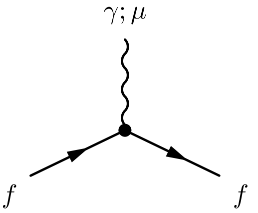
对于各种费米子，相关的系数如下，

$$Q_{u_i} = \frac{2}{3}, \qquad Q_{d_i} = -\frac{1}{3}, \qquad Q_{\nu_i} = 0, \qquad Q_{\ell_i} = -1; \tag{113}$$
$$g_\mathrm{V}^{u_i} = \frac{1}{2} - \frac{4}{3}s_\mathrm{W}^2, \quad g_\mathrm{A}^{u_i} = \frac{1}{2}; \quad g_\mathrm{V}^{d_i} = -\frac{1}{2} + \frac{2}{3}s_\mathrm{W}^2, \quad g_\mathrm{A}^{d_i} = -\frac{1}{2}; \tag{114}$$
$$g_\mathrm{V}^{\nu_i} = \frac{1}{2}, \quad g_\mathrm{A}^{\nu_i} = \frac{1}{2}; \quad g_\mathrm{V}^{\ell_i} = -\frac{1}{2} + 2s_\mathrm{W}^2, \quad g_\mathrm{A}^{\ell_i} = -\frac{1}{2}; \tag{115}$$
$$g_\mathrm{L}^{u_i} = \frac{1}{2} - \frac{2}{3}s_\mathrm{W}^2, \quad g_\mathrm{R}^{u_i} = -\frac{2}{3}s_\mathrm{W}^2; \quad g_\mathrm{L}^{d_i} = -\frac{1}{2} + \frac{1}{3}s_\mathrm{W}^2, \quad g_\mathrm{R}^{d_i} = \frac{1}{3}s_\mathrm{W}^2; \tag{116}$$
$$g_\mathrm{L}^{\nu_i} = \frac{1}{2}, \quad g_\mathrm{R}^{\nu_i} = 0; \quad g_\mathrm{L}^{\ell_i} = -\frac{1}{2} + s_\mathrm{W}^2, \quad g_\mathrm{R}^{\ell_i} = s_\mathrm{W}^2. \tag{117}$$

可以看到，电磁流耦合与 QED 耦合完全相同，由此辨认出 A_μ 是电磁场，e 是单位电荷量，由 $Q = T^3 + Y$ 定义的 Q 确实是电荷。为了保持电荷守恒，指定复矢量场 $W_\mu^+(x)$ 携带 $Q = +1$ 的电荷，从而 $W^\pm$ 玻色子的电荷为 ± 1。不同代夸克之间的相互作用只发生在弱带电流耦合中，源自 CKM 矩阵 V 的非对角元。

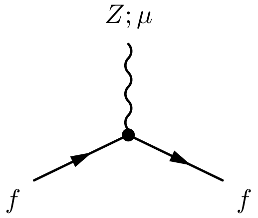
下面给出费米子电弱规范相互作用顶点的费曼规则。

QED 耦合:



$$= \mathrm{i}Q_f e\gamma^\mu \quad \text{（对于电子，}Q_e=-1\text{）} \tag{118}$$

费米子与 Z 玻色子的耦合:

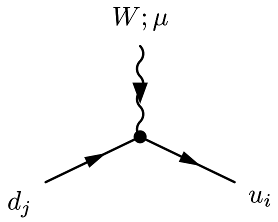


$$= \frac{\mathrm{i}g}{2c_\mathrm{W}}\,\gamma^\mu(g_\mathrm{V}^f - g_\mathrm{A}^f\gamma^5) \tag{119}$$

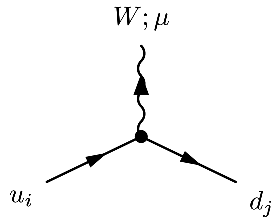
$$g_\mathrm{V}^{u_i} = \frac{1}{2} - \frac{4}{3}s_\mathrm{W}^2,\quad g_\mathrm{A}^{u_i} = \frac{1}{2};\quad g_\mathrm{V}^{d_i} = -\frac{1}{2} + \frac{2}{3}s_\mathrm{W}^2,\quad g_\mathrm{A}^{d_i} = -\frac{1}{2}; \tag{120}$$

$$g_\mathrm{V}^{\nu_i} = \frac{1}{2},\quad g_\mathrm{A}^{\nu_i} = \frac{1}{2};\quad g_\mathrm{V}^{\ell_i} = -\frac{1}{2} + 2s_\mathrm{W}^2,\quad g_\mathrm{A}^{\ell_i} = -\frac{1}{2}. \tag{121}$$

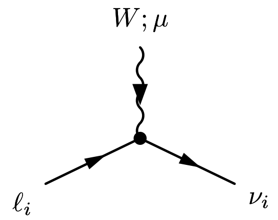
费米子与 $W^\pm$ 玻色子的耦合:



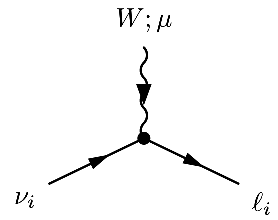
$$= \frac{\mathrm{i}g}{\sqrt{2}}\,V_{ij}\gamma^\mu P_\mathrm{L}$$



$$= \frac{\mathrm{i}g}{\sqrt{2}}\,V_{ji}^\dagger\gamma^\mu P_\mathrm{L} \tag{122}$$



$$= \frac{\mathrm{i}g}{\sqrt{2}}\,\gamma^\mu P_\mathrm{L}$$



$$= \frac{\mathrm{i}g}{\sqrt{2}}\,\gamma^\mu P_\mathrm{L} \tag{123}$$

4.3 电弱规范场的自相互作用

电弱规范场自相互作用的拉氏量是

$$\mathcal{L}_\mathrm{EWG} = -\frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{124}$$

其中

$$W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g\varepsilon^{abc}W^{b\mu}W^{c\nu},\quad B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu. \tag{125}$$

利用 (63) 和 (67) 式，推出

$$
\begin{aligned}
W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 &= \frac{\mathrm{i}}{\sqrt{2}}[(W_\mu^+ - W_\mu^-)(s_\mathrm{W} A_\nu + c_\mathrm{W} Z_\nu) - (s_\mathrm{W} A_\mu + c_\mathrm{W} Z_\mu)(W_\nu^+ - W_\nu^-)] \\
&= \frac{\mathrm{i}}{\sqrt{2}}[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
&\qquad - s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)], \qquad (126) \\
W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 &= \frac{1}{\sqrt{2}}[(s_\mathrm{W} A_\mu + c_\mathrm{W} Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_\mathrm{W} A_\nu + c_\mathrm{W} Z_\nu)] \\
&= -\frac{1}{\sqrt{2}}[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
&\qquad + s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]. \qquad (127)
\end{aligned}
$$

从而

$$
\begin{aligned}
W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g\varepsilon^{1bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 + g W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2 \\
&= \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^2 W_\nu^3 - g W_\mu^3 W_\nu^2) \\
&= \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + \mathrm{i}g[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad + \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - \mathrm{i}g[s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{1}{\sqrt{2}}(F_{\mu\nu}^+ + F_{\mu\nu}^-), \qquad (128)
\end{aligned}
$$

其中，

$$
F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + \mathrm{i}e(W_\mu^+ A_\nu - A_\mu W_\nu^+) + \mathrm{i}gc_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \qquad (129)
$$

$$
F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- - \mathrm{i}e(W_\mu^- A_\nu - A_\mu W_\nu^-) - \mathrm{i}gc_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \qquad (130)
$$

另一方面，

$$
\begin{aligned}
W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 + g\varepsilon^{2bc} W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g W_\mu^1 W_\nu^3 + g W_\mu^3 W_\nu^1 \\
&= \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\
&= \frac{\mathrm{i}}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + \mathrm{i}g[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\} \\
&\quad - \frac{\mathrm{i}}{\sqrt{2}}\{\partial_\mu W_\nu^- - \partial_\nu W_\mu^- - \mathrm{i}g[s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]\} \\
&= \frac{\mathrm{i}}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-). \qquad (131)
\end{aligned}
$$

因此

$$
\begin{aligned}
& -\frac{1}{4}W^1_{\mu\nu}W^{1\mu\nu}-\frac{1}{4}W^2_{\mu\nu}W^{2\mu\nu} \\
= & -\frac{1}{8}(F^+_{\mu\nu}+F^-_{\mu\nu})(F^{+\mu\nu}+F^{-\mu\nu})+\frac{1}{8}(F^+_{\mu\nu}-F^-_{\mu\nu})(F^{+\mu\nu}-F^{-\mu\nu})=-\frac{1}{2}F^+_{\mu\nu}F^{-\mu\nu} \\
= & -\frac{1}{2}[\partial_\mu W^+_\nu-\partial_\nu W^+_\mu+\mathrm{i}e(W^+_\mu A_\nu-A_\mu W^+_\nu)+\mathrm{i}gc_\mathrm{W}(W^+_\mu Z_\nu-Z_\mu W^+_\nu)] \\
& \times[\partial^\mu W^{-\nu}-\partial^\nu W^{-\mu}-\mathrm{i}e(W^{-\mu}A^\nu-A^\mu W^{-\nu})-\mathrm{i}gc_\mathrm{W}(W^{-\mu}Z^\nu-Z^\mu W^{-\nu})] \\
= & -(\partial_\mu W^+_\nu)(\partial^\mu W^{-\nu})+(\partial_\mu W^+_\nu)(\partial^\nu W^{-\mu}) \\
& +\mathrm{i}e[(\partial_\mu W^+_\nu)W^{-\mu}A^\nu-(\partial_\mu W^+_\nu)W^{-\nu}A^\mu-W^+_\mu(\partial^\mu W^{-\nu})A_\nu+W^+_\nu(\partial^\mu W^{-\nu})A_\mu] \\
& +\mathrm{i}gc_\mathrm{W}[(\partial_\mu W^+_\nu)W^{-\mu}Z^\nu-(\partial_\mu W^+_\nu)W^{-\nu}Z^\mu-W^+_\mu(\partial^\mu W^{-\nu})Z_\nu+W^+_\nu(\partial^\mu W^{-\nu})Z_\mu] \\
& +e^2(W^+_\mu W^{-\nu}A_\nu A^\mu-W^+_\mu W^{-\mu}A_\nu A^\nu)+g^2c^2_\mathrm{W}(W^+_\mu W^{-\nu}Z_\nu Z^\mu-W^+_\mu W^{-\mu}Z_\nu Z^\nu) \\
& +egc_\mathrm{W}(W^+_\mu W^{-\nu}A_\nu Z^\mu+W^+_\mu W^{-\nu}A^\mu Z_\nu-2W^+_\mu W^{-\mu}A_\nu Z^\nu).
\end{aligned}
\tag{132}
$$

由

$$
\begin{aligned}
W^1_\mu W^2_\nu-W^2_\mu W^1_\nu &= \frac{\mathrm{i}}{2}(W^+_\mu+W^-_\mu)(W^+_\nu-W^-_\nu)-\frac{\mathrm{i}}{2}(W^+_\mu-W^-_\mu)(W^+_\nu+W^-_\nu) \\
&= -\mathrm{i}(W^+_\mu W^-_\nu-W^-_\mu W^+_\nu),
\end{aligned}
\tag{133}
$$

得到

$$
\begin{aligned}
W^3_{\mu\nu} &= \partial_\mu W^3_\nu-\partial_\nu W^3_\mu+g\varepsilon^{3bc}W^b_\mu W^c_\nu=\partial_\mu W^3_\nu-\partial_\nu W^3_\mu+gW^1_\mu W^2_\nu-gW^2_\mu W^1_\nu \\
&= s_\mathrm{W}\partial_\mu A_\nu+c_\mathrm{W}\partial_\mu Z_\nu-s_\mathrm{W}\partial_\nu A_\mu+c_\mathrm{W}\partial_\nu Z_\mu+g(W^1_\mu W^2_\nu-W^2_\mu W^1_\nu) \\
&= s_\mathrm{W}(\partial_\mu A_\nu-\partial_\nu A_\mu)+c_\mathrm{W}(\partial_\mu Z_\nu-\partial_\nu Z_\mu)-\mathrm{i}g(W^+_\mu W^-_\nu-W^-_\mu W^+_\nu),
\end{aligned}
\tag{134}
$$

$$
\begin{aligned}
B_{\mu\nu} &= \partial_\mu(c_\mathrm{W}A_\nu-s_\mathrm{W}Z_\nu)-\partial_\nu(c_\mathrm{W}A_\mu-s_\mathrm{W}Z_\mu) \\
&= c_\mathrm{W}(\partial_\mu A_\nu-\partial_\nu A_\mu)-s_\mathrm{W}(\partial_\mu Z_\nu-\partial_\nu Z_\mu).
\end{aligned}
\tag{135}
$$

于是

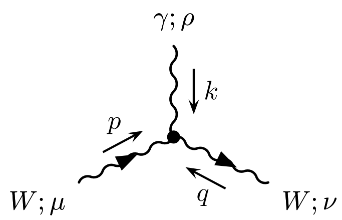
$$
\begin{aligned}
& -\frac{1}{4}W^3_{\mu\nu}W^{3\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
= & -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu)-(\partial_\mu A_\nu)(\partial^\nu A^\mu)]-\frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu)-(\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
& +\mathrm{i}e[W^{+\mu}W^{-\nu}(\partial_\mu A_\nu)-W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)]+\mathrm{i}gc_\mathrm{W}[W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu)-W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)] \\
& +\frac{g^2}{2}(W^+_\mu W^{+\mu}W^-_\nu W^{-\nu}-W^+_\mu W^{+\nu}W^-_\nu W^{-\mu}).
\end{aligned}
\tag{136}
$$

综合起来，有

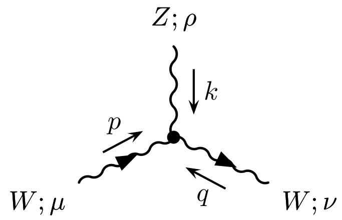
$$
\mathcal{L}_\mathrm{EWG} = \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu)-(\partial_\mu A_\nu)(\partial^\mu A^\nu)]+\frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu)-(\partial_\mu Z_\nu)(\partial^\mu Z^\nu)]
$$

$$
\begin{aligned}
&+(\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu})-(\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu})+\frac{g^2}{2}(W_\mu^+W^{+\mu}W_\nu^-W^{-\nu}-W_\mu^+W^{+\nu}W_\nu^-W^{-\mu})\\
&+\mathrm{i}e[(\partial_\mu W_\nu^+)W^{-\mu}A^\nu-(\partial_\mu W_\nu^+)W^{-\nu}A^\mu-W^{+\mu}(\partial_\mu W_\nu^-)A^\nu+W^{+\nu}(\partial_\mu W_\nu^-)A^\mu\\
&\qquad+W^{+\mu}W^{-\nu}(\partial_\mu A_\nu)-W^{+\nu}W^{-\mu}(\partial_\mu A_\nu)]+e^2(W_\mu^+W^{-\nu}A_\nu A^\mu-W_\mu^+W^{-\mu}A_\nu A^\nu)\\
&+\mathrm{i}gc_\mathrm{W}[(\partial_\mu W_\nu^+)W^{-\mu}Z^\nu-(\partial_\mu W_\nu^+)W^{-\nu}Z^\mu-W^{+\mu}(\partial_\mu W_\nu^-)Z^\nu+W^{+\nu}(\partial_\mu W_\nu^-)Z^\mu\\
&\qquad\quad+W^{+\mu}W^{-\nu}(\partial_\mu Z_\nu)-W^{+\nu}W^{-\mu}(\partial_\mu Z_\nu)]\\
&+g^2c_\mathrm{W}^2(W_\mu^+W^{-\nu}Z_\nu Z^\mu-W_\mu^+W^{-\mu}Z_\nu Z^\nu)\\
&+egc_\mathrm{W}(W_\mu^+W^{-\nu}A_\nu Z^\mu+W_\mu^+W^{-\nu}A^\mu Z_\nu-2W_\mu^+W^{-\mu}A_\nu Z^\nu).
\end{aligned}
\tag{137}
$$

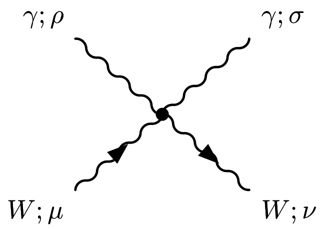
下面是电弱规范玻色子自耦合的费曼规则:



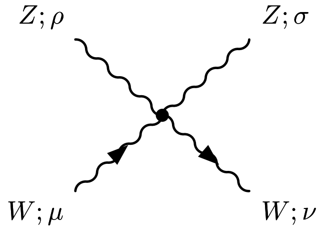
$$
= -\mathrm{i}e[g^{\mu\nu}(p-q)^\rho+g^{\nu\rho}(q-k)^\mu+g^{\rho\mu}(k-p)^\nu] \tag{138}
$$



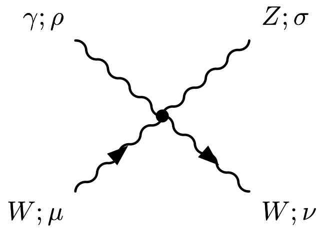
$$
= -\mathrm{i}gc_\mathrm{W}[g^{\mu\nu}(p-q)^\rho+g^{\nu\rho}(q-k)^\mu+g^{\rho\mu}(k-p)^\nu] \tag{139}
$$



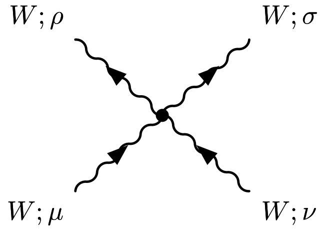
$$
= \mathrm{i}e^2(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma}) \tag{140}
$$



$$
= \mathrm{i}g^2c_\mathrm{W}^2(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma}) \tag{141}
$$



$$
= \mathrm{i}egc_\mathrm{W}(g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}-2g^{\mu\nu}g^{\rho\sigma}) \tag{142}
$$



$$= -\mathrm{i}g^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{143}$$

5 R$_\xi$ 规范下电弱拉氏量和费曼规则

在 $\mathbf{R}_{\boldsymbol{\xi}}$ 规范下，将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \dfrac{1}{\sqrt{2}}[v + H(x) + \mathrm{i}\chi(x)] \end{pmatrix}, \tag{144}$$

其中 ϕ^+ 和 χ 是 Nambu-Goldstone 标量场。那么，$\tilde{\Phi}(x)$ 的形式是

$$\tilde{\Phi}(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\sqrt{2}}[v + H(x) - \mathrm{i}\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \tag{145}$$

利用

$$\Phi^\dagger\Phi = \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \tag{146}$$

$$(\Phi^\dagger\Phi)^2 = \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \tag{147}$$

推出 Higgs 场势能项

$$\begin{aligned}
&-V_{\mathrm{H}}(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \\
=\ & \frac{\mu^2}{2}(v^2 + H^2 + 2vH + \chi^2) + \mu^2|\phi^+|^2 - \frac{\lambda}{4}(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda|\phi^+|^4 \\
& -\lambda|\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\
=\ & \frac{1}{2}\left(\mu^2 - \frac{\lambda}{2}v^2\right)v^2 + \frac{1}{2}(\mu^2 - 3\lambda v^2)H^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)\chi^2 - \frac{\lambda}{4}H^4 - \frac{\lambda}{4}\chi^4 \\
& -\lambda vH^3 - \frac{\lambda}{2}H^2\chi^2 - \lambda vH\chi^2 + (\mu^2 - \lambda v^2)|\phi^+|^2 - \lambda|\phi^+|^4 - \lambda|\phi^+|^2(H^2 + 2vH + \chi^2) \\
=\ & \frac{\lambda}{4}v^4 - \lambda v^2H^2 - \frac{\lambda}{4}H^4 - \frac{\lambda}{4}\chi^4 - \lambda vH^3 - \frac{\lambda}{2}H^2\chi^2 - \lambda vH\chi^2 \\
& -\lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2) \\
=\ & \frac{1}{8}m_H^2v^2 - \frac{1}{2}m_H^2H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4 - \frac{m_H^2}{2v}H\chi^2 - \frac{m_H^2}{4v^2}H^2\chi^2 - \frac{m_H^2}{8v^2}\chi^4 \\
& -\frac{m_H^2}{2v^2}\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2).
\end{aligned} \tag{148}$$

由 $V=U_u^\dagger U_d$ 得到 $V^\dagger=U_d^\dagger U_u$ 和 $U_d=U_uV$，$U_u=U_dV^\dagger$，则

$$\tilde{y}_d=U_d y_{\rm D}K_d^\dagger=U_uVy_{\rm D}K_d^\dagger,\quad \tilde{y}_u=U_u y_{\rm U}K_u^\dagger=U_dV^\dagger y_{\rm U}K_u^\dagger, \tag{149}$$

故

$$\tilde{y}_{d,ij}\bar{u}'_{i\rm L}d'_{j\rm R}=\bar{u}'_{i\rm L}(U_uVy_{\rm D}K_d^\dagger)_{ij}d'_{j\rm R}=\bar{u}'_{i\rm L}U_{u,ik}V_{kl}y_{d_l}(K_d^\dagger)_{lj}d'_{m\rm R}=y_{d_j}\bar{u}_{i\rm L}V_{ij}d_{j\rm R}, \tag{150}$$

$$\tilde{y}_{u,ij}\bar{d}'_{i\rm L}u'_{j\rm R}=\bar{d}'_{i\rm L}(U_dV^\dagger y_{\rm U}K_u^\dagger)_{ij}u'_{j\rm R}=\bar{d}'_{i\rm L}U_{d,ik}V^\dagger_{kl}y_{u_l}(K_u^\dagger)_{lj}u'_{j\rm R}=y_{u_i}\bar{d}_{j\rm L}V^\dagger_{ji}u_{i\rm R}. \tag{151}$$

结合 (84) 和 (85) 式，得

$$\begin{aligned}\tilde{y}_{d,ij}\bar{Q}_{i\rm L}d'_{j\rm R}\Phi &= \tilde{y}_{d,ij}\left[\bar{u}'_{i\rm L}d'_{j\rm R}\phi^+ +\frac{1}{\sqrt{2}}\,\bar{d}'_{i\rm L}d'_{j\rm R}(v+H+\mathrm{i}\chi)\right]\\ &= y_{d_j}\bar{u}_{i\rm L}V_{ij}d_{j\rm R}\phi^+ +\frac{y_{d_i}}{\sqrt{2}}\,\bar{d}_{i\rm L}d_{i\rm R}(v+H+\mathrm{i}\chi),\end{aligned} \tag{152}$$

$$\begin{aligned}\tilde{y}_{u,ij}\bar{Q}_{i\rm L}u'_{j\rm R}\tilde{\Phi} &= \tilde{y}_{u,ij}\left[\frac{1}{\sqrt{2}}\,\bar{u}'_{i\rm L}u'_{j\rm R}(v+H-\mathrm{i}\chi)-\bar{d}'_{i\rm L}u'_{j\rm R}\phi^-\right]\\ &= \frac{y_{u_i}}{\sqrt{2}}\,\bar{u}_{i\rm L}u_{j\rm R}(v+H-\mathrm{i}\chi)-y_{u_i}\bar{d}_{j\rm L}V^\dagger_{ji}u_{i\rm R}\phi^-.\end{aligned} \tag{153}$$

从而，Yukawa 相互作用拉氏量化为

$$\begin{aligned}&\mathcal{L}_{\rm Y}=-\tilde{y}_{d,ij}\bar{Q}_{i\rm L}d'_{j\rm R}\Phi-\tilde{y}_{u,ij}\bar{Q}_{i\rm L}u'_{j\rm R}\tilde{\Phi}-y_{\ell_i}\bar{L}_{i\rm L}\ell_{i\rm R}\Phi+\text{H.c.}\\ =\ &-y_{d_j}\bar{u}_{i\rm L}V_{ij}d_{j\rm R}\phi^+-\frac{y_{d_i}}{\sqrt{2}}\,\bar{d}_{i\rm L}d_{i\rm R}(v+H+\mathrm{i}\chi)-\frac{y_{u_i}}{\sqrt{2}}\,\bar{u}_{i\rm L}u_{i\rm R}(v+H-\mathrm{i}\chi)+y_{u_i}\bar{d}_{j\rm L}V^\dagger_{ji}u_{i\rm R}\phi^-\\ &-y_{\ell_i}\bar{\nu}_{i\rm L}\ell_{i\rm R}\phi^+-\frac{y_{\ell_i}}{\sqrt{2}}\,\bar{\ell}_{i\rm L}\ell_{i\rm R}(v+H+\mathrm{i}\chi)+\text{H.c.}\\ =\ &-m_{d_i}\bar{d}_{i\rm L}d_{i\rm R}-m_{u_i}\bar{u}_{i\rm L}u_{i\rm R}-m_{\ell_i}\bar{\ell}_{i\rm L}\ell_{i\rm R}-\frac{m_{d_i}}{v}\,\bar{d}_{i\rm L}d_{i\rm R}(H+\mathrm{i}\chi)-\frac{m_{u_i}}{v}\,\bar{u}_{i\rm L}u_{i\rm R}(H-\mathrm{i}\chi)\\ &-\frac{m_{\ell_i}}{v}\,\bar{\ell}_{i\rm L}\ell_{i\rm R}(H+\mathrm{i}\chi)-\frac{\sqrt{2}m_{d_j}}{v}\,\bar{u}_{i\rm L}V_{ij}d_{j\rm R}\phi^++\frac{\sqrt{2}m_{u_i}}{v}\,\bar{d}_{j\rm L}V^\dagger_{ji}u_{i\rm R}\phi^--\frac{\sqrt{2}m_{\ell_i}}{v}\,\bar{\nu}_{i\rm L}\ell_{i\rm R}\phi^++\text{H.c.}\\ =\ &-m_{d_i}\bar{d}_id_i-m_{u_i}\bar{u}_iu_i-m_{\ell_i}\bar{\ell}_i\ell_i-\frac{m_{d_i}}{v}\,H\bar{d}_id_i-\frac{m_{u_i}}{v}\,H\bar{u}_iu_i-\frac{m_{\ell_i}}{v}\,H\bar{\ell}_i\ell_i\\ &-\frac{m_{d_i}}{v}\,\chi\bar{d}_i\mathrm{i}\gamma^5d_i+\frac{m_{u_i}}{v}\,\chi\bar{u}_i\mathrm{i}\gamma^5u_i-\frac{m_{\ell_i}}{v}\,\chi\bar{\ell}_i\mathrm{i}\gamma^5\ell_i+\frac{\sqrt{2}V_{ij}}{v}\,\phi^+\bar{u}_i(m_{u_i}P_{\rm L}-m_{d_j}P_{\rm R})d_j\\ &-\frac{\sqrt{2}V^\dagger_{ji}}{v}\,\phi^-\bar{d}_j(m_{d_j}P_{\rm L}-m_{u_i}P_{\rm R})u_i-\frac{\sqrt{2}m_{\ell_i}}{v}\,(\phi^+\bar{\nu}_iP_{\rm R}\ell_i+\phi^-\bar{\ell}_iP_{\rm L}\nu_i).\end{aligned} \tag{154}$$

利用

$$D_\mu\Phi=\begin{pmatrix}\partial_\mu-\mathrm{i}eA_\mu-\dfrac{\mathrm{i}g}{2c_{\rm W}}(c_{\rm W}^2-s_{\rm W}^2)Z_\mu & -\dfrac{\mathrm{i}g}{\sqrt{2}}\,W_\mu^+\\ -\dfrac{\mathrm{i}g}{\sqrt{2}}\,W_\mu^- & \partial_\mu+\dfrac{\mathrm{i}g}{2c_{\rm W}}\,Z_\mu\end{pmatrix}\begin{pmatrix}\phi^+\\ \dfrac{1}{\sqrt{2}}(v+H+\mathrm{i}\chi)\end{pmatrix}$$

$$
= \begin{pmatrix} \partial_\mu\phi^+ - \mathrm{i}\left[eA_\mu + \dfrac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ - \dfrac{\mathrm{i}g}{2}W_\mu^+(H+\mathrm{i}\chi) - \mathrm{i}m_W W_\mu^+ \\ \dfrac{1}{\sqrt{2}}\left[\partial_\mu(H+\mathrm{i}\chi) - \mathrm{i}gW_\mu^-\phi^+ + \dfrac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu(H+\mathrm{i}\chi) + \mathrm{i}m_Z Z_\mu\right] \end{pmatrix}, \tag{155}
$$

将 Higgs 场协变动能项化为

$$
\begin{aligned}
&(D^\mu\Phi)^\dagger D_\mu\Phi \\
&= \left|\partial_\mu\phi^+ - \mathrm{i}\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ - \frac{\mathrm{i}g}{2}W_\mu^+(H+\mathrm{i}\chi) - \mathrm{i}m_W W_\mu^+\right|^2 \\
&\quad + \frac{1}{2}\left|\partial_\mu(H+\mathrm{i}\chi) - \mathrm{i}gW_\mu^-\phi^+ + \frac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu(H+\mathrm{i}\chi) + \mathrm{i}m_Z Z_\mu\right|^2 \\
&= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi) \\
&\quad + \left(-\mathrm{i}\partial^\mu\phi^-\left\{\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H+\mathrm{i}\chi) + m_W W_\mu^+\right\} + \mathrm{H.c.}\right) \\
&\quad + \left\{-\frac{\mathrm{i}}{2}\partial^\mu(H-\mathrm{i}\chi)\left[gW_\mu^-\phi^+ - \frac{g}{2c_\mathrm{W}}Z_\mu(H+\mathrm{i}\chi) - m_Z Z_\mu\right] + \mathrm{H.c.}\right\} \\
&\quad + \left|\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H+\mathrm{i}\chi) + m_W W_\mu^+\right|^2 \\
&\quad + \frac{1}{2}\left|gW_\mu^-\phi^+ - \frac{g}{2c_\mathrm{W}}Z_\mu(H+\mathrm{i}\chi) - m_Z Z_\mu\right|^2 \\
&= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi) \\
&\quad + m_W^2 W^{-\mu}W_\mu^+ + \frac{1}{2}m_Z^2 Z^\mu Z_\mu + gm_W H W_\mu^+ W^{-\mu} + \frac{gm_Z}{2c_\mathrm{W}}HZ^\mu Z_\mu \\
&\quad + \frac{g}{2}[\mathrm{i}W_\mu^+\phi^-\overleftrightarrow{\partial^\mu}(H+\mathrm{i}\chi) + \mathrm{H.c.}] + \mathrm{i}eA_\mu\phi^-\overleftrightarrow{\partial^\mu}\phi^+ + \frac{g}{2c_\mathrm{W}}Z_\mu[-\chi\overleftrightarrow{\partial^\mu}H + \mathrm{i}(c_\mathrm{W}^2 - s_\mathrm{W}^2)\phi^-\overleftrightarrow{\partial^\mu}\phi^+] \\
&\quad + \frac{g^2}{4}W_\mu^+ W^{-\mu}(2\phi^+\phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu\phi^+\phi^- \\
&\quad + \frac{g^2}{4c_\mathrm{W}^2}Z_\mu Z^\mu\left[(c_\mathrm{W}^2 - s_\mathrm{W}^2)^2\phi^+\phi^- + \frac{1}{2}H^2 + \frac{1}{2}\chi^2\right] \\
&\quad + \left[\frac{eg}{2}W_\mu^+ A^\mu\phi^-(H+\mathrm{i}\chi) - \frac{g^2 s_\mathrm{W}^2}{2c_\mathrm{W}}W_\mu^+ Z^\mu\phi^-(H+\mathrm{i}\chi) + \mathrm{H.c.}\right] + \frac{eg(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{c_\mathrm{W}}A_\mu Z^\mu\phi^+\phi^- \\
&\quad + (em_W A^\mu\phi^+ W_\mu^- - gs_\mathrm{W}^2 m_Z Z^\mu\phi^+ W_\mu^- + \mathrm{H.c.}) + \mathcal{L}_{\mathrm{b1}},
\end{aligned} \tag{156}
$$

其中

$$
\mathcal{L}_{\mathrm{b1}} = -\mathrm{i}m_W(\partial^\mu\phi^-)W_\mu^+ + \mathrm{i}m_W(\partial^\mu\phi^+)W_\mu^- + m_Z(\partial^\mu\chi)Z_\mu. \tag{157}
$$

在 R$_\xi$ 规范下，将规范固定函数设为

$$
G^\pm = \frac{1}{\sqrt{\xi_W}}(\partial^\mu W_\mu^\pm \mp \mathrm{i}\xi_W m_W\phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi_Z}}(\partial^\mu Z_\mu - \xi_Z m_Z\chi), \quad G^\gamma = \frac{1}{\sqrt{\xi_\gamma}}\partial^\mu A_\mu, \tag{158}
$$

它们在路径积分量子化中的泛函积分形式为

$$
\begin{aligned}
&\int \mathcal{D}\omega^+ \int \mathcal{D}\omega^- \int \mathcal{D}\omega^Z \int \mathcal{D}\omega^\gamma \exp\left[-\mathrm{i}\int d^4x \left(\omega^+\omega^- + \frac{1}{2}\omega^Z\omega^Z + \frac{1}{2}\omega^\gamma\omega^\gamma\right)\right] \\
&\quad \times\delta(G^+ - \omega^+)\delta(G^- - \omega^-)\delta(G^Z - \omega^Z)\delta(G^\gamma - \omega^\gamma) \\
=\ & \exp\left[-\mathrm{i}\int \mathrm{d}^4x \left(G^+G^- + \frac{1}{2}G^ZG^Z + \frac{1}{2}G^\gamma G^\gamma\right)\right]. \qquad (159)
\end{aligned}
$$

由此得到拉氏量中的规范固定项

$$
\begin{aligned}
&\mathcal{L}_{\mathrm{EW,GF}} = -G^+G^- - \frac{1}{2}(G^Z)^2 - \frac{1}{2}(G^\gamma)^2 \\
=\ & -\frac{1}{\xi_W}(\partial^\mu W_\mu^+ - \mathrm{i}\xi_W m_W \phi^+)(\partial^\nu W_\nu^- + \mathrm{i}\xi_W m_W \phi^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu - \xi_Z m_Z \chi)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2 \\
=\ & -\frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2 - \xi_W m_W^2 \phi^+\phi^- - \frac{1}{2}\xi_Z m_Z^2 \chi^2 + \mathcal{L}_{\mathrm{b2}}. \qquad (160)
\end{aligned}
$$

可见，Nambu-Goldstone 玻色子 $\phi^\pm$ 和 χ 在 R$_\xi$ 规范下具有依赖于 ξ_W 和 ξ_Z 的非物理质量，

$$
m_\phi = \sqrt{\xi_W}\, m_W, \quad m_\chi = \sqrt{\xi_Z}\, m_Z. \qquad (161)
$$

这里

$$
\mathcal{L}_{\mathrm{b2}} = -\mathrm{i}m_W \phi^-(\partial^\mu W_\mu^+) + \mathrm{i}m_W \phi^+ \partial^\mu W_\mu^- + m_Z \chi \partial^\mu Z_\mu. \qquad (162)
$$

由于

$$
\mathcal{L}_{\mathrm{b1}} + \mathcal{L}_{\mathrm{b2}} = -\mathrm{i}m_W \partial^\mu(\phi^- W_\mu^+) + \mathrm{i}m_W \partial^\mu(\phi^+ W_\mu^-) + m_Z \partial^\mu(\chi Z_\mu), \qquad (163)
$$

这两项体现为全散度，不会有物理效应。可见，协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 $\mathcal{L}_{\mathrm{b1}}$ 被规范固定项中的 $\mathcal{L}_{\mathrm{b2}}$ 抵消掉，这就是如此选取规范固定函数的目的。

这样一来，电弱规范场传播子相关拉氏量变成

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EW,prop}} =\ & (\partial_\mu W_\nu^+)(\partial^\nu W^{-\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) - \frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) + m_W^2 W^{-\mu}W_\mu^+ \\
& + \frac{1}{2}\left[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - \frac{1}{\xi_Z}(\partial^\mu Z_\mu)^2 + m_Z^2 Z^\mu Z_\mu\right] \\
& + \frac{1}{2}\left[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu) - \frac{1}{\xi_\gamma}(\partial^\mu A_\mu)^2\right] \\
\to\ & W_\mu^+\left[g^{\mu\nu}(\partial^2 + m_W^2) - \left(1 - \frac{1}{\xi_W}\right)\partial^\mu\partial^\nu\right]W_\nu^- \\
& + \frac{1}{2}Z_\mu\left[g^{\mu\nu}(\partial^2 + m_Z^2) - \left(1 - \frac{1}{\xi_Z}\right)\partial^\mu\partial^\nu\right]Z_\nu \\
& + \frac{1}{2}A_\mu\left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi_\gamma}\right)\partial^\mu\partial^\nu\right]A_\nu. \qquad (164)
\end{aligned}
$$

于是，光子的传播子与胶子形式类似，为

$$\frac{-\mathrm{i}}{p^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}(1-\xi_\gamma)\right]. \tag{165}$$

将 $W^\pm$ 传播子相关拉氏量变换到动量空间，得

$$-g^{\mu\nu}(p^2-m_W^2)+\left(1-\frac{1}{\xi_W}\right)p^\mu p^\nu=-\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)(p^2-m_W^2)-\frac{p^\mu p^\nu}{p^2}\frac{p^2-\xi_W m_W^2}{\xi_W}, \tag{166}$$

它的逆矩阵是

$$-\frac{1}{p^2-m_W^2}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)-\frac{\xi_W}{p^2-\xi_W m_W^2}\frac{p_\mu p_\nu}{p^2}=-\frac{1}{p^2-m_W^2}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi_W m_W^2}(1-\xi_W)\right], \tag{167}$$

这是因为由

$$\left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)\frac{p^\mu p^\nu}{p^2}=\frac{p_\rho p^\nu}{p^2}-\frac{p_\rho p^\nu}{p^2}=0,\quad \left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)=\delta_\rho{}^\nu-\frac{p_\rho p^\nu}{p^2} \tag{168}$$

得

$$\begin{aligned}&\left[-\frac{1}{p^2-m_W^2}\left(g_{\rho\mu}-\frac{p_\rho p_\mu}{p^2}\right)-\frac{\xi_W}{p^2-\xi_W m_W^2}\frac{p_\rho p_\mu}{p^2}\right]\\&\times\left[-\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)(p^2-m_W^2)-\frac{p^\mu p^\nu}{p^2}\frac{p^2-\xi_W m_W^2}{\xi}{}_W\right]\\=\ &\delta_\rho{}^\nu-\frac{p_\rho p^\nu}{p^2}+\frac{p_\rho p^\nu}{p^2}=\delta_\rho{}^\nu.\end{aligned} \tag{169}$$

从而，$W^\pm$ 传播子的形式为

$$\frac{-\mathrm{i}}{p^2-m_W^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi_W m_W^2}(1-\xi_W)\right]. \tag{170}$$

同理，Z 传播子的形式为

$$\frac{-\mathrm{i}}{p^2-m_Z^2+\mathrm{i}\epsilon}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2-\xi_Z m_Z^2}(1-\xi_Z)\right]. \tag{171}$$

电弱规范场的无穷小规范变换形式是

$$\delta W_\mu^a=\frac{1}{g}\,\partial_\mu\alpha^a+\varepsilon^{abc}W_\mu^b\alpha^c,\quad \delta B_\mu=\frac{1}{g'}\,\partial_\mu\alpha^Y. \tag{172}$$

定义

$$\alpha^\pm\equiv\frac{1}{\sqrt{2}}(\alpha^1\mp\mathrm{i}\alpha^2),\quad \alpha^Z\equiv\alpha^3-\alpha^Y,\quad \alpha^\gamma\equiv s_\mathrm{W}^2\alpha^3+c_\mathrm{W}^2\alpha^Y, \tag{173}$$

利用

$$\varepsilon^{1bc}W_\mu^b\alpha^c = W_\mu^2\alpha^3 - W_\mu^3\alpha^2, \quad \varepsilon^{2bc}W_\mu^b\alpha^c = -W_\mu^1\alpha^3 + W_\mu^3\alpha^1, \tag{174}$$

$$\pm\mathrm{i}\sqrt{2}\alpha^\pm = \pm\mathrm{i}\alpha^1 + \alpha^2, \qquad \pm\mathrm{i}\sqrt{2}W_\mu^\pm = \pm\mathrm{i}W_\mu^1 + W_\mu^2, \tag{175}$$

有

$$\begin{aligned}\varepsilon^{1bc}W_\mu^b\alpha^c \mp \mathrm{i}\varepsilon^{2bc}W_\mu^b\alpha^c &= (W_\mu^2\alpha^3 - W_\mu^3\alpha^2) \mp \mathrm{i}(-W_\mu^1\alpha^3 + W_\mu^3\alpha^1)\\ &= (W_\mu^2 \pm \mathrm{i}W_\mu^1)\alpha^3 - W_\mu^3(\alpha^2 \pm \mathrm{i}\alpha^1)\\ &= \pm\mathrm{i}\sqrt{2}W_\mu^\pm(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma) \mp \mathrm{i}\sqrt{2}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^\pm, \end{aligned}\tag{176}$$

$$\begin{aligned}\varepsilon^{3bc}W_\mu^b\alpha^c &= W_\mu^1\alpha^2 - W_\mu^2\alpha^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-)\frac{\mathrm{i}}{\sqrt{2}}(\alpha^+ - \alpha^-) - \frac{\mathrm{i}}{\sqrt{2}}(W_\mu^+ - W_\mu^-)\frac{1}{\sqrt{2}}(\alpha^+ + \alpha^-)\\ &= -\mathrm{i}(W_\mu^+\alpha^- - W_\mu^-\alpha^+). \end{aligned}\tag{177}$$

因此,

$$\begin{aligned}\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1 - \mathrm{i}\delta W_\mu^2) = \frac{1}{\sqrt{2}g}\,\partial_\mu(\alpha^1 - \mathrm{i}\alpha^2) + \frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c - \mathrm{i}\varepsilon^{2bc}W_\mu^b\alpha^c)\\ &= \frac{1}{g}\,\partial_\mu\alpha^+ - \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^+ + \mathrm{i}W_\mu^+(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma), \end{aligned}\tag{178}$$

$$\delta W_\mu^- = (\delta W_\mu^+)^\dagger = \frac{1}{g}\,\partial_\mu\alpha^- + \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^- - \mathrm{i}W_\mu^-(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma), \tag{179}$$

$$\begin{aligned}\delta Z_\mu^a &= c_\mathrm{W}\delta W_\mu^3 - s_\mathrm{W}\delta B_\mu = \frac{c_\mathrm{W}}{g}\,\partial_\mu\alpha^3 + c_\mathrm{W}\varepsilon^{3bc}W_\mu^b\alpha^c - \frac{s_\mathrm{W}}{g'}\,\partial_\mu\alpha^Y\\ &= \frac{c_\mathrm{W}}{g}\,\partial_\mu\alpha^Z - \mathrm{i}c_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+), \end{aligned}\tag{180}$$

$$\begin{aligned}\delta A_\mu &= s_\mathrm{W}\delta W_\mu^3 + c_\mathrm{W}\delta B_\mu = \frac{s_\mathrm{W}}{g}\,\partial_\mu\alpha^3 + s_\mathrm{W}\varepsilon^{3bc}W_\mu^b\alpha^c + \frac{c_\mathrm{W}}{g'}\,\partial_\mu\alpha^Y\\ &= \frac{1}{e}\,\partial_\mu\alpha^\gamma - \mathrm{i}s_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+). \end{aligned}\tag{181}$$

另一方面，根据

$$\begin{aligned}\alpha^aT^a + \alpha^YY_H &= \frac{1}{2}(\alpha^a\sigma^a + \alpha^Y) = \frac{1}{2}\begin{pmatrix}\alpha^3 + \alpha^Y & \alpha^1 - \mathrm{i}\alpha^2\\ \alpha^1 + \mathrm{i}\alpha^2 & -\alpha^3 + \alpha^Y\end{pmatrix}\\ &= \frac{1}{2}\begin{pmatrix}2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z & \sqrt{2}\alpha^+\\ \sqrt{2}\alpha^- & -\alpha^Z\end{pmatrix}, \end{aligned}\tag{182}$$

可知 Higgs 场的无穷小规范变换形式为

$$\delta\Phi = \mathrm{i}(\alpha^aT^a + \alpha^YY_H)\Phi = \frac{\mathrm{i}}{2}\begin{pmatrix}2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z & \sqrt{2}\alpha^+\\ \sqrt{2}\alpha^- & -\alpha^Z\end{pmatrix}\begin{pmatrix}\phi^+\\ \frac{1}{\sqrt{2}}(v + H + \mathrm{i}\chi)\end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{\mathrm{i}}{2}[\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H+\mathrm{i}\chi)\alpha^+] \\ \dfrac{1}{\sqrt{2}}\left[\mathrm{i}\phi^+\alpha^- - \dfrac{1}{2}(\mathrm{i}v + \mathrm{i}H - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \dfrac{1}{\sqrt{2}}(\delta H + \mathrm{i}\delta\chi) \end{pmatrix}. \tag{183}$$

利用

$$\mathrm{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \mathrm{Im}(\phi^+\alpha^-) = -\frac{\mathrm{i}}{2}(\phi^+\alpha^- - \phi^-\alpha^+), \tag{184}$$

推出

$$\delta\phi^+ = \frac{\mathrm{i}}{2}\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H+\mathrm{i}\chi)\alpha^+\}, \tag{185}$$

$$\delta\phi^- = -\frac{\mathrm{i}}{2}\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H-\mathrm{i}\chi)\alpha^-\}, \tag{186}$$

$$\delta H = \frac{1}{2}[\mathrm{i}(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v+H)\alpha^Z]. \tag{187}$$

于是，规范固定函数的无穷小规范变换为

$$\begin{aligned} \sqrt{\xi_W}\,\delta G^+ &= \partial^\mu\delta W_\mu^+ - \mathrm{i}\xi_W m_W \delta\phi^+ \\ &= \partial^\mu\left[\frac{1}{g}\,\partial_\mu\alpha^+ - \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^+ + \mathrm{i}W_\mu^+(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad + \frac{1}{2}\xi_W m_W\left\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H+\mathrm{i}\chi)\alpha^+\right\}, \end{aligned} \tag{188}$$

$$\begin{aligned} \sqrt{\xi_W}\,\delta G^- &= \partial^\mu\delta W_\mu^- + \mathrm{i}\xi_W m_W \delta\phi^- \\ &= \partial^\mu\left[\frac{1}{g}\,\partial_\mu\alpha^- + \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^- - \mathrm{i}W_\mu^-(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma)\right] \\ &\quad + \frac{1}{2}\xi_W m_W\left\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v+H-\mathrm{i}\chi)\alpha^-\right\}, \end{aligned} \tag{189}$$

$$\begin{aligned} \sqrt{\xi_Z}\,\delta G^Z &= \partial^\mu\delta Z_\mu - \xi_Z m_Z \delta\chi \\ &= \partial^\mu\left[\frac{c_\mathrm{W}}{g}\,\partial_\mu\alpha^Z - \mathrm{i}c_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right] \\ &\quad -\frac{1}{2}\xi_Z m_Z\left[\phi^+\alpha^- + \phi^-\alpha^+ - (v+H)\alpha^Z\right], \end{aligned} \tag{190}$$

$$\sqrt{\xi_\gamma}\,\delta G^\gamma = \partial^\mu\delta A_\mu = \partial^\mu\left[\frac{1}{e}\,\partial_\mu\alpha^\gamma - \mathrm{i}s_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right]. \tag{191}$$

因此,

$$\sqrt{\xi_W}g\,\frac{\delta G^+}{\delta\alpha^+} = \partial^2 + \xi_W m_W^2 - \mathrm{i}e\partial^\mu A_\mu - \mathrm{i}gc_\mathrm{W}\partial^\mu Z_\mu + \frac{1}{2}g\xi_W m_W(H+\mathrm{i}\chi), \tag{192}$$

$$\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\,\frac{\delta G^+}{\delta\alpha^Z} = \mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^+ + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\,\phi^+, \tag{193}$$

$$\sqrt{\xi_W}e\,\frac{\delta G^+}{\delta\alpha^\gamma} = \mathrm{i}e\partial^\mu W_\mu^+ + e\xi_W m_W\phi^+, \tag{194}$$

$$\sqrt{\xi_W}g\,\frac{\delta G^-}{\delta\alpha^-} = \partial^2 + \xi_W m_W^2 + \mathrm{i}e\partial^\mu A_\mu + \mathrm{i}gc_\mathrm{W}\partial^\mu Z_\mu + \frac{1}{2}g\xi_W m_W(H-\mathrm{i}\chi), \tag{195}$$

$$
\begin{aligned}
\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\frac{\delta G^-}{\delta\alpha^Z} &= -\mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^- + \frac{g(c_\mathrm{W}^2-s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\phi^-, &(196)\\
\sqrt{\xi_W}e\,\frac{\delta G^-}{\delta\alpha^\gamma} &= -\mathrm{i}e\partial^\mu W_\mu^- + e\xi_W m_W\phi^-, &(197)\\
\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^+} &= \mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^- - \frac{1}{2}g\xi_Z m_Z\phi^-, &(198)\\
\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^-} &= -\mathrm{i}gc_\mathrm{W}\partial^\mu W_\mu^+ - \frac{1}{2}g\xi_Z m_Z\phi^+, &(199)\\
\frac{\sqrt{\xi_Z}g}{c_\mathrm{W}}\frac{\delta G^Z}{\delta\alpha^Z} &= \partial^2 + \xi_Z m_Z^2 + \frac{g\xi_Z m_Z}{2c_\mathrm{W}}H, &(200)\\
\sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^+} &= \mathrm{i}e\partial^\mu W_\mu^-,\quad \sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^-} = -\mathrm{i}e\partial^\mu W_\mu^+,\quad \sqrt{\xi_\gamma}e\,\frac{\delta G^\gamma}{\delta\alpha^\gamma} = \partial^2. &(201)
\end{aligned}
$$

最后，得到以下 Faddeev-Popov 鬼场拉氏量，

$$
\begin{aligned}
\mathcal{L}_\mathrm{EWG,FP} &= -\bar\eta^+\left(\sqrt{\xi_W}g\,\frac{\delta G^+}{\delta\alpha^+}\right)\eta^+ - \bar\eta^Z\left(\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^+}\right)\eta^+ - \bar\eta^\gamma\left(\sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^+}\right)\eta^+\\
&\quad -\bar\eta^-\left(\sqrt{\xi_W}g\,\frac{\delta G^-}{\delta\alpha^-}\right)\eta^- - \bar\eta^Z\left(\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^-}\right)\eta^- - \bar\eta^\gamma\left(\sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^-}\right)\eta^-\\
&\quad -\bar\eta^Z\left(\frac{\sqrt{\xi_Z}g}{c_\mathrm{W}}\frac{\delta G^Z}{\delta\alpha^Z}\right)\eta^Z - \bar\eta^+\left(\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\frac{\delta G^+}{\delta\alpha^Z}\right)\eta^Z - \bar\eta^-\left(\frac{\sqrt{\xi_W}g}{c_\mathrm{W}}\frac{\delta G^-}{\delta\alpha^Z}\right)\eta^Z\\
&\quad -\bar\eta^\gamma\left(\sqrt{\xi_\gamma}e\,\frac{\delta G^\gamma}{\delta\alpha^\gamma}\right)\eta^\gamma - \bar\eta^+\left(\sqrt{\xi_W}e\,\frac{\delta G^+}{\delta\alpha^\gamma}\right)\eta^\gamma - \bar\eta^-\left(\sqrt{\xi_W}e\,\frac{\delta G^-}{\delta\alpha^\gamma}\right)\eta^\gamma\\
&= \bar\eta^+\left[-\partial^2 - \xi_W m_W^2 - \mathrm{i}e\overleftarrow{\partial^\mu}A_\mu - \mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi_W m_W(H+\mathrm{i}\chi)\right]\eta^+\\
&\quad +\bar\eta^Z\left(\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^- + \frac{1}{2}g\xi_Z m_Z\phi^-\right)\eta^+ + \mathrm{i}e(\partial^\mu\bar\eta^\gamma)W_\mu^-\eta^+\\
&\quad +\bar\eta^-\left[-\partial^2 - \xi_W m_W^2 + \mathrm{i}e\overleftarrow{\partial^\mu}A_\mu + \mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi_W m_W(H-\mathrm{i}\chi)\right]\eta^-\\
&\quad +\bar\eta^Z\left(-\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^+ + \frac{1}{2}g\xi_Z m_Z\phi^+\right)\eta^- - \mathrm{i}e(\partial^\mu\bar\eta^\gamma)W_\mu^+\eta^-\\
&\quad +\bar\eta^Z\left(-\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_\mathrm{W}}H\right)\eta^Z\\
&\quad +\bar\eta^+\left(\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^+ - \frac{g(c_\mathrm{W}^2-s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\phi^+\right)\eta^Z\\
&\quad +\bar\eta^-\left(-\mathrm{i}gc_\mathrm{W}\overleftarrow{\partial^\mu}W_\mu^- - \frac{g(c_\mathrm{W}^2-s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}}\phi^-\right)\eta^Z\\
&\quad -\bar\eta^\gamma\partial^2\eta^\gamma + \bar\eta^+(\mathrm{i}e\overleftarrow{\partial^\mu}W_\mu^+ - e\xi_W m_W\phi^+)\eta^\gamma + \bar\eta^-(-\mathrm{i}e\overleftarrow{\partial^\mu}W_\mu^- - e\xi_W m_W\phi^-)\eta^\gamma. \qquad (202)
\end{aligned}
$$

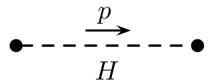
鬼粒子的质量为

$$
m_{\eta^+} = m_{\eta^-} = \sqrt{\xi_W}\,m_W,\quad m_{\eta^Z} = \sqrt{\xi_Z}\,m_Z,\quad m_{\eta^\gamma} = 0. \qquad (203)
$$

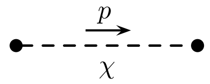
下面给出 R$_\xi$ 规范下的费曼规则。$\xi_i = 1$ 对应 Feynman-'t Hooft 规范，$\xi_i = 0$ 对应 Landau

规范，$\xi_W, \xi_Z \to \infty$ 对应幺正规范。在树图计算中，常取 $\xi_\gamma = 1$ 和 $\xi_W, \xi_Z \to \infty$。在圈图计算中，常取 $\xi_\gamma = \xi_W = \xi_Z = 1$。

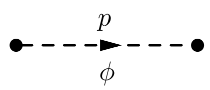
传播子：



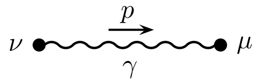
$$= \frac{\mathrm{i}}{p^2 - m_H^2 + \mathrm{i}\epsilon} \tag{204}$$



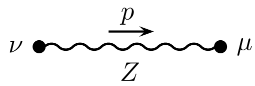
$$= \frac{\mathrm{i}}{p^2 - \xi_Z m_Z^2 + \mathrm{i}\epsilon} \tag{205}$$



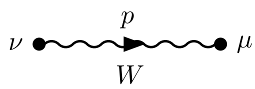
$$= \frac{\mathrm{i}}{p^2 - \xi_W m_W^2 + \mathrm{i}\epsilon} \tag{206}$$



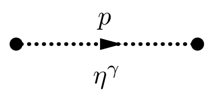
$$= \frac{-\mathrm{i}}{p^2 + \mathrm{i}\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \xi_\gamma) \right] \tag{207}$$



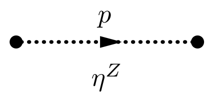
$$= \frac{-\mathrm{i}}{p^2 - m_Z^2 + \mathrm{i}\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2} (1 - \xi_Z) \right] \tag{208}$$



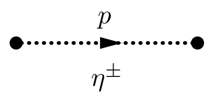
$$= \frac{-\mathrm{i}}{p^2 - m_W^2 + \mathrm{i}\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2} (1 - \xi_W) \right] \tag{209}$$



$$= \frac{\mathrm{i}}{p^2 + i\epsilon} \tag{210}$$

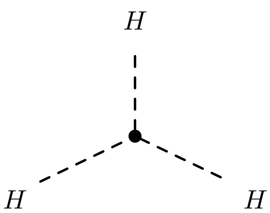


$$= \frac{\mathrm{i}}{p^2 - \xi_Z m_Z^2 + \mathrm{i}\epsilon} \tag{211}$$

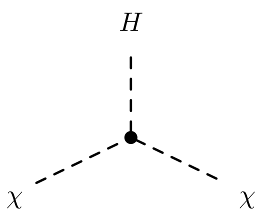


$$= \frac{\mathrm{i}}{p^2 - \xi_W m_W^2 + \mathrm{i}\epsilon} \tag{212}$$

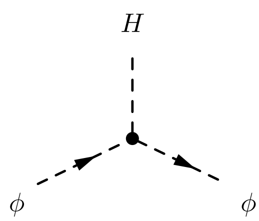
标量玻色子三线性耦合：



$$= -\frac{3\mathrm{i}m_H^2}{v} = -6\mathrm{i}\lambda v$$

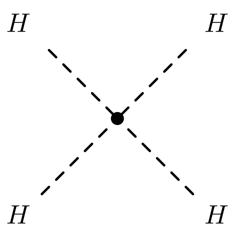


$$= -\frac{\mathrm{i}m_H^2}{v} = -2\mathrm{i}\lambda v$$

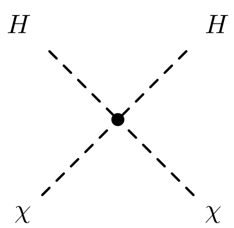


$$= -\frac{\mathrm{i}m_H^2}{v} = -2\mathrm{i}\lambda v \tag{213}$$

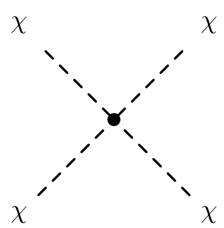
标量玻色子四线性耦合：



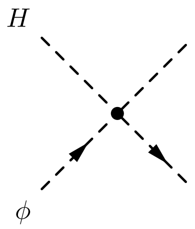
$$= -\frac{3\mathrm{i}m_H^2}{v^2} = -6\mathrm{i}\lambda$$



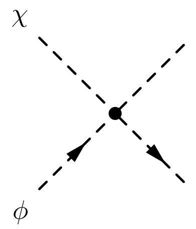
$$= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda$$



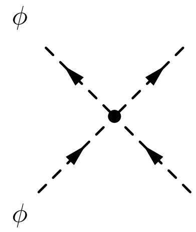
$$= -\frac{3\mathrm{i}m_H^2}{v^2} = -6\mathrm{i}\lambda \tag{214}$$



$$= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda$$

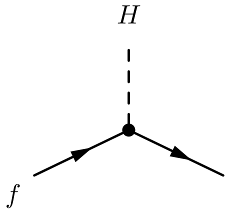


$$= -\frac{\mathrm{i}m_H^2}{v^2} = -2\mathrm{i}\lambda$$

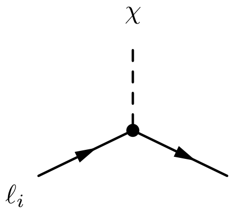


$$= -\frac{2\mathrm{i}m_H^2}{v^2} = -4\mathrm{i}\lambda \tag{215}$$

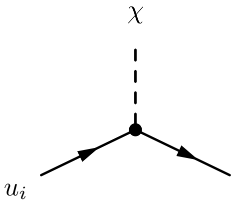
Yukawa 耦合:



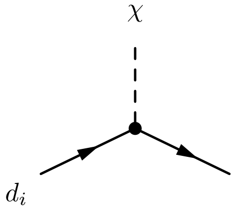
$$= -\frac{\mathrm{i}m_f}{v} = -\frac{\mathrm{i}y_f}{\sqrt{2}}$$



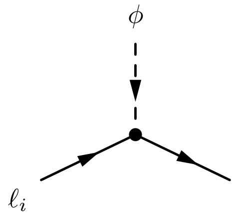
$$= \frac{m_{\ell_i}}{v}\,\gamma^5 = \frac{y_{\ell_i}}{\sqrt{2}}\,\gamma^5 \tag{216}$$



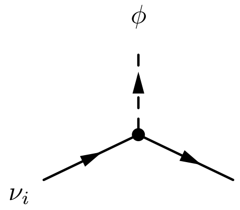
$$= -\frac{m_{u_i}}{v}\,\gamma^5 = -\frac{y_{u_i}}{\sqrt{2}}\,\gamma^5$$



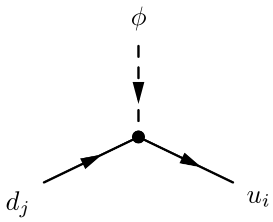
$$= \frac{m_{d_i}}{v}\,\gamma^5 = \frac{y_{d_i}}{\sqrt{2}}\,\gamma^5 \tag{217}$$



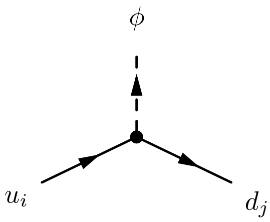
$$= -\frac{\mathrm{i}\sqrt{2}m_{\ell_i}}{v}\,P_\mathrm{R} = -\mathrm{i}y_{\ell_i}P_\mathrm{R}$$



$$= -\frac{\mathrm{i}\sqrt{2}m_{\ell_i}}{v}\,P_\mathrm{L} = -\mathrm{i}y_{\ell_i}P_\mathrm{L} \tag{218}$$

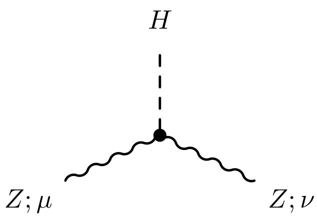


$$= \frac{\mathrm{i}\sqrt{2}V_{ij}}{v}(m_{u_i}P_\mathrm{L} - m_{d_j}P_\mathrm{R}) = \mathrm{i}V_{ij}(y_{u_i}P_\mathrm{L} - y_{d_j}P_\mathrm{R})$$

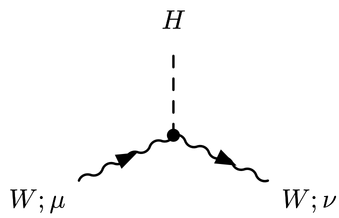


$$= -\frac{\mathrm{i}\sqrt{2}V_{ji}^\dagger}{v}(m_{d_j}P_\mathrm{L} - m_{u_i}P_\mathrm{R}) = -\mathrm{i}V_{ji}^\dagger(y_{d_j}P_\mathrm{L} - y_{u_i}P_\mathrm{R}) \tag{219}$$

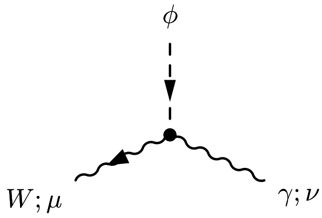
标量玻色子与电弱规范玻色子的三线性耦合:



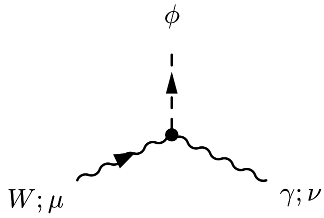
$$= \frac{\mathrm{i}gm_Z}{c_\mathrm{W}}\,g^{\mu\nu} = \frac{\mathrm{i}g^2v}{2c_\mathrm{W}^2}\,g^{\mu\nu}$$



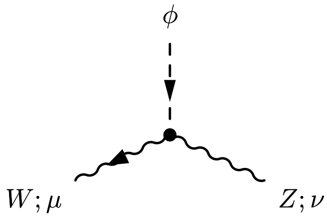
$$= \mathrm{i}gm_W g^{\mu\nu} = \frac{\mathrm{i}g^2v}{2}\,g^{\mu\nu} \tag{220}$$



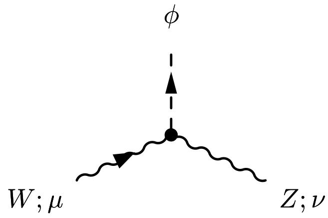
$$= \mathrm{i}em_W g^{\mu\nu} = \frac{\mathrm{i}egv}{2}\, g^{\mu\nu}$$



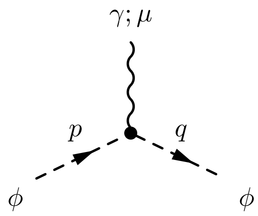
$$= \mathrm{i}em_W g^{\mu\nu} = \frac{\mathrm{i}egv}{2}\, g^{\mu\nu} \tag{221}$$



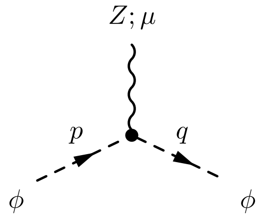
$$= -\mathrm{i}gs_{\mathrm{W}}^2 m_Z g^{\mu\nu} = -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2 v}{2c_{\mathrm{W}}}\, g^{\mu\nu}$$



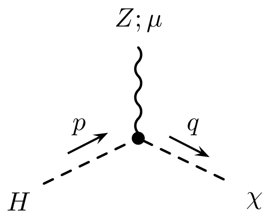
$$= -\mathrm{i}gs_{\mathrm{W}}^2 m_Z g^{\mu\nu} = -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2 v}{2c_{\mathrm{W}}}\, g^{\mu\nu} \tag{222}$$



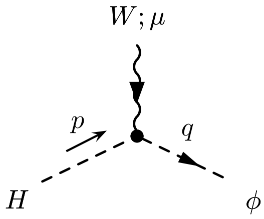
$$= \mathrm{i}e(p+q)^\mu \tag{223}$$



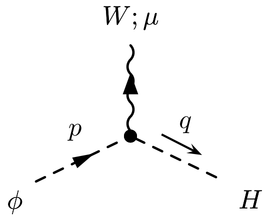
$$= \frac{\mathrm{i}g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)}{2c_{\mathrm{W}}}\,(p+q)^\mu$$



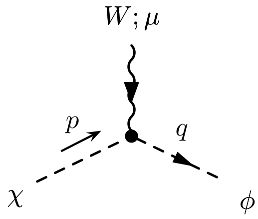
$$= -\frac{g}{2c_{\mathrm{W}}}\,(p+q)^\mu \tag{224}$$



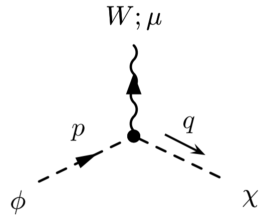
$$= \frac{\mathrm{i}g}{2}\,(p+q)^\mu$$



$$= \frac{\mathrm{i}g}{2}\,(p+q)^\mu \tag{225}$$

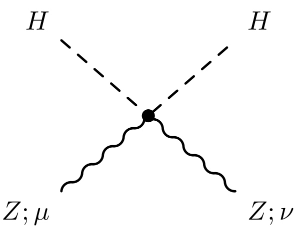


$$= -\frac{g}{2}\,(p+q)^\mu$$

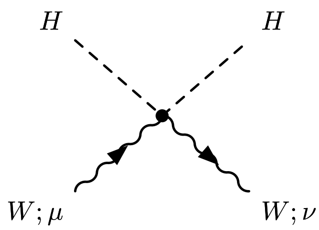


$$= \frac{g}{2}\,(p+q)^\mu \tag{226}$$

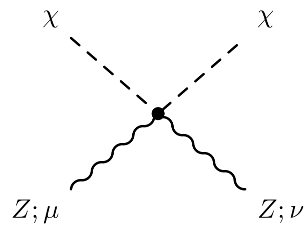
标量玻色子与电弱规范玻色子的四线性耦合:



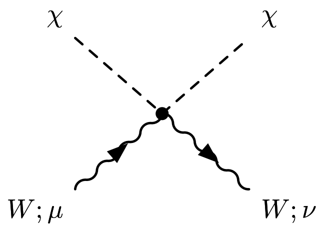
$$= \frac{\mathrm{i}g^2}{2c_{\mathrm{W}}^2}\, g^{\mu\nu}$$



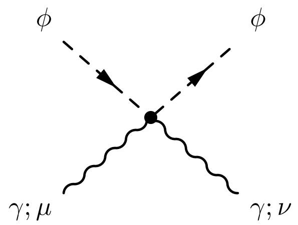
$$= \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu} \tag{227}$$



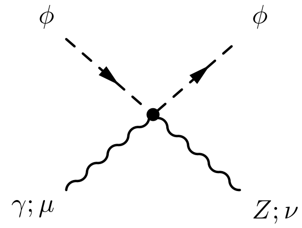
$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}\, g^{\mu\nu}$$



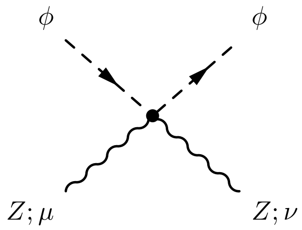
$$= \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu} \tag{228}$$



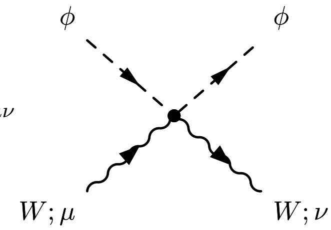
$$= 2\mathrm{i}e^2 g^{\mu\nu}$$



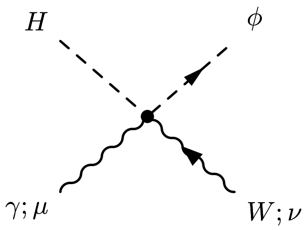
$$= \frac{\mathrm{i}eg(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{c_\mathrm{W}}\, g^{\mu\nu} \tag{229}$$



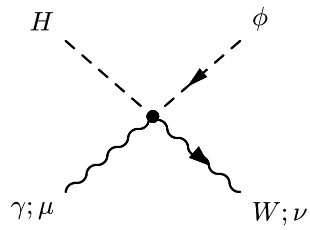
$$= \frac{\mathrm{i}g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)^2}{2c_\mathrm{W}^2}\, g^{\mu\nu}$$



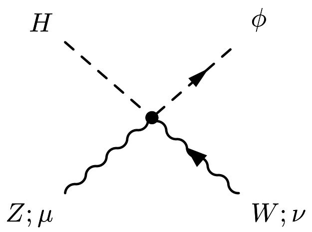
$$= \frac{\mathrm{i}g^2}{2}\, g^{\mu\nu} \tag{230}$$



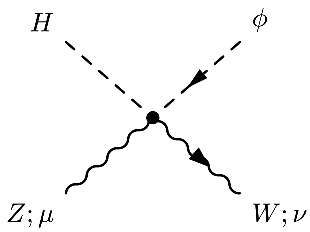
$$= \frac{\mathrm{i}eg}{2}\, g^{\mu\nu}$$



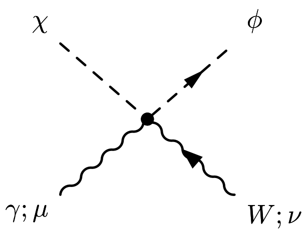
$$= \frac{\mathrm{i}eg}{2}\, g^{\mu\nu} \tag{231}$$



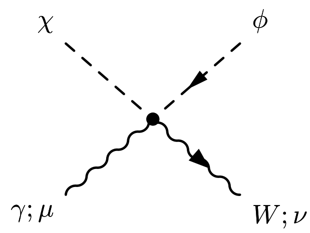
$$= -\frac{\mathrm{i}g^2 s_\mathrm{W}^2}{2c_\mathrm{W}}\, g^{\mu\nu}$$



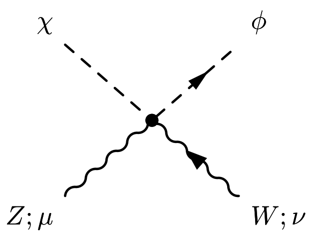
$$= -\frac{\mathrm{i}g^2 s_\mathrm{W}^2}{2c_\mathrm{W}}\, g^{\mu\nu} \tag{232}$$



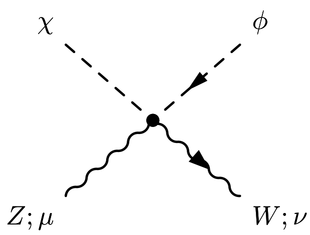
$$= -\frac{eg}{2}\, g^{\mu\nu}$$



$$= \frac{eg}{2}\, g^{\mu\nu} \tag{233}$$

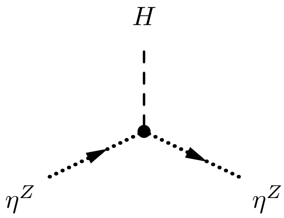


$$= \frac{g^2 s_\mathrm{W}^2}{2c_\mathrm{W}}\, g^{\mu\nu}$$

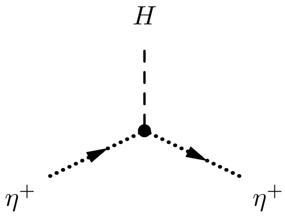


$$= -\frac{g^2 s_\mathrm{W}^2}{2c_\mathrm{W}}\, g^{\mu\nu} \tag{234}$$

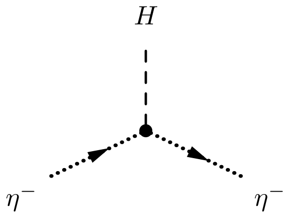
鬼粒子与标量玻色子的耦合:



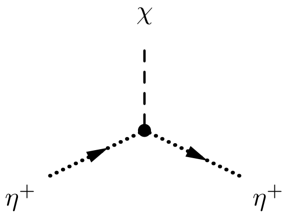
$$= -\frac{\mathrm{i}g\xi_Z m_Z}{2c_\mathrm{W}} = -\frac{\mathrm{i}g^2\xi_Z v}{4c_\mathrm{W}^2} \tag{235}$$



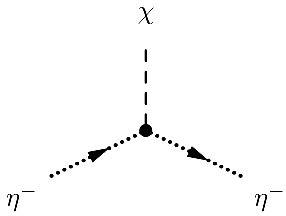
$$= -\frac{\mathrm{i}g\xi_W m_W}{2} = -\frac{\mathrm{i}g^2\xi_W v}{4}$$



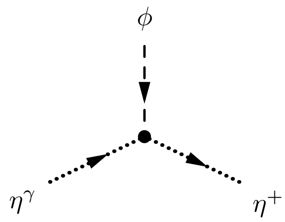
$$= -\frac{\mathrm{i}g\xi_W m_W}{2} = -\frac{\mathrm{i}g^2\xi_W v}{4} \tag{236}$$



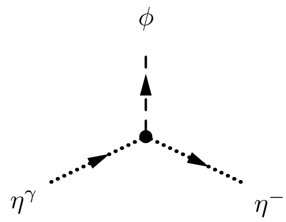
$$= \frac{g\xi_W m_W}{2} = \frac{g^2\xi_W v}{4}$$



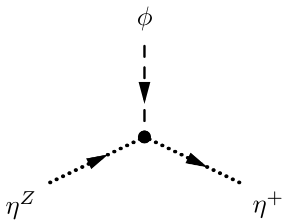
$$= -\frac{g\xi_W m_W}{2} = -\frac{g^2\xi_W v}{4} \tag{237}$$



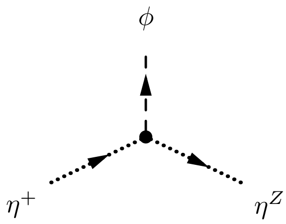
$$= -\mathrm{i}e\xi_W m_W = -\frac{\mathrm{i}eg\xi_W v}{2}$$



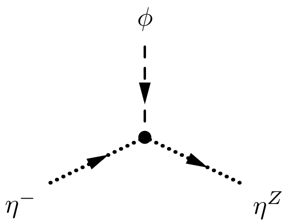
$$= -\mathrm{i}e\xi_W m_W = -\frac{\mathrm{i}eg\xi_W v}{2} \tag{238}$$



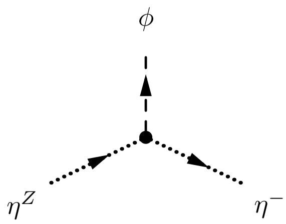
$$= -\frac{\mathrm{i}g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}} = -\frac{\mathrm{i}g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W v}{4c_\mathrm{W}}$$



$$= \frac{\mathrm{i}g\xi_Z m_Z}{2} = \frac{\mathrm{i}g^2\xi_Z v}{4c_\mathrm{W}} \tag{239}$$

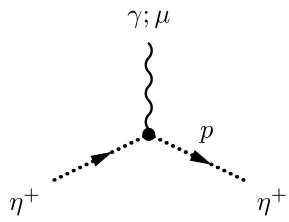


$$= \frac{\mathrm{i}g\xi_Z m_Z}{2} = \frac{\mathrm{i}g^2\xi_Z v}{4c_\mathrm{W}}$$

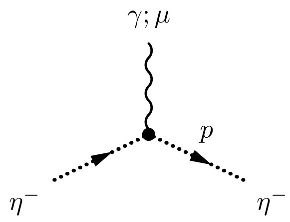


$$= -\frac{\mathrm{i}g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}} = -\frac{\mathrm{i}g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W v}{4c_\mathrm{W}} \tag{240}$$

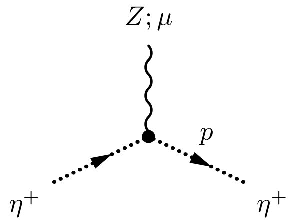
鬼粒子与电弱规范玻色子的耦合:



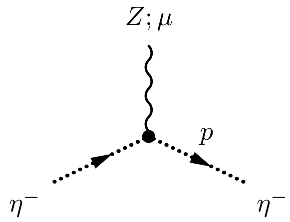
$= \mathrm{i}ep^\mu$



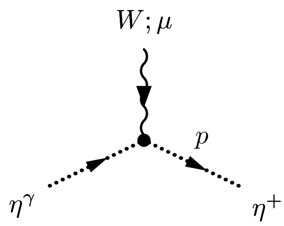
$$= -\mathrm{i}ep^\mu \tag{241}$$



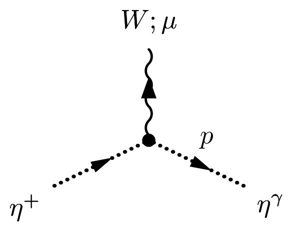
$= \mathrm{i}gc_\mathrm{W}p^\mu$



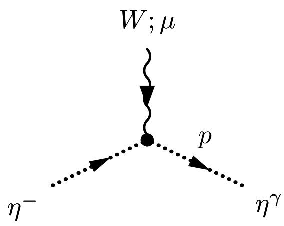
$$= -\mathrm{i}gc_\mathrm{W}p^\mu \tag{242}$$



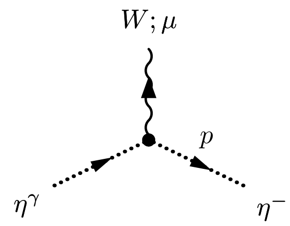
$= -\mathrm{i}ep^\mu$



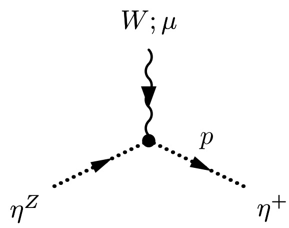
$$= -\mathrm{i}ep^\mu \tag{243}$$



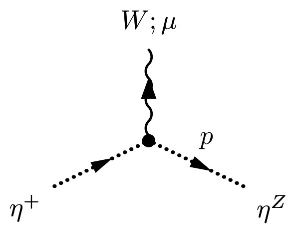
$= \mathrm{i}ep^\mu$



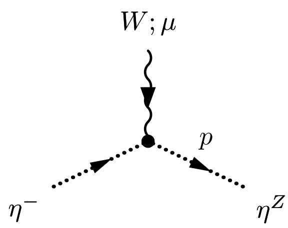
$$= \mathrm{i}ep^\mu \tag{244}$$



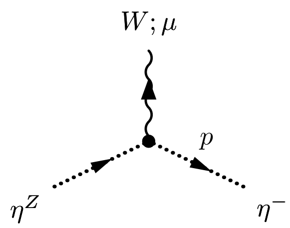
$= -\mathrm{i}gc_\mathrm{W}p^\mu$



$$= -\mathrm{i}gc_\mathrm{W}p^\mu \tag{245}$$



$= \mathrm{i}gc_\mathrm{W}p^\mu$

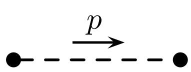


$$= \mathrm{i}gc_\mathrm{W}p^\mu \tag{246}$$

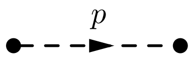
6 内外线一般费曼规则

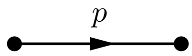
本节列出一些通用的内外线费曼规则。

内线费曼规则如下。

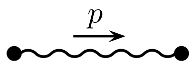
- 实标量玻色子传播子: 

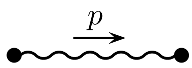
$= \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$

- 复标量玻色子传播子： $= \dfrac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$

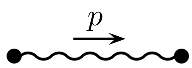
- Dirac 费米子传播子： $= \dfrac{\mathrm{i}(\not{p} + m)}{p^2 - m^2 + \mathrm{i}\epsilon}$

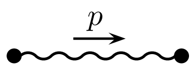
- 无质量实矢量玻色子传播子：

ν  $\mu \ = \dfrac{-\mathrm{i}g^{\mu\nu}}{p^2 + \mathrm{i}\epsilon}$ （Feynman 规范）

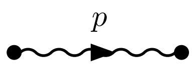
ν  $\mu \ = \dfrac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu / p^2)}{p^2 + \mathrm{i}\epsilon}$ （Landau 规范）

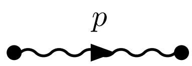
- 有质量实矢量玻色子传播子：

ν  $\mu \ = \dfrac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + \mathrm{i}\epsilon}$ （幺正规范）

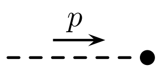
ν  $\mu \ = \dfrac{-\mathrm{i}g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\epsilon}$ （Feynman 规范）

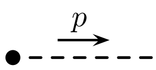
- 有质量复矢量玻色子传播子：

ν  $\mu \ = \dfrac{-\mathrm{i}(g^{\mu\nu} - p^\mu p^\nu / m^2)}{p^2 - m^2 + \mathrm{i}\epsilon}$ （幺正规范）

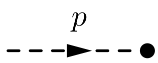
ν  $\mu \ = \dfrac{-\mathrm{i}g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\epsilon}$ （Feynman 规范）

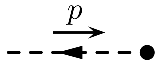
实标量场外线费曼规则如下。

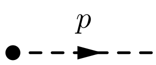
- 实标量玻色子入射外线： $= 1$

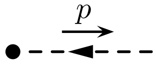
- 实标量玻色子出射外线： $= 1$

复标量场外线费曼规则如下。

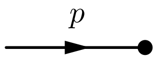
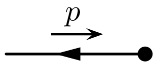
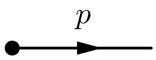
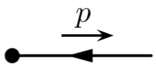
- 正标量玻色子入射外线： $= 1$

- 反标量玻色子入射外线： $= 1$

- 正标量玻色子出射外线： $= 1$

- 反标量玻色子出射外线： $= 1$

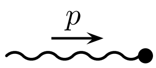
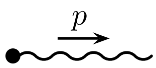
以 λ 代表自旋极化指标（如螺旋度），**Dirac 旋量场**外线费曼规则如下。

- Dirac 正费米子入射外线： λ  $= u(\mathbf{p}, \lambda)$
- Dirac 反费米子入射外线： λ  $= \bar{v}(\mathbf{p}, \lambda)$
- Dirac 正费米子出射外线：  λ $= \bar{u}(\mathbf{p}, \lambda)$
- Dirac 反费米子出射外线：  λ $= v(\mathbf{p}, \lambda)$

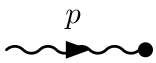
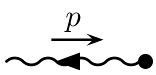
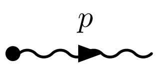
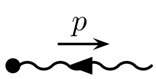
在计算非极化振幅模方时，可利用自旋求和关系

$$\sum_{\lambda} u(\mathbf{p}, \lambda)\bar{u}(\mathbf{p}, \lambda) = \not{p} + m, \quad \sum_{\lambda} v(\mathbf{p}, \lambda)\bar{v}(\mathbf{p}, \lambda) = \not{p} - m. \tag{247}$$

以 λ 代表自旋极化指标，**实矢量场**外线费曼规则如下。

- 实矢量玻色子入射外线： $\lambda; \mu$  $= \varepsilon^{\mu}(\mathbf{p}, \lambda)$
- 实矢量玻色子出射外线：  $\lambda; \mu$ $= \varepsilon^{\mu *}(\mathbf{p}, \lambda)$

复矢量场外线费曼规则如下。

- 正矢量玻色子入射外线： $\lambda; \mu$  $= \varepsilon^{\mu}(\mathbf{p}, \lambda)$
- 反矢量玻色子入射外线： $\lambda; \mu$  $= \varepsilon^{\mu}(\mathbf{p}, \lambda)$
- 正矢量玻色子出射外线：  $\lambda; \mu$ $= \varepsilon^{\mu *}(\mathbf{p}, \lambda)$
- 反矢量玻色子出射外线：  $\lambda; \mu$ $= \varepsilon^{\mu *}(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，若包含无质量矢量玻色子外线，可利用极化求和替换关系

$$\sum_{\lambda} \varepsilon_{\mu}^{*}(\mathbf{p}, \lambda)\varepsilon_{\nu}(\mathbf{p}, \lambda) \to -g_{\mu\nu}\,; \tag{248}$$

若包含有质量矢量玻色子外线，可利用极化求和关系

$$\sum_{\lambda} \varepsilon_{\mu}^{*}(\mathbf{p}, \lambda)\varepsilon_{\nu}(\mathbf{p}, \lambda) = -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{m^2}. \tag{249}$$

7 常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2020 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中，光速、约化 Planck 常数和真空介电常数均取为 1，即 $c = \hbar = \varepsilon_0 = 1$。从而，速度没有量纲 (dimension)；长度量纲与时间量纲相同，是能量量纲的倒数；能量、质量和动量具有相同的量纲；精细结构常数表达为

$$\alpha = \frac{e^2}{4\pi}, \tag{250}$$

而单位电荷量 $e = \sqrt{4\pi\alpha}$ 是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系为

$$1 \;=\; c = 2.99792458 \times 10^{10}\ \mathrm{cm} \cdot \mathrm{s}^{-1}, \tag{251}$$

$$1 \;=\; \hbar = 6.582119569 \times 10^{-25}\ \mathrm{GeV} \cdot \mathrm{s}, \tag{252}$$

$$1 \;=\; \hbar c = 1.973269804 \times 10^{-14}\ \mathrm{GeV} \cdot \mathrm{cm}, \tag{253}$$

$$1 \;=\; (\hbar c)^2 = 3.893793721 \times 10^{8}\ \mathrm{GeV}^2 \cdot \mathrm{pb}, \tag{254}$$

由此得到

$$1\ \mathrm{s} = 2.997925 \times 10^{10}\ \mathrm{cm}, \qquad 1\ \mathrm{cm} = 3.335641 \times 10^{-11}\ \mathrm{s}, \tag{255}$$

$$1\ \mathrm{s} = 1.519267 \times 10^{24}\ \mathrm{GeV}^{-1}, \quad 1\ \mathrm{GeV}^{-1} = 6.582120 \times 10^{-25}\ \mathrm{s}, \tag{256}$$

$$1\ \mathrm{cm} = 5.067731 \times 10^{13}\ \mathrm{GeV}^{-1}, \quad 1\ \mathrm{GeV}^{-1} = 1.973270 \times 10^{-14}\ \mathrm{cm}, \tag{257}$$

$$1\ \mathrm{cm}^2 = 2.568189 \times 10^{27}\ \mathrm{GeV}^{-2}, \quad 1\ \mathrm{GeV}^{-2} = 3.893794 \times 10^{-28}\ \mathrm{cm}^2, \tag{258}$$

$$1\ \mathrm{cm}^3 \cdot \mathrm{s}^{-1} = 8.566558 \times 10^{16}\ \mathrm{GeV}^{-2}, \quad 1\ \mathrm{GeV}^{-2} = 1.167330 \times 10^{-17}\ \mathrm{cm}^3 \cdot \mathrm{s}^{-1}. \tag{259}$$

靶 (barn) 是散射截面的常用单位，记作 b，满足

$$1\ \mathrm{b} \;=\; 10^{-24}\ \mathrm{cm}^2 = 10^{9}\ \mathrm{nb} = 10^{12}\ \mathrm{pb} = 10^{15}\ \mathrm{fb} = 10^{18}\ \mathrm{ab}, \tag{260}$$

$$1\ \mathrm{pb} \;=\; 10^{-36}\ \mathrm{cm}^2 = 2.568189 \times 10^{-9}\ \mathrm{GeV}^{-2}, \quad 1\ \mathrm{GeV}^{-2} = 3.893794 \times 10^{8}\ \mathrm{pb}. \tag{261}$$

Fermi 耦合常数是

$$G_\mathrm{F} = 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2}, \tag{262}$$

括号内数字代表测量值的 1σ 不确定度，由树图阶关系式

$$\frac{G_\mathrm{F}}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_\mathrm{W}^2}, \tag{263}$$

得到 Higgs 场真空期待值为

$$v = (\sqrt{2}G_{\rm F})^{-1/2} = 246.2197\ \text{GeV}. \tag{264}$$

在低能标（Thomson 极限）处，精细结构常数为

$$\alpha = 7.2973525693(11) \times 10^{-3} = \frac{1}{137.035999084(21)}; \tag{265}$$

在 $\overline{\rm MS}$ 重整化方案（以 ^ 为标志）中，α^{-1} 跑动到 $\mu = m_Z$ 能标处的数值是

$$\widehat{\alpha}^{-1}(m_Z) = 127.952 \pm 0.009\,. \tag{266}$$

在 $\overline{\rm MS}$ 方案中，$\mu = m_Z$ 能标处强耦合常数 $\alpha_{\rm s} = g_{\rm s}^2/(4\pi)$ 的数值为

$$\widehat{\alpha}_{\rm s}(m_Z) = 0.1179 \pm 0.0010, \tag{267}$$

Weinberg 角 $\theta_{\rm W}$ 的数值对应于

$$\widehat{s}_{\rm W}^2 = \sin^2\widehat{\theta}_{\rm W}(m_Z) = 0.23121 \pm 0.00004\,. \tag{268}$$

在标准模型中，光子、胶子和中微子没有质量，其它基本粒子的质量为

$$m_W = 80.379 \pm 0.012\ \text{GeV}, \qquad m_Z = 91.1876 \pm 0.0021\ \text{GeV}, \tag{269}$$

$$m_H = 125.10 \pm 0.14\ \text{GeV}, \qquad m_e = 0.51099895000(15)\ \text{MeV}, \tag{270}$$

$$m_\mu = 105.6583745(24)\ \text{MeV}, \qquad m_\tau = 1776.86 \pm 0.12\ \text{MeV}, \tag{271}$$

$$m_u = 2.16^{+0.49}_{-0.26}\ \text{MeV}, \qquad m_d = 4.67^{+0.48}_{-0.17}\ \text{MeV}, \tag{272}$$

$$m_s = 93^{+11}_{-5}\ \text{MeV}, \qquad m_c = 1.67 \pm 0.07\ \text{GeV}, \tag{273}$$

$$m_b = 4.78 \pm 0.06\ \text{GeV}, \qquad m_t = 172.76 \pm 0.30\ \text{GeV}. \tag{274}$$

这里，u、d、s 夸克的质量是 $\mu \simeq 2$ GeV 能标处的流夸克质量 (current-quark mass)，其余粒子的质量均为极点质量 (pole mass)。c、b 夸克在 $\overline{\rm MS}$ 方案中的跑动质量 (running mass) 为

$$\bar{m}_c(\bar{m}_c) = 1.27 \pm 0.02\ \text{GeV}, \quad \bar{m}_b(\bar{m}_b) = 4.18^{+0.03}_{-0.02}\ \text{GeV}. \tag{275}$$

质子和中子的质量为

$$m_p = 938.27208816(29)\ \text{MeV}, \quad m_n = 939.56542052(54)\ \text{MeV}. \tag{276}$$

在电弱能标附近作领头阶计算时，可将单位电荷量取为

$$e = \sqrt{4\pi\widehat{\alpha}(m_Z)} = 0.3133873, \tag{277}$$

将强耦合常数取为

$$g_{\mathrm{s}} = \sqrt{4\pi\widehat{\alpha}_{\mathrm{s}}(m_Z)} = 1.217200\,. \tag{278}$$

从树图阶关系计算 Higgs 场四线性耦合常数 λ 和 Yukawa 耦合常数 y_t、y_b、y_τ、y_c，得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1290741, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9922828, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.745492\times 10^{-2}, \tag{279}$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020576\times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 9.591991\times 10^{-3}. \tag{280}$$

耦合常数 g 和 g' 有以下两种取值方式。

1. 根据树图阶关系 $\sin^2\theta_{\mathrm{W}} = 1 - m_W^2/m_Z^2$ 计算 Weinberg 角，得

$$s_{\mathrm{W}}^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2230132, \quad c_{\mathrm{W}}^2 = 1 - s_{\mathrm{W}}^2 = 0.7769868, \tag{281}$$

$$s_{\mathrm{W}} = \sqrt{s_{\mathrm{W}}^2} = 0.4722428, \qquad c_{\mathrm{W}} = \sqrt{c_{\mathrm{W}}^2} = 0.8814685, \tag{282}$$

   故

$$g = \frac{e}{s_{\mathrm{W}}} = 0.6636148, \quad g' = \frac{e}{c_{\mathrm{W}}} = 0.3555286\,. \tag{283}$$

2. 根据 $\overline{\mathrm{MS}}$ 方案中 Weinberg 角的数值 (268) 计算 g 和 g'，得

$$c_{\mathrm{W}}^2 = 1 - \widehat{s}_{\mathrm{W}}^2 = 0.76879, \quad s_{\mathrm{W}} = \sqrt{\widehat{s}_{\mathrm{W}}^2} = 0.4808430, \quad c_{\mathrm{W}} = \sqrt{c_{\mathrm{W}}^2} = 0.8768067, \tag{284}$$

$$g = \frac{e}{s_{\mathrm{W}}} = 0.6517456, \quad g' = \frac{e}{c_{\mathrm{W}}} = 0.3574189\,. \tag{285}$$

CKM 矩阵元 V_{ij} 之模的测量值为

$$|V_{ij}| = \begin{pmatrix} 0.97401\pm 0.00011 & 0.22650\pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636\pm 0.00048 & 0.97320\pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.00024}_{-0.00035} \end{pmatrix}. \tag{286}$$

如果只近似地讨论第一、二代夸克的混合，可利用 Cabibbo 转动角 θ_{C} 将 CKM 矩阵表达为

$$V \simeq \begin{pmatrix} \cos\theta_{\mathrm{C}} & \sin\theta_{\mathrm{C}} & \\ -\sin\theta_{\mathrm{C}} & \cos\theta_{\mathrm{C}} & \\ & & 1 \end{pmatrix}, \quad \sin\theta_{\mathrm{C}} = |V_{12}| = 0.22650\,. \tag{287}$$

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