Dirac/Majorana $\leftrightarrow$ Weyl

Ref: S. P. Martin, hep-ph/9709356

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}, \quad P_L = \frac{1}{2}(1-\gamma_5) = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \quad P_R = \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$$

$$\text{Dirac spinor}: \quad \psi_D = \begin{pmatrix} \xi \\ \eta^\dagger \end{pmatrix}, \quad \bar\psi_D = \psi^\dagger\gamma^0 = \begin{pmatrix} \xi^\dagger & \eta \end{pmatrix}\begin{pmatrix} & 1 \\ 1 & \end{pmatrix} = \begin{pmatrix} \eta & \xi^\dagger \end{pmatrix}$$

$$\xi\eta = \eta\xi, \quad \xi^\dagger\bar\sigma^\mu\eta = (\eta^\dagger\bar\sigma^\mu\xi)^\dagger = -\eta\sigma^\mu\xi^\dagger$$

$$\mathcal{L}_{\text{Dirac}} = \bar\psi_D(i\gamma^\mu\partial_\mu - m)\psi_D = \begin{pmatrix} \eta & \xi^\dagger \end{pmatrix}\begin{pmatrix} -m & i\sigma^\mu\partial_\mu \\ i\bar\sigma^\mu\partial_\mu & -m \end{pmatrix}\begin{pmatrix} \xi \\ \eta^\dagger \end{pmatrix}$$

$$= i\eta\sigma^\mu\partial_\mu\eta^\dagger + i\xi^\dagger\bar\sigma^\mu\partial_\mu\xi - m(\eta\xi + \xi^\dagger\eta^\dagger)$$

$$= i\partial_\mu(\eta\sigma^\mu\eta^\dagger) - i(\partial_\mu\eta)\sigma^\mu\eta^\dagger + i\xi^\dagger\bar\sigma^\mu\partial_\mu\xi - m(\eta\xi + \xi^\dagger\eta^\dagger)$$

$$\to i\xi^\dagger\bar\sigma^\mu\partial_\mu\xi + i\eta^\dagger\bar\sigma^\mu\partial_\mu\eta - m(\xi\eta + \xi^\dagger\eta^\dagger)$$

$$\text{Majorana spinor}: \quad \psi_M = \begin{pmatrix} \xi \\ \xi^\dagger \end{pmatrix}, \quad \bar\psi_M = \begin{pmatrix} \xi & \xi^\dagger \end{pmatrix}$$

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2}\bar\psi_M(i\gamma^\mu\partial_\mu - m)\psi_M \to i\xi^\dagger\bar\sigma^\mu\partial_\mu\xi - \frac{1}{2}m(\xi\xi + \xi^\dagger\xi^\dagger)$$

$$(\chi_M)_j = \begin{pmatrix} \chi_j^0 \\ (\chi_j^0)^\dagger \end{pmatrix}, \quad (\bar\chi_M)_i = \begin{pmatrix} \chi_i^0 & (\chi_i^0)^\dagger \end{pmatrix}$$

$$(\bar\chi_M)_i\gamma^\mu(\chi_M)_j = \begin{pmatrix} \chi_i^0 & (\chi_i^0)^\dagger \end{pmatrix}\begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix}\begin{pmatrix} \chi_j^0 \\ (\chi_j^0)^\dagger \end{pmatrix}$$

$$= \chi_i^0\sigma^\mu(\chi_j^0)^\dagger + (\chi_i^0)^\dagger\bar\sigma^\mu\chi_j^0 = -(\chi_j^0)^\dagger\bar\sigma^\mu\chi_i^0 + (\chi_i^0)^\dagger\bar\sigma^\mu\chi_j^0$$

$$(\bar\chi_M)_i\gamma^\mu(\chi_M)_i = 0$$

$$(\bar\chi_M)_i\gamma^\mu\gamma_5(\chi_M)_j = \begin{pmatrix} \chi_i^0 & (\chi_i^0)^\dagger \end{pmatrix}\begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix}\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}\begin{pmatrix} \chi_j^0 \\ (\chi_j^0)^\dagger \end{pmatrix}$$

$$= \chi_i^0\sigma^\mu(\chi_j^0)^\dagger - (\chi_i^0)^\dagger\bar\sigma^\mu\chi_j^0 = -(\chi_j^0)^\dagger\bar\sigma^\mu\chi_i^0 - (\chi_i^0)^\dagger\bar\sigma^\mu\chi_j^0$$

$$(\bar\chi_M)_i\gamma^\mu\gamma_5(\chi_M)_i = -2(\chi_i^0)^\dagger\bar\sigma^\mu\chi_i^0$$

$$(\bar\chi_M)_i(\chi_M)_j = \begin{pmatrix} \chi_i^0 & (\chi_i^0)^\dagger \end{pmatrix}\begin{pmatrix} \chi_j^0 \\ (\chi_j^0)^\dagger \end{pmatrix} = \chi_i^0\chi_j^0 + (\chi_i^0)^\dagger(\chi_j^0)^\dagger$$

$$(\bar\chi_M)_i i\gamma_5(\chi_M)_j = \begin{pmatrix} \chi_i^0 & (\chi_i^0)^\dagger \end{pmatrix}\begin{pmatrix} -i & \\ & i \end{pmatrix}\begin{pmatrix} \chi_j^0 \\ (\chi_j^0)^\dagger \end{pmatrix} = -i\chi_i^0\chi_j^0 + i(\chi_i^0)^\dagger(\chi_j^0)^\dagger$$

Triplet-quadruplet fermionic dark matter

2-component Weyl spinors: T, Q_1, Q_2

$$\mathcal{L}_{\mathrm{T}} = iT^\dagger\bar{\sigma}^\mu D_\mu T - \frac{1}{2}m_T(TT + T^\dagger T^\dagger), \quad \mathcal{L}_{\mathrm{Q}} = iQ_1^\dagger\bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger\bar{\sigma}^\mu D_\mu Q_2 - m_Q(Q_1Q_2 + Q_1^\dagger Q_2^\dagger)$$

$$\mathcal{L}_{\mathrm{HTQ}} = y_1 Q_1 TH - y_2 Q_2 TH^\dagger + h.c.$$

$$\left(\mathbf{2}, \frac{1}{2}\right): \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}; \quad (\mathbf{3}, 0): \quad T = \begin{pmatrix} T^+ \\ T^0 \\ T^- \end{pmatrix}, \quad \left(\mathbf{4}, -\frac{1}{2}\right): \quad Q_1 = \begin{pmatrix} Q_1^+ \\ Q_1^0 \\ Q_1^- \\ Q_1^{--} \end{pmatrix}, \quad \left(\mathbf{4}, \frac{1}{2}\right): \quad Q_2 = \begin{pmatrix} Q_2^{++} \\ Q_2^+ \\ Q_2^0 \\ Q_2^- \end{pmatrix}$$

$$T^i_j = u^i v_j - \frac{1}{2}\delta^i_j u^k v_k$$

$$T^+ = T^1_2, \quad T^- = T^2_1, \quad T^0 = \sqrt{2}T^1_1 = -\sqrt{2}T^2_2$$

$$(T^+)^\dagger = (T^\dagger)^2_1, \quad (T^-)^\dagger = (T^\dagger)^1_2, \quad (T^0)^\dagger = \sqrt{2}(T^\dagger)^1_1 = -\sqrt{2}(T^\dagger)^2_2$$

$$TT = T^j_i T^i_j = T^2_1 T^1_2 + T^1_2 T^2_1 + T^1_1 T^1_1 + T^2_2 T^2_2 = 2T^-T^+ + T^0T^0$$

$$\left(\begin{array}{l} \text{Note: since } \varepsilon^{12} = +1, \varepsilon_{12} = -1, \\ \varepsilon^{il}\varepsilon_{jk} T^j_i T^k_l = \varepsilon^{12}\varepsilon_{12} T^1_1 T^2_2 + \varepsilon^{12}\varepsilon_{21} T^2_1 T^1_2 + \varepsilon^{21}\varepsilon_{12} T^1_2 T^2_1 + \varepsilon^{21}\varepsilon_{21} T^2_2 T^1_1 = -T^1_1 T^2_2 + T^2_1 T^1_2 + T^1_2 T^2_1 - T^2_2 T^1_1 = 2T^-T^+ + T^0T^0 = T^j_i T^i_j \end{array}\right)$$

$$-\frac{1}{2}m_T(TT + T^\dagger T^\dagger) = -m_T T^-T^+ - \frac{1}{2}m_T T^0T^0 + h.c.$$

$$iT^\dagger\bar{\sigma}^\mu\partial_\mu T = i(T^\dagger)^j_i\bar{\sigma}^\mu\partial_\mu T^i_j = i(T^\dagger)^2_1\bar{\sigma}^\mu\partial_\mu T^1_2 + i(T^\dagger)^1_2\bar{\sigma}^\mu\partial_\mu T^2_1 + i(T^\dagger)^1_1\bar{\sigma}^\mu\partial_\mu T^1_1 + i(T^\dagger)^2_2\bar{\sigma}^\mu\partial_\mu T^2_2$$

$$= i(T^+)^\dagger\bar{\sigma}^\mu\partial_\mu T^+ + i(T^-)^\dagger\bar{\sigma}^\mu\partial_\mu T^- + i(T^0)^\dagger\bar{\sigma}^\mu\partial_\mu T^0$$

$$Q^{ij}_k = \frac{1}{2}\left(T'^i_k w^j + T'^j_k w^i - \frac{1}{3}\delta^i_k T'^j_l w^l - \frac{1}{3}\delta^j_k T'^i_l w^l\right)$$

$$Q_1^+ = (Q_1)^{11}_2, \quad Q_1^0 = \sqrt{3}(Q_1)^{11}_1 = -\sqrt{3}(Q_1)^{12}_2 = -\sqrt{3}(Q_1)^{21}_2, \quad Q_1^- = \sqrt{3}(Q_1)^{22}_2 = -\sqrt{3}(Q_1)^{12}_1 = -\sqrt{3}(Q_1)^{21}_1, \quad Q_1^{--} = (Q_1)^{22}_1$$

$$Q_2^{++} = (Q_2)^{11}_2, \quad Q_2^+ = \sqrt{3}(Q_2)^{11}_1 = -\sqrt{3}(Q_2)^{12}_2 = -\sqrt{3}(Q_2)^{21}_2, \quad Q_2^0 = \sqrt{3}(Q_2)^{22}_2 = -\sqrt{3}(Q_2)^{12}_1 = -\sqrt{3}(Q_2)^{21}_1, \quad Q_2^- = (Q_2)^{22}_1$$

$$\varepsilon_{il}(Q_1)^{ij}_k(Q_2)^{lk}_j = \varepsilon_{12}(Q_1)^{11}_1(Q_2)^{21}_1 + \varepsilon_{12}(Q_1)^{11}_2(Q_2)^{22}_1 + \varepsilon_{12}(Q_1)^{12}_1(Q_2)^{21}_2 + \varepsilon_{12}(Q_1)^{12}_2(Q_2)^{22}_2$$

$$+\varepsilon_{21}(Q_1)^{21}_1(Q_2)^{11}_1 + \varepsilon_{21}(Q_1)^{21}_2(Q_2)^{12}_1 + \varepsilon_{21}(Q_1)^{22}_1(Q_2)^{11}_2 + \varepsilon_{21}(Q_1)^{22}_2(Q_2)^{12}_2$$

$$= -(Q_1)^{11}_1(Q_2)^{21}_1 - (Q_1)^{11}_2(Q_2)^{22}_1 - (Q_1)^{12}_1(Q_2)^{21}_2 - (Q_1)^{12}_2(Q_2)^{22}_2 + (Q_1)^{21}_1(Q_2)^{11}_1 + (Q_1)^{21}_2(Q_2)^{12}_1 + (Q_1)^{22}_1(Q_2)^{11}_2 + (Q_1)^{22}_2(Q_2)^{12}_2$$

$$= \frac{1}{3}Q_1^0Q_2^0 - Q_1^+Q_2^- - \frac{1}{3}Q_1^-Q_2^+ + \frac{1}{3}Q_1^0Q_2^0 - \frac{1}{3}Q_1^-Q_2^+ + \frac{1}{3}Q_1^0Q_2^0 + Q_1^{--}Q_2^{++} - \frac{1}{3}Q_1^-Q_2^+$$

$$= Q_1^{--}Q_2^{++} - Q_1^-Q_2^+ + Q_1^0Q_2^0 - Q_1^+Q_2^-$$

$$-m_Q(Q_1Q_2 + Q_1^\dagger Q_2^\dagger) = -m_Q\varepsilon_{il}(Q_1)^{ij}_k(Q_2)^{lk}_j + h.c. = -m_Q(Q_1^{--}Q_2^{++} - Q_1^-Q_2^+ + Q_1^0Q_2^0 - Q_1^+Q_2^-) + h.c.$$

$$= -m_Q(Q_1^{--}Q_2^{++} - Q_1^-Q_2^+ - Q_1^+Q_2^-) - \frac{1}{2}m_Q(Q_1^0Q_2^0 + Q_2^0Q_1^0) + h.c.$$

$$iQ_1^\dagger\bar{\sigma}^\mu\partial_\mu Q_1 = i(Q_1^\dagger)^k_{ij}\bar{\sigma}^\mu\partial_\mu(Q_1)^{ij}_k = i(Q_1^{--})^\dagger\bar{\sigma}^\mu\partial_\mu Q_1^{--} + i(Q_1^-)^\dagger\bar{\sigma}^\mu\partial_\mu Q_1^- + i(Q_1^0)^\dagger\bar{\sigma}^\mu\partial_\mu Q_1^0 + i(Q_1^+)^\dagger\bar{\sigma}^\mu\partial_\mu Q_1^+$$

$$H^i=\begin{pmatrix}H^+\\H^0\end{pmatrix},\quad H_i^\dagger=\begin{pmatrix}H^-\\H^{0*}\end{pmatrix}$$

$$Q_1TH=\varepsilon_{jl}(Q_1)_i^{jk}T_k^iH^l=\varepsilon_{12}[(Q_1)_1^{11}T_1^1+(Q_1)_1^{12}T_2^1+(Q_1)_2^{11}T_1^2+(Q_1)_2^{12}T_2^2]H^2+\varepsilon_{21}[(Q_1)_1^{21}T_1^1+(Q_1)_1^{22}T_2^1+(Q_1)_2^{21}T_1^2+(Q_1)_2^{22}T_2^2]H^1$$

$$=-\left(\frac{1}{\sqrt6}Q_1^0T^0-\frac{1}{\sqrt3}Q_1^-T^++Q_1^+T^-+\frac{1}{\sqrt6}Q_1^0T^0\right)H^0+\left(-\frac{1}{\sqrt6}Q_1^-T^0+Q_1^{--}T^+-\frac{1}{\sqrt3}Q_1^0T^--\frac{1}{\sqrt6}Q_1^-T^0\right)H^+$$

$$=\left(\frac{1}{\sqrt3}Q_1^-T^+-\frac{2}{\sqrt6}Q_1^0T^0-Q_1^+T^-\right)H^0+\left(Q_1^{--}T^+-\frac{2}{\sqrt6}Q_1^-T^0-\frac{1}{\sqrt3}Q_1^0T^-\right)H^+$$

$$y_1Q_1TH\to\frac{1}{\sqrt6}y_1Q_1^-T^+(v+h)-\frac{1}{\sqrt3}y_1Q_1^0T^0(v+h)-\frac{1}{\sqrt2}y_1Q_1^+T^-(v+h)$$

$$Q_2TH^\dagger=(Q_2)_i^{jk}T_k^iH_j^\dagger=[(Q_2)_1^{11}T_1^1+(Q_2)_1^{12}T_2^1+(Q_2)_2^{11}T_1^2+(Q_2)_2^{12}T_2^2]H_1^\dagger+[(Q_2)_1^{21}T_1^1+(Q_2)_1^{22}T_2^1+(Q_2)_2^{21}T_1^2+(Q_2)_2^{22}T_2^2]H_2^\dagger$$

$$=\left(\frac{1}{\sqrt6}Q_2^+T^0-\frac{1}{\sqrt3}Q_2^0T^++Q_2^{++}T^-+\frac{1}{\sqrt6}Q_2^+T^0\right)H_1^\dagger+\left(-\frac{1}{\sqrt6}Q_2^0T^0+Q_2^-T^+-\frac{1}{\sqrt3}Q_2^+T^--\frac{1}{\sqrt6}Q_2^0T^0\right)H_2^\dagger$$

$$=\left(Q_2^{++}T^-+\frac{2}{\sqrt6}Q_2^+T^0-\frac{1}{\sqrt3}Q_2^0T^+\right)H^-+\left(-\frac{2}{\sqrt6}Q_2^0T^0-\frac{1}{\sqrt3}Q_2^+T^-+Q_2^-T^+\right)H^{0*}$$

$$-y_2Q_2TH^\dagger\to\frac{1}{\sqrt3}y_2Q_2^0T^0(v+h)+\frac{1}{\sqrt6}y_2Q_2^+T^-(v+h)-\frac{1}{\sqrt2}y_2Q_2^-T^+(v+h)$$

$$-\frac12\begin{pmatrix}T^0 & Q_1^0 & Q_2^0\end{pmatrix}\begin{pmatrix}m_T & \frac{1}{\sqrt3}y_1v & -\frac{1}{\sqrt3}y_2v\\ \frac{1}{\sqrt3}y_1v & 0 & m_Q\\ -\frac{1}{\sqrt3}y_2v & m_Q & 0\end{pmatrix}\begin{pmatrix}T^0\\Q_1^0\\Q_2^0\end{pmatrix}=-\frac12\begin{pmatrix}\chi_1^0 & \chi_2^0 & \chi_3^0\end{pmatrix}\begin{pmatrix}m_{\chi_1^0} & & \\ & m_{\chi_2^0} & \\ & & m_{\chi_3^0}\end{pmatrix}\begin{pmatrix}\chi_1^0\\\chi_2^0\\\chi_3^0\end{pmatrix}$$

$$-\begin{pmatrix}T^- & Q_1^- & Q_2^-\end{pmatrix}\begin{pmatrix}m_T & \frac{1}{\sqrt2}y_1v & -\frac{1}{\sqrt6}y_2v\\ -\frac{1}{\sqrt6}y_1v & 0 & -m_Q\\ \frac{1}{\sqrt2}y_2v & -m_Q & 0\end{pmatrix}\begin{pmatrix}T^+\\Q_1^+\\Q_2^+\end{pmatrix}=-\begin{pmatrix}\chi_1^- & \chi_2^- & \chi_3^-\end{pmatrix}\begin{pmatrix}m_{\chi_1^\pm} & & \\ & m_{\chi_2^\pm} & \\ & & m_{\chi_3^\pm}\end{pmatrix}\begin{pmatrix}\chi_1^+\\\chi_2^+\\\chi_3^+\end{pmatrix}$$

$$\mathcal{M}_N=\begin{pmatrix}m_T & \frac{1}{\sqrt3}y_1v & -\frac{1}{\sqrt3}y_2v\\ \frac{1}{\sqrt3}y_1v & 0 & m_Q\\ -\frac{1}{\sqrt3}y_2v & m_Q & 0\end{pmatrix},\quad \mathcal{M}_C=\begin{pmatrix}m_T & \frac{1}{\sqrt2}y_1v & -\frac{1}{\sqrt6}y_2v\\ -\frac{1}{\sqrt6}y_1v & 0 & -m_Q\\ \frac{1}{\sqrt2}y_2v & -m_Q & 0\end{pmatrix}$$

$$\mathcal{N}^{\mathrm T}\mathcal{M}_N\mathcal{N}=\tilde{\mathcal{M}}_N=\mathrm{diag}(m_{\chi_1^0},m_{\chi_2^0},m_{\chi_3^0}),\quad \mathcal{C}_R^{\mathrm T}\mathcal{M}_C\mathcal{C}_L=\tilde{\mathcal{M}}_C=\mathrm{diag}(m_{\chi_1^\pm},m_{\chi_2^\pm},m_{\chi_3^\pm})$$

$$\mathcal{N}^{-1}=\mathcal{N}^\dagger,\quad \mathcal{C}_L^{-1}=\mathcal{C}_L^\dagger,\quad \mathcal{C}_R^{-1}=\mathcal{C}_R^\dagger$$

$$\mathcal{C}_R^{\mathrm T}\mathcal{M}_C\mathcal{M}_C^\dagger\mathcal{C}_R^*=(\mathcal{C}_R^{\mathrm T}\mathcal{M}_C\mathcal{C}_L)(\mathcal{C}_L^\dagger\mathcal{M}_C^\dagger\mathcal{C}_R^*)=\mathrm{diag}(m^2_{\chi_1^\pm},m^2_{\chi_2^\pm},m^2_{\chi_3^\pm})$$

$$\mathcal{C}_L^\dagger\mathcal{M}_C^\dagger\mathcal{M}_C\mathcal{C}_L=(\mathcal{C}_L^\dagger\mathcal{M}_C^\dagger\mathcal{C}_R^*)(\mathcal{C}_R^{\mathrm T}\mathcal{M}_C\mathcal{C}_L)=\mathrm{diag}(m^2_{\chi_1^\pm},m^2_{\chi_2^\pm},m^2_{\chi_3^\pm})$$

$$\begin{pmatrix}T^0\\Q_1^0\\Q_2^0\end{pmatrix}=\mathcal{N}\begin{pmatrix}\chi_1^0\\\chi_2^0\\\chi_3^0\end{pmatrix},\quad \begin{pmatrix}T^+\\Q_1^+\\Q_2^+\end{pmatrix}=\mathcal{C}_L\begin{pmatrix}\chi_1^+\\\chi_2^+\\\chi_3^+\end{pmatrix},\quad \begin{pmatrix}T^-\\Q_1^-\\Q_2^-\end{pmatrix}=\mathcal{C}_R\begin{pmatrix}\chi_1^-\\\chi_2^-\\\chi_3^-\end{pmatrix}$$

$$D_\mu T = (\partial_\mu - igW_\mu^a t_T^a)T$$

$$t_T^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} & 1 & \\ 1 & & -1 \\ & -1 & \end{pmatrix},\quad t_T^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} & -i & \\ i & & i \\ & -i & \end{pmatrix},\quad t_T^3 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix}$$

$$\begin{bmatrix} \text{Ref: Hambye et al., 0903.4010} \\ T_{(3)} = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix},\quad T_{(3)} t_T^a T_{(3)}^{-1} = -(t_T^a)^*,\quad \mathcal{T} \in \mathbf{2} \quad \Rightarrow \quad \tilde{\mathcal{T}} = T_{(3)}\mathcal{T}^* \in \mathbf{2} \\ \tilde{\mathcal{T}} = \mathcal{T} \quad \Leftrightarrow \quad \mathcal{T} \text{ is real} \end{bmatrix}$$

$$gW_\mu^a t_T^a = \begin{pmatrix} gW_\mu^3 & g(W_\mu^1 - iW_\mu^2)/\sqrt{2} & 0 \\ g(W_\mu^1 + iW_\mu^2)/\sqrt{2} & 0 & -g(W_\mu^1 - iW_\mu^2)/\sqrt{2} \\ 0 & -g(W_\mu^1 + iW_\mu^2)/\sqrt{2} & -gW_\mu^3 \end{pmatrix} = \begin{pmatrix} eA_\mu + gc_W Z_\mu & gW_\mu^+ & 0 \\ gW_\mu^- & 0 & -gW_\mu^+ \\ 0 & -gW_\mu^- & -eA_\mu - gc_W Z_\mu \end{pmatrix}$$

$$D_\mu Q_i = (\partial_\mu - ig' B_\mu Y_{Q_i} - igW_\mu^a t_Q^a)Q_i$$

$$Y_{Q_1} = -\frac{1}{2},\quad Y_{Q_2} = \frac{1}{2},\quad t_Q^1 = \begin{pmatrix} & \sqrt{3}/2 & & \\ \sqrt{3}/2 & & 1 & \\ & 1 & & \sqrt{3}/2 \\ & & \sqrt{3}/2 & \end{pmatrix},\quad t_Q^2 = \begin{pmatrix} & -\sqrt{3}i/2 & & \\ \sqrt{3}i/2 & & -i & \\ & i & & -\sqrt{3}i/2 \\ & & \sqrt{3}i/2 & \end{pmatrix},\quad t_Q^3 = \begin{pmatrix} 3/2 & & & \\ & 1/2 & & \\ & & -1/2 & \\ & & & -3/2 \end{pmatrix}$$

(Ref: 1310.8152)

$$g' = \frac{e}{c_W} = \frac{gs_W}{c_W}$$

$$\frac{1}{2}(-g'B_\mu + NgW_\mu^3) = -\frac{1}{2}g'(c_W A_\mu - s_W Z_\mu) + \frac{N}{2}g(s_W A_\mu + c_W Z_\mu) = \frac{N-1}{2}eA_\mu + \frac{g}{2c_W}(s_W^2 + Nc_W^2)Z_\mu$$

$$g'B_\mu Y_{Q_1} + gW_\mu^a t_Q^a = \begin{pmatrix} (-g'B_\mu + 3gW_\mu^3)/2 & \sqrt{3}g(W_\mu^1 - iW_\mu^2)/2 & 0 & 0 \\ \sqrt{3}g(W_\mu^1 + iW_\mu^2)/2 & (-g'B_\mu + gW_\mu^3)/2 & g(W_\mu^1 - iW_\mu^2) & 0 \\ 0 & g(W_\mu^1 + iW_\mu^2) & (-g'B_\mu - gW_\mu^3)/2 & \sqrt{3}g(W_\mu^1 - iW_\mu^2)/2 \\ 0 & 0 & \sqrt{3}g(W_\mu^1 + iW_\mu^2)/2 & (-g'B_\mu - 3gW_\mu^3)/2 \end{pmatrix}$$

$$= \begin{pmatrix} eA_\mu + \frac{g}{2c_W}(s_W^2 + 3c_W^2)Z_\mu & \frac{\sqrt{6}}{2}gW_\mu^+ & 0 & 0 \\ \frac{\sqrt{6}}{2}gW_\mu^- & \frac{g}{2c_W}Z_\mu & \sqrt{2}gW_\mu^+ & 0 \\ 0 & \sqrt{2}gW_\mu^- & -eA_\mu + \frac{g}{2c_W}(s_W^2 - c_W^2)Z_\mu & \frac{\sqrt{6}}{2}gW_\mu^+ \\ 0 & 0 & \frac{\sqrt{6}}{2}gW_\mu^- & -2eA_\mu + \frac{g}{2c_W}(s_W^2 - 3c_W^2)Z_\mu \end{pmatrix}$$

$$\frac{1}{2}(g'B_\mu + NgW_\mu^3) = \frac{1}{2}g'(c_W A_\mu - s_W Z_\mu) + \frac{N}{2}g(s_W A_\mu + c_W Z_\mu) = \frac{N+1}{2}eA_\mu + \frac{g}{2c_W}(Nc_W^2 - s_W^2)Z_\mu$$

$$g'B_\mu Y_{Q_2} + gW_\mu^a t_Q^a = \begin{pmatrix} (g'B_\mu + 3gW_\mu^3)/2 & \sqrt{3}g(W_\mu^1 - iW_\mu^2)/2 & 0 & 0 \\ \sqrt{3}g(W_\mu^1 + iW_\mu^2)/2 & (g'B_\mu + gW_\mu^3)/2 & g(W_\mu^1 - iW_\mu^2) & 0 \\ 0 & g(W_\mu^1 + iW_\mu^2) & (g'B_\mu - gW_\mu^3)/2 & \sqrt{3}g(W_\mu^1 - iW_\mu^2)/2 \\ 0 & 0 & \sqrt{3}g(W_\mu^1 + iW_\mu^2)/2 & (g'B_\mu - 3gW_\mu^3)/2 \end{pmatrix}$$

$$= \begin{pmatrix} 2eA_\mu + \frac{g}{2c_W}(3c_W^2 - s_W^2)Z_\mu & \frac{\sqrt{6}}{2}gW_\mu^+ & 0 & 0 \\ \frac{\sqrt{6}}{2}gW_\mu^- & eA_\mu + \frac{g}{2c_W}(c_W^2 - s_W^2)Z_\mu & \sqrt{2}gW_\mu^+ & 0 \\ 0 & \sqrt{2}gW_\mu^- & -\frac{g}{2c_W}Z_\mu & \frac{\sqrt{6}}{2}gW_\mu^+ \\ 0 & 0 & \frac{\sqrt{6}}{2}gW_\mu^- & -eA_\mu - \frac{g}{2c_W}(3c_W^2 + s_W^2)Z_\mu \end{pmatrix}$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{T}} &\supset T^{\dagger}\bar{\sigma}^{\mu} g W_{\mu}^{a} t_{T}^{a} T \\
&= (eA_{\mu} + g c_{W} Z_{\mu})(T^{+})^{\dagger}\bar{\sigma}^{\mu}T^{+} + gW_{\mu}^{+}(T^{+})^{\dagger}\bar{\sigma}^{\mu}T^{0} \\
&\quad + gW_{\mu}^{-}(T^{0})^{\dagger}\bar{\sigma}^{\mu}T^{+} - gW_{\mu}^{+}(T^{0})^{\dagger}\bar{\sigma}^{\mu}T^{-} \\
&\quad - gW_{\mu}^{-}(T^{-})^{\dagger}\bar{\sigma}^{\mu}T^{0} - (eA_{\mu} + g c_{W} Z_{\mu})(T^{-})^{\dagger}\bar{\sigma}^{\mu}T^{-}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Q}} &\supset Q_{1}^{\dagger}\bar{\sigma}^{\mu}(g' B_{\mu} Y_{Q_1} + g W_{\mu}^{a} t_{Q}^{a})Q_{1} + Q_{2}^{\dagger}\bar{\sigma}^{\mu}(g' B_{\mu} Y_{Q_2} + g W_{\mu}^{a} t_{Q}^{a})Q_{2} \\
&= \left[eA_{\mu} + \frac{g}{2c_W}(s_W^2 + 3c_W^2)Z_{\mu} \right](Q_1^{+})^{\dagger}\bar{\sigma}^{\mu}Q_1^{+} + \left[2eA_{\mu} + \frac{g}{2c_W}(3c_W^2 - s_W^2)Z_{\mu} \right](Q_2^{++})^{\dagger}\bar{\sigma}^{\mu}Q_2^{++} + \frac{\sqrt{6}}{2} g W_{\mu}^{+}[(Q_1^{+})^{\dagger}\bar{\sigma}^{\mu}Q_1^{0} + (Q_2^{++})^{\dagger}\bar{\sigma}^{\mu}Q_2^{+}] \\
&\quad + \frac{\sqrt{6}}{2} g W_{\mu}^{-}[(Q_1^{0})^{\dagger}\bar{\sigma}^{\mu}Q_1^{+} + (Q_2^{+})^{\dagger}\bar{\sigma}^{\mu}Q_2^{++}] + \frac{g}{2c_W} Z_{\mu}(Q_1^{0})^{\dagger}\bar{\sigma}^{\mu}Q_1^{0} + \left[eA_{\mu} + \frac{g}{2c_W}(c_W^2 - s_W^2)Z_{\mu} \right](Q_2^{+})^{\dagger}\bar{\sigma}^{\mu}Q_2^{+} + \sqrt{2} g W_{\mu}^{+}[(Q_1^{0})^{\dagger}\bar{\sigma}^{\mu}Q_1^{-} + (Q_2^{+})^{\dagger}\bar{\sigma}^{\mu}Q_2^{0}] \\
&\quad + \sqrt{2} g W_{\mu}^{-}[(Q_1^{-})^{\dagger}\bar{\sigma}^{\mu}Q_1^{0} + (Q_2^{0})^{\dagger}\bar{\sigma}^{\mu}Q_2^{+}] + \left[-eA_{\mu} + \frac{g}{2c_W}(s_W^2 - c_W^2)Z_{\mu} \right](Q_1^{-})^{\dagger}\bar{\sigma}^{\mu}Q_1^{-} - \frac{g}{2c_W} Z_{\mu}(Q_2^{0})^{\dagger}\bar{\sigma}^{\mu}Q_2^{0} + \frac{\sqrt{6}}{2} g W_{\mu}^{+}[(Q_1^{-})^{\dagger}\bar{\sigma}^{\mu}Q_1^{--} + (Q_2^{0})^{\dagger}\bar{\sigma}^{\mu}Q_2^{-}] \\
&\quad + \frac{\sqrt{6}}{2} g W_{\mu}^{-}[(Q_1^{--})^{\dagger}\bar{\sigma}^{\mu}Q_1^{-} + (Q_2^{-})^{\dagger}\bar{\sigma}^{\mu}Q_2^{0}] + \left[-2eA_{\mu} + \frac{g}{2c_W}(s_W^2 - 3c_W^2)Z_{\mu} \right](Q_1^{--})^{\dagger}\bar{\sigma}^{\mu}Q_1^{--} + \left[-eA_{\mu} - \frac{g}{2c_W}(3c_W^2 + s_W^2)Z_{\mu} \right](Q_2^{-})^{\dagger}\bar{\sigma}^{\mu}Q_2^{-}
\end{aligned}
$$

$$
T^{0} = \mathcal{N}_{1i}\chi_i^0, \quad Q_1^0 = \mathcal{N}_{2i}\chi_i^0, \quad Q_2^0 = \mathcal{N}_{3i}\chi_i^0
$$

$$
\mathcal{L}_{Z\chi_i^0\chi_j^0} = \frac{g}{2c_W} Z_{\mu}(Q_1^0)^{\dagger}\bar{\sigma}^{\mu}Q_1^0 - \frac{g}{2c_W} Z_{\mu}(Q_2^0)^{\dagger}\bar{\sigma}^{\mu}Q_2^0 = \frac{g}{2c_W} Z_{\mu}(\chi_i^0)^{\dagger}\bar{\sigma}^{\mu}\chi_j^0(\mathcal{N}_{2i}^{*}\mathcal{N}_{2j} - \mathcal{N}_{3i}^{*}\mathcal{N}_{3j})
$$

$$
\mathcal{L}_{Z\chi_1^0\chi_1^0} = \frac{g}{2c_W} Z_{\mu}(\chi_1^0)^{\dagger}\bar{\sigma}^{\mu}\chi_1^0(|\mathcal{N}_{21}|^2 - |\mathcal{N}_{31}|^2)
$$

$$
\mathcal{L}_{h\chi_i^0\chi_j^0} = -\frac{1}{\sqrt{3}} y_1 h Q_1^0 T^0 + \frac{1}{\sqrt{3}} y_2 h Q_2^0 T^0 + h.c. = -\frac{1}{\sqrt{3}} h \chi_i^0 \chi_j^0 (y_1 \mathcal{N}_{2i}\mathcal{N}_{1j} - y_2 \mathcal{N}_{3i}\mathcal{N}_{1j}) + h.c.
$$

$$
\mathcal{L}_{h\chi_1^0\chi_1^0} = -\frac{1}{\sqrt{3}} h \chi_1^0 \chi_1^0 (y_1 \mathcal{N}_{21} - y_2 \mathcal{N}_{31})\mathcal{N}_{11} + h.c.
$$

$$
= -\frac{1}{\sqrt{3}} h \{ \chi_1^0 \chi_1^0 \,\mathrm{Re}[(y_1 \mathcal{N}_{21} - y_2 \mathcal{N}_{31})\mathcal{N}_{11}] + i \chi_1^0 \chi_1^0 \,\mathrm{Im}[(y_1 \mathcal{N}_{21} - y_2 \mathcal{N}_{31})\mathcal{N}_{11}] \} + h.c.
$$

$$
= -\frac{1}{\sqrt{3}} h \{ \mathrm{Re}[(y_1 \mathcal{N}_{21} - y_2 \mathcal{N}_{31})\mathcal{N}_{11}](\bar{\chi}_M)_1^0(\chi_M)_1^0 - \mathrm{Im}[(y_1 \mathcal{N}_{21} - y_2 \mathcal{N}_{31})\mathcal{N}_{11}](\bar{\chi}_M)_1^0 i\gamma_5 (\chi_M)_1^0 \}
$$

For $y_1, y_2 \in \mathbf{R}$, $\quad \mathrm{Im}[(y_1 \mathcal{N}_{21} - y_2 \mathcal{N}_{31})\mathcal{N}_{11}] = 0$

$y = y_1 = y_2 \quad \Rightarrow \quad$ Custodial $SU(2)_R$ global symmetry

$$(\mathcal{Q}^A)^{ij}_k = \begin{pmatrix} (Q_1)^{ij}_k \\ (Q_2)^{ij}_k \end{pmatrix}, \quad (\mathcal{H}^A)_i = \begin{pmatrix} H^\dagger_i \\ H_i \end{pmatrix}, \quad H_i \equiv \varepsilon_{ij} H^j \quad (A \text{ is an } SU(2)_R \text{ indice})$$

$$\mathcal{L}_{\rm HTQ} = y[\varepsilon_{jl}(Q_1)^{jk}_i T^i_k H^l - (Q_2)^{jk}_i T^i_k H^\dagger_j] + h.c. = y[(Q_1)^{jk}_i T^i_k H_j - (Q_2)^{jk}_i T^i_k H^\dagger_j] + h.c. = -y\varepsilon_{AB}(\mathcal{Q}^A)^{jk}_i T^i_k (\mathcal{H}^B)_j + h.c.$$

$$\mathcal{L}_{\rm Q} = i(Q_1^\dagger)^k_{ij}\bar{\sigma}^\mu D_\mu (Q_1)^{ij}_k + i(Q_2^\dagger)^k_{ij}\bar{\sigma}^\mu D_\mu (Q_2)^{ij}_k - m_Q(\varepsilon_{il}(Q_1)^{ij}_k (Q_2)^{lk}_j + h.c.)$$

$$= i(\mathcal{Q}_A^\dagger)^k_{ij}\bar{\sigma}^\mu D_\mu (\mathcal{Q}^A)^{ij}_k + \frac{1}{2} m_Q [\varepsilon_{AB}\varepsilon_{il}(\mathcal{Q}^A)^{ij}_k (\mathcal{Q}^B)^{lk}_j + h.c.]$$

$$y = y_1 = y_2 \text{ and } m_T > m_Q \quad \Rightarrow \quad \chi^0_1 = \frac{1}{\sqrt{2}}(Q^0_1 + Q^0_2) \text{ and } \begin{cases} m_{\chi^0_1} = m_{\chi^\pm_1} = m_Q \\ m_{\chi^0_2} = m_{\chi^\pm_2} = \frac{1}{2}\left[\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} + m_Q - m_T\right] \\ m_{\chi^0_3} = m_{\chi^\pm_3} = \frac{1}{2}\left[\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} - m_Q + m_T\right] \\ m_{\chi^{\pm\pm}} = m_Q \end{cases}$$

$$m_{\chi^0_2} - m_{\chi^0_1} = \frac{1}{2}\left[\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} - m_Q - m_T\right]$$

$$\frac{\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} + m_Q + m_T}{2yv/\sqrt{3}} = \frac{8y^2v^2/3}{2yv/\sqrt{3}\left[\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} - m_Q - m_T\right]} = \frac{4yv/\sqrt{3}}{\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} - m_Q - m_T} = \frac{2}{a}$$

$$a \equiv \frac{m_{\chi^0_2} - m_{\chi^0_1}}{yv/\sqrt{3}} = \frac{\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} - m_Q - m_T}{2yv/\sqrt{3}}, \quad b \equiv \sqrt{2+a^2}$$

$$\mathcal{N} = \begin{pmatrix} 0 & \frac{ai}{b} & -\frac{\sqrt{2}}{b} \\ \frac{1}{\sqrt{2}} & -\frac{i}{b} & -\frac{a}{\sqrt{2}b} \\ \frac{1}{\sqrt{2}} & \frac{i}{b} & \frac{a}{\sqrt{2}b} \end{pmatrix}, \quad \mathcal{C}_L = \begin{pmatrix} 0 & \frac{a}{b} & -\frac{\sqrt{2}i}{b} \\ \frac{i}{2} & -\frac{\sqrt{6}}{2b} & -\frac{\sqrt{3}ai}{2b} \\ \frac{\sqrt{3}i}{2} & \frac{\sqrt{2}}{2b} & \frac{ai}{2b} \end{pmatrix}, \quad \mathcal{C}_R = \begin{pmatrix} 0 & -\frac{a}{b} & \frac{\sqrt{2}i}{b} \\ \frac{\sqrt{3}i}{2} & -\frac{\sqrt{2}}{2b} & -\frac{ai}{2b} \\ \frac{i}{2} & \frac{\sqrt{6}}{2b} & \frac{\sqrt{3}ai}{2b} \end{pmatrix}$$

$$\mathcal{L}_{Z\chi^0_1\chi^0_1} = \frac{g}{2c_W} Z_\mu (\chi^0_1)^\dagger \bar{\sigma}^\mu \chi^0_1 (|\mathcal{N}_{21}|^2 - |\mathcal{N}_{31}|^2) = 0$$

$$\mathcal{L}_{h\chi^0_1\chi^0_1} = -\frac{1}{\sqrt{3}} h\chi^0_1\chi^0_1 (y_1\mathcal{N}_{21} - y_2\mathcal{N}_{31})\mathcal{N}_{11} + h.c. = 0$$

$$y = y_1 = y_2 \text{ and } m_T < m_Q \quad \Rightarrow \quad \begin{cases} m_{\chi^0_1} = m_{\chi^\pm_1} = \frac{1}{2}\left[\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} - m_Q + m_T\right] \\ m_{\chi^0_2} = m_{\chi^\pm_2} = m_Q \\ m_{\chi^0_3} = m_{\chi^\pm_3} = \frac{1}{2}\left[\sqrt{8y^2v^2/3 + (m_Q+m_T)^2} + m_Q - m_T\right] \\ m_{\chi^{\pm\pm}} = m_Q \end{cases}$$

$$X^{++}=\begin{pmatrix}\chi_L^{++}\\(\chi_R^{--})^\dagger\end{pmatrix}=\begin{pmatrix}Q_2^{++}\\(Q_1^{--})^\dagger\end{pmatrix},\quad \bar{X}^{++}=\begin{pmatrix}\chi_R^{--} & (\chi_L^{++})^\dagger\end{pmatrix}=\begin{pmatrix}Q_1^{--} & (Q_2^{++})^\dagger\end{pmatrix}$$

$$X_i^0=\begin{pmatrix}\chi_{iL}^0\\(\chi_{iR}^0)^\dagger\end{pmatrix},\quad \chi_L^0=\chi_R^0=\mathcal{N}^\dagger\psi_L^0=\mathcal{N}^\dagger\psi_R^0=\begin{pmatrix}\chi_1^0 & \chi_2^0 & \chi_3^0\end{pmatrix}^{\mathrm{T}},\quad \bar{X}_i^0=\begin{pmatrix}\chi_{iR}^0 & (\chi_{iL}^0)^\dagger\end{pmatrix}$$

$$X_i^+=\begin{pmatrix}\chi_{iL}^+\\(\chi_{iR}^-)^\dagger\end{pmatrix},\quad \chi_L^+=\mathcal{C}_L^\dagger\psi_L^+=\begin{pmatrix}\chi_1^+ & \chi_2^+ & \chi_3^+\end{pmatrix}^{\mathrm{T}},\quad \chi_R^-=\mathcal{C}_R^\dagger\psi_R^-=\begin{pmatrix}\chi_1^- & \chi_2^- & \chi_3^-\end{pmatrix}^{\mathrm{T}},\quad \bar{X}_i^+=\begin{pmatrix}\chi_{iR}^- & (\chi_{iL}^+)^\dagger\end{pmatrix}$$

$$\Psi_i^0=\begin{pmatrix}\psi_{iL}^0\\(\psi_{iR}^0)^\dagger\end{pmatrix},\quad \psi_L^0=\psi_R^0=\mathcal{N}\chi_L^0=\mathcal{N}\chi_R^0=\begin{pmatrix}T^0 & Q_1^0 & Q_2^0\end{pmatrix}^{\mathrm{T}},\quad \bar{\Psi}_i^0=\begin{pmatrix}\psi_{iR}^0 & (\psi_{iL}^0)^\dagger\end{pmatrix}$$

$$\Psi_i^+=\begin{pmatrix}\psi_{iL}^+\\(\psi_{iR}^-)^\dagger\end{pmatrix},\quad \psi_L^+=\mathcal{C}_L\chi_L^+=\begin{pmatrix}T^+ & Q_1^+ & Q_2^+\end{pmatrix}^{\mathrm{T}},\quad \psi_R^-=\mathcal{C}_R\chi_R^-=\begin{pmatrix}T^- & Q_1^- & Q_2^-\end{pmatrix}^{\mathrm{T}},\quad \bar{\Psi}_i^+=\begin{pmatrix}\psi_{iR}^- & (\psi_{iL}^+)^\dagger\end{pmatrix}$$

$$-\frac{1}{2}\bar{\Psi}_{iR}^0(\mathcal{M}_N)_{ij}\Psi_{jL}^0-\frac{1}{2}\bar{\Psi}_{iL}^0(\mathcal{M}_N^{\mathrm{T}})_{ij}\Psi_{jR}^0=-\frac{1}{2}\psi_{iR}^0(\mathcal{M}_N)_{ij}\psi_{jL}^0-\frac{1}{2}(\psi_{iL}^0)^\dagger(\mathcal{M}_N^{\mathrm{T}})_{ij}(\psi_{jR}^0)^\dagger$$
$$=-\frac{1}{2}(\psi_R^0)^{\mathrm{T}}\mathcal{M}_N\psi_L^0-\frac{1}{2}[(\psi_L^0)^\dagger]^{\mathrm{T}}\mathcal{M}_N^{\mathrm{T}}(\psi_R^0)^\dagger=-\frac{1}{2}(\chi_R^0)^{\mathrm{T}}\mathcal{N}^{\mathrm{T}}\mathcal{M}_N\mathcal{N}\chi_L^0-\frac{1}{2}[(\chi_L^0)^\dagger]^{\mathrm{T}}\mathcal{N}^\dagger\mathcal{M}_N^{\mathrm{T}}\mathcal{N}^*(\chi_R^0)^\dagger$$
$$=-\frac{1}{2}(\chi_R^0)^{\mathrm{T}}\tilde{\mathcal{M}}_N\chi_L^0-\frac{1}{2}[(\chi_L^0)^\dagger]^{\mathrm{T}}\tilde{\mathcal{M}}_N(\chi_R^0)^\dagger=-\frac{1}{2}m_{\chi_i^0}\bar{X}_i^0X_i^0$$

$$-\bar{\Psi}_{iR}^+(\mathcal{M}_C)_{ij}\Psi_{jL}^+-\bar{\Psi}_{iL}^+(\mathcal{M}_C^{\mathrm{T}})_{ij}\Psi_{jR}^+=-\psi_{iR}^-(\mathcal{M}_C)_{ij}\psi_{jL}^+-(\psi_{iL}^+)^\dagger(\mathcal{M}_C^{\mathrm{T}})_{ij}(\psi_{jR}^-)^\dagger$$
$$=-(\psi_R^-)^{\mathrm{T}}\mathcal{M}_C\psi_L^+-[(\psi_L^+)^\dagger]^{\mathrm{T}}\mathcal{M}_C^{\mathrm{T}}(\psi_R^-)^\dagger=-(\chi_R^-)^{\mathrm{T}}\mathcal{C}_R^{\mathrm{T}}\mathcal{M}_N\mathcal{C}_L\chi_L^+-[(\chi_L^+)^\dagger]^{\mathrm{T}}\mathcal{C}_L^\dagger\mathcal{M}_C^{\mathrm{T}}\mathcal{C}_R^*(\chi_R^-)^\dagger$$
$$=-(\chi_R^-)^{\mathrm{T}}\tilde{\mathcal{M}}_C\chi_L^+-[(\chi_L^+)^\dagger]^{\mathrm{T}}\tilde{\mathcal{M}}_C(\chi_R^-)^\dagger=-m_{\chi_i^\pm}\bar{X}_i^+X_i^+$$

$$\Psi_{iL}^0=\begin{pmatrix}\psi_{iL}^0\\0\end{pmatrix}=\begin{pmatrix}(\mathcal{N}\chi_L^0)_i\\0\end{pmatrix}=\mathcal{N}_{ij}X_{jL}^0,\quad \Psi_{iR}^0=\begin{pmatrix}0\\(\psi_{iR}^0)^\dagger\end{pmatrix}=\begin{pmatrix}0\\(\mathcal{N}\chi_R^0)_i^\dagger\end{pmatrix}=\mathcal{N}_{ij}^*X_{jR}^0$$
$$\bar{\Psi}_{iL}^0=\begin{pmatrix}0 & (\psi_{iL}^0)^\dagger\end{pmatrix}=\mathcal{N}_{ij}^*\bar{X}_{jL}^0,\quad \bar{\Psi}_{iR}^0=\begin{pmatrix}\psi_{iR}^0 & 0\end{pmatrix}=\mathcal{N}_{ij}\bar{X}_{jR}^0$$
$$\Psi_{iL}^+=\begin{pmatrix}\psi_{iL}^+\\0\end{pmatrix}=\begin{pmatrix}(\mathcal{C}_L\chi_L^+)_i\\0\end{pmatrix}=(\mathcal{C}_L)_{ij}X_{jL}^+,\quad \Psi_{iR}^+=\begin{pmatrix}0\\(\psi_{iR}^-)^\dagger\end{pmatrix}=\begin{pmatrix}0\\(\mathcal{C}_R\chi_R^-)_i^\dagger\end{pmatrix}=(\mathcal{C}_R)_{ij}^*X_{jR}^+$$
$$\bar{\Psi}_{iL}^+=\begin{pmatrix}0 & (\psi_{iL}^+)^\dagger\end{pmatrix}=(\mathcal{C}_L)_{ij}^*\bar{X}_{jL}^+,\quad \bar{\Psi}_{iR}^+=\begin{pmatrix}\psi_{iR}^- & 0\end{pmatrix}=(\mathcal{C}_R)_{ij}\bar{X}_{jR}^+$$

$$\bar{\Psi}^+_{iL}\gamma^\mu\Psi^+_{iL} = \begin{pmatrix} 0 & (\psi^+_{iL})^\dagger \end{pmatrix}\begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}\begin{pmatrix} \psi^+_{iL} \\ 0 \end{pmatrix} = (\psi^+_{iL})^\dagger\bar{\sigma}^\mu\psi^+_{iL}$$

$$\bar{\Psi}^+_{iR}\gamma^\mu\Psi^+_{iR} = \begin{pmatrix} \psi^-_{iR} & 0 \end{pmatrix}\begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}\begin{pmatrix} 0 \\ (\psi^-_{iR})^\dagger \end{pmatrix} = \psi^-_{iR}\sigma^\mu(\psi^-_{iR})^\dagger = -(\psi^-_{iR})^\dagger\bar{\sigma}^\mu\psi^-_{iR}$$

$$\mathcal{L}_{A\Psi^+_i\Psi^+_i} = a_{A\Psi^+_i\Psi^+_i}A_\mu\bar{\Psi}^+_{iL}\gamma^\mu\Psi^+_{iL} + b_{A\Psi^+_i\Psi^+_i}A_\mu\bar{\Psi}^+_{iR}\gamma^\mu\Psi^+_{iR}$$

$$= a_{A\Psi^+_k\Psi^+_k}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_L)_{kj}A_\mu\bar{X}^+_{iL}\gamma^\mu X^+_{jL} + b_{A\Psi^+_k\Psi^+_k}(\mathcal{C}_R)_{ki}(\mathcal{C}_R)^*_{kj}A_\mu\bar{X}^+_{iR}\gamma^\mu X^+_{jR} = a_{AX^+_iX^+_j}A_\mu\bar{X}^+_{iL}\gamma^\mu X^+_{jL} + b_{AX^+_iX^+_j}A_\mu\bar{X}^+_{iR}\gamma^\mu X^+_{jR}$$

$$a_{A\Psi^+_1\Psi^+_1} = b_{A\Psi^+_1\Psi^+_1} = a_{A\Psi^+_2\Psi^+_2} = b_{A\Psi^+_2\Psi^+_2} = a_{A\Psi^+_3\Psi^+_3} = b_{A\Psi^+_3\Psi^+_3} = e$$

$$a_{AX^+_iX^+_j} = a_{A\Psi^+_k\Psi^+_k}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_L)_{kj} = e\delta_{ij}, \quad b_{AX^+_iX^+_j} = b_{A\Psi^+_k\Psi^+_k}(\mathcal{C}_R)_{ki}(\mathcal{C}_R)^*_{kj} = e\delta_{ij}$$

$$\mathcal{L}_{Z\Psi^+_i\Psi^+_i} = a_{Z\Psi^+_i\Psi^+_i}Z_\mu\bar{\Psi}^+_{iL}\gamma^\mu\Psi^+_{iL} + b_{Z\Psi^+_i\Psi^+_i}Z_\mu\bar{\Psi}^+_{iR}\gamma^\mu\Psi^+_{iR}$$

$$= a_{Z\Psi^+_k\Psi^+_k}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_L)_{kj}Z_\mu\bar{X}^+_{iL}\gamma^\mu X^+_{jL} + b_{Z\Psi^+_k\Psi^+_k}(\mathcal{C}_R)_{ki}(\mathcal{C}_R)^*_{kj}Z_\mu\bar{X}^+_{iR}\gamma^\mu X^+_{jR} = a_{ZX^+_iX^+_j}Z_\mu\bar{X}^+_{iL}\gamma^\mu X^+_{jL} + b_{ZX^+_iX^+_j}Z_\mu\bar{X}^+_{iR}\gamma^\mu X^+_{jR}$$

$$a_{Z\Psi^+_1\Psi^+_1} = b_{Z\Psi^+_1\Psi^+_1} = gc_W, \quad a_{Z\Psi^+_2\Psi^+_2} = \frac{g}{2c_W}(3c_W^2 + s_W^2), \quad b_{Z\Psi^+_2\Psi^+_2} = \frac{g}{2c_W}(c_W^2 - s_W^2), \quad a_{Z\Psi^+_3\Psi^+_3} = \frac{g}{2c_W}(c_W^2 - s_W^2), \quad b_{Z\Psi^+_3\Psi^+_3} = \frac{g}{2c_W}(3c_W^2 + s_W^2)$$

$$a_{ZX^+_iX^+_j} = a_{Z\Psi^+_k\Psi^+_k}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_L)_{kj}, \quad b_{ZX^+_iX^+_j} = b_{Z\Psi^+_k\Psi^+_k}(\mathcal{C}_R)_{ki}(\mathcal{C}_R)^*_{kj}$$

$$\bar{\Psi}^0_{iL}\gamma^\mu\Psi^0_{iL} = (\psi^0_{iL})^\dagger\bar{\sigma}^\mu\psi^0_{iL}, \quad \bar{\Psi}^0_{iR}\gamma^\mu\Psi^0_{iR} = -(\psi^0_{iR})^\dagger\bar{\sigma}^\mu\psi^0_{iR} = -(\psi^0_{iL})^\dagger\bar{\sigma}^\mu\psi^0_{iL}$$

$$\mathcal{L}_{Z\Psi^0_i\Psi^0_i} = \frac{1}{2}a_{Z\Psi^0_i\Psi^0_i}Z_\mu\bar{\Psi}^0_{iL}\gamma^\mu\Psi^0_{iL} + \frac{1}{2}b_{Z\Psi^0_i\Psi^0_i}Z_\mu\bar{\Psi}^0_{iR}\gamma^\mu\Psi^0_{iR}$$

$$= \frac{1}{2}a_{Z\Psi^0_k\Psi^0_k}\mathcal{N}^*_{ki}\mathcal{N}_{kj}Z_\mu\bar{X}^0_{iL}\gamma^\mu X^0_{jL} + \frac{1}{2}b_{Z\Psi^0_k\Psi^0_k}\mathcal{N}_{ki}\mathcal{N}^*_{kj}Z_\mu\bar{X}^0_{iR}\gamma^\mu X^0_{jR} = \frac{1}{2}(a_{ZX^0_iX^0_j}Z_\mu\bar{X}^0_{iL}\gamma^\mu X^0_{jL} + b_{ZX^0_iX^0_j}Z_\mu\bar{X}^0_{iR}\gamma^\mu X^0_{jR})$$

$$a_{Z\Psi^0_1\Psi^0_1} = b_{Z\Psi^0_1\Psi^0_1} = 0, \quad a_{Z\Psi^0_2\Psi^0_2} = -b_{Z\Psi^0_2\Psi^0_2} = \frac{g}{2c_W}, \quad a_{Z\Psi^0_3\Psi^0_3} = -b_{Z\Psi^0_3\Psi^0_3} = -\frac{g}{2c_W}$$

$$a_{ZX^0_iX^0_j} = a_{Z\Psi^0_k\Psi^0_k}\mathcal{N}^*_{ki}\mathcal{N}_{kj}, \quad b_{ZX^0_iX^0_j} = b_{Z\Psi^0_k\Psi^0_k}\mathcal{N}_{ki}\mathcal{N}^*_{kj}$$

$$\left(\begin{array}{l}
a_{ZX^0_jX^0_i} = a^*_{ZX^0_iX^0_j}, \quad b_{ZX^0_jX^0_i} = b^*_{ZX^0_iX^0_j}, \quad a_{ZX^0_iX^0_j} = -b_{ZX^0_jX^0_i} \\
C^{-1} = -C, \quad X^0_i = C(\bar{X}^0_i)^{\mathrm{T}}, \quad \bar{X}^0_i = (X^0_i)^{\mathrm{T}}C \\
C^{-1}(\gamma^\mu P_L)^{\mathrm{T}}C = \frac{1}{2}C^{-1}(\gamma^\mu)^{\mathrm{T}}C - \frac{1}{2}C^{-1}(\gamma^\mu\gamma_5)^{\mathrm{T}}C = -\frac{1}{2}\gamma^\mu - \frac{1}{2}\gamma^\mu\gamma_5 = -\gamma^\mu P_R \\
a_{ZX^0_1X^0_2}\bar{X}^0_{1L}\gamma^\mu X^0_{2L} = a_{ZX^0_1X^0_2}(\bar{X}^0_1\gamma^\mu P_L X^0_2)^{\mathrm{T}} = -a_{ZX^0_1X^0_2}(X^0_2)^{\mathrm{T}}CC^{-1}(\gamma^\mu P_L)^{\mathrm{T}}C^{-1}C(\bar{X}^0_1)^{\mathrm{T}} = -a_{ZX^0_1X^0_2}\bar{X}^0_2C^{-1}(\gamma^\mu P_L)^{\mathrm{T}}C^{-1}X^0_1 \\
= a_{ZX^0_1X^0_2}\bar{X}^0_2C^{-1}(\gamma^\mu P_L)^{\mathrm{T}}CX^0_1 = -a_{ZX^0_1X^0_2}\bar{X}^0_2\gamma^\mu P_R X^0_1 = b_{ZX^0_2X^0_1}\bar{X}^0_{2R}\gamma^\mu X^0_{1R} \\
\frac{1}{2}a_{ZX^0_1X^0_2}Z_\mu\bar{X}^0_{1L}\gamma^\mu X^0_{2L} \text{ and } \frac{1}{2}b_{ZX^0_2X^0_1}Z_\mu\bar{X}^0_{2R}\gamma^\mu X^0_{1R} \text{ give an identical vertex!}
\end{array}\right)$$

$$\bar{\Psi}^{+}_{iL}\gamma^{\mu}\Psi^{0}_{iL}=\begin{pmatrix}0 & (\psi^{+}_{iL})^{\dagger}\end{pmatrix}\begin{pmatrix} & \sigma^{\mu}\\ \bar{\sigma}^{\mu} & \end{pmatrix}\begin{pmatrix}\psi^{0}_{iL}\\ 0\end{pmatrix}=(\psi^{+}_{iL})^{\dagger}\bar{\sigma}^{\mu}\psi^{0}_{iL}$$

$$\bar{\Psi}^{+}_{iR}\gamma^{\mu}\Psi^{0}_{iR}=\begin{pmatrix}\psi^{-}_{iR} & 0\end{pmatrix}\begin{pmatrix} & \sigma^{\mu}\\ \bar{\sigma}^{\mu} & \end{pmatrix}\begin{pmatrix}0\\ (\psi^{0}_{iR})^{\dagger}\end{pmatrix}=\psi^{-}_{iR}\sigma^{\mu}(\psi^{0}_{iR})^{\dagger}=-(\psi^{0}_{iR})^{\dagger}\bar{\sigma}^{\mu}\psi^{-}_{iR}$$

$$\mathcal{L}_{W\Psi^{+}_{i}\Psi^{0}_{i}}=a_{W\Psi^{+}_{i}\Psi^{0}_{i}}(W^{+}_{\mu}\bar{\Psi}^{+}_{iL}\gamma^{\mu}\Psi^{0}_{iL}+h.c.)+b_{W\Psi^{+}_{i}\Psi^{0}_{i}}(W^{+}_{\mu}\bar{\Psi}^{+}_{iR}\gamma^{\mu}\Psi^{0}_{iR}+h.c.)$$

$$=a_{W\Psi^{+}_{k}\Psi^{0}_{k}}[(\mathcal{C}_{L})^{*}_{ki}\mathcal{N}_{kj}W^{+}_{\mu}\bar{X}^{+}_{iL}\gamma^{\mu}X^{0}_{jL}+h.c.]+b_{W\Psi^{+}_{k}\Psi^{0}_{k}}[(\mathcal{C}_{R})_{ki}\mathcal{N}^{*}_{kj}W^{+}_{\mu}\bar{X}^{+}_{iR}\gamma^{\mu}X^{0}_{jR}+h.c.]$$

$$=a_{WX^{+}_{i}X^{0}_{j}}W^{+}_{\mu}\bar{X}^{+}_{iL}\gamma^{\mu}X^{0}_{jL}+a^{*}_{WX^{+}_{i}X^{0}_{j}}W^{-}_{\mu}\bar{X}^{0}_{jL}\gamma^{\mu}X^{+}_{iL}+b_{WX^{+}_{i}X^{0}_{j}}W^{+}_{\mu}\bar{X}^{+}_{iR}\gamma^{\mu}X^{0}_{jR}+b^{*}_{WX^{+}_{i}X^{0}_{j}}W^{-}_{\mu}\bar{X}^{0}_{jR}\gamma^{\mu}X^{+}_{iR}$$

$$a_{W\Psi^{+}_{1}\Psi^{0}_{1}}=b_{W\Psi^{+}_{1}\Psi^{0}_{1}}=g,\quad a_{W\Psi^{+}_{2}\Psi^{0}_{2}}=\frac{\sqrt{6}}{2}g,\quad b_{W\Psi^{+}_{2}\Psi^{0}_{2}}=-\sqrt{2}g,\quad a_{W\Psi^{+}_{3}\Psi^{0}_{3}}=\sqrt{2}g,\quad b_{W\Psi^{+}_{3}\Psi^{0}_{3}}=-\frac{\sqrt{6}}{2}g$$

$$a_{WX^{+}_{i}X^{0}_{j}}=a_{W\Psi^{+}_{k}\Psi^{0}_{k}}(\mathcal{C}_{L})^{*}_{ki}\mathcal{N}_{kj},\quad b_{WX^{+}_{i}X^{0}_{j}}=b_{W\Psi^{+}_{k}\Psi^{0}_{k}}(\mathcal{C}_{R})_{ki}\mathcal{N}^{*}_{kj}$$

$$\bar{X}^{++}_{L}\gamma^{\mu}\Psi^{+}_{iL}=(Q^{++}_{2})^{\dagger}\bar{\sigma}^{\mu}\psi^{+}_{iL},\quad \bar{X}^{++}_{R}\gamma^{\mu}\Psi^{+}_{iR}=-(Q^{--}_{1})^{\dagger}\bar{\sigma}^{\mu}\psi^{-}_{iR}$$

$$\mathcal{L}_{WX^{++}\Psi^{+}_{i}}=a_{WX^{++}\Psi^{+}_{i}}(W^{+}_{\mu}\bar{X}^{++}_{L}\gamma^{\mu}\Psi^{+}_{iL}+h.c.)+b_{WX^{++}\Psi^{+}_{i}}(W^{+}_{\mu}\bar{X}^{++}_{R}\gamma^{\mu}\Psi^{+}_{iR}+h.c.)$$

$$=a_{WX^{++}\Psi^{+}_{j}}[(\mathcal{C}_{L})_{ji}W^{+}_{\mu}\bar{X}^{++}_{L}\gamma^{\mu}X^{+}_{iL}+h.c.]+b_{WX^{++}\Psi^{+}_{j}}[(\mathcal{C}_{R})^{*}_{ji}W^{+}_{\mu}\bar{X}^{++}_{R}\gamma^{\mu}X^{+}_{iR}+h.c.]$$

$$=a_{WX^{++}X^{+}_{i}}W^{+}_{\mu}\bar{X}^{++}_{L}\gamma^{\mu}X^{+}_{iL}+a^{*}_{WX^{++}X^{+}_{i}}W^{-}_{\mu}\bar{X}^{+}_{iL}\gamma^{\mu}X^{++}_{L}+b_{WX^{++}X^{+}_{i}}\bar{X}^{++}_{R}\gamma^{\mu}X^{+}_{iR}+b^{*}_{WX^{++}X^{+}_{i}}\bar{X}^{+}_{iR}\gamma^{\mu}X^{++}_{R}$$

$$a_{WX^{++}\Psi^{+}_{1}}=b_{WX^{++}\Psi^{+}_{1}}=0,\quad a_{WX^{++}\Psi^{+}_{2}}=0,\quad b_{WX^{++}\Psi^{+}_{2}}=-\frac{\sqrt{6}}{2}g,\quad a_{WX^{++}\Psi^{+}_{3}}=\frac{\sqrt{6}}{2}g,\quad b_{WX^{++}\Psi^{+}_{3}}=0$$

$$a_{WX^{++}X^{+}_{i}}=a_{WX^{++}\Psi^{+}_{j}}(\mathcal{C}_{L})_{ji},\quad b_{WX^{++}X^{+}_{i}}=b_{WX^{++}\Psi^{+}_{j}}(\mathcal{C}_{R})^{*}_{ji}$$

$$\bar{X}^{++}_{L}\gamma^{\mu}X^{++}_{L}=(Q^{++}_{2})^{\dagger}\bar{\sigma}^{\mu}Q^{++}_{2},\quad \bar{X}^{++}_{R}\gamma^{\mu}X^{++}_{R}=-(Q^{--}_{1})^{\dagger}\bar{\sigma}^{\mu}Q^{--}_{1}$$

$$\mathcal{L}_{AX^{++}X^{++}}=a_{AX^{++}X^{++}}A_{\mu}\bar{X}^{++}_{L}\gamma^{\mu}X^{++}_{L}+b_{AX^{++}X^{++}}A_{\mu}\bar{X}^{++}_{R}\gamma^{\mu}X^{++}_{R}$$

$$a_{AX^{++}X^{++}}=b_{AX^{++}X^{++}}=2e$$

$$\mathcal{L}_{ZX^{++}X^{++}}=a_{ZX^{++}X^{++}}Z_{\mu}\bar{X}^{++}_{L}\gamma^{\mu}X^{++}_{L}+b_{ZX^{++}X^{++}}Z_{\mu}\bar{X}^{++}_{R}\gamma^{\mu}X^{++}_{R}$$

$$a_{ZX^{++}X^{++}}=b_{ZX^{++}X^{++}}=\frac{g}{2c_{W}}(3c^{2}_{W}-s^{2}_{W})$$

$$\mathcal{L}_{\mathrm{HTQ}} = y_1 Q_1 T H - y_2 Q_2 T H^\dagger + h.c.$$

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} G^+(x) \\ \dfrac{1}{\sqrt{2}}[v + h(x) + iG^0(x)] \end{pmatrix}, \quad H^+ = G^+, \quad H^- = G^-, \quad H^0 = \frac{1}{\sqrt{2}}(v + h + iG^0), \quad H^{0*} = \frac{1}{\sqrt{2}}(v + h - iG^0)$$

$$y_1 Q_1 T H = y_1 G^+ \left(Q_1^{--} T^+ - \frac{2}{\sqrt{6}} Q_1^- T^0 - \frac{1}{\sqrt{3}} Q_1^0 T^- \right) + y_1 (v + h + iG^0) \left(\frac{1}{\sqrt{6}} Q_1^- T^+ - \frac{1}{\sqrt{3}} Q_1^0 T^0 - \frac{1}{\sqrt{2}} Q_1^+ T^- \right)$$

$$-y_2 Q_2 T H^\dagger = y_2 G^- \left(-Q_2^{++} T^- - \frac{2}{\sqrt{6}} Q_2^+ T^0 + \frac{1}{\sqrt{3}} Q_2^0 T^+ \right) + y_2 (v + h - iG^0) \left(\frac{1}{\sqrt{3}} Q_2^0 T^0 + \frac{1}{\sqrt{6}} Q_2^+ T^- - \frac{1}{\sqrt{2}} Q_2^- T^+ \right)$$

$$\bar{\Psi}^0_{iR} \Psi^0_{jL} = \begin{pmatrix} \psi^0_{iR} & 0 \end{pmatrix} \begin{pmatrix} \psi^0_{jL} \\ 0 \end{pmatrix} = \psi^0_{iR} \psi^0_{jL}, \quad \bar{\Psi}^0_{iL} \Psi^0_{jR} = \begin{pmatrix} 0 & (\psi^0_{iL})^\dagger \end{pmatrix} \begin{pmatrix} 0 \\ (\psi^0_{jR})^\dagger \end{pmatrix} = (\psi^0_{iL})^\dagger (\psi^0_{jR})^\dagger$$

$$\mathcal{L}_{h\Psi^0_i\Psi^0_j} = \frac{1}{2} a_{h\Psi^0_i\Psi^0_j} h \bar{\Psi}^0_{iR} \Psi^0_{jL} + \frac{1}{2} b_{h\Psi^0_i\Psi^0_j} h \bar{\Psi}^0_{iL} \Psi^0_{jR}$$

$$= \frac{1}{2} a_{h\Psi^0_k\Psi^0_l} \mathcal{N}_{ki} \mathcal{N}_{lj} h \bar{X}^0_{iR} X^0_{jL} + \frac{1}{2} b_{h\Psi^0_k\Psi^0_l} \mathcal{N}^*_{ki} \mathcal{N}^*_{lj} h \bar{X}^0_{iL} X^0_{jR} = \frac{1}{2} (a_{hX^0_iX^0_j} h \bar{X}^0_{iR} X^0_{jL} + b_{hX^0_iX^0_j} h \bar{X}^0_{iL} X^0_{jR})$$

$$a_{h\Psi^0_1\Psi^0_2} = b_{h\Psi^0_1\Psi^0_2} = -\frac{y_1}{\sqrt{3}} = a_{h\Psi^0_2\Psi^0_1} = b_{h\Psi^0_2\Psi^0_1}, \quad a_{h\Psi^0_1\Psi^0_3} = b_{h\Psi^0_1\Psi^0_3} = \frac{y_2}{\sqrt{3}} = a_{h\Psi^0_3\Psi^0_1} = b_{h\Psi^0_3\Psi^0_1}$$

$$a_{hX^0_iX^0_j} = a_{h\Psi^0_k\Psi^0_l} \mathcal{N}_{ki} \mathcal{N}_{lj}, \quad b_{hX^0_iX^0_j} = b_{h\Psi^0_k\Psi^0_l} \mathcal{N}^*_{ki} \mathcal{N}^*_{lj}$$

$$\left(\begin{array}{l} a_{hX^0_jX^0_i} = a_{h\Psi^0_k\Psi^0_l} \mathcal{N}_{kj} \mathcal{N}_{li} = a_{h\Psi^0_l\Psi^0_k} \mathcal{N}_{kj} \mathcal{N}_{li} = a_{hX^0_iX^0_j}, \quad b_{hX^0_jX^0_i} = b_{hX^0_iX^0_j}, \quad a_{hX^0_iX^0_j} = b^*_{hX^0_iX^0_j} \\ C^{-1}(P_L)^{\mathrm{T}} C = \frac{1}{2} C^{-1}(1-\gamma_5)^{\mathrm{T}} C = \frac{1}{2}(1-\gamma_5) = P_L \\ a_{hX^0_1X^0_2} \bar{X}^0_{1R} X^0_{2L} = a_{hX^0_1X^0_2} (\bar{X}^0_1 P_L X^0_2)^{\mathrm{T}} = -a_{hX^0_1X^0_2} (X^0_2)^{\mathrm{T}} (P_L)^{\mathrm{T}} (\bar{X}^0_1)^{\mathrm{T}} = -a_{hX^0_1X^0_2} (X^0_2)^{\mathrm{T}} C C^{-1} (P_L)^{\mathrm{T}} C^{-1} C (\bar{X}^0_1)^{\mathrm{T}} = -a_{hX^0_1X^0_2} \bar{X}^0_2 C^{-1} (P_L)^{\mathrm{T}} C^{-1} X^0_1 \\ = a_{hX^0_1X^0_2} \bar{X}^0_2 C^{-1} (P_L)^{\mathrm{T}} C X^0_1 = a_{hX^0_1X^0_2} \bar{X}^0_2 P_L X^0_1 = a_{hX^0_2X^0_1} \bar{X}^0_{2R} X^0_{1L} \\ \frac{1}{2} a_{hX^0_1X^0_2} h \bar{X}^0_{1R} X^0_{2L} \left(\frac{1}{2} b_{hX^0_1X^0_2} h \bar{X}^0_{1L} X^0_{2R} \right) \text{ and } \frac{1}{2} a_{hX^0_2X^0_1} h \bar{X}^0_{2R} X^0_{1L} \left(\frac{1}{2} b_{hX^0_2X^0_1} h \bar{X}^0_{2L} X^0_{1R} \right) \text{ give an identical vertex!} \end{array} \right)$$

$$\mathcal{L}_{G^0\Psi^0_i\Psi^0_j} = \frac{1}{2} a_{G^0\Psi^0_i\Psi^0_j} G^0 \bar{\Psi}^0_{iR} \Psi^0_{jL} + \frac{1}{2} b_{G^0\Psi^0_i\Psi^0_j} G^0 \bar{\Psi}^0_{iL} \Psi^0_{jR}$$

$$= \frac{1}{2} a_{G^0\Psi^0_k\Psi^0_l} \mathcal{N}_{ki} \mathcal{N}_{lj} G^0 \bar{X}^0_{iR} X^0_{jL} + \frac{1}{2} b_{G^0\Psi^0_k\Psi^0_l} \mathcal{N}^*_{ki} \mathcal{N}^*_{lj} G^0 \bar{X}^0_{iL} X^0_{jR} = \frac{1}{2} (a_{G^0X^0_iX^0_j} G^0 \bar{X}^0_{iR} X^0_{jL} + b_{G^0X^0_iX^0_j} G^0 \bar{X}^0_{iL} X^0_{jR})$$

$$a_{G^0\Psi^0_1\Psi^0_2} = -b_{G^0\Psi^0_1\Psi^0_2} = -\frac{y_1}{\sqrt{3}} i = a_{G^0\Psi^0_2\Psi^0_1} = -b_{G^0\Psi^0_2\Psi^0_1}, \quad a_{G^0\Psi^0_1\Psi^0_3} = -b_{G^0\Psi^0_1\Psi^0_3} = -\frac{y_2}{\sqrt{3}} i = a_{G^0\Psi^0_3\Psi^0_1} = -b_{G^0\Psi^0_3\Psi^0_1}$$

$$a_{G^0X^0_iX^0_j} = a_{G^0\Psi^0_k\Psi^0_l} \mathcal{N}_{ki} \mathcal{N}_{lj}, \quad b_{G^0X^0_iX^0_j} = b_{G^0\Psi^0_k\Psi^0_l} \mathcal{N}^*_{ki} \mathcal{N}^*_{lj}$$

$$\left(\begin{array}{l} a_{G^0X^0_jX^0_i} = a_{G^0X^0_iX^0_j}, \quad b_{G^0X^0_jX^0_i} = b_{G^0X^0_iX^0_j}, \quad a_{G^0X^0_iX^0_j} = b^*_{G^0X^0_iX^0_j} \\ \frac{1}{2} a_{G^0X^0_1X^0_2} G^0 \bar{X}^0_{1R} X^0_{2L} \left(\frac{1}{2} b_{G^0X^0_1X^0_2} G^0 \bar{X}^0_{1L} X^0_{2R} \right) \text{ and } \frac{1}{2} a_{G^0X^0_2X^0_1} G^0 \bar{X}^0_{2R} X^0_{1L} \left(\frac{1}{2} b_{G^0X^0_2X^0_1} G^0 \bar{X}^0_{2L} X^0_{1R} \right) \text{ give an identical vertex!} \end{array} \right)$$

$$\bar{\Psi}^+_{iR}\Psi^+_{jL} = \begin{pmatrix}\psi^-_{iR} & 0\end{pmatrix}\begin{pmatrix}\psi^+_{jL} \\ 0\end{pmatrix} = \psi^-_{iR}\psi^+_{jL}, \quad \bar{\Psi}^+_{iL}\Psi^+_{jR} = \begin{pmatrix}0 & (\psi^+_{iL})^\dagger\end{pmatrix}\begin{pmatrix}0 \\ (\psi^-_{jR})^\dagger\end{pmatrix} = (\psi^+_{iL})^\dagger(\psi^-_{jR})^\dagger$$

$$\mathcal{L}_{h\Psi^+_i\Psi^+_j} = a_{h\Psi^+_i\Psi^+_j} h\bar{\Psi}^+_{iR}\Psi^+_{jL} + b_{h\Psi^+_i\Psi^+_j} h\bar{\Psi}^+_{iL}\Psi^+_{jR}$$

$$= a_{h\Psi^+_k\Psi^+_l}(\mathcal{C}_R)_{ki}(\mathcal{C}_L)_{lj} h\bar{X}^+_{iR}X^+_{jL} + b_{h\Psi^+_k\Psi^+_l}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_R)^*_{lj} h\bar{X}^+_{iL}X^+_{jR} = a_{hX^+_iX^+_j} h\bar{X}^+_{iR}X^+_{jL} + b_{hX^+_iX^+_j} h\bar{X}^+_{iL}X^+_{jR}$$

$$a_{h\Psi^+_1\Psi^+_2} = b_{h\Psi^+_2\Psi^+_1} = -\frac{y_1}{\sqrt{2}}, \quad a_{h\Psi^+_2\Psi^+_1} = b_{h\Psi^+_1\Psi^+_2} = \frac{y_1}{\sqrt{6}}, \quad a_{h\Psi^+_1\Psi^+_3} = b_{h\Psi^+_3\Psi^+_1} = \frac{y_2}{\sqrt{6}}, \quad a_{h\Psi^+_3\Psi^+_1} = b_{h\Psi^+_1\Psi^+_3} = -\frac{y_2}{\sqrt{2}}$$

$$a_{hX^+_iX^+_j} = a_{h\Psi^+_k\Psi^+_l}(\mathcal{C}_R)_{ki}(\mathcal{C}_L)_{lj}, \quad b_{hX^+_iX^+_j} = b_{h\Psi^+_k\Psi^+_l}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_R)^*_{lj}$$

$$\mathcal{L}_{G^0\Psi^+_i\Psi^+_j} = a_{G^0\Psi^+_i\Psi^+_j} G^0\bar{\Psi}^+_{iR}\Psi^+_{jL} + b_{G^0\Psi^+_i\Psi^+_j} G^0\bar{\Psi}^+_{iL}\Psi^+_{jR}$$

$$= a_{G^0\Psi^+_k\Psi^+_l}(\mathcal{C}_R)_{ki}(\mathcal{C}_L)_{lj} G^0\bar{X}^+_{iR}X^+_{jL} + b_{G^0\Psi^+_k\Psi^+_l}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_R)^*_{lj} G^0\bar{X}^+_{iL}X^+_{jR} = a_{G^0X^+_iX^+_j} G^0\bar{X}^+_{iR}X^+_{jL} + b_{G^0X^+_iX^+_j} G^0\bar{X}^+_{iL}X^+_{jR}$$

$$a_{G^0\Psi^+_1\Psi^+_2} = -b_{G^0\Psi^+_2\Psi^+_1} = -\frac{y_1}{\sqrt{2}}i, \quad a_{G^0\Psi^+_2\Psi^+_1} = -b_{G^0\Psi^+_1\Psi^+_2} = \frac{y_1}{\sqrt{6}}i, \quad a_{G^0\Psi^+_1\Psi^+_3} = -b_{G^0\Psi^+_3\Psi^+_1} = -\frac{y_2}{\sqrt{6}}i, \quad a_{G^0\Psi^+_3\Psi^+_1} = -b_{G^0\Psi^+_1\Psi^+_3} = \frac{y_2}{\sqrt{2}}i$$

$$a_{G^0X^+_iX^+_j} = a_{G^0\Psi^+_k\Psi^+_l}(\mathcal{C}_R)_{ki}(\mathcal{C}_L)_{lj}, \quad b_{G^0X^+_iX^+_j} = b_{G^0\Psi^+_k\Psi^+_l}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_R)^*_{lj}$$

$$\bar{\Psi}^+_{iR}\Psi^0_{jL} = \psi^-_{iR}\psi^0_{jL}, \quad \bar{\Psi}^+_{iL}\Psi^0_{jR} = (\psi^+_{iL})^\dagger(\psi^0_{jR})^\dagger$$

$$\mathcal{L}_{G^\pm\Psi^+_i\Psi^0_j} = a_{G^\pm\Psi^+_i\Psi^0_j}(G^+\bar{\Psi}^+_{iR}\Psi^0_{jL} + h.c.) + b_{G^\pm\Psi^+_i\Psi^0_j}(G^+\bar{\Psi}^+_{iL}\Psi^0_{jR} + h.c.)$$

$$= a_{G^\pm\Psi^+_k\Psi^0_l}[(\mathcal{C}_R)_{ki}\mathcal{N}_{lj}G^+\bar{X}^+_{iR}X^0_{jL} + h.c.] + b_{G^\pm\Psi^+_k\Psi^0_l}[(\mathcal{C}_L)^*_{ki}\mathcal{N}^*_{lj}G^+\bar{X}^+_{iL}X^0_{jR} + h.c.]$$

$$= a_{G^\pm X^+_iX^0_j}G^+\bar{X}^+_{iR}X^0_{jL} + a^*_{G^\pm X^+_iX^0_j}G^-\bar{X}^0_{jL}X^+_{iR} + b_{G^\pm X^+_iX^0_j}G^+\bar{X}^+_{iL}X^0_{jR} + b^*_{G^\pm X^+_iX^0_j}G^-\bar{X}^0_{jR}X^+_{iL}$$

$$a_{G^\pm\Psi^+_1\Psi^0_2} = -\frac{y_1}{\sqrt{3}}, \quad a_{G^\pm\Psi^+_2\Psi^0_1} = -\frac{2y_1}{\sqrt{6}}, \quad b_{G^\pm\Psi^+_1\Psi^0_3} = \frac{y_2}{\sqrt{3}}, \quad b_{G^\pm\Psi^+_3\Psi^0_1} = -\frac{2y_2}{\sqrt{6}}$$

$$a_{G^\pm X^+_iX^0_j} = a_{G^\pm\Psi^+_k\Psi^0_l}(\mathcal{C}_R)_{ki}\mathcal{N}_{lj}, \quad b_{G^\pm X^+_iX^0_j} = b_{G^\pm\Psi^+_k\Psi^0_l}(\mathcal{C}_L)^*_{ki}\mathcal{N}^*_{lj}$$

$$\bar{X}^{++}_R\Psi^+_{iL} = Q^{--}_1\psi^+_{iL}, \quad \bar{X}^{++}_L\Psi^+_{iR} = (Q^{++}_2)^\dagger(\psi^-_{iR})^\dagger$$

$$\mathcal{L}_{G^\pm X^{++}\Psi^+_i} = a_{G^\pm X^{++}\Psi^+_i}(G^+\bar{X}^{++}_R\Psi^+_{iL} + h.c.) + b_{G^\pm X^{++}\Psi^+_i}(G^+\bar{X}^{++}_L\Psi^+_{iR} + h.c.)$$

$$= a_{G^\pm X^{++}\Psi^+_j}[(\mathcal{C}_L)_{ji}G^+\bar{X}^{++}_RX^+_{iL} + h.c.] + b_{G^\pm X^{++}\Psi^+_j}[(\mathcal{C}_R)^*_{ji}G^+\bar{X}^{++}_LX^+_{iR} + h.c.]$$

$$= a_{G^\pm X^{++}X^+_i}G^+\bar{X}^{++}_RX^+_{iL} + a^*_{G^\pm X^{++}X^+_i}G^-\bar{X}^+_{iL}X^{++}_R + b_{G^\pm X^{++}X^+_i}G^+\bar{X}^{++}_LX^+_{iR} + b^*_{G^\pm X^{++}X^+_i}G^-\bar{X}^+_{iR}X^{++}_L$$

$$a_{G^\pm X^{++}\Psi^+_1} = y_1, \quad b_{G^\pm X^{++}\Psi^+_1} = -y_2$$

$$a_{G^\pm X^{++}X^+_i} = a_{G^\pm X^{++}\Psi^+_j}(\mathcal{C}_L)_{ji}, \quad b_{G^\pm X^{++}X^+_i} = b_{G^\pm X^{++}\Psi^+_j}(\mathcal{C}_R)^*_{ji}$$

Ref: Baro & Boudjema, 0906.1665

$$X_i = \begin{pmatrix} \chi_i^L \\ (\chi_i^R)^\dagger \end{pmatrix}$$

$$\mathcal{L}_X = \bar{X}_i i\gamma^\mu \partial_\mu X_i - \bar{X}_i \tilde{\mathcal{M}}_{ij} X_j = \bar{X}_i i\gamma^\mu \partial_\mu X_i - \tilde{\mathcal{M}}_{ij} \bar{X}_i^R X_j^L - \tilde{\mathcal{M}}_{ji}^* \bar{X}_i^L X_j^R$$

$$= (\chi_i^L)^\dagger \bar{\sigma}^\mu i\partial_\mu \chi_i^L + (\chi_i^R)^\dagger \bar{\sigma}^\mu i\partial_\mu \chi_i^R - m_{\chi_i}[\chi_i^R \chi_i^L + (\chi_i^L)^\dagger (\chi_i^R)^\dagger]$$

$$\chi^L = D_L \psi^L, \quad \chi^R = D_R \psi^R, \quad \tilde{\mathcal{M}} = D_R^* \mathcal{M} D_L^\dagger = \mathrm{diag}(m_{\chi_1}, m_{\chi_2}, m_{\chi_3})$$

$$\chi_{i,0}^L = \sqrt{Z_{ij}^L}\chi_j^L = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}^L\right)\chi_j^L, \quad \chi_{i,0}^R = \sqrt{Z_{ij}^R}\chi_j^R = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}^R\right)\chi_j^R, \quad X_{i,0} = X_i + \frac{1}{2}(\delta Z_{ij}^L P_L + \delta Z_{ij}^{R*} P_R) X_j$$

$$\mathcal{M}_{ij,0} = \mathcal{M}_{ij} + \delta\mathcal{M}_{ij}, \quad \tilde{\mathcal{M}}_{ij,0} = \tilde{\mathcal{M}}_{ij} + \delta\tilde{\mathcal{M}}_{ij} = m_{\chi_i}\delta_{ij} + \delta\tilde{\mathcal{M}}_{ij}$$

Fixing $\delta D_{L,R} = 0 \quad \Rightarrow \quad \tilde{\mathcal{M}}_0 = D_R^* \mathcal{M}_0 D_L^\dagger = D_R^* \mathcal{M} D_L^\dagger + D_R^* \delta\mathcal{M} D_L^\dagger \quad \Rightarrow \quad \delta\tilde{\mathcal{M}} = D_R^* \delta\mathcal{M} D_L^\dagger$

$$\bar{X}_{i,0} i\gamma^\mu \partial_\mu X_{j,0} = \bar{X}_i \left[\delta_{ki} + \frac{1}{2}(\delta Z_{ki}^{L*} P_R + \delta Z_{ki}^R P_L)\right] i\gamma^\mu \partial_\mu \left[\delta_{kj} + \frac{1}{2}(\delta Z_{kj}^L P_L + \delta Z_{kj}^{R*} P_R)\right] X_j$$

$$= \bar{X}_i i\gamma^\mu \partial_\mu X_i + \frac{1}{2}\bar{X}_i (\delta Z_{ij}^L \gamma^\mu P_L + \delta Z_{ij}^{R*}\gamma^\mu P_R + \delta Z_{ji}^{L*} P_R \gamma^\mu + \delta Z_{ji}^R P_L \gamma^\mu) i\partial_\mu X_j$$

$$-\tilde{\mathcal{M}}_{0,ij} \bar{X}_{i,0}^R X_{j,0}^L = -(m_{\chi_k}\delta_{kl} + \delta\tilde{\mathcal{M}}_{kl})\left(\delta_{ki} + \frac{1}{2}\delta Z_{ki}^R\right)\left(\delta_{lj} + \frac{1}{2}\delta Z_{lj}^L\right)\bar{X}_i^R X_j^L$$

$$= -m_{\chi_i}\bar{X}_i^R X_i^L - \delta\tilde{\mathcal{M}}_{ij}\bar{X}_i^R X_j^L - \frac{1}{2}(m_{\chi_i}\delta Z_{ij}^L + m_{\chi_j}\delta Z_{ji}^R)\bar{X}_i^R X_j^L$$

$$-\tilde{\mathcal{M}}_{ji,0}^* \bar{X}_{i,0}^L X_{j,0}^R = -(m_{\chi_l}\delta_{lk} + \delta\tilde{\mathcal{M}}_{lk}^*)\left(\delta_{ki} + \frac{1}{2}\delta Z_{ki}^{L*}\right)\left(\delta_{lj} + \frac{1}{2}\delta Z_{lj}^{R*}\right)\bar{X}_i^L X_j^R$$

$$= -m_{\chi_i}\bar{X}_i^L X_i^R - \delta\tilde{\mathcal{M}}_{ji}^*\bar{X}_i^L X_j^R - \frac{1}{2}(m_{\chi_j}\delta Z_{ji}^{L*} + m_{\chi_i}\delta Z_{ij}^{R*})\bar{X}_i^L X_j^R$$

$$\mathcal{L}_{X,0} = \mathcal{L}_X + \frac{1}{2}\bar{X}_i(\delta Z_{ij}^L\gamma^\mu P_L + \delta Z_{ij}^{R*}\gamma^\mu P_R + \delta Z_{ji}^{L*} P_R\gamma^\mu + \delta Z_{ji}^R P_L\gamma^\mu) i\partial_\mu X_j - \delta\tilde{\mathcal{M}}_{ij}\bar{X}_i P_L X_j - \delta\tilde{\mathcal{M}}_{ji}^*\bar{X}_i P_R X_j$$
$$-\frac{1}{2}\bar{X}_i[m_{\chi_i}(\delta Z_{ij}^L P_L + \delta Z_{ij}^{R*} P_R) + m_{\chi_j}(\delta Z_{ji}^{L*} P_R + \delta Z_{ji}^R P_L)] X_j$$

$$\hat{\Sigma}_{X_i X_j}(q) = (\not{q} - m_{\chi_i})\delta_{ij} + \Sigma_{X_i X_j}(q) - P_L\delta\tilde{\mathcal{M}}_{ij} - P_R\delta\tilde{\mathcal{M}}_{ji}^* + \frac{1}{2}(\not{q} - m_{\chi_i})(\delta Z_{ij}^L P_L + \delta Z_{ij}^{R*} P_R) + \frac{1}{2}(\delta Z_{ji}^{L*} P_R + \delta Z_{ji}^R P_L)(\not{q} - m_{\chi_j})$$

$$\Sigma_{X_i X_j}(q) = P_L \Sigma_{X_i X_j}^{LS}(q^2) + P_R \Sigma_{X_i X_j}^{RS}(q^2) + \not{q} P_L \Sigma_{X_i X_j}^{LV}(q^2) + \not{q} P_R \Sigma_{X_i X_j}^{RV}(q^2)$$

$$\hat{\Sigma}_{X_iX_j}(q) = (\not{q} - m_{\chi_i})\delta_{ij} + \Sigma_{X_iX_j}(q) - P_L\delta\tilde{\mathcal{M}}_{ij} - P_R\delta\tilde{\mathcal{M}}^*_{ji} + \frac{1}{2}(\not{q} - m_{\chi_i})(\delta Z^L_{ij}P_L + \delta Z^{R*}_{ij}P_R) + \frac{1}{2}(\delta Z^{L*}_{ji}P_R + \delta Z^R_{ji}P_L)(\not{q} - m_{\chi_i})$$

$$\Sigma_{X_iX_j}(q) = P_L\Sigma^{LS}_{X_iX_j}(q^2) + P_R\Sigma^{RS}_{X_iX_j}(q^2) + \not{q}P_L\Sigma^{LV}_{X_iX_j}(q^2) + \not{q}P_R\Sigma^{RV}_{X_iX_j}(q^2)$$

$$\bar{u}_{X_i}(q)\Sigma_{X_iX_j}(q)u_{X_j}(q) = [\bar{u}_{X_i}(q)\Sigma_{X_iX_j}(q)u_{X_j}(q)]^* = \bar{u}_{X_j}(q)[P_R\Sigma^{LS*}_{X_iX_j}(q^2) + P_L\Sigma^{RS*}_{X_iX_j}(q^2) + P_R\not{q}\Sigma^{LV*}_{X_iX_j}(q^2) + P_L\not{q}\Sigma^{RV*}_{X_iX_j}(q^2)]u_{X_i}(q)$$

$$= \bar{u}_{X_i}(q)[P_R\Sigma^{LS*}_{X_jX_i}(q^2) + P_L\Sigma^{RS*}_{X_jX_i}(q^2) + \not{q}P_L\Sigma^{LV*}_{X_jX_i}(q^2) + \not{q}P_R\Sigma^{RV*}_{X_jX_i}(q^2)]u_{X_j}(q)$$

$$\Rightarrow \quad \Sigma^{RS}_{X_iX_j}(q^2) = \Sigma^{LS*}_{X_jX_i}(q^2), \quad \Sigma^{LV}_{X_iX_j}(q^2) = \Sigma^{LV*}_{X_jX_i}(q^2), \quad \Sigma^{RV}_{X_iX_j}(q^2) = \Sigma^{RV*}_{X_jX_i}(q^2)$$

$$v_{X_j}(q) = C\bar{u}^{\mathrm{T}}_{X_j}(q), \quad \bar{u}_{X_i}(q) = v^{\mathrm{T}}_{X_i}(q)C$$

For Majorana fermions [Ref: Denner et al., Nucl.Phys. B387, 467 (1992)]

$$\bar{u}_{X_i}(q)\Sigma_{X_iX_j}(q)u_{X_j}(q) = -\bar{v}_{X_j}(q)\Sigma_{X_jX_i}(-q)v_{X_i}(q) = -u^{\mathrm{T}}_{X_j}(q)C\Sigma_{X_jX_i}(-q)C\bar{u}^{\mathrm{T}}_{X_i}(q) = -\bar{u}_{X_i}(q)C^{\mathrm{T}}\Sigma^{\mathrm{T}}_{X_jX_i}(-q)C^{\mathrm{T}}u_{X_j}(q)$$

$$= \bar{u}_{X_i}(q)C^{-1}[P_L\Sigma^{LS}_{X_jX_i}(q^2) + P_R\Sigma^{RS}_{X_jX_i}(q^2) - q^\mu(\gamma_\mu P_L)^{\mathrm{T}}\Sigma^{LV}_{X_jX_i}(q^2) - q^\mu(\gamma_\mu P_R)^{\mathrm{T}}\Sigma^{RV}_{X_jX_i}(q^2)]Cu_{X_j}(q)$$

$$= \bar{u}_{X_i}(q)[P_L\Sigma^{LS}_{X_jX_i}(q^2) + P_R\Sigma^{RS}_{X_jX_i}(q^2) + \not{q}P_R\Sigma^{LV}_{X_jX_i}(q^2) + \not{q}P_L\Sigma^{RV}_{X_jX_i}(q^2)]u_{X_j}(q)$$

$$\Rightarrow \quad \Sigma^{LS}_{X_iX_j}(q^2) = \Sigma^{LS}_{X_jX_i}(q^2), \quad \Sigma^{RS}_{X_iX_j}(q^2) = \Sigma^{RS}_{X_jX_i}(q^2), \quad \Sigma^{LV}_{X_iX_j}(q^2) = \Sigma^{RV}_{X_jX_i}(q^2)$$

$\widetilde{\mathrm{Re}}$ takes the real part of the loop integrals, but leaves the couplings alone.

$$0=\lim_{q^2\to m_{\chi_j}^2}\widetilde{\mathrm{Re}}\hat{\Sigma}_{X_iX_j}(q)u_{X_j}(q)$$

$$=\lim_{q^2\to m_{\chi_j}^2}\widetilde{\mathrm{Re}}[(\not{q}-m_{\chi_i})\delta_{ij}+P_L\Sigma^{LS}_{X_iX_j}(q^2)+P_R\Sigma^{RS}_{X_iX_j}(q^2)+\not{q}P_L\Sigma^{LV}_{X_iX_j}(q^2)+\not{q}P_R\Sigma^{RV}_{X_iX_j}(q^2)$$

$$-P_L\delta\tilde{\mathcal{M}}_{ij}-P_R\delta\tilde{\mathcal{M}}^*_{ji}+\frac{1}{2}(\not{q}-m_{\chi_i})(\delta Z^L_{ij}P_L+\delta Z^{R*}_{ij}P_R)+\frac{1}{2}(\delta Z^{L*}_{ji}P_R+\delta Z^R_{ji}P_L)(\not{q}-m_{\chi_j})]u_{X_j}(q)$$

$$\Rightarrow\quad \widetilde{\mathrm{Re}}[P_L\Sigma^{LS}_{X_iX_j}(m^2_{\chi_j})+P_R\Sigma^{RS}_{X_iX_j}(m^2_{\chi_j})+m_{\chi_j}P_R\Sigma^{LV}_{X_iX_j}(m^2_{\chi_j})+m_{\chi_j}P_L\Sigma^{RV}_{X_iX_j}(m^2_{\chi_j})$$

$$-P_L\delta\tilde{\mathcal{M}}_{ij}-P_R\delta\tilde{\mathcal{M}}^*_{ji}+\frac{1}{2}(m_{\chi_j}\delta Z^{R*}_{ij}-m_{\chi_i}\delta Z^L_{ij})P_L+\frac{1}{2}(m_{\chi_j}\delta Z^{L}_{ij}-m_{\chi_i}\delta Z^{R*}_{ij})P_R]=0$$

$$\Rightarrow\quad\begin{cases}\widetilde{\mathrm{Re}}\Sigma^{LS}_{X_iX_j}(m^2_{\chi_j})+m_{\chi_j}\widetilde{\mathrm{Re}}\Sigma^{RV}_{X_iX_j}(m^2_{\chi_j})-\delta\tilde{\mathcal{M}}_{ij}+\frac{1}{2}(m_{\chi_j}\delta Z^{R*}_{ij}-m_{\chi_i}\delta Z^L_{ij})=0\\ \widetilde{\mathrm{Re}}\Sigma^{RS}_{X_iX_j}(m^2_{\chi_j})+m_{\chi_j}\widetilde{\mathrm{Re}}\Sigma^{LV}_{X_iX_j}(m^2_{\chi_j})-\delta\tilde{\mathcal{M}}^*_{ji}+\frac{1}{2}(m_{\chi_j}\delta Z^{L}_{ij}-m_{\chi_i}\delta Z^{R*}_{ij})=0\end{cases}$$

$$i=j\quad\Rightarrow\quad\begin{cases}\widetilde{\mathrm{Re}}\Sigma^{LS}_{X_iX_i}(m^2_{\chi_i})+m_{\chi_i}\widetilde{\mathrm{Re}}\Sigma^{RV}_{X_iX_i}(m^2_{\chi_i})-\delta m_{\chi_i}+\frac{1}{2}m_{\chi_i}(\delta Z^{R*}_{ii}-\delta Z^L_{ii})=0\\ \widetilde{\mathrm{Re}}\Sigma^{RS}_{X_iX_i}(m^2_{\chi_i})+m_{\chi_i}\widetilde{\mathrm{Re}}\Sigma^{LV}_{X_iX_i}(m^2_{\chi_i})-\delta m_{\chi_i}+\frac{1}{2}m_{\chi_i}(\delta Z^{L}_{ii}-\delta Z^{R*}_{ii})=0\end{cases}$$

$$\Rightarrow\quad\delta m_{\chi_i}=\frac{1}{2}\widetilde{\mathrm{Re}}[\Sigma^{LS}_{X_iX_i}(m^2_{\chi_i})+\Sigma^{RS}_{X_iX_i}(m^2_{\chi_i})+m_{\chi_i}\Sigma^{LV}_{X_iX_i}(m^2_{\chi_i})+m_{\chi_i}\Sigma^{RV}_{X_iX_i}(m^2_{\chi_i})]$$

$$i\neq j\quad\Rightarrow\quad\begin{cases}\widetilde{\mathrm{Re}}\Sigma^{LS}_{X_iX_j}(m^2_{\chi_j})+m_{\chi_j}\widetilde{\mathrm{Re}}\Sigma^{RV}_{X_iX_j}(m^2_{\chi_j})-\delta\tilde{\mathcal{M}}_{ij}+\frac{1}{2}(m_{\chi_j}\delta Z^{R*}_{ij}-m_{\chi_i}\delta Z^L_{ij})=0\\ \{i\leftrightarrow j\}^*\quad\Rightarrow\quad\widetilde{\mathrm{Re}}\Sigma^{RS*}_{X_jX_i}(m^2_{\chi_i})+m_{\chi_i}\widetilde{\mathrm{Re}}\Sigma^{LV*}_{X_jX_i}(m^2_{\chi_i})-\delta\tilde{\mathcal{M}}_{ij}+\frac{1}{2}(m_{\chi_i}\delta Z^{L*}_{ji}-m_{\chi_j}\delta Z^{R}_{ji})=0\end{cases}$$

$$\delta Z^{L*}_{ji}=\delta Z^L_{ij}(m^2_{\chi_i}\leftrightarrow m^2_{\chi_j}),\quad\delta Z^R_{ji}=\delta Z^{R*}_{ij}(m^2_{\chi_i}\leftrightarrow m^2_{\chi_j})\quad\text{[Ref: Denner, 0709.1075, Eq. (3.21)]}$$

$$\Rightarrow\quad\begin{cases}\widetilde{\mathrm{Re}}\Sigma^{LS}_{X_iX_j}(m^2_{\chi_j})+m_{\chi_j}\widetilde{\mathrm{Re}}\Sigma^{RV}_{X_iX_j}(m^2_{\chi_j})-\delta\tilde{\mathcal{M}}_{ij}+\frac{1}{2}(m_{\chi_j}\delta Z^{R*}_{ij}-m_{\chi_i}\delta Z^L_{ij})=0\\ \widetilde{\mathrm{Re}}\Sigma^{RS*}_{X_jX_i}(m^2_{\chi_i})+m_{\chi_i}\widetilde{\mathrm{Re}}\Sigma^{LV*}_{X_jX_i}(m^2_{\chi_i})-\delta\tilde{\mathcal{M}}_{ij}+\frac{1}{2}(m_{\chi_i}\delta Z^{L}_{ij}-m_{\chi_j}\delta Z^{R*}_{ij})=0\end{cases}$$

$$\Rightarrow\quad\delta\tilde{\mathcal{M}}_{ij}=\frac{1}{2}[\widetilde{\mathrm{Re}}\Sigma^{RS*}_{X_jX_i}(m^2_{\chi_i})+\widetilde{\mathrm{Re}}\Sigma^{LS}_{X_iX_j}(m^2_{\chi_j})+m_{\chi_i}\widetilde{\mathrm{Re}}\Sigma^{LV*}_{X_jX_i}(m^2_{\chi_i})+m_{\chi_j}\widetilde{\mathrm{Re}}\Sigma^{RV}_{X_iX_j}(m^2_{\chi_j})]$$

$$\Sigma^{RS}_{X_iX_j}(q^2)=\Sigma^{LS*}_{X_jX_i}(q^2),\quad\Sigma^{LV}_{X_iX_j}(q^2)=\Sigma^{LV*}_{X_jX_i}(q^2)\quad\Rightarrow\quad\delta\tilde{\mathcal{M}}_{ij}=\frac{1}{2}\widetilde{\mathrm{Re}}[\Sigma^{LS}_{X_iX_j}(m^2_{\chi_i})+\Sigma^{LS}_{X_iX_j}(m^2_{\chi_j})+m_{\chi_i}\Sigma^{LV}_{X_iX_j}(m^2_{\chi_i})+m_{\chi_j}\Sigma^{RV}_{X_iX_j}(m^2_{\chi_j})]$$

Renormalization scheme

Ref: Baro & Boudjema, 0906.1665

$$\mathcal{M}_N = \begin{pmatrix} m_T & \frac{1}{\sqrt{3}} y_1 v & -\frac{1}{\sqrt{3}} y_2 v \\ \frac{1}{\sqrt{3}} y_1 v & 0 & m_Q \\ -\frac{1}{\sqrt{3}} y_2 v & m_Q & 0 \end{pmatrix}, \quad \mathcal{M}_C = \begin{pmatrix} m_T & \frac{1}{\sqrt{2}} y_1 v & -\frac{1}{\sqrt{6}} y_2 v \\ -\frac{1}{\sqrt{6}} y_1 v & 0 & -m_Q \\ \frac{1}{\sqrt{2}} y_2 v & -m_Q & 0 \end{pmatrix}$$

$$\delta(\mathcal{M}_N)_{11} = \delta(\mathcal{M}_C)_{11} = \delta m_T, \quad \delta(\mathcal{M}_N)_{23} = \delta(\mathcal{M}_N)_{32} = \delta m_Q, \quad \delta(\mathcal{M}_C)_{23} = \delta(\mathcal{M}_C)_{32} = -\delta m_Q$$

$$\delta(\mathcal{M}_N)_{12} = \delta(\mathcal{M}_N)_{21} = \frac{1}{\sqrt{3}} v\delta y_1, \quad \delta(\mathcal{M}_N)_{13} = \delta(\mathcal{M}_N)_{31} = -\frac{1}{\sqrt{3}} v\delta y_2$$

$$\delta(\mathcal{M}_C)_{12} = \frac{1}{\sqrt{2}} v\delta y_1, \quad \delta(\mathcal{M}_C)_{21} = -\frac{1}{\sqrt{6}} v\delta y_1, \quad \delta(\mathcal{M}_C)_{13} = -\frac{1}{\sqrt{6}} v\delta y_2, \quad \delta(\mathcal{M}_C)_{31} = \frac{1}{\sqrt{2}} v\delta y_2$$

$$\mathcal{N}^{\mathrm{T}} \mathcal{M}_N \mathcal{N} = \tilde{\mathcal{M}}_N = \mathrm{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}), \quad \mathcal{C}_R^{\mathrm{T}} \mathcal{M}_C \mathcal{C}_L = \tilde{\mathcal{M}}_C = \mathrm{diag}(m_{\chi_1^\pm}, m_{\chi_2^\pm}, m_{\chi_3^\pm})$$

$$(\delta\mathcal{M}_N)_{ij} = (\mathcal{N}^* \delta\tilde{\mathcal{M}}_N \mathcal{N}^\dagger)_{ij} = \mathcal{N}_{ik}^* (\delta\tilde{\mathcal{M}}_N)_{kl} \mathcal{N}_{lj}^\dagger = \mathcal{N}_{ik}^* \mathcal{N}_{jl}^* (\delta\tilde{\mathcal{M}}_N)_{kl}$$

$$\delta m_T = \mathcal{N}_{1k}^* \mathcal{N}_{1l}^* (\delta\tilde{\mathcal{M}}_N)_{kl}$$

$$\delta m_Q = \mathcal{N}_{2k}^* \mathcal{N}_{3l}^* (\delta\tilde{\mathcal{M}}_N)_{kl} = \mathcal{N}_{3k}^* \mathcal{N}_{2l}^* (\delta\tilde{\mathcal{M}}_N)_{kl}$$

$$v\delta y_1 = \sqrt{3}\mathcal{N}_{1k}^* \mathcal{N}_{2l}^* (\delta\tilde{\mathcal{M}}_N)_{kl} = \sqrt{3}\mathcal{N}_{2k}^* \mathcal{N}_{1l}^* (\delta\tilde{\mathcal{M}}_N)_{kl}$$

$$v\delta y_2 = -\sqrt{3}\mathcal{N}_{1k}^* \mathcal{N}_{3l}^* (\delta\tilde{\mathcal{M}}_N)_{kl} = -\sqrt{3}\mathcal{N}_{3k}^* \mathcal{N}_{1l}^* (\delta\tilde{\mathcal{M}}_N)_{kl}$$

$$(\delta\tilde{\mathcal{M}}_N)_{ij} = \frac{1}{2}[\widetilde{\mathrm{Re}}\Sigma^{LS}_{X_i^0 X_j^0}(m^2_{\chi_i^0}) + \widetilde{\mathrm{Re}}\Sigma^{LS}_{X_i^0 X_j^0}(m^2_{\chi_j^0}) + m_{\chi_i^0}\widetilde{\mathrm{Re}}\Sigma^{LV}_{X_i^0 X_j^0}(m^2_{\chi_i^0}) + m_{\chi_j^0}\widetilde{\mathrm{Re}}\Sigma^{RV}_{X_i^0 X_j^0}(m^2_{\chi_j^0})]$$

$$(\delta\tilde{\mathcal{M}}_N)_{ii} = \delta m_{\chi_i^0} = \frac{1}{2}\widetilde{\mathrm{Re}}[\Sigma^{LS}_{X_i^0 X_i^0}(m^2_{\chi_i^0}) + \Sigma^{RS}_{X_i^0 X_i^0}(m^2_{\chi_i^0}) + m_{\chi_i^0}\Sigma^{LV}_{X_i^0 X_i^0}(m^2_{\chi_i^0}) + m_{\chi_i^0}\Sigma^{RV}_{X_i^0 X_i^0}(m^2_{\chi_i^0})]$$

$$m^{\mathrm{phys}}_{\chi_i^0} = m_{\chi_i^0}$$

$$m^{\mathrm{phys}}_{\chi_i^\pm} = m_{\chi_i^\pm} + (\delta\tilde{\mathcal{M}}_C)_{ii} - \frac{1}{2}\widetilde{\mathrm{Re}}[\Sigma^{LS}_{X_i^+ X_i^+}(m^2_{\chi_i^\pm}) + \Sigma^{RS}_{X_i^+ X_i^+}(m^2_{\chi_i^\pm}) + m_{\chi_i^\pm}\Sigma^{LV}_{X_i^+ X_i^+}(m^2_{\chi_i^\pm}) + m_{\chi_i^\pm}\Sigma^{RV}_{X_i^+ X_i^+}(m^2_{\chi_i^\pm})]$$

$$\begin{aligned}(\delta\tilde{\mathcal{M}}_C)_{ii} &= (\mathcal{C}_R^{\mathrm{T}} \delta\mathcal{M}_C \mathcal{C}_L)_{ii} = (\mathcal{C}_R^{\mathrm{T}})_{ij} (\delta\mathcal{M}_C)_{jk} (\mathcal{C}_L)_{ki} = (\mathcal{C}_R)_{ji} (\mathcal{C}_L)_{ki} (\delta\mathcal{M}_C)_{jk} \\ &= (\mathcal{C}_R)_{1i}(\mathcal{C}_L)_{1i}\delta m_T - [(\mathcal{C}_R)_{2i}(\mathcal{C}_L)_{3i} + (\mathcal{C}_R)_{3i}(\mathcal{C}_L)_{2i}]\delta m_Q \\ &\quad + \frac{1}{\sqrt{6}} v\delta y_1[\sqrt{3}(\mathcal{C}_R)_{1i}(\mathcal{C}_L)_{2i} - (\mathcal{C}_R)_{2i}(\mathcal{C}_L)_{1i}] + \frac{1}{\sqrt{6}} v\delta y_2[\sqrt{3}(\mathcal{C}_R)_{3i}(\mathcal{C}_L)_{1i} - (\mathcal{C}_R)_{1i}(\mathcal{C}_L)_{3i}]\end{aligned}$$

$$m^{\mathrm{phys}}_{\chi^{\pm\pm}} = m_Q + \delta m_Q - \frac{1}{2}\widetilde{\mathrm{Re}}[\Sigma^{LS}_{X^{++}}(m^2_{\chi^{\pm\pm}}) + \Sigma^{RS}_{X^{++}}(m^2_{\chi^{\pm\pm}}) + m_Q\Sigma^{LV}_{X^{++}}(m^2_{\chi^{\pm\pm}}) + m_Q\Sigma^{RV}_{X^{++}}(m^2_{\chi^{\pm\pm}})]$$

Ref: Denner, 0709.1075; Denner et al., Nucl.Phys. B387, 467 (1992)

$$\Sigma_{X_iX_j}(p) = P_L\Sigma^{LS}_{X_iX_j}(p^2) + P_R\Sigma^{RS}_{X_iX_j}(p^2) + \not{p}P_L\Sigma^{LV}_{X_iX_j}(p) + \not{p}P_R\Sigma^{RV}_{X_iX_j}(p^2)$$

$$B_0(p^2,m_1^2,m_2^2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{1}{[q^2-m_1^2+i\varepsilon][(p+q)^2-m_2^2+i\varepsilon]}$$

$$p_\mu B_1(p^2,m_1^2,m_2^2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{q_\mu}{[q^2-m_1^2+i\varepsilon][(p+q)^2-m_2^2+i\varepsilon]}$$

$\overline{\mathrm{DR}}$ regularization scheme: $\gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu,\quad \gamma^\mu\gamma_\mu = 4$

$$\bar{u}_{X_i^0}(p)i\Sigma_{X_i^0-W^-X_j^+-X_k^0}(p^2)u_{X_k^0}(p)$$

$$= \mu^{4-D}\sum_j\int\frac{d^Dq}{(2\pi)^D}\bar{u}_{X_i^0}(p)i\gamma^\mu(a^*_{WX_j^+X_i^0}P_L + b^*_{WX_j^+X_i^0}P_R)\frac{i(-\not{q}+m_{\chi_j^\pm})}{q^2-m^2_{\chi_j^\pm}+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2-m_W^2+i\varepsilon}i\gamma^\nu(a_{WX_j^+X_k^0}P_L + b_{WX_j^+X_k^0}P_R)u_{X_k^0}(p)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\bar{u}_{X_i^0}(p)(a^*_{WX_j^+X_i^0}P_R + b^*_{WX_j^+X_i^0}P_L)\frac{-(2\not{q}+4m_{\chi_j^\pm})}{[q^2-m^2_{\chi_j^\pm}+i\varepsilon][(p+q)^2-m_W^2+i\varepsilon]}(a_{WX_j^+X_k^0}P_L + b_{WX_j^+X_k^0}P_R)u_{X_k^0}(p)$$

$$16\pi^2\Sigma^{LV}_{X_i^0-W^-X_j^+-X_k^0}(p^2) = -2\sum_j a^*_{WX_j^+X_i^0}a_{WX_j^+X_k^0}B_1(p^2,m^2_{\chi_j^\pm},m_W^2),\quad 16\pi^2\Sigma^{RV}_{X_i^0-W^-X_j^+-X_k^0}(p^2) = -2\sum_j b^*_{WX_j^+X_i^0}b_{WX_j^+X_k^0}B_1(p^2,m^2_{\chi_j^\pm},m_W^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^0-W^-X_j^+-X_k^0}(p^2) = -4\sum_j b^*_{WX_j^+X_i^0}a_{WX_j^+X_k^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_W^2),\quad 16\pi^2\Sigma^{RS}_{X_i^0-W^-X_j^+-X_k^0}(p^2) = -4\sum_j a^*_{WX_j^+X_i^0}b_{WX_j^+X_k^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_W^2)$$

$$C[\gamma^\mu(a_{WX_j^+X_i^0}P_L + b_{WX_j^+X_i^0}P_R)]^{\mathrm{T}}C^{-1} = -\gamma^\mu(a_{WX_j^+X_i^0}P_R + b_{WX_j^+X_i^0}P_L)$$

$$\bar{u}_{X_i^0}(p)i\Sigma_{X_i^0-W^+X_j^--X_k^0}(p^2)u_{X_k^0}(p)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\bar{u}_{X_i^0}(p)iC[\gamma^\mu(a_{WX_j^+X_i^0}P_L + b_{WX_j^+X_i^0}P_R)]^{\mathrm{T}}C^{-1}\frac{i(-\not{q}+m_{\chi_j^\pm})}{q^2-m^2_{\chi_j^\pm}+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2-m_W^2+i\varepsilon}iC[\gamma^\nu(a^*_{WX_j^+X_k^0}P_L + b^*_{WX_j^+X_k^0}P_R)]^{\mathrm{T}}C^{-1}u_{X_k^0}(p)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\bar{u}_{X_i^0}(p)i\gamma^\mu(a_{WX_j^+X_i^0}P_R + b_{WX_j^+X_i^0}P_L)\frac{i(-\not{q}+m_{\chi_j^\pm})}{q^2-m^2_{\chi_j^\pm}+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2-m_W^2+i\varepsilon}i\gamma^\nu(a^*_{WX_j^+X_k^0}P_R + b^*_{WX_j^+X_k^0}P_L)u_{X_k^0}(p)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\bar{u}_{X_i^0}(p)(a_{WX_j^+X_i^0}P_L + b_{WX_j^+X_i^0}P_R)\frac{-(2\not{q}+4m_{\chi_j^\pm})}{[q^2-m^2_{\chi_j^\pm}+i\varepsilon][(p+q)^2-m_W^2+i\varepsilon]}(a^*_{WX_j^+X_k^0}P_R + b^*_{WX_j^+X_k^0}P_L)u_{X_k^0}(p)$$

$$16\pi^2\Sigma^{LV}_{X_i^0-W^+X_j^--X_k^0}(p^2) = -2\sum_j b_{WX_j^+X_i^0}b^*_{WX_j^+X_k^0}B_1(p^2,m^2_{\chi_j^\pm},m_W^2),\quad 16\pi^2\Sigma^{RV}_{X_i^0-W^+X_j^--X_k^0}(p^2) = -2\sum_j a_{WX_j^+X_i^0}a^*_{WX_j^+X_k^0}B_1(p^2,m^2_{\chi_j^\pm},m_W^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^0-W^+X_j^--X_k^0}(p^2) = -4\sum_j a_{WX_j^+X_i^0}b^*_{WX_j^+X_k^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_W^2),\quad 16\pi^2\Sigma^{RS}_{X_i^0-W^+X_j^--X_k^0}(p^2) = -4\sum_j b_{WX_j^+X_i^0}a^*_{WX_j^+X_k^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_W^2)$$

$$i\Sigma_{X_i^0-G^-X_j^+-X_k^0}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dqi(a^*_{G^\pm X_j^+X_i^0}P_R+b^*_{G^\pm X_j^+X_i^0}P_L)\frac{i(-\not{q}+m_{\chi_j^\pm})}{q^2-m^2_{\chi_j^\pm}+i\varepsilon}\frac{i}{(p+q)^2-m_W^2+i\varepsilon}i(a_{G^\pm X_j^+X_k^0}P_L+b_{G^\pm X_j^+X_k^0}P_R)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a^*_{G^\pm X_j^+X_i^0}P_R+b^*_{G^\pm X_j^+X_i^0}P_L)\frac{-\not{q}+m_{\chi_j^\pm}}{[q^2-m^2_{\chi_j^\pm}+i\varepsilon][(p+q)^2-m_W^2+i\varepsilon]}(a_{G^\pm X_j^+X_k^0}P_L+b_{G^\pm X_j^+X_k^0}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^0-G^-X_j^+-X_k^0}(p^2)=-\sum_j a^*_{G^\pm X_j^+X_i^0}a_{G^\pm X_j^+X_k^0}B_1(p^2,m^2_{\chi_j^\pm},m_W^2),\quad 16\pi^2\Sigma^{RV}_{X_i^0-G^-X_j^+-X_k^0}(p^2)=-\sum_j b^*_{G^\pm X_j^+X_i^0}b_{G^\pm X_j^+X_k^0}B_1(p^2,m^2_{\chi_j^\pm},m_W^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^0-G^-X_j^+-X_k^0}(p^2)=\sum_j b^*_{G^\pm X_j^+X_i^0}a_{G^\pm X_j^+X_k^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_W^2),\quad 16\pi^2\Sigma^{RS}_{X_i^0-G^-X_j^+-X_k^0}(p^2)=\sum_j a^*_{G^\pm X_j^+X_i^0}b_{G^\pm X_j^+X_k^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_W^2)$$

$$C(a^*_{G^\pm X_j^+X_i^0}P_R+b^*_{G^\pm X_j^+X_i^0}P_L)^{\mathrm{T}}C^{-1}=a^*_{G^\pm X_j^+X_i^0}P_R+b^*_{G^\pm X_j^+X_i^0}P_L$$

$$i\Sigma_{X_i^0-G^+X_j^--X_k^0}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^DqiC(a_{G^\pm X_j^+X_i^0}P_L+b_{G^\pm X_j^+X_i^0}P_R)^{\mathrm{T}}C^{-1}\frac{i(-\not{q}+m_{\chi_j^\pm})}{q^2-m^2_{\chi_j^\pm}+i\varepsilon}\frac{i}{(p+q)^2-m_W^2+i\varepsilon}iC(a^*_{G^\pm X_j^+X_k^0}P_R+b^*_{G^\pm X_j^+X_k^0}P_L)^{\mathrm{T}}C^{-1}$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{G^\pm X_j^+X_i^0}P_L+b_{G^\pm X_j^+X_i^0}P_R)\frac{-\not{q}+m_{\chi_j^\pm}}{[q^2-m^2_{\chi_j^\pm}+i\varepsilon][(p+q)^2-m_W^2+i\varepsilon]}(a^*_{G^\pm X_j^+X_k^0}P_R+b^*_{G^\pm X_j^+X_k^0}P_L)$$

$$16\pi^2\Sigma^{LV}_{X_i^0-G^-X_j^+-X_k^0}(p^2)=-\sum_j b_{G^\pm X_j^+X_i^0}b^*_{G^\pm X_j^+X_k^0}B_1(p^2,m^2_{\chi_j^\pm},m_W^2),\quad 16\pi^2\Sigma^{RV}_{X_i^0-G^-X_j^+-X_k^0}(p^2)=-\sum_j a_{G^\pm X_j^+X_i^0}a^*_{G^\pm X_j^+X_k^0}B_1(p^2,m^2_{\chi_j^\pm},m_W^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^0-G^-X_j^+-X_k^0}(p^2)=\sum_j a_{G^\pm X_j^+X_i^0}b^*_{G^\pm X_j^+X_k^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_W^2),\quad 16\pi^2\Sigma^{RS}_{X_i^0-G^-X_j^+-X_k^0}(p^2)=\sum_j b_{G^\pm X_j^+X_i^0}a^*_{G^\pm X_j^+X_k^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_W^2)$$

$$i\Sigma_{X_i^0-ZX_j^0-X_k^0}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq i\gamma^\mu(a_{ZX_i^0X_j^0}P_L+b_{ZX_i^0X_j^0}P_R)\frac{i(-\not{q}+m_{\chi_j^0})}{q^2-m_{\chi_j^0}^2+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2-m_Z^2+i\varepsilon}i\gamma^\nu(a_{ZX_j^0X_k^0}P_L+b_{ZX_j^0X_k^0}P_R)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{ZX_i^0X_j^0}P_R+b_{ZX_i^0X_j^0}P_L)\frac{-(2\not{q}+4m_{\chi_j^0})}{[q^2-m_{\chi_j^0}^2+i\varepsilon][(p+q)^2-m_Z^2+i\varepsilon]}(a_{ZX_j^0X_k^0}P_L+b_{ZX_j^0X_k^0}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^0-ZX_j^0-X_k^0}(p^2)=-2\sum_j a_{ZX_i^0X_j^0}a_{ZX_j^0X_k^0}B_1(p^2,m_{\chi_j^0}^2,m_Z^2),\quad 16\pi^2\Sigma^{RV}_{X_i^0-ZX_j^0-X_k^0}(p^2)=-2\sum_j b_{ZX_i^0X_j^0}b_{ZX_j^0X_k^0}B_1(p^2,m_{\chi_j^0}^2,m_Z^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^0-ZX_j^0-X_k^0}(p^2)=-4\sum_j b_{ZX_i^0X_j^0}a_{ZX_j^0X_k^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_Z^2),\quad 16\pi^2\Sigma^{RS}_{X_i^0-ZX_j^0-X_k^0}(p^2)=-4\sum_j a_{ZX_i^0X_j^0}b_{ZX_j^0X_k^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_Z^2)$$

$$i\Sigma_{X_i^0-hX_j^0-X_k^0}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq i(a_{hX_i^0X_j^0}P_L+b_{hX_i^0X_j^0}P_R)\frac{i(-\not{q}+m_{\chi_j^0})}{q^2-m_{\chi_j^0}^2+i\varepsilon}\frac{i}{(p+q)^2-m_h^2+i\varepsilon}i(a_{hX_j^0X_k^0}P_L+b_{hX_j^0X_k^0}P_R)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{hX_i^0X_j^0}P_L+b_{hX_i^0X_j^0}P_R)\frac{-\not{q}+m_{\chi_j^0}}{[q^2-m_{\chi_j^0}^2+i\varepsilon][(p+q)^2-m_h^2+i\varepsilon]}(a_{hX_j^0X_k^0}P_L+b_{hX_j^0X_k^0}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^0-hX_j^0-X_k^0}(p^2)=-\sum_j b_{hX_i^0X_j^0}a_{hX_j^0X_k^0}B_1(p^2,m_{\chi_j^0}^2,m_h^2),\quad 16\pi^2\Sigma^{RV}_{X_i^0-hX_j^0-X_k^0}(p^2)=-\sum_j a_{hX_i^0X_j^0}b_{hX_j^0X_k^0}B_1(p^2,m_{\chi_j^0}^2,m_h^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^0-hX_j^0-X_k^0}(p^2)=\sum_j a_{hX_i^0X_j^0}a_{hX_j^0X_k^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_h^2),\quad 16\pi^2\Sigma^{RS}_{X_i^0-hX_j^0-X_k^0}(p^2)=\sum_j b_{hX_i^0X_j^0}b_{hX_j^0X_k^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_h^2)$$

$$i\Sigma_{X_i^0-G^0X_j^0-X_k^0}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{G^0X_i^0X_j^0}P_L+b_{G^0X_i^0X_j^0}P_R)\frac{-\not{q}+m_{\chi_j^0}}{[q^2-m_{\chi_j^0}^2+i\varepsilon][(p+q)^2-m_Z^2+i\varepsilon]}(a_{G^0X_j^0X_k^0}P_L+b_{G^0X_j^0X_k^0}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^0-G^0X_j^0-X_k^0}(p^2)=-\sum_j b_{G^0X_i^0X_j^0}a_{G^0X_j^0X_k^0}B_1(p^2,m_{\chi_j^0}^2,m_Z^2),\quad 16\pi^2\Sigma^{RV}_{X_i^0-G^0X_j^0-X_k^0}(p^2)=-\sum_j a_{G^0X_i^0X_j^0}b_{G^0X_j^0X_k^0}B_1(p^2,m_{\chi_j^0}^2,m_Z^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^0-G^0X_j^0-X_k^0}(p^2)=\sum_j a_{G^0X_i^0X_j^0}a_{G^0X_j^0X_k^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_Z^2),\quad 16\pi^2\Sigma^{RS}_{X_i^0-G^0X_j^0-X_k^0}(p^2)=\sum_j b_{G^0X_i^0X_j^0}b_{G^0X_j^0X_k^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_Z^2)$$

$$\bar{u}_{X_i^+}(p)i\Sigma_{X_i^+-W^+X_j^0-X_k^+}(p^2)u_{X_k^+}(p)$$

$$=\mu^{4-D}\sum_j\int\frac{d^Dq}{(2\pi)^D}\bar{u}_{X_i^+}(p)i\gamma^\mu(a_{WX_i^+X_j^0}P_L+b_{WX_i^+X_j^0}P_R)\frac{i(-\not{q}+m_{\chi_j^0})}{q^2-m_{\chi_j^0}^2+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2-m_W^2+i\varepsilon}i\gamma^\nu(a^*_{WX_k^+X_j^0}P_L+b^*_{WX_k^+X_j^0}P_R)u_{X_k^+}(p)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\bar{u}_{X_i^+}(p)(a_{WX_i^+X_j^0}P_R+b_{WX_i^+X_j^0}P_L)\frac{-(2\not{q}+4m_{\chi_j^0})}{[q^2-m_{\chi_j^0}^2+i\varepsilon][(p+q)^2-m_W^2+i\varepsilon]}(a^*_{WX_k^+X_j^0}P_L+b^*_{WX_k^+X_j^0}P_R)u_{X_k^+}(p)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-W^+X_j^0-X_k^+}(p^2)=-2\sum_j a_{WX_i^+X_j^0}a^*_{WX_k^+X_j^0}B_1(p^2,m_{\chi_j^0}^2,m_W^2),\quad 16\pi^2\Sigma^{RV}_{X_i^+-W^+X_j^0-X_k^+}(p^2)=-2\sum_j b_{WX_i^+X_j^0}b^*_{WX_k^+X_j^0}B_1(p^2,m_{\chi_j^0}^2,m_W^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-W^+X_j^0-X_k^+}(p^2)=-4\sum_j b_{WX_i^+X_j^0}a^*_{WX_k^+X_j^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_W^2),\quad 16\pi^2\Sigma^{RS}_{X_i^+-W^+X_j^0-X_k^+}(p^2)=-4\sum_j a_{WX_i^+X_j^0}b^*_{WX_k^+X_j^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_W^2)$$

$$i\Sigma_{X_i^+-G^+X_j^0-X_k^+}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\,i(a_{G^\pm X_i^+X_j^0}P_L+b_{G^\pm X_i^+X_j^0}P_R)\frac{i(-\not{q}+m_{\chi_j^0})}{q^2-m_{\chi_j^0}^2+i\varepsilon}\frac{i}{(p+q)^2-m_W^2+i\varepsilon}i(a^*_{G^\pm X_k^+X_j^0}P_R+b^*_{G^\pm X_k^+X_j^0}P_L)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{G^\pm X_i^+X_j^0}P_L+b_{G^\pm X_i^+X_j^0}P_R)\frac{-\not{q}+m_{\chi_j^0}}{[q^2-m_{\chi_j^0}^2+i\varepsilon][(p+q)^2-m_W^2+i\varepsilon]}(a^*_{G^\pm X_k^+X_j^0}P_R+b^*_{G^\pm X_k^+X_j^0}P_L)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-G^+X_j^0-X_k^+}(p^2)=-\sum_j b_{G^\pm X_i^+X_j^0}b^*_{G^\pm X_k^+X_j^0}B_1(p^2,m_{\chi_j^0}^2,m_W^2),\quad 16\pi^2\Sigma^{RV}_{X_i^+-G^+X_j^0-X_k^+}(p^2)=-\sum_j a_{G^\pm X_i^+X_j^0}a^*_{G^\pm X_k^+X_j^0}B_1(p^2,m_{\chi_j^0}^2,m_W^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-G^+X_j^0-X_k^+}(p^2)=\sum_j a_{G^\pm X_i^+X_j^0}b^*_{G^\pm X_k^+X_j^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_W^2),\quad 16\pi^2\Sigma^{RS}_{X_i^+-G^+X_j^0-X_k^+}(p^2)=\sum_j b_{G^\pm X_i^+X_j^0}a^*_{G^\pm X_k^+X_j^0}m_{\chi_j^0}B_0(p^2,m_{\chi_j^0}^2,m_W^2)$$

$$i\Sigma_{X_i^+-AX_j^+-X_k^+}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\,i\gamma^\mu(a_{AX_i^+X_j^+}P_L+b_{AX_i^+X_j^+}P_R)\frac{i(-\not{q}+m_{\chi_j^\pm})}{q^2-m_{\chi_j^\pm}^2+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2+i\varepsilon}i\gamma^\nu(a_{AX_j^+X_k^+}P_L+b_{AX_j^+X_k^+}P_R)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{AX_i^+X_j^+}P_R+b_{AX_i^+X_j^+}P_L)\frac{-(2\not{q}+4m_{\chi_j^\pm})}{[q^2-m_{\chi_j^\pm}^2+i\varepsilon][(p+q)^2+i\varepsilon]}(a_{AX_j^+X_k^+}P_L+b_{AX_j^+X_k^+}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-AX_j^+-X_k^+}(p^2)=-2\sum_j a_{AX_i^+X_j^+}a_{AX_j^+X_k^+}B_1(p^2,m_{\chi_j^\pm}^2,0),\quad 16\pi^2\Sigma^{RV}_{X_i^+-AX_j^+-X_k^+}(p^2)=-2\sum_j b_{AX_i^+X_j^+}b_{AX_j^+X_k^+}B_1(p^2,m_{\chi_j^\pm}^2,0)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-AX_j^+-X_k^+}(p^2)=-4\sum_j b_{AX_i^+X_j^+}a_{AX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m_{\chi_j^\pm}^2,0),\quad 16\pi^2\Sigma^{RS}_{X_i^+-AX_j^+-X_k^+}(p^2)=-4\sum_j a_{AX_i^+X_j^+}b_{AX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m_{\chi_j^\pm}^2,0)$$

$$i\Sigma_{X_i^+-ZX_j^+-X_k^+}(p^2)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\, i\gamma^\mu(a_{ZX_i^+X_j^+}P_L + b_{ZX_i^+X_j^+}P_R)\frac{i(-\not{q}+m_{\chi_j^\pm})}{q^2-m_{\chi_j^\pm}^2+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2-m_Z^2+i\varepsilon}i\gamma^\nu(a_{ZX_j^+X_k^+}P_L + b_{ZX_j^+X_k^+}P_R)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{ZX_i^+X_j^+}P_R + b_{ZX_i^+X_j^+}P_L)\frac{-(2\not{q}+4m_{\chi_j^\pm})}{[q^2-m_{\chi_j^\pm}^2+i\varepsilon][(p+q)^2-m_Z^2+i\varepsilon]}(a_{ZX_j^+X_k^+}P_L + b_{ZX_j^+X_k^+}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-ZX_j^+-X_k^+}(p^2) = -2\sum_j a_{ZX_i^+X_j^+}a_{ZX_j^+X_k^+}B_1(p^2,m_{\chi_j^\pm}^2,m_Z^2),\quad 16\pi^2\Sigma^{RV}_{X_i^+-ZX_j^+-X_k^+}(p^2) = -2\sum_j b_{ZX_i^+X_j^+}b_{ZX_j^+X_k^+}B_1(p^2,m_{\chi_j^\pm}^2,m_Z^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-ZX_j^+-X_k^+}(p^2) = -4\sum_j b_{ZX_i^+X_j^+}a_{ZX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m_{\chi_j^\pm}^2,m_Z^2),\quad 16\pi^2\Sigma^{RS}_{X_i^+-ZX_j^+-X_k^+}(p^2) = -4\sum_j a_{ZX_i^+X_j^+}b_{ZX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m_{\chi_j^\pm}^2,m_Z^2)$$

$$i\Sigma_{X_i^+-hX_j^+-X_k^+}(p^2)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq\, i(a_{hX_i^+X_j^+}P_L + b_{hX_i^+X_j^+}P_R)\frac{i(-\not{q}+m_{\chi_j^\pm})}{q^2-m_{\chi_j^\pm}^2+i\varepsilon}\frac{i}{(p+q)^2-m_h^2+i\varepsilon}i(a_{hX_j^+X_k^+}P_L + b_{hX_j^+X_k^+}P_R)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{hX_i^+X_j^+}P_L + b_{hX_i^+X_j^+}P_R)\frac{-\not{q}+m_{\chi_j^\pm}}{[q^2-m_{\chi_j^\pm}^2+i\varepsilon][(p+q)^2-m_h^2+i\varepsilon]}(a_{hX_j^+X_k^+}P_L + b_{hX_j^+X_k^+}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-hX_j^+-X_k^+}(p^2) = -\sum_j b_{hX_i^+X_j^+}a_{hX_j^+X_k^+}B_1(p^2,m_{\chi_j^\pm}^2,m_h^2),\quad 16\pi^2\Sigma^{RV}_{X_i^+-hX_j^+-X_k^+}(p^2) = -\sum_j a_{hX_i^+X_j^+}b_{hX_j^+X_k^+}B_1(p^2,m_{\chi_j^\pm}^2,m_h^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-hX_j^+-X_k^+}(p^2) = \sum_j a_{hX_i^+X_j^+}a_{hX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m_{\chi_j^\pm}^2,m_h^2),\quad 16\pi^2\Sigma^{RS}_{X_i^+-hX_j^+-X_k^+}(p^2) = \sum_j b_{hX_i^+X_j^+}b_{hX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m_{\chi_j^\pm}^2,m_h^2)$$

$$i\Sigma_{X_i^+-G^0X_j^+-X_k^+}(p^2)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{G^0X_i^+X_j^+}P_L + b_{G^0X_i^+X_j^+}P_R)\frac{-\not{q}+m_{\chi_j^\pm}}{[q^2-m_{\chi_j^\pm}^2+i\varepsilon][(p+q)^2-m_Z^2+i\varepsilon]}(a_{G^0X_j^+X_k^+}P_L + b_{G^0X_j^+X_k^+}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-G^0X_j^+-X_k^+}(p^2) = -\sum_j b_{G^0X_i^+X_j^+}a_{G^0X_j^+X_k^+}B_1(p^2,m_{\chi_j^\pm}^2,m_Z^2),\quad 16\pi^2\Sigma^{RV}_{X_i^+-G^0X_j^+-X_k^+}(p^2) = -\sum_j a_{G^0X_i^+X_j^+}b_{G^0X_j^+X_k^+}B_1(p^2,m_{\chi_j^\pm}^2,m_Z^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-G^0X_j^+-X_k^+}(p^2) = \sum_j a_{G^0X_i^+X_j^+}a_{G^0X_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m_{\chi_j^\pm}^2,m_Z^2),\quad 16\pi^2\Sigma^{RS}_{X_i^+-G^0X_j^+-X_k^+}(p^2) = \sum_j b_{G^0X_i^+X_j^+}b_{G^0X_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m_{\chi_j^\pm}^2,m_Z^2)$$

$$i\Sigma_{X_i^+-W^-X^{++}-X_k^+}(p^2)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q i\gamma^\mu (a^*_{WX^{++}X_i^+}P_L + b^*_{WX^{++}X_i^+}P_R)\frac{i(-\not q + m_{\chi^{\pm\pm}})}{q^2 - m^2_{\chi^{\pm\pm}} + i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2 - m_W^2 + i\varepsilon} i\gamma^\nu (a_{WX^{++}X_k^+}P_L + b_{WX^{++}X_k^+}P_R)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q (a^*_{WX^{++}X_i^+}P_R + b^*_{WX^{++}X_i^+}P_L)\frac{-(2\not q + 4m_{\chi^{\pm\pm}})}{[q^2 - m^2_{\chi^{\pm\pm}} + i\varepsilon][(p+q)^2 - m_W^2 + i\varepsilon]}(a_{WX^{++}X_k^+}P_L + b_{WX^{++}X_k^+}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-W^-X^{++}-X_k^+}(p^2) = -2a^*_{WX^{++}X_i^+}a_{WX^{++}X_k^+}B_1(p^2, m^2_{\chi^{\pm\pm}}, m_W^2), \quad 16\pi^2\Sigma^{RV}_{X_i^+-W^-X^{++}-X_k^+}(p^2) = -2b^*_{WX^{++}X_i^+}b_{WX^{++}X_k^+}B_1(p^2, m^2_{\chi^{\pm\pm}}, m_W^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-W^-X^{++}-X_k^+}(p^2) = -4b^*_{WX^{++}X_i^+}a_{WX^{++}X_k^+}m_{\chi^{\pm\pm}}B_0(p^2, m^2_{\chi^{\pm\pm}}, m_W^2), \quad 16\pi^2\Sigma^{RS}_{X_i^+-W^-X^{++}-X_k^+}(p^2) = -4a^*_{WX^{++}X_i^+}b_{WX^{++}X_k^+}m_{\chi^{\pm\pm}}B_0(p^2, m^2_{\chi^{\pm\pm}}, m_W^2)$$

$$i\Sigma_{X_i^+-G^-X^{++}-X_k^+}(p^2)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q i(a^*_{G^\pm X^{++}X_i^+}P_R + b^*_{G^\pm X^{++}X_i^+}P_L)\frac{i(-\not q + m_{\chi^{\pm\pm}})}{q^2 - m^2_{\chi^{\pm\pm}} + i\varepsilon}\frac{i}{(p+q)^2 - m_W^2 + i\varepsilon} i(a_{G^\pm X^{++}X_k^+}P_L + b_{G^\pm X^{++}X_k^+}P_R)$$

$$= \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q (a^*_{G^\pm X^{++}X_i^+}P_R + b^*_{G^\pm X^{++}X_i^+}P_L)\frac{-\not q + m_{\chi^{\pm\pm}}}{[q^2 - m^2_{\chi^{\pm\pm}} + i\varepsilon][(p+q)^2 - m_W^2 + i\varepsilon]}(a_{G^\pm X^{++}X_k^+}P_L + b_{G^\pm X^{++}X_k^+}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-G^-X^{++}-X_k^+}(p^2) = -a^*_{G^\pm X^{++}X_i^+}a_{G^\pm X^{++}X_k^+}B_1(p^2, m^2_{\chi^{\pm\pm}}, m_W^2), \quad 16\pi^2\Sigma^{RV}_{X_i^+-G^-X^{++}-X_k^+}(p^2) = -b^*_{G^\pm X^{++}X_i^+}b_{G^\pm X^{++}X_k^+}B_1(p^2, m^2_{\chi^{\pm\pm}}, m_W^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-G^-X^{++}-X_k^+}(p^2) = b^*_{G^\pm X^{++}X_i^+}a_{G^\pm X^{++}X_k^+}m_{\chi^{\pm\pm}}B_0(p^2, m^2_{\chi^{\pm\pm}}, m_W^2), \quad 16\pi^2\Sigma^{RS}_{X_i^+-G^-X^{++}-X_k^+}(p^2) = a^*_{G^\pm X^{++}X_i^+}b_{G^\pm X^{++}X_k^+}m_{\chi^{\pm\pm}}B_0(p^2, m^2_{\chi^{\pm\pm}}, m_W^2)$$

$$\bar{u}_{X^{++}}(p) i\Sigma_{X^{++}-W^{+}X_i^{+}-X^{++}}(p^2) u_{X^{++}}(p)$$

$$= \mu^{4-D}\sum_j \int \frac{d^D q}{(2\pi)^D} \bar{u}_{X^{++}}(p) i\gamma^{\mu}(a_{WX^{++}X_i^{+}} P_L + b_{WX^{++}X_i^{+}} P_R) \frac{i(-\not{q} + m_{\chi_i^{+}})}{q^2 - m_{\chi_i^{+}}^2 + i\varepsilon} \frac{-ig_{\mu\nu}}{(p+q)^2 - m_W^2 + i\varepsilon} i\gamma^{\nu}(a^*_{WX^{++}X_i^{+}} P_L + b^*_{WX^{++}X_i^{+}} P_R) u_{X^{++}}(p)$$

$$= \frac{i}{16\pi^2} \frac{(2\pi\mu)^{4-D}}{i\pi^2} \sum_j \int d^D q \bar{u}_{X_i^{+}}(p)(a_{WX^{++}X_i^{+}} P_R + b_{WX^{++}X_i^{+}} P_L) \frac{-(2\not{q} + 4m_{\chi_i^{+}})}{[q^2 - m_{\chi_i^{+}}^2 + i\varepsilon][(p+q)^2 - m_W^2 + i\varepsilon]} (a^*_{WX^{++}X_i^{+}} P_L + b^*_{WX^{++}X_i^{+}} P_R) u_{X_i^{+}}(p)$$

$$16\pi^2 \Sigma^{LV}_{X^{++}-W^{+}X_i^{+}-X^{++}}(p^2) = -2\sum_i |a_{WX^{++}X_i^{+}}|^2 B_1(p^2, m_{\chi_i^{+}}^2, m_W^2), \quad 16\pi^2 \Sigma^{RV}_{X^{++}-W^{+}X_i^{+}-X^{++}}(p^2) = -2\sum_i |b_{WX^{++}X_i^{+}}|^2 B_1(p^2, m_{\chi_i^{+}}^2, m_W^2)$$

$$16\pi^2 \Sigma^{LS}_{X^{++}-W^{+}X_i^{+}-X^{++}}(p^2) = -4\sum_i b_{WX^{++}X_i^{+}} a^*_{WX^{++}X_i^{+}} m_{\chi_i^{+}} B_0(p^2, m_{\chi_i^{+}}^2, m_W^2), \quad 16\pi^2 \Sigma^{RS}_{X^{++}-W^{+}X_i^{+}-X^{++}}(p^2) = -4\sum_j a_{WX^{++}X_i^{+}} b^*_{WX^{++}X_i^{+}} m_{\chi_i^{+}} B_0(p^2, m_{\chi_i^{+}}^2, m_W^2)$$

$$i\Sigma_{X^{++}-G^{+}X_i^{+}-X^{++}}(p^2)$$

$$= \frac{i}{16\pi^2} \frac{(2\pi\mu)^{4-D}}{i\pi^2} \sum_j \int d^D q\, i(a_{G^{\pm}X^{++}X_i^{+}} P_L + b_{G^{\pm}X^{++}X_i^{+}} P_R) \frac{i(-\not{q} + m_{\chi_i^{+}})}{q^2 - m_{\chi_i^{+}}^2 + i\varepsilon} \frac{i}{(p+q)^2 - m_W^2 + i\varepsilon} i(a^*_{G^{\pm}X^{++}X_i^{+}} P_R + b^*_{G^{\pm}X^{++}X_i^{+}} P_L)$$

$$= \frac{i}{16\pi^2} \frac{(2\pi\mu)^{4-D}}{i\pi^2} \sum_j \int d^D q (a_{G^{\pm}X^{++}X_i^{+}} P_L + b_{G^{\pm}X^{++}X_i^{+}} P_R) \frac{-\not{q} + m_{\chi_i^{+}}}{[q^2 - m_{\chi_i^{+}}^2 + i\varepsilon][(p+q)^2 - m_W^2 + i\varepsilon]} (a^*_{G^{\pm}X^{++}X_i^{+}} P_R + b^*_{G^{\pm}X^{++}X_i^{+}} P_L)$$

$$16\pi^2 \Sigma^{LV}_{X^{++}-G^{+}X_i^{+}-X^{++}}(p^2) = -\sum_i |b_{G^{\pm}X^{++}X_i^{+}}|^2 B_1(p^2, m_{\chi_i^{+}}^2, m_W^2), \quad 16\pi^2 \Sigma^{RV}_{X^{++}-G^{+}X_i^{+}-X^{++}}(p^2) = -\sum_i |a_{G^{\pm}X^{++}X_i^{+}}|^2 B_1(p^2, m_{\chi_i^{+}}^2, m_W^2)$$

$$16\pi^2 \Sigma^{LS}_{X^{++}-G^{+}X_i^{+}-X^{++}}(p^2) = \sum_i a_{G^{\pm}X^{++}X_i^{+}} b^*_{G^{\pm}X^{++}X_i^{+}} m_{\chi_i^{+}} B_0(p^2, m_{\chi_i^{+}}^2, m_W^2), \quad 16\pi^2 \Sigma^{RS}_{X^{++}-G^{+}X_i^{+}-X^{++}}(p^2) = \sum_i b_{G^{\pm}X^{++}X_i^{+}} a^*_{G^{\pm}X^{++}X_i^{+}} m_{\chi_i^{+}} B_0(p^2, m_{\chi_i^{+}}^2, m_W^2)$$

$$i\Sigma_{X^{++}-AX^{++}-X^{++}}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dqi\gamma^\mu(a_{AX^{++}X^{++}}P_L+b_{AX^{++}X^{++}}P_R)\frac{i(-\not{q}+m_{\chi^{\pm\pm}})}{q^2-m^2_{\chi^{\pm\pm}}+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2+i\varepsilon}i\gamma^\nu(a_{AX^{++}X^{++}}P_L+b_{AX^{++}X^{++}}P_R)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{AX^{++}X^{++}}P_R+b_{AX^{++}X^{++}}P_L)\frac{-(2\not{q}+4m_{\chi^{\pm\pm}})}{[q^2-m^2_{\chi^{\pm\pm}}+i\varepsilon][(p+q)^2+i\varepsilon]}(a_{AX^{++}X^{++}}P_L+b_{AX^{++}X^{++}}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-AX_j^+-X_i^+}(p^2)=-2a^2_{AX^{++}X^{++}}B_1(p^2,m^2_{\chi^{\pm\pm}},0),\quad 16\pi^2\Sigma^{RV}_{X_i^+-AX_j^+-X_i^+}(p^2)=-2b^2_{AX^{++}X^{++}}B_1(p^2,m^2_{\chi^{\pm\pm}},0)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-AX_j^+-X_i^+}(p^2)=-4b_{AX^{++}X^{++}}a_{AX^{++}X^{++}}m_{\chi^{\pm\pm}}B_0(p^2,m^2_{\chi^{\pm\pm}},0),\quad 16\pi^2\Sigma^{RS}_{X_i^+-AX_j^+-X_i^+}(p^2)=-4a_{AX^{++}X^{++}}b_{AX^{++}X^{++}}m_{\chi^{\pm\pm}}B_0(p^2,m^2_{\chi^{\pm\pm}},0)$$

$$i\Sigma_{X^{++}-ZX^{++}-X^{++}}(p^2)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dqi\gamma^\mu(a_{ZX^{++}X^{++}}P_L+b_{ZX^{++}X^{++}}P_R)\frac{i(-\not{q}+m_{\chi^{\pm\pm}})}{q^2-m^2_{\chi^{\pm\pm}}+i\varepsilon}\frac{-ig_{\mu\nu}}{(p+q)^2-m_Z^2+i\varepsilon}i\gamma^\nu(a_{ZX^{++}X^{++}}P_L+b_{ZX^{++}X^{++}}P_R)$$

$$=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\sum_j\int d^Dq(a_{ZX^{++}X^{++}}P_R+b_{ZX^{++}X^{++}}P_L)\frac{-(2\not{q}+4m_{\chi^{\pm\pm}})}{[q^2-m^2_{\chi^{\pm\pm}}+i\varepsilon][(p+q)^2-m_Z^2+i\varepsilon]}(a_{ZX^{++}X^{++}}P_L+b_{ZX^{++}X^{++}}P_R)$$

$$16\pi^2\Sigma^{LV}_{X_i^+-ZX_j^+-X_i^+}(p^2)=-2a^2_{ZX^{++}X^{++}}B_1(p^2,m^2_{\chi^{\pm\pm}},m_Z^2),\quad 16\pi^2\Sigma^{RV}_{X_i^+-ZX_j^+-X_i^+}(p^2)=-2b^2_{ZX^{++}X^{++}}B_1(p^2,m^2_{\chi^{\pm\pm}},m_Z^2)$$

$$16\pi^2\Sigma^{LS}_{X_i^+-ZX_j^+-X_i^+}(p^2)=-4b_{ZX^{++}X^{++}}a_{ZX^{++}X^{++}}m_{\chi^{\pm\pm}}B_0(p^2,m^2_{\chi^{\pm\pm}},m_Z^2),\quad 16\pi^2\Sigma^{RS}_{X_i^+-ZX_j^+-X_i^+}(p^2)=-4a_{ZX^{++}X^{++}}b_{ZX^{++}X^{++}}m_{\chi^{\pm\pm}}B_0(p^2,m^2_{\chi^{\pm\pm}},m_Z^2)$$

$$B_0 \sim \Delta, \quad B_1 \sim -\frac{1}{2}\Delta, \quad \Delta = \frac{2}{4-D} - \gamma_E + \ln 4\pi$$

$$16\pi^2 \Sigma^{LV}_{X_i^0 X_k^0}(p^2) = \sum_j \Big(-2a^*_{WX_j^+X_i^0} a_{WX_j^+X_k^0} - 2b_{WX_j^+X_i^0} b^*_{WX_j^+X_k^0} - a^*_{G^\pm X_j^+X_i^0} a_{G^\pm X_j^+X_k^0} - b_{G^\pm X_j^+X_i^0} b^*_{G^\pm X_j^+X_k^0} \Big) B_1(p^2, m^2_{\chi_j^\pm}, m_W^2)$$
$$- \sum_j b_{hX_i^0X_j^0} a_{hX_j^0X_k^0} B_1(p^2, m^2_{\chi_j^0}, m_h^2) + \sum_j \Big(-2a_{ZX_i^0X_j^0} a_{ZX_j^0X_k^0} - b_{G^0X_i^0X_j^0} a_{G^0X_j^0X_k^0} \Big) B_1(p^2, m^2_{\chi_j^0}, m_Z^2)$$

$$16\pi^2 \Sigma^{RV}_{X_i^0 X_k^0}(p^2) = \sum_j \Big(-2b^*_{WX_j^+X_i^0} b_{WX_j^+X_k^0} - 2a_{WX_j^+X_i^0} a^*_{WX_j^+X_k^0} - b^*_{G^\pm X_j^+X_i^0} b_{G^\pm X_j^+X_k^0} - a_{G^\pm X_j^+X_i^0} a^*_{G^\pm X_j^+X_k^0} \Big) B_1(p^2, m^2_{\chi_j^\pm}, m_W^2)$$
$$- \sum_j a_{hX_i^0X_j^0} b_{hX_j^0X_k^0} B_1(p^2, m^2_{\chi_j^0}, m_h^2) + \sum_j \Big(-2b_{ZX_i^0X_j^0} b_{ZX_j^0X_k^0} - a_{G^0X_i^0X_j^0} b_{G^0X_j^0X_k^0} \Big) B_1(p^2, m^2_{\chi_j^0}, m_Z^2)$$

$$16\pi^2 \Sigma^{LS}_{X_i^0 X_k^0}(p^2) = \sum_j \Big(-4b^*_{WX_j^+X_i^0} a_{WX_j^+X_k^0} - 4a_{WX_j^+X_i^0} b^*_{WX_j^+X_k^0} + b^*_{G^\pm X_j^+X_i^0} a_{G^\pm X_j^+X_k^0} + a_{G^\pm X_j^+X_i^0} b^*_{G^\pm X_j^+X_k^0} \Big) m_{\chi_j^\pm} B_0(p^2, m^2_{\chi_j^\pm}, m_W^2)$$
$$+ \sum_j a_{hX_i^0X_j^0} a_{hX_j^0X_k^0} m_{\chi_j^0} B_0(p^2, m^2_{\chi_j^0}, m_h^2) + \sum_j \Big(-4b_{ZX_i^0X_j^0} a_{ZX_j^0X_k^0} + a_{G^0X_i^0X_j^0} a_{G^0X_j^0X_k^0} \Big) m_{\chi_j^0} B_0(p^2, m^2_{\chi_j^0}, m_Z^2)$$

$$16\pi^2 \Sigma^{RS}_{X_i^0 X_k^0}(p^2) = \sum_j \Big(-4a^*_{WX_j^+X_i^0} b_{WX_j^+X_k^0} - 4b_{WX_j^+X_i^0} a^*_{WX_j^+X_k^0} + a^*_{G^\pm X_j^+X_i^0} b_{G^\pm X_j^+X_k^0} + b_{G^\pm X_j^+X_i^0} a^*_{G^\pm X_j^+X_k^0} \Big) m_{\chi_j^\pm} B_0(p^2, m^2_{\chi_j^\pm}, m_W^2)$$
$$+ \sum_j b_{hX_i^0X_j^0} b_{hX_j^0X_k^0} m_{\chi_j^0} B_0(p^2, m^2_{\chi_j^0}, m_h^2) + \sum_j \Big(-4a_{ZX_i^0X_j^0} b_{ZX_j^0X_k^0} + b_{G^0X_i^0X_j^0} b_{G^0X_j^0X_k^0} \Big) m_{\chi_j^0} B_0(p^2, m^2_{\chi_j^0}, m_Z^2)$$

$$16\pi^2 \Sigma^{LV}_{X^{++}}(p^2) = \sum_i \Big(-2\,| a_{WX^{++}X_i^+} |^2 - | b_{G^\pm X^{++}X_i^+} |^2 \Big) B_1(p^2, m^2_{\chi_i^+}, m_W^2)$$
$$- 2a^2_{AX^{++}X^{++}} B_1(p^2, m^2_{\chi^{\pm\pm}}, 0) - 2a^2_{ZX^{++}X^{++}} B_1(p^2, m^2_{\chi^{\pm\pm}}, m_Z^2)$$

$$16\pi^2 \Sigma^{RV}_{X^{++}}(p^2) = \sum_i \Big(-2\,| b_{WX^{++}X_i^+} |^2 - | a_{G^\pm X^{++}X_i^+} |^2 \Big) B_1(p^2, m^2_{\chi_i^+}, m_W^2)$$
$$- 2b^2_{AX^{++}X^{++}} B_1(p^2, m^2_{\chi^{\pm\pm}}, 0) - 2b^2_{ZX^{++}X^{++}} B_1(p^2, m^2_{\chi^{\pm\pm}}, m_Z^2)$$

$$16\pi^2 \Sigma^{LS}_{X^{++}}(p^2) = \sum_i \Big(-4b_{WX^{++}X_i^+} a^*_{WX^{++}X_i^+} + a_{G^\pm X^{++}X_i^+} b^*_{G^\pm X^{++}X_i^+} \Big) m_{\chi_i^+} B_0(p^2, m^2_{\chi_i^+}, m_W^2)$$
$$- 4b_{AX^{++}X^{++}} a_{AX^{++}X^{++}} m_{\chi^{\pm\pm}} B_0(p^2, m^2_{\chi^{\pm\pm}}, 0) - 4b_{ZX^{++}X^{++}} a_{ZX^{++}X^{++}} m_{\chi^{\pm\pm}} B_0(p^2, m^2_{\chi^{\pm\pm}}, m_Z^2)$$

$$16\pi^2 \Sigma^{RS}_{X^{++}}(p^2) = \sum_i \Big(-4a_{WX^{++}X_i^+} b^*_{WX^{++}X_i^+} + b_{G^\pm X^{++}X_i^+} a^*_{G^\pm X^{++}X_i^+} \Big) m_{\chi_i^+} B_0(p^2, m^2_{\chi_i^+}, m_W^2)$$
$$- 4a_{AX^{++}X^{++}} b_{AX^{++}X^{++}} m_{\chi^{\pm\pm}} B_0(p^2, m^2_{\chi^{\pm\pm}}, 0) - 4a_{ZX^{++}X^{++}} b_{ZX^{++}X^{++}} m_{\chi^{\pm\pm}} B_0(p^2, m^2_{\chi^{\pm\pm}}, m_Z^2)$$

$$16\pi^2\Sigma^{LV}_{X_i^+X_k^+}(p^2)=\sum_j\left(-2a_{WX_i^+X_j^0}a^*_{WX_k^+X_j^0}-b_{G^\pm X_i^+X_j^0}b^*_{G^\pm X_k^+X_j^0}\right)B_1(p^2,m^2_{\chi_j^0},m_W^2)-2\sum_j a_{AX_i^+X_j^+}a_{AX_j^+X_k^+}B_1(p^2,m^2_{\chi_j^\pm},0)$$
$$+\sum_j\left(-2a_{ZX_i^+X_j^+}a_{ZX_j^+X_k^+}-b_{G^0X_i^+X_j^+}a_{G^0X_j^+X_k^+}\right)B_1(p^2,m^2_{\chi_j^\pm},m_Z^2)-\sum_j b_{hX_i^+X_j^+}a_{hX_j^+X_k^+}B_1(p^2,m^2_{\chi_j^\pm},m_h^2)$$
$$+\left(-2a^*_{WX^{++}X_i^+}a_{WX^{++}X_k^+}-a^*_{G^\pm X^{++}X_i^+}a_{G^\pm X^{++}X_k^+}\right)B_1(p^2,m^2_{\chi^{\pm\pm}},m_W^2)$$
$$16\pi^2\Sigma^{RV}_{X_i^+X_k^+}(p^2)=\sum_j\left(-2b_{WX_i^+X_j^0}b^*_{WX_k^+X_j^0}-a_{G^\pm X_i^+X_j^0}a^*_{G^\pm X_k^+X_j^0}\right)B_1(p^2,m^2_{\chi_j^0},m_W^2)-2\sum_j b_{AX_i^+X_j^+}b_{AX_j^+X_k^+}B_1(p^2,m^2_{\chi_j^\pm},0)$$
$$+\sum_j\left(-2b_{ZX_i^+X_j^+}b_{ZX_j^+X_k^+}-a_{G^0X_i^+X_j^+}b_{G^0X_j^+X_k^+}\right)B_1(p^2,m^2_{\chi_j^\pm},m_Z^2)-\sum_j a_{hX_i^+X_j^+}b_{hX_j^+X_k^+}B_1(p^2,m^2_{\chi_j^\pm},m_h^2)$$
$$+\left(-2b^*_{WX^{++}X_i^+}b_{WX^{++}X_k^+}-b^*_{G^\pm X^{++}X_i^+}b_{G^\pm X^{++}X_k^+}\right)B_1(p^2,m^2_{\chi^{\pm\pm}},m_W^2)$$
$$16\pi^2\Sigma^{LS}_{X_i^+X_k^+}(p^2)=\sum_j\left(-4b_{WX_i^+X_j^0}a^*_{WX_k^+X_j^0}+a_{G^\pm X_i^+X_j^0}b^*_{G^\pm X_k^+X_j^0}\right)m_{\chi_j^0}B_0(p^2,m^2_{\chi_j^0},m_W^2)-4\sum_j b_{AX_i^+X_j^+}a_{AX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},0)$$
$$+\sum_j\left(-4b_{ZX_i^+X_j^+}a_{ZX_j^+X_k^+}+a_{G^0X_i^+X_j^+}a_{G^0X_j^+X_k^+}\right)m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_Z^2)+\sum_j a_{hX_i^+X_j^+}a_{hX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_h^2)$$
$$+\left(-4b^*_{WX^{++}X_i^+}a_{WX^{++}X_k^+}+b^*_{G^\pm X^{++}X_i^+}a_{G^\pm X^{++}X_k^+}\right)m_{\chi^{\pm\pm}}B_0(p^2,m^2_{\chi^{\pm\pm}},m_W^2)$$
$$16\pi^2\Sigma^{RS}_{X_i^+X_k^+}(p^2)=\sum_j\left(-4a_{WX_i^+X_j^0}b^*_{WX_k^+X_j^0}+b_{G^\pm X_i^+X_j^0}a^*_{G^\pm X_k^+X_j^0}\right)m_{\chi_j^0}B_0(p^2,m^2_{\chi_j^0},m_W^2)-4\sum_j a_{AX_i^+X_j^+}b_{AX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},0)$$
$$+\sum_j\left(-4a_{ZX_i^+X_j^+}b_{ZX_j^+X_k^+}+b_{G^0X_i^+X_j^+}b_{G^0X_j^+X_k^+}\right)m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_Z^2)+\sum_j b_{hX_i^+X_j^+}b_{hX_j^+X_k^+}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_j^\pm},m_h^2)$$
$$+\left(-4a^*_{WX^{++}X_i^+}b_{WX^{++}X_k^+}+a^*_{G^\pm X^{++}X_i^+}b_{G^\pm X^{++}X_k^+}\right)m_{\chi^{\pm\pm}}B_0(p^2,m^2_{\chi^{\pm\pm}},m_W^2)$$

Oblique electroweak corrections

Ref: Peskin & Takeuchi, PRD 46, 381 (1992)

$$i\Pi_{IJ}^{\mu\nu}(p^2) = \int d^4x e^{-ipx} \left\langle J_I^\mu(x) J_J^\nu(0) \right\rangle \equiv i g^{\mu\nu}\Pi_{IJ}(p^2) + (p^\mu p^\nu \text{ terms})$$

$$\Pi_{AA}(p^2) = e^2\Pi_{QQ}(p^2), \quad \Pi_{ZA}(p^2) = \frac{e^2}{s_W c_W}[\Pi_{3Q}(p^2) - s_W^2\Pi_{QQ}(p^2)]$$

$$\Pi_{ZZ}(p^2) = \frac{e^2}{s_W^2 c_W^2}[\Pi_{33}(p^2) - 2s_W^2\Pi_{3Q}(p^2) + s_W^4\Pi_{QQ}(p^2)], \quad \Pi_{WW}(p^2) = \frac{e^2}{s_W^2}\Pi_{11}(p^2)$$

$$\Pi_{IJ}(p^2) \equiv \Pi_{IJ}(0) + p^2\Pi'_{IJ}(p^2) = \Pi_{IJ}(0) + p^2\Pi'_{IJ}(0) + \mathcal{O}(p^4)$$

$$\alpha S \equiv 4e^2[\Pi'_{33}(0) - \Pi'_{3Q}(0)], \quad \alpha T \equiv \frac{e^2}{s_W^2 c_W^2 m_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)], \quad \alpha U \equiv 4e^2[\Pi'_{11}(0) - \Pi'_{33}(0)]$$

They are contributed from operators like:

$$\frac{1}{\Lambda^2} B^{\mu\nu}(H^\dagger W_{\mu\nu}^a \sigma^a H) \quad \to \quad S, \qquad \frac{1}{\Lambda^2} H^\dagger H (D^\mu H)^\dagger D_\mu H \quad \to \quad T, \qquad \frac{1}{\Lambda^4}(H^\dagger W^{a\mu\nu}\sigma^a H)(H^\dagger W_{\mu\nu}^b \sigma^b H) \quad \to \quad U$$

$$\Pi_{QQ}(p^2) = \frac{1}{e^2}\Pi_{AA}(p^2), \quad \Pi_{11}(p^2) = \frac{s_W^2}{e^2}\Pi_{WW}(p^2)$$

$$\Pi_{3Q}(p^2) = \frac{s_W c_W}{e^2}\Pi_{ZA}(p^2) + s_W^2\Pi_{QQ}(p^2) = \frac{1}{e^2}[s_W c_W \Pi_{ZA}(p^2) + s_W^2\Pi_{AA}(p^2)]$$

$$\Pi_{33}(p^2) = \frac{s_W^2 c_W^2}{e^2}\Pi_{ZZ}(p^2) + 2s_W^2\Pi_{3Q}(p^2) - s_W^4\Pi_{QQ}(p^2) = \frac{1}{e^2}[s_W^2 c_W^2 \Pi_{ZZ}(p^2) + 2s_W^3 c_W \Pi_{ZA}(p^2) + s_W^4\Pi_{AA}(p^2)]$$

$$\frac{1}{4}\alpha S = e^2[\Pi'_{33}(0) - \Pi'_{3Q}(0)] = s_W^2 c_W^2 \Pi'_{ZZ}(0) + s_W c_W (2s_W^2 - 1)\Pi'_{ZA}(0) - s_W^2 c_W^2 \Pi'_{AA}(0)$$

$$s_W^2 c_W^2 m_Z^2 \alpha T = e^2[\Pi_{11}(0) - \Pi_{33}(0)] = s_W^2\Pi_{WW}(0) - s_W^2 c_W^2\Pi_{ZZ}(0) - 2s_W^3 c_W \Pi_{ZA}(0) - s_W^4\Pi_{AA}(0)$$

$$\frac{1}{4}\alpha U = e^2[\Pi'_{11}(0) - \Pi'_{33}(0)] = s_W^2\Pi'_{WW}(0) - s_W^2 c_W^2\Pi'_{ZZ}(0) - 2s_W^3 c_W \Pi'_{ZA}(0) - s_W^4\Pi'_{AA}(0)$$

$$g_{\mu\nu}B_{00}(p^2,m_1^2,m_2^2)+p_\mu p_\nu B_{11}(p^2,m_1^2,m_2^2)=\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{q_\mu q_\nu}{[q^2-m_1^2+i\varepsilon][(p+q)^2-m_2^2+i\varepsilon]}$$

$$B_{00}(p^2,m_1^2,m_2^2)\sim\frac{1}{6(D-4)}(p^2-3m_1^2-3m_2^2)$$

$$DB_{00}(p^2,m_1^2,m_2^2)=4B_{00}(p^2,m_1^2,m_2^2)+(D-4)B_{00}(p^2,m_1^2,m_2^2)=4B_{00}(p^2,m_1^2,m_2^2)+\frac{1}{6}(p^2-3m_1^2-3m_2^2)$$

$$4B_{00}(p^2,m_1^2,m_2^2)+p^2B_{11}(p^2,m_1^2,m_2^2)=A_0(m_2^2)+m_1^2B_0(p^2,m_1^2,m_2^2)+\frac{1}{6}(3m_1^2+3m_2^2-p^2)$$

[Note: the equation above can be verified with FeynCalc in Mathematica]

$$\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{q^2}{[q^2-m_1^2+i\varepsilon][(p+q)^2-m_2^2+i\varepsilon]}=g^{\mu\nu}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{q_\mu q_\nu}{[q^2-m_1^2+i\varepsilon][(p+q)^2-m_2^2+i\varepsilon]}$$

$$=DB_{00}(p^2,m_1^2,m_2^2)+p^2B_{11}(p^2,m_1^2,m_2^2)$$

$$=4B_{00}(p^2,m_1^2,m_2^2)+p^2B_{11}(p^2,m_1^2,m_2^2)+\frac{1}{6}(p^2-3m_1^2-3m_2^2)$$

$$=A_0(m_2^2)+m_1^2B_0(p^2,m_1^2,m_2^2)$$

$$2p\cdot q=[(q+p)^2-m_2^2+i\varepsilon]-(q^2-m_1^2+i\varepsilon)-(p^2+m_1^2-m_2^2)$$

$$2p^2B_1(p,m_1,m_2)=\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{2p\cdot q}{(q^2-m_1^2+i\varepsilon)[(q+p)^2-m_2^2+i\varepsilon]}$$

$$=\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\left\{\frac{1}{q^2-m_1^2+i\varepsilon}-\frac{1}{(q+p)^2-m_2^2+i\varepsilon}-\frac{p^2+m_1^2-m_2^2}{(q^2-m_1^2+i\varepsilon)[(q+p)^2-m_2^2+i\varepsilon]}\right\}$$

$$=A_0(m_1)-A_0(m_2)-(p^2+m_1^2-m_2^2)B_0(p,m_1,m_2)$$

$$B_1(p^2,m_1^2,m_2^2)=\frac{1}{2p^2}[A_0(m_1^2)-A_0(m_2^2)-(p^2+m_1^2-m_2^2)B_0(p^2,m_1^2,m_2^2)]$$

$$p^2B_1(p^2,m_1^2,m_2^2)+A_0(m_2^2)+m_1^2B_0(p^2,m_1^2,m_2^2)$$

$$=\frac{1}{2}[A_0(m_1^2)-A_0(m_2^2)-(p^2+m_1^2-m_2^2)B_0(p^2,m_1^2,m_2^2)]+A_0(m_2^2)+m_1^2B_0(p^2,m_1^2,m_2^2)$$

$$=\frac{1}{2}[A_0(m_1^2)+A_0(m_2^2)-(p^2-m_1^2-m_2^2)B_0(p^2,m_1^2,m_2^2)]$$

$$q_\rho(p+q)_\sigma \mathrm{Tr}(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma) = 4q_\rho(p+q)_\sigma(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\rho\nu}) = 4[p^\mu q^\nu + p^\nu q^\mu + 2q^\mu q^\nu - g^{\mu\nu}(p\cdot q + q^2)]$$

$$\mathrm{Tr}[\gamma^\mu(\not q + m_1)\gamma^\nu(\not p + \not q + m_2)] = q_\rho(p+q)_\sigma \mathrm{Tr}(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma) + m_1 m_2 \mathrm{Tr}(\gamma^\mu\gamma^\nu)$$

$$= 4p^\mu q^\nu + 4p^\nu q^\mu + 8q^\mu q^\nu - 4g^{\mu\nu}(p\cdot q + q^2) + 4m_1 m_2 g^{\mu\nu}$$

$$\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q \frac{-\mathrm{Tr}[\gamma^\mu(\not q + m_1)\gamma^\nu(\not p + \not q + m_2)]}{[q^2 - m_1^2 + i\varepsilon][(p+q)^2 - m_2^2 + i\varepsilon]}$$

$$= -8p^\mu p^\nu B_1(p^2, m_1^2, m_2^2) - 8[g^{\mu\nu}B_{00}(p^2, m_1^2, m_2^2) + p^\mu p^\nu B_{11}(p^2, m_1^2, m_2^2)]$$

$$\quad + 4g^{\mu\nu}[p^2 B_1(p^2, m_1^2, m_2^2) + A_0(m_2^2) + m_1^2 B_0(p^2, m_1^2, m_2^2)] - 4m_1 m_2 g^{\mu\nu} B_0(p^2, m_1^2, m_2^2)$$

$$= 2g^{\mu\nu}[A_0(m_1^2) + A_0(m_2^2) - (p^2 - m_1^2 - m_2^2)B_0(p^2, m_1^2, m_2^2) - 4B_{00}(p^2, m_1^2, m_2^2) - 2m_1 m_2 B_0(p^2, m_1^2, m_2^2)] + (p^\mu p^\nu \text{ terms})$$

$$\xrightarrow{m=m_1=m_2} 2g^{\mu\nu}[2A_0(m^2) - p^2 B_0(p^2, m^2, m^2) - 4B_{00}(p^2, m^2, m^2)] + (p^\mu p^\nu \text{ terms})$$

$$\mathrm{Tr}[\gamma^\mu(a_1 P_L + b_1 P_R)(\not q + m_1)\gamma^\nu(a_2 P_L + b_2 P_R)(\not p + \not q + m_2)]$$

$$= \frac{1}{4}\mathrm{Tr}\{\gamma^\mu[(a_1+b_1) + (b_1 - a_1)\gamma_5](\not q + m_1)\gamma^\nu[(a_2+b_2) + (b_2 - a_2)\gamma_5](\not p + \not q + m_2)\}$$

$$= \frac{1}{4}[(a_1+b_1)(a_2+b_2) + (b_1-a_1)(b_2-a_2)]q_\rho(p+q)_\sigma \mathrm{Tr}(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma) + \frac{1}{4}[(a_1+b_1)(a_2+b_2) - (b_1-a_1)(b_2-a_2)]m_1 m_2 \mathrm{Tr}(\gamma^\mu\gamma^\nu)$$

$$\quad + \frac{1}{4}[-(b_1-a_1)(a_2+b_2) - (a_1+b_1)(b_2-a_2)]q_\rho(p+q)_\sigma \mathrm{Tr}(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma\gamma_5)$$

$$= 2(a_1 a_2 + b_1 b_2)[p^\mu q^\nu + p^\nu q^\mu + 2q^\mu q^\nu - g^{\mu\nu}(p\cdot q + q^2)] + 2(a_1 b_2 + b_1 a_2)m_1 m_2 g^{\mu\nu} + 2i(a_1 a_2 - b_1 b_2)\varepsilon^{\mu\nu\rho\sigma}q_\rho p_\sigma$$

$$\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q \frac{-\mathrm{Tr}[\gamma^\mu(a_1 P_L + b_1 P_R)(\not q + m_1)\gamma^\nu(a_2 P_L + b_2 P_R)(\not p + \not q + m_2)]}{[q^2 - m_1^2 + i\varepsilon][(p+q)^2 - m_2^2 + i\varepsilon]}$$

$$= -2(a_1 a_2 + b_1 b_2)\{2g^{\mu\nu}B_{00}(p^2, m_1^2, m_2^2) - g^{\mu\nu}[p^2 B_1(p^2, m_1^2, m_2^2) + A_0(m_2^2) + m_1^2 B_0(p^2, m_1^2, m_2^2)]\}$$

$$\quad - 2(a_1 b_2 + b_1 a_2)m_1 m_2 g^{\mu\nu}B_0(p^2, m_1^2, m_2^2) + (p^\mu p^\nu \text{ terms})$$

$$= g^{\mu\nu}\{(a_1 a_2 + b_1 b_2)[A_0(m_1^2) + A_0(m_2^2) - (p^2 - m_1^2 - m_2^2)B_0(p^2, m_1^2, m_2^2) - 4B_{00}(p^2, m_1^2, m_2^2)]$$

$$\qquad - 2(a_1 b_2 + b_1 a_2)m_1 m_2 B_0(p^2, m_1^2, m_2^2)\} + (p^\mu p^\nu \text{ terms})$$

$$= g^{\mu\nu}[(a_1 a_2 + b_1 b_2)J_1(p^2, m_1^2, m_2^2) - 2(a_1 b_2 + b_1 a_2)m_1 m_2 B_0(p^2, m_1^2, m_2^2)] + (p^\mu p^\nu \text{ terms})$$

$$\xrightarrow{e=a_2=b_2,\quad m=m_1=m_2} e(a_1 + b_1)g^{\mu\nu}J_2(p^2, m^2) + (p^\mu p^\nu \text{ terms})$$

$$J_1(p^2, m_1^2, m_2^2) \equiv A_0(m_1^2) + A_0(m_2^2) - (p^2 - m_1^2 - m_2^2)B_0(p^2, m_1^2, m_2^2) - 4B_{00}(p^2, m_1^2, m_2^2)$$

$$J_2(p^2, m^2) \equiv 2A_0(m^2) - p^2 B_0(p^2, m^2, m^2) - 4B_{00}(p^2, m^2, m^2)$$

$$i\Pi^{\mu\nu}_{A-X_i^+X_i^--A}(p^2)=\mu^{4-D}\sum_i\int\frac{d^Dq}{(2\pi)^D}\frac{-\mathrm{Tr}[ie\gamma^\mu i(\not q+m_{\chi_i^\pm})ie\gamma^\nu i(\not p+\not q+m_{\chi_i^\pm})]}{[q^2-m^2_{\chi_i^\pm}+i\varepsilon][(p+q)^2-m^2_{\chi_i^\pm}+i\varepsilon]}$$

$$=\frac{i}{16\pi^2}\sum_i\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-e^2\mathrm{Tr}[\gamma^\mu(\not q+m_{\chi_i^\pm})\gamma^\nu(\not p+\not q+m_{\chi_i^\pm})]}{[q^2-m^2_{\chi_i^\pm}+i\varepsilon][(p+q)^2-m^2_{\chi_i^\pm}+i\varepsilon]}$$

$$=\frac{i}{16\pi^2}\sum_i 2e^2g^{\mu\nu}J_2(p^2,m^2_{\chi_i^\pm})+(p^\mu p^\nu\text{ terms})$$

$$16\pi^2\Pi_{A-X_i^+X_i^--A}(p^2)=2e^2\sum_i J_2(p^2,m^2_{\chi_i^\pm})$$

$$i\Pi^{\mu\nu}_{A-X^{++}X^{--}-A}(p^2)=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-\mathrm{Tr}[2ie\gamma^\mu i(\not q+m_{\chi^{\pm\pm}})2ie\gamma^\nu i(\not p+\not q+m_{\chi^{\pm\pm}})]}{[q^2-m^2_{\chi^{\pm\pm}}+i\varepsilon][(p+q)^2-m^2_{\chi^{\pm\pm}}+i\varepsilon]}$$

$$=\frac{i}{16\pi^2}8e^2g^{\mu\nu}J_2(p^2,m^2_{\chi^{\pm\pm}})+(p^\mu p^\nu\text{ terms})$$

$$16\pi^2\Pi_{A-X^{++}X^{--}-A}(p^2)=8e^2J_2(p^2,m^2_{\chi^{\pm\pm}})$$

$$i\Pi^{\mu\nu}_{Z-X_i^+X_i^--A}(p^2)=\frac{i}{16\pi^2}\sum_i\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-\mathrm{Tr}[i\gamma^\mu(a_{ZX_i^+X_i^+}P_L+b_{ZX_i^+X_i^+}P_R)i(\not q+m_{\chi_i^\pm})ie\gamma^\nu i(\not p+\not q+m_{\chi_i^\pm})]}{[q^2-m^2_{\chi_i^\pm}+i\varepsilon][(p+q)^2-m^2_{\chi_i^\pm}+i\varepsilon]}$$

$$=\frac{i}{16\pi^2}\sum_i e(a_{ZX_i^+X_i^+}+b_{ZX_i^+X_i^+})g^{\mu\nu}J_2(p^2,m^2_{\chi_i^\pm})+(p^\mu p^\nu\text{ terms})$$

$$16\pi^2\Pi_{Z-X_i^+X_i^--A}(p^2)=e\sum_i(a_{ZX_i^+X_i^+}+b_{ZX_i^+X_i^+})J_2(p^2,m^2_{\chi_i^\pm})$$

$$i\Pi^{\mu\nu}_{Z-X^{++}X^{--}-A}(p^2)=\frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-\mathrm{Tr}[ia_{ZX^{++}X^{++}}\gamma^\mu i(\not q+m_{\chi^{\pm\pm}})2ie\gamma^\nu i(\not p+\not q+m_{\chi^{\pm\pm}})]}{[q^2-m^2_{\chi^{\pm\pm}}+i\varepsilon][(p+q)^2-m^2_{\chi^{\pm\pm}}+i\varepsilon]}$$

$$=\frac{i}{16\pi^2}\frac{2eg(3c_W^2-s_W^2)}{c_W}g^{\mu\nu}J_2(p^2,m^2_{\chi^{\pm\pm}})+(p^\mu p^\nu\text{ terms})$$

$$16\pi^2\Pi_{Z-X^{++}X^{--}-A}(p^2)=\frac{2eg(3c_W^2-s_W^2)}{c_W}J_2(p^2,m^2_{\chi^{\pm\pm}})$$

Note: for neutral Majorana particle loops, $\begin{cases} i = j, & \frac{1}{2} \text{ for combinatorial factor} \\ i \neq j, & \frac{1}{2} \text{ for avoiding double counting in the index sum} \end{cases}$

$$i\Pi^{\mu\nu}_{Z-X_i^0X_j^0-Z}(p^2) = \frac{i}{16\pi^2}\sum_{ij}\frac{1}{2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-\mathrm{Tr}[i\gamma^\mu(a_{ZX_j^0X_i^0}P_L + b_{ZX_j^0X_i^0}P_R)i(\not{q}+m_{\chi_i^0})i\gamma^\nu(a_{ZX_i^0X_j^0}P_L + b_{ZX_i^0X_j^0}P_R)i(\not{p}+\not{q}+m_{\chi_j^0})]}{[q^2-m^2_{\chi_i^0}+i\varepsilon][(p+q)^2-m^2_{\chi_j^0}+i\varepsilon]}$$

$$16\pi^2\Pi_{Z-X_i^0X_j^0-Z}(p^2) = \frac{1}{2}\sum_{ij}[(a_{ZX_j^0X_i^0}a_{ZX_i^0X_j^0} + b_{ZX_j^0X_i^0}b_{ZX_i^0X_j^0})J_1(p^2,m^2_{\chi_i^0},m^2_{\chi_j^0}) - 2(a_{ZX_j^0X_i^0}b_{ZX_i^0X_j^0} + b_{ZX_j^0X_i^0}a_{ZX_i^0X_j^0})m_{\chi_i^0}m_{\chi_j^0}B_0(p^2,m^2_{\chi_i^0},m^2_{\chi_j^0})]$$

$$i\Pi^{\mu\nu}_{Z-X_i^+X_j^--Z}(p^2) = \frac{i}{16\pi^2}\sum_{ij}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-\mathrm{Tr}[i\gamma^\mu(a_{ZX_j^+X_i^+}P_L + b_{ZX_j^+X_i^+}P_R)i(\not{q}+m_{\chi_i^\pm})i\gamma^\nu(a_{ZX_i^+X_j^+}P_L + b_{ZX_i^+X_j^+}P_R)i(\not{p}+\not{q}+m_{\chi_j^\pm})]}{[q^2-m^2_{\chi_i^\pm}+i\varepsilon][(p+q)^2-m^2_{\chi_j^\pm}+i\varepsilon]}$$

$$16\pi^2\Pi_{Z-X_i^+X_j^--Z}(p^2) = \sum_{ij}[(a_{ZX_j^+X_i^+}a_{ZX_i^+X_j^+} + b_{ZX_j^+X_i^+}b_{ZX_i^+X_j^+})J_1(p^2,m^2_{\chi_i^\pm},m^2_{\chi_j^\pm}) - 2(a_{ZX_j^+X_i^+}b_{ZX_i^+X_j^+} + b_{ZX_j^+X_i^+}a_{ZX_i^+X_j^+})m_{\chi_i^\pm}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_i^\pm},m^2_{\chi_j^\pm})]$$

$$i\Pi^{\mu\nu}_{Z-X^{++}X^{--}-Z}(p^2) = \frac{i}{16\pi^2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-\mathrm{Tr}[ia_{ZX^{++}X^{++}}\gamma^\mu i(\not{q}+m_{\chi^{\pm\pm}})ia_{ZX^{++}X^{++}}\gamma^\nu i(\not{p}+\not{q}+m_{\chi^{\pm\pm}})]}{[q^2-m^2_{\chi^{\pm\pm}}+i\varepsilon][(p+q)^2-m^2_{\chi^{\pm\pm}}+i\varepsilon]}$$

$$= \frac{i}{16\pi^2}\frac{g^2(3c_W^2-s_W^2)^2}{2c_W^2}g^{\mu\nu}J_2(p^2,m^2_{\chi^{\pm\pm}}) + (p^\mu p^\nu \text{ terms})$$

$$16\pi^2\Pi_{Z-X^{++}X^{--}-Z}(p^2) = \frac{g^2(3c_W^2-s_W^2)^2}{2c_W^2}J_2(p^2,m^2_{\chi^{\pm\pm}})$$

$$i\Pi^{\mu\nu}_{W-X_i^0X_j^+-W}(p^2) = \frac{i}{16\pi^2}\sum_{ij}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-\mathrm{Tr}[i\gamma^\mu(a_{WX_j^+X_i^0}P_L + b_{WX_j^+X_i^0}P_R)i(\not{q}+m_{\chi_i^0})i\gamma^\nu(a^*_{WX_j^+X_i^0}P_L + b^*_{WX_j^+X_i^0}P_R)i(\not{p}+\not{q}+m_{\chi_j^\pm})]}{[q^2-m^2_{\chi_i^0}+i\varepsilon][(p+q)^2-m^2_{\chi_j^\pm}+i\varepsilon]}$$

$$16\pi^2\Pi_{W-X_i^0X_j^+-W}(p^2) = \sum_{ij}[(|a_{WX_j^+X_i^0}|^2 + |b_{WX_j^+X_i^0}|^2)J_1(p^2,m^2_{\chi_i^0},m^2_{\chi_j^\pm}) - 2(a_{WX_j^+X_i^0}b^*_{WX_j^+X_i^0} + b_{WX_j^+X_i^0}a^*_{WX_j^+X_i^0})m_{\chi_i^0}m_{\chi_j^\pm}B_0(p^2,m^2_{\chi_i^0},m^2_{\chi_j^\pm})]$$

$$i\Pi^{\mu\nu}_{W-X_i^-X^{++}-W}(p^2) = \frac{i}{16\pi^2}\sum_{i}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{-\mathrm{Tr}[i\gamma^\mu(a_{WX^{++}X_i^+}P_L + b_{WX^{++}X_i^+}P_R)i(\not{q}+m_{\chi_i^\pm})i\gamma^\nu(a^*_{WX^{++}X_i^+}P_L + b^*_{WX^{++}X_i^+}P_R)i(\not{p}+\not{q}+m_{\chi^{\pm\pm}})]}{[q^2-m^2_{\chi_i^\pm}+i\varepsilon][(p+q)^2-m^2_{\chi^{\pm\pm}}+i\varepsilon]}$$

$$16\pi^2\Pi_{W-X_i^-X^{++}-W}(p^2) = \sum_{i}[(|a_{WX^{++}X_i^+}|^2 + |b_{WX^{++}X_i^+}|^2)J_1(p^2,m^2_{\chi_i^\pm},m^2_{\chi^{\pm\pm}}) - 2(a_{WX^{++}X_i^+}b^*_{WX^{++}X_i^+} + b_{WX^{++}X_i^+}a^*_{WX^{++}X_i^+})m_{\chi_i^\pm}m_{\chi^{\pm\pm}}B_0(p^2,m^2_{\chi_i^\pm},m^2_{\chi^{\pm\pm}})]$$

$$A_0(m^2) \sim m^2\Delta, \quad B_0(p^2,m_1^2,m_2^2) \sim \Delta, \quad B_1(p^2,m_1^2,m_2^2) \sim -\frac{1}{2}\Delta, \quad B_{00}(p^2,m_1^2,m_2^2) \sim -\frac{1}{12}(p^2-3m_1^2-3m_2^2)\Delta$$

$$J_2(p^2,m^2) = 2A_0(m^2) - p^2 B_0(p^2,m^2,m^2) - 4B_{00}(p^2,m^2,m^2) \sim -\frac{2}{3}p^2\Delta$$

$$J_1(p^2,m^2_{\chi_i^\pm},m^2_{\chi_j^\pm}) \sim m_1^2\Delta + m_2^2\Delta - (p^2-m_1^2-m_2^2)\Delta + \frac{1}{3}(p^2-3m_1^2-3m_2^2)\Delta = \left(m_1^2+m_2^2-\frac{2}{3}p^2\right)\Delta$$

$$16\pi^2\Pi_{AA}(p^2) \sim (2\cdot 3e^2+8e^2)\left(-\frac{2}{3}p^2\Delta\right) = -\frac{28}{3}e^2p^2\Delta$$

$$\sum_i (a_{ZX_i^+X_i^+} + b_{ZX_i^+X_i^+}) = \sum_{ik}\left[a_{Z\Psi_k^+\Psi_k^+}(\mathcal{C}_L)^*_{ki}(\mathcal{C}_L)_{ki} + b_{Z\Psi_k^+\Psi_k^+}(\mathcal{C}_R)_{ki}(\mathcal{C}_R)^*_{ki}\right]$$

$$= \sum_k (a_{Z\Psi_k^+\Psi_k^+} + b_{Z\Psi_k^+\Psi_k^+})\delta_{kk} = 2\left[gc_W + \frac{g}{2c_W}(3c_W^2+s_W^2) + \frac{g}{2c_W}(c_W^2-s_W^2)\right] = 6gc_W$$

$$16\pi^2\Pi_{ZA}(p^2) \sim -\frac{2}{3}p^2\Delta\left[e\sum_i(a_{ZX_i^+X_i^+} + b_{ZX_i^+X_i^+}) + \frac{2eg(3c_W^2-s_W^2)}{c_W}\right] = -4eg\,\frac{7c_W^2-1}{3c_W}\,p^2\Delta$$

$$16\pi^2\Pi_{ZZ}(p^2) \sim \frac{1}{2}\sum_{ij}[(a_{ZX_j^0X_i^0}a_{ZX_i^0X_j^0} + b_{ZX_j^0X_i^0}b_{ZX_i^0X_j^0})\left(m^2_{\chi_i^0}+m^2_{\chi_j^0}-\frac{2}{3}p^2\right)\Delta - 2(a_{ZX_j^0X_i^0}b_{ZX_i^0X_j^0} + b_{ZX_j^0X_i^0}a_{ZX_i^0X_j^0})m_{\chi_i^0}m_{\chi_j^0}\Delta]$$

$$+\sum_{ij}[(a_{ZX_j^+X_i^+}a_{ZX_i^+X_j^+} + b_{ZX_j^+X_i^+}b_{ZX_i^+X_j^+})\left(m^2_{\chi_i^\pm}+m^2_{\chi_j^\pm}-\frac{2}{3}p^2\right)\Delta - 2(a_{ZX_j^+X_i^+}b_{ZX_i^+X_j^+} + b_{ZX_j^+X_i^+}a_{ZX_i^+X_j^+})m_{\chi_i^\pm}m_{\chi_j^\pm}\Delta] - \frac{g^2(3c_W^2-s_W^2)^2}{3c_W^2}p^2\Delta$$

$$16\pi^2\Pi_{WW}(p^2) \sim \sum_{ij}[(|a_{WX_j^+X_i^0}|^2 + |b_{WX_j^+X_i^0}|^2)\left(m^2_{\chi_i^0}+m^2_{\chi_j^\pm}-\frac{2}{3}p^2\right)\Delta - 2(a_{WX_j^+X_i^0}b^*_{WX_j^+X_i^0} + b_{WX_j^+X_i^0}a^*_{WX_j^+X_i^0})m_{\chi_i^0}m_{\chi_j^\pm}\Delta]$$

$$+\sum_i[(|a_{WX^{++}X_i^+}|^2 + |b_{WX^{++}X_i^+}|^2)\left(m^2_{\chi_i^\pm}+m^2_{\chi^{\pm\pm}}-\frac{2}{3}p^2\right)\Delta - 2(a_{WX^{++}X_i^+}b^*_{WX^{++}X_i^+} + b_{WX^{++}X_i^+}a^*_{WX^{++}X_i^+})m_{\chi_i^\pm}m_{\chi^{\pm\pm}}\Delta]$$

n-dim Gaussian distribution

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{n/2} |\mathbf{V}|^{1/2}} e^{-Q/2}$$

Quadratic form: $Q = (\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}} \mathbf{V}^{-1} (\mathbf{x}-\boldsymbol{\mu}) = (x_i - \mu_i) V_{ij}^{-1} (x_j - \mu_j)$

$\mathbf{V}$ is the positive definite symmetric covariance matrix

$$V_{ij} = \mathrm{cov}(X_i, X_j) = \begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & \\ \rho_{1n}\sigma_1\sigma_n & \cdots & & \sigma_n^2 \end{pmatrix}$$

Correlation coefficients: $\rho_{ij} = \dfrac{\mathrm{cov}(X_i, X_j)}{\sqrt{V(X_i)V(X_j)}}$

Probability inside the *n*-dim ellipsoid $Q = Q_0$: $P(Q < Q_0) = F_{\chi^2(n)}(Q_0)$

$F_{\chi^2(n)}(x)$ is the cumulative χ^2 distribution function with *n* d.o.f.

$F_{\chi^2(1)}(1) = 68.3\%$, $F_{\chi^2(1)}(4) = 95.4\%$, $F_{\chi^2(1)}(9) = 99.7\%$

$F_{\chi^2(2)}(2.295749) = 68.3\%$, $F_{\chi^2(2)}(6.180074) = 95.4\%$, $F_{\chi^2(2)}(11.82916) = 99.7\%$

$F_{\chi^2(3)}(3.526741) = 68.3\%$, $F_{\chi^2(3)}(8.024882) = 95.4\%$, $F_{\chi^2(3)}(14.15641) = 99.7\%$

2-dim Gaussian distribution

$$V_{ij} = \mathrm{cov}(X_i, X_j) = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}, \quad V_{ij}^{-1} = \frac{1}{(1-\rho^2)\sigma_1^2\sigma_2^2} \begin{pmatrix} \sigma_2^2 & -\rho\sigma_1\sigma_2 \\ -\rho\sigma_1\sigma_2 & \sigma_1^2 \end{pmatrix}$$

$$f(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-Q/2}$$

$$Q = \frac{1}{1-\rho^2}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2 - 2\rho\frac{x_1-\mu_1}{\sigma_1}\frac{x_2-\mu_2}{\sigma_2}\right]$$

Ellipse parametric equation with parameter *t*:

$$\begin{cases} x_1 = \mu_1 + a\cos\phi\cos t - b\sin\phi\sin t \\ x_2 = \mu_2 + a\sin\phi\cos t + b\cos\phi\sin t \end{cases}$$

$$\phi = \frac{1}{2}\tan^{-1}\left(\frac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}\right), \quad a = \frac{\sigma_1\sigma_2\sqrt{Q(1-\rho^2)}}{\sqrt{\sigma_2^2\cos^2\phi - 2\rho\sigma_1\sigma_2\sin\phi\cos\phi + \sigma_1^2\sin^2\phi}}, \quad b = \frac{\sigma_1\sigma_2\sqrt{Q(1-\rho^2)}}{\sqrt{\sigma_2^2\sin^2\phi + 2\rho\sigma_1\sigma_2\sin\phi\cos\phi + \sigma_1^2\cos^2\phi}}$$

Constraints on S, T, U parameters (Ref: Gfitter, 1407.3792):

$S = 0.05 \pm 0.11$, $T = 0.09 \pm 0.13$, $U = 0.01 \pm 0.11$

$\rho_{ST} = +0.90$, $\rho_{SU} = -0.59$, $\rho_{TU} = -0.83$

Fixing U to 0:

$S = 0.06 \pm 0.09$, $T = 0.10 \pm 0.07$, $\rho_{ST} = +0.91$

Direct detection

$$a_{h\Psi_i^0\Psi_j^0} = b_{h\Psi_i^0\Psi_j^0}$$

$$\mathcal{L}_{hX_i^0X_j^0} = \frac{1}{2}(a_{hX_i^0X_j^0} h\bar{X}_{iR}^0 X_{jL}^0 + b_{hX_i^0X_j^0} h\bar{X}_{iL}^0 X_{jR}^0) = \frac{1}{2} h\bar{X}_i^0 (a_{hX_i^0X_j^0} P_L + b_{hX_i^0X_j^0} P_R) X_j^0$$

$$= \frac{1}{4}(a_{hX_i^0X_j^0} + b_{hX_i^0X_j^0}) h\bar{X}_i^0 X_j^0 + \frac{1}{4}(b_{hX_i^0X_j^0} - a_{hX_i^0X_j^0}) h\bar{X}_i^0 \gamma_5 X_j^0$$

$$= \frac{1}{4} a_{h\Psi_k^0\Psi_l^0} (\mathcal{N}_{ki}\mathcal{N}_{lj} + \mathcal{N}_{ki}^*\mathcal{N}_{lj}^*) h\bar{X}_i^0 X_j^0 + \frac{1}{4} a_{h\Psi_k^0\Psi_l^0} (\mathcal{N}_{ki}^*\mathcal{N}_{lj}^* - \mathcal{N}_{ki}\mathcal{N}_{lj}) h\bar{X}_i^0 \gamma_5 X_j^0$$

$$= \frac{1}{2} a_{h\Psi_k^0\Psi_l^0} \operatorname{Re}(\mathcal{N}_{ki}\mathcal{N}_{lj}) h\bar{X}_i^0 X_j^0 - \frac{1}{2} a_{h\Psi_k^0\Psi_l^0} \operatorname{Im}(\mathcal{N}_{ki}\mathcal{N}_{lj}) h\bar{X}_i^0 i\gamma_5 X_j^0$$

$$\operatorname{Im}(\mathcal{N}_{ki}\mathcal{N}_{li}) = 0$$

$$\mathcal{L}_{hX_1^0X_1^0} = \frac{1}{2} a_{h\Psi_k^0\Psi_l^0} \operatorname{Re}(\mathcal{N}_{k1}\mathcal{N}_{l1}) h\bar{X}_1^0 X_1^0 - \frac{1}{2} a_{h\Psi_k^0\Psi_l^0} \operatorname{Im}(\mathcal{N}_{k1}\mathcal{N}_{l1}) h\bar{X}_1^0 i\gamma_5 X_1^0$$

$$= [a_{h\Psi_1^0\Psi_2^0} \operatorname{Re}(\mathcal{N}_{11}\mathcal{N}_{21}) + a_{h\Psi_1^0\Psi_3^0} \operatorname{Re}(\mathcal{N}_{11}\mathcal{N}_{31})] h\bar{X}_1^0 X_1^0$$

$$= \frac{1}{\sqrt{3}}[-y_1 \operatorname{Re}(\mathcal{N}_{11}\mathcal{N}_{21}) + y_2 \operatorname{Re}(\mathcal{N}_{11}\mathcal{N}_{31})] h\bar{X}_1^0 X_1^0$$

$$\equiv \frac{1}{2} g_{hX_1^0X_1^0} h\bar{X}_1^0 X_1^0$$

$$g_{hX_1^0X_1^0} = \frac{1}{2}(a_{hX_1^0X_1^0} + b_{hX_1^0X_1^0}) = \frac{2}{\sqrt{3}}[-y_1 \operatorname{Re}(\mathcal{N}_{11}\mathcal{N}_{21}) + y_2 \operatorname{Re}(\mathcal{N}_{11}\mathcal{N}_{31})]$$

$$\mathcal{L}_{S,q} = \sum_q G_{S,q} \bar{X}_1^0 X_1^0 \bar{q} q, \quad \mathcal{L}_{S,N} = \sum_{N=p,n} G_{S,N} \bar{X}_1^0 X_1^0 \bar{N} N$$

$$G_{S,N} = m_N \left(\sum_{q=u,d,s} \frac{G_{S,q}}{m_q} f_q^N + \sum_{q=c,b,t} \frac{G_{S,q}}{m_q} f_Q^N \right), \quad f_Q^N = \frac{2}{27}\left(1 - \sum_{q=u,d,s} f_q^N \right)$$

hep-ph/0001005:

$$f_u^p = 0.020 \pm 0.004, \quad f_d^p = 0.026 \pm 0.005, \quad f_u^n = 0.014 \pm 0.003, \quad f_d^n = 0.036 \pm 0.008, \quad f_s^p = f_s^n = 0.118 \pm 0.062$$

$$\Rightarrow \quad f_Q^p = 0.0619, \quad f_Q^n = 0.0616$$

$$G_{S,q} = -\frac{g_{hX_1^0X_1^0} m_q}{2 v m_h^2}, \quad G_{S,N} = -\frac{g_{hX_1^0X_1^0} m_N}{2 v m_h^2} \left(\sum_{q=u,d,s} f_q^N + 3 f_Q^N \right) \quad \Rightarrow \quad G_{S,n} \simeq G_{S,p}$$

$$\sigma_{\chi p}^{\mathrm{SI}} = \frac{4}{\pi} \mu_{\chi p}^2 G_{S,p}^2, \quad \mu_{\chi p} \equiv \frac{m_\chi m_p}{m_\chi + m_p}$$

$$a_{Z\Psi_k^0\Psi_k^0} = -b_{Z\Psi_k^0\Psi_k^0}$$

$$\mathcal{L}_{ZX_i^0X_j^0} = \frac{1}{2}(a_{ZX_i^0X_j^0} Z_\mu \bar{X}_{iL}^0 \gamma^\mu X_{jL}^0 + b_{ZX_i^0X_j^0} Z_\mu \bar{X}_{iR}^0 \gamma^\mu X_{jR}^0) = \frac{1}{2}(a_{ZX_i^0X_j^0} Z_\mu \bar{X}_i^0 \gamma^\mu P_L X_j^0 + b_{ZX_i^0X_j^0} Z_\mu \bar{X}_i^0 \gamma^\mu P_R X_j^0)$$

$$= \frac{1}{4}(a_{ZX_i^0X_j^0} + b_{ZX_i^0X_j^0}) Z_\mu \bar{X}_i^0 \gamma^\mu X_j^0 + \frac{1}{4}(b_{ZX_i^0X_j^0} - a_{ZX_i^0X_j^0}) Z_\mu \bar{X}_i^0 \gamma^\mu \gamma_5 X_j^0$$

$$= \frac{1}{4}(a_{Z\Psi_k^0\Psi_k^0} \mathcal{N}_{ki}^* \mathcal{N}_{kj} + b_{Z\Psi_k^0\Psi_k^0} \mathcal{N}_{ki} \mathcal{N}_{kj}^*) Z_\mu \bar{X}_i^0 \gamma^\mu X_j^0 + \frac{1}{4}(b_{Z\Psi_k^0\Psi_k^0} \mathcal{N}_{ki} \mathcal{N}_{kj}^* - a_{Z\Psi_k^0\Psi_k^0} \mathcal{N}_{ki}^* \mathcal{N}_{kj}) Z_\mu \bar{X}_i^0 \gamma^\mu \gamma_5 X_j^0$$

$$= \frac{1}{4} a_{Z\Psi_k^0\Psi_k^0} (\mathcal{N}_{ki}^* \mathcal{N}_{kj} - \mathcal{N}_{ki} \mathcal{N}_{kj}^*) Z_\mu \bar{X}_i^0 \gamma^\mu X_j^0 - \frac{1}{4} a_{Z\Psi_k^0\Psi_k^0} (\mathcal{N}_{ki} \mathcal{N}_{kj}^* + \mathcal{N}_{ki}^* \mathcal{N}_{kj}) Z_\mu \bar{X}_i^0 \gamma^\mu \gamma_5 X_j^0$$

$$\mathcal{L}_{ZX_1^0X_1^0} = -\frac{1}{2} a_{Z\Psi_k^0\Psi_k^0} |\mathcal{N}_{k1}|^2 Z_\mu \bar{X}_1^0 \gamma^\mu \gamma_5 X_1^0 \equiv \frac{1}{2} g_{ZX_1^0X_1^0} Z_\mu \bar{X}_1^0 \gamma^\mu \gamma_5 X_1^0$$

$$g_{ZX_1^0X_1^0} = \frac{1}{2}(b_{ZX_1^0X_1^0} - a_{ZX_1^0X_1^0}) = \frac{g}{2c_W}(|\mathcal{N}_{31}|^2 - |\mathcal{N}_{21}|^2)$$

$$\mathcal{L}_{A,q} = \sum_q G_{A,q} \bar{X}_1^0 \gamma^\mu \gamma_5 X_1^0 \bar{q} \gamma^\mu \gamma_5 q, \quad \mathcal{L}_{A,N} = \sum_{N=p,n} G_{A,N} \bar{X}_1^0 \gamma^\mu \gamma_5 X_1^0 \bar{N} \gamma^\mu \gamma_5 N$$

$$G_{A,N} = \sum_{q=u,d,s} G_{A,q} \Delta_q^N$$

hep-ex/0609039 :

$$\Delta_u^p = \Delta_d^n = 0.842 \pm 0.012, \quad \Delta_d^p = \Delta_u^n = -0.427 \pm 0.013, \quad \Delta_s^p = \Delta_s^n = -0.085 \pm 0.018$$

$$G_{A,q} = \frac{g g_A^q g_{ZX_1^0X_1^0}}{4 c_W m_Z^2}, \quad g_A^u = \frac{1}{2}, \quad g_A^d = g_A^s = -\frac{1}{2}$$

$$\sigma_{\chi N}^{\mathrm{SD}} = \frac{12}{\pi} \mu_{\chi N}^2 G_{A,N}^2, \quad \mu_{\chi N} \equiv \frac{m_\chi m_N}{m_\chi + m_N}$$

Annihilation cross sections

$$i(p_1)+\bar{i}(p_2)\to f(k_1)+\bar{f}(k_2)$$

Center-of-mass frame:

$$s=(p_1+p_2)^2=(k_1+k_2)^2,\quad p_1^0=p_2^0=k_1^0=k_2^0=\frac{\sqrt{s}}{2},\quad \beta_{i,f}\equiv\sqrt{1-4m_{i,f}^2/s}$$

$$|\mathbf{p}_1|=|\mathbf{p}_2|=\sqrt{\frac{s}{4}-m_i^2}=\frac{\sqrt{s}}{2}\beta_i,\quad |\mathbf{k}_1|=|\mathbf{k}_2|=\sqrt{\frac{s}{4}-m_f^2}=\frac{\sqrt{s}}{2}\beta_f$$

$$p_1\cdot p_2=\frac{s}{2}-m_i^2,\quad k_1\cdot k_2=\frac{s}{2}-m_f^2$$

$$p_1\cdot k_1=p_2\cdot k_2=p_1^0k_1^0-|\mathbf{p}_1||\mathbf{k}_1|\cos\theta=\frac{s}{4}(1-\beta_i\beta_f\cos\theta)$$

$$p_1\cdot k_2=p_2\cdot k_1=p_1^0k_1^0+|\mathbf{p}_1||\mathbf{k}_1|\cos\theta=\frac{s}{4}(1+\beta_i\beta_f\cos\theta)$$

$$t=(p_1-k_1)^2=(p_2-k_2)^2=m_i^2+m_f^2-2p_1\cdot k_1=m_i^2+m_f^2-\frac{s}{2}(1-\beta_i\beta_f\cos\theta)$$

$$u=(p_1-k_2)^2=(p_2-k_1)^2=m_i^2+m_f^2-2p_1\cdot k_2=m_i^2+m_f^2-\frac{s}{2}(1+\beta_i\beta_f\cos\theta)$$

$$\frac{d\sigma_{\text{ann}}}{d\Omega}=\frac{1}{2p_1^0 2p_2^0|\mathbf{v}_1-\mathbf{v}_2|}\frac{|\mathbf{k}_1|}{(2\pi)^2 4E_{\text{CM}}}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{1}{v}\frac{\beta_f}{32\pi^2 s}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2,\quad v\equiv|\mathbf{v}_1-\mathbf{v}_2|=2\beta_i$$

$$s\simeq 4m_i^2+m_i^2v^2,\quad v\simeq\sqrt{s/m_i^2-4}$$

$$s\to 4m_i^2,\quad \beta_i\to 0,\quad \beta_f\to\sqrt{1-m_f^2/m_i^2}$$

$$\sigma_{\text{ann}}v\simeq 4\pi\frac{\beta_f}{32\pi^2 4m_i^2}\lim_{\beta_i\to 0}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{\beta_f}{32\pi m_i^2}\lim_{\beta_i\to 0}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2$$

$$\chi_1^0(p_1)+\chi_1^0(p_2)\to W^+(k_1)+W^-(k_2)$$

$$i\mathcal{M}_{t,i}=\bar{v}(p_1)i\gamma^\mu(a^*_{WX_i^+X_1^0}P_L+b^*_{WX_i^+X_1^0}P_R)\frac{i(-\not{p}_1+\not{k}_1+m_{\chi_i^\pm})}{(-p_1+k_1)^2-m^2_{\chi_i^\pm}}i\gamma^\nu(a_{WX_i^+X_1^0}P_L+b_{WX_i^+X_1^0}P_R)u(p_2)\varepsilon^*_\mu(k_1)\varepsilon^*_\nu(k_2)$$

$$=\frac{-i}{t-m^2_{\chi_i^\pm}}\bar{v}(p_1)\gamma^\mu(a^*_{WX_i^+X_1^0}P_L+b^*_{WX_i^+X_1^0}P_R)(-\not{p}_1+\not{k}_1+m_{\chi_i^\pm})\gamma^\nu(a_{WX_i^+X_1^0}P_L+b_{WX_i^+X_1^0}P_R)u(p_2)\varepsilon^*_\mu(k_1)\varepsilon^*_\nu(k_2)$$

$$(i\mathcal{M}_{t,j})^*=\frac{i}{t-m^2_{\chi_j^\pm}}\bar{u}(p_2)\gamma^\sigma(a^*_{WX_j^+X_1^0}P_L+b^*_{WX_j^+X_1^0}P_R)(-\not{p}_1+\not{k}_1+m_{\chi_j^\pm})\gamma^\rho(a_{WX_j^+X_1^0}P_L+b_{WX_j^+X_1^0}P_R)v(p_1)\varepsilon_\rho(k_1)\varepsilon_\sigma(k_2)$$

$$i\mathcal{M}_{u,i}=\bar{v}(p_2)i\gamma^\mu(a^*_{WX_i^+X_1^0}P_L+b^*_{WX_i^+X_1^0}P_R)\frac{i(\not{p}_1-\not{k}_2+m_{\chi_i^\pm})}{(p_1-k_2)^2-m^2_{\chi_i^\pm}}i\gamma^\nu(a_{WX_i^+X_1^0}P_L+b_{WX_i^+X_1^0}P_R)u(p_1)\varepsilon^*_\mu(k_1)\varepsilon^*_\nu(k_2)$$

$$=\frac{-i}{u-m^2_{\chi_i^\pm}}\bar{v}(p_2)\gamma^\mu(a^*_{WX_i^+X_1^0}P_L+b^*_{WX_i^+X_1^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_i^\pm})\gamma^\nu(a_{WX_i^+X_1^0}P_L+b_{WX_i^+X_1^0}P_R)u(p_1)\varepsilon^*_\mu(k_1)\varepsilon^*_\nu(k_2)$$

$$(i\mathcal{M}_{u,j})^*=\frac{i}{u-m^2_{\chi_j^\pm}}\bar{u}(p_1)\gamma^\sigma(a^*_{WX_j^+X_1^0}P_L+b^*_{WX_j^+X_1^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_j^\pm})\gamma^\rho(a_{WX_j^+X_1^0}P_L+b_{WX_j^+X_1^0}P_R)v(p_2)\varepsilon_\rho(k_1)\varepsilon_\sigma(k_2)$$

$$\frac{1}{4}\sum_{ij}\sum_{\text{spins}}\mathcal{M}_{t,i}\mathcal{M}^*_{t,j} = \frac{1}{4}\sum_{ij}\frac{1}{(t-m^2_{\chi_i^\pm})(t-m^2_{\chi_j^\pm})}\mathrm{Tr}[(\not p_1 - m_{\chi_1^0})\gamma^\mu(a^*_{WX_i^+X_1^0}P_L + b^*_{WX_i^+X_1^0}P_R)(-\not p_1+\not k_1+m_{\chi_i^\pm})\gamma^\nu(a_{WX_i^+X_1^0}P_L+b_{WX_i^+X_1^0}P_R)$$

$$\times(\not p_2+m_{\chi_1^0})\gamma^\sigma(a^*_{WX_j^+X_1^0}P_L+b^*_{WX_j^+X_1^0}P_R)(-\not p_1+\not k_1+m_{\chi_j^\pm})\gamma^\rho(a_{WX_j^+X_1^0}P_L+b_{WX_j^+X_1^0}P_R)]\left(-g_{\mu\rho}+\frac{k_{1\mu}k_{1\rho}}{m_W^2}\right)\left(-g_{\nu\sigma}+\frac{k_{2\nu}k_{2\sigma}}{m_W^2}\right)$$

$$\frac{1}{4}\sum_{ij}\sum_{\text{spins}}\mathcal{M}_{u,i}\mathcal{M}^*_{u,j} = \frac{1}{4}\sum_{ij}\frac{1}{(u-m^2_{\chi_i^\pm})(u-m^2_{\chi_j^\pm})}\mathrm{Tr}[(\not p_2 - m_{\chi_1^0})\gamma^\mu(a^*_{WX_i^+X_1^0}P_L + b^*_{WX_i^+X_1^0}P_R)(\not p_1-\not k_2+m_{\chi_i^\pm})\gamma^\nu(a_{WX_i^+X_1^0}P_L+b_{WX_i^+X_1^0}P_R)$$

$$\times(\not p_1+m_{\chi_1^0})\gamma^\sigma(a^*_{WX_j^+X_1^0}P_L+b^*_{WX_j^+X_1^0}P_R)(\not p_1-\not k_2+m_{\chi_j^\pm})\gamma^\rho(a_{WX_j^+X_1^0}P_L+b_{WX_j^+X_1^0}P_R)]\left(-g_{\mu\rho}+\frac{k_{1\mu}k_{1\rho}}{m_W^2}\right)\left(-g_{\nu\sigma}+\frac{k_{2\nu}k_{2\sigma}}{m_W^2}\right)$$

$$u(p)=C\bar v(p)^{\mathrm T},\quad v(p)=C\bar u(p)^{\mathrm T},\quad \bar v(p)=u(p)^{\mathrm T}C,\quad \bar u(p)=v(p)^{\mathrm T}C$$

$$-\frac{1}{4}\sum_{ij}\sum_{\text{spins}}\mathcal{M}_{t,i}\mathcal{M}^*_{u,j} = -\frac{1}{4}\sum_{ij}\frac{1}{(t-m^2_{\chi_i^\pm})(u-m^2_{\chi_j^\pm})}\mathrm{Tr}[(\not p_1 - m_{\chi_1^0})\gamma^\mu(a^*_{WX_i^+X_1^0}P_L + b^*_{WX_i^+X_1^0}P_R)(-\not p_1+\not k_1+m_{\chi_i^\pm})\gamma^\nu(a_{WX_i^+X_1^0}P_L+b_{WX_i^+X_1^0}P_R)$$

$$\times(\not p_2+m_{\chi_1^0})C^{\mathrm T}(a_{WX_j^+X_1^0}P_L+b_{WX_j^+X_1^0}P_R)^{\mathrm T}\gamma^{\rho\mathrm T}(\not p_1-\not k_2+m_{\chi_j^\pm})^{\mathrm T}(a^*_{WX_j^+X_1^0}P_L+b^*_{WX_j^+X_1^0}P_R)^{\mathrm T}\gamma^{\sigma\mathrm T}C^{\mathrm T}]\left(-g_{\mu\rho}+\frac{k_{1\mu}k_{1\rho}}{m_W^2}\right)\left(-g_{\nu\sigma}+\frac{k_{2\nu}k_{2\sigma}}{m_W^2}\right)$$

$$=+\frac{1}{4}\sum_{ij}\frac{1}{(t-m^2_{\chi_i^\pm})(u-m^2_{\chi_j^\pm})}\mathrm{Tr}[(\not p_1 - m_{\chi_1^0})\gamma^\mu(a^*_{WX_i^+X_1^0}P_L + b^*_{WX_i^+X_1^0}P_R)(-\not p_1+\not k_1+m_{\chi_i^\pm})\gamma^\nu(a_{WX_i^+X_1^0}P_L+b_{WX_i^+X_1^0}P_R)$$

$$\times(\not p_2+m_{\chi_1^0})(a_{WX_j^+X_1^0}P_L+b_{WX_j^+X_1^0}P_R)\gamma^\rho(-\not p_1+\not k_2+m_{\chi_j^\pm})(a^*_{WX_j^+X_1^0}P_L+b^*_{WX_j^+X_1^0}P_R)\gamma^\sigma]\left(-g_{\mu\rho}+\frac{k_{1\mu}k_{1\rho}}{m_W^2}\right)\left(-g_{\nu\sigma}+\frac{k_{2\nu}k_{2\sigma}}{m_W^2}\right)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{1}{4}\sum_{ij}\sum_{\text{spins}}(\mathcal{M}_{t,i}-\mathcal{M}_{u,i})(\mathcal{M}^*_{t,j}-\mathcal{M}^*_{u,j})=\frac{1}{4}\sum_{ij}\sum_{\text{spins}}(\mathcal{M}_{t,i}\mathcal{M}^*_{t,j}+\mathcal{M}_{u,i}\mathcal{M}^*_{u,j}-\mathcal{M}_{t,i}\mathcal{M}^*_{u,j}-\mathcal{M}_{u,i}\mathcal{M}^*_{t,j})$$

$$\lim_{\beta_i\to 0}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}(\chi_i^\pm)|^2=\frac{2m_\chi^2}{m_W^4(m_\chi^2+m^2_{\chi_i^\pm}-m_W^2)^2}[2m_W^4(m_\chi^2-m_W^2)(|a_{WX_i^+X_1^0}|^2+|b_{WX_i^+X_1^0}|^2)^2-m^2_{\chi_i^\pm}(4m_\chi^4-4m_\chi^2m_W^2+3m_W^4)(a^*_{WX_i^+X_1^0}b_{WX_i^+X_1^0}-b^*_{WX_i^+X_1^0}a_{WX_i^+X_1^0})^2]$$

$$\chi_1^0(p_1)+\chi_1^0(p_2)\to Z(k_1)+Z(k_2)$$

$$i\mathcal{M}_{t,i}=\bar{v}(p_2)i\gamma^{\mu}(a_{ZX_1^0X_i^0}P_L+b_{ZX_1^0X_i^0}P_R)\frac{i(\not{p}_1-\not{k}_1+m_{\chi_i^0})}{(p_1-k_1)^2-m^2_{\chi_i^0}}i\gamma^{\nu}(a_{ZX_i^0X_1^0}P_L+b_{ZX_i^0X_1^0}P_R)u(p_1)\varepsilon^*_{\mu}(k_2)\varepsilon^*_{\nu}(k_1)$$

$$=\frac{-i}{t-m^2_{\chi_i^0}}\bar{v}(p_2)\gamma^{\mu}(a_{ZX_1^0X_i^0}P_L+b_{ZX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_1+m_{\chi_i^0})\gamma^{\nu}(a_{ZX_i^0X_1^0}P_L+b_{ZX_i^0X_1^0}P_R)u(p_1)\varepsilon^*_{\mu}(k_2)\varepsilon^*_{\nu}(k_1)$$

$$(i\mathcal{M}_{t,i})^*=\frac{i}{t-m^2_{\chi_i^0}}\bar{u}(p_1)\gamma^{\sigma}(a^*_{ZX_i^0X_1^0}P_L+b^*_{ZX_i^0X_1^0}P_R)(\not{p}_1-\not{k}_1+m_{\chi_i^0})\gamma^{\rho}(a^*_{ZX_1^0X_i^0}P_L+b^*_{ZX_1^0X_i^0}P_R)v(p_2)\varepsilon_{\rho}(k_2)\varepsilon_{\sigma}(k_1)$$

$$i\mathcal{M}_{u,i}=\bar{v}(p_2)i\gamma^{\mu}(a_{ZX_1^0X_i^0}P_L+b_{ZX_1^0X_i^0}P_R)\frac{i(\not{p}_1-\not{k}_2+m_{\chi_i^0})}{(p_1-k_2)^2-m^2_{\chi_i^0}}i\gamma^{\nu}(a_{ZX_i^0X_1^0}P_L+b_{ZX_i^0X_1^0}P_R)u(p_1)\varepsilon^*_{\mu}(k_1)\varepsilon^*_{\nu}(k_2)$$

$$=\frac{-i}{u-m^2_{\chi_i^0}}\bar{v}(p_2)\gamma^{\mu}(a_{ZX_1^0X_i^0}P_L+b_{ZX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_i^0})\gamma^{\nu}(a_{ZX_i^0X_1^0}P_L+b_{ZX_i^0X_1^0}P_R)u(p_1)\varepsilon^*_{\mu}(k_1)\varepsilon^*_{\nu}(k_2)$$

$$(i\mathcal{M}_{u,i})^*=\frac{i}{u-m^2_{\chi_i^0}}\bar{u}(p_1)\gamma^{\sigma}(a^*_{ZX_i^0X_1^0}P_L+b^*_{ZX_i^0X_1^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_i^0})\gamma^{\rho}(a^*_{ZX_1^0X_i^0}P_L+b^*_{ZX_1^0X_i^0}P_R)v(p_2)\varepsilon_{\rho}(k_1)\varepsilon_{\sigma}(k_2)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_{t,i}|^2=\frac{1}{4}\frac{1}{(t-m^2_{\chi_i^0})^2}\mathrm{Tr}[(\not{p}_2-m_{\chi_1^0})\gamma^{\mu}(a_{ZX_1^0X_i^0}P_L+b_{ZX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_1+m_{\chi_i^0})\gamma^{\nu}(a_{ZX_i^0X_1^0}P_L+b_{ZX_i^0X_1^0}P_R)$$

$$\times(\not{p}_1+m_{\chi_1^0})\gamma^{\sigma}(a^*_{ZX_i^0X_1^0}P_L+b^*_{ZX_i^0X_1^0}P_R)(\not{p}_1-\not{k}_1+m_{\chi_i^0})\gamma^{\rho}(a^*_{ZX_1^0X_i^0}P_L+b^*_{ZX_1^0X_i^0}P_R)]\left(-g_{\mu\rho}+\frac{k_{2\mu}k_{2\rho}}{m_Z^2}\right)\left(-g_{\nu\sigma}+\frac{k_{1\nu}k_{1\sigma}}{m_Z^2}\right)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_{u,i}|^2=\frac{1}{4}\frac{1}{(u-m^2_{\chi_i^0})^2}\mathrm{Tr}[(\not{p}_2-m_{\chi_1^0})\gamma^{\mu}(a_{ZX_1^0X_i^0}P_L+b_{ZX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_i^0})\gamma^{\nu}(a_{ZX_i^0X_1^0}P_L+b_{ZX_i^0X_1^0}P_R)$$

$$\times(\not{p}_1+m_{\chi_1^0})\gamma^{\sigma}(a^*_{ZX_i^0X_1^0}P_L+b^*_{ZX_i^0X_1^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_i^0})\gamma^{\rho}(a^*_{ZX_1^0X_i^0}P_L+b^*_{ZX_1^0X_i^0}P_R)]\left(-g_{\mu\rho}+\frac{k_{1\mu}k_{1\rho}}{m_Z^2}\right)\left(-g_{\nu\sigma}+\frac{k_{2\nu}k_{2\sigma}}{m_Z^2}\right)$$

$$\frac{1}{4}\sum_{\text{spins}}\mathcal{M}_{t,i}\mathcal{M}^*_{u,i}=\frac{1}{4}\frac{1}{(t-m^2_{\chi_i^0})(u-m^2_{\chi_i^0})}\mathrm{Tr}[(\not{p}_2-m_{\chi_1^0})\gamma^{\mu}(a_{ZX_1^0X_i^0}P_L+b_{ZX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_1+m_{\chi_i^0})\gamma^{\nu}(a_{ZX_i^0X_1^0}P_L+b_{ZX_i^0X_1^0}P_R)$$

$$\times(\not{p}_1+m_{\chi_1^0})\gamma^{\sigma}(a^*_{ZX_i^0X_1^0}P_L+b^*_{ZX_i^0X_1^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_i^0})\gamma^{\rho}(a^*_{ZX_1^0X_i^0}P_L+b^*_{ZX_1^0X_i^0}P_R)]\left(-g_{\mu\sigma}+\frac{k_{2\mu}k_{2\sigma}}{m_Z^2}\right)\left(-g_{\rho\nu}+\frac{k_{1\rho}k_{1\nu}}{m_Z^2}\right)$$

$$\lim_{\beta_i\to 0}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}(\chi_i^0)|^2=\frac{2m_\chi^2}{m_Z^4(m_\chi^2+m^2_{\chi_i^0}-m_Z^2)^2}[m^2_{\chi_i^0}(4m_\chi^4-4m_\chi^2m_Z^2+3m_Z^4)|a_{ZX_1^0X_i^0}+a_{ZX_i^0X_1^0}|^2|a_{ZX_1^0X_i^0}-a_{ZX_i^0X_1^0}|^2+8m_Z^4(m_\chi^2-m_Z^2)|a_{ZX_1^0X_i^0}|^2|a_{ZX_i^0X_1^0}|^2]$$

$$\chi_1^0(p_1)+\chi_1^0(p_2)\to h(k_1)+h(k_2)$$

$$i\mathcal{M}_{t,i}=\bar{v}(p_2)i(a_{hX_1^0X_i^0}P_L+b_{hX_1^0X_i^0}P_R)\frac{i(\not{p}_1-\not{k}_1+m_{\chi_i^0})}{(p_1-k_1)^2-m_{\chi_i^0}^2}i(a_{hX_i^0X_1^0}P_L+b_{hX_i^0X_1^0}P_R)u(p_1)$$

$$=\frac{-i}{t-m_{\chi_i^0}^2}\bar{v}(p_2)(a_{hX_1^0X_i^0}P_L+b_{hX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_1+m_{\chi_i^0})(a_{hX_i^0X_1^0}P_L+b_{hX_i^0X_1^0}P_R)u(p_1)$$

$$(i\mathcal{M}_{t,i})^*=\frac{i}{t-m_{\chi_i^0}^2}\bar{u}(p_1)(a^*_{hX_i^0X_1^0}P_R+b^*_{hX_i^0X_1^0}P_L)(\not{p}_1-\not{k}_1+m_{\chi_i^0})(a^*_{hX_1^0X_i^0}P_R+b^*_{hX_1^0X_i^0}P_L)v(p_2)$$

$$i\mathcal{M}_{u,i}=\bar{v}(p_2)i(a_{hX_1^0X_i^0}P_L+b_{hX_1^0X_i^0}P_R)\frac{i(\not{p}_1-\not{k}_2+m_{\chi_i^0})}{(p_1-k_2)^2-m_{\chi_i^0}^2}i(a_{hX_i^0X_1^0}P_L+b_{hX_i^0X_1^0}P_R)u(p_1)$$

$$=\frac{-i}{u-m_{\chi_i^0}^2}\bar{v}(p_2)(a_{hX_1^0X_i^0}P_L+b_{hX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_i^0})(a_{hX_i^0X_1^0}P_L+b_{hX_i^0X_1^0}P_R)u(p_1)$$

$$(i\mathcal{M}_{u,i})^*=\frac{i}{u-m_{\chi_i^0}^2}\bar{u}(p_1)(a^*_{hX_i^0X_1^0}P_R+b^*_{hX_i^0X_1^0}P_L)(\not{p}_1-\not{k}_2+m_{\chi_i^0})(a^*_{hX_1^0X_i^0}P_R+b^*_{hX_1^0X_i^0}P_L)v(p_2)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_{t,i}|^2=\frac{1}{4}\frac{1}{(t-m_{\chi_i^0}^2)^2}\mathrm{Tr}[(\not{p}_2-m_{\chi_1^0})(a_{hX_1^0X_i^0}P_L+b_{hX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_1+m_{\chi_i^0})(a_{hX_i^0X_1^0}P_L+b_{hX_i^0X_1^0}P_R)$$
$$\times(\not{p}_1+m_{\chi_1^0})(a^*_{hX_i^0X_1^0}P_R+b^*_{hX_i^0X_1^0}P_L)(\not{p}_1-\not{k}_1+m_{\chi_i^0})(a^*_{hX_1^0X_i^0}P_R+b^*_{hX_1^0X_i^0}P_L)]$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_{u,i}|^2=\frac{1}{4}\frac{1}{(u-m_{\chi_i^0}^2)^2}\mathrm{Tr}[(\not{p}_2-m_{\chi_1^0})(a_{hX_1^0X_i^0}P_L+b_{hX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_2+m_{\chi_i^0})(a_{hX_i^0X_1^0}P_L+b_{hX_i^0X_1^0}P_R)$$
$$\times(\not{p}_1+m_{\chi_1^0})(a^*_{hX_i^0X_1^0}P_R+b^*_{hX_i^0X_1^0}P_L)(\not{p}_1-\not{k}_2+m_{\chi_i^0})(a^*_{hX_1^0X_i^0}P_R+b^*_{hX_1^0X_i^0}P_L)]$$

$$\frac{1}{4}\sum_{\text{spins}}\mathcal{M}_{t,i}\mathcal{M}^*_{u,i}=\frac{1}{4}\frac{1}{(t-m_{\chi_i^0}^2)(u-m_{\chi_i^0}^2)}\mathrm{Tr}[(\not{p}_2-m_{\chi_1^0})(a_{hX_1^0X_i^0}P_L+b_{hX_1^0X_i^0}P_R)(\not{p}_1-\not{k}_1+m_{\chi_i^0})(a_{hX_i^0X_1^0}P_L+b_{hX_i^0X_1^0}P_R)$$
$$\times(\not{p}_1+m_{\chi_1^0})(a^*_{hX_i^0X_1^0}P_R+b^*_{hX_i^0X_1^0}P_L)(\not{p}_1-\not{k}_2+m_{\chi_i^0})(a^*_{hX_1^0X_i^0}P_R+b^*_{hX_1^0X_i^0}P_L)]$$

$$\lim_{\beta_i\to 0}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}(\chi_i^0)|^2=\frac{2m_\chi^2m_{\chi_i^0}^2}{(m_\chi^2+m_{\chi_i^0}^2-m_h^2)^2}|a_{hX_1^0X_i^0}a_{hX_i^0X_1^0}-b_{hX_1^0X_i^0}b_{hX_i^0X_1^0}|^2=0$$

$$\mathcal{L}_{\mathrm{T}} = iT^\dagger \bar{\sigma}^\mu D_\mu T - \frac{1}{2}(m_T TT + h.c.)$$
$$\mathcal{L}_{\mathrm{Q}} = iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 - (m_Q Q_1 Q_2 + h.c.)$$
$$\mathcal{L}_{\mathrm{HTQ}} = y_1 Q_1 TH - y_2 Q_2 TH^\dagger + h.c.$$

$$m_T \to m_T e^{i\phi_T}, \quad m_Q \to m_Q e^{i\phi_Q}, \quad y_1 \to y_1 e^{i\phi_1}, \quad y_2 \to y_2 e^{i\phi_2}$$
$$\Rightarrow \begin{cases} \mathcal{L}_{\mathrm{T}} = iT^\dagger \bar{\sigma}^\mu D_\mu T - \frac{1}{2}(m_T e^{i\phi_T} TT + h.c.) \\ \mathcal{L}_{\mathrm{Q}} = iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 - (m_Q e^{i\phi_Q} Q_1 Q_2 + h.c.) \\ \mathcal{L}_{\mathrm{HTQ}} = y_1 e^{i\phi_1} Q_1 TH - y_2 e^{i\phi_2} Q_2 TH^\dagger + h.c. \end{cases}$$

$$T \to e^{-i\phi_T/2} T, \quad Q_1 \to e^{-i\phi_1} Q_1, \quad Q_2 \to e^{-i\phi_2} Q_2$$
$$\Rightarrow \begin{cases} \mathcal{L}_{\mathrm{T}} = iT^\dagger \bar{\sigma}^\mu D_\mu T - \frac{1}{2}(m_T TT + h.c.) \\ \mathcal{L}_{\mathrm{Q}} = iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 - (m_Q e^{i(\phi_Q - \phi_1 - \phi_2)} Q_1 Q_2 + h.c.) \\ \mathcal{L}_{\mathrm{HTQ}} = y_1 Q_1 TH - y_2 Q_2 TH^\dagger + h.c. \end{cases}$$

[By choosing appropriate field redefinitions, m_T, y_1, and y_2 can be made to be real, and the phase of m_Q is the only source of CP violation arising from the dark sector]

$$m_T \to -m_T, \quad m_Q \to -m_Q$$
$$\Rightarrow \begin{cases} \mathcal{L}_{\mathrm{T}} \to iT^\dagger \bar{\sigma}^\mu D_\mu T + \frac{1}{2}(m_T TT + h.c.) \\ \mathcal{L}_{\mathrm{Q}} \to iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 + (m_Q Q_1 Q_2 + h.c.) \\ \mathcal{L}_{\mathrm{HTQ}} \to y_1 Q_1 TH - y_2 Q_2 TH^\dagger + h.c. \end{cases}$$

[$T \to iT, \quad Q_1 \to -iQ_1, \quad Q_2 \to -iQ_2$ gives the original Lagrangian]

$$m_T \to -m_T, \quad y_2 \to -y_2$$
$$\Rightarrow \begin{cases} \mathcal{L}_{\mathrm{T}} \to iT^\dagger \bar{\sigma}^\mu D_\mu T + \frac{1}{2}(m_T TT + h.c.) \\ \mathcal{L}_{\mathrm{Q}} \to iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 - (m_Q Q_1 Q_2 + h.c.) \\ \mathcal{L}_{\mathrm{HTQ}} \to y_1 Q_1 TH + y_2 Q_2 TH^\dagger + h.c. \end{cases}$$

[$T \to iT, \quad Q_1 \to -iQ_1, \quad Q_2 \to iQ_2$ gives the original Lagrangian]