Renormalization group equations in the standard model (SM)

Ref: Peskin & Schroeder, an Introduction to Quantum Field Theory;
Sher, Phys. Rept. 179, 273 (1989); Quiros, hep-ph/9901312; Langacker, Phys. Rept. 72, 185 (1981)
Bardin & Passarino, the Standard Model in the Making; Denner, 0709.1075

In an instructive ϕ^4 theory, a bare Green n-point function $\langle\Omega|T\phi_0(x_1)\cdots\phi_0(x_n)|\Omega\rangle$ is given by the bare coupling constant λ_0 and the cutoff Λ, independent of the renormalization scale $\mu_{\rm R}$. The dependence of $\mu_{\rm R}$ enters when we remove the cutoff dependence by rescaling the fields and eliminating λ_0 in favor of the renormalized coupling λ. The renormalized Green function $\langle\Omega|T\phi(x_1)\cdots\phi(x_n)|\Omega\rangle = Z^{-n/2}\langle\Omega|T\phi_0(x_1)\cdots\phi_0(x_n)|\Omega\rangle$ depends on $\mu_{\rm R}$, and could be defined equally well with a new λ', Z' at another scale $\mu'_{\rm R}$.

Connected n-point Green function: $G_{\rm c}^{(n)}(x_1,\cdots,x_n) = \langle\Omega|T\phi(x_1)\cdots\phi(x_n)|\Omega\rangle_{\rm connected}$

An infinitesimal shift of $\mu_{\rm R}$, $\mu_{\rm R}\to\mu_{\rm R}+\delta\mu_{\rm R}$, leads to

$$\lambda\to\lambda+\delta\lambda,\quad \phi\to(1+\delta\eta)\phi,\quad G_{\rm c}^{(n)}\to(1+n\delta\eta)G_{\rm c}^{(n)}$$

$$\Rightarrow\quad dG_{\rm c}^{(n)} = \frac{\partial G_{\rm c}^{(n)}}{\partial\mu_{\rm R}}\delta\mu_{\rm R} + \frac{\partial G_{\rm c}^{(n)}}{\partial\lambda}\delta\lambda = n\delta\eta G_{\rm c}^{(n)}$$

$$\beta\equiv\frac{\mu_{\rm R}}{\delta\mu_{\rm R}}\delta\lambda,\quad \gamma\equiv-\frac{\mu_{\rm R}}{\delta\mu_{\rm R}}\delta\eta$$

$$\Rightarrow\quad \text{Callan-Symanzik equation:}\quad \left[\mu_{\rm R}\frac{\partial}{\partial\mu_{\rm R}}+\beta(\lambda)\frac{\partial}{\partial\lambda}+n\gamma(\lambda)\right]G_{\rm c}^{(n)}(\{x_i\};\mu_{\rm R},\lambda)=0$$

In a renormalizable massless scalar field theory, the 2-point Green function in the mometum space is

$$G_{\rm c}^{(2)}(p) = \begin{pmatrix}\text{Tree-level}\\ \text{propagator}\end{pmatrix} + \begin{pmatrix}\text{1PI loop}\\ \text{diagrams}\end{pmatrix} + \begin{pmatrix}\text{2-point}\\ \text{counterterm}\end{pmatrix} = \frac{i}{p^2}+\frac{i}{p^2}\left(A\ln\frac{\Lambda^2}{-p^2}+\text{finite}\right)+\frac{i}{p^2}(ip^2\delta_Z)\frac{i}{p^2}$$

$$\delta_Z = A\ln\frac{\Lambda^2}{\mu_{\rm R}^2}+\text{finite}$$

The β term in the Callan-Symanzik equation to $G_{\rm c}^{(2)}(p^2)$ is smaller by at least one power of λ than the γ term

$$\Rightarrow\quad -\frac{i}{p^2}\mu_{\rm R}\frac{\partial}{\partial\mu_{\rm R}}\delta_Z + 2\gamma(\lambda)\frac{i}{p^2}=0 \quad\Rightarrow\quad \gamma(\lambda)=\frac{1}{2}\mu_{\rm R}\frac{\partial}{\partial\mu_{\rm R}}\delta_Z = \frac{1}{2}\frac{\partial\delta_Z}{\partial\ln\mu_{\rm R}} = -A \quad \text{(lowest order)}$$

For a generic dimensionless coupling g, associated with an n-point vertex, the n-point Green function is

$$G_{\rm c}^{(n)}(\{p_i\}) = \begin{pmatrix}\text{Tree-level}\\ \text{propagator}\end{pmatrix} + \begin{pmatrix}\text{1PI loop}\\ \text{diagrams}\end{pmatrix} + \begin{pmatrix}\text{Vertex}\\ \text{counterterm}\end{pmatrix} + \begin{pmatrix}\text{External leg}\\ \text{corrections}\end{pmatrix}$$

$$=\left(\prod_i\frac{i}{p_i^2}\right)\left[-ig-iB\ln\frac{\Lambda^2}{-p^2}-i\delta_g+(-ig)\sum_i\left(A_i\ln\frac{\Lambda^2}{-p_i^2}-\delta_{Z_i}\right)\right]+\text{finite}$$

$p^2 :=$ a typical invariant built from $\{p_i\}$, $\quad \delta_g = -B\ln\frac{\Lambda^2}{\mu_{\rm R}^2}+\text{finite},\quad \delta_{Z_i}=A_i\ln\frac{\Lambda^2}{\mu_{\rm R}^2}+\text{finite}$

$$\text{Callan-Symanzik equation}\quad\Rightarrow\quad \mu_{\rm R}\frac{\partial}{\partial\mu_{\rm R}}\left(\delta_g - g\sum_i\delta_{Z_i}\right)+\beta(g)+g\sum_i\left(\frac{1}{2}\mu_{\rm R}\frac{\partial}{\partial\mu_{\rm R}}\delta_{Z_i}\right)=0\quad\text{(lowest order)}$$

$$\Rightarrow\quad \beta(g)=\mu_{\rm R}\frac{\partial}{\partial\mu_{\rm R}}\left(-\delta_g+\frac{1}{2}g\sum_i\delta_{Z_i}\right)=\frac{\partial}{\partial\ln\mu_{\rm R}}\left(-\delta_g+\frac{1}{2}g\sum_i\delta_{Z_i}\right)=-2B-g\sum_i A_i\quad\text{(lowest order)}$$

The renormalization group equation (RGE) $\dfrac{d\bar{g}}{d\ln\mu_{\rm R}}=\beta(g)$ controls the running coupling constant $\bar{g}$

1) β functions for gauge couplings

An SU(N) gauge theory with $n_{\mathrm{D}}(r)$ Dirac fermions, $n_{\mathrm{W}}(r)$ Weyl fermions, and $n_{\mathrm{S}}(r)$ complex scalars in representations r [§16.6 in Peskin & Schroeder, an Introduction to QFT]:

$$\beta_g = -\frac{g^3}{16\pi^2}\frac{1}{3}\left\{11C_2(G) - \sum_r [4n_{\mathrm{D}}(r) + 2n_{\mathrm{W}}(r) + n_{\mathrm{S}}(r)]C(r)\right\}$$

$$\mathrm{Tr}(t^a t^b) = C(r)\delta^{ab}, \quad t^a t^a = C_2(r)\cdot\mathbf{1}, \quad C(N) = \frac{1}{2}, \quad C_2(G) = N$$

A U(1) gauge theory with $n_{\mathrm{D}}(Q)$ Dirac fermions, $n_{\mathrm{W}}(Q)$ Weyl fermions, and $n_{\mathrm{S}}(Q)$ complex scalars with charges Q: $\beta_e = \frac{e^3}{16\pi^2}\frac{1}{3}\sum_Q [4n_{\mathrm{D}}(Q) + 2n_{\mathrm{W}}(Q) + n_{\mathrm{S}}(Q)]Q^2$

$\mathrm{SU(3)_C}$: 6 quarks $\to$ 6 color triplets

$$\beta_{g_s} = -\frac{g_{\mathrm{s}}^3}{16\pi^2}\frac{1}{3}\left(11\cdot 3 - 4\cdot 6\cdot\frac{1}{2}\right) = -\frac{7g_{\mathrm{s}}^3}{16\pi^2} = b_{\mathrm{s}}g_{\mathrm{s}}^3, \quad b_{\mathrm{s}} = -\frac{7}{16\pi^2}$$

$\mathrm{SU(2)_L}$: 3 fermion generations $\left(\text{each}\begin{cases}\text{1 left-handed lepton doublet}\\ \text{1 left-handed quark doublet with 3 colors}\end{cases}\right)$, 1 Higgs doublet

$$\beta_g = -\frac{g^3}{16\pi^2}\frac{1}{3}\left[11\cdot 2 - 3\cdot 2\cdot(1+3)\cdot\frac{1}{2} - \frac{1}{2}\right] = -\frac{g^3}{16\pi^2}\frac{1}{3}\left(22 - 12 - \frac{1}{2}\right) = -\frac{19g^3}{96\pi^2} = b_2 g^3, \quad b_2 = -\frac{19}{96\pi^2}$$

$\mathrm{U(1)_Y}$: $Y_{L_{i\mathrm{L}}} = -\frac{1}{2}, \quad Y_{\ell_{i\mathrm{R}}} = -1, \quad 3 \text{ colors}\left(Y_{Q_{i\mathrm{L}}} = \frac{1}{6}, \quad Y_{u_{i\mathrm{R}}} = \frac{2}{3}, \quad Y_{d_{i\mathrm{R}}} = -\frac{1}{3}\right), \quad Y_H = \frac{1}{2}$

$$\beta_{g'} = \frac{g'^3}{16\pi^2}\frac{1}{3}\left\{3\cdot 2\cdot\left[2\cdot\left(-\frac{1}{2}\right)^2 + (-1)^2 + 3\cdot 2\cdot\left(\frac{1}{6}\right)^2 + 3\cdot\left(\frac{2}{3}\right)^2 + 3\cdot\left(-\frac{1}{3}\right)^2\right] + 2\cdot\left(\frac{1}{2}\right)^2\right\} = \frac{41g'^3}{96\pi^2} = b'g'^3, \quad b' = \frac{41}{96\pi^2}$$

$$g_1 \equiv \sqrt{\frac{5}{3}}g', \quad \beta_{g_1} = \frac{41g_1^3}{160\pi^2} = b_1 g_1^3, \quad b_1 = \frac{3}{5}b'$$

$\mathrm{U(1)_{EM}}$: $Q_{\ell_i} = -1, \quad 3 \text{ colors}\left(Q_{u_i} = \frac{2}{3}, \quad Q_{d_i} = -\frac{1}{3}\right), \quad Q_{G^+} = 1$ (charged Goldstone boson)

$$\beta_e = \frac{e^3}{16\pi^2}\frac{1}{3}\left\{3\cdot 4\cdot\left[(-1)^2 + 3\cdot\left(\frac{2}{3}\right)^2 + 3\cdot\left(-\frac{1}{3}\right)^2\right] + 1^2\right\} = \frac{11e^3}{16\pi^2} = be^3, \quad b = \frac{11}{16\pi^2}$$

$$\alpha_{\mathrm{s}} = \frac{g_{\mathrm{s}}^2}{4\pi}, \quad \alpha_2 = \frac{g^2}{4\pi}, \quad \alpha' = \frac{g'}{4\pi}, \quad \alpha = \frac{e^2}{4\pi}, \quad \alpha_1 = \frac{5}{3}\alpha' : \quad \alpha_i^{-1}(Q^2) = \alpha_i^{-1}(\mu_{\mathrm{R}}^2) - 4\pi b_i \ln\frac{Q^2}{\mu_{\mathrm{R}}^2}$$

$$\left[\begin{array}{l} \dfrac{dg_i}{d\ln\mu_{\mathrm{R}}} = \beta_{g_i} = b_i e^3 \;\Rightarrow\; \dfrac{d\alpha_i}{d\ln\mu_{\mathrm{R}}} = \dfrac{2g_i}{4\pi}\dfrac{dg_i}{d\ln\mu_{\mathrm{R}}} = \dfrac{2g_i}{4\pi}bg_i^3 = 8\pi b\alpha_i^2 \;\Rightarrow\; \dfrac{d\alpha_i}{\alpha_i^2} = 8\pi b_i\, d\ln\mu_{\mathrm{R}} \\ \quad\Rightarrow\; -[\alpha_i^{-1}(Q) - \alpha_i^{-1}(\mu_{\mathrm{R}})] = 8\pi b_i(\ln Q - \ln\mu_{\mathrm{R}}) = 4\pi b_i \ln\dfrac{Q^2}{\mu_{\mathrm{R}}^2} \end{array}\right]$$

(2)

2) β functions for Higgs self-couplings

β functions for Higgs self-couplings can be derived by analyzing the scale dependence of the Coleman-Weinberg effective potential, following Sher, Phys. Rept. 179, 273 (1989)

⊙ Effective action and effective potential

Generating functional $Z[J] = \exp(iW[J]) = \int \mathcal{D}\phi \exp\left[i \int d^4x (\mathcal{L}[\phi] + J\phi) \right]$

Connected generating functional $W[J] \equiv -i \ln Z[J]$

$$\frac{\delta W[J]}{\delta J(x)} = -i \frac{\delta \ln Z}{\delta J(x)} = \frac{\int \mathcal{D}\phi \exp\left[i \int d^4y (\mathcal{L}[\phi] + J\phi) \right] \phi(x)}{\int \mathcal{D}\phi \exp\left[i \int d^4y (\mathcal{L}[\phi] + J\phi) \right]} = \langle \Omega | \phi(x) | \Omega \rangle_J$$

Classical field $\phi_{\mathrm{c}}(x) \equiv \langle \Omega | \phi(x) | \Omega \rangle_J = \dfrac{\delta W[J]}{\delta J(x)}$

Effective action $\Gamma[\phi_{\mathrm{c}}] \equiv W[J] - \int d^4x J(x) \phi_{\mathrm{c}}(x)$, $\quad \dfrac{\delta \Gamma[\phi_{\mathrm{c}}]}{\delta \phi_{\mathrm{c}}(x)} = -J(x)$

$$Z[j] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \cdots d^4x_n J(x_1) \cdots J(x_n) G^{(n)}(x_1, \cdots, x_n)$$

$G^{(n)}(x_1, \cdots, x_n) :=$ n-point Green function

$$W[j] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \cdots d^4x_n J(x_1) \cdots J(x_n) G_{\mathrm{c}}^{(n)}(x_1, \cdots, x_n)$$

$G_{\mathrm{c}}^{(n)}(x_1, \cdots, x_n) :=$ connected n-point Green function

$$\Gamma[\phi_{\mathrm{c}}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n \phi_{\mathrm{c}}(x_1) \cdots \phi_{\mathrm{c}}(x_n) \Gamma^{(n)}(x_1, \cdots, x_n)$$

$\Gamma^{(n)}(x_1, \cdots, x_n) :=$ one-particle irreducible (1PI) n-point Green function

Fourier transformations

$$\Gamma^{(n)}(x_1, \cdots, x_n) = \prod_{i=1}^{n} \left[\int \frac{d^4p_i}{(2\pi)^4} e^{ip_i \cdot x_i} \right] (2\pi)^4 \delta^{(4)}(p_1 + \cdots + p_n) \tilde{\Gamma}^{(n)}(p_1, \cdots, p_n)$$

$$\phi_{\mathrm{c}}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \tilde{\phi}_{\mathrm{c}}(p), \quad \int d^4x e^{ip\cdot x} = (2\pi)^4 \delta^{(4)}(p), \quad \int d^4x = (2\pi)^4 \delta^{(4)}(0)$$

$$\begin{aligned}
\Gamma[\phi_{\mathrm{c}}] &= \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int \frac{d^4p_i}{(2\pi)^4} e^{ip_i \cdot x_i} \right] \left[\prod_{j=1}^{n} \int d^4x_j \phi_{\mathrm{c}}(x_j) \right] (2\pi)^4 \delta^{(4)}(p_1 + \cdots + p_n) \tilde{\Gamma}^{(n)}(p_1, \cdots, p_n) \\
&= \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int \frac{d^4p_i}{(2\pi)^4} \right] \left[\prod_{j=1}^{n} \int d^4x_j \int \frac{d^4p'_j}{(2\pi)^4} e^{i(p_i - p'_j)\cdot x_j} \tilde{\phi}_{\mathrm{c}}(p'_j) \right] (2\pi)^4 \delta^{(4)}(p_1 + \cdots + p_n) \tilde{\Gamma}^{(n)}(p_1, \cdots, p_n) \\
&= \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int \frac{d^4p_i}{(2\pi)^4} \tilde{\phi}_{\mathrm{c}}(p_i) \right] (2\pi)^4 \delta^{(4)}(p_1 + \cdots + p_n) \tilde{\Gamma}^{(n)}(p_1, \cdots, p_n)
\end{aligned}$$

In a translationally invariant theory, $\phi_{\mathrm{c}}(x) = \phi_{\mathrm{c}}$ is constant, $\tilde{\phi}_{\mathrm{c}}(p) = \int d^4x e^{ip\cdot x} \phi_{\mathrm{c}}(x) = (2\pi)^4 \delta^{(4)}(p) \phi_{\mathrm{c}}$

Define effective potential $V_{\mathrm{eff}}(\phi_{\mathrm{c}})$ as $\Gamma[\phi_{\mathrm{c}}] \equiv -\int d^4x\, V_{\mathrm{eff}}(\phi_{\mathrm{c}}) = -V_{\mathrm{eff}}(\phi_{\mathrm{c}}) \int d^4x = -(2\pi)^4 \delta^{(4)}(0) V_{\mathrm{eff}}(\phi_{\mathrm{c}})$

$$\Gamma[\phi_{\mathrm{c}}] = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int \frac{d^4p_i}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p_i) \phi_{\mathrm{c}} \right] (2\pi)^4 \delta^{(4)}(p_1 + \cdots + p_n) \tilde{\Gamma}^{(n)}(p_1, \cdots, p_n) = \sum_{n=0}^{\infty} \frac{1}{n!} \phi_{\mathrm{c}}^n (2\pi)^4 \delta^{(4)}(0) \tilde{\Gamma}^{(n)}(p_i = 0)$$

$\Rightarrow \quad V_{\mathrm{eff}}(\phi_{\mathrm{c}}) = i \sum_{n=0}^{\infty} \frac{1}{n!} \phi_{\mathrm{c}}^n i \Gamma^{(n)}(p_i = 0)$ is given by the sum of 1PI diagrams with zero external momenta

⊙ 1-loop SM effective potential in the Landau gauge with the $\overline{\text{MS}}$ scheme

$$\mathcal{L}_{\rm H}=(D^\mu H)^\dagger D_\mu H - V_{\rm H}(H),\quad V_{\rm H}=-\mu^2 H^\dagger H+\lambda(H^\dagger H)^2,\quad \text{SM Higgs doublet } H(x)=\begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}[v+h(x)+iG^0(x)]\end{pmatrix}$$

Minimalization of $V_{\rm H}$ at tree level $\Rightarrow$ $\mu^2=\lambda v^2,\quad m_h^2=-\mu^2+3\lambda v^2=2\lambda v^2,\quad m_G^2=-\mu^2+\lambda v^2=0$

$\phi_{\rm c}$-dependent Higgs doublet $H_{\rm c}(x)=\begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}[\phi_{\rm c}+h(x)+iG^0(x)]\end{pmatrix}$

$$-V_{\rm H}^{\rm c}(H_{\rm c})=\frac{1}{2}\mu^2\phi_{\rm c}^2-\frac{1}{4}\lambda\phi_{\rm c}^4+(\mu^2-\lambda\phi_{\rm c}^2)\phi_{\rm c}h-\frac{1}{2}(-\mu^2+3\lambda\phi_{\rm c}^2)h^2-\frac{1}{2}(-\mu^2+\lambda\phi_{\rm c}^2)(G^0)^2-(-\mu^2+\lambda\phi_{\rm c}^2)|G^+|^2$$
$$-\lambda\phi_{\rm c}h^3-\lambda\phi_{\rm c}h(G^0)^2-\frac{1}{4}\lambda h^4-\frac{1}{4}\lambda(G^0)^4-\lambda|G^+|^4-\frac{1}{2}\lambda h^2(G^0)^2-\lambda|G^+|^2[h^2+2\phi_{\rm c}h+(G^0)^2]$$

Shifted mass-square: $\overline{m}_h^2(\phi_{\rm c})=-\mu^2+3\lambda\phi_{\rm c}^2,\quad \overline{m}_G^2(\phi_{\rm c})=-\mu^2+\lambda\phi_{\rm c}^2$

$\phi_{\rm c}^2h^2$ vertex $=-6i\lambda,\quad \ln(1+x)=\sum_{n=1}^\infty\frac{1}{n}(-1)^{n-1}x^n,\quad \ln(1-x)=-\sum_{n=1}^\infty\frac{1}{n}x^n$

Contribution from 1-loops of h (1PI diagrams as Fig. 1 in Quiros, hep-ph/9901312):

$$V_{{\rm eff},h}(\phi_{\rm c})=i\sum_{n=1}^\infty\int\frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left[\frac{1}{2}\phi_{\rm c}^2(-6i\lambda)\frac{i}{p^2-(-\mu^2)+i\varepsilon}\right]^n=i\sum_{n=1}^\infty\int\frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left(\frac{3\lambda\phi_{\rm c}^2}{p^2+\mu^2+i\varepsilon}\right)^n=-\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\ln\left(1-\frac{3\lambda\phi_{\rm c}^2}{p^2+\mu^2+i\varepsilon}\right)$$

Wick rotation to Euclidean space: $p^0=ip_{\rm E}^0,\quad p_{\rm E}^\mu=(-ip^0,\mathbf{p}),\quad p^2=(p^0)^2-|\mathbf{p}|^2=-p_{\rm E}^2,\quad d^4p=id^4p_{\rm E}$

$$V_{{\rm eff},h}(\phi_{\rm c})=\frac{1}{2}\int\frac{d^4p_{\rm E}}{(2\pi)^4}\ln\left(1+\frac{3\lambda\phi_{\rm c}^2}{p_{\rm E}^2-\mu^2}\right)=\frac{1}{2}\int\frac{d^4p_{\rm E}}{(2\pi)^4}\ln\frac{p_{\rm E}^2-\mu^2+3\lambda\phi_{\rm c}^2}{p_{\rm E}^2-\mu^2}$$
$$=\frac{1}{2}\int\frac{d^4p_{\rm E}}{(2\pi)^4}\ln[p_{\rm E}^2+\overline{m}_h^2(\phi_{\rm c})]+\phi_{\rm c}\text{-independent term (neglected below!)}$$

$$\int\frac{d^dp_{\rm E}}{(2\pi)^d}\frac{1}{p_{\rm E}^2+K}=\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}K^{1-d/2}},\quad \Gamma(z+1)=z\Gamma(z)$$

Dimensional regularization: $V_{{\rm eff},h}(\phi_{\rm c})=\frac{1}{2}(\mu_{\rm R}^2)^{2-d/2}\int\frac{d^dp_{\rm E}}{(2\pi)^d}\ln[p_{\rm E}^2+\overline{m}_h^2(\phi_{\rm c})]$

$$\frac{\partial^2V_{{\rm eff},h}}{\partial^2\overline{m}_h^2}=\frac{1}{2}(\mu_{\rm R}^2)^{2-d/2}\int\frac{d^dp_{\rm E}}{(2\pi)^d}\frac{1}{p_{\rm E}^2+\overline{m}_h^2}=\frac{1}{2}(\mu_{\rm R}^2)^{2-d/2}\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}(\overline{m}_h^2)^{1-d/2}}$$

$$\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}K^{2-d/2}}=\frac{1}{16\pi^2}\left[\frac{2}{4-d}-\ln K-\gamma_{\rm E}+\ln 4\pi+\mathcal{O}(4-d)\right]$$

$$\frac{1}{1-d/2}=-\frac{1}{1-(2-d/2)}=-\left\{1+\frac{4-d}{2}+\mathcal{O}[(4-d)^2]\right\},\quad \frac{1}{d}=\frac{1}{4}\frac{1}{1-(2-d/2)/2}=\frac{1}{4}\left\{1+\frac{1}{2}\frac{4-d}{2}+\mathcal{O}[(4-d)^2]\right\}$$

$$1+\frac{1}{2}=\frac{3}{2},\quad \frac{1}{d(1-d/2)}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}K^{2-d/2}}=-\frac{1}{64\pi^2}\left[\frac{2}{4-d}-\ln K-\gamma_{\rm E}+\ln 4\pi+\frac{3}{2}+\mathcal{O}(4-d)\right]$$

$$V_{{\rm eff},h}(\phi_{\rm c})=\int_0^{\overline{m}_h^2}dK\frac{\partial^2V_{{\rm eff},h}}{\partial^2\overline{m}_h^2}=\frac{1}{2}(\mu_{\rm R}^2)^{2-d/2}\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}d/2}(\overline{m}_h^2)^{d/2}=\frac{\overline{m}_h^4}{d(1-d/2)}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(m_h^2/\mu_{\rm R}^2)^{2-d/2}}$$
$$=-\frac{\overline{m}_h^4(\phi_{\rm c})}{64\pi^2}\left[\frac{2}{4-d}-\ln\frac{\overline{m}_h^2(\phi_{\rm c})}{\mu_{\rm R}^2}-\gamma_{\rm E}+\ln 4\pi+\frac{3}{2}+\mathcal{O}(4-d)\right]$$

$\overline{\text{MS}}$ renormalization scheme: subtracting the term proportional to $\left(\frac{2}{4-d}-\gamma_{\rm E}+\ln 4\pi\right)$

$$\Rightarrow\quad V_{{\rm eff},h}(\phi_{\rm c})=\frac{1}{64\pi^2}\overline{m}_h^4(\phi_{\rm c})\left[\ln\frac{\overline{m}_h^2(\phi_{\rm c})}{\mu_{\rm R}^2}-\frac{3}{2}\right]$$

$\phi_c^2(G^0)^2$ vertex $=-2i\lambda$, $\phi_c^2 G^+G^-$ vertex $=-2i\lambda$

Contribution from 1-loops of G^0 and $G^\pm$:

$$V_{\text{eff},G^0}(\phi_c)=i\sum_{n=1}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left[\frac{1}{2}\phi_c^2(-2i\lambda)\frac{i}{p^2-(-\mu^2)+i\varepsilon}\right]^n=\frac{1}{2}\int\frac{d^4p_E}{(2\pi)^4}\ln\frac{p_E^2-\mu^2+\lambda\phi_c^2}{p_E^2-\mu^2}$$

$$\to\frac{1}{2}\int\frac{d^4p_E}{(2\pi)^4}\ln[p_E^2+\overline{m}_G^2(\phi_c)]\to\frac{1}{64\pi^2}\overline{m}_G^4(\phi_c)\left[\ln\frac{\overline{m}_G^2(\phi_c)}{\mu_R^2}-\frac{3}{2}\right]$$

$$V_{\text{eff},G^\pm}(\phi_c)=i\sum_{n=1}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{n}\left[\frac{1}{2}\phi_c^2(-2i\lambda)\frac{i}{p^2-(-\mu^2)+i\varepsilon}\right]^n\to\frac{2}{64\pi^2}\overline{m}_G^4(\phi_c)\left[\ln\frac{\overline{m}_G^2(\phi_c)}{\mu_R^2}-\frac{3}{2}\right]$$

Total scalar contribution

$$V_{\text{eff,S}}(\phi_c)=V_{\text{eff},h}+V_{\text{eff},G^0}+V_{\text{eff},G^\pm}=\frac{1}{64\pi^2}\overline{m}_h^4(\phi_c)\left[\ln\frac{\overline{m}_h^2(\phi_c)}{\mu_R^2}-\frac{3}{2}\right]+\frac{3}{64\pi^2}\overline{m}_G^4(\phi_c)\left[\ln\frac{\overline{m}_G^2(\phi_c)}{\mu_R^2}-\frac{3}{2}\right]$$

Neglecting all Yukawa couplings expect the top Yukawa coupling

$\mathcal{L}(\phi_c)\supset-\frac{y_t}{\sqrt{2}}\phi_c\overline{t}t$, $\phi_c\overline{t}t$ vertex $=-i\frac{y_t}{\sqrt{2}}$, Shifted mass-square $\overline{m}_t^2(\phi_c)=\frac{1}{2}y_t^2\phi_c^2$

$$\text{Tr}[(\not{p}\not{p})^n]=\text{Tr}[(\not{p}\not{p})^{n-1}p^\mu p^\nu(\gamma_\mu\gamma_\nu)]=\text{Tr}[(\not{p}\not{p})^{n-1}p^\mu p^\nu(2g_{\mu\nu}-\gamma_\nu\gamma_\mu)]=2\text{Tr}[(\not{p}\not{p})^{n-1}]p^2-\text{Tr}[(\not{p}\not{p})^n]$$

$$\Rightarrow\quad\text{Tr}[(\not{p}\not{p})^n]=\text{Tr}[(\not{p}\not{p})^{n-1}]p^2=\text{Tr}[(\not{p}\not{p})^{n-2}]p^4=\cdots=\text{Tr}(\not{p}\not{p})p^{2n-2}=\text{Tr}(1)p^{2n}=4p^{2n}$$

Tr(1) counts the number of degrees of freedom of a fermion $\begin{cases}\text{Tr}(1)=4 \text{ for a Dirac fermion}\\ \text{Tr}(1)=2 \text{ for a Weyl fermion}\end{cases}$

Contribution from 1-loops of t (1PI diagrams as Fig. 2 in Quiros, hep-ph/9901312):

$$V_{\text{eff},t}(\phi_c)=3\cdot i\sum_{n=1}^{\infty}\int\frac{d^4p}{(2\pi)^4}(-1)\frac{1}{2n}\text{Tr}\left\{\left[\phi_c^2\left(-i\frac{y_t}{\sqrt{2}}\right)^2\frac{i\not{p}}{p^2+i\varepsilon}\frac{i\not{p}}{p^2+i\varepsilon}\right]^n\right\}$$

$$=-3i\,\text{Tr}(1)\sum_{n=1}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left[\phi_c^2\frac{y_t^2}{2}\frac{p^2}{(p^2+i\varepsilon)^2}\right]^n=-12i\sum_{n=1}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left(\frac{y_t^2\phi_c^2/2}{p^2+i\varepsilon}\right)^n$$

$$=6i\int\frac{d^4p}{(2\pi)^4}\ln\left(1-\frac{y_t^2\phi_c^2/2}{p^2+i\varepsilon}\right)\to-6\int\frac{d^4p_E}{(2\pi)^4}\ln[p_E^2+\overline{m}_t^2(\phi_c)]$$

$$\to-\frac{12}{64\pi^2}\overline{m}_t^4(\phi_c)\left[\ln\frac{\overline{m}_t^2(\phi_c)}{\mu_R^2}-\frac{3}{2}\right]$$

Fermionic contribution

$$V_{\text{eff,F}}(\phi_c)=V_{\text{eff},t}(\phi_c)=-\frac{12}{64\pi^2}\overline{m}_t^4(\phi_c)\left[\ln\frac{\overline{m}_t^2(\phi_c)}{\mu_R^2}-\frac{3}{2}\right]$$

$$\mathcal{L}(\phi_c) \supset \frac{1}{4} g^2 \phi_c^2 W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2 + g'^2)\phi_c^2 Z_\mu Z^\mu, \quad \phi_c^2 W^+ W^- \text{ vertex} = \frac{i}{2} g^2, \quad \phi_c^2 ZZ \text{ vertex} = \frac{i}{2}(g^2 + g'^2)$$

Shifted mass-square $\bar{m}_W^2(\phi_c) = \frac{1}{4} g^2 \phi_c^2, \quad \bar{m}_Z^2(\phi_c) = \frac{1}{4}(g^2 + g'^2)\phi_c^2$

Gauge boson propagator in the Landau gauge: $\dfrac{-iP^\mu{}_\nu}{p^2 + i\varepsilon}, \quad P^\mu{}_\nu \equiv g^\mu{}_\nu - \dfrac{p^\mu p_\nu}{p^2}$

$$p_\mu P^\mu{}_\nu = 0, \quad P^n = P\ [(P^\mu{}_\nu)^n = P^\mu{}_\nu], \quad \mathrm{Tr}(P^n) = \mathrm{Tr}(P) = P^\mu{}_\mu = \left(g^\mu{}_\mu - \frac{p^\mu p_\mu}{p^2} \right) = g^\mu{}_\mu - 1 = d - 1$$

$\mathrm{Tr}(P) = d - 1 \to 3$ counts the number of degrees of freedom of a massive vector boson

$$d - 1 = 3\left\{1 - \frac{2}{3}\frac{4-d}{2} + \mathcal{O}[(4-d)^2]\right\}, \quad \frac{3}{2} - \frac{2}{3} = \frac{5}{6}$$

$$\frac{d-1}{d(1-d/2)} \frac{\Gamma(2-d/2)}{(4\pi)^{d/2} K^{2-d/2}} = -\frac{3}{64\pi^2}\left[\frac{2}{4-d} - \ln K - \gamma_\mathrm{E} + \ln 4\pi + \frac{5}{6} + \mathcal{O}(4-d)\right]$$

Contribution from 1-loops of $W^\pm$ (1PI diagrams as Fig. 3 in Quiros, hep-ph/9901312):

$$V_{\mathrm{eff},W}(\phi_c) = i\sum_{n=1}^\infty \int \frac{d^4 p}{(2\pi)^4} \frac{1}{n} \mathrm{Tr}\left\{\left[\frac{1}{2}\phi_c^2 \frac{i}{2} g^2 \frac{-iP}{p^2 + i\varepsilon}\right]^n\right\} = i\,\mathrm{Tr}(P) \sum_{n=1}^\infty \int \frac{d^4 p}{(2\pi)^4} \frac{1}{n}\left[\frac{g^2\phi_c^2/4}{p^2 + i\varepsilon}\right]^n$$

$$= -i\,\mathrm{Tr}(P) \int \frac{d^4 p}{(2\pi)^4} \ln\left(1 - \frac{g^2\phi_c^2/4}{p^2 + i\varepsilon}\right) \to \mathrm{Tr}(P) \int \frac{d^4 p_\mathrm{E}}{(2\pi)^4} \ln[p_\mathrm{E}^2 + \bar{m}_W^2(\phi_c)]$$

$$\to \frac{2(d-1)\bar{m}_W^4}{d(1-d/2)} \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(m_h^2/\mu_\mathrm{R}^2)^{2-d/2}} \to \frac{6}{64\pi^2}\bar{m}_W^4(\phi_c)\left[\ln\frac{\bar{m}_W^2(\phi_c)}{\mu_\mathrm{R}^2} - \frac{5}{6}\right]$$

Contribution from 1-loops of Z:

$$V_{\mathrm{eff},Z}(\phi_c) = i\sum_{n=1}^\infty \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2n} \mathrm{Tr}\left\{\left[\frac{1}{2}\phi_c^2 \frac{i}{2}(g^2 + g'^2) \frac{-iP}{p^2 + i\varepsilon}\right]^n\right\} = -\frac{i}{2}\mathrm{Tr}(P) \int \frac{d^4 p}{(2\pi)^4} \ln\left[1 - \frac{(g^2 + g'^2)\phi_c^2/4}{p^2 + i\varepsilon}\right]$$

$$\to \frac{1}{2}\mathrm{Tr}(P) \int \frac{d^4 p_\mathrm{E}}{(2\pi)^4} \ln[p_\mathrm{E}^2 + \bar{m}_Z^2(\phi_c)] \to \frac{3}{64\pi^2}\bar{m}_Z^4(\phi_c)\left[\ln\frac{\bar{m}_Z^2(\phi_c)}{\mu_\mathrm{R}^2} - \frac{5}{6}\right]$$

Electroweak guage boson constribution

$$V_{\mathrm{eff,V}}(\phi_c) = V_{\mathrm{eff},W} + V_{\mathrm{eff},Z} = \frac{6}{64\pi^2}\bar{m}_W^4(\phi_c)\left[\ln\frac{\bar{m}_W^2(\phi_c)}{\mu_\mathrm{R}^2} - \frac{5}{6}\right] + \frac{3}{64\pi^2}\bar{m}_Z^4(\phi_c)\left[\ln\frac{\bar{m}_Z^2(\phi_c)}{\mu_\mathrm{R}^2} - \frac{5}{6}\right]$$

In total, $V_{\rm eff}(\phi_{\rm c}) = V_0(\phi_{\rm c}) + V_1(\phi_{\rm c})$

Tree level $V_0 = -\frac{1}{2}\mu^2\phi_{\rm c}^2 + \frac{1}{4}\lambda\phi_{\rm c}^4$, 1-loop level $V_1 = V_{\rm eff,S} + V_{\rm eff,F} + V_{\rm eff,V}$

Explicit expression for the 1-loop effective potential in the Landau gauge with the $\overline{\rm MS}$ scheme:

$$V_{\rm eff}(\phi_{\rm c}) = -\frac{1}{2}\mu^2\phi_{\rm c}^2 + \frac{1}{4}\lambda\phi_{\rm c}^4 + \frac{1}{64\pi^2}\bar{m}_h^4(\phi_{\rm c})\left[\ln\frac{\bar{m}_h^2(\phi_{\rm c})}{\mu_{\rm R}^2} - \frac{3}{2}\right] + \frac{3}{64\pi^2}\bar{m}_G^4(\phi_{\rm c})\left[\ln\frac{\bar{m}_G^2(\phi_{\rm c})}{\mu_{\rm R}^2} - \frac{3}{2}\right]$$

$$-\frac{12}{64\pi^2}\bar{m}_t^4(\phi_{\rm c})\left[\ln\frac{\bar{m}_t^2(\phi_{\rm c})}{\mu_{\rm R}^2} - \frac{3}{2}\right] + \frac{6}{64\pi^2}\bar{m}_W^4(\phi_{\rm c})\left[\ln\frac{\bar{m}_W^2(\phi_{\rm c})}{\mu_{\rm R}^2} - \frac{5}{6}\right] + \frac{3}{64\pi^2}\bar{m}_Z^4(\phi_{\rm c})\left[\ln\frac{\bar{m}_Z^2(\phi_{\rm c})}{\mu_{\rm R}^2} - \frac{5}{6}\right]$$

$$\bar{m}_h^2(\phi_{\rm c}) = -\mu^2 + 3\lambda\phi_{\rm c}^2, \quad \bar{m}_G^2(\phi_{\rm c}) = -\mu^2 + \lambda\phi_{\rm c}^2, \quad \bar{m}_t^2(\phi_{\rm c}) = \frac{1}{2}y_t^2\phi_{\rm c}^2, \quad \bar{m}_W^2(\phi_{\rm c}) = \frac{1}{4}g^2\phi_{\rm c}^2, \quad \bar{m}_Z^2(\phi_{\rm c}) = \frac{1}{4}(g^2 + g'^2)\phi_{\rm c}^2$$

The effective potential should not depend on the renormalization scale: $\frac{dV_{\rm eff}(\phi_{\rm c})}{d\mu_{\rm R}} = 0$

$$\Rightarrow \quad \left(\mu_{\rm R}\frac{\partial}{\partial\mu_{\rm R}} + \beta_g\frac{\partial}{\partial g} + \beta_{g'}\frac{\partial}{\partial g'} + \beta_{y_t}\frac{\partial}{\partial y_t} + \beta_\lambda\frac{\partial}{\partial\lambda} + \beta_{\mu^2}\mu^2\frac{\partial}{\partial\mu^2} - \gamma\phi_{\rm c}\frac{\partial}{\partial\phi_{\rm c}}\right)V_{\rm eff}(\phi_{\rm c}) = 0$$

$$\beta_g = \mu_{\rm R}\frac{dg}{d\mu_{\rm R}}, \quad \beta_{g'} = \mu_{\rm R}\frac{dg'}{d\mu_{\rm R}}, \quad \beta_{y_t} = \mu_{\rm R}\frac{dy_t}{d\mu_{\rm R}}, \quad \beta_\lambda = \mu_{\rm R}\frac{d\lambda}{d\mu_{\rm R}}, \quad \beta_{\mu^2} = \frac{\mu_{\rm R}}{\mu^2}\frac{d\mu^2}{d\mu_{\rm R}}, \quad \gamma = -\frac{\mu_{\rm R}}{\phi_{\rm c}}\frac{d\phi_{\rm c}}{d\mu_{\rm R}}$$

$\beta_i \sim \mathcal{O}(\hbar), \quad \gamma \sim \mathcal{O}(\hbar), \quad V_0 \sim \mathcal{O}(1), \quad V_1 \sim \mathcal{O}(\hbar)$

$\mathcal{O}(\hbar)$: $\left(\beta_\lambda\frac{\partial}{\partial\lambda} + \beta_{\mu^2}\mu^2\frac{\partial}{\partial\mu^2} - \gamma\phi_{\rm c}\frac{\partial}{\partial\phi_{\rm c}}\right)V_0 = -\mu_{\rm R}\frac{\partial V_1}{\partial\mu_{\rm R}}$ [Ref: Sher, Phys.Rept. 179, 273 (1989)]

Note: at the leading order, β_i's are independent of gauge choices, but γ and V_1 are gauge-dependent!

$$\left(\beta_\lambda\frac{\partial}{\partial\lambda} + \beta_{\mu^2}\mu^2\frac{\partial}{\partial\mu^2} - \gamma\phi_{\rm c}\frac{\partial}{\partial\phi_{\rm c}}\right)V_0 = \frac{1}{4}\beta_\lambda\phi_{\rm c}^4 - \frac{1}{2}\beta_{\mu^2}\mu^2\phi_{\rm c}^2 + \gamma\mu^2\phi_{\rm c}^2 - \gamma\lambda\phi_{\rm c}^4 = \left(\frac{1}{4}\beta_\lambda - \gamma\lambda\right)\phi_{\rm c}^4 + \left(-\frac{1}{2}\beta_{\mu^2} + \gamma\right)\mu^2\phi_{\rm c}^2$$

$$-\mu_{\rm R}\frac{\partial V_1}{\partial\mu_{\rm R}} = -\frac{\partial V_1}{\partial\ln\mu_{\rm R}} = 2\left[\frac{6}{64\pi^2}\bar{m}_W^4(\phi_{\rm c}) + \frac{3}{64\pi^2}\bar{m}_Z^4(\phi_{\rm c}) - \frac{12}{64\pi^2}\bar{m}_t^4(\phi_{\rm c}) + \frac{1}{64\pi^2}\bar{m}_h^4(\phi_{\rm c}) + \frac{3}{64\pi^2}\bar{m}_G^4(\phi_{\rm c})\right]$$

$$= \frac{1}{32\pi^2}\left[\frac{3}{8}g^4\phi_{\rm c}^4 + \frac{3}{16}(g^2 + g'^2)^2\phi_{\rm c}^4 - 3y_t^4\phi_{\rm c}^4 + (-\mu^2 + 3\lambda\phi_{\rm c}^2)^2 + 3(-\mu^2 + \lambda\phi_{\rm c}^2)^2\right]$$

$$= \frac{1}{32\pi^2}[(12\lambda^2 + B)\phi_{\rm c}^4 - 12\lambda\mu^2\phi_{\rm c}^2 + 4\mu^4]$$

$$B \equiv \frac{3}{16}(3g^4 + 2g^2g'^2 + g'^4) - 3y_t^4$$

$$\Rightarrow \quad \frac{1}{4}\beta_\lambda - \gamma\lambda = \frac{1}{32\pi^2}(12\lambda^2 + B), \quad -\frac{1}{2}\beta_{\mu^2} + \gamma = -\frac{3}{8\pi^2}\lambda$$

$$\Rightarrow \quad \beta_\lambda = 4\gamma\lambda + \frac{1}{8\pi^2}(12\lambda^2 + B), \quad \beta_{\mu^2} = 2\gamma + \frac{3}{4\pi^2}\lambda$$

[Note: these relations only hold for the Landau gauge!]

$\odot$ Calculation for the anomalous dimension $\gamma = \gamma_h$ in the Landau gauge

h self-energy $h-(\text{1PI})-h = i\Pi_h(p^2) = i\Pi_h^{\text{fermion}} + i\Pi_h^{\text{gauge}} + i\Pi_h^{\text{ghost}} + i\Pi_h^{\text{scalar}}$

h-h counter term $h-\otimes-h = i(p^2\delta_h - \delta_{m_h})$

$i(\Pi_h + p^2\delta_h - \delta_{m_h})$ is finite $\Rightarrow$ $\dfrac{\partial \Pi_h}{\partial p^2} + \delta_h$ is finite

h anomalous dimension $\gamma_h = \dfrac{1}{2}\mu_{\text{R}}\dfrac{\partial \delta_h}{\partial \mu_{\text{R}}} = \dfrac{1}{2}\dfrac{\partial \delta_h}{\partial \ln \mu_{\text{R}}} = \dfrac{\partial \delta_h}{\partial \ln \mu_{\text{R}}^2}$

$$\frac{1}{K^{2-d/2}} = 1 - \frac{4-d}{2}\ln K + \mathcal{O}[(4-d)^2], \quad \frac{1}{(4\pi)^{d/2}} = \frac{1}{16\pi^2}(4\pi)^{2-d/2} = \frac{1}{16\pi^2}\left\{1 + \frac{4-d}{2}\ln 4\pi + \mathcal{O}[(4-d)^2]\right\}$$

$$\Gamma(z+1) = z\Gamma(z), \quad \Gamma(2-d/2) = \frac{2}{4-d} - \gamma_{\text{E}} + \mathcal{O}(4-d)$$

$$\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}K^{2-d/2}} = \frac{1}{16\pi^2}\left[\frac{2}{4-d} - \ln K - \gamma_{\text{E}} + \ln 4\pi + \mathcal{O}(4-d)\right]$$

$$\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{\ell^2 - K} = -\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K^{1-d/2}}, \quad \int\frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2 - K)^2} = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K^{2-d/2}}, \quad \int\frac{d^d\ell}{(2\pi)^d}\frac{\ell^2}{(\ell^2-K)^2} = -\frac{d}{2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K^{1-d/2}}$$

$$x[(p+q)^2 - m_t^2] + (1-x)(q^2 - m_t^2) = xp^2 + 2xp\cdot q + q^2 - m_t^2 = (q+xp)^2 + x(1-x)p^2 - m_t^2 = \ell^2 - K_t$$

$$\ell = q + xp, \quad K_t = -x(1-x)p^2 + m_t^2$$

$$\text{Tr}[(\not p + \not q + m_t)(\not q + m_t)] = \text{Tr}[(\not p + \not q)\not q] + m_t^2\text{Tr}(1)$$
$$= \text{Tr}(\gamma^\mu\gamma^\nu)(p+q)_\mu q_\nu + 4m_t^2 = 4g^{\mu\nu}(p+q)_\mu q_\nu + 4m_t^2 = 4(p+q)\cdot q + 4m_t^2$$
$$= 4[\ell + (1-x)p]\cdot(\ell - xp) + 4m_t^2 \to 4[\ell^2 - x(1-x)p^2 + m_t^2] = 4(\ell^2 + K_t)$$

$$\frac{1}{(p+q)^2 - m_t^2}\frac{1}{q^2 - m_t^2} = \int_0^1 dx\frac{1}{\{x[(p+q)^2 - m_t^2] + (1-x)(q^2 - m_t^2)\}^2} = \int_0^1 dx\frac{1}{(\ell^2 - K_t)^2}$$

$$\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2 - m_t^2}\frac{1}{q^2 - m_t^2} = \int_0^1 dx\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2 - K_t)^2} = \int_0^1 dx\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_t^{2-d/2}}$$

$$i\Pi_h^{\text{fermion}} = h - \left(\begin{smallmatrix} t \\ \\ \bar{t}\end{smallmatrix}\right) - h = (-1)\cdot 3\int\frac{d^dq}{(2\pi)^d}\left(-i\frac{y_t}{\sqrt{2}}\right)^2\text{Tr}\left[\frac{i(\not p + \not q + m_t)}{(p+q)^2 - m_t^2}\frac{i(\not q + m_t)}{q^2 - m_t^2}\right] = -3\frac{y_t^2}{2}\int\frac{d^dq}{(2\pi)^d}\frac{\text{Tr}[(\not p + \not q + m_t)(\not q + m_t)]}{[(p+q)^2 - m_t^2](q^2 - m_t^2)}$$
$$= -6y_t^2\int_0^1 dx\int\frac{d^d\ell}{(2\pi)^d}\frac{\ell^2 + K_t}{(\ell^2 - K_t)^2} = -6y_t^2\int_0^1 dx\left\{-\frac{d}{2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K_t^{1-d/2}} + [-x(1-x)p^2 + m_t^2]\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_t^{2-d/2}}\right\}$$

$$\frac{\partial}{\partial p^2}\left[\frac{i\Gamma(n-d/2)}{(4\pi)^{d/2}K^{n-d/2}}\right] = \frac{i\Gamma(n-d/2)}{(4\pi)^{d/2}K^{n+1-d/2}}[-(n-d/2)]\frac{\partial K}{\partial p^2} = -\frac{i\Gamma(n+1-d/2)}{(4\pi)^{d/2}K^{n+1-d/2}}\frac{\partial K}{\partial p^2} \text{ is finite for } n \geq 2$$

$$\frac{\partial}{\partial p^2}\left[\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K_t^{1-d/2}}\right] = -\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_t^{2-d/2}}\frac{\partial K_t}{\partial p^2} = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_t^{2-d/2}}x(1-x), \quad \int_0^1 dx x(1-x) = \frac{1}{6}$$

$$\frac{\partial(i\Pi_h^{\text{fermion}})}{\partial p^2} = -6y_t^2\int_0^1 dx\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_t^{2-d/2}}\left[-\frac{d}{2}x(1-x) - x(1-x)\right]$$
$$= -6y_t^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left(-\frac{d}{2}\frac{1}{6} - \frac{1}{6}\right) + \text{finite} = 3y_t^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_h^{\text{fermion}} = -3y_t^2\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\text{R}}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}3y_t^2\ln\mu_{\text{R}}^2 + \cdots$$

$$\gamma_h^{\text{fermion}} = \frac{\partial\delta_h^{\text{fermion}}}{\partial\ln\mu_{\text{R}}^2} = \frac{1}{16\pi^2}3y_t^2$$

Landau gauge: $\xi = 0, \quad i\Pi_h^{\text{ghost}} = 0$

$$P_{\mu\nu}(q) \equiv g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad g_{\mu\nu}g^{\nu\mu} = d$$

$$x[(p+q)^2 - m_Z^2] + (1-x)(q^2 - m_Z^2) = \ell^2 - K_Z, \quad \ell = q + xp, \quad K_Z = -x(1-x)p^2 + m_Z^2$$

$$\frac{1}{(p+q)^2 - m_Z^2}\frac{1}{q^2 - m_Z^2} = \int_0^1 dx \frac{1}{\{x[(p+q)^2 - m_Z^2] + (1-x)(q^2 - m_Z^2)\}^2} = \int_0^1 dx \frac{1}{(\ell^2 - K_Z)^2}$$

$$x[(p+q)^2 - m_Z^2] + y(q^2 - m_Z^2) + z(p+q)^2 + wq^2 = q^2 + 2(x+z)p\cdot q + (x+z)p^2 - (x+y)m_Z^2$$
$$= \ell_1^2 + (x+z)(1-x-z)p^2 - (x+y)m_Z^2$$

$$\ell_1 = q + (x+z)p, \quad K_{Z1} = -(x+z)(1-x-z)p^2 + (x+y)m_Z^2$$

$$\frac{1}{(p+q)^2 - m_Z^2}\frac{1}{q^2 - m_Z^2}\frac{1}{(p+q)^2}\frac{1}{q^2} = \int_0^1 dxdydzdw \frac{2\delta(x+y+z+w-1)}{\{x[(p+q)^2 - m_Z^2] + y(q^2 - m_Z^2) + z(p+q)^2 + wq^2\}^2}$$
$$= 2\int_0^1 dxdydz \frac{\delta(x+y+z+w-1)}{(\ell_1^2 - K_{Z1})^4}$$

$$P_{\mu\nu}(p+q)P^{\nu\mu}(q) = \left[g_{\mu\nu} - \frac{(p+q)_\mu (p+q)_\nu}{(p+q)^2}\right]\left(g^{\nu\mu} - \frac{q^\nu q^\mu}{q^2}\right) = d - 2 + \frac{(p\cdot q + q^2)^2}{(p+q)^2 q^2}$$

$$(p\cdot q + q^2)^2 = \{p\cdot[\ell_1 - (x+z)p] + [\ell_1 - (x+z)p]^2\}^2 = [\ell_1^2 + (1-2x-2z)p\cdot\ell_1 + (x+z)(x+z-1)p^2]^2$$
$$\to \ell_1^4 + (1-2x-2z)^2 p_\mu p_\nu \ell_1^\mu \ell_1^\nu + 2(x+z)(x+z-1)p^2\ell_1^2 + (x+z)^2(x+z-1)^2 p^4$$

$$i\Pi_h^{\text{gauge},Z,1} = h - \overset{Z}{\underset{Z}{(\quad)}} - h = \frac{1}{2}\left(i\frac{gm_Z}{c_{\text{W}}}\right)^2 \int \frac{d^dq}{(2\pi)^d} \frac{-iP_{\mu\nu}(p+q)}{(p+q)^2 - m_Z^2}\frac{-iP^{\nu\mu}(q)}{q^2 - m_Z^2}$$
$$= \frac{g^2 m_Z^2}{2c_{\text{W}}^2}\int \frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2 - m_Z^2}\frac{1}{q^2 - m_Z^2}\left[d - 2 + \frac{(p\cdot q + q^2)^2}{(p+q)^2 q^2}\right]$$
$$= \frac{g^2 m_Z^2}{2c_{\text{W}}^2}\left[\int_0^1 dx \int \frac{d^d\ell}{(2\pi)^d}\frac{d-2}{(\ell^2 - K_Z)^2} + I_1\right] = \frac{g^2 m_Z^2}{2c_W^2}\left[\int_0^1 dx \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_Z^{2-d/2}}(d-2) + I_1\right]$$

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2 - K)^4} = \frac{1}{6}\frac{i\Gamma(4-d/2)}{(4\pi)^{d/2}K^{4-d/2}}, \quad \int \frac{d^d\ell}{(2\pi)^d}\frac{\ell^2}{(\ell^2 - K)^4} = -\frac{d}{12}\frac{i\Gamma(3-d/2)}{(4\pi)^{d/2}K^{3-d/2}}$$

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{\ell^\mu \ell^\nu}{(\ell^2 - K)^4} = -\frac{g^{\mu\nu}}{12}\frac{i\Gamma(3-d/2)}{(4\pi)^{d/2}K^{3-d/2}}, \quad \int \frac{d^d\ell}{(2\pi)^d}\frac{(\ell^2)^2}{(\ell^2 - K)^4} = \frac{d(d+2)}{24}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K^{2-d/2}}$$

$$I_1 \equiv 2\int_0^1 dxdydzdw\delta(x+y+z+w-1)\int \frac{d^d\ell_1}{(2\pi)^d}\frac{1}{(\ell_1^2 - K_{Z1})^4}$$
$$\times[\ell_1^4 + (1-2x-2z)^2 p_\mu p_\nu \ell_1^\mu \ell_1^\nu + 2(x+z)(x+z-1)p^2\ell_1^2 + (x+z)^2(x+z-1)^2 p^4]$$
$$= 2\int_0^1 dxdydzdw\delta(x+y+z+w-1)\left[\frac{d(d+2)}{24}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_{Z1}^{2-d/2}} - \frac{1}{12}(1-2x-2z)^2 p^2 \frac{i\Gamma(3-d/2)}{(4\pi)^{d/2}K_{Z1}^{3-d/2}}\right.$$
$$\left. - \frac{d}{6}(x+z)(x+z-1)p^2 \frac{i\Gamma(3-d/2)}{(4\pi)^{d/2}K_{Z1}^{3-d/2}} + \frac{1}{6}(x+z)^2(x+z-1)^2 p^4 \frac{i\Gamma(4-d/2)}{(4\pi)^{d/2}K_{Z1}^{4-d/2}}\right]$$

$\dfrac{\partial(i\Pi_h^{\text{gauge},Z,1})}{\partial p^2}$ is finite

$$i\Pi_h^{\text{gauge},Z,2} = \begin{array}{c} Z \\ (\quad) \\ h- \quad -\vee- \quad -h \end{array} = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d}\frac{-i}{q^2-m_Z^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right) i\frac{g^2}{2c_{\text{W}}^2}g^{\mu\nu}$$

$$= \frac{g^2}{4c_{\text{W}}^2}(d-1)\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2-m_Z^2} = -\frac{g^2}{4c_{\text{W}}^2}(d-1)\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}(m_Z^2)^{1-d/2}}$$

$\dfrac{\partial(i\Pi_h^{\text{gauge},Z,2})}{\partial p^2}$ is finite

$$x[(p+q)^2-m_Z^2]+(1-x)q^2 = \ell^2-\tilde{K}_Z, \quad \ell = q+xp, \quad \tilde{K}_Z = -x(1-x)p^2+xm_Z^2$$

$$x[(p+q)^2-m_Z^2]+yq^2+z(p+q)^2 = q^2+2(x+z)p\cdot q+(x+z)p^2-xm_Z^2$$

$$= q^2+2(1-y)p\cdot q+(1-y)p^2-xm_Z^2 = \ell_2^2+y(1-y)p^2-xm_Z^2$$

$$\ell_2 = q+(1-y)p, \quad K_{Z2} = -y(1-y)p^2+xm_Z^2$$

$$\frac{1}{(p+q)^2-m_Z^2}\frac{1}{q^2}\frac{1}{(p+q)^2} = \int_0^1 dxdydz\frac{2\delta(x+y+z-1)}{\{x[(p+q)^2-m_Z^2]+yq^2+z(p+q)^2\}^2} = 2\int_0^1 dxdydz\frac{\delta(x+y+z-1)}{(\ell_2^2-K_{Z2})^3}$$

$$P_{\mu\nu}(p+q)(q-p)^\mu(q-p)^\nu = \left[g_{\mu\nu}-\frac{(p+q)_\mu(p+q)_\nu}{(p+q)^2}\right](q-p)^\mu(q-p)^\nu$$

$$= (q-p)^2-\frac{[(p+q)\cdot(q-p)]^2}{(p+q)^2} = (q-p)^2-\frac{(q^2-p^2)^2}{(p+q)^2}$$

$$(q-p)^2 = [\ell-(1+x)p]^2 \to \ell^2+(1+x)^2p^2$$

$$(q^2-p^2)^2 = \{[\ell_2-(1-y)p]^2-p^2\}^2 = [\ell_2^2-2(1-y)p\cdot\ell_2+y(y-2)p^2]^2$$

$$\to \ell_2^4+4(1-y)^2p_\mu p_\nu\ell_2^\mu\ell_2^\nu+2y(y-2)p^2\ell_2^2+y^2(y-2)^2p^4$$

$$i\Pi_h^{\text{gauge},Z,3} = h-\begin{array}{c} Z \\ (\quad) \\ G^0 \end{array}-h = \left(-\frac{g}{2c_{\text{W}}}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{-iP_{\mu\nu}(p+q)}{(p+q)^2-m_Z^2}(p-q)^\mu\frac{i}{q^2}(q-p)^\nu$$

$$= -\frac{g^2}{4c_{\text{W}}^2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2-m_Z^2}\frac{1}{q^2}P_{\mu\nu}(p+q)(q-p)^\mu(q-p)^\nu = -\frac{g^2}{4c_{\text{W}}^2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2-m_Z^2}\frac{1}{q^2}\left[(q-p)^2-\frac{(q^2-p^2)^2}{(p+q)^2}\right]$$

$$= -\frac{g^2}{4c_{\text{W}}^2}\left[\int_0^1 dx\int\frac{d^d\ell}{(2\pi)^d}\frac{\ell^2+(1+x)^2p^2}{(\ell^2-\tilde{K}_Z)^2}+I_2\right] = -\frac{g^2}{4c_{\text{W}}^2}\left\{\int_0^1 dx\left[-\frac{d}{2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}\tilde{K}_Z^{1-d/2}}+(1+x)^2p^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}\tilde{K}_Z^{2-d/2}}\right]+I_2\right\}$$

$$\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2-K)^3} = -\frac{1}{2}\frac{i\Gamma(3-d/2)}{(4\pi)^{d/2}K^{3-d/2}}, \quad \int\frac{d^d\ell}{(2\pi)^d}\frac{\ell^2}{(\ell^2-K)^3} = \frac{d}{4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K^{2-d/2}}$$

$$\int\frac{d^d\ell}{(2\pi)^d}\frac{\ell^\mu\ell^\nu}{(\ell^2-K)^3} = \frac{g^{\mu\nu}}{4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K^{2-d/2}}, \quad \int\frac{d^d\ell}{(2\pi)^d}\frac{(\ell^2)^2}{(\ell^2-K)^3} = -\frac{d(d+2)}{8}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K^{1-d/2}}$$

$$I_2 \equiv -2\int_0^1 dxdydz\delta(x+y+z-1)\int\frac{d^d\ell_2}{(2\pi)^d}\frac{1}{(\ell_2^2-K_{Z2})^3}[\ell_2^4+4(1-y)^2p_\mu p_\nu\ell_2^\mu\ell_2^\nu+2y(y-2)p^2\ell_2^2+y^2(y-2)^2p^4]$$

$$= -2\int_0^1 dxdydz\delta(x+y+z-1)\left[-\frac{d(d+2)}{8}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K_{Z2}^{1-d/2}}+(1-y)^2p^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_{Z2}^{2-d/2}}\right.$$

$$\left.+\frac{d}{2}y(y-2)p^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_{Z2}^{2-d/2}}-\frac{1}{2}y^2(y-2)^2p^4\frac{i\Gamma(3-d/2)}{(4\pi)^{d/2}K_{Z2}^{3-d/2}}\right]$$

$$\frac{\partial I_2}{\partial p^2} = -2\int_0^1 dxdydz\delta(x+y+z-1)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_{Z2}^{2-d/2}}\left[-\frac{d(d+2)}{8}y(1-y)+(1-y)^2+\frac{d}{2}y(y-2)\right]+\text{finite}$$
$$=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\int_0^1 dxdydz\delta(x+y+z-1)[6y(1-y)-2(1-y)^2-4y(y-2)]+\text{finite}$$
$$=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\int_0^1 dx\int_0^{1-x}dy(-12y^2+18y-2)+\text{finite}=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\int_0^1 dx(4x^3-3x^2-4x+3)+\text{finite}$$
$$=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$\int_0^1 dx(1+x)^2=\frac{7}{3}$$

$$\frac{\partial(i\Pi_h^{\text{gauge},Z,3})}{\partial p^2}=-\frac{g^2}{4c_{\text{W}}^2}\left\{\int_0^1 dx\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}\tilde{K}_Z^{2-d/2}}\left[-2x(1-x)+(1+x)^2\right]+I_2\right\}+\text{finite}$$
$$=-\frac{g^2}{4c_{\text{W}}^2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left(-2\frac{1}{6}+\frac{7}{3}+1\right)+\text{finite}=-\frac{3g^2}{4c_{\text{W}}^2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Pi_h^{\text{gauge},W,1}=h-\overset{W^+}{\underset{W^-}{(\quad)}}-h=(igm_W)^2\int\frac{d^dq}{(2\pi)^d}\frac{-iP_{\mu\nu}(p+q)}{(p+q)^2-m_W^2}\frac{-iP^{\nu\mu}(q)}{q^2-m_W^2}$$
$$=g^2m_W^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2-m_W^2}\frac{1}{q^2-m_W^2}\left[d-2+\frac{(p\cdot q+q^2)^2}{(p+q)^2q^2}\right]$$
$\dfrac{\partial(i\Pi_h^{\text{gauge},W,1})}{\partial p^2}$ is finite

$$i\Pi_h^{\text{gauge},W,2}=\overset{W}{(\quad)}=\int\frac{d^dq}{(2\pi)^d}\frac{-i}{q^2-m_W^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)i\frac{g^2}{2}g^{\mu\nu}=-\frac{g^2}{2}(d-1)\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}(m_W^2)^{1-d/2}}$$
$h- \quad -\vee- \quad -h$

$\dfrac{\partial(i\Pi_h^{\text{gauge},W,2})}{\partial p^2}$ is finite

$$i\Pi_h^{\text{gauge},W,3}=h-\overset{W^+}{\underset{G^-}{(\quad)}}-h=\left(i\frac{g}{2}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{-iP_{\mu\nu}(p+q)}{(p+q)^2-m_W^2}(q-p)^\mu\frac{i}{q^2}(q-p)^\nu$$
$$=-\frac{g^2}{4}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2-m_W^2}\frac{1}{q^2}P_{\mu\nu}(p+q)(q-p)^\mu(q-p)^\nu$$
$$=-\frac{g^2}{4}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2-m_W^2}\frac{1}{q^2}\left[(q-p)^2-\frac{(q^2-p^2)^2}{(p+q)^2}\right]$$
$$\frac{\partial(i\Pi_h^{\text{gauge},W,3})}{\partial p^2}=-\frac{3g^2}{4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$x[(q-p)^2-m_W^2]+(1-x)q^2=xp^2-2xp\cdot q+q^2-xm_W^2=(q-xp)^2+x(1-x)p^2-xm_W^2=\tilde{\ell}^2-\tilde{K}_W$$

$$\tilde{\ell}=q-xp,\quad \tilde{K}_W=-x(1-x)p^2+xm_W^2,\quad \frac{1}{(q-p)^2-m_W^2}\frac{1}{q^2}=\int_0^1 dx\frac{1}{(\tilde{\ell}^2-\tilde{K}_W)^2}$$

$$\begin{aligned}x[(q-p)^2-m_W^2]+yq^2+z(p-q)^2&=q^2-2(x+z)p\cdot q+(x+z)p^2-xm_W^2\\&=q^2-2(1-y)p\cdot q+(1-y)p^2-xm_W^2=\tilde{\ell}_2^2+y(1-y)p^2-xm_W^2=\tilde{\ell}_2^2-K_{W2}\end{aligned}$$

$$\tilde{\ell}_2=q-(1-y)p,\quad K_{W2}=-y(1-y)p^2+xm_W^2,\quad \frac{1}{(q-p)^2-m_W^2}\frac{1}{q^2}\frac{1}{(q-p)^2}=2\int_0^1 dxdydz\frac{\delta(x+y+z-1)}{(\tilde{\ell}_2^2-K_{W2})^3}$$

$$\begin{aligned}P_{\mu\nu}(q-p)(p+q)^\mu(p+q)^\nu&=\left[g_{\mu\nu}-\frac{(q-p)_\mu(q-p)_\nu}{(q-p)^2}\right](p+q)^\mu(p+q)^\nu\\&=(p+q)^2-\frac{[(q-p)\cdot(p+q)]^2}{(q-p)^2}=(p+q)^2-\frac{(q^2-p^2)^2}{(q-p)^2}\end{aligned}$$

$$(p+q)^2=[\tilde{\ell}+(1+x)p]^2\to\tilde{\ell}^2+(1+x)^2p^2$$

$$\begin{aligned}(q^2-p^2)^2&=\{[\tilde{\ell}_2+(1-y)p]^2-p^2\}^2\to[\tilde{\ell}_2^2+2(1-y)p\cdot\tilde{\ell}_2+y(y-2)p^2]^2\\&\to\tilde{\ell}_2^4+4(1-y)^2p_\mu p_\nu\tilde{\ell}_2^\mu\tilde{\ell}_2^\nu+2y(y-2)p^2\tilde{\ell}_2^2+y^2(y-2)^2p^4\end{aligned}$$

$$\begin{aligned}i\Pi_h^{\mathrm{gauge},W,4}&=h-\underset{G^+}{\overset{W^-}{(\quad)}}-h=\left(i\frac{g}{2}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{-iP_{\mu\nu}(q-p)}{(q-p)^2-m_W^2}(p+q)^\mu\frac{i}{q^2}(p+q)^\nu\\&=-\frac{g^2}{4}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q-p)^2-m_W^2}\frac{1}{q^2}P_{\mu\nu}(q-p)(p+q)^\mu(p+q)^\nu\\&=-\frac{g^2}{4}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q-p)^2-m_W^2}\frac{1}{q^2}\left[(p+q)^2-\frac{(q^2-p^2)^2}{(q-p)^2}\right]\\&=-\frac{g^2}{4}\left\{\int_0^1dx\left[-\frac{d}{2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}\tilde{K}_W^{1-d/2}}+(1+x)^2p^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}\tilde{K}_W^{2-d/2}}\right]+I_3\right\}\end{aligned}$$

$$I_3\equiv-2\int_0^1dxdydz\delta(x+y+z-1)\int\frac{d^d\ell_2}{(2\pi)^d}\frac{1}{(\tilde{\ell}_2^2-K_{W2})^3}[\tilde{\ell}_2^4+4(1-y)^2p_\mu p_\nu\tilde{\ell}_2^\mu\tilde{\ell}_2^\nu+2y(y-2)p^2\tilde{\ell}_2^2+y^2(y-2)^2p^4]$$

$$\frac{\partial I_3}{\partial p^2}=\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite},\quad \frac{\partial(i\Pi_h^{\mathrm{gauge},W,4})}{\partial p^2}=-\frac{3g^2}{4}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$c_{\mathrm{W}}^2=\frac{g^2}{g^2+g'^2},\quad \frac{g^2}{2c_{\mathrm{W}}^2}+g^2=g^2\left(\frac{1}{2c_{\mathrm{W}}^2}+1\right)=g^2\left(\frac{g^2+g'^2}{2g^2}+1\right)=g^2\frac{3g^2+g'^2}{2g^2}=\frac{1}{2}(3g^2+g'^2)$$

$$\begin{aligned}\frac{\partial(i\Pi_h^{\mathrm{gauge}})}{\partial p^2}&=\frac{\partial}{\partial p^2}(i\Pi_h^{\mathrm{gauge},Z,3}+i\Pi_h^{\mathrm{gauge},W,3}+i\Pi_h^{\mathrm{gauge},W,4})+\text{finite}\\&=\left(-\frac{3g^2}{4c_{\mathrm{W}}^2}-\frac{3g^2}{4}-\frac{3g^2}{4}\right)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}=-\frac{3}{2}\left(\frac{g^2}{2c_{\mathrm{W}}^2}+g^2\right)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}\\&=-\frac{3}{4}(3g^2+g'^2)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}\end{aligned}$$

$$\delta_h^{\mathrm{gauge}}=\frac{3}{4}(3g^2+g'^2)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}+\text{finite}=-\frac{1}{16\pi^2}\frac{3}{4}(3g^2+g'^2)\ln\mu_{\mathrm{R}}^2+\cdots$$

$$\gamma_h^{\mathrm{gauge}}=\frac{\partial\delta_h^{\mathrm{gauge}}}{\partial\ln\mu_{\mathrm{R}}^2}=-\frac{1}{16\pi^2}\frac{3}{4}(3g^2+g'^2)$$

(12)

$$K_h = -x(1-x)p^2 + m_h^2$$

$$i\Pi_h^{\text{scalar},1} = h - \overset{h}{\underset{h}{(\quad)}} - h = \frac{1}{2}(-6i\lambda v)^2 \int \frac{d^d q}{(2\pi)^d} \frac{i}{(p+q)^2 - m_h^2} \frac{i}{q^2 - m_h^2}$$

$$= 18\lambda^2 v^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p+q)^2 - m_h^2} \frac{1}{q^2 - m_h^2} = 18\lambda^2 v^2 \int_0^1 dx \int \frac{d^d \ell}{(2\pi)^d} \frac{1}{(\ell^2 - K_h)^2}$$

$$= 18\lambda^2 v^2 \int_0^1 dx \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2} K_h^{2-d/2}}$$

$$x[(p+q)^2 + i\varepsilon] + (1-x)(q^2 + i\varepsilon) = xp^2 + 2xp\cdot q + q^2 + i\varepsilon = (q+xp)^2 + x(1-x)p^2 + i\varepsilon = \ell^2 - K_\varepsilon$$

$$\ell = q + xp, \quad K_\varepsilon = -x(1-x)p^2 - i\varepsilon, \quad \frac{1}{(p+q)^2 + i\varepsilon} \frac{1}{q^2 + i\varepsilon} = \int_0^1 dx \frac{1}{(\ell^2 - K_\varepsilon)^2}$$

$$i\Pi_h^{\text{scalar},2} = h - \overset{G^0}{\underset{G^0}{(\quad)}} - h = \frac{1}{2}(-2i\lambda v)^2 \int \frac{d^d q}{(2\pi)^d} \frac{i}{(p+q)^2} \frac{i}{q^2} = 2\lambda^2 v^2 \int_0^1 dx \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2} K_\varepsilon^{2-d/2}}$$

$$i\Pi_h^{\text{scalar},3} = h - \overset{G^+}{\underset{G^-}{(\quad)}} - h = (-2i\lambda v)^2 \int \frac{d^d q}{(2\pi)^d} \frac{i}{(p+q)^2} \frac{i}{q^2} = 4\lambda^2 v^2 \int_0^1 dx \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2} K_\varepsilon^{2-d/2}}$$

$$i\Pi_h^{\text{scalar},4} = \underset{h-\quad -\vee-\quad -h}{\overset{h}{(\quad)}} = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}(-6i\lambda)\frac{i}{q^2 - m_h^2} = 3\lambda \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_h^2} = 3\lambda \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(1-d/2)}{(m_h^2)^{1-d/2}}$$

$$i\Pi_h^{\text{scalar},5} = \underset{h-\quad -\vee-\quad -h}{\overset{G^0}{(\quad)}} = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}(-2i\lambda)\frac{i}{q^2 + i\varepsilon} = \lambda \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(1-d/2)}{(-i\varepsilon)^{1-d/2}}$$

$$i\Pi_h^{\text{scalar},6} = \underset{h-\quad -\vee-\quad -h}{\overset{G^\pm}{(\quad)}} = \int \frac{d^d q}{(2\pi)^d}(-2i\lambda)\frac{i}{q^2 + i\varepsilon} = 2\lambda \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(1-d/2)}{(-i\varepsilon)^{1-d/2}}$$

$$\frac{\partial(i\Pi_h^{\text{scalar}})}{\partial p^2} \text{ is finite} \quad \Rightarrow \quad \gamma_h^{\text{scalar}} = 0$$

Summing all contributions:

$$\gamma = \gamma_h = \gamma_h^{\text{fermion}} + \gamma_h^{\text{gauge}} = \frac{1}{16\pi^2} 3y_t^2 - \frac{1}{16\pi^2}\frac{3}{4}(3g^2 + g'^2) = \frac{1}{64\pi^2}(12y_t^2 - 9g^2 - 3g'^2)$$

⊙ Explicit, gauge-independent expressions for the β functions of λ and μ^2

$$\beta_\lambda = 4\gamma\lambda + \frac{1}{8\pi^2}(12\lambda^2 + B) = \frac{1}{16\pi^2}\lambda(12y_t^2 - 9g^2 - 3g'^2) + \frac{1}{8\pi^2}\left[12\lambda^2 + \frac{3}{16}(3g^4 + 2g^2 g'^2 + g'^4) - 3y_t^4\right]$$

$$= \frac{1}{16\pi^2}\left[24\lambda^2 + \lambda(12y_t^2 - 9g^2 - 3g'^2) - 6y_t^4 + \frac{3}{8}(3g^4 + 2g^2 g'^2 + g'^4)\right]$$

$$\beta_{\mu^2} = 2\gamma + \frac{3}{4\pi^2}\lambda = \frac{1}{32\pi^2}(12y_t^2 - 9g^2 - 3g'^2) + \frac{3}{4\pi^2}\lambda = \frac{1}{16\pi^2}\left(12\lambda + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2\right)$$

3) β function for the top Yukawa coupling

h-t-t vertex $h-\!\!\begin{smallmatrix} \nearrow t \\ \nwarrow t\end{smallmatrix} = -i\dfrac{y_t}{\sqrt{2}}$

h self-energy $h-(\mathrm{1PI})-h = i\Pi_h(p^2) = i\Pi_h^{\mathrm{fermion}} + i\Pi_h^{\mathrm{gauge}} + i\Pi_h^{\mathrm{ghost}} + i\Pi_h^{\mathrm{scalar}}$

h-h counter term $h-\otimes-h = i(p^2\delta_h - \delta_{m_h})$, $i(\Pi_h + p^2\delta_h - \delta_{m_h})$ is finite $\Rightarrow$ $\dfrac{\partial \Pi_h}{\partial p^2} + \delta_h$ is finite

t self-energy $t-(\mathrm{1PI})-t = i\Pi_t(p) = i\Pi_t^{\mathrm{gauge}} + i\Pi_t^{\mathrm{scalar}}$

$$\frac{\partial \Pi_t}{\partial \boldsymbol{p}} = \frac{\partial \Pi_{t,\mathrm{V}}}{\partial \boldsymbol{p}} + \frac{\partial \Pi_{t,\mathrm{A}}}{\partial \boldsymbol{p}}\gamma_5, \quad \Pi_t = \boldsymbol{p}\frac{\partial \Pi_t}{\partial \boldsymbol{p}} + \cdots = \boldsymbol{p}\frac{\partial \Pi_{t,\mathrm{V}}}{\partial \boldsymbol{p}} + \boldsymbol{p}\frac{\partial \Pi_{t,\mathrm{A}}}{\partial \boldsymbol{p}}\gamma_5 + \cdots$$

t-t counter term $t-\otimes-t \supset i\boldsymbol{p}(\delta_{t,\mathrm{V}} + \gamma_5\delta_{t,\mathrm{A}})$, $\dfrac{\partial \Pi_{t,\mathrm{V}}}{\partial \boldsymbol{p}} + \delta_{t,\mathrm{V}}$ and $\dfrac{\partial \Pi_{t,\mathrm{A}}}{\partial \boldsymbol{p}} + \delta_{t,\mathrm{A}}$ are finite

h-t-t vertex correction $h-\langle \mathrm{1PI}\big|_{-t}^{-t} = i\Sigma_{y_t}(p_1,p_2,p_3)$

h-t-t counter term $h-\otimes\begin{smallmatrix} -t \\ -t\end{smallmatrix} = -i\dfrac{\delta_{y_t}}{\sqrt{2}}$, $\Sigma_{y_t} - \dfrac{\delta_{y_t}}{\sqrt{2}}$ is finite

The h-t-t vertex does not have a γ_5 structure!

h-t-t Green function $G_{\mathrm{c,V}}^{(3)}(\{p_i\}) = \dfrac{i}{p_1^2}\dfrac{i}{\boldsymbol{p}_2}\left\{-i\dfrac{y_t}{\sqrt{2}} - iB\ln\dfrac{\Lambda^2}{-p^2} - i\dfrac{\delta_{y_t}}{\sqrt{2}} - i\dfrac{y_t}{\sqrt{2}}\left[\sum_{i=1}^{3}\left(A_i\ln\dfrac{\Lambda^2}{-p_i^2}\right) - 2\delta_{t,\mathrm{V}} - \delta_h\right]\right\}\dfrac{i}{\boldsymbol{p}_3}$

Callan-Symanzik equation $\left[\dfrac{\partial}{\partial \ln\mu_{\mathrm{R}}} + \beta_{y_t}\dfrac{\partial}{\partial y_t} + \dfrac{1}{2}\dfrac{\partial(2\delta_{t,\mathrm{V}} + \delta_h)}{\partial \ln\mu_{\mathrm{R}}}\right]G_{\mathrm{c,V}}^{(3)} = 0$

$$\Rightarrow \quad \frac{\partial}{\partial \ln\mu_{\mathrm{R}}}\left[-i\frac{\delta_{y_t}}{\sqrt{2}} - i\frac{y_t}{\sqrt{2}}(-2\delta_{t,\mathrm{V}} - \delta_h)\right] - i\frac{1}{\sqrt{2}}\beta_{y_t} - i\frac{y_t}{\sqrt{2}}\frac{1}{2}\frac{\partial(2\delta_{t,\mathrm{V}} + \delta_h)}{\partial \ln\mu_{\mathrm{R}}} = 0 \quad \text{(lowest order)}$$

$$\Rightarrow \quad \frac{\partial}{\partial \ln\mu_{\mathrm{R}}}(\delta_{y_t} - 2y_t\delta_{t,\mathrm{V}} - y_t\delta_h) + \beta_{y_t} + y_t\frac{1}{2}\frac{\partial(2\delta_{t,\mathrm{V}} + \delta_h)}{\partial \ln\mu_{\mathrm{R}}} = 0$$

β function for y_t : $\beta_{y_t} = \dfrac{\partial}{\partial \ln\mu_{\mathrm{R}}}\left(-\delta_{y_t} + y_t\delta_{t,\mathrm{V}} + \dfrac{1}{2}y_t\delta_h\right) = -\dfrac{\partial \delta_{y_t}}{\partial \ln\mu_{\mathrm{R}}} + y_t\dfrac{\partial \delta_{t,\mathrm{V}}}{\partial \ln\mu_{\mathrm{R}}} + \dfrac{1}{2}y_t\dfrac{\partial \delta_h}{\partial \ln\mu_{\mathrm{R}}}$

The calculation below will be performed in the Feynman-t' Hooft gauge: $\xi = 1$

⊙ Calculation for $\dfrac{\partial \delta_h}{\partial \ln\mu_{\mathrm{R}}}$ in the Feynman-t' Hooft gauge

$\delta_h^{\mathrm{fermion}}$ is as the same as in the Landau gauge: $\delta_h^{\mathrm{fermion}} = -3y_t^2\dfrac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}} + \text{finite} = \dfrac{1}{16\pi^2}3y_t^2\ln\mu_{\mathrm{R}}^2 + \cdots$

$$\frac{\partial \delta_h^{\mathrm{fermion}}}{\partial \ln\mu_{\mathrm{R}}} = \frac{1}{16\pi^2}6y_t^2$$

$$x[(p+q)^2 - m_Z^2] + (1-x)(q^2 - m_Z^2) = \ell^2 - K_Z, \quad \ell = q + xp, \quad K_Z = -x(1-x)p^2 + m_Z^2$$

$$\frac{1}{(p+q)^2 - m_Z^2}\frac{1}{q^2 - m_Z^2} = \int_0^1 dx\frac{1}{\{x[(p+q)^2 - m_Z^2] + (1-x)(q^2 - m_Z^2)\}^2} = \int_0^1 dx\frac{1}{(\ell^2 - K_Z)^2}$$

$$i\Pi_h^{\mathrm{gauge},Z,1} = h-\left(\begin{smallmatrix} Z \\ \\ Z\end{smallmatrix}\right)-h = \frac{1}{2}\left(i\frac{gm_Z}{c_{\mathrm{W}}}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}}{(p+q)^2 - m_Z^2}\frac{-ig^{\nu\mu}}{q^2 - m_Z^2} = \frac{g^2m_Z^2}{2c_W^2}d\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2 - m_Z^2}\frac{1}{q^2 - m_Z^2}$$

$$= \frac{g^2m_Z^2}{2c_{\mathrm{W}}^2}d\int_0^1 dx\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2 - K_Z)^2} = \frac{g^2m_Z^2d}{2c_W^2}\int_0^1 dx\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_Z^{2-d/2}}$$

$\dfrac{\partial(i\Pi_h^{\mathrm{gauge},Z,1})}{\partial p^2}$ is finite

$$i\Pi_h^{\text{gauge},Z,2} = \begin{array}{c} Z \\ (\quad) \\ h- \ -\vee- \ -h \end{array} = \frac{1}{2}\int\frac{d^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}}{q^2-m_Z^2}\, i\frac{g^2}{2c_{\mathrm{W}}^2}\, g^{\mu\nu} = \frac{g^2}{4c_{\mathrm{W}}^2}\, d\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2-m_Z^2} = -\frac{g^2 d}{4c_{\mathrm{W}}^2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}(m_Z^2)^{1-d/2}}$$

$\dfrac{\partial(i\Pi_h^{\text{gauge},Z,2})}{\partial p^2}$ is finite

$$(q-p)^2 = [\ell-(1+x)p]^2 \to \ell^2+(1+x)^2p^2$$

$$i\Pi_h^{\text{gauge},Z,3} = h-\begin{array}{c}Z\\(\quad)\\G^0\end{array}-h = \left(-\frac{g}{2c_{\mathrm{W}}}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}}{(p+q)^2-m_Z^2}(p-q)^\mu\frac{i}{q^2-m_Z^2}(q-p)^\nu$$

$$= -\frac{g^2}{4c_{\mathrm{W}}^2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2-m_Z^2}\frac{1}{q^2-m_Z^2}(q-p)^2 = -\frac{g^2}{4c_{\mathrm{W}}^2}\int_0^1dx\int\frac{d^d\ell}{(2\pi)^d}\frac{\ell^2+(1+x)^2p^2}{(\ell^2-K_Z)^2}$$

$$= -\frac{g^2}{4c_{\mathrm{W}}^2}\int_0^1 dx\left[-\frac{d}{2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K_Z^{1-d/2}}+(1+x)^2p^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_Z^{2-d/2}}\right]$$

$$\frac{\partial(i\Pi_h^{\text{gauge},Z,3})}{\partial p^2} = -\frac{g^2}{4c_{\mathrm{W}}^2}\int_0^1dx\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_Z^{2-d/2}}\left[-\frac{d}{2}x(1-x)+(1+x)^2\right]$$

$$= -\frac{g^2}{4c_{\mathrm{W}}^2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left(-\frac{d}{2}\frac{1}{6}+\frac{7}{3}\right)+\text{finite} = -\frac{g^2}{2c_{\mathrm{W}}^2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Pi_h^{\text{ghost},Z} = h-\begin{array}{c}\eta^Z\\(\quad)\\\eta^Z\end{array}-h = (-1)\left(-i\frac{gm_Z}{2c_{\mathrm{W}}}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{(p+q)^2-m_Z^2}\frac{i}{q^2-m_Z^2} = -\frac{g^2m_Z^2}{4c_{\mathrm{W}}^2}\int_0^1dx\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_Z^{2-d/2}}$$

$\dfrac{\partial(i\Pi_h^{\text{ghost},Z})}{\partial p^2}$ is finite

$$K_W = -x(1-x)p^2+m_W^2$$

$$i\Pi_h^{\text{gauge},W,1} = h-\begin{array}{c}W^+\\(\quad)\\W^-\end{array}-h = (igm_W)^2\int\frac{d^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}}{(p+q)^2-m_W^2}\frac{-ig^{\nu\mu}}{q^2-m_W^2} = g^2m_W^2\, d\int_0^1dx\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_W^{2-d/2}}$$

$\dfrac{\partial(i\Pi_h^{\text{gauge},W,1})}{\partial p^2}$ is finite

$$i\Pi_h^{\text{gauge},W,2} = \begin{array}{c} W \\ (\quad) \\ h- \ -\vee- \ -h \end{array} = \int\frac{d^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}}{q^2-m_W^2}\, i\frac{g^2}{2}\, g^{\mu\nu} = -\frac{g^2d}{2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}(m_W^2)^{1-d/2}}$$

$\dfrac{\partial(i\Pi_h^{\text{gauge},W,2})}{\partial p^2}$ is finite

$$x[(q-p)^2-m_W^2]+(1-x)(q^2-m_W^2) = xp^2-2xp\cdot q+q^2-m_W^2 = (q-xp)^2+x(1-x)p^2-m_W^2 = \tilde\ell^2-K_W$$

$$\tilde\ell = q-xp, \quad (p+q)^2 = [\tilde\ell+(1+x)p]^2 \to \tilde\ell^2+(1+x)^2p^2$$

$$i\Pi_h^{\text{gauge},W,3} = h-\begin{array}{c}W^+\\(\quad)\\G^-\end{array}-h \ + \ h-\begin{array}{c}W^-\\(\quad)\\G^+\end{array}-h = \left(i\frac{g}{2}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{q^2-m_W^2}\left[\frac{-ig_{\mu\nu}(q-p)^\mu(q-p)^\nu}{(p+q)^2-m_W^2}+\frac{-ig_{\mu\nu}(p+q)^\mu(p+q)^\nu}{(q-p)^2-m_W^2}\right]$$

$$= -\frac{g^2}{4}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2-m_W^2}\left[\frac{(q-p)^2}{(p+q)^2-m_W^2}+\frac{(p+q)^2}{(q-p)^2-m_W^2}\right] = -\frac{g^2}{4}\int_0^1dx\left[\int\frac{d^d\ell}{(2\pi)^d}\frac{\ell^2+(1+x)^2p^2}{(\ell^2-K_W)^2}+\int\frac{d^d\tilde\ell}{(2\pi)^d}\frac{\tilde\ell^2+(1+x)^2p^2}{(\tilde\ell^2-K_W)^2}\right]$$

$$= -\frac{g^2}{2}\int_0^1dx\left[-\frac{d}{2}\frac{i\Gamma(1-d/2)}{(4\pi)^{d/2}K_W^{1-d/2}}+(1+x)^2p^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_W^{2-d/2}}\right]$$

$$\frac{\partial(i\Pi_h^{\text{gauge},W,3})}{\partial p^2} = -\frac{g^2}{2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left(-\frac{d}{2}\frac{1}{6}+\frac{7}{3}\right)+\text{finite} = -g^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}+\text{finite}$$

$$i\Pi_h^{\mathrm{ghost},W} = h-\underset{\eta^+}{\overset{\eta^+}{(\quad)}}-h \;+\; h-\underset{\eta^-}{\overset{\eta^-}{(\quad)}}-h = (-1)\cdot 2\left(-i\frac{gm_W}{2}\right)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{(p+q)^2-m_W^2}\frac{i}{q^2-m_W^2} = -\frac{g^2m_W^2}{2}\int_0^1 dx\,\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_W^{2-d/2}}$$

$\dfrac{\partial(i\Pi_h^{\mathrm{ghost},W})}{\partial p^2}$ is finite

$$\frac{\partial(i\Pi_h^{\mathrm{gauge}})}{\partial p^2} = \frac{\partial}{\partial p^2}(i\Pi_h^{\mathrm{gauge},Z,3}+i\Pi_h^{\mathrm{gauge},W,3}) = -\left(\frac{g^2}{2c_{\mathrm{W}}^2}+g^2\right)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} = -\frac{1}{2}(3g^2+g'^2)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}$$

$$\delta_h^{\mathrm{gauge}} = \frac{1}{2}(3g^2+g'^2)\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}+\text{finite} = -\frac{1}{16\pi^2}\frac{1}{2}(3g^2+g'^2)\ln\mu_{\mathrm{R}}^2+\cdots$$

$$\frac{\partial\delta_h^{\mathrm{gauge}}}{\partial\ln\mu_{\mathrm{R}}} = -\frac{1}{16\pi^2}(3g^2+g'^2)$$

$$K_h = -x(1-x)p^2+m_h^2$$

$$i\Pi_h^{\mathrm{scalar},1} = h-\underset{h}{\overset{h}{(\quad)}}-h = \frac{1}{2}(-6i\lambda v)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{(p+q)^2-m_h^2}\frac{i}{q^2-m_h^2} = 18\lambda^2v^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2-m_h^2}\frac{1}{q^2-m_h^2}$$

$$= 18\lambda^2v^2\int_0^1 dx\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2-K_4)^2} \quad = 18\lambda^2v^2\int_0^1 dx\,\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_h^{2-d/2}}$$

$$i\Pi_h^{\mathrm{scalar},2} = h-\underset{G^0}{\overset{G^0}{(\quad)}}-h = \frac{1}{2}(-2i\lambda v)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{(p+q)^2-m_Z^2}\frac{i}{q^2-m_Z^2} = 2\lambda^2v^2\int_0^1 dx\,\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_h^{2-d/2}}$$

$$i\Pi_h^{\mathrm{scalar},3} = h-\underset{G^-}{\overset{G^+}{(\quad)}}-h = (-2i\lambda v)^2\int\frac{d^dq}{(2\pi)^d}\frac{i}{(p+q)^2-m_W^2}\frac{i}{q^2-m_W^2} = 4\lambda^2v^2\int_0^1 dx\,\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_h^{2-d/2}}$$

$$i\Pi_h^{\mathrm{scalar},4} = \underset{h-\;\;-\vee-\;\;-h}{\overset{h}{(\quad)}} = \frac{1}{2}\int\frac{d^dq}{(2\pi)^d}(-6i\lambda)\frac{i}{q^2-m_h^2} = 3\lambda\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2-m_h^2} = 3\lambda\frac{i}{(4\pi)^{d/2}}\frac{\Gamma(1-d/2)}{(m_h^2)^{1-d/2}}$$

$$i\Pi_h^{\mathrm{scalar},5} = \underset{h-\;\;-\vee-\;\;-h}{\overset{G^0}{(\quad)}} = \frac{1}{2}\int\frac{d^dq}{(2\pi)^d}(-2i\lambda)\frac{i}{q^2-m_Z^2} = \lambda\frac{i}{(4\pi)^{d/2}}\frac{\Gamma(1-d/2)}{(m_Z^2)^{1-d/2}}$$

$$i\Pi_h^{\mathrm{scalar},6} = \underset{h-\;\;-\vee-\;\;-h}{\overset{G^\pm}{(\quad)}} = \int\frac{d^dq}{(2\pi)^d}(-2i\lambda)\frac{i}{q^2-m_W^2} = 2\lambda\frac{i}{(4\pi)^{d/2}}\frac{\Gamma(1-d/2)}{(m_W^2)^{1-d/2}}$$

$\dfrac{\partial(i\Pi_h^{\mathrm{scalar}})}{\partial p^2} = \dfrac{\partial}{\partial p^2}\displaystyle\sum_{j=1}^{6}(i\Pi_h^{\mathrm{scalar},j})$ is finite

In total, $\dfrac{\partial\delta_h}{\partial\ln\mu_{\mathrm{R}}} = \dfrac{\partial\delta_h^{\mathrm{fermion}}}{\partial\ln\mu_{\mathrm{R}}}+\dfrac{\partial\delta_h^{\mathrm{gauge}}}{\partial\ln\mu_{\mathrm{R}}} = \dfrac{1}{16\pi^2}(6y_t^2-3g^2-g'^2)$

⊙ Calculation for $\dfrac{\partial \delta_{t,\mathrm{V}}}{\partial \ln \mu_{\mathrm{R}}}$ in the <u>Feynman-t' Hooft gauge</u>

As the field strength renormalization is independent of the related masses,
the calculation can be simplified by setting the masses to zero

$$t^a t^a = C_2(r)\cdot \mathbf{1};\quad \mathrm{SU(3)_C} \to C_2(r=\mathbf{3}) = \frac{4}{3}$$

$$\gamma^\mu\gamma^\nu\gamma_\mu = -(d-2)\gamma^\nu,\quad \gamma^\mu(\slashed{p}+\slashed{q})\gamma_\mu = (p+q)_\nu\gamma^\mu\gamma^\nu\gamma_\mu = -(d-2)(\slashed{p}+\slashed{q})$$

$$x(p+q)^2+(1-x)q^2 = xp^2+2xp\cdot q+q^2 = (q+xp)^2+x(1-x)p^2 = \ell^2-K_0$$

$$\ell = q+xp,\quad K_0=-x(1-x)p^2,\quad \slashed{p}+\slashed{q} = \slashed{\ell}+(1-x)\slashed{p} \to (1-x)\slashed{p}$$

$$\frac{1}{(p+q)^2}\frac{1}{q^2} = \int_0^1 dx\frac{1}{[x(p+q)^2+(1-x)q^2]^2} = \int_0^1 dx\frac{1}{(\ell^2-K_0)^2}$$

$$i\Pi_t^{\text{gauge},g} = \begin{array}{c} g \\ (\qquad) \\ t\quad --t--\quad t \end{array} = \int\frac{d^dq}{(2\pi)^d}(ig_{\mathrm{s}}\gamma^\mu t^a)\frac{i(\slashed{p}+\slashed{q})}{(p+q)^2}(ig_{\mathrm{s}}\gamma^\nu t^b)\frac{-ig_{\mu\nu}\delta^{ab}}{q^2} = -g_{\mathrm{s}}^2C_2(\mathbf{3})\int\frac{d^dq}{(2\pi)^d}\frac{\gamma^\mu(\slashed{p}+\slashed{q})\gamma_\mu}{(p+q)^2q^2}$$

$$= \frac{4}{3}g_{\mathrm{s}}^2(d-2)\int\frac{d^dq}{(2\pi)^d}\frac{\slashed{p}+\slashed{q}}{(p+q)^2q^2} = \frac{4}{3}g_{\mathrm{s}}^2(d-2)\int_0^1dx\int\frac{d^d\ell}{(2\pi)^d}\frac{(1-x)\slashed{p}}{(\ell^2-K_0)^2} = \frac{4}{3}g_{\mathrm{s}}^2(d-2)\int_0^1dx(1-x)\slashed{p}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}$$

$$\int_0^1dx(1-x) = \frac{1}{2}$$

$$\frac{\partial(i\Pi_t^{\text{gauge},g})}{\partial\slashed{p}} = \frac{4}{3}g_{\mathrm{s}}^2(d-2)\int_0^1dx(1-x)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}} + \text{finite} = \frac{4}{3}g_{\mathrm{s}}^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$Q_t = \frac{2}{3},\quad e = gs_{\mathrm{W}}$$

$$i\Pi_t^{\text{gauge},\gamma} = \begin{array}{c} \gamma \\ (\qquad) \\ t\quad --t--\quad t \end{array} = (iQ_te)^2\int\frac{d^dq}{(2\pi)^d}\gamma^\mu\frac{i(\slashed{p}+\slashed{q})}{(p+q)^2}\gamma^\nu\frac{-ig_{\mu\nu}}{q^2}$$

$$= -Q_t^2e^2\int\frac{d^dq}{(2\pi)^d}\frac{\gamma^\mu(\slashed{p}+\slashed{q})\gamma_\mu}{(p+q)^2q^2} = \frac{4}{9}g^2s_{\mathrm{W}}^2(d-2)\int_0^1dx(1-x)\slashed{p}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}$$

$$\frac{\partial(i\Pi_t^{\text{gauge},\gamma})}{\partial\slashed{p}} = \frac{4}{9}g^2s_{\mathrm{W}}^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\gamma^\mu(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5)(\slashed{p}+\slashed{q})\gamma_\mu(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5) = -(d-2)(g_{\mathrm{V}}^t+g_{\mathrm{A}}^t\gamma_5)(\slashed{p}+\slashed{q})(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5)$$

$$= -(d-2)(\slashed{p}+\slashed{q})(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5)(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5) = -(d-2)(\slashed{p}+\slashed{q})[(g_{\mathrm{V}}^t)^2+(g_{\mathrm{A}}^t)^2-2g_{\mathrm{V}}^tg_{\mathrm{A}}^t\gamma_5]$$

$$i\Pi_t^{\text{gauge},Z} = \begin{array}{c} Z \\ (\qquad) \\ t\quad --t--\quad t \end{array} = \int\frac{d^dq}{(2\pi)^d}\left[i\frac{g}{2c_{\mathrm{W}}}\gamma^\mu(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5)\right]\frac{i(\slashed{p}+\slashed{q})}{(p+q)^2}\left[i\frac{g}{2c_{\mathrm{W}}}\gamma^\nu(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5)\right]\frac{-ig_{\mu\nu}}{q^2}$$

$$= -\frac{g^2}{4c_{\mathrm{W}}^2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p+q)^2q^2}[\gamma^\mu(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5)(\slashed{p}+\slashed{q})\gamma_\mu(g_{\mathrm{V}}^t-g_{\mathrm{A}}^t\gamma_5)]$$

$$= \frac{g^2}{4c_{\mathrm{W}}^2}(d-2)\int\frac{d^dq}{(2\pi)^d}\frac{\slashed{p}+\slashed{q}}{(p+q)^2q^2}[(g_{\mathrm{V}}^t)^2+(g_{\mathrm{A}}^t)^2-2g_{\mathrm{V}}^tg_{\mathrm{A}}^t\gamma_5]$$

$$= \frac{g^2}{4c_{\mathrm{W}}^2}(d-2)\int_0^1dx(1-x)\slashed{p}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}[(g_{\mathrm{V}}^t)^2+(g_{\mathrm{A}}^t)^2-2g_{\mathrm{V}}^tg_{\mathrm{A}}^t\gamma_5]$$

$$\frac{\partial(i\Pi_{t,\mathrm{V}}^{\text{gauge},Z})}{\partial\slashed{p}} = \frac{g^2}{4c_{\mathrm{W}}^2}\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}[(g_{\mathrm{V}}^t)^2+(g_{\mathrm{A}}^t)^2] + \text{finite}$$

Simplify the CKM matrix as an identity matrix

$$P_{\mathrm{L}} \equiv \frac{1-\gamma_5}{2}, \quad P_{\mathrm{R}} \equiv \frac{1+\gamma_5}{2}$$

$$\gamma^{\mu} P_{\mathrm{L}} (\not{p}+\not{q}) \gamma_{\mu} P_{\mathrm{L}} = -(d-2) P_{\mathrm{R}} (\not{p}+\not{q}) P_{\mathrm{L}} = -(d-2)(\not{p}+\not{q}) P_{\mathrm{L}} = -\frac{1}{2}(d-2)(\not{p}+\not{q})(1-\gamma_5)$$

$$i\Pi_t^{\mathrm{gauge},W} = \underset{t \quad --b-- \quad t}{\overset{W^+}{(\qquad)}} = \int \frac{d^d q}{(2\pi)^d} \left(i\frac{g}{\sqrt{2}} \gamma^{\mu} P_{\mathrm{L}} \right) \frac{i(\not{p}+\not{q})}{(p+q)^2} \left(i\frac{g}{\sqrt{2}} \gamma^{\nu} P_{\mathrm{L}} \right) \frac{-ig_{\mu\nu}}{q^2}$$

$$= -\frac{1}{2} g^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p+q)^2 q^2} [\gamma^{\mu} P_{\mathrm{L}} (\not{p}+\not{q}) \gamma_{\mu} P_{\mathrm{L}}] = \frac{1}{4} g^2 (d-2) \int \frac{d^d q}{(2\pi)^d} \frac{\not{p}+\not{q}}{(p+q)^2 q^2} (1-\gamma_5)$$

$$= \frac{1}{4} g^2 (d-2) \int_0^1 dx (1-x) \not{p} \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2} K_0^{2-d/2}} (1-\gamma_5)$$

$$\frac{\partial (i\Pi_{t,\mathrm{V}}^{\mathrm{gauge},W})}{\partial \not{p}} = \frac{1}{4} g^2 \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\frac{\partial (i\Pi_{t,\mathrm{V}}^{\mathrm{gauge}})}{\partial \not{p}} = \frac{\partial}{\partial \not{p}} (i\Pi_t^{\mathrm{gauge},g} + i\Pi_t^{\mathrm{gauge},\gamma} + i\Pi_{t,\mathrm{V}}^{\mathrm{gauge},Z} + i\Pi_{t,\mathrm{V}}^{\mathrm{gauge},W})$$

$$= \left\{ \frac{4}{3} g_{\mathrm{s}}^2 + \frac{4}{9} g^2 s_{\mathrm{W}}^2 + \frac{g^2}{4c_{\mathrm{W}}^2} [(g_{\mathrm{V}}^t)^2 + (g_{\mathrm{A}}^t)^2] + \frac{1}{4} g^2 \right\} \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_{t,\mathrm{V}}^{\mathrm{gauge}} = -\left\{ \frac{4}{3} g_{\mathrm{s}}^2 + \frac{4}{9} g^2 s_{\mathrm{W}}^2 + \frac{g^2}{4c_{\mathrm{W}}^2} [(g_{\mathrm{V}}^t)^2 + (g_{\mathrm{A}}^t)^2] + \frac{1}{4} g^2 \right\} \frac{\Gamma(2-d/2)}{(4\pi)^{d/2} (\mu_{\mathrm{R}}^2)^{2-d/2}} + \text{finite}$$

$$= \frac{1}{16\pi^2} \left\{ \frac{4}{3} g_{\mathrm{s}}^2 + \frac{4}{9} g^2 s_{\mathrm{W}}^2 + \frac{g^2}{4c_{\mathrm{W}}^2} [(g_{\mathrm{V}}^t)^2 + (g_{\mathrm{A}}^t)^2] + \frac{1}{4} g^2 \right\} \ln \mu_{\mathrm{R}}^2 + \cdots$$

$$\frac{\partial \delta_{t,\mathrm{V}}^{\mathrm{gauge}}}{\partial \ln \mu_{\mathrm{R}}} = \frac{1}{16\pi^2} \left\{ \frac{8}{3} g_{\mathrm{s}}^2 + \frac{8}{9} g^2 s_{\mathrm{W}}^2 + \frac{g^2}{2c_{\mathrm{W}}^2} [(g_{\mathrm{V}}^t)^2 + (g_{\mathrm{A}}^t)^2] + \frac{1}{2} g^2 \right\}$$

$$i\Pi_t^{\mathrm{scalar},h} = \underset{t \quad --t-- \quad t}{\overset{h}{(\qquad)}} = \int \frac{d^d q}{(2\pi)^d} \left(-i\frac{y_t}{\sqrt{2}} \right) \frac{i(\not{p}+\not{q})}{(p+q)^2} \left(-i\frac{y_t}{\sqrt{2}} \right) \frac{i}{q^2} = \frac{1}{2} y_t^2 \int \frac{d^d q}{(2\pi)^d} \frac{\not{p}+\not{q}}{(p+q)^2 q^2}$$

$$= \frac{1}{2} y_t^2 \int_0^1 dx \int \frac{d^d \ell}{(2\pi)^d} \frac{(1-x)\not{p}}{(\ell^2 - K_0)^2} = \frac{1}{2} y_t^2 \int_0^1 dx (1-x) \not{p} \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2} K_0^{2-d/2}}$$

$$\frac{\partial (i\Pi_t^{\mathrm{scalar},h})}{\partial \not{p}} = \frac{1}{4} y_t^2 \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi_t^{\mathrm{scalar},G^0} = \underset{t \quad --t-- \quad t}{\overset{G^0}{(\qquad)}} = \int \frac{d^d q}{(2\pi)^d} \left(-\frac{y_t}{\sqrt{2}} \gamma_5 \right) \frac{i(\not{p}+\not{q})}{(p+q)^2} \left(-\frac{y_t}{\sqrt{2}} \gamma_5 \right) \frac{i}{q^2} = \frac{1}{2} y_t^2 \int \frac{d^d q}{(2\pi)^d} \frac{\not{p}+\not{q}}{(p+q)^2 q^2}$$

$$= \frac{1}{2} y_t^2 \int_0^1 dx (1-x) \not{p} \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2} K_0^{2-d/2}}$$

$$\frac{\partial (i\Pi_t^{\mathrm{scalar},G^0})}{\partial \not{p}} = \frac{1}{4} y_t^2 \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Pi_t^{\mathrm{scalar},G^+} = \begin{matrix} & G^+ & \\ & (\qquad) & \\ t & --b-- & t \end{matrix} = \int \frac{d^dq}{(2\pi)^d}(iy_tP_\mathrm{R})\frac{i(\not p+\not q)}{(p+q)^2}(iy_tP_\mathrm{L})\frac{i}{q^2} = y_t^2\int\frac{d^dq}{(2\pi)^d}\frac{\not p+\not q}{(p+q)^2q^2}\frac{1-\gamma_5}{2}$$

$$= \frac{1}{2}y_t^2\int_0^1 dx(1-x)\not p\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}K_0^{2-d/2}}(1-\gamma_5)$$

$$\frac{\partial(i\Pi_{t,\mathrm{V}}^{\mathrm{scalar},G^+})}{\partial\not p} = \frac{1}{4}y_t^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\frac{\partial(i\Pi_{t,\mathrm{V}}^{\mathrm{scalar}})}{\partial\not p} = \frac{\partial}{\partial\not p}(i\Pi_t^{\mathrm{scalar},h} + i\Pi_t^{\mathrm{scalar},G^0} + i\Pi_{t,\mathrm{V}}^{\mathrm{scalar},G^+}) = \frac{3}{4}y_t^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\delta_{t,\mathrm{V}}^{\mathrm{scalar}} = -\frac{3}{4}y_t^2\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_\mathrm{R}^2)^{2-d/2}} + \text{finite} = \frac{1}{16\pi^2}\frac{3}{4}y_t^2\ln\mu_\mathrm{R}^2 + \cdots, \qquad \frac{\partial\delta_{t,\mathrm{V}}^{\mathrm{scalar}}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}\frac{3}{2}y_t^2$$

In total, $\displaystyle \frac{\partial\delta_{t,\mathrm{V}}}{\partial\ln\mu_\mathrm{R}} = \frac{\partial\delta_{t,\mathrm{V}}^{\mathrm{scalar}}}{\partial\ln\mu_\mathrm{R}} + \frac{\partial\delta_{t,\mathrm{V}}^{\mathrm{gauge}}}{\partial\ln\mu_\mathrm{R}} = \frac{1}{16\pi^2}\left\{\frac{3}{2}y_t^2 + \frac{8}{3}g_\mathrm{s}^2 + \frac{8}{9}g^2s_\mathrm{W}^2 + \frac{g^2}{2c_\mathrm{W}^2}[(g_\mathrm{V}^t)^2 + (g_\mathrm{A}^t)^2] + \frac{1}{2}g^2\right\}$

⊙ Calculation for $\dfrac{\partial\delta_{y_t}}{\partial\ln\mu_\mathrm{R}}$ in the <u>Feynman-t' Hooft gauge</u>

All external momenta can be neglected for computing $\dfrac{\partial\delta_{y_t}}{\partial\ln\mu_\mathrm{R}}$

$$\not q\not q = q^\mu q^\nu\gamma_\mu\gamma_\nu = \frac{1}{2}q^\mu q^\nu(\gamma_\mu\gamma_\nu + \gamma_\mu\gamma_\nu) = q^\mu q^\nu g_{\mu\nu} = q^2, \quad \gamma^\mu\gamma_\mu = d$$

$$i\Sigma_{y_t}^{ttg} = \begin{matrix} & h & \\ & t/\quad\backslash t & \\ t & --g-- & t \end{matrix} = \int\frac{d^dq}{(2\pi)^d}(ig_\mathrm{s}\gamma^\mu t^a)\frac{i\not q}{q^2}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i\not q}{q^2}(ig_\mathrm{s}\gamma_\mu t^a)\frac{-i}{q^2} = -\frac{1}{\sqrt{2}}y_tg_\mathrm{s}^2C_2(\mathbf{3})\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^3}\gamma^\mu\not q\not q\gamma_\mu$$

$$= -\frac{4}{3\sqrt{2}}y_tg_\mathrm{s}^2d\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = -\frac{16}{3\sqrt{2}}y_tg_\mathrm{s}^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma_{y_t}^{tt\gamma} = \begin{matrix} & h & \\ & t/\quad\backslash t & \\ t & --\gamma-- & t \end{matrix} = \int\frac{d^dq}{(2\pi)^d}(iQ_te\gamma^\mu)\frac{i\not q}{q^2}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i\not q}{q^2}(iQ_te\gamma_\mu)\frac{-i}{q^2} = -\frac{1}{\sqrt{2}}Q_t^2y_te^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^3}\gamma^\mu\not q\not q\gamma_\mu$$

$$= -\frac{4}{9\sqrt{2}}y_tg^2s_\mathrm{W}^2d\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = -\frac{16}{9\sqrt{2}}y_tg^2s_\mathrm{W}^2\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$\gamma^\mu(g_\mathrm{V}^t - g_\mathrm{A}^t\gamma_5)\not q\not q\gamma_\mu(g_\mathrm{V}^t - g_\mathrm{A}^t\gamma_5) = d\cdot q^2(g_\mathrm{V}^t + g_\mathrm{A}^t\gamma_5)(g_\mathrm{V}^t - g_\mathrm{A}^t\gamma_5) = d\cdot q^2[(g_\mathrm{V}^t)^2 - (g_\mathrm{A}^t)^2]$$

$$i\Sigma_{y_t}^{ttZ} = \begin{matrix} & h & \\ & t/\quad\backslash t & \\ t & --Z-- & t \end{matrix} = \int\frac{d^dq}{(2\pi)^d}\left[i\frac{g}{2c_\mathrm{W}}\gamma^\mu(g_\mathrm{V}^t - g_\mathrm{A}^t\gamma_5)\right]\frac{i\not q}{q^2}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i\not q}{q^2}\left[i\frac{g}{2c_\mathrm{W}}\gamma_\mu(g_\mathrm{V}^t - g_\mathrm{A}^t\gamma_5)\right]\frac{-i}{q^2}$$

$$= -\frac{1}{4\sqrt{2}c_\mathrm{W}^2}y_tg^2[(g_\mathrm{V}^t)^2 - (g_\mathrm{A}^t)^2]d\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = -\frac{1}{\sqrt{2}c_\mathrm{W}^2}y_tg^2[(g_\mathrm{V}^t)^2 - (g_\mathrm{A}^t)^2]\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma_{y_t}^{tth} = \begin{matrix} & h & \\ & t/\quad\backslash t & \\ t & --h-- & t \end{matrix} = \int\frac{d^dq}{(2\pi)^d}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i\not q}{q^2}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i\not q}{q^2}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i}{q^2} = \frac{1}{2\sqrt{2}}y_t^3\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^3}\not q\not q$$

$$= \frac{1}{2\sqrt{2}}y_t^3\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2} = \frac{1}{2\sqrt{2}}y_t^3\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma^{ttG^0}_{y_t} = \begin{array}{c} h \\ t\,/\quad \backslash\, t \\ t\quad --G^0--\quad t \end{array} = \int \frac{d^d q}{(2\pi)^d}\left(-\frac{y_t}{\sqrt{2}}\gamma_5\right)\frac{i\not{q}}{q^2}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i\not{q}}{q^2}\left(-\frac{y_t}{\sqrt{2}}\gamma_5\right)\frac{i}{q^2} = -\frac{1}{2\sqrt{2}}y_t^3\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^3}\gamma_5\not{q}\not{q}\gamma_5$$

$$= -\frac{1}{2\sqrt{2}}y_t^3\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^2} = -\frac{1}{2\sqrt{2}}y_t^3\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma^{hht}_{y_t} = \begin{array}{c} h \\ h\,/\quad \backslash\, h \\ t\quad --t--\quad t \end{array} = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i\not{q}}{q^2}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i}{q^2}(-6i\lambda v)\frac{i}{q^2} = 0$$

$$i\Sigma^{G^0G^0t}_{y_t} = \begin{array}{c} h \\ G^0\,/\quad \backslash\, G^0 \\ t\quad --t--\quad t \end{array} = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\left(-\frac{y_t}{\sqrt{2}}\gamma_5\right)\frac{i\not{q}}{q^2}\left(-\frac{y_t}{\sqrt{2}}\gamma_5\right)\frac{i}{q^2}(-2i\lambda v)\frac{i}{q^2} = 0$$

$$i\Sigma^{G^+G^+b}_{y_t} = \begin{array}{c} h \\ G^+\,/\quad \backslash\, G^+ \\ t\quad --b--\quad t \end{array} = \int \frac{d^d q}{(2\pi)^d}(iy_tP_{\mathrm{L}})\frac{i\not{q}}{q^2}(iy_tP_{\mathrm{R}})\frac{i}{q^2}(-2i\lambda v)\frac{i}{q^2} = 0$$

$$i\Sigma^{ZZt}_{y_t} = \begin{array}{c} h \\ Z\,/\quad \backslash\, Z \\ t\quad --t--\quad t \end{array} = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\left[i\frac{g}{2c_{\mathrm{W}}}\gamma^\mu(g^t_{\mathrm{V}} - g^t_{\mathrm{A}}\gamma_5)\right]\frac{i\not{q}}{q^2}\left[i\frac{g}{2c_{\mathrm{W}}}\gamma^\nu(g^t_{\mathrm{V}} - g^t_{\mathrm{A}}\gamma_5)\right]\frac{-ig_{\mu\rho}}{q^2}\left(i\frac{gm_Z}{c_{\mathrm{W}}}g^{\rho\sigma}\right)\frac{-ig_{\sigma\nu}}{q^2} = 0$$

$$i\Sigma^{WWb}_{y_t} = \begin{array}{c} h \\ W\,/\quad \backslash\, W \\ t\quad --b--\quad t \end{array} = \int \frac{d^d q}{(2\pi)^d}\left(i\frac{g}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}}\right)\frac{i\not{q}}{q^2}\left(i\frac{g}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}}\right)\frac{-ig_{\mu\rho}}{q^2}(igm_W g^{\rho\sigma})\frac{-ig_{\sigma\nu}}{q^2} = 0$$

$$i\Sigma^{ZG^0t}_{y_t} = \begin{array}{c} h \\ G^0\,/\quad \backslash\, Z \\ t\quad --t--\quad t \end{array} = \int \frac{d^d q}{(2\pi)^d}\left[i\frac{g}{2c_{\mathrm{W}}}\gamma^\mu(g^t_{\mathrm{V}} - g^t_{\mathrm{A}}\gamma_5)\right]\frac{i\not{q}}{q^2}\left(-\frac{y_t}{\sqrt{2}}\gamma_5\right)\frac{-ig_{\mu\nu}}{q^2}\left(-\frac{g}{2c_{\mathrm{W}}}q^\nu\right)\frac{i}{q^2}$$

$$= -\frac{1}{4\sqrt{2}c^2_{\mathrm{W}}}y_tg^2\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^3}[\not{q}(g^t_{\mathrm{V}} - g^t_{\mathrm{A}}\gamma_5)\not{q}\gamma_5] = -\frac{1}{4\sqrt{2}c^2_{\mathrm{W}}}y_tg^2\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^2}(g^t_{\mathrm{A}} + g^t_{\mathrm{V}}\gamma_5)$$

$$= -\frac{1}{4\sqrt{2}c^2_{\mathrm{W}}}y_tg^2(g^t_{\mathrm{A}} + g^t_{\mathrm{V}}\gamma_5)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma^{G^0Zt}_{y_t} = \begin{array}{c} h \\ Z\,/\quad \backslash\, G^0 \\ t\quad --t--\quad t \end{array} = \int \frac{d^d q}{(2\pi)^d}\left(-\frac{y_t}{\sqrt{2}}\gamma_5\right)\frac{i\not{q}}{q^2}\left[i\frac{g}{2c_{\mathrm{W}}}\gamma^\mu(g^t_{\mathrm{V}} - g^t_{\mathrm{A}}\gamma_5)\right]\frac{i}{q^2}\left(\frac{g}{2c_{\mathrm{W}}}q^\nu\right)\frac{-ig_{\mu\nu}}{q^2}$$

$$= \frac{1}{4\sqrt{2}c^2_{\mathrm{W}}}y_tg^2\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^3}\gamma_5\not{q}\not{q}(g^t_{\mathrm{V}} - g^t_{\mathrm{A}}\gamma_5) = -\frac{1}{4\sqrt{2}c^2_{\mathrm{W}}}y_tg^2\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^2}(g^t_{\mathrm{A}} - g^t_{\mathrm{V}}\gamma_5)$$

$$= -\frac{1}{4\sqrt{2}c^2_{\mathrm{W}}}y_tg^2(g^t_{\mathrm{A}} - g^t_{\mathrm{V}}\gamma_5)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma^{W^+G^+t}_{y_t} = \begin{array}{c} h \\ G^+ \nearrow\!\searrow W^+ \\ t\quad ---b---\quad t \end{array} = \int \frac{d^d q}{(2\pi)^d}\left(i\frac{g}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}}\right)\frac{i\not{q}}{q^2}(iy_tP_{\mathrm{R}})\frac{-ig_{\mu\nu}}{q^2}\left(-i\frac{g}{2}q^\nu\right)\frac{i}{q^2}$$

$$= -\frac{1}{2\sqrt{2}}y_tg^2\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^3}\not{q}P_{\mathrm{L}}\not{q}P_{\mathrm{R}} = -\frac{1}{2\sqrt{2}}y_tg^2\int\frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^2}P_{\mathrm{R}} = -\frac{1}{4\sqrt{2}}y_tg^2(1+\gamma_5)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

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$$i\Sigma_{y_t}^{G^+W^+t} = \begin{array}{c} h \\ W^+ \nearrow\searrow G^+ \\ t \quad ---b--- \quad t \end{array} = \int \frac{d^d q}{(2\pi)^d}(iy_t P_{\mathrm{L}})\frac{i\not{q}}{q^2}\left(i\frac{g}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}}\right)\frac{i}{q^2}\left(-i\frac{g}{2}q^\nu\right)\frac{-ig_{\mu\nu}}{q^2}$$

$$= -\frac{1}{2\sqrt{2}}y_t g^2 \int \frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^3}P_{\mathrm{L}}\not{q}\not{q}P_{\mathrm{L}} = -\frac{1}{2\sqrt{2}}y_t g^2 \int \frac{d^d q}{(2\pi)^d}\frac{1}{(q^2)^2}P_{\mathrm{L}} = -\frac{1}{4\sqrt{2}}y_t g^2(1-\gamma_5)\frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}} + \text{finite}$$

$$i\Sigma_{y_t} = \sum_i (i\Sigma_{y_t}^i) = \frac{i\Gamma(2-d/2)}{(4\pi)^{d/2}}\left[-\frac{16}{3\sqrt{2}}y_t g_{\mathrm{s}}^2 - \frac{16}{9\sqrt{2}}y_t g^2 s_{\mathrm{W}}^2 - \frac{1}{\sqrt{2}c_{\mathrm{W}}^2}y_t g^2[(g_{\mathrm{V}}^t)^2 - (g_{\mathrm{A}}^t)^2] + \frac{1}{2\sqrt{2}}y_t^3 - \frac{1}{2\sqrt{2}}y_t^3 \right.$$

$$\left. -\frac{1}{4\sqrt{2}c_{\mathrm{W}}^2}y_t g^2 g_{\mathrm{A}}^t - \frac{1}{4\sqrt{2}c_{\mathrm{W}}^2}y_t g^2 g_{\mathrm{A}}^t - \frac{1}{4\sqrt{2}}y_t g^2 - \frac{1}{4\sqrt{2}}y_t g^2\right] + \text{finite}$$

$\Sigma_{y_t} - \dfrac{\delta_{y_t}}{\sqrt{2}}$ is finite

$\Downarrow$

$$\delta_{y_t} = \sqrt{2}\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}(\mu_{\mathrm{R}}^2)^{2-d/2}}\left[-\frac{16}{3\sqrt{2}}y_t g_{\mathrm{s}}^2 - \frac{16}{9\sqrt{2}}y_t g^2 s_{\mathrm{W}}^2 - \frac{1}{\sqrt{2}c_{\mathrm{W}}^2}y_t g^2[(g_{\mathrm{V}}^t)^2 - (g_{\mathrm{A}}^t)^2] - \frac{1}{2\sqrt{2}c_{\mathrm{W}}^2}y_t g^2 g_{\mathrm{A}}^t - \frac{1}{2\sqrt{2}}y_t g^2\right] + \text{finite}$$

$$= -\ln\mu_{\mathrm{R}}^2 \frac{1}{16\pi^2}\left(-\frac{16}{3}y_t g_{\mathrm{s}}^2 - \frac{16}{9}y_t g^2 s_{\mathrm{W}}^2 - \frac{1}{c_{\mathrm{W}}^2}y_t g^2[(g_{\mathrm{V}}^t)^2 - (g_{\mathrm{A}}^t)^2] - \frac{1}{2c_{\mathrm{W}}^2}y_t g^2 g_{\mathrm{A}}^t - \frac{1}{2}y_t g^2\right) + \text{finite}$$

$$\frac{\partial\delta_{y_t}}{\partial\ln\mu_{\mathrm{R}}} = \frac{1}{16\pi^2}y_t\left\{\frac{32}{3}g_{\mathrm{s}}^2 + \frac{32}{9}g^2 s_{\mathrm{W}}^2 + \frac{2g^2}{c_{\mathrm{W}}^2}[(g_{\mathrm{V}}^t)^2 - (g_{\mathrm{A}}^t)^2] + \frac{g^2}{c_{\mathrm{W}}^2}g_{\mathrm{A}}^t + g^2\right\}$$

⊙ Explicit, gauge-independent expression for the β functions of y_t

$$g_{\mathrm{V}}^t = \frac{1}{2} - \frac{4}{3}s_{\mathrm{W}}^2, \quad g_{\mathrm{A}}^t = \frac{1}{2}, \quad \frac{g^2}{c_{\mathrm{W}}^2} = g^2 + g'^2, \quad \frac{g^2 s_{\mathrm{W}}^2}{c_{\mathrm{W}}^2} = g'^2$$

$$-\frac{3}{2}(g_{\mathrm{V}}^t)^2 + \frac{5}{2}(g_{\mathrm{A}}^t)^2 - g_{\mathrm{A}}^t = -\frac{3}{2}\left(\frac{1}{2} - \frac{4}{3}s_{\mathrm{W}}^2\right)^2 + \frac{5}{8} - \frac{1}{2} = -\frac{1}{4} + 2s_{\mathrm{W}}^2 - \frac{8}{3}s_{\mathrm{W}}^4$$

$$-\frac{8}{3}g^2 s_{\mathrm{W}}^2 + \frac{g^2}{c_{\mathrm{W}}^2}\left[-\frac{3}{2}(g_{\mathrm{V}}^t)^2 + \frac{5}{2}(g_{\mathrm{A}}^t)^2 - g_{\mathrm{A}}^t\right] = -\frac{8}{3}g^2 s_{\mathrm{W}}^2 + \frac{g^2}{c_{\mathrm{W}}^2}\left(-\frac{1}{4} + 2s_{\mathrm{W}}^2 - \frac{8}{3}s_{\mathrm{W}}^4\right)$$

$$= -\frac{1}{4}\frac{g^2}{c_{\mathrm{W}}^2} + \frac{g^2 s_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}\left(-\frac{8}{3}c_{\mathrm{W}}^2 + 2 - \frac{8}{3}s_{\mathrm{W}}^2\right) = -\frac{1}{4}\frac{g^2}{c_{\mathrm{W}}^2} - \frac{2}{3}\frac{g^2 s_{\mathrm{W}}^2}{c_{\mathrm{W}}^2} = -\frac{1}{4}g^2 - \frac{11}{12}g'^2$$

$$\frac{16\pi^2}{y_t}\beta_{y_t} = 16\pi^2\left(-\frac{1}{y_t}\frac{\partial\delta_{y_t}}{\partial\ln\mu_{\mathrm{R}}} + \frac{\partial\delta_{t,\mathrm{V}}}{\partial\ln\mu_{\mathrm{R}}} + \frac{1}{2}\frac{\partial\delta_h}{\partial\ln\mu_{\mathrm{R}}}\right)$$

$$= -\frac{32}{3}g_{\mathrm{s}}^2 - \frac{32}{9}g^2 s_{\mathrm{W}}^2 - \frac{2g^2}{c_{\mathrm{W}}^2}[(g_{\mathrm{V}}^t)^2 - (g_{\mathrm{A}}^t)^2] - \frac{g^2}{c_{\mathrm{W}}^2}g_{\mathrm{A}}^t - g^2$$

$$+\left\{\frac{3}{2}y_t^2 + \frac{8}{3}g_{\mathrm{s}}^2 + \frac{8}{9}g^2 s_{\mathrm{W}}^2 + \frac{g^2}{2c_{\mathrm{W}}^2}[(g_{\mathrm{V}}^t)^2 + (g_{\mathrm{A}}^t)^2] + \frac{1}{2}g^2\right\} + \left(3y_t^2 - \frac{3}{2}g^2 - \frac{1}{2}g'^2\right)$$

$$= \frac{9}{2}y_t^2 - 8g_{\mathrm{s}}^2 - 2g^2 - \frac{1}{2}g'^2 - \frac{8}{3}g^2 s_{\mathrm{W}}^2 + \frac{g^2}{c_{\mathrm{W}}^2}\left[-\frac{3}{2}(g_{\mathrm{V}}^t)^2 + \frac{5}{2}(g_{\mathrm{A}}^t)^2 - g_{\mathrm{A}}^t\right]$$

$$= \frac{9}{2}y_t^2 - 8g_{\mathrm{s}}^2 - 2g^2 - \frac{1}{2}g'^2 - \frac{1}{4}g^2 - \frac{11}{12}g'^2 = \frac{9}{2}y_t^2 - 8g_{\mathrm{s}}^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2$$

$$\beta_{y_t} = \frac{1}{16\pi^2}y_t\left(\frac{9}{2}y_t^2 - 8g_{\mathrm{s}}^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right)$$