标准模型的拉氏量和 Feynman 规则

余钊焕

中山大学物理学院

https://yzhxxzxy.github.io

2022 年 9 月 8 日

目　录

1 约定

本文采用有理化的自然单位制，推导过程参考文献 [1, 2, 3, 4]，协变导数的约定与 *Review of Particle Physics* [5] 第 9、10、11 章一致。

Minkowski 度规张量

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{1}$$

Pauli 矩阵

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -\mathrm{i} \\ \mathrm{i} & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \tag{2}$$

$$\sigma^\mu \equiv (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma}). \tag{3}$$

Weyl 表象中的 Dirac 矩阵

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbf{1} & \\ & \mathbf{1} \end{pmatrix}. \tag{4}$$

左右手投影算符

$$P_\mathrm{L} \equiv \frac{1}{2}(1 - \gamma^5) = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{0} \end{pmatrix}, \quad P_\mathrm{R} \equiv \frac{1}{2}(1 + \gamma^5) = \begin{pmatrix} \mathbf{0} & \\ & \mathbf{1} \end{pmatrix} \tag{5}$$

用于定义左手旋量场 $\psi_\mathrm{L} \equiv P_\mathrm{L}\psi$ 和右手旋量场 $\psi_\mathrm{R} \equiv P_\mathrm{R}\psi$。Levi-Civita 符号的约定取

$$\varepsilon^{0123} = \varepsilon^{123} = +1. \tag{6}$$

Feynman 规则约定如下。

- 对于指向相互作用顶点的动量 p，时空导数 ∂_μ 在动量空间 Feynman 规则里贡献一个 $-\mathrm{i}p_\mu$ 因子。

- 实线表示费米子，实线上的箭头表示费米子数流动的方向。

- 虚线表示标量玻色子，虚线上的箭头表示玻色子数流动的方向。

- 螺旋线表示胶子；波浪线表示其它规范玻色子，波浪线上的箭头表示玻色子数流动的方向。

- 点线表示鬼粒子，点线上的箭头表示鬼粒子数流动的方向。

- 如果没有额外箭头标记，动量方向与粒子线上的箭头方向一致；否则与额外箭头方向一致。

2 标准模型概述

粒子物理标准模型是一个 $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$ 规范理论。模型中有三代费米子，包括三代中微子 $\nu_i = \nu_e, \nu_\mu, \nu_\tau$，三代带电轻子 $\ell_i = e, \mu, \tau$，三代上型夸克 $u_i = u, c, t$ 和三代下型夸克 $d_i = d, s, b$ ($i = 1, 2, 3$)。规范玻色子传递费米子之间的规范相互作用。

$\mathrm{SU(3)_C}$ 部分描述夸克的强相互作用，称为量子色动力学 (Quantum Chromodynamics, QCD)，相应的规范玻色子是胶子。$\mathrm{SU(2)_L \times U(1)_Y}$ 部分统一描述夸克和轻子的电磁和弱相互作用，称为电弱规范理论。理论中有一个 Higgs 二重态，通过 Brout–Englert–Higgs (BEH) 机制引发规范群的自发对称性破缺，使 $\mathrm{SU(2)_L \times U(1)_Y}$ 群破缺为 $\mathrm{U(1)_{EM}}$ 群。$\mathrm{U(1)_{EM}}$ 规范理论称为量子电动力学 (Quantum Electrodynamics, QED)。

破缺前，理论中存在 4 个无质量的规范玻色子和 4 个 Higgs 自由度；左手费米子和右手费米子都没有质量，具有不同的量子数。

破缺后，3 个规范玻色子与 3 个 Higgs 自由度结合，从而获得质量，成为 $W^\pm$ 和 Z^0 玻色子，传递弱相互作用。剩下的 1 个无质量规范玻色子是光子，即是 $\mathrm{U(1)_{EM}}$ 群的规范玻色子，传递电磁相互作用。剩下的 1 个中性 Higgs 自由度称为 Higgs 玻色子。费米子与 Higgs 二重态的 Yukawa 耦合导致左手费米子和右手费米子获得质量，组合成 Dirac 费米子。

理论中的中微子没有右手分量，因而没有获得质量。1998 年实验发现中微子振荡，证明中微子具有质量，所以需要扩充标准模型才能正确描述中微子物理。

3 量子色动力学

QCD 的拉氏量表达为

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{q}(\mathrm{i}\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}, \quad q = u, d, s, c, b, t, \quad a = 1, \cdots, 8, \tag{7}$$

其中

$$D_\mu = \partial_\mu + \mathrm{i}g_\mathrm{s}G^a_\mu t^a, \quad G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_\mathrm{s}f^{abc}G^b_\mu G^c_\nu. \tag{8}$$

q 为夸克旋量场 $\mathrm{SU(3)_C}$ 三重态，$\mathrm{SU(3)_C}$ 规范场 G^a_μ 对应于胶子 g，g_s 是 $\mathrm{SU(3)_C}$ 规范耦合常数。$t^a = \lambda^a/2$ 是 $\mathrm{SU(3)_C}$ 群基础表示的生成元，其中 λ^a 为 Gell-Mann 矩阵。$\mathrm{SU(3)_C}$ 生成元对易关系为 $[t^a, t^b] = \mathrm{i}f^{abc}t^c$，结构常数 f^{abc} 是全反对称的，非零分量为

$$f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}. \tag{9}$$

由

$$-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} = -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_\mathrm{s}f^{abc}G^b_\mu G^c_\nu)(\partial^\mu G^{a,\nu} - \partial^\nu G^{a,\mu} + g_\mathrm{s}f^{ade}G^{d,\mu}G^{e,\nu})$$

$$= -\frac{1}{2}[(\partial_\mu G^a_\nu)(\partial^\mu G^{a,\nu}) - (\partial_\mu G^a_\nu)(\partial^\nu G^{a,\mu})] + g_\mathrm{s}f^{abc}(\partial_\mu G^a_\nu)G^{b,\mu}G^{c,\nu}$$

$$-\frac{g_{\mathrm{s}}^2}{4} f^{abc} f^{ade} G_\mu^b G_\nu^c G^{d,\mu} G^{e,\nu}, \tag{10}$$

推出

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q [\bar{q}(\mathrm{i}\gamma^\mu \partial_\mu - m_q)q - g_{\mathrm{s}} G_\mu^a \bar{q}\gamma^\mu t^a q] + \frac{1}{2}[(\partial_\mu G_\nu^a)(\partial^\nu G^{a,\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a,\nu})]$$

$$+ g_{\mathrm{s}} f^{abc}(\partial_\mu G_\nu^a) G^{b,\mu} G^{c,\nu} - \frac{g_{\mathrm{s}}^2}{4} f^{abc} f^{ade} G_\mu^b G_\nu^c G^{d,\mu} G^{e,\nu}. \tag{11}$$

设用于固定胶子场规范的函数 $G^a(x) = \partial^\mu G_\mu^a(x) - \omega^a(x)$，其中 $\omega^a(x)$ 是某个任意函数，规范固定条件是 $G^a(x) = 0$。这是 Lorenz 规范的推广，$\omega^a(x) = 0$ 对应于 Lorenz 规范。在路径积分量子化中，以中心为 $\omega^a(x) = 0$ 的 Gauss 权重对 $\omega^a(x)$ 作泛函积分，有

$$\int \mathcal{D}\omega^a \exp\left[-\mathrm{i}\int \mathrm{d}^4 x \frac{1}{2\xi}(\omega^a)^2\right] \delta(G^a) = \exp\left[-\mathrm{i}\int \mathrm{d}^4 x \frac{1}{2\xi}(\partial^\mu G_\mu^a)^2\right]. \tag{12}$$

可见，拉氏量中的规范固定项为

$$\mathcal{L}_{\mathrm{QCD,GF}} = -\frac{1}{2\xi}(\partial^\mu G_\mu^a)^2. \tag{13}$$

ξ 的任何一个取值对应于一种规范。$\xi = 1$ 称为 Feynman-'t Hooft 规范，$\xi = 0$ 称为 Landau 规范。于是，胶子传播子相关拉氏量为

$$\mathcal{L}_{\mathrm{QCD,prop}} = \frac{1}{2}\left[(\partial_\mu G_\nu^a)(\partial^\nu G^{a,\mu}) - (\partial_\mu G_\nu^a)(\partial^\mu G^{a,\nu}) - \frac{1}{\xi}(\partial^\mu G_\mu^a)^2\right]$$

$$\rightarrow \frac{1}{2} G_\mu^a\left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu\right] G_\nu^a. \tag{14}$$

这里 $\rightarrow$ 代表丢弃一些全散度项。变换到动量空间，得

$$-g^{\mu\nu}p^2 + \left(1 - \frac{1}{\xi}\right)p^\mu p^\nu, \tag{15}$$

它的逆矩阵是

$$-\frac{1}{p^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi)\right], \tag{16}$$

这是因为

$$-\frac{1}{p^2}\left[g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}(1 - \xi)\right]\left[-g^{\mu\nu}p^2 + \left(1 - \frac{1}{\xi}\right)p^\mu p^\nu\right]$$

$$= \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2}\left(1 - \frac{1}{\xi}\right) - \frac{p_\rho p^\nu}{p^2}(1 - \xi) + \frac{p_\rho p^\nu}{p^2}(1 - \xi)\left(1 - \frac{1}{\xi}\right) = \delta_\rho{}^\nu. \tag{17}$$

从而胶子传播子的形式为

$$\frac{-\mathrm{i}\delta^{ab}}{p^2 + \mathrm{i}\epsilon} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi) \right]. \tag{18}$$

$\mathrm{SU}(3)_{\mathrm{C}}$ 规范变换为

$$q' = Uq, \quad (G_\mu^a t^a)' = U G_\mu^a t^a U^\dagger - \frac{\mathrm{i}}{g_\mathrm{s}} U \partial_\mu U^\dagger, \tag{19}$$

其中 $U(x) = \exp[\mathrm{i}\alpha^a(x)t^a]$。胶子场的无穷小规范变换形式是

$$\begin{aligned}
(G_\mu^a t^a)' &= (1 + \mathrm{i}\alpha^a t^a) G_\mu^b t^b (1 - \mathrm{i}\alpha^c t^c) - \frac{\mathrm{i}}{g_\mathrm{s}}(1 + \mathrm{i}\alpha^a t^a)\partial_\mu(1 - \mathrm{i}\alpha^c t^c) \\
&= G_\mu^b t^b + \mathrm{i}\alpha^c G_\mu^b [t^c, t^b] - \frac{1}{g_\mathrm{s}}(\partial_\mu \alpha^c)t^c + \mathcal{O}(\alpha^a \alpha^b) \\
&= G_\mu^a t^a - f^{cba}\alpha^c G_\mu^b t^a - \frac{1}{g_\mathrm{s}}(\partial_\mu \alpha^a)t^a + \mathcal{O}(\alpha^a \alpha^b) \\
&= \left(G_\mu^a + f^{abc} G_\mu^b \alpha^c - \frac{1}{g_\mathrm{s}}\partial_\mu \alpha^a \right) t^a + \mathcal{O}(\alpha^a \alpha^b),
\end{aligned} \tag{20}$$

即 G_μ^a 的无穷小变化为

$$\delta G_\mu^a = (G_\mu^a)' - G_\mu^a = -\frac{1}{g_\mathrm{s}}\partial_\mu \alpha^a + f^{abc} G_\mu^b \alpha^c = -\frac{1}{g_\mathrm{s}}(D_\mu \alpha)^a = -\frac{1}{g_\mathrm{s}} D_\mu^{ac}\alpha^c, \tag{21}$$

其中 $(D_\mu \alpha)^a = \partial_\mu \alpha^a - g_\mathrm{s} f^{abc} G_\mu^b \alpha^c$ 是 $\mathrm{SU}(3)_{\mathrm{C}}$ 伴随表示中的协变导数，而

$$D_\mu^{ac} = \delta^{ac}\partial_\mu - g_\mathrm{s} f^{abc} G_\mu^b. \tag{22}$$

因此，规范固定函数 G^a 的无穷小变化为

$$\delta G^a = \partial^\mu \delta G_\mu^a = -\frac{1}{g_\mathrm{s}}\partial^\mu D_\mu^{ac}\alpha^c, \tag{23}$$

故

$$\frac{\delta G^a}{\delta \alpha^c} = -\frac{1}{g_\mathrm{s}}\partial^\mu D_\mu^{ac} = -\frac{1}{g_\mathrm{s}}\delta^{ac}\partial^2 + f^{abc}\partial^\mu G_\mu^b. \tag{24}$$

根据 Grassmann 数的积分式

$$\left(\prod_i \int \mathrm{d}\theta_i^* \mathrm{d}\theta_i \right) \exp(-\theta_i^* B_{ij} \theta_j) = \det(B), \tag{25}$$

$\delta G^a / \delta \alpha^c$ 的行列式可用 Faddeev-Popov 鬼场 η_g^a 和 $\bar{\eta}_g^a$ 表达为

$$\det\left(\frac{\delta G^a}{\delta \alpha^c}\right) = \det\left(-\frac{1}{g_\mathrm{s}}\partial^\mu D_\mu^{ac}\right) = \int \mathcal{D}\eta_g^a \, \mathcal{D}\bar{\eta}_g^c \exp\left[\mathrm{i}\int \mathrm{d}^4 x \, \bar{\eta}_g^a(-\partial^\mu D_\mu^{ac})\eta_g^c\right], \tag{26}$$

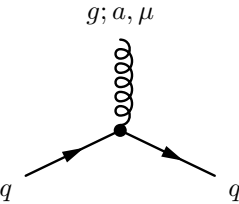这里 $-1/g_{\rm s}$ 因子被吸收到鬼场 η_g^a 和 $\bar\eta_g^a$ 的归一化中。注意到

$$- \partial^\mu D_\mu^{ac} = g_{\rm s}\frac{\delta G^a}{\delta\alpha^c} = -\delta^{ac}\partial^2 + g_{\rm s}f^{abc}\partial^\mu G_\mu^b, \tag{27}$$
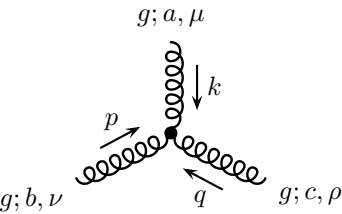
Faddeev-Popov 鬼场在拉氏量中的贡献是

$$\begin{aligned}
\mathcal{L}_{\rm QCD,FP} &= \bar\eta_g^a(-\partial^\mu D_\mu^{ac})\eta_g^c = \bar\eta_g^a\left(g_{\rm s}\frac{\delta G^a}{\delta\alpha^c}\right)\eta_g^c = \bar\eta_g^a(-\delta^{ac}\partial^2 + g_{\rm s}f^{abc}\partial^\mu G_\mu^b)\eta_g^c \\
&\rightarrow -\bar\eta_g^a\delta^{ab}\partial^2\eta_g^b - g_{\rm s}f^{abc}(\partial^\mu\bar\eta_g^a)G_\mu^b\eta_g^c.
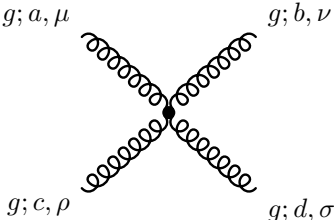\end{aligned} \tag{28}$$

下面列出 QCDFeynman 规则。

QCD 耦合顶点:

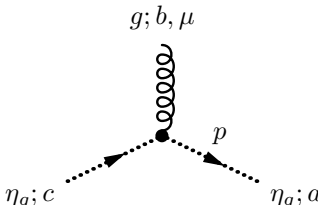$$= -{\rm i}g_{\rm s}\gamma^\mu t^a \tag{29}$$

$$= -g_{\rm s}f^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu] \tag{30}$$
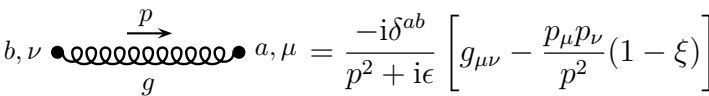

$$\begin{aligned}
= -{\rm i}g_{\rm s}^2[&f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\
&+ f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\
&+ f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]
\end{aligned} \tag{31}$$

$$= g_{\rm s}f^{abc}p^\mu \tag{32}$$

胶子传播子:

$$= \frac{-{\rm i}\delta^{ab}}{p^2 + {\rm i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)\right] \tag{33}$$

鬼粒子传播子:

$$= \frac{{\rm i}\delta^{ab}}{p^2 + {\rm i}\epsilon} \tag{34}$$

4 电弱规范理论

电弱规范理论的规范群是 $\mathrm{SU(2)_L \times U(1)_Y}$，每一代左手旋量场构成 2 个 $\mathrm{SU(2)_L}$ 二重态

$$L_{i\mathrm{L}} = \begin{pmatrix} P_\mathrm{L}\nu_i \\ P_\mathrm{L}\ell_i \end{pmatrix} = \begin{pmatrix} \nu_{i\mathrm{L}} \\ \ell_{i\mathrm{L}} \end{pmatrix}, \quad Q_{i\mathrm{L}} = \begin{pmatrix} P_\mathrm{L}u'_i \\ P_\mathrm{L}d'_i \end{pmatrix} = \begin{pmatrix} u'_{i\mathrm{L}} \\ d'_{i\mathrm{L}} \end{pmatrix}, \quad i = 1, 2, 3. \tag{35}$$

它们的协变导数是

$$D_\mu = \partial_\mu + \mathrm{i}gW_\mu^a \tau^a + \mathrm{i}g'B_\mu Y, \tag{36}$$

其中 $W_\mu^a(x)$ $(a = 1, 2, 3)$ 是 $\mathrm{SU(2)_L}$ 规范场，$B_\mu(x)$ 是 $\mathrm{U(1)_Y}$ 规范场，g 和 g' 分别是 $\mathrm{SU(2)_L}$ 和 $\mathrm{U(1)_Y}$ 的规范耦合常数，取 $g > 0$ 且 $g' > 0$。

$$\tau^a = \frac{\sigma^a}{2} \tag{37}$$

是 $\mathrm{SU(2)_L}$ 群 2 维表示的生成元，对应于弱同位旋。生成元 τ^3 的本征值是弱同位旋第 3 分量，记为 T^3。Y 是弱超荷。各代右手旋量场 $\ell_{i\mathrm{R}} = P_\mathrm{R}\ell_i$、$u'_{i\mathrm{R}} = P_\mathrm{R}u'_i$ 和 $d'_{i\mathrm{R}} = P_\mathrm{R}d'_i$ 是 $\mathrm{SU(2)_L}$ 单态，协变导数为

$$D_\mu = \partial_\mu + \mathrm{i}g'B_\mu Y. \tag{38}$$

表 1 列出费米子场的电荷 Q、弱同位旋第 3 分量 T^3、弱超荷 Y、重子数 B 和轻子数 $L_e/L_\mu/L_\tau$，其中电荷 Q 由 T^3 和 Y 定义，

$$Q \equiv T^3 + Y. \tag{39}$$

表 1: 标准模型费米子场的量子数。

| 统一记号	第一代	第二代	第三代	Q	T^3	Y	B	$L_e/L_\mu/L_\tau$
$L_{i\mathrm{L}} = \begin{pmatrix} \nu_{i\mathrm{L}} \\ \ell_{i\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} \nu_{e\mathrm{L}} \\ e_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$	$\begin{matrix} 0 \\ -1 \end{matrix}$	$\begin{matrix} 1/2 \\ -1/2 \end{matrix}$	$\begin{matrix} -1/2 \\ -1/2 \end{matrix}$	$\begin{matrix} 0 \\ 0 \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$
$Q_{i\mathrm{L}} = \begin{pmatrix} u'_{i\mathrm{L}} \\ d'_{i\mathrm{L}} \end{pmatrix}$	$\begin{pmatrix} u'_\mathrm{L} \\ d'_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} c'_\mathrm{L} \\ s'_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} t'_\mathrm{L} \\ b'_\mathrm{L} \end{pmatrix}$	$\begin{matrix} 2/3 \\ -1/3 \end{matrix}$	$\begin{matrix} 1/2 \\ -1/2 \end{matrix}$	$\begin{matrix} 1/6 \\ 1/6 \end{matrix}$	$\begin{matrix} 1/3 \\ 1/3 \end{matrix}$	$\begin{matrix} 0 \\ 0 \end{matrix}$
$\ell_{i\mathrm{R}}$	e_R	μ_R	τ_R	-1	0	-1	0	1
$u'_{i\mathrm{R}}$	u'_R	c'_R	t'_R	$2/3$	0	$2/3$	$1/3$	0
$d'_{i\mathrm{R}}$	d'_R	s'_R	b'_R	$-1/3$	0	$-1/3$	$1/3$	0
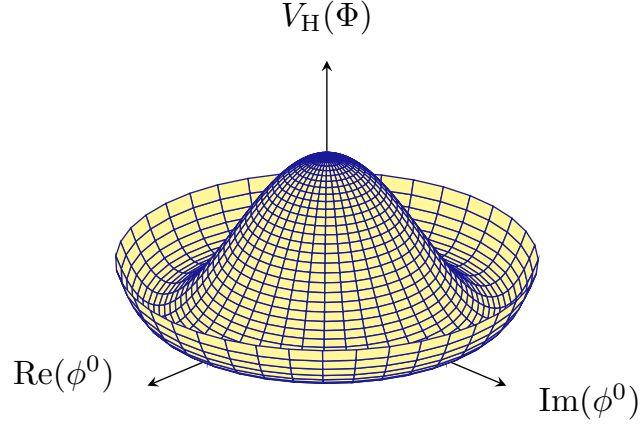
图 1: Higgs 场势能示意图。这里压缩掉 $\mathrm{Re}(\phi^+)$ 和 $\mathrm{Im}(\phi^+)$ 两个维度。

4.1 Brout-Englert-Higgs 机制

由于左手费米子和右手费米子参与不同的 $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 规范相互作用，耦合左右手费米子场的质量项会破坏规范对称性。另一方面，规范对称性也禁止规范玻色子具有质量。为了让费米子和弱规范玻色子获得质量，需要引入 BEH 机制，使 $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ 规范对称性自发破缺。因此，引入 Higgs 标量场

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}, \tag{40}$$

其中 ϕ^+ 和 ϕ^0 都是复标量场。Φ 是 $\mathrm{SU}(2)_\mathrm{L}$ 二重态，弱超荷是

$$Y_H = \frac{1}{2}. \tag{41}$$

Higgs 场的协变动能项和势能项为

$$\mathcal{L}_\mathrm{H} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V_\mathrm{H}(\Phi), \quad V_\mathrm{H}(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \tag{42}$$

其中协变导数为

$$D_\mu \Phi = (\partial_\mu + \mathrm{i}g' B_\mu Y_H + \mathrm{i}g W_\mu^a \tau^a)\Phi. \tag{43}$$

当 $\lambda > 0$ 且 $\mu^2 > 0$ 时，Higgs 场势能 $V_\mathrm{H}(\Phi)$ 呈现出图 1 所示墨西哥草帽状的形式，势能最小值位于方程

$$\Phi^\dagger \Phi = [\mathrm{Re}(\phi^+)]^2 + [\mathrm{Im}(\phi^+)]^2 + [\mathrm{Re}(\phi^0)]^2 + [\mathrm{Im}(\phi^0)]^2 = \frac{v^2}{2} \tag{44}$$

对应的 3 维球面上，其中 $v \equiv \sqrt{\mu^2/\lambda}$，满足

$$\mu^2 = \lambda v^2. \tag{45}$$

Higgs 场的真空期待值位于这个 3 维球面上的某一点，不失一般性，可将它取为

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{46}$$

其它真空期待值可通过 $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ 整体变换

$$\langle\Phi\rangle \to \exp(\mathrm{i}\alpha^a\tau^a)\exp(\mathrm{i}\alpha^Y Y_H)\langle\Phi\rangle \tag{47}$$

得到，因为 $\langle\Phi^\dagger\Phi\rangle$ 在这样的变换下保持不变。若 $\alpha^1 = \alpha^2 = 0$ 且 $\alpha^3 = \alpha^Y$，则

$$\exp(\mathrm{i}\alpha^a\tau^a)\exp(\mathrm{i}\alpha^Y Y_H) = \exp[\mathrm{i}\alpha^3(\sigma^3 + \mathbf{1})/2] = \exp\left[\mathrm{i}\alpha^3\begin{pmatrix} 1 & \\ & 0 \end{pmatrix}\right] = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\alpha^3} & \\ & 1 \end{pmatrix}, \tag{48}$$

而 $\langle\Phi\rangle$ 在此变换下不变。因此，有 1 个方向的规范对称性没有受到破坏，只有 3 个方向的规范对称性发生自发破缺。根据 Goldstone 定理，破缺后存在 3 个无质量的 Nambu-Goldstone 玻色子。最终，有 3 个规范玻色子结合 Nambu-Goldstone 玻色子，通过 BEH 机制获得质量。

以 $\langle\Phi\rangle$ 为基础，将 Higgs 场参数化为

$$\Phi(x) = \exp\left[-\mathrm{i}\,\frac{\chi^a(x)}{v}\,\tau^a\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{49}$$

其中 $\chi^a(x)$ 和 $H(x)$ 都是实标量场。$\exp[-\mathrm{i}\chi^a(x)\tau^a/v]$ 因子能够通过 $\mathrm{SU(2)_L}$ 规范变换消去，因而可将 $\Phi(x)$ 直接取为

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \Phi^\dagger\Phi = \frac{1}{2}(v + H)^2. \tag{50}$$

此时 Higgs 场只剩下一个物理自由度 $H(x)$，对应于 Higgs 玻色子，这种取法称为**幺正规范**。

在幺正规范下，势能项化为

$$
\begin{aligned}
-V_\mathrm{H}(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 &= \frac{\mu^2}{2}(v + H)^2 - \frac{\lambda}{4}(v + H)^4 \\
&= \frac{\mu^2}{2}(v^2 + H^2 + 2vH) - \frac{\lambda}{4}(v^4 + 4v^2H^2 + H^4 + 4v^3H + 2v^2H^2 + 4vH^3) \\
&= \frac{1}{4}\mu^2 v^2 + \frac{1}{4}(\mu^2 - \lambda v^2)v^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)H^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4 \\
&= \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{1}{2}\frac{m_H^2}{v}H^3 - \frac{1}{8}\frac{m_H^2}{v^2}H^4,
\end{aligned} \tag{51}
$$

其中 Higgs 玻色子的质量为

$$m_H \equiv \sqrt{2}\mu, \quad m_H^2 = 2\mu^2 = 2\lambda v^2. \tag{52}$$

由于

$$g'B_\mu Y_H + gW_\mu^a \tau^a = \frac{1}{2} \begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g(W_\mu^1 + \mathrm{i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix}, \tag{53}$$

Higgs 场真空期待值 v 对协变导数 $D_\mu\Phi$ 的贡献为

$$\begin{aligned} D_\mu\Phi &\supset \mathrm{i}(g'B_\mu Y_H + gW_\mu^a \tau^a)\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= \frac{\mathrm{i}v}{2\sqrt{2}} \begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g(W_\mu^1 + \mathrm{i}W_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \supset \frac{\mathrm{i}v}{2\sqrt{2}} \begin{pmatrix} g(W_\mu^1 - \mathrm{i}W_\mu^2) \\ g'B_\mu - gW_\mu^3 \end{pmatrix}, \end{aligned} \tag{54}$$

故协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$ 中正比于 v^2 的项是

$$(D^\mu\Phi)^\dagger(D_\mu\Phi) \supset \frac{v^2}{8}[g^2|W_\mu^1 - \mathrm{i}W_\mu^2|^2 + (g'B_\mu - gW_\mu^3)^2] = \frac{v^2}{8}(g^2 W^{a,\mu}W_\mu^a + g'^2 B^\mu B_\mu - 2gg'B^\mu W_\mu^3). \tag{55}$$

这些项是规范玻色子的质量项，重新表达为

$$\mathcal{L}_{\mathrm{GBM}} = \frac{1}{2}m_W^2(W^{1,\mu}W_\mu^1 + W^{2,\mu}W_\mu^2) + \frac{1}{2}\begin{pmatrix} W^{3,\mu} & B^\mu \end{pmatrix} M_{W^3 B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{56}$$

其中

$$m_W \equiv \frac{1}{2}gv \tag{57}$$

是 W_μ^1 和 W_μ^2 获得的质量，而 $W^{3\mu}$ 和 B^μ 的质量平方矩阵为

$$M_{W^3 B}^2 \equiv \frac{v^2}{4}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \tag{58}$$

为了使 $M_{W^3 B}^2$ 矩阵对角化，定义

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{59}$$

其中

$$s_{\mathrm{W}} \equiv \sin\theta_{\mathrm{W}} = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_{\mathrm{W}} \equiv \cos\theta_{\mathrm{W}} = \frac{g}{\sqrt{g^2 + g'^2}}, \tag{60}$$

θ_{W} 称为 Weinberg 角，也称为弱混合角。从后面的讨论可以看出 $A_\mu(x)$ 就是电磁场，对应于光子。$Z_\mu(x)$ 对应于矢量玻色子 Z。反过来，有

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \tag{61}$$

由

$$M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} = \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}}^2 & -s_{\mathrm{W}}c_{\mathrm{W}} \\ -s_{\mathrm{W}}c_{\mathrm{W}} & s_{\mathrm{W}}^2 \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix}$$
$$= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}} & 0 \\ -s_{\mathrm{W}} & 0 \end{pmatrix} \tag{62}$$

得

$$\begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} = \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & 0 \\ -s_{\mathrm{W}} & 0 \end{pmatrix}$$
$$= \frac{(g^2 + g'^2)v^2}{4} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \tag{63}$$

因此

$$\frac{1}{2} \begin{pmatrix} W^{3,\mu} & B^\mu \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \frac{1}{2} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} M_{W^3B}^2 \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$
$$= \frac{(g^2 + g'^2)v^2}{8} \begin{pmatrix} Z^\mu & A^\mu \end{pmatrix} \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2} m_Z^2 Z^\mu Z_\mu, \quad (64)$$

其中

$$m_Z \equiv \frac{1}{2}\sqrt{g^2 + g'^2}\, v = \frac{gv}{2c_{\mathrm{W}}} = \frac{m_W}{c_{\mathrm{W}}} \tag{65}$$

是 Z 玻色子的质量，而光子没有质量。另一方面，用质量相同的实矢量场 W_μ^1 和 W_μ^2 线性组合出复矢量场

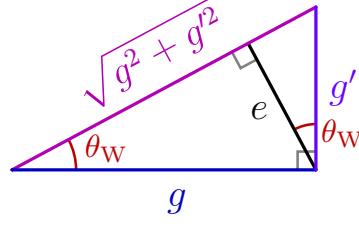$$W_\mu^+ \equiv \frac{1}{\sqrt{2}}(W_\mu^1 - \mathrm{i}W_\mu^2), \tag{66}$$

它的厄米共轭为

$$W_\mu^- \equiv (W_\mu^+)^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 + \mathrm{i}W_\mu^2), \tag{67}$$

则

$$W_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad W_\mu^2 = \frac{\mathrm{i}}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \tag{68}$$

从而

$$\frac{1}{2}(W^{1,\mu}W_\mu^1 + W^{2,\mu}W_\mu^2) = \frac{1}{4}[(W^{+,\mu} + W^{-,\mu})(W_\mu^+ + W_\mu^-) - (W^{+,\mu} - W^{-,\mu})(W_\mu^+ - W_\mu^-)]$$
$$= W^{+,\mu}W_\mu^-, \tag{69}$$

图 2: g、g'、e 和 θ_{W} 的关系。

([56](#)) 式化为

$$\mathcal{L}_{\mathrm{GBM}} = m_W^2 W^{+,\mu} W_\mu^- + \frac{1}{2} m_Z^2 Z^\mu Z_\mu. \tag{70}$$

复矢量场 $W_\mu^\pm$ 描述一对正反矢量玻色子 $W^\pm$，质量为 m_W。可见，BEH 机制使传递弱相互作用的规范玻色子 $W^\pm$ 和 Z 获得了质量。

接下来用质量本征态 $W_\mu^\pm$ 和 Z_μ 表达协变动能项 $(D^\mu\Phi)^\dagger(D_\mu\Phi)$。注意到

$$A_\mu = s_{\mathrm{W}} W_\mu^3 + c_{\mathrm{W}} B_\mu, \quad Z_\mu = c_{\mathrm{W}} W_\mu^3 - s_{\mathrm{W}} B_\mu, \tag{71}$$

$$B_\mu = c_{\mathrm{W}} A_\mu - s_{\mathrm{W}} Z_\mu, \quad W_\mu^3 = s_{\mathrm{W}} A_\mu + c_{\mathrm{W}} Z_\mu, \tag{72}$$

有

$$g' B_\mu + g W_\mu^3 = g'(c_{\mathrm{W}} A_\mu - s_{\mathrm{W}} Z_\mu) + g(s_{\mathrm{W}} A_\mu + c_{\mathrm{W}} Z_\mu) = \frac{2gg'}{\sqrt{g^2 + g'^2}} A_\mu + \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}} Z_\mu$$

$$= 2e A_\mu + \frac{g}{c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2) Z_\mu, \tag{73}$$

其中

$$e \equiv \frac{gg'}{\sqrt{g^2 + g'^2}} = g s_{\mathrm{W}} = g' c_{\mathrm{W}}. \tag{74}$$

后面的讨论将表明 e 就是单位电荷量。g、g'、e 和 θ_{W} 的关系如图 [2](#) 所示。再利用

$$g' B_\mu - g W_\mu^3 = g'(c_{\mathrm{W}} A_\mu - s_{\mathrm{W}} Z_\mu) - g(s_{\mathrm{W}} A_\mu + c_{\mathrm{W}} Z_\mu) = -\left(\frac{g s_{\mathrm{W}}^2}{c_{\mathrm{W}}} + g c_{\mathrm{W}}\right) Z_\mu = -\frac{g}{c_{\mathrm{W}}} Z_\mu, \tag{75}$$

得

$$g' B_\mu Y_H + g W_\mu^a \tau^a = \frac{1}{2}\begin{pmatrix} g' B_\mu + g W_\mu^3 & g(W_\mu^1 - \mathrm{i} W_\mu^2) \\ g(W_\mu^1 + \mathrm{i} W_\mu^2) & g' B_\mu - g W_\mu^3 \end{pmatrix}$$

$$= \begin{pmatrix} e A_\mu + \dfrac{g}{2c_{\mathrm{W}}}(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2) Z_\mu & \dfrac{g}{\sqrt{2}} W_\mu^+ \\ \dfrac{g}{\sqrt{2}} W_\mu^- & -\dfrac{g}{2c_{\mathrm{W}}} Z_\mu \end{pmatrix}. \tag{76}$$

在幺正规范下，

$$(D^\mu\Phi)^\dagger(D_\mu\Phi) = \left|\left(\begin{matrix}\partial_\mu + \mathrm{i}eA_\mu + \dfrac{\mathrm{i}g}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & \dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^+ \\[2mm] \dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^- & \partial_\mu - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu\end{matrix}\right)\dfrac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v+H\end{pmatrix}\right|^2$$

$$= \frac{1}{2}\left(-\frac{\mathrm{i}g}{\sqrt{2}}W^{-,\mu}(v+H) \quad \partial^\mu H + \frac{\mathrm{i}g}{2c_\mathrm{W}}Z^\mu(v+H)\right)\begin{pmatrix}\dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^+(v+H) \\[2mm] \partial_\mu H - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu(v+H)\end{pmatrix}$$

$$= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + (v+H)^2\left(\frac{g^2}{4}W_\mu^+W^{-,\mu} + \frac{g^2}{8c_\mathrm{W}^2}Z_\mu Z^\mu\right)$$

$$= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + m_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu$$

$$\quad + gm_W HW_\mu^+ W^{-,\mu} + \frac{gm_Z}{2c_\mathrm{W}}HZ_\mu Z^\mu + \frac{g^2}{4}H^2 W_\mu^+ W^{-,\mu} + \frac{g^2}{8c_\mathrm{W}^2}H^2 Z_\mu Z^\mu. \tag{77}$$

除了 $W^\pm$ 和 Z 玻色子的质量项之外，还出现了 Higgs 玻色子 H 与 $W^\pm$、Z 的三线性和四线性耦合项。

Higgs 场 $\Phi(x)$ 的弱超荷为 $+1/2$。引入 $\Phi(x)$ 的共轭态

$$\tilde{\Phi}(x) = \mathrm{i}\sigma^2\Phi^*(x) = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix}\phi^-(x) \\ \phi^{0*}(x)\end{pmatrix} = \begin{pmatrix}\phi^{0*}(x) \\ -\phi^-(x)\end{pmatrix}, \tag{78}$$

其中 $\phi^- \equiv (\phi^+)^*$，则 $\tilde{\Phi}(x)$ 是弱超荷为 $-1/2$ 的 $\mathrm{SU(2)_L}$ 二重态。在幺正规范下，$\tilde{\Phi}(x)$ 化为

$$\tilde{\Phi}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\begin{pmatrix}0 \\ v+H(x)\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}v+H(x) \\ 0\end{pmatrix}. \tag{79}$$

用 $\Phi(x)$、$\tilde{\Phi}(x)$ 和费米子场组成满足 $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ 规范对称性的 Yukawa 相互作用拉氏量

$$\mathcal{L}_\mathrm{Y} = -\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - \tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{H.c.}, \tag{80}$$

其中 H.c. 表示厄米共轭，Yukawa 耦合常数 $\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 联系着不同代的夸克场，而 y_{ℓ_i} 只联系同一代的轻子场。在幺正规范下，利用

$$\bar{Q}_{i\mathrm{L}}\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{u}'_{i\mathrm{L}} & \bar{d}'_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}0 \\ v+H\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{d}'_{i\mathrm{L}}, \tag{81}$$

$$\bar{Q}_{i\mathrm{L}}\tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}\bar{u}'_{i\mathrm{L}} & \bar{d}'_{i\mathrm{L}}\end{pmatrix}\begin{pmatrix}v+H \\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{u}'_{i\mathrm{L}}, \tag{82}$$

$$\bar{L}_{i\mathrm{L}}\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_{i\mathrm{L}} & \bar{\ell}_{i\mathrm{L}} \end{pmatrix} \begin{pmatrix} 0 \\ v+H \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\bar{\ell}_{i\mathrm{L}}, \tag{83}$$

推出

$$\mathcal{L}_{\mathrm{Y}} = -\frac{1}{\sqrt{2}}(v+H)(\tilde{y}_{d,ij}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}} + \tilde{y}_{u,ij}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}} + y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + \mathrm{H.c.}). \tag{84}$$

$\tilde{y}_{d,ij}$ 和 $\tilde{y}_{u,ij}$ 可看作 3×3 矩阵 $\tilde{y}_d$ 和 $\tilde{y}_u$ 的元素。$\tilde{y}_d\tilde{y}_d^\dagger$ 和 $\tilde{y}_u\tilde{y}_u^\dagger$ 是厄米矩阵，必定可以分别通过幺正矩阵 U_d 和 U_u 对角化成 y_{D}^2 和 y_{U}^2 两个对角元为实数的对角矩阵，满足 $U_d^\dagger\tilde{y}_d\tilde{y}_d^\dagger U_d = y_{\mathrm{D}}^2$ 和 $U_u^\dagger\tilde{y}_u\tilde{y}_u^\dagger U_u = y_{\mathrm{U}}^2$，即

$$\tilde{y}_d\tilde{y}_d^\dagger = U_d y_{\mathrm{D}}^2 U_d^\dagger, \quad \tilde{y}_u\tilde{y}_u^\dagger = U_u y_{\mathrm{U}}^2 U_u^\dagger. \tag{85}$$

符合这两条式子的 $\tilde{y}_d$ 和 $\tilde{y}_u$ 可以表达为

$$\tilde{y}_d = U_d y_{\mathrm{D}} K_d^\dagger, \quad \tilde{y}_u = U_u y_{\mathrm{U}} K_u^\dagger, \tag{86}$$

其中对角矩阵 y_{D} 和 y_{U} 满足 $y_{\mathrm{D}}y_{\mathrm{D}} = y_{\mathrm{D}}^2$ 和 $y_{\mathrm{U}}y_{\mathrm{U}} = y_{\mathrm{U}}^2$，而 $K_d^\dagger$ 和 $K_u^\dagger$ 是两个幺正矩阵。

将 y_{D} 和 y_{U} 表示成

$$y_{\mathrm{D}} = \begin{pmatrix} y_{d_1} & & \\ & y_{d_2} & \\ & & y_{d_3} \end{pmatrix} = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix}, \quad y_{\mathrm{U}} = \begin{pmatrix} y_{u_1} & & \\ & y_{u_2} & \\ & & y_{u_3} \end{pmatrix} = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}. \tag{87}$$

通过幺正变换定义

$$d_{i\mathrm{L}} \equiv (U_d^\dagger)_{ij}d'_{j\mathrm{L}}, \quad d_{i\mathrm{R}} \equiv (K_d^\dagger)_{ij}d'_{j\mathrm{R}}, \quad u_{i\mathrm{L}} \equiv (U_u^\dagger)_{ij}u'_{j\mathrm{L}}, \quad u_{i\mathrm{R}} \equiv (K_u^\dagger)_{ij}u'_{j\mathrm{R}}, \tag{88}$$

则 $\bar{d}_{i\mathrm{L}} = \bar{d}'_{j\mathrm{L}}U_{d,ji}$，$\bar{u}_{i\mathrm{L}} = \bar{u}'_{j\mathrm{L}}U_{u,ji}$，从而

$$\tilde{y}_{d,ij}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}(U_d y_d K_d^\dagger)_{ij}d'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}U_{d,ik}y_{d_k}(K_d^\dagger)_{kj}d'_{j\mathrm{R}} = y_{d_k}\bar{d}_{k\mathrm{L}}d_{k\mathrm{R}} = y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}, \tag{89}$$

$$\tilde{y}_{u,ij}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}}(U_u y_u K_u^\dagger)_{ij}u'_{j\mathrm{R}} = y_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}, \tag{90}$$

故

$$\begin{aligned} \mathcal{L}_{\mathrm{Y}} &= -\frac{1}{\sqrt{2}}(v+H)(y_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} + y_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} + y_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} + \mathrm{H.c.}) \\ &= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i, \end{aligned} \tag{91}$$
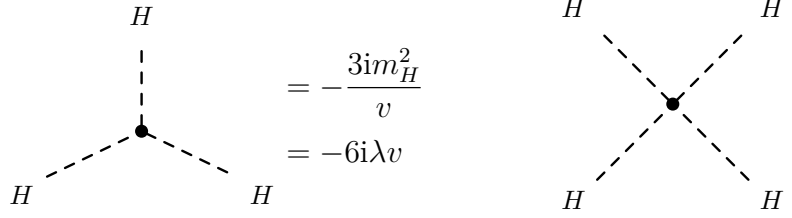
其中前三项是费米子质量项，后三项是 Higgs 玻色子与费米子的 Yukawa 耦合项。于是，三代夸克和带电轻子获得了质量

$$m_{d_i} \equiv \frac{1}{\sqrt{2}}y_{d_i}v, \quad m_{u_i} \equiv \frac{1}{\sqrt{2}}y_{u_i}v, \quad m_{\ell_i} \equiv \frac{1}{\sqrt{2}}y_{\ell_i}v. \tag{92}$$

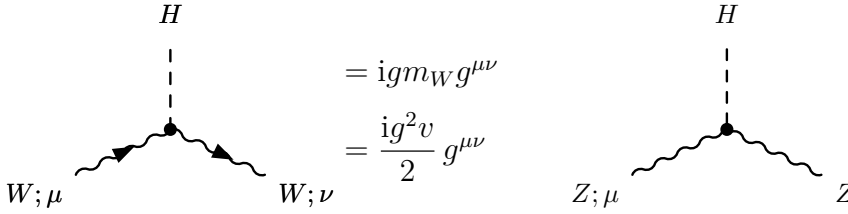d'_{iL}、d'_{iR}、u'_{iL} 和 u'_{iR} 称为规范本征态，d_{iL}、d_{iR}、u_{iL} 和 u_{iR} 称为质量本征态。
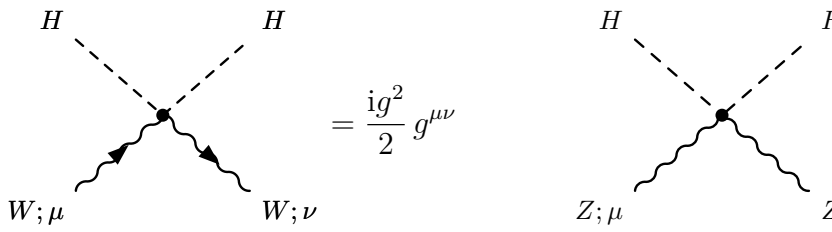
下面给出幺正规范下 Higgs 场的顶点 Feynman 规则。

Higgs 玻色子自耦合：

$$= -\frac{3\mathrm{i}m_H^2}{v}$$
$$= -6\mathrm{i}\lambda v$$

$$= -\frac{3\mathrm{i}m_H^2}{v^2}$$
$$= -6\mathrm{i}\lambda \tag{93}$$

Higgs 玻色子与电弱规范玻色子的耦合：
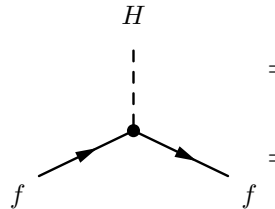
$$= \mathrm{i}gm_W g^{\mu\nu}$$
$$= \frac{\mathrm{i}g^2 v}{2} g^{\mu\nu}$$

$$= \frac{\mathrm{i}gm_Z}{c_W} g^{\mu\nu}$$
$$= \frac{\mathrm{i}g^2 v}{2c_W^2} g^{\mu\nu} \tag{94}$$

$$= \frac{\mathrm{i}g^2}{2} g^{\mu\nu}$$

$$= \frac{\mathrm{i}g^2}{2c_W^2} g^{\mu\nu} \tag{95}$$

Higgs 玻色子与费米子 $f = d_i, u_i, \ell_i$ 的耦合：

$$= -\frac{\mathrm{i}m_f}{v}$$
$$= -\frac{\mathrm{i}y_f}{\sqrt{2}} \tag{96}$$

4.2 费米子电弱规范相互作用

(88) 式意味着

$$d'_{iL} = U_{d,ij}d_{jL}, \quad d'_{iR} = K_{d,ij}d_{jR}, \quad u'_{iL} = U_{u,ij}u_{jL}, \quad u'_{iR} = K_{u,ij}u_{jR}, \tag{97}$$

从而

$$\bar{d}'_{iL}\gamma^\mu d'_{iL} = \bar{d}_{jL}(U_d^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{kL} = \bar{d}_{jL}\delta_{jk}\gamma^\mu d_{kL} = \bar{d}_{iL}\gamma^\mu d_{iL} \tag{98}$$

同理有

$$\bar{u}'_{iL}\gamma^\mu u'_{iL} = \bar{u}_{iL}\gamma^\mu u_{iL}, \quad \bar{d}'_{iR}\gamma^\mu d'_{iR} = \bar{d}_{iR}\gamma^\mu d_{iR}, \quad \bar{u}'_{iR}\gamma^\mu u'_{iR} = \bar{u}_{iR}\gamma^\mu u_{iR}. \tag{99}$$

另一方面，

$$\bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} = \bar{u}_{j\mathrm{L}}(U_u^\dagger)_{ji}\gamma^\mu U_{d,ik}d_{k\mathrm{L}} = \bar{u}_{i\mathrm{L}}\gamma^\mu V_{ij}d_{j\mathrm{L}} \tag{100}$$

$$\bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} = \bar{d}_{j\mathrm{L}}(U_d^\dagger)_{ji}\gamma^\mu U_{u,ik}u_{k\mathrm{L}} = \bar{d}_{j\mathrm{L}}V_{ji}^\dagger\gamma^\mu u_{i\mathrm{L}} \tag{101}$$

其中

$$V \equiv U_u^\dagger U_d \tag{102}$$

称为 Cabibbo-Kobayashi-Maskawa (CKM) 矩阵，其厄米共轭矩阵为 $V^\dagger = U_d^\dagger U_u$。

SU(2)$_\mathrm{L}$ × U(1)$_\mathrm{Y}$ 规范不变的费米子协变动能项为

$$\mathcal{L}_{\mathrm{EWF}} = \bar{Q}_{i\mathrm{L}}\mathrm{i}\slashed{D}Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}}\mathrm{i}\slashed{D}u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}}\mathrm{i}\slashed{D}d'_{i\mathrm{R}} + \bar{L}_{i\mathrm{L}}\mathrm{i}\slashed{D}L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}}\mathrm{i}\slashed{D}\ell_{i\mathrm{R}}. \tag{103}$$

根据 Q 的定义 (39)，有

$$\begin{aligned}
g'YB_\mu + gT^3W_\mu^3 &= g'Y(c_\mathrm{W}A_\mu - s_\mathrm{W}Z_\mu) + gT^3(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu) \\
&= e(Y+T^3)A_\mu + \left(gc_\mathrm{W}T^3 - \frac{gs_\mathrm{W}}{c_\mathrm{W}}s_\mathrm{W}Y\right)Z_\mu = QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3c_\mathrm{W}^2 - Ys_\mathrm{W}^2)Z_\mu \\
&= QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu, \tag{104}
\end{aligned}$$

故

$$\begin{aligned}
D_\mu Q_{i\mathrm{L}} &= (\partial_\mu + \mathrm{i}g'B_\mu Y + \mathrm{i}gW_\mu^a\tau^a)Q_{i\mathrm{L}} \\
&= \partial_\mu Q_{i\mathrm{L}} + \mathrm{i}\begin{pmatrix} g'YB_\mu + gT^3W_\mu^3 & \frac{g}{2}\left(W_\mu^1 - \mathrm{i}W_\mu^2\right) \\ \frac{g}{2}\left(W_\mu^1 + \mathrm{i}W_\mu^2\right) & g'YB_\mu + gT^3W_\mu^3 \end{pmatrix}Q_{i\mathrm{L}} \\
&= \partial_\mu Q_{i\mathrm{L}} + \mathrm{i}\begin{pmatrix} QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu & \frac{g}{\sqrt{2}}W_\mu^+ \\ \frac{g}{\sqrt{2}}W_\mu^- & QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu \end{pmatrix}Q_{i\mathrm{L}} \\
&= \partial_\mu Q_{i\mathrm{L}} + \mathrm{i}\begin{pmatrix} \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]u'_{i\mathrm{L}} + \frac{g}{\sqrt{2}}W_\mu^+ d'_{i\mathrm{L}} \\ \frac{g}{\sqrt{2}}W_\mu^- u'_{i\mathrm{L}} + \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)\right]d'_{i\mathrm{L}} \end{pmatrix}. \tag{105}
\end{aligned}$$

于是

$$\begin{aligned}
\bar{Q}_{i\mathrm{L}}\mathrm{i}\slashed{D}Q_{i\mathrm{L}} &\supset -\left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{u}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} - \left[QeA_\mu + \frac{g}{c_\mathrm{W}}(T^3 - Qs_\mathrm{W}^2)Z_\mu\right]\bar{d}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} \\
&\quad -\frac{g}{\sqrt{2}}W_\mu^+ \bar{u}'_{i\mathrm{L}}\gamma^\mu d'_{i\mathrm{L}} - \frac{g}{\sqrt{2}}W_\mu^- \bar{d}'_{i\mathrm{L}}\gamma^\mu u'_{i\mathrm{L}} \\
&= -\left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{u}_i\gamma^\mu\frac{1-\gamma^5}{2}u_i - \left(QeA_\mu + \frac{g}{c_\mathrm{W}}g_\mathrm{L}Z_\mu\right)\bar{d}_i\gamma^\mu\frac{1-\gamma^5}{2}d_i
\end{aligned}$$

$$-\frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu P_{\mathrm{L}} V_{ij} d_j - \frac{g}{\sqrt{2}} W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_{\mathrm{L}} u_i, \tag{106}$$

其中左手耦合系数

$$g_{\mathrm{L}} \equiv T^3 - Q s_{\mathrm{W}}^2. \tag{107}$$

另一方面,

$$\begin{aligned}
D_\mu d'_{i\mathrm{R}} &= (\partial_\mu + \mathrm{i} g' B_\mu Y) d'_{i\mathrm{R}} = \partial_\mu d'_{i\mathrm{R}} + \mathrm{i} g' Q (c_{\mathrm{W}} A_\mu - s_{\mathrm{W}} Z_\mu) d'_{i\mathrm{R}} \\
&= \partial_\mu d'_{i\mathrm{R}} + \mathrm{i} Q e A_\mu d'_{i\mathrm{R}} - \frac{\mathrm{i} g}{c_{\mathrm{W}}} Q s_{\mathrm{W}}^2 Z_\mu d'_{i\mathrm{R}},
\end{aligned} \tag{108}$$

则

$$\begin{aligned}
&\bar{u}'_{i\mathrm{R}} \mathrm{i} \slashed{D} u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} \mathrm{i} \slashed{D} d'_{i\mathrm{R}} \\
&\supset -\left(Q e A_\mu - \frac{g}{c_{\mathrm{W}}} Q s_{\mathrm{W}}^2 Z_\mu \right) \bar{u}'_{i\mathrm{R}} \gamma^\mu u'_{i\mathrm{R}} - \left(Q e A_\mu - \frac{g}{c_{\mathrm{W}}} Q s_{\mathrm{W}}^2 Z_\mu \right) \bar{d}'_{i\mathrm{R}} \gamma^\mu d'_{i\mathrm{R}} \\
&= -\left(Q e A_\mu + \frac{g}{c_{\mathrm{W}}} g_{\mathrm{R}} Z_\mu \right) \bar{u}_i \gamma^\mu \frac{1 + \gamma^5}{2} u_i - \left(Q e A_\mu + \frac{g}{c_{\mathrm{W}}} g_{\mathrm{R}} Z_\mu \right) \bar{d}_i \gamma^\mu \frac{1 + \gamma^5}{2} d_i,
\end{aligned} \tag{109}$$

其中右手耦合系数

$$g_{\mathrm{R}} \equiv -Q s_{\mathrm{W}}^2. \tag{110}$$

引入矢量流和轴矢量流耦合系数

$$g_{\mathrm{V}} \equiv g_{\mathrm{L}} + g_{\mathrm{R}} = T^3 - 2 Q s_{\mathrm{W}}^2, \quad g_{\mathrm{A}} \equiv g_{\mathrm{L}} - g_{\mathrm{R}} = T^3, \tag{111}$$

得

$$\begin{aligned}
&\bar{Q}_{i\mathrm{L}} \mathrm{i} \slashed{D} Q_{i\mathrm{L}} + \bar{u}'_{i\mathrm{R}} \mathrm{i} \slashed{D} u'_{i\mathrm{R}} + \bar{d}'_{i\mathrm{R}} \mathrm{i} \slashed{D} d'_{i\mathrm{R}} \\
&\supset -Q e \bar{u}_i \gamma^\mu u_i A_\mu - Q e \bar{d}_i \gamma^\mu d_i A_\mu - \frac{g}{2 c_{\mathrm{W}}} \bar{u}_i \gamma^\mu (g_{\mathrm{V}} - g_{\mathrm{A}} \gamma^5) u_i Z_\mu - \frac{g}{2 c_{\mathrm{W}}} \bar{d}_i \gamma^\mu (g_{\mathrm{V}} - g_{\mathrm{A}} \gamma^5) d_i Z_\mu \\
&\quad - \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu P_{\mathrm{L}} V_{ij} d_j - \frac{g}{\sqrt{2}} W_\mu^- \bar{d}_j V_{ji}^\dagger \gamma^\mu P_{\mathrm{L}} u_i.
\end{aligned} \tag{112}$$

同理, 有

$$\begin{aligned}
\bar{L}_{i\mathrm{L}} \mathrm{i} \slashed{D} L_{i\mathrm{L}} + \bar{\ell}_{i\mathrm{R}} \mathrm{i} \slashed{D} \ell_{i\mathrm{R}} &\supset -Q e \bar{\ell}_i \gamma^\mu \ell_i A_\mu - \frac{g}{2 c_{\mathrm{W}}} \bar{\ell}_i \gamma^\mu (g_{\mathrm{V}} - g_{\mathrm{A}} \gamma^5) \ell_i Z_\mu - \frac{g}{2 c_{\mathrm{W}}} \bar{\nu}_i \gamma^\mu (g_{\mathrm{V}} - g_{\mathrm{A}} \gamma^5) \nu_i Z_\mu \\
&\quad - \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_i \gamma^\mu P_{\mathrm{L}} \ell_i - \frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_i \gamma^\mu P_{\mathrm{L}} \nu_i.
\end{aligned} \tag{113}$$

总结起来, 将费米子电弱规范相互作用写成流耦合的形式,

$$\mathcal{L}_{\mathrm{EWF}} \supset -A_\mu J_{\mathrm{EM}}^\mu - Z_\mu J_Z^\mu - W_\mu^+ J_W^{+,\mu} - W_\mu^- J_W^{-,\mu}, \tag{114}$$

其中 f 代表任意费米子场, 电磁流

$$J_{\text{EM}}^{\mu} \equiv \sum_f Q_f e \bar{f} \gamma^{\mu} f, \tag{115}$$

弱中性流

$$J_Z^{\mu} \equiv \frac{g}{2c_{\text{W}}} \sum_f \bar{f} \gamma^{\mu} (g_{\text{V}}^f - g_{\text{A}}^f \gamma^5) f = \frac{g}{c_{\text{W}}} \sum_f (g_{\text{L}}^f \bar{f}_{\text{L}} \gamma^{\mu} f_{\text{L}} + g_{\text{R}}^f \bar{f}_{\text{R}} \gamma^{\mu} f_{\text{R}}), \tag{116}$$

弱带电流

$$J_W^{+,\mu} \equiv \frac{g}{\sqrt{2}} (\bar{u}_{i\text{L}} \gamma^{\mu} V_{ij} d_{j\text{L}} + \bar{\nu}_{i\text{L}} \gamma^{\mu} \ell_{i\text{L}}), \quad J_W^{-,\mu} \equiv (J_W^{+\mu})^{\dagger} = \frac{g}{\sqrt{2}} (\bar{d}_{j\text{L}} V_{ji}^{\dagger} \gamma^{\mu} u_{i\text{L}} + \bar{\ell}_{i\text{L}} \gamma^{\mu} \nu_{i\text{L}}). \tag{117}$$

对于各种费米子, 相关的系数如下,

$$Q_{u_i} = \frac{2}{3}, \qquad Q_{d_i} = -\frac{1}{3}, \qquad Q_{\nu_i} = 0, \qquad Q_{\ell_i} = -1; \tag{118}$$

$$g_{\text{V}}^{u_i} = \frac{1}{2} - \frac{4}{3} s_{\text{W}}^2, \quad g_{\text{A}}^{u_i} = \frac{1}{2}; \qquad g_{\text{V}}^{d_i} = -\frac{1}{2} + \frac{2}{3} s_{\text{W}}^2, \quad g_{\text{A}}^{d_i} = -\frac{1}{2}; \tag{119}$$

$$g_{\text{V}}^{\nu_i} = \frac{1}{2}, \qquad g_{\text{A}}^{\nu_i} = \frac{1}{2}; \qquad g_{\text{V}}^{\ell_i} = -\frac{1}{2} + 2s_{\text{W}}^2, \quad g_{\text{A}}^{\ell_i} = -\frac{1}{2}; \tag{120}$$
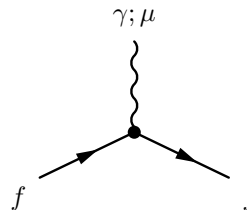
$$g_{\text{L}}^{u_i} = \frac{1}{2} - \frac{2}{3} s_{\text{W}}^2, \quad g_{\text{R}}^{u_i} = -\frac{2}{3} s_{\text{W}}^2; \quad g_{\text{L}}^{d_i} = -\frac{1}{2} + \frac{1}{3} s_{\text{W}}^2, \quad g_{\text{R}}^{d_i} = \frac{1}{3} s_{\text{W}}^2; \tag{121}$$

$$g_{\text{L}}^{\nu_i} = \frac{1}{2}, \qquad g_{\text{R}}^{\nu_i} = 0; \qquad g_{\text{L}}^{\ell_i} = -\frac{1}{2} + s_{\text{W}}^2, \quad g_{\text{R}}^{\ell_i} = s_{\text{W}}^2. \tag{122}$$

可以看到, 电磁流耦合与 QED 耦合完全相同, 由此辩认出 A_{μ} 是电磁场, e 是单位电荷量, 由 $Q = T^3 + Y$ 定义的 Q 确实是电荷。为了保持电荷守恒, 指定复矢量场 $W_{\mu}^+(x)$ 携带 $Q = +1$ 的电荷, 从而 $W^{\pm}$ 玻色子的电荷为 ± 1。不同代夸克之间的相互作用只发生在弱带电流耦合中, 源自 CKM 矩阵 V 的非对角元。
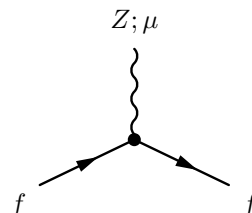
下面给出费米子电弱规范相互作用顶点的 Feynman 规则。

QED 耦合:

$$= -\text{i} Q_f e \gamma^{\mu} \tag{123}$$

费米子与 Z 玻色子的耦合:

$$= -\frac{\text{i}g}{2c_{\text{W}}} \gamma^{\mu} (g_{\text{V}}^f - g_{\text{A}}^f \gamma^5) \tag{124}$$

费米子与 $W^\pm$ 玻色子的耦合:

$$= -\frac{\mathrm{i}g}{\sqrt{2}}\, V_{ij}\gamma^\mu P_\mathrm{L} \qquad\qquad\qquad = -\frac{\mathrm{i}g}{\sqrt{2}}\, V_{ji}^\dagger\gamma^\mu P_\mathrm{L} \qquad (125)$$

$$= -\frac{\mathrm{i}g}{\sqrt{2}}\, \gamma^\mu P_\mathrm{L} \qquad\qquad\qquad\qquad = -\frac{\mathrm{i}g}{\sqrt{2}}\, \gamma^\mu P_\mathrm{L} \qquad (126)$$

4.3 电弱规范场的自相互作用

电弱规范场自相互作用的拉氏量是

$$\mathcal{L}_{\mathrm{EWG}} = -\frac{1}{4}\, W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}\, B_{\mu\nu}B^{\mu\nu}, \qquad (127)$$

其中

$$W_{\mu\nu}^a \equiv \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\varepsilon^{abc}W_\mu^b W_\nu^c, \quad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu. \qquad (128)$$

利用 (68) 和 (72) 式，推出

$$
\begin{aligned}
W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2 &= \frac{\mathrm{i}}{\sqrt{2}}[(W_\mu^+ - W_\mu^-)(s_\mathrm{W}A_\nu + c_\mathrm{W}Z_\nu) - (s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)(W_\nu^+ - W_\nu^-)] \\
&= \frac{\mathrm{i}}{\sqrt{2}}[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
&\qquad -s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) - c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)], \qquad (129) \\
W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3 &= \frac{1}{\sqrt{2}}[(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)(W_\nu^+ + W_\nu^-) - (W_\mu^+ + W_\mu^-)(s_\mathrm{W}A_\nu + c_\mathrm{W}Z_\nu)] \\
&= -\frac{1}{\sqrt{2}}[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+) \\
&\qquad +s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)]. \qquad (130)
\end{aligned}
$$

从而

$$
\begin{aligned}
W_{\mu\nu}^1 &= \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 - g\varepsilon^{1bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 - g(W_\mu^2 W_\nu^3 - W_\mu^3 W_\nu^2) \\
&= \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) + \frac{1}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - g(W_\mu^2 W_\nu^3 - gW_\mu^3 W_\nu^2) \\
&= \frac{1}{\sqrt{2}}\{\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - \mathrm{i}g[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)]\}
\end{aligned}
$$

$$+ \frac{1}{\sqrt{2}} \{ \partial_\mu W_\nu^- - \partial_\nu W_\mu^- + \mathrm{i}g[s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)] \}$$

$$= \frac{1}{\sqrt{2}}(F_{\mu\nu}^+ + F_{\mu\nu}^-), \tag{131}$$

其中，

$$F_{\mu\nu}^+ \equiv \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - \mathrm{i}e(W_\mu^+ A_\nu - A_\mu W_\nu^+) - \mathrm{i}g c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+), \tag{132}$$

$$F_{\mu\nu}^- \equiv \partial_\mu W_\nu^- - \partial_\nu W_\mu^- + \mathrm{i}e(W_\mu^- A_\nu - A_\mu W_\nu^-) + \mathrm{i}g c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-). \tag{133}$$

另一方面，

$$\begin{aligned}
W_{\mu\nu}^2 &= \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g\varepsilon^{2bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\
&= \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - g(W_\mu^3 W_\nu^1 - W_\mu^1 W_\nu^3) \\
&= \frac{\mathrm{i}}{\sqrt{2}}\{ \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - \mathrm{i}g[s_\mathrm{W}(W_\mu^+ A_\nu - A_\mu W_\nu^+) + c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \} \\
&\quad - \frac{\mathrm{i}}{\sqrt{2}}\{ \partial_\mu W_\nu^- - \partial_\nu W_\mu^- + \mathrm{i}g[s_\mathrm{W}(W_\mu^- A_\nu - A_\mu W_\nu^-) + c_\mathrm{W}(W_\mu^- Z_\nu - Z_\mu W_\nu^-)] \} \\
&= \frac{\mathrm{i}}{\sqrt{2}}(F_{\mu\nu}^+ - F_{\mu\nu}^-). \tag{134}
\end{aligned}$$

因此

$$\begin{aligned}
&-\frac{1}{4}W_{\mu\nu}^1 W^{1,\mu\nu} - \frac{1}{4}W_{\mu\nu}^2 W^{2,\mu\nu} \\
&= -\frac{1}{8}(F_{\mu\nu}^+ + F_{\mu\nu}^-)(F^{+,\mu\nu} + F^{-,\mu\nu}) + \frac{1}{8}(F_{\mu\nu}^+ - F_{\mu\nu}^-)(F^{+,\mu\nu} - F^{-,\mu\nu}) = -\frac{1}{2}F_{\mu\nu}^+ F^{-,\mu\nu} \\
&= -\frac{1}{2}[\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ - \mathrm{i}e(W_\mu^+ A_\nu - A_\mu W_\nu^+) - \mathrm{i}g c_\mathrm{W}(W_\mu^+ Z_\nu - Z_\mu W_\nu^+)] \\
&\quad \times [\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu} + \mathrm{i}e(W^{-,\mu} A^\nu - A^\mu W^{-,\nu}) + \mathrm{i}g c_\mathrm{W}(W^{-,\mu} Z^\nu - Z^\mu W^{-,\nu})] \\
&= -(\partial_\mu W_\nu^+)(\partial^\mu W^{-,\nu}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{-,\mu}) \\
&\quad -\mathrm{i}e[(\partial_\mu W_\nu^+)W^{-,\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-,\nu}A^\mu - W_\mu^+(\partial^\mu W^{-,\nu})A_\nu + W_\nu^+(\partial^\mu W^{-,\nu})A_\mu] \\
&\quad -\mathrm{i}g c_\mathrm{W}[(\partial_\mu W_\nu^+)W^{-,\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-,\nu}Z^\mu - W_\mu^+(\partial^\mu W^{-,\nu})Z_\nu + W_\nu^+(\partial^\mu W^{-,\nu})Z_\mu] \\
&\quad +e^2(W_\mu^+ W^{-,\nu}A_\nu A^\mu - W_\mu^+ W^{-,\mu}A_\nu A^\nu) + g^2 c_\mathrm{W}^2(W_\mu^+ W^{-,\nu}Z_\nu Z^\mu - W_\mu^+ W^{-,\mu}Z_\nu Z^\nu) \\
&\quad +egc_\mathrm{W}(W_\mu^+ W^{-,\nu}A_\nu Z^\mu + W_\mu^+ W^{-,\nu}A^\mu Z_\nu - 2W_\mu^+ W^{-,\mu}A_\nu Z^\nu). \tag{135}
\end{aligned}$$

由

$$\begin{aligned}
W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1 &= \frac{\mathrm{i}}{2}(W_\mu^+ + W_\mu^-)(W_\nu^+ - W_\nu^-) - \frac{\mathrm{i}}{2}(W_\mu^+ - W_\mu^-)(W_\nu^+ + W_\nu^-) \\
&= -\mathrm{i}(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \tag{136}
\end{aligned}$$

得到

$$
\begin{aligned}
W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g\varepsilon^{3bc}W_\mu^b W_\nu^c = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
&= s_{\mathrm{W}}\partial_\mu A_\nu + c_{\mathrm{W}}\partial_\mu Z_\nu - s_{\mathrm{W}}\partial_\nu A_\mu + c_{\mathrm{W}}\partial_\nu Z_\mu - g(W_\mu^1 W_\nu^2 - W_\mu^2 W_\nu^1) \\
&= s_{\mathrm{W}}(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_{\mathrm{W}}(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + ig(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+), \\
B_{\mu\nu} &= \partial_\mu(c_{\mathrm{W}}A_\nu - s_{\mathrm{W}}Z_\nu) - \partial_\nu(c_{\mathrm{W}}A_\mu - s_{\mathrm{W}}Z_\mu) \\
&= c_{\mathrm{W}}(\partial_\mu A_\nu - \partial_\nu A_\mu) - s_{\mathrm{W}}(\partial_\mu Z_\nu - \partial_\nu Z_\mu).
\end{aligned}
$$

(137)
(138)
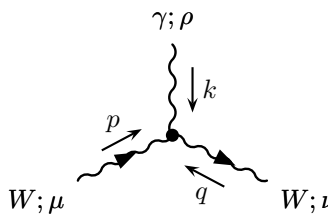
于是

$$
\begin{aligned}
&-\frac{1}{4}W_{\mu\nu}^3 W^{3,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
=& -\frac{1}{2}[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A_\nu)(\partial^\nu A^\mu)] - \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - (\partial_\mu Z_\nu)(\partial^\nu Z^\mu)] \\
&-ie[W^{+,\mu}W^{-,\nu}(\partial_\mu A_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu A_\nu)] - igc_{\mathrm{W}}[W^{+,\mu}W^{-,\nu}(\partial_\mu Z_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu Z_\nu)] \\
&+\frac{g^2}{2}(W_\mu^+ W^{+,\mu}W_\nu^- W^{-,\nu} - W_\mu^+ W^{+,\nu}W_\nu^- W^{-,\mu}).
\end{aligned}
$$

(139)
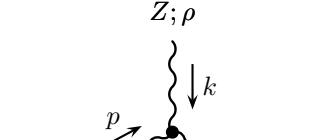
综合起来，有

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EWG}} =\ & \frac{1}{2}[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu)] + \frac{1}{2}[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu)] \\
& +(\partial_\mu W_\nu^+)(\partial^\nu W^{-,\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-,\nu}) + \frac{g^2}{2}(W_\mu^+ W^{+,\mu}W_\nu^- W^{-,\nu} - W_\mu^+ W^{+,\nu}W_\nu^- W^{-,\mu}) \\
& -ie[(\partial_\mu W_\nu^+)W^{-,\mu}A^\nu - (\partial_\mu W_\nu^+)W^{-,\nu}A^\mu - W^{+,\mu}(\partial_\mu W_\nu^-)A^\nu + W^{+,\nu}(\partial_\mu W_\nu^-)A^\mu \\
& \qquad +W^{+,\mu}W^{-,\nu}(\partial_\mu A_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu A_\nu)] \\
& -igc_{\mathrm{W}}[(\partial_\mu W_\nu^+)W^{-,\mu}Z^\nu - (\partial_\mu W_\nu^+)W^{-,\nu}Z^\mu - W^{+,\mu}(\partial_\mu W_\nu^-)Z^\nu + W^{+,\nu}(\partial_\mu W_\nu^-)Z^\mu \\
& \qquad + W^{+,\mu}W^{-,\nu}(\partial_\mu Z_\nu) - W^{+,\nu}W^{-,\mu}(\partial_\mu Z_\nu)] \\
& +e^2(W_\mu^+ W^{-,\nu}A_\nu A^\mu - W_\mu^+ W^{-,\mu}A_\nu A^\nu) + g^2 c_{\mathrm{W}}^2(W_\mu^+ W^{-,\nu}Z_\nu Z^\mu - W_\mu^+ W^{-,\mu}Z_\nu Z^\nu) \\
& +egc_{\mathrm{W}}(W_\mu^+ W^{-,\nu}A_\nu Z^\mu + W_\mu^+ W^{-,\nu}A^\mu Z_\nu - 2W_\mu^+ W^{-,\mu}A_\nu Z^\nu).
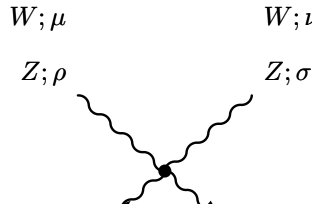\end{aligned}
$$

(140)

下面是电弱规范玻色子自耦合的 Feynman 规则：

$$
= ie[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu]
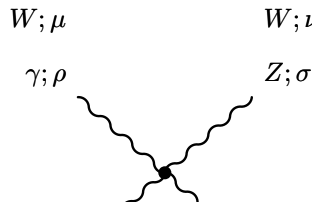$$

(141)

$$= \mathrm{i}gc_{\mathrm{W}}[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu + g^{\rho\mu}(k-p)^\nu] \tag{142}$$
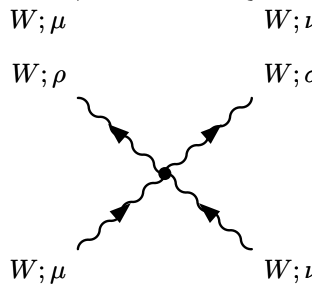
$$= \mathrm{i}e^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{143}$$

$$= \mathrm{i}g^2c_{\mathrm{W}}^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{144}$$

$$= \mathrm{i}egc_{\mathrm{W}}(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{145}$$

$$= -\mathrm{i}g^2(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - 2g^{\mu\nu}g^{\rho\sigma}) \tag{146}$$

5   R$_\xi$ 规范下电弱拉氏量和 Feynman 规则

在 **R$_\xi$** 规范下，将 Higgs 场参数化为

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \dfrac{1}{\sqrt{2}}[v + H(x) + \mathrm{i}\chi(x)] \end{pmatrix}, \tag{147}$$

其中 ϕ^+ 和 χ 是 Nambu-Goldstone 标量场。那么，$\tilde{\Phi}(x)$ 的形式是

$$\tilde{\Phi}(x) = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\sqrt{2}}[v + H(x) - \mathrm{i}\chi(x)] \\ -\phi^-(x) \end{pmatrix}. \tag{148}$$

利用

$$\Phi^\dagger\Phi = \frac{1}{2}(v^2 + H^2 + 2vH + \chi^2) + |\phi^+|^2, \tag{149}$$

$$(\Phi^\dagger\Phi)^2 = \frac{1}{4}(v^2 + H^2 + 2vH + \chi^2)^2 + |\phi^+|^4 + |\phi^+|^2(v^2 + H^2 + 2vH + \chi^2), \tag{150}$$

推出 Higgs 场势能项

$$\begin{aligned}
-V_{\mathrm{H}}(\Phi) &= \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \\
&= \frac{\mu^2}{2}(v^2 + H^2 + 2vH + \chi^2) + \mu^2|\phi^+|^2 - \frac{\lambda}{4}(v^2 + H^2 + 2vH + \chi^2)^2 - \lambda|\phi^+|^4 \\
&\quad -\lambda|\phi^+|^2(v^2 + H^2 + 2vH + \chi^2) \\
&= \frac{1}{2}\left(\mu^2 - \frac{\lambda}{2}v^2\right)v^2 + \frac{1}{2}(\mu^2 - 3\lambda v^2)H^2 + (\mu^2 - \lambda v^2)vH + \frac{1}{2}(\mu^2 - \lambda v^2)\chi^2 - \frac{\lambda}{4}H^4 - \frac{\lambda}{4}\chi^4 \\
&\quad -\lambda vH^3 - \frac{\lambda}{2}H^2\chi^2 - \lambda vH\chi^2 + (\mu^2 - \lambda v^2)|\phi^+|^2 - \lambda|\phi^+|^4 - \lambda|\phi^+|^2(H^2 + 2vH + \chi^2) \\
&= \frac{\lambda}{4}v^4 - \lambda v^2 H^2 - \frac{\lambda}{4}H^4 - \frac{\lambda}{4}\chi^4 - \lambda vH^3 - \frac{\lambda}{2}H^2\chi^2 - \lambda vH\chi^2 \\
&\quad -\lambda\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2) \\
&= \frac{1}{8}m_H^2 v^2 - \frac{1}{2}m_H^2 H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4 - \frac{m_H^2}{2v}H\chi^2 - \frac{m_H^2}{4v^2}H^2\chi^2 - \frac{m_H^2}{8v^2}\chi^4 \\
&\quad -\frac{m_H^2}{2v^2}\phi^+\phi^-(\phi^+\phi^- + H^2 + 2vH + \chi^2). \tag{151}
\end{aligned}$$

由 $V = U_u^\dagger U_d$ 得到 $V^\dagger = U_d^\dagger U_u$ 和 $U_d = U_u V$, $U_u = U_d V^\dagger$, 则

$$\tilde{y}_d = U_d y_{\mathrm{D}} K_d^\dagger = U_u V y_{\mathrm{D}} K_d^\dagger, \quad \tilde{y}_u = U_u y_{\mathrm{U}} K_u^\dagger = U_d V^\dagger y_{\mathrm{U}} K_u^\dagger, \tag{152}$$

故

$$\tilde{y}_{d,ij}\bar{u}'_{i\mathrm{L}}d'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}}(U_u V y_{\mathrm{D}} K_d^\dagger)_{ij}d'_{j\mathrm{R}} = \bar{u}'_{i\mathrm{L}}U_{u,ik}V_{kl}y_{d_l}(K_d^\dagger)_{lj}d'_{m\mathrm{R}} = y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}, \tag{153}$$

$$\tilde{y}_{u,ij}\bar{d}'_{i\mathrm{L}}u'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}(U_d V^\dagger y_{\mathrm{U}} K_u^\dagger)_{ij}u'_{j\mathrm{R}} = \bar{d}'_{i\mathrm{L}}U_{d,ik}V_{kl}^\dagger y_{u_l}(K_u^\dagger)_{lj}u'_{j\mathrm{R}} = y_{u_i}\bar{d}_{j\mathrm{L}}V_{ji}^\dagger u_{i\mathrm{R}}. \tag{154}$$

结合 (89) 和 (90) 式, 得

$$\begin{aligned}
\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi &= \tilde{y}_{d,ij}\left[\bar{u}'_{i\mathrm{L}}d'_{j\mathrm{R}}\phi^+ + \frac{1}{\sqrt{2}}\bar{d}'_{i\mathrm{L}}d'_{j\mathrm{R}}(v + H + \mathrm{i}\chi)\right] \\
&= y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{y_{d_i}}{\sqrt{2}}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v + H + \mathrm{i}\chi), \tag{155}
\end{aligned}$$

$$\begin{aligned}
\tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} &= \tilde{y}_{u,ij}\left[\frac{1}{\sqrt{2}}\bar{u}'_{i\mathrm{L}}u'_{j\mathrm{R}}(v + H - \mathrm{i}\chi) - \bar{d}'_{i\mathrm{L}}u'_{j\mathrm{R}}\phi^-\right] \\
&= \frac{y_{u_i}}{\sqrt{2}}\bar{u}_{i\mathrm{L}}u_{j\mathrm{R}}(v + H - \mathrm{i}\chi) - y_{u_i}\bar{d}_{j\mathrm{L}}V_{ji}^\dagger u_{i\mathrm{R}}\phi^-. \tag{156}
\end{aligned}$$

从而，Yukawa 相互作用拉氏量化为

$$\mathcal{L}_\mathrm{Y} = -\tilde{y}_{d,ij}\bar{Q}_{i\mathrm{L}}d'_{j\mathrm{R}}\Phi - \tilde{y}_{u,ij}\bar{Q}_{i\mathrm{L}}u'_{j\mathrm{R}}\tilde{\Phi} - y_{\ell_i}\bar{L}_{i\mathrm{L}}\ell_{i\mathrm{R}}\Phi + \mathrm{H.c.}$$

$$= -y_{d_j}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ - \frac{y_{d_i}}{\sqrt{2}}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(v+H+\mathrm{i}\chi) - \frac{y_{u_i}}{\sqrt{2}}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(v+H-\mathrm{i}\chi) + y_{u_i}\bar{d}_{j\mathrm{L}}V^\dagger_{ji}u_{i\mathrm{R}}\phi^-$$

$$\quad - y_{\ell_i}\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ - \frac{y_{\ell_i}}{\sqrt{2}}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(v+H+\mathrm{i}\chi) + \mathrm{H.c.}$$

$$= -m_{d_i}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}} - m_{u_i}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}} - m_{\ell_i}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}} - \frac{m_{d_i}}{v}\bar{d}_{i\mathrm{L}}d_{i\mathrm{R}}(H+\mathrm{i}\chi) - \frac{m_{u_i}}{v}\bar{u}_{i\mathrm{L}}u_{i\mathrm{R}}(H-\mathrm{i}\chi)$$

$$\quad - \frac{m_{\ell_i}}{v}\bar{\ell}_{i\mathrm{L}}\ell_{i\mathrm{R}}(H+\mathrm{i}\chi) - \frac{\sqrt{2}m_{d_j}}{v}\bar{u}_{i\mathrm{L}}V_{ij}d_{j\mathrm{R}}\phi^+ + \frac{\sqrt{2}m_{u_i}}{v}\bar{d}_{j\mathrm{L}}V^\dagger_{ji}u_{i\mathrm{R}}\phi^- - \frac{\sqrt{2}m_{\ell_i}}{v}\bar{\nu}_{i\mathrm{L}}\ell_{i\mathrm{R}}\phi^+ + \mathrm{H.c.}$$

$$= -m_{d_i}\bar{d}_i d_i - m_{u_i}\bar{u}_i u_i - m_{\ell_i}\bar{\ell}_i\ell_i - \frac{m_{d_i}}{v}H\bar{d}_i d_i - \frac{m_{u_i}}{v}H\bar{u}_i u_i - \frac{m_{\ell_i}}{v}H\bar{\ell}_i\ell_i$$

$$\quad - \frac{m_{d_i}}{v}\chi\bar{d}_i\mathrm{i}\gamma^5 d_i + \frac{m_{u_i}}{v}\chi\bar{u}_i\mathrm{i}\gamma^5 u_i - \frac{m_{\ell_i}}{v}\chi\bar{\ell}_i\mathrm{i}\gamma^5\ell_i + \frac{\sqrt{2}V_{ij}}{v}\phi^+\bar{u}_i(m_{u_i}P_\mathrm{L} - m_{d_j}P_\mathrm{R})d_j$$

$$\quad - \frac{\sqrt{2}V^\dagger_{ji}}{v}\phi^-\bar{d}_j(m_{d_j}P_\mathrm{L} - m_{u_i}P_\mathrm{R})u_i - \frac{\sqrt{2}m_{\ell_i}}{v}(\phi^+\bar{\nu}_i P_\mathrm{R}\ell_i + \phi^-\bar{\ell}_i P_\mathrm{L}\nu_i). \tag{157}$$

利用

$$D_\mu\Phi = \begin{pmatrix} \partial_\mu + \mathrm{i}eA_\mu + \dfrac{\mathrm{i}g}{2c_\mathrm{W}}(c_\mathrm{W}^2 - s_\mathrm{W}^2)Z_\mu & \dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^+ \\ \dfrac{\mathrm{i}g}{\sqrt{2}}W_\mu^- & \partial_\mu - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu \end{pmatrix}\begin{pmatrix} \phi^+ \\ \dfrac{1}{\sqrt{2}}(v+H+\mathrm{i}\chi) \end{pmatrix}$$

$$= \begin{pmatrix} \partial_\mu\phi^+ + \mathrm{i}\left[eA_\mu + \dfrac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \dfrac{\mathrm{i}g}{2}W_\mu^+(H+\mathrm{i}\chi) + \mathrm{i}m_W W_\mu^+ \\ \dfrac{1}{\sqrt{2}}\left[\partial_\mu(H+\mathrm{i}\chi) + \mathrm{i}gW_\mu^-\phi^+ - \dfrac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu(H+\mathrm{i}\chi) - \mathrm{i}m_Z Z_\mu\right] \end{pmatrix}, \tag{158}$$

将 Higgs 场协变动能项化为

$$(D^\mu\Phi)^\dagger D_\mu\Phi$$

$$= \left|\partial_\mu\phi^+ + \mathrm{i}\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{\mathrm{i}g}{2}W_\mu^+(H+\mathrm{i}\chi) + \mathrm{i}m_W W_\mu^+\right|^2$$

$$\quad + \frac{1}{2}\left|\partial_\mu(H+\mathrm{i}\chi) + \mathrm{i}gW_\mu^-\phi^+ - \frac{\mathrm{i}g}{2c_\mathrm{W}}Z_\mu(H+\mathrm{i}\chi) - \mathrm{i}m_Z Z_\mu\right|^2$$

$$= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi)$$

$$\quad + \left(\mathrm{i}\partial^\mu\phi^-\left\{\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H+\mathrm{i}\chi) + m_W W_\mu^+\right\} + \mathrm{H.c.}\right)$$

$$\quad + \left\{\frac{\mathrm{i}}{2}\partial^\mu(H-\mathrm{i}\chi)\left[gW_\mu^-\phi^+ - \frac{g}{2c_\mathrm{W}}Z_\mu(H+\mathrm{i}\chi) - m_Z Z_\mu\right] + \mathrm{H.c.}\right\}$$

$$\quad + \left|\left[eA_\mu + \frac{g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}Z_\mu\right]\phi^+ + \frac{g}{2}W_\mu^+(H+\mathrm{i}\chi) + m_W W_\mu^+\right|^2$$

$$+\frac{1}{2}\left|gW_\mu^-\phi^+ - \frac{g}{2c_W}Z_\mu(H+\mathrm{i}\chi) - m_Z Z_\mu\right|^2$$

$$= (\partial^\mu\phi^+)(\partial_\mu\phi^-) + \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{2}(\partial^\mu\chi)(\partial_\mu\chi)$$

$$+m_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu + gm_W H W_\mu^+ W^{-,\mu} + \frac{gm_Z}{2c_W} H Z_\mu Z^\mu$$

$$-\frac{g}{2}[\mathrm{i}W_\mu^+\phi^-\overleftrightarrow{\partial^\mu}(H+\mathrm{i}\chi) + \mathrm{H.c.}] - \mathrm{i}eA_\mu\phi^-\overleftrightarrow{\partial^\mu}\phi^+ - \frac{g}{2c_W}Z_\mu[-\chi\overleftrightarrow{\partial^\mu}H + \mathrm{i}(c_W^2 - s_W^2)\phi^-\overleftrightarrow{\partial^\mu}\phi^+]$$

$$+\frac{g^2}{4}W_\mu^+ W^{-,\mu}(2\phi^+\phi^- + H^2 + \chi^2) + e^2 A_\mu A^\mu \phi^+\phi^-$$

$$+\frac{g^2}{4c_W^2}Z_\mu Z^\mu\left[(c_W^2 - s_W^2)^2\phi^+\phi^- + \frac{1}{2}H^2 + \frac{1}{2}\chi^2\right]$$

$$+\left[\frac{eg}{2}W_\mu^+ A^\mu\phi^-(H+\mathrm{i}\chi) - \frac{g^2 s_W^2}{2c_W}W_\mu^+ Z^\mu\phi^-(H+\mathrm{i}\chi) + \mathrm{H.c.}\right] + \frac{eg(c_W^2 - s_W^2)}{c_W}A_\mu Z^\mu\phi^+\phi^-$$

$$+(em_W A^\mu\phi^+ W_\mu^- - gs_W^2 m_Z Z^\mu\phi^+ W_\mu^- + \mathrm{H.c.}) + \mathcal{L}_{\mathrm{b1}}, \tag{159}$$

其中

$$\mathcal{L}_{\mathrm{b1}} = \mathrm{i}m_W(\partial^\mu\phi^-)W_\mu^+ - \mathrm{i}m_W(\partial^\mu\phi^+)W_\mu^- - m_Z(\partial^\mu\chi)Z_\mu. \tag{160}$$

在 R_ξ 规范下，将规范固定函数设为

$$G^\pm = \frac{1}{\sqrt{\xi_W}}(\partial^\mu W_\mu^\pm \pm \mathrm{i}\xi_W m_W\phi^\pm), \quad G^Z = \frac{1}{\sqrt{\xi_Z}}(\partial^\mu Z_\mu + \xi_Z m_Z\chi), \quad G^\gamma = \frac{1}{\sqrt{\xi_\gamma}}\partial^\mu A_\mu, \tag{161}$$

它们在路径积分量子化中的泛函积分形式为

$$\int\mathcal{D}\omega^+\int\mathcal{D}\omega^-\int\mathcal{D}\omega^Z\int\mathcal{D}\omega^\gamma\exp\left[-\mathrm{i}\int d^4x\left(\omega^+\omega^- + \frac{1}{2}\omega^Z\omega^Z + \frac{1}{2}\omega^\gamma\omega^\gamma\right)\right]$$

$$\times\delta(G^+ - \omega^+)\delta(G^- - \omega^-)\delta(G^Z - \omega^Z)\delta(G^\gamma - \omega^\gamma)$$

$$= \exp\left[-\mathrm{i}\int\mathrm{d}^4x\left(G^+ G^- + \frac{1}{2}G^Z G^Z + \frac{1}{2}G^\gamma G^\gamma\right)\right]. \tag{162}$$

由此得到拉氏量中的规范固定项

$$\mathcal{L}_{\mathrm{EW,GF}} = -G^+ G^- - \frac{1}{2}(G^Z)^2 - \frac{1}{2}(G^\gamma)^2$$

$$= -\frac{1}{\xi_W}(\partial^\mu W_\mu^+ + \mathrm{i}\xi_W m_W\phi^+)(\partial^\nu W_\nu^- - \mathrm{i}\xi_W m_W\phi^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu + \xi_Z m_Z\chi)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2$$

$$= -\frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi_\gamma}(\partial^\mu A_\mu)^2 - \xi_W m_W^2\phi^+\phi^- - \frac{1}{2}\xi_Z m_Z^2\chi^2 + \mathcal{L}_{\mathrm{b2}}. \tag{163}$$

可见，Nambu-Goldstone 玻色子 $\phi^\pm$ 和 χ 在 R_ξ 规范下具有依赖于 ξ_W 和 ξ_Z 的非物理质量，

$$m_\phi = \sqrt{\xi_W}\,m_W, \quad m_\chi = \sqrt{\xi_Z}\,m_Z. \tag{164}$$

这里

$$\mathcal{L}_{\mathrm{b2}} = \mathrm{i}m_W \phi^-(\partial^\mu W_\mu^+) - \mathrm{i}m_W \phi^+ \partial^\mu W_\mu^- - m_Z \chi \partial^\mu Z_\mu. \tag{165}$$

$\mathcal{L}_{\mathrm{b1}}$ 与 $\mathcal{L}_{\mathrm{b2}}$ 之和

$$\mathcal{L}_{\mathrm{b1}} + \mathcal{L}_{\mathrm{b2}} = \mathrm{i}m_W \partial^\mu(\phi^- W_\mu^+) - \mathrm{i}m_W \partial^\mu(\phi^+ W_\mu^-) - m_Z \partial^\mu(\chi Z_\mu) \tag{166}$$

是全散度, 不会有物理效应. 可见, 协变动能项中规范场与 Nambu-Goldstone 标量场之间的双线性耦合项 $\mathcal{L}_{\mathrm{b1}}$ 被规范固定项中的 $\mathcal{L}_{\mathrm{b2}}$ 抵消掉, 这就是如此选取规范固定函数的目的.

这样一来, 电弱规范场传播子相关拉氏量变成

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EW,prop}} &= (\partial_\mu W_\nu^+)(\partial^\nu W^{-,\mu}) - (\partial_\mu W_\nu^+)(\partial^\mu W^{-,\nu}) - \frac{1}{\xi_W}(\partial^\mu W_\mu^+)(\partial^\nu W_\nu^-) + m_W^2 W_\mu^+ W^{-,\mu} \\
&\quad + \frac{1}{2}\left[(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) - (\partial_\mu Z_\nu)(\partial^\mu Z^\nu) - \frac{1}{\xi_Z}(\partial^\mu Z_\mu)^2 + m_Z^2 Z_\mu Z^\mu \right] \\
&\quad + \frac{1}{2}\left[(\partial_\mu A_\nu)(\partial^\nu A^\mu) - (\partial_\mu A_\nu)(\partial^\mu A^\nu) - \frac{1}{\xi_\gamma}(\partial^\mu A_\mu)^2 \right] \\
&\to W_\mu^+ \left[g^{\mu\nu}(\partial^2 + m_W^2) - \left(1 - \frac{1}{\xi_W}\right)\partial^\mu \partial^\nu \right] W_\nu^- \\
&\quad + \frac{1}{2} Z_\mu \left[g^{\mu\nu}(\partial^2 + m_Z^2) - \left(1 - \frac{1}{\xi_Z}\right)\partial^\mu \partial^\nu \right] Z_\nu \\
&\quad + \frac{1}{2} A_\mu \left[g^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi_\gamma}\right)\partial^\mu \partial^\nu \right] A_\nu. \tag{167}
\end{aligned}
$$

于是, 光子的传播子与胶子形式类似, 为

$$\frac{-\mathrm{i}}{p^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi_\gamma) \right]. \tag{168}$$

将 $W^\pm$ 传播子相关拉氏量变换到动量空间, 得

$$-g^{\mu\nu}(p^2 - m_W^2) + \left(1 - \frac{1}{\xi_W}\right)p^\mu p^\nu = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2}\frac{p^2 - \xi_W m_W^2}{\xi_W}, \tag{169}$$

它的逆矩阵是

$$-\frac{1}{p^2 - m_W^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - \frac{\xi_W}{p^2 - \xi_W m_W^2}\frac{p_\mu p_\nu}{p^2} = -\frac{1}{p^2 - m_W^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W) \right], \tag{170}$$

这是因为由

$$\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right)\frac{p^\mu p^\nu}{p^2} = \frac{p_\rho p^\nu}{p^2} - \frac{p_\rho p^\nu}{p^2} = 0, \quad \left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2}\right)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) = \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2} \tag{171}$$

得

$$
\begin{aligned}
&\left[-\frac{1}{p^2 - m_W^2}\left(g_{\rho\mu} - \frac{p_\rho p_\mu}{p^2} \right) - \frac{\xi_W}{p^2 - \xi_W m_W^2}\frac{p_\rho p_\mu}{p^2} \right] \\
&\times \left[-\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right)(p^2 - m_W^2) - \frac{p^\mu p^\nu}{p^2}\frac{p^2 - \xi_W m_W^2}{\xi}_W \right] \\
&= \delta_\rho{}^\nu - \frac{p_\rho p^\nu}{p^2} + \frac{p_\rho p^\nu}{p^2} = \delta_\rho{}^\nu .
\end{aligned} \tag{172}
$$

从而, $W^\pm$ 传播子的形式为

$$
\frac{-\mathrm{i}}{p^2 - m_W^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W) \right]. \tag{173}
$$

同理, Z 传播子的形式为

$$
\frac{-\mathrm{i}}{p^2 - m_Z^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2}(1 - \xi_Z) \right]. \tag{174}
$$

电弱规范场的无穷小规范变换形式是

$$
\delta W_\mu^a = -\frac{1}{g}\,\partial_\mu \alpha^a + \varepsilon^{abc}W_\mu^b\alpha^c, \quad \delta B_\mu = -\frac{1}{g'}\,\partial_\mu\alpha^Y. \tag{175}
$$

定义

$$
\alpha^\pm \equiv \frac{1}{\sqrt{2}}(\alpha^1 \mp \mathrm{i}\alpha^2), \quad \alpha^Z \equiv \alpha^3 - \alpha^Y, \quad \alpha^\gamma \equiv s_W^2\alpha^3 + c_W^2\alpha^Y, \tag{176}
$$

利用

$$
\varepsilon^{1bc}W_\mu^b\alpha^c = W_\mu^2\alpha^3 - W_\mu^3\alpha^2, \quad \varepsilon^{2bc}W_\mu^b\alpha^c = -W_\mu^1\alpha^3 + W_\mu^3\alpha^1, \tag{177}
$$

$$
\pm\mathrm{i}\sqrt{2}\alpha^\pm = \pm\mathrm{i}\alpha^1 + \alpha^2, \qquad \pm\mathrm{i}\sqrt{2}W_\mu^\pm = \pm\mathrm{i}W_\mu^1 + W_\mu^2, \tag{178}
$$

有

$$
\begin{aligned}
\varepsilon^{1bc}W_\mu^b\alpha^c \mp \mathrm{i}\varepsilon^{2bc}W_\mu^b\alpha^c &= (W_\mu^2\alpha^3 - W_\mu^3\alpha^2) \mp \mathrm{i}(-W_\mu^1\alpha^3 + W_\mu^3\alpha^1) \\
&= (W_\mu^2 \pm \mathrm{i}W_\mu^1)\alpha^3 - W_\mu^3(\alpha^2 \pm \mathrm{i}\alpha^1) \\
&= \pm\mathrm{i}\sqrt{2}W_\mu^\pm(c_W^2\alpha^Z + \alpha^\gamma) \mp \mathrm{i}\sqrt{2}(s_W A_\mu + c_W Z_\mu)\alpha^\pm,
\end{aligned} \tag{179}
$$

$$
\begin{aligned}
\varepsilon^{3bc}W_\mu^b\alpha^c &= W_\mu^1\alpha^2 - W_\mu^2\alpha^1 = \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-)\frac{\mathrm{i}}{\sqrt{2}}(\alpha^+ - \alpha^-) - \frac{\mathrm{i}}{\sqrt{2}}(W_\mu^+ - W_\mu^-)\frac{1}{\sqrt{2}}(\alpha^+ + \alpha^-) \\
&= -\mathrm{i}(W_\mu^+\alpha^- - W_\mu^-\alpha^+).
\end{aligned} \tag{180}
$$

因此,

$$
\begin{aligned}
\delta W_\mu^+ &= \frac{1}{\sqrt{2}}(\delta W_\mu^1 - \mathrm{i}\delta W_\mu^2) = -\frac{1}{\sqrt{2}g}\partial_\mu(\alpha^1 - \mathrm{i}\alpha^2) + \frac{1}{\sqrt{2}}(\varepsilon^{1bc}W_\mu^b\alpha^c - \mathrm{i}\varepsilon^{2bc}W_\mu^b\alpha^c) \\
&= -\frac{1}{g}\partial_\mu\alpha^+ - \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^+ + \mathrm{i}W_\mu^+(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma),
\end{aligned}
\tag{181}
$$

$$
\delta W_\mu^- = (\delta W_\mu^+)^\dagger = -\frac{1}{g}\partial_\mu\alpha^- + \mathrm{i}(s_\mathrm{W}A_\mu + c_\mathrm{W}Z_\mu)\alpha^- - \mathrm{i}W_\mu^-(c_\mathrm{W}^2\alpha^Z + \alpha^\gamma),
\tag{182}
$$

$$
\begin{aligned}
\delta Z_\mu^a &= c_\mathrm{W}\delta W_\mu^3 - s_\mathrm{W}\delta B_\mu = -\frac{c_\mathrm{W}}{g}\partial_\mu\alpha^3 + c_\mathrm{W}\varepsilon^{3bc}W_\mu^b\alpha^c + \frac{s_\mathrm{W}}{g'}\partial_\mu\alpha^Y \\
&= -\frac{c_\mathrm{W}}{g}\partial_\mu\alpha^Z - \mathrm{i}c_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+),
\end{aligned}
\tag{183}
$$

$$
\begin{aligned}
\delta A_\mu &= s_\mathrm{W}\delta W_\mu^3 + c_\mathrm{W}\delta B_\mu = -\frac{s_\mathrm{W}}{g}\partial_\mu\alpha^3 + s_\mathrm{W}\varepsilon^{3bc}W_\mu^b\alpha^c - \frac{c_\mathrm{W}}{g'}\partial_\mu\alpha^Y \\
&= -\frac{1}{e}\partial_\mu\alpha^\gamma - \mathrm{i}s_\mathrm{W}(W_\mu^+\alpha^- - W_\mu^-\alpha^+).
\end{aligned}
\tag{184}
$$

另一方面, 根据

$$
\begin{aligned}
\alpha^a T^a + \alpha^Y Y_H &= \frac{1}{2}(\alpha^a\sigma^a + \alpha^Y) = \frac{1}{2}\begin{pmatrix} \alpha^3 + \alpha^Y & \alpha^1 - \mathrm{i}\alpha^2 \\ \alpha^1 + \mathrm{i}\alpha^2 & -\alpha^3 + \alpha^Y \end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix} 2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix},
\end{aligned}
\tag{185}
$$

可知 Higgs 场的无穷小规范变换形式为

$$
\begin{aligned}
\delta\Phi &= \mathrm{i}(\alpha^a T^a + \alpha^Y Y_H)\Phi = \frac{\mathrm{i}}{2}\begin{pmatrix} 2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z & \sqrt{2}\alpha^+ \\ \sqrt{2}\alpha^- & -\alpha^Z \end{pmatrix}\begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + \mathrm{i}\chi) \end{pmatrix} \\
&= \begin{pmatrix} \frac{\mathrm{i}}{2}\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H + \mathrm{i}\chi)\alpha^+\} \\ \frac{1}{\sqrt{2}}\left[\mathrm{i}\phi^+\alpha^- - \frac{1}{2}(\mathrm{i}v + \mathrm{i}H - \chi)\alpha^Z\right] \end{pmatrix} = \begin{pmatrix} \delta\phi^+ \\ \frac{1}{\sqrt{2}}(\delta H + \mathrm{i}\delta\chi) \end{pmatrix}.
\end{aligned}
\tag{186}
$$

利用

$$
\mathrm{Re}(\phi^+\alpha^-) = \frac{1}{2}(\phi^+\alpha^- + \phi^-\alpha^+), \quad \mathrm{Im}(\phi^+\alpha^-) = -\frac{\mathrm{i}}{2}(\phi^+\alpha^- - \phi^-\alpha^+),
\tag{187}
$$

推出

$$
\delta\phi^+ = \frac{\mathrm{i}}{2}\{\phi^+[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H + \mathrm{i}\chi)\alpha^+\},
\tag{188}
$$

$$
\delta\phi^- = -\frac{\mathrm{i}}{2}\{\phi^-[2\alpha^\gamma + (c_\mathrm{W}^2 - s_\mathrm{W}^2)\alpha^Z] + (v + H - \mathrm{i}\chi)\alpha^-\},
\tag{189}
$$

$$
\delta H = \frac{1}{2}[\mathrm{i}(\phi^+\alpha^- - \phi^-\alpha^+) + \chi\alpha^Z], \quad \delta\chi = \frac{1}{2}[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z].
\tag{190}
$$

于是，规范固定函数的无穷小规范变换为

$$
\begin{aligned}
\sqrt{\xi_W}\,\delta G^+ &= \partial^\mu \delta W_\mu^+ + \mathrm{i}\xi_W m_W \delta\phi^+ \\
&= \partial^\mu\left[-\frac{1}{g}\,\partial_\mu\alpha^+ - \mathrm{i}(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu)\alpha^+ + \mathrm{i}W_\mu^+(c_{\mathrm{W}}^2\alpha^Z + \alpha^\gamma)\right] \\
&\quad -\frac{1}{2}\xi_W m_W\left\{\phi^+[2\alpha^\gamma + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\alpha^Z] + (v + H + \mathrm{i}\chi)\alpha^+\right\},
\end{aligned}
\tag{191}
$$

$$
\begin{aligned}
\sqrt{\xi_W}\,\delta G^- &= \partial^\mu \delta W_\mu^- - \mathrm{i}\xi_W m_W \delta\phi^- \\
&= \partial^\mu\left[-\frac{1}{g}\,\partial_\mu\alpha^- + \mathrm{i}(s_{\mathrm{W}}A_\mu + c_{\mathrm{W}}Z_\mu)\alpha^- - \mathrm{i}W_\mu^-(c_{\mathrm{W}}^2\alpha^Z + \alpha^\gamma)\right] \\
&\quad -\frac{1}{2}\xi_W m_W\left\{\phi^-[2\alpha^\gamma + (c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\alpha^Z] + (v + H - \mathrm{i}\chi)\alpha^-\right\},
\end{aligned}
\tag{192}
$$

$$
\begin{aligned}
\sqrt{\xi_Z}\,\delta G^Z &= \partial^\mu \delta Z_\mu + \xi_Z m_Z \delta\chi \\
&= \partial^\mu\left[-\frac{c_{\mathrm{W}}}{g}\,\partial_\mu\alpha^Z - \mathrm{i}c_{\mathrm{W}}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right] \\
&\quad +\frac{1}{2}\xi_Z m_Z\left[\phi^+\alpha^- + \phi^-\alpha^+ - (v + H)\alpha^Z\right],
\end{aligned}
\tag{193}
$$

$$
\sqrt{\xi_\gamma}\,\delta G^\gamma = \partial^\mu \delta A_\mu = \partial^\mu\left[-\frac{1}{e}\,\partial_\mu\alpha^\gamma - \mathrm{i}s_{\mathrm{W}}(W_\mu^+\alpha^- - W_\mu^-\alpha^+)\right].
\tag{194}
$$

因此，

$$
\sqrt{\xi_W}\,g\,\frac{\delta G^+}{\delta\alpha^+} = -\partial^2 - \xi_W m_W^2 - \mathrm{i}e\partial^\mu A_\mu - \mathrm{i}gc_{\mathrm{W}}\partial^\mu Z_\mu - \frac{1}{2}\,g\xi_W m_W(H + \mathrm{i}\chi),
\tag{195}
$$

$$
\frac{\sqrt{\xi_W}\,g}{c_{\mathrm{W}}}\,\frac{\delta G^+}{\delta\alpha^Z} = \mathrm{i}gc_{\mathrm{W}}\partial^\mu W_\mu^+ - \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}\,\phi^+,
\tag{196}
$$

$$
\sqrt{\xi_W}\,e\,\frac{\delta G^+}{\delta\alpha^\gamma} = \mathrm{i}e\partial^\mu W_\mu^+ - e\xi_W m_W\phi^+,
\tag{197}
$$

$$
\sqrt{\xi_W}\,g\,\frac{\delta G^-}{\delta\alpha^-} = -\partial^2 - \xi_W m_W^2 + \mathrm{i}e\partial^\mu A_\mu + \mathrm{i}gc_{\mathrm{W}}\partial^\mu Z_\mu - \frac{1}{2}\,g\xi_W m_W(H - \mathrm{i}\chi),
\tag{198}
$$

$$
\frac{\sqrt{\xi_W}\,g}{c_{\mathrm{W}}}\,\frac{\delta G^-}{\delta\alpha^Z} = -\mathrm{i}gc_{\mathrm{W}}\partial^\mu W_\mu^- - \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}\,\phi^-,
\tag{199}
$$

$$
\sqrt{\xi_W}\,e\,\frac{\delta G^-}{\delta\alpha^\gamma} = -\mathrm{i}e\partial^\mu W_\mu^- - e\xi_W m_W\phi^-,
\tag{200}
$$

$$
\sqrt{\xi_Z}\,g\,\frac{\delta G^Z}{\delta\alpha^+} = \mathrm{i}gc_{\mathrm{W}}\partial^\mu W_\mu^- + \frac{1}{2}\,g\xi_Z m_Z\phi^-,
\tag{201}
$$

$$
\sqrt{\xi_Z}\,g\,\frac{\delta G^Z}{\delta\alpha^-} = -\mathrm{i}gc_{\mathrm{W}}\partial^\mu W_\mu^+ + \frac{1}{2}\,g\xi_Z m_Z\phi^+,
\tag{202}
$$

$$
\frac{\sqrt{\xi_Z}\,g}{c_{\mathrm{W}}}\,\frac{\delta G^Z}{\delta\alpha^Z} = -\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_{\mathrm{W}}}H,
\tag{203}
$$

$$
\sqrt{\xi_\gamma}\,g\,\frac{\delta G^\gamma}{\delta\alpha^+} = \mathrm{i}e\partial^\mu W_\mu^-, \quad \sqrt{\xi_\gamma}\,g\,\frac{\delta G^\gamma}{\delta\alpha^-} = -\mathrm{i}e\partial^\mu W_\mu^+, \quad \sqrt{\xi_\gamma}\,e\,\frac{\delta G^\gamma}{\delta\alpha^\gamma} = -\partial^2.
\tag{204}
$$

最后，得到以下 Faddeev-Popov 鬼场拉氏量,

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EWG,FP}} =\ & \bar{\eta}^+\left(\sqrt{\xi_W}g\,\frac{\delta G^+}{\delta\alpha^+}\right)\eta^+ + \bar{\eta}^Z\left(\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^+}\right)\eta^+ + \bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^+}\right)\eta^+ \\
& +\bar{\eta}^-\left(\sqrt{\xi_W}g\,\frac{\delta G^-}{\delta\alpha^-}\right)\eta^- + \bar{\eta}^Z\left(\sqrt{\xi_Z}g\,\frac{\delta G^Z}{\delta\alpha^-}\right)\eta^- + \bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}g\,\frac{\delta G^\gamma}{\delta\alpha^-}\right)\eta^- \\
& +\bar{\eta}^Z\left(\frac{\sqrt{\xi_Z}g}{c_{\mathrm{W}}}\,\frac{\delta G^Z}{\delta\alpha^Z}\right)\eta^Z + \bar{\eta}^+\left(\frac{\sqrt{\xi_W}g}{c_{\mathrm{W}}}\,\frac{\delta G^+}{\delta\alpha^Z}\right)\eta^Z + \bar{\eta}^-\left(\frac{\sqrt{\xi_W}g}{c_{\mathrm{W}}}\,\frac{\delta G^-}{\delta\alpha^Z}\right)\eta^Z \\
& +\bar{\eta}^\gamma\left(\sqrt{\xi_\gamma}e\,\frac{\delta G^\gamma}{\delta\alpha^\gamma}\right)\eta^\gamma + \bar{\eta}^+\left(\sqrt{\xi_W}e\,\frac{\delta G^+}{\delta\alpha^\gamma}\right)\eta^\gamma + \bar{\eta}^-\left(\sqrt{\xi_W}e\,\frac{\delta G^-}{\delta\alpha^\gamma}\right)\eta^\gamma \\
\rightarrow\ & \bar{\eta}^+\left[-\partial^2 - \xi_W m_W^2 + \mathrm{i}e\overleftarrow{\partial^\mu}A_\mu + \mathrm{i}gc_{\mathrm{W}}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi_W m_W(H + \mathrm{i}\chi)\right]\eta^+ \\
& +\bar{\eta}^Z\left(-\mathrm{i}gc_{\mathrm{W}}\overleftarrow{\partial^\mu}W_\mu^- + \frac{1}{2}g\xi_Z m_Z\phi^-\right)\eta^+ - \mathrm{i}e(\partial^\mu\bar{\eta}^\gamma)W_\mu^-\eta^+ \\
& +\bar{\eta}^-\left[-\partial^2 - \xi_W m_W^2 - \mathrm{i}e\overleftarrow{\partial^\mu}A_\mu - \mathrm{i}gc_{\mathrm{W}}\overleftarrow{\partial^\mu}Z_\mu - \frac{1}{2}g\xi_W m_W(H - \mathrm{i}\chi)\right]\eta^- \\
& +\bar{\eta}^Z\left(\mathrm{i}gc_{\mathrm{W}}\overleftarrow{\partial^\mu}W_\mu^+ + \frac{1}{2}g\xi_Z m_Z\phi^+\right)\eta^- + \mathrm{i}e(\partial^\mu\bar{\eta}^\gamma)W_\mu^+\eta^- \\
& +\bar{\eta}^Z\left(-\partial^2 - \xi_Z m_Z^2 - \frac{g\xi_Z m_Z}{2c_{\mathrm{W}}}H\right)\eta^Z \\
& +\bar{\eta}^+\left(-\mathrm{i}gc_{\mathrm{W}}\overleftarrow{\partial^\mu}W_\mu^+ - \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}\phi^+\right)\eta^Z \\
& +\bar{\eta}^-\left(\mathrm{i}gc_{\mathrm{W}}\overleftarrow{\partial^\mu}W_\mu^- - \frac{g(c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2)\xi_W m_W}{2c_{\mathrm{W}}}\phi^-\right)\eta^Z \\
& -\bar{\eta}^\gamma\partial^2\eta^\gamma + \bar{\eta}^+(-\mathrm{i}e\overleftarrow{\partial^\mu}W_\mu^+ - e\xi_W m_W\phi^+)\eta^\gamma + \bar{\eta}^-(\mathrm{i}e\overleftarrow{\partial^\mu}W_\mu^- - e\xi_W m_W\phi^-)\eta^\gamma. \quad (205)
\end{aligned}
$$

可以认为这里通过鬼场 $\eta^\pm$、η^Z、η^γ 的归一化吸收了 $-1/g$、$-c_{\mathrm{W}}/g$、$-1/e$ 因子, 通过鬼场 $\bar{\eta}^\pm$、$\bar{\eta}^Z$、$\bar{\eta}^\gamma$ 的归一化吸收了 $1/\sqrt{\xi_W}$、$1/\sqrt{\xi_Z}$、$1/\sqrt{\xi_\gamma}$ 因子。鬼粒子的质量为

$$
m_{\eta^+} = m_{\eta^-} = \sqrt{\xi_W}\,m_W, \quad m_{\eta^Z} = \sqrt{\xi_Z}\,m_Z, \quad m_{\eta^\gamma} = 0. \quad (206)
$$

下面给出 R$_\xi$ 规范下的 Feynman 规则。$\xi_i = 1$ 对应 Feynman-'t Hooft 规范, $\xi_i = 0$ 对应 Landau 规范, $\xi_W, \xi_Z \to \infty$ 对应幺正规范。在树图计算中, 常取 $\xi_\gamma = 1$ 和 $\xi_W, \xi_Z \to \infty$。在圈图计算中, 常取 $\xi_\gamma = \xi_W = \xi_Z = 1$。

传播子:

$$
\overset{\overset{\textstyle p}{\longrightarrow}}{\underset{H}{\bullet\text{-}\text{-}\text{-}\text{-}\text{-}\bullet}} = \frac{\mathrm{i}}{p^2 - m_H^2 + \mathrm{i}\epsilon} \quad (207)
$$

$$
\overset{\overset{\textstyle p}{\longrightarrow}}{\underset{\chi}{\bullet\text{-}\text{-}\text{-}\text{-}\text{-}\bullet}} = \frac{\mathrm{i}}{p^2 - \xi_Z m_Z^2 + \mathrm{i}\epsilon} \quad (208)
$$

$$
\overset{\overset{\textstyle p}{\longrightarrow}}{\underset{\phi}{\bullet\text{-}\text{-}\text{-}\text{-}\text{-}\bullet}} = \frac{\mathrm{i}}{p^2 - \xi_W m_W^2 + \mathrm{i}\epsilon} \quad (209)
$$

$$\nu \;\xrightarrow[\gamma]{p}\; \mu \;=\; \frac{-\mathrm{i}}{p^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1 - \xi_\gamma)\right] \tag{210}$$

$$\nu \;\xrightarrow[Z]{p}\; \mu \;=\; \frac{-\mathrm{i}}{p^2 - m_Z^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2}(1 - \xi_Z)\right] \tag{211}$$

$$\nu \;\xrightarrow[W]{p}\; \mu \;=\; \frac{-\mathrm{i}}{p^2 - m_W^2 + \mathrm{i}\epsilon}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2 - \xi_W m_W^2}(1 - \xi_W)\right] \tag{212}$$

$$\xrightarrow[\eta^\gamma]{p} \;=\; \frac{\mathrm{i}}{p^2 + \mathrm{i}\epsilon} \tag{213}$$

$$\xrightarrow[\eta^Z]{p} \;=\; \frac{\mathrm{i}}{p^2 - \xi_Z m_Z^2 + \mathrm{i}\epsilon} \tag{214}$$

$$\xrightarrow[\eta^\pm]{p} \;=\; \frac{\mathrm{i}}{p^2 - \xi_W m_W^2 + \mathrm{i}\epsilon} \tag{215}$$

标量玻色子三线性耦合:

$$
\begin{aligned}
&= -\frac{3\mathrm{i}m_H^2}{v} & &= -\frac{\mathrm{i}m_H^2}{v} & &= -\frac{\mathrm{i}m_H^2}{v}\\
&= -6\mathrm{i}\lambda v & &= -2\mathrm{i}\lambda v & &= -2\mathrm{i}\lambda v
\end{aligned}
\tag{216}
$$

标量玻色子四线性耦合:

$$
\begin{aligned}
&= -\frac{3\mathrm{i}m_H^2}{v^2} & &= -\frac{\mathrm{i}m_H^2}{v^2} & &= -\frac{3\mathrm{i}m_H^2}{v^2}\\
&= -6\mathrm{i}\lambda & &= -2\mathrm{i}\lambda & &= -6\mathrm{i}\lambda
\end{aligned}
\tag{217}
$$

$$
\begin{aligned}
&= -\frac{\mathrm{i}m_H^2}{v^2} & &= -\frac{\mathrm{i}m_H^2}{v^2} & &= -\frac{2\mathrm{i}m_H^2}{v^2}\\
&= -2\mathrm{i}\lambda & &= -2\mathrm{i}\lambda & &= -4\mathrm{i}\lambda
\end{aligned}
\tag{218}
$$

Yukawa 耦合:

$$
\begin{aligned}
&= -\frac{\mathrm{i}m_f}{v} & &= \frac{m_{\ell_i}}{v}\gamma^5\\
&= -\frac{\mathrm{i}y_f}{\sqrt{2}} & &= \frac{y_{\ell_i}}{\sqrt{2}}\gamma^5
\end{aligned}
\tag{219}
$$

$$\chi$$

$$= -\frac{m_{u_i}}{v}\gamma^5$$

$$u_i \qquad u_i \qquad = -\frac{y_{u_i}}{\sqrt{2}}\gamma^5$$

$$\chi$$

$$= \frac{m_{d_i}}{v}\gamma^5$$

$$d_i \qquad d_i \qquad = \frac{y_{d_i}}{\sqrt{2}}\gamma^5 \tag{220}$$

$$\phi$$

$$= -\frac{\mathrm{i}\sqrt{2}m_{\ell_i}}{v}P_{\mathrm{R}}$$

$$\ell_i \qquad \nu_i \qquad = -\mathrm{i}y_{\ell_i}P_{\mathrm{R}}$$

$$\phi$$

$$= -\frac{\mathrm{i}\sqrt{2}m_{\ell_i}}{v}P_{\mathrm{L}}$$

$$\nu_i \qquad \ell_j \qquad = -\mathrm{i}y_{\ell_i}P_{\mathrm{L}} \tag{221}$$

$$\phi$$

$$= \frac{\mathrm{i}\sqrt{2}V_{ij}}{v}(m_{u_i}P_{\mathrm{L}} - m_{d_j}P_{\mathrm{R}})$$

$$d_j \qquad u_i \qquad = \mathrm{i}V_{ij}(y_{u_i}P_{\mathrm{L}} - y_{d_j}P_{\mathrm{R}})$$

$$\phi$$

$$= -\frac{\mathrm{i}\sqrt{2}V_{ji}^\dagger}{v}(m_{d_j}P_{\mathrm{L}} - m_{u_i}P_{\mathrm{R}})$$

$$u_i \qquad d_j \qquad = -\mathrm{i}V_{ji}^\dagger(y_{d_j}P_{\mathrm{L}} - y_{u_i}P_{\mathrm{R}}) \tag{222}$$

标量玻色子与电弱规范玻色子的三线性耦合：

$$H$$

$$= \frac{\mathrm{i}gm_Z}{c_{\mathrm{W}}}g^{\mu\nu}$$

$$Z;\mu \qquad Z;\nu \qquad = \frac{\mathrm{i}g^2 v}{2c_{\mathrm{W}}^2}g^{\mu\nu}$$

$$H$$

$$= \mathrm{i}gm_W g^{\mu\nu}$$

$$W;\mu \qquad W;\nu \qquad = \frac{\mathrm{i}g^2 v}{2}g^{\mu\nu} \tag{223}$$

$$\phi$$

$$= \mathrm{i}em_W g^{\mu\nu}$$

$$W;\mu \qquad \gamma;\nu \qquad = \frac{\mathrm{i}egv}{2}g^{\mu\nu}$$

$$\phi$$

$$= \mathrm{i}em_W g^{\mu\nu}$$

$$W;\mu \qquad \gamma;\nu \qquad = \frac{\mathrm{i}egv}{2}g^{\mu\nu} \tag{224}$$

$$\phi$$

$$= -\mathrm{i}gs_{\mathrm{W}}^2 m_Z g^{\mu\nu}$$

$$W;\mu \qquad Z;\nu \qquad = -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2 v}{2c_{\mathrm{W}}}g^{\mu\nu}$$

$$\phi$$

$$= -\mathrm{i}gs_{\mathrm{W}}^2 m_Z g^{\mu\nu}$$

$$W;\mu \qquad Z;\nu \qquad = -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2 v}{2c_{\mathrm{W}}}g^{\mu\nu} \tag{225}$$

$$\gamma;\mu$$

$$p \qquad q \qquad = -\mathrm{i}e(p+q)^\mu$$

$$\phi \qquad \phi \tag{226}$$

$$= -\frac{\mathrm{i}g(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{2c_\mathrm{W}}\,(p+q)^\mu \qquad\qquad = \frac{g}{2c_\mathrm{W}}\,(p+q)^\mu \qquad (227)$$
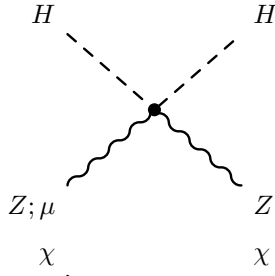
$$= -\frac{\mathrm{i}g}{2}\,(p+q)^\mu \qquad\qquad = -\frac{\mathrm{i}g}{2}\,(p+q)^\mu \qquad (228)$$
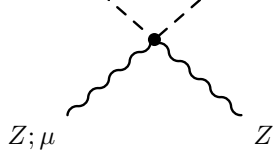
$$= \frac{g}{2}\,(p+q)^\mu \qquad\qquad = -\frac{g}{2}\,(p+q)^\mu \qquad (229)$$
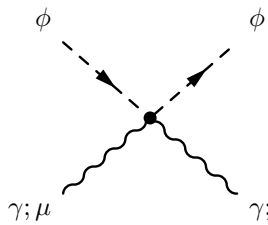
标量玻色子与电弱规范玻色子的四线性耦合:

$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}\,g^{\mu\nu} \qquad\qquad = \frac{\mathrm{i}g^2}{2}\,g^{\mu\nu} \qquad (230)$$
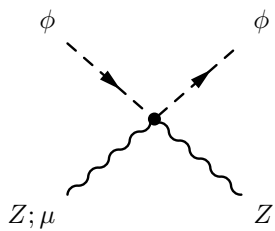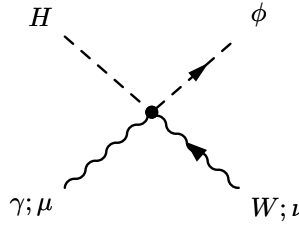
$$= \frac{\mathrm{i}g^2}{2c_\mathrm{W}^2}\,g^{\mu\nu} \qquad\qquad = \frac{\mathrm{i}g^2}{2}\,g^{\mu\nu} \qquad (231)$$
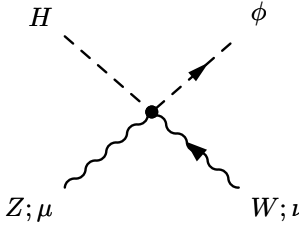
$$= 2\mathrm{i}e^2 g^{\mu\nu} \qquad\qquad = \frac{\mathrm{i}eg(c_\mathrm{W}^2 - s_\mathrm{W}^2)}{c_\mathrm{W}}\,g^{\mu\nu} \qquad (232)$$
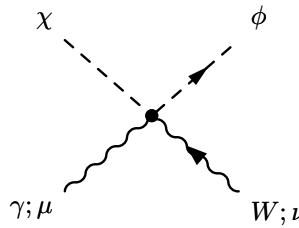
$$= \frac{\mathrm{i}g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)^2}{2c_\mathrm{W}^2}\,g^{\mu\nu} \qquad\qquad = \frac{\mathrm{i}g^2}{2}\,g^{\mu\nu} \qquad (233)$$
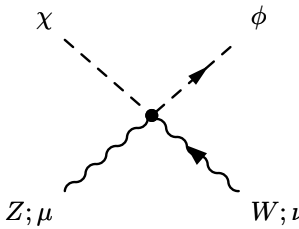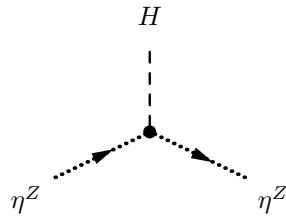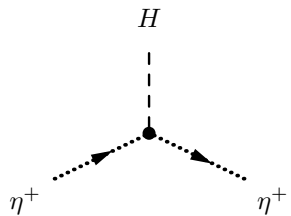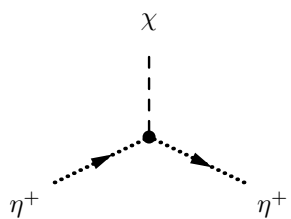
$$H \quad \phi$$
$$\gamma;\mu \quad W;\nu \qquad = \frac{\mathrm{i}eg}{2}\,g^{\mu\nu} \qquad (234)$$

$$H \quad \phi$$
$$\gamma;\mu \quad W;\nu \qquad = \frac{\mathrm{i}eg}{2}\,g^{\mu\nu}$$

$$H \quad \phi$$
$$Z;\mu \quad W;\nu \qquad = -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\,g^{\mu\nu} \qquad (235)$$

$$H \quad \phi$$
$$Z;\mu \quad W;\nu \qquad = -\frac{\mathrm{i}g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\,g^{\mu\nu}$$

$$\chi \quad \phi$$
$$\gamma;\mu \quad W;\nu \qquad = -\frac{eg}{2}\,g^{\mu\nu} \qquad (236)$$

$$\chi \quad \phi$$
$$\gamma;\mu \quad W;\nu \qquad = \frac{eg}{2}\,g^{\mu\nu}$$

$$\chi \quad \phi$$
$$Z;\mu \quad W;\nu \qquad = \frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\,g^{\mu\nu} \qquad (237)$$

$$\chi \quad \phi$$
$$Z;\mu \quad W;\nu \qquad = -\frac{g^2 s_{\mathrm{W}}^2}{2c_{\mathrm{W}}}\,g^{\mu\nu}$$

鬼粒子与标量玻色子的耦合:

$$H$$
$$\eta^Z \qquad \eta^Z \qquad
\begin{aligned}
&= -\frac{\mathrm{i}g\xi_Z m_Z}{2c_{\mathrm{W}}}\\
&= -\frac{\mathrm{i}g^2\xi_Z v}{4c_{\mathrm{W}}^2}
\end{aligned} \qquad (238)$$

$$H$$
$$\eta^+ \qquad \eta^+ \qquad
\begin{aligned}
&= -\frac{\mathrm{i}g\xi_W m_W}{2}\\
&= -\frac{\mathrm{i}g^2\xi_W v}{4}
\end{aligned}$$

$$H$$
$$\eta^- \qquad \eta^- \qquad
\begin{aligned}
&= -\frac{\mathrm{i}g\xi_W m_W}{2}\\
&= -\frac{\mathrm{i}g^2\xi_W v}{4}
\end{aligned} \qquad (239)$$

$$\chi$$
$$\eta^+ \qquad \eta^+ \qquad
\begin{aligned}
&= \frac{g\xi_W m_W}{2}\\
&= \frac{g^2\xi_W v}{4}
\end{aligned}$$

$$\chi$$
$$\eta^- \qquad \eta^- \qquad
\begin{aligned}
&= -\frac{g\xi_W m_W}{2}\\
&= -\frac{g^2\xi_W v}{4}
\end{aligned} \qquad (240)$$

$$
\begin{aligned}
&= -\mathrm{i}e\xi_W m_W \\
&= -\frac{\mathrm{i}eg\xi_W v}{2}
\end{aligned}
\qquad (241)
$$

$$
\begin{aligned}
&= -\mathrm{i}e\xi_W m_W \\
&= -\frac{\mathrm{i}eg\xi_W v}{2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\mathrm{i}g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}} \\
&= -\frac{\mathrm{i}g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W v}{4c_\mathrm{W}}
\end{aligned}
\qquad (242)
$$

$$
\begin{aligned}
&= \frac{\mathrm{i}g\xi_Z m_Z}{2} \\
&= \frac{\mathrm{i}g^2\xi_Z v}{4c_\mathrm{W}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\mathrm{i}g\xi_Z m_Z}{2} \\
&= \frac{\mathrm{i}g^2\xi_Z v}{4c_\mathrm{W}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\mathrm{i}g(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W m_W}{2c_\mathrm{W}} \\
&= -\frac{\mathrm{i}g^2(c_\mathrm{W}^2 - s_\mathrm{W}^2)\xi_W v}{4c_\mathrm{W}}
\end{aligned}
$$

$$(243)$$

鬼粒子与电弱规范玻色子的耦合:

$$
= -\mathrm{i}ep^\mu \qquad\qquad = \mathrm{i}ep^\mu \qquad (244)
$$

$$
= -\mathrm{i}gc_\mathrm{W}p^\mu \qquad\qquad = \mathrm{i}gc_\mathrm{W}p^\mu \qquad (245)
$$

$$
= \mathrm{i}ep^\mu \qquad\qquad = \mathrm{i}ep^\mu \qquad (246)
$$

$$
= -\mathrm{i}ep^\mu \qquad\qquad = -\mathrm{i}ep^\mu \qquad (247)
$$

$$= \mathrm{i}gc_{\mathrm{W}}p^{\mu} \tag{248}$$

$$= \mathrm{i}gc_{\mathrm{W}}p^{\mu}$$

$$= -\mathrm{i}gc_{\mathrm{W}}p^{\mu} \tag{249}$$

$$= -\mathrm{i}gc_{\mathrm{W}}p^{\mu}$$

6   内外线一般 Feynman 规则

本节列出一些通用的内外线 Feynman 规则。

内线 Feynman 规则如下。

- 实标量玻色子传播子：$\displaystyle = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$

- 复标量玻色子传播子：$\displaystyle = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\epsilon}$

- Dirac 费米子传播子：$\displaystyle = \frac{\mathrm{i}(\not{p} + m)}{p^2 - m^2 + \mathrm{i}\epsilon}$

- 无质量实矢量玻色子传播子：

$$\nu \quad \mu \quad = \frac{-\mathrm{i}g^{\mu\nu}}{p^2 + \mathrm{i}\epsilon} \quad \text{（Feynman 规范）}$$

$$\nu \quad \mu \quad = \frac{-\mathrm{i}(g^{\mu\nu} - p^{\mu}p^{\nu}/p^2)}{p^2 + \mathrm{i}\epsilon} \quad \text{（Landau 规范）}$$

- 有质量实矢量玻色子传播子：

$$\nu \quad \mu \quad = \frac{-\mathrm{i}(g^{\mu\nu} - p^{\mu}p^{\nu}/m^2)}{p^2 - m^2 + \mathrm{i}\epsilon} \quad \text{（幺正规范）}$$

$$\nu \quad \mu \quad = \frac{-\mathrm{i}g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\epsilon} \quad \text{（Feynman 规范）}$$

- 有质量复矢量玻色子传播子：

$$\nu \quad \mu \quad = \frac{-\mathrm{i}(g^{\mu\nu} - p^{\mu}p^{\nu}/m^2)}{p^2 - m^2 + \mathrm{i}\epsilon} \quad \text{（幺正规范）}$$

$$\nu \quad \mu \quad = \frac{-\mathrm{i}g^{\mu\nu}}{p^2 - m^2 + \mathrm{i}\epsilon} \quad \text{（Feynman 规范）}$$

实标量场外线 Feynman 规则如下。

- 实标量玻色子入射外线: $- - \overset{p}{\dashrightarrow} - \bullet = 1$

- 实标量玻色子出射外线: $\bullet - \overset{p}{\dashrightarrow} - - = 1$

复标量场外线 Feynman 规则如下。

- 正标量玻色子入射外线: $- - \overset{p}{\blacktriangleright} - - \bullet = 1$

- 反标量玻色子入射外线: $- - \overset{p}{\blacktriangleleft} - - \bullet = 1$

- 正标量玻色子出射外线: $\bullet - - \overset{p}{\blacktriangleright} - - = 1$

- 反标量玻色子出射外线: $\bullet - - \overset{p}{\blacktriangleleft} - - = 1$

以 λ 代表自旋极化指标（如螺旋度），**Dirac 旋量场**外线 Feynman 规则如下。

- Dirac 正费米子入射外线: $\lambda \overset{p}{\longrightarrow} \bullet = u(\mathbf{p}, \lambda)$

- Dirac 反费米子入射外线: $\lambda \overset{p}{\longleftarrow} \bullet = \bar{v}(\mathbf{p}, \lambda)$

- Dirac 正费米子出射外线: $\bullet \overset{p}{\longrightarrow} \lambda = \bar{u}(\mathbf{p}, \lambda)$

- Dirac 反费米子出射外线: $\bullet \overset{p}{\longleftarrow} \lambda = v(\mathbf{p}, \lambda)$

在计算非极化振幅模方时，可利用自旋求和关系

$$\sum_\lambda u(\mathbf{p}, \lambda)\bar{u}(\mathbf{p}, \lambda) = \not{p} + m, \quad \sum_\lambda v(\mathbf{p}, \lambda)\bar{v}(\mathbf{p}, \lambda) = \not{p} - m. \tag{250}$$

以 λ 代表自旋极化指标，**实矢量场**外线 Feynman 规则如下。

- 实矢量玻色子入射外线: $\lambda; \mu \overset{p}{\sim\!\!\sim\!\!\sim} \bullet = \varepsilon^\mu(\mathbf{p}, \lambda)$

- 实矢量玻色子出射外线: $\bullet \overset{p}{\sim\!\!\sim\!\!\sim} \lambda; \mu = \varepsilon^{\mu*}(\mathbf{p}, \lambda)$

复矢量场外线 Feynman 规则如下。

- 正矢量玻色子入射外线：$\lambda;\mu$ $\sim\!\!\!\!\!\!\rightarrow\!\!\!\bullet$ $= \varepsilon^\mu(\mathbf{p},\lambda)$

- 反矢量玻色子入射外线：$\lambda;\mu$ $\sim\!\!\!\leftarrow\!\!\!\bullet$ $= \varepsilon^\mu(\mathbf{p},\lambda)$

- 正矢量玻色子出射外线：$\bullet\!\!\!\rightarrow\!\!\!\sim$ $\lambda;\mu$ $= \varepsilon^{\mu*}(\mathbf{p},\lambda)$

- 反矢量玻色子出射外线：$\bullet\!\!\!\leftarrow\!\!\!\sim$ $\lambda;\mu$ $= \varepsilon^{\mu*}(\mathbf{p},\lambda)$

在计算非极化振幅模方时，若包含无质量矢量玻色子外线，可利用极化求和替换关系

$$\sum_\lambda \varepsilon_\mu^*(\mathbf{p},\lambda)\varepsilon_\nu(\mathbf{p},\lambda) \to -g_{\mu\nu}\,; \tag{251}$$

若包含有质量矢量玻色子外线，可利用极化求和关系

$$\sum_\lambda \varepsilon_\mu^*(\mathbf{p},\lambda)\varepsilon_\nu(\mathbf{p},\lambda) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}. \tag{252}$$

7   常用单位和标准模型参数

本节数据来自 Particle Data Group 发布的 2022 版 *Review of Particle Physics* [5]。

在有理化的自然单位制中，光速、约化 Planck 常数和真空介电常数均取为 1，即 $c = \hbar = \varepsilon_0 = 1$。从而，速度没有量纲 (dimension)；长度量纲与时间量纲相同，是能量量纲的倒数；能量、质量和动量具有相同的量纲；精细结构常数表达为

$$\alpha = \frac{e^2}{4\pi}, \tag{253}$$

而单位电荷量 $e = \sqrt{4\pi\alpha}$ 是没有量纲的。可以将能量单位电子伏特 (eV) 视作上述有量纲物理量的基本单位。

单位间转换关系为

$$1 = c = 2.99792458 \times 10^{10}\ \mathrm{cm \cdot s^{-1}}, \tag{254}$$

$$1 = \hbar = 6.582119569 \times 10^{-25}\ \mathrm{GeV \cdot s}, \tag{255}$$

$$1 = \hbar c = 1.973269804 \times 10^{-14}\ \mathrm{GeV \cdot cm}, \tag{256}$$

$$1 = (\hbar c)^2 = 3.893793721 \times 10^{8}\ \mathrm{GeV^2 \cdot pb}, \tag{257}$$

由此得到

$$1\ \mathrm{s} = 2.997925 \times 10^{10}\ \mathrm{cm}, \qquad\qquad 1\ \mathrm{cm} = 3.335641 \times 10^{-11}\ \mathrm{s}, \tag{258}$$

$$1 \text{ s} = 1.519267 \times 10^{24} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 6.582120 \times 10^{-25} \text{ s}, \tag{259}$$

$$1 \text{ cm} = 5.067731 \times 10^{13} \text{ GeV}^{-1}, \quad 1 \text{ GeV}^{-1} = 1.973270 \times 10^{-14} \text{ cm}, \tag{260}$$

$$1 \text{ cm}^2 = 2.568189 \times 10^{27} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^{-28} \text{ cm}^2, \tag{261}$$

$$1 \text{ cm}^3 \cdot \text{s}^{-1} = 8.566558 \times 10^{16} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 1.167330 \times 10^{-17} \text{ cm}^3 \cdot \text{s}^{-1}. \tag{262}$$

靶 (barn) 是散射截面的常用单位，记作 b，满足

$$1 \text{ b} = 10^{-24} \text{ cm}^2 = 10^9 \text{ nb} = 10^{12} \text{ pb} = 10^{15} \text{ fb} = 10^{18} \text{ ab}, \tag{263}$$

$$1 \text{ pb} = 10^{-36} \text{ cm}^2 = 2.568189 \times 10^{-9} \text{ GeV}^{-2}, \quad 1 \text{ GeV}^{-2} = 3.893794 \times 10^8 \text{ pb}. \tag{264}$$

Fermi 耦合常数是

$$G_{\text{F}} = 1.1663788(6) \times 10^{-5} \text{ GeV}^{-2}, \tag{265}$$

括号内数字代表测量值的 1σ 不确定度，由树图阶关系式

$$\frac{G_{\text{F}}}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8m_{\text{W}}^2}, \tag{266}$$

得到 Higgs 场真空期待值为

$$v = (\sqrt{2}G_{\text{F}})^{-1/2} = 246.2196 \text{ GeV}. \tag{267}$$

在低能标（Thomson 极限）处，精细结构常数为

$$\alpha = \frac{1}{137.035999180(10)}; \tag{268}$$

在 $\overline{\text{MS}}$ 重整化方案（以 ˆ 为标志）中，α^{-1} 跑动到 $\mu = m_Z$ 能标处的数值是

$$\widehat{\alpha}^{-1}(m_Z) = 127.951 \pm 0.009. \tag{269}$$

在 $\overline{\text{MS}}$ 方案中，$\mu = m_Z$ 能标处强耦合常数 $\alpha_{\text{s}} = g_{\text{s}}^2/(4\pi)$ 的数值为

$$\widehat{\alpha}_{\text{s}}(m_Z) = 0.1179 \pm 0.0009, \tag{270}$$

Weinberg 角 θ_{W} 的数值对应于

$$\widehat{s}_{\text{W}}^2 = \sin^2 \widehat{\theta}_{\text{W}}(m_Z) = 0.23122 \pm 0.00004. \tag{271}$$

在标准模型中，光子、胶子和中微子没有质量，其它基本粒子的质量为

$$m_W = 80.377 \pm 0.012 \text{ GeV}, \qquad m_Z = 91.1876 \pm 0.0021 \text{ GeV}, \tag{272}$$

$$m_H = 125.25 \pm 0.17 \text{ GeV}, \qquad m_e = 0.51099895000(15) \text{ MeV}, \qquad (273)$$

$$m_\mu = 105.6583755(23) \text{ MeV}, \quad m_\tau = 1776.86 \pm 0.12 \text{ MeV}, \qquad (274)$$

$$m_u = 2.16^{+0.49}_{-0.26} \text{ MeV}, \qquad m_d = 4.67^{+0.48}_{-0.17} \text{ MeV}, \qquad (275)$$

$$m_s = 93.4^{+8.6}_{-3.4} \text{ MeV}, \qquad m_c = 1.27 \pm 0.02 \text{ GeV}, \qquad (276)$$

$$m_b = 4.18^{+0.03}_{-0.02} \text{ GeV}, \qquad m_t = 172.69 \pm 0.30 \text{ GeV}. \qquad (277)$$

这里，u、d、s 夸克的质量是 $\mu = 2$ GeV 能标处的 $\overline{\text{MS}}$ 质量，c 和 b 夸克的质量分别是 $\mu = m_c$ 和 $\mu = m_b$ 能标处的 $\overline{\text{MS}}$ 质量，其余粒子的质量均为极点质量 (pole mass)。相应地，计算出来的 c、b 夸克极点质量为

$$m_c^{\text{pole}} = 1.67 \pm 0.07 \text{ GeV}, \quad m_b^{\text{pole}} = 4.78 \pm 0.06 \text{ GeV}. \qquad (278)$$

质子和中子的质量为

$$m_p = 938.27208816(29) \text{ MeV}, \quad m_n = 939.56542052(54) \text{ MeV}. \qquad (279)$$

在电弱能标附近作领头阶计算时，可将单位电荷量取为

$$e = \sqrt{4\pi\widehat{\alpha}(m_Z)} = 0.3133885, \qquad (280)$$

将强耦合常数取为

$$g_{\text{s}} = \sqrt{4\pi\widehat{\alpha}_{\text{s}}(m_Z)} = 1.217200 . \qquad (281)$$

从树图阶关系计算 Higgs 场四线性耦合常数 λ 和 Yukawa 耦合常数 y_t、y_b、y_τ、y_c，得

$$\lambda = \frac{m_H^2}{2v^2} = 0.1293839, \quad y_t = \frac{\sqrt{2}m_t}{v} = 0.9918808, \quad y_b = \frac{\sqrt{2}m_b}{v} = 2.400870 \times 10^{-2}, \quad (282)$$

$$y_\tau = \frac{\sqrt{2}m_\tau}{v} = 1.020576 \times 10^{-2}, \quad y_c = \frac{\sqrt{2}m_c}{v} = 7.294508 \times 10^{-3}. \qquad (283)$$

耦合常数 g 和 g' 有以下两种取值方式。

1. 根据树图阶关系 $\sin^2\theta_{\text{W}} = 1 - m_W^2/m_Z^2$ 计算 Weinberg 角，得

$$s_{\text{W}}^2 = 1 - \frac{m_W^2}{m_Z^2} = 0.2230519, \quad c_{\text{W}}^2 = 1 - s_{\text{W}}^2 = 0.7769481, \qquad (284)$$

$$s_{\text{W}} = \sqrt{s_{\text{W}}^2} = 0.4722837, \qquad c_{\text{W}} = \sqrt{c_{\text{W}}^2} = 0.8814466, \qquad (285)$$

故

$$g = \frac{e}{s_{\text{W}}} = 0.6635599, \quad g' = \frac{e}{c_{\text{W}}} = 0.3555389 . \qquad (286)$$

2. 根据 $\overline{\text{MS}}$ 方案中 Weinberg 角的数值 (271) 计算 g 和 g', 得

$$c_{\text{W}}^2 = 1 - \widehat{s}_{\text{W}}^2 = 0.76878, \quad s_{\text{W}} = \sqrt{\widehat{s}_{\text{W}}^2} = 0.4808534, \quad c_{\text{W}} = \sqrt{c_{\text{W}}^2} = 0.8768010, \quad (287)$$

$$g = \frac{e}{s_{\text{W}}} = 0.6517340, \quad g' = \frac{e}{c_{\text{W}}} = 0.3574226\,. \quad (288)$$

将 CKM 矩阵参数化为

$$V = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ & 1 & \\ -s_{13}\mathrm{e}^{\mathrm{i}\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}, \quad (289)$$

其中 $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$。实验拟合值为

$$s_{12} = 0.22500 \pm 0.00067, \qquad s_{13} = 0.00369 \pm 0.00011, \quad (290)$$

$$s_{23} = 0.04182^{+0.00085}_{-0.00074}, \qquad \delta = 1.144 \pm 0.027\,. \quad (291)$$

如果只讨论第一、二代夸克的混合, 可利用 Cabibbo 转动角 θ_{C} 将 CKM 矩阵近似地表达为

$$V \simeq \begin{pmatrix} \cos\theta_{\text{C}} & \sin\theta_{\text{C}} & \\ -\sin\theta_{\text{C}} & \cos\theta_{\text{C}} & \\ & & 1 \end{pmatrix}, \quad \sin\theta_{\text{C}} = s_{12} = 0.225\,. \quad (292)$$

参考文献

[1] M. E. Peskin and D. V. Schroeder, "An Introduction to Quantum Field Theory," Reading, USA: Addison-Wesley (1995), 842 pages.

[2] T. P. Cheng and L. F. Li, "Gauge Theory of Elementary Particle Physics," Oxford, UK: Clarendon (1984), 536 pages.

[3] A. Denner, "Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200," Fortsch. Phys. **41**, 307 (1993) [arXiv:0709.1075 [hep-ph]].

[4] 杜东生，杨茂志，《粒子物理导论》，中国北京：科学出版社 (2014)，402 页。

[5] R. L. Workman [Particle Data Group], "Review of Particle Physics," PTEP **2022**, 083C01 (2022)