CP-conserved Lagrangians

(1) CP-even scalar ϕ, Majarana fermion χ

$$\mathcal{L}_{\mathrm{M1}} = \frac{1}{\Lambda}\phi(k_1 B_{\mu\nu}B^{\mu\nu} + k_2 W^a_{\mu\nu}W^{a\mu\nu} + k_3 G^a_{\mu\nu}G^{a\mu\nu}) + \frac{1}{2}g_\chi \phi\bar{\chi}\chi - \frac{1}{2}m_\phi \phi^2 - \frac{1}{2}m_\chi \bar{\chi}\chi$$

(2) CP-odd scalar ϕ, Majarana fermion χ

$$\mathcal{L}_{\mathrm{M2}} = \frac{1}{\Lambda}\phi(k_1 B_{\mu\nu}\tilde{B}^{\mu\nu} + k_2 W^a_{\mu\nu}\tilde{W}^{a\mu\nu} + k_3 G^a_{\mu\nu}\tilde{G}^{a\mu\nu}) + \frac{1}{2}g_\chi \phi\bar{\chi}i\gamma_5\chi - \frac{1}{2}m_\phi \phi^2 - \frac{1}{2}m_\chi \bar{\chi}\chi$$

(3) CP-even scalar ϕ, real scalar χ

$$\mathcal{L}_{\mathrm{S}} = \frac{1}{\Lambda}\phi(k_1 B_{\mu\nu}B^{\mu\nu} + k_2 W^a_{\mu\nu}W^{a\mu\nu} + k_3 G^a_{\mu\nu}G^{a\mu\nu}) + \frac{1}{2}g_\chi \phi\chi^2 - \frac{1}{2}m_\phi^2 \phi^2 - \frac{1}{2}m_\chi^2 \chi^2$$

(4) CP-even scalar ϕ, real vector χ

$$\mathcal{L}_{\mathrm{V}} = \frac{1}{\Lambda}\phi(k_1 B_{\mu\nu}B^{\mu\nu} + k_2 W^a_{\mu\nu}W^{a\mu\nu} + k_3 G^a_{\mu\nu}G^{a\mu\nu}) + \frac{1}{2}g_\chi \phi\chi^\mu\chi_\mu - \frac{1}{2}m_\phi^2 \phi^2 + \frac{1}{2}m_\chi^2 \chi^\mu\chi_\mu$$

$$B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu, \quad W^{a\mu\nu} \equiv \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g_2 \varepsilon^{abc} W^{b\mu} W^{c\nu}$$

$$B_\mu = c_W A_\mu - s_W Z_\mu, \quad W^3_\mu = s_W A_\mu + c_W Z_\mu, \quad W^\pm_\mu \equiv \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$$

$$c_W \equiv \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \quad s_W \equiv \sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = c_W A_{\mu\nu} - s_W Z_{\mu\nu}$$

$$W^3_{\mu\nu} = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu - g_2 \varepsilon^{3bc} W^b_\mu W^c_\nu = s_W A_{\mu\nu} + c_W Z_{\mu\nu} - g_2 W^1_\mu W^2_\nu + g_2 W^2_\mu W^1_\nu$$

$$A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \quad Z_{\mu\nu} \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu$$

$$B_{\mu\nu}B^{\mu\nu} = c_W^2 A_{\mu\nu}A^{\mu\nu} - 2 s_W c_W A_{\mu\nu} Z^{\mu\nu} + s_W^2 Z_{\mu\nu} Z^{\mu\nu}$$

$$W^3_{\mu\nu}W^{3\mu\nu} \supset s_W^2 A_{\mu\nu}A^{\mu\nu} + 2 s_W c_W A_{\mu\nu} Z^{\mu\nu} + c_W^2 Z_{\mu\nu} Z^{\mu\nu}$$

$$k_1 B_{\mu\nu}B^{\mu\nu} + k_2 W^3_{\mu\nu}W^{3\mu\nu} \supset k_{\mathrm{AA}} A_{\mu\nu}A^{\mu\nu} + k_{\mathrm{AZ}} A_{\mu\nu}Z^{\mu\nu} + k_{\mathrm{ZZ}} Z_{\mu\nu}Z^{\mu\nu}$$

$$k_{\mathrm{AA}} \equiv k_1 c_W^2 + k_2 s_W^2, \quad k_{\mathrm{AZ}} \equiv 2 s_W c_W (k_2 - k_1), \quad k_{\mathrm{ZZ}} \equiv k_1 s_W^2 + k_2 c_W^2$$

$$A_{\mu\nu}A^{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) = 2(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu)$$

$$A_{\mu\nu}Z^{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) = 2(\partial_\mu A_\nu \partial^\mu Z^\nu - \partial_\mu A_\nu \partial^\nu Z^\mu)$$

$$Z_{\mu\nu}Z^{\mu\nu} = (\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) = 2(\partial_\mu Z_\nu \partial^\mu Z^\nu - \partial_\mu Z_\nu \partial^\nu Z^\mu)$$

$$W^1_{\mu\nu}W^{1\mu\nu} + W^2_{\mu\nu}W^{2\mu\nu} = \frac{1}{2}(F^+_{\mu\nu} + F^-_{\mu\nu})(F^{+\mu\nu} + F^{-\mu\nu}) - \frac{1}{2}(F^+_{\mu\nu} - F^-_{\mu\nu})(F^{+\mu\nu} - F^{-\mu\nu}) = 2F^+_{\mu\nu}F^{-\mu\nu}$$

$$k_2(W^1_{\mu\nu}W^{1\mu\nu} + W^2_{\mu\nu}W^{2\mu\nu}) = 2k_2 F^+_{\mu\nu}F^{-\mu\nu} \supset 2k_2(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu})$$

$$= 4k_2(\partial_\mu W^+_\nu \partial^\mu W^{-\nu} - \partial_\mu W^+_\nu \partial^\nu W^{-\mu})$$

$$k_3 G^a_{\mu\nu}G^{a\mu\nu} \supset 2k_3(\partial_\mu G^a_\nu \partial^\mu G^{a\nu} - \partial_\mu G^a_\nu \partial^\nu G^{a\mu})$$

$\phi(q) \to \chi(p_1) + \chi(p_2)$ decay width

(1) Majorana fermion χ, CP-even scalar ϕ, $\quad \mathcal{L}_{\rm S} \supset \frac{1}{2} g_\chi \phi \bar{\chi}\chi$

$$i\mathcal{M} = i g_\chi \bar{u}(p_1) v(p_2), \quad (i\mathcal{M})^* = i g_\chi \bar{v}(p_2) u(p_1)$$

$$\sum_{\rm spins} |\mathcal{M}|^2 = g_\chi^2 \mathrm{Tr}[(\not{p}_1 + m_\chi)(\not{p}_2 - m_\chi)] = 4 g_\chi^2 (p_1 \cdot p_2 - m_\chi^2) = 4 g_\chi^2 \left[\frac{1}{2}(m_\phi^2 - 2m_\chi^2) - m_\chi^2 \right]$$

$$= 2 g_\chi^2 (m_\phi^2 - 4m_\chi^2) = 2 g_\chi^2 m_\phi^2 \eta_\chi^2$$

$$\Gamma(\phi \to \chi\chi) = \frac{1}{2} \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\rm spins} |\mathcal{M}|^2 = \frac{1}{2} \frac{1}{8\pi} \frac{1}{m_\phi^2} \frac{m_\phi}{2} \eta_\chi 2 g_\chi^2 m_\phi^2 \eta_\chi^2 = \frac{\eta_\chi^3 g_\chi^2 m_\phi}{16\pi}$$

$$\eta_\chi \equiv \sqrt{1 - 4m_\chi^2 / m_\phi^2}$$

(2) Majorana fermion χ, CP-odd scalar ϕ, $\quad \mathcal{L}_{\rm S} \supset \frac{1}{2} g_\chi \phi \bar{\chi} i\gamma_5 \chi$

$$i\mathcal{M} = -g_\chi \bar{u}(p_1)\gamma_5 v(p_2), \quad (i\mathcal{M})^* = g_\chi \bar{v}(p_2)\gamma_5 u(p_1)$$

$$\sum_{\rm spins} |\mathcal{M}|^2 = -g_\chi^2 \mathrm{Tr}[(\not{p}_1 + m_\chi)\gamma_5(\not{p}_2 - m_\chi)\gamma_5] = 4 g_\chi^2 (p_1 \cdot p_2 + m_\chi^2) = 4 g_\chi^2 \left[\frac{1}{2}(m_\phi^2 - 2m_\chi^2) + m_\chi^2 \right] = 2 g_\chi^2 m_\phi^2$$

$$\Gamma(\phi \to \chi\chi) = \frac{1}{2} \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\rm spins} |\mathcal{M}|^2 = \frac{1}{2} \frac{1}{8\pi} \frac{1}{m_\phi^2} \frac{m_\phi}{2} \eta_\chi 2 g_\chi^2 m_\phi^2 = \frac{\eta_\chi g_\chi^2 m_\phi}{16\pi}$$

(3) Real scalar χ, $\quad \mathcal{L}_{\rm S} \supset \frac{1}{2} g_\chi \phi \chi^2$

$$i\mathcal{M} = i g_\chi, \quad \sum_{\rm spins} |\mathcal{M}|^2 = g_\chi^2$$

$$\Gamma(\phi \to \chi\chi) = \frac{1}{2} \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\rm spins} |\mathcal{M}|^2 = \frac{1}{2} \frac{1}{8\pi} \frac{1}{m_\phi^2} \frac{m_\phi}{2} \eta_\chi g_\chi^2 = \frac{\eta_\chi g_\chi^2}{32\pi m_\phi}$$

(4) Real vector χ, $\quad \mathcal{L}_{\rm V} \supset \frac{1}{2} g_\chi \phi \chi^\mu \chi_\mu$

$$i\mathcal{M} = i g_\chi g^{\mu\nu} \varepsilon_\mu^*(p_1) \varepsilon_\nu^*(p_2), \quad (i\mathcal{M})^* = -i g_\chi g^{\rho\sigma} \varepsilon_\rho(p_1) \varepsilon_\sigma(p_2)$$

$$\sum_{\rm spins} |\mathcal{M}|^2 = g_\chi^2 g^{\mu\nu} g^{\rho\sigma} \left(-g_{\mu\rho} + \frac{p_{1\mu} p_{1\rho}}{m_\chi^2} \right) \left(-g_{\nu\sigma} + \frac{p_{2\nu} p_{2\sigma}}{m_\chi^2} \right) = \frac{g_\chi^2}{m_\chi^4} [(p_1 \cdot p_2)^2 - m_\chi^2 (p_1^2 + p_2^2 - 4m_\chi^2)]$$

$$= \frac{g_\chi^2}{m_\chi^4} \left[\frac{1}{4}(m_\phi^2 - 2m_\chi^2)^2 + 2m_\chi^4 \right] = \frac{g_\chi^2}{4m_\chi^4} (m_\phi^4 - 4m_\phi^2 m_\chi^2 + 12 m_\chi^4) = \frac{g_\chi^2 m_\phi^4}{4m_\chi^4} (1 - 4\xi_\chi^2 + 12\xi_\chi^4)$$

$$\Gamma(\phi \to \chi\chi) = \frac{1}{2} \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\rm spins} |\mathcal{M}|^2 = \frac{1}{2} \frac{1}{8\pi} \frac{1}{m_\phi^2} \frac{m_\phi}{2} \eta_\chi \frac{g_\chi^2 m_\phi^4}{4m_\chi^4} (1 - 4\xi_\chi^2 + 12\xi_\chi^4) = \frac{\eta_\chi g_\chi^2 m_\phi^3}{128\pi m_\chi^4} (1 - 4\xi_\chi^2 + 12\xi_\chi^4)$$

$$\xi_\chi \equiv \frac{m_\chi}{m_\phi}$$

CP-even ϕ Feynman rules

$$\mathcal{L} \supset \frac{1}{\Lambda}\phi(k_1 B_{\mu\nu}B^{\mu\nu} + k_2 W^a_{\mu\nu}W^{a\mu\nu} + k_3 G^a_{\mu\nu}G^{a\mu\nu})$$

$$\supset \frac{\phi}{\Lambda}[2k_{\mathrm{AA}}(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu) + 2k_{\mathrm{AZ}}(\partial_\mu A_\nu \partial^\mu Z^\nu - \partial_\mu A_\nu \partial^\nu Z^\mu)$$

$$+ 2k_{\mathrm{ZZ}}(\partial_\mu Z_\nu \partial^\mu Z^\nu - \partial_\mu Z_\nu \partial^\nu Z^\mu) + 4k_2(\partial_\mu W^+_\nu \partial^\mu W^{-\nu} - \partial_\mu W^+_\nu \partial^\nu W^{-\mu})$$

$$+ 2k_3(\partial_\mu G^a_\nu \partial^\mu G^{a\nu} - \partial_\mu G^a_\nu \partial^\nu G^{a\mu})]$$

For momenta pointing into the vertex: $\partial_\mu \to -ip_\mu$

$\phi(q) - X_{1\mu}(p_1) - X_{2\nu}(p_2)$ Feynman rules

$$\phi A_\mu(p_1) A_\nu(p_2) \to 2k_{\mathrm{AA}}\frac{\phi}{\Lambda}(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu)$$

$$\to 2k_{\mathrm{AA}}\frac{\phi}{\Lambda}(g^{\rho\sigma}g^{\mu\nu}\partial_\rho A_\mu \partial_\sigma A_\nu + g^{\rho\sigma}g^{\nu\mu}\partial_\rho A_\nu \partial_\sigma A_\mu - g^{\rho\nu}g^{\mu\sigma}\partial_\rho A_\mu \partial_\sigma A_\nu - g^{\rho\mu}g^{\nu\sigma}\partial_\rho A_\nu \partial_\sigma A_\mu)$$

$$\to \frac{2ik_{\mathrm{AA}}}{\Lambda}[g^{\rho\sigma}g^{\mu\nu}(-ip_{1\rho})(-ip_{2\sigma}) + g^{\rho\sigma}g^{\nu\mu}(-ip_{2\rho})(-ip_{1\sigma}) - g^{\rho\nu}g^{\mu\sigma}(-ip_{1\rho})(-ip_{2\sigma}) - g^{\rho\mu}g^{\nu\sigma}(-ip_{2\rho})(-ip_{1\sigma})]$$

$$= -\frac{4ik_{\mathrm{AA}}}{\Lambda}(g^{\mu\nu}p_1 \cdot p_2 - p_2^\mu p_1^\nu)$$

$$\phi Z_\mu(p_1) Z_\nu(p_2) \to -\frac{4ik_{\mathrm{ZZ}}}{\Lambda}(g^{\mu\nu}p_1 \cdot p_2 - p_2^\mu p_1^\nu)$$

$$\phi A_\mu(p_1) Z_\nu(p_2) \to 2k_{\mathrm{AZ}}\frac{\phi}{\Lambda}(\partial_\mu A_\nu \partial^\mu Z^\nu - \partial_\mu A_\nu \partial^\nu Z^\mu) = 2k_{\mathrm{AZ}}\frac{\phi}{\Lambda}(g^{\rho\sigma}g^{\mu\nu} - g^{\rho\nu}g^{\mu\sigma})\partial_\rho A_\mu \partial_\sigma Z_\nu$$

$$\to \frac{2ik_{\mathrm{AZ}}}{\Lambda}(g^{\rho\sigma}g^{\mu\nu} - g^{\rho\nu}g^{\mu\sigma})(-ip_{1\rho})(-ip_{2\sigma}) = -\frac{2ik_{\mathrm{AZ}}}{\Lambda}(g^{\mu\nu}p_1 \cdot p_2 - p_2^\mu p_1^\nu)$$

$$\phi W^+_\mu(p_1) W^-_\nu(p_2) \to -\frac{4ik_2}{\Lambda}(g^{\mu\nu}p_1 \cdot p_2 - p_2^\mu p_1^\nu)$$

$$\phi G^a_\mu(p_1) G^a_\nu(p_2) \to -\frac{4ik_3}{\Lambda}(g^{\mu\nu}p_1 \cdot p_2 - p_2^\mu p_1^\nu)$$

CP-even ϕ Decay widths

$\phi(q) \to X_1(p_1) + X_2(p_2)$ kinematics

$$m_\phi^2 = q^2 = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2p_1 \cdot p_2$$

$$p_1 \cdot p_2 = \frac{1}{2}(m_\phi^2 - m_1^2 - m_2^2)$$

$$|\mathbf{p}_1| = \frac{1}{2m_\phi}\sqrt{\left[m_\phi^2 - (m_1 + m_2)^2\right]\left[m_\phi^2 - (m_1 - m_2)^2\right]}$$

$$m_1 = m_2 = m_X \quad \to \quad p_1 \cdot p_2 = \frac{1}{2}(m_\phi^2 - 2m_X^2), \quad |\mathbf{p}_1| = |\mathbf{p}_2| = \frac{m_\phi}{2}\sqrt{1 - 4m_X^2 / m_\phi^2} = \frac{m_\phi}{2}\eta_X, \quad \eta_X \equiv \sqrt{1 - 4m_X^2 / m_\phi^2}$$

$$m_2 = 0 \quad \to \quad p_1 \cdot p_2 = \frac{1}{2}(m_\phi^2 - m_1^2) = \frac{m_\phi^2}{2}(1 - \xi_1^2), \quad |\mathbf{p}_1| = |\mathbf{p}_2| = \frac{1}{2m_\phi}(m_\phi^2 - m_1^2) = \frac{m_\phi}{2}(1 - \xi_1^2), \quad \xi_1 \equiv \frac{m_1}{m_\phi}$$

$$m_1 = m_2 = 0 \quad \to \quad p_1 \cdot p_2 = \frac{m_\phi^2}{2}, \quad |\mathbf{p}_1| = |\mathbf{p}_2| = \frac{m_\phi}{2}$$

$$\Gamma(\phi \to X_1 X_2) = n_{\text{id}} \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\text{spins}} |\mathcal{M}|^2, \quad n_{\text{id}} = \begin{cases} 1, & X_1 \neq X_2 \\ \frac{1}{2}, & X_1 = X_2 \end{cases}$$

$\phi(q) \to \gamma(p_1) + \gamma(p_2)$

$$i\mathcal{M} = -\frac{4ik_{\text{AA}}}{\Lambda}(g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$(i\mathcal{M})^* = \frac{4ik_{\text{AA}}}{\Lambda}(g^{\rho\sigma} p_1 \cdot p_2 - p_2^\rho p_1^\sigma)\varepsilon_\rho(p_1)\varepsilon_\sigma(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{16k_{\text{AA}}^2}{\Lambda^2}(g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu)(g^{\rho\sigma} p_1 \cdot p_2 - p_2^\rho p_1^\sigma)\sum_{\text{spins}} \varepsilon_\mu^*(p_1)\varepsilon_\rho(p_1)\varepsilon_\nu^*(p_2)\varepsilon_\sigma(p_2)$$

$$= \frac{16k_{\text{AA}}^2}{\Lambda^2}(g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu)(g^{\rho\sigma} p_1 \cdot p_2 - p_2^\rho p_1^\sigma)(-g_{\mu\rho})(-g_{\nu\sigma})$$

$$= \frac{16k_{\text{AA}}^2}{\Lambda^2}(g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu)(g_{\mu\nu} p_1 \cdot p_2 - p_{2\mu} p_{1\nu}) = \frac{16k_{\text{AA}}^2}{\Lambda^2}[2(p_1 \cdot p_2)^2 + p_1^2 p_2^2]$$

$$= \frac{32k_{\text{AA}}^2}{\Lambda^2}(p_1 \cdot p_2)^2 = \frac{8k_{\text{AA}}^2 m_\phi^4}{\Lambda^2}$$

$$\Gamma(\phi \to \gamma\gamma) = \frac{1}{2}\frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_\phi^2}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{2}\frac{1}{8\pi}\frac{1}{m_\phi^2}\frac{m_\phi}{2}\frac{8k_{\text{AA}}^2 m_\phi^4}{\Lambda^2} = \frac{k_{\text{AA}}^2 m_\phi^3}{4\pi\Lambda^2}$$

$$\phi(q) \to Z(p_1) + Z(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{16k_{ZZ}^2}{\Lambda^2}(g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu)(g^{\rho\sigma} p_1 \cdot p_2 - p_2^\rho p_1^\sigma)\left(-g_{\mu\rho} + \frac{p_{1\mu} p_{1\rho}}{m_Z^2}\right)\left(-g_{\nu\sigma} + \frac{p_{2\nu} p_{2\sigma}}{m_Z^2}\right)$$

$$= \frac{16k_{ZZ}^2}{\Lambda^2}[2(p_1 \cdot p_2)^2 + p_1^2 p_2^2] = \frac{16k_{ZZ}^2}{\Lambda^2}\left[2\frac{1}{4}(m_\phi^2 - 2m_Z^2)^2 + m_Z^4\right] = \frac{8k_{ZZ}^2}{\Lambda^2}(m_\phi^4 - 4m_\phi^2 m_Z^2 + 6m_Z^4) = \frac{8k_{ZZ}^2 m_\phi^4}{\Lambda^2}(1 - 4\xi_Z^2 + 6\xi_Z^4)$$

$$\Gamma(\phi \to ZZ) = \frac{1}{2}\frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_\phi^2}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{1}{2}\frac{1}{8\pi}\frac{1}{m_\phi^2}\frac{m_\phi}{2}\eta_Z \frac{8k_{ZZ}^2 m_\phi^4}{\Lambda^2}(1 - 4\xi_Z^2 + 6\xi_Z^4) = \frac{k_{ZZ}^2 m_\phi^3}{4\pi\Lambda^2}\eta_Z(1 - 4\xi_Z^2 + 6\xi_Z^4)$$

$$\eta_X \equiv \sqrt{1 - 4m_X^2 / m_\phi^2}, \quad \xi_X \equiv m_X / m_\phi$$

$$\phi(q) \to \gamma(p_1) + Z(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{4k_{\mathrm{AZ}}^2}{\Lambda^2}(g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu)(g^{\rho\sigma} p_1 \cdot p_2 - p_2^\rho p_1^\sigma)(-g_{\mu\rho})\left(-g_{\nu\sigma} + \frac{p_{2\nu} p_{2\sigma}}{m_Z^2}\right)$$

$$= \frac{4k_{\mathrm{AZ}}^2}{\Lambda^2}[2(p_1 \cdot p_2)^2 + p_1^2 p_2^2] = \frac{4k_{\mathrm{AZ}}^2}{\Lambda^2} 2\frac{1}{4}(m_\phi^2 - m_Z^2)^2 = \frac{2k_{\mathrm{AZ}}^2}{\Lambda^2}(m_\phi^2 - m_Z^2)^2 = \frac{2k_{\mathrm{AZ}}^2 m_\phi^4}{\Lambda^2}(1 - \xi_Z^2)^2$$

$$\Gamma(\phi \to \gamma Z) = \frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_\phi^2}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{1}{8\pi}\frac{1}{m_\phi^2}\frac{m_\phi}{2}(1 - \xi_Z^2)\frac{2k_{\mathrm{AZ}}^2 m_\phi^4}{\Lambda^2}(1 - \xi_Z^2)^2 = \frac{k_{\mathrm{AZ}}^2 m_\phi^3}{8\pi\Lambda^2}(1 - \xi_Z^2)^3$$

$$\phi(q) \to W^+(p_1) + W^-(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{16k_2^2}{\Lambda^2}(g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu)(g^{\rho\sigma} p_1 \cdot p_2 - p_2^\rho p_1^\sigma)\left(-g_{\mu\rho} + \frac{p_{1\mu} p_{1\rho}}{m_W^2}\right)\left(-g_{\nu\sigma} + \frac{p_{2\nu} p_{2\sigma}}{m_W^2}\right) = \frac{8k_2^2 m_\phi^4}{\Lambda^2}(1 - 4\xi_W^2 + 6\xi_W^4)$$

$$\Gamma(\phi \to W^+W^-) = \frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_\phi^2}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{k_2^2 m_\phi^3}{2\pi\Lambda^2}\eta_W(1 - 4\xi_W^2 + 6\xi_W^4)$$

$$\phi(q) \to g(p_1) + g(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{16k_3^2}{\Lambda^2}(g^{\mu\nu} p_1 \cdot p_2 - p_2^\mu p_1^\nu)(g^{\rho\sigma} p_1 \cdot p_2 - p_2^\rho p_1^\sigma)(-g_{\mu\rho})(-g_{\nu\sigma}) = \frac{8k_3^2 m_\phi^4}{\Lambda^2}$$

$$\Gamma(\phi \to gg) = 8\frac{1}{2}\frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_\phi^2}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{2k_3^2 m_\phi^3}{\pi\Lambda^2}$$

CP-odd ϕ Feynman rules

$$k_1 B_{\mu\nu}\tilde{B}^{\mu\nu} + k_2 W^3_{\mu\nu}\tilde{W}^{3\mu\nu} \supset k_{\mathrm{AA}} A_{\mu\nu}\tilde{A}^{\mu\nu} + k_{\mathrm{AZ}} A_{\mu\nu}\tilde{Z}^{\mu\nu} + k_{\mathrm{ZZ}} Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$

$$A_{\mu\nu}\tilde{Z}^{\mu\nu} \equiv \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} A_{\mu\nu} Z_{\rho\sigma} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\rho Z_\sigma - \partial_\sigma Z_\rho) = 2\varepsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu \partial_\rho Z_\sigma$$

$$W^1_{\mu\nu}\tilde{W}^{1\mu\nu} + W^2_{\mu\nu}\tilde{W}^{2\mu\nu} = \frac{1}{2}(F^+_{\mu\nu} + F^-_{\mu\nu})(\tilde{F}^{+\mu\nu} + \tilde{F}^{-\mu\nu}) - \frac{1}{2}(F^+_{\mu\nu} - F^-_{\mu\nu})(\tilde{F}^{+\mu\nu} - \tilde{F}^{-\mu\nu}) = 2F^+_{\mu\nu}\tilde{F}^{-\mu\nu}$$

$$\supset 2\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial_\rho W^-_\sigma - \partial_\sigma W^-_\rho) = 4\varepsilon^{\mu\nu\rho\sigma}\partial_\mu W^+_\nu \partial_\rho W^-_\sigma$$

$$\mathcal{L} \supset \frac{1}{\Lambda}\phi(k_1 B_{\mu\nu}\tilde{B}^{\mu\nu} + k_2 W^a_{\mu\nu}\tilde{W}^{a\mu\nu} + k_3 G^a_{\mu\nu}\tilde{G}^{a\mu\nu})$$

$$\supset \frac{\phi}{\Lambda}(2k_{\mathrm{AA}}\varepsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu \partial_\rho A_\sigma + 2k_{\mathrm{AZ}}\varepsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu \partial_\rho Z_\sigma + 2k_{\mathrm{ZZ}}\varepsilon^{\mu\nu\rho\sigma}\partial_\mu Z_\nu \partial_\rho Z_\sigma$$
$$+ 4k_2\varepsilon^{\mu\nu\rho\sigma}\partial_\mu W^+_\nu \partial_\rho W^-_\sigma + 2k_3\varepsilon^{\mu\nu\rho\sigma}\partial_\mu G^a_\nu \partial_\rho G^a_\sigma)$$

For momenta pointing into the vertex: $\partial_\mu \to -ip_\mu$

$\phi(q) - X_{1\mu}(p_1) - X_{2\nu}(p_2)$ Feynman rules

$$\phi A_\mu(p_1) A_\nu(p_2) \to 2k_{\mathrm{AA}}\frac{\phi}{\Lambda}\varepsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu \partial_\rho A_\sigma \to 2k_{\mathrm{AA}}\frac{\phi}{\Lambda}(\varepsilon^{\rho\mu\sigma\nu}\partial_\rho A_\mu \partial_\sigma A_\nu + \varepsilon^{\rho\nu\sigma\mu}\partial_\rho A_\nu \partial_\sigma A_\mu)$$

$$\to \frac{2ik_{\mathrm{AA}}}{\Lambda}[\varepsilon^{\rho\mu\sigma\nu}(-ip_{1\rho})(-ip_{2\sigma}) + \varepsilon^{\rho\nu\sigma\mu}(-ip_{2\rho})(-ip_{1\sigma})]$$

$$= -\frac{4ik_{\mathrm{AA}}}{\Lambda}\varepsilon^{\rho\mu\sigma\nu} p_{1\rho}p_{2\sigma} = \frac{4ik_{\mathrm{AA}}}{\Lambda}\varepsilon^{\mu\nu\rho\sigma} p_{1\rho}p_{2\sigma}$$

$$\phi Z_\mu(p_1) Z_\nu(p_2) \to \frac{4ik_{\mathrm{ZZ}}}{\Lambda}\varepsilon^{\mu\nu\rho\sigma} p_{1\rho}p_{2\sigma}$$

$$\phi A_\mu(p_1) Z_\nu(p_2) \to 2k_{\mathrm{AZ}}\frac{\phi}{\Lambda}\varepsilon^{\rho\mu\sigma\nu}\partial_\rho A_\mu \partial_\sigma Z_\nu \to -\frac{2ik_{\mathrm{AZ}}}{\Lambda}\varepsilon^{\rho\mu\sigma\nu} p_{1\rho}p_{2\sigma} = \frac{2ik_{\mathrm{AZ}}}{\Lambda}\varepsilon^{\mu\nu\rho\sigma} p_{1\rho}p_{2\sigma}$$

$$\phi W^+_\mu(p_1) W^-_\nu(p_2) \to \frac{4ik_2}{\Lambda}\varepsilon^{\mu\nu\rho\sigma} p_{1\rho}p_{2\sigma}$$

$$\phi G^a_\mu(p_1) G^a_\nu(p_2) \to \frac{4ik_3}{\Lambda}\varepsilon^{\mu\nu\rho\sigma} p_{1\rho}p_{2\sigma}$$

CP-odd ϕ Decay widths

$$\phi(q) \to \gamma(p_1) + \gamma(p_2)$$

$$i\mathcal{M} = \frac{4ik_{\text{AA}}}{\Lambda}\varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$(i\mathcal{M})^* = -\frac{4ik_{\text{AA}}}{\Lambda}\varepsilon^{\rho\sigma\gamma\delta} p_{1\gamma} p_{2\delta} \varepsilon_\rho(p_1)\varepsilon_\sigma(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{16k_{\text{AA}}^2}{\Lambda^2}\varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \varepsilon^{\rho\sigma\gamma\delta} p_{1\gamma} p_{2\delta} \sum_{\text{spins}} \varepsilon_\mu^*(p_1)\varepsilon_\rho(p_1)\varepsilon_\nu^*(p_2)\varepsilon_\sigma(p_2)$$

$$= \frac{16k_{\text{AA}}^2}{\Lambda^2}\varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \varepsilon^{\rho\sigma\gamma\delta} p_{1\gamma} p_{2\delta}(-g_{\mu\rho})(-g_{\nu\sigma})$$

$$= \frac{16k_{\text{AA}}^2}{\Lambda^2} 2[(p_1 \cdot p_2)^2 - p_1^2 p_2^2] = \frac{32k_{\text{AA}}^2}{\Lambda^2}(p_1 \cdot p_2)^2 = \frac{8k_{\text{AA}}^2 m_\phi^4}{\Lambda^2}$$

$$\Gamma(\phi \to \gamma\gamma) = \frac{1}{2}\frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_\phi^2}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{2}\frac{1}{8\pi}\frac{1}{m_\phi^2}\frac{m_\phi}{2}\frac{8k_{\text{AA}}^2 m_\phi^4}{\Lambda^2} = \frac{k_{\text{AA}}^2 m_\phi^3}{4\pi\Lambda^2}$$

$$\phi(q) \to Z(p_1) + Z(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{16k_{\text{ZZ}}^2}{\Lambda^2}\varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \varepsilon^{\rho\sigma\gamma\delta} p_{1\gamma} p_{2\delta}\left(-g_{\mu\rho} + \frac{p_{1\mu}p_{1\rho}}{m_Z^2}\right)\left(-g_{\nu\sigma} + \frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$= \frac{16k_{\text{ZZ}}^2}{\Lambda^2} 2[(p_1 \cdot p_2)^2 - p_1^2 p_2^2] = \frac{16k_{\text{ZZ}}^2}{\Lambda^2} 2\left[\frac{1}{4}(m_\phi^2 - 2m_Z^2)^2 - m_Z^4\right] = \frac{8k_{\text{ZZ}}^2}{\Lambda^2}(m_\phi^4 - 4m_\phi^2 m_Z^2) = \frac{8k_{\text{ZZ}}^2 m_\phi^4}{\Lambda^2}\eta_Z^2$$

$$\Gamma(\phi \to ZZ) = \frac{1}{2}\frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_\phi^2}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{2}\frac{1}{8\pi}\frac{1}{m_\phi^2}\frac{m_\phi}{2}\eta_Z \frac{8k_{\text{ZZ}}^2 m_\phi^4}{\Lambda^2}\eta_Z^2 = \frac{k_{\text{ZZ}}^2 m_\phi^3}{4\pi\Lambda^2}\eta_Z^3$$

$$\phi(q) \to \gamma(p_1) + Z(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{4k_{\text{AZ}}^2}{\Lambda^2} \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \varepsilon^{\rho\sigma\gamma\delta} p_{1\gamma} p_{2\delta} (-g_{\mu\rho}) \left(-g_{\nu\sigma} + \frac{p_{2\nu} p_{2\sigma}}{m_Z^2} \right)$$

$$= \frac{4k_{\text{AZ}}^2}{\Lambda^2} 2[(p_1 \cdot p_2)^2 - p_1^2 p_2^2] = \frac{4k_{\text{AZ}}^2}{\Lambda^2} 2 \frac{1}{4} (m_\phi^2 - m_Z^2)^2 = \frac{2k_{\text{AZ}}^2}{\Lambda^2} (m_\phi^2 - m_Z^2)^2 = \frac{2k_{\text{AZ}}^2 m_\phi^4}{\Lambda^2} (1 - \xi_Z^2)^2$$

$$\Gamma(\phi \to \gamma Z) = \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{8\pi} \frac{1}{m_\phi^2} \frac{m_\phi}{2} (1 - \xi_Z^2) \frac{2k_{\text{AZ}}^2 m_\phi^4}{\Lambda^2} (1 - \xi_Z^2)^2 = \frac{k_{\text{AZ}}^2 m_\phi^3}{8\pi\Lambda^2} (1 - \xi_Z^2)^3$$

$$\phi(q) \to W^+(p_1) + W^-(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{16k_2^2}{\Lambda^2} \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \varepsilon^{\rho\sigma\gamma\delta} p_{1\gamma} p_{2\delta} \left(-g_{\mu\rho} + \frac{p_{1\mu} p_{1\rho}}{m_W^2} \right) \left(-g_{\nu\sigma} + \frac{p_{2\nu} p_{2\sigma}}{m_W^2} \right) = \frac{8k_2^2 m_\phi^4}{\Lambda^2} \eta_W^2$$

$$\Gamma(\phi \to W^+ W^-) = \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{k_2^2 m_\phi^3}{2\pi\Lambda^2} \eta_W^3$$

$$\phi(q) \to g(p_1) + g(p_2)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{16k_3^2}{\Lambda^2} \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \varepsilon^{\rho\sigma\gamma\delta} p_{1\gamma} p_{2\delta} (-g_{\mu\rho})(-g_{\nu\sigma}) = \frac{8k_3^2 m_\phi^4}{\Lambda^2}$$

$$\Gamma(\phi \to gg) = 8 \frac{1}{2} \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{2k_3^2 m_\phi^3}{\pi\Lambda^2}$$

DM annihilation cross section

$$\langle \sigma_{\rm ann} v_{\rm Møl} \rangle^{\rm comoving} = \langle \sigma_{\rm ann} v_{\rm lab} \rangle^{\rm lab}$$

Laboratory frame:

$$s = 4m_\chi^2 + m_\chi^2 v^2 + \frac{3}{4} m_\chi^2 v^4 + \mathcal{O}(v^6), \quad v \equiv v_{\rm lab} = \frac{\sqrt{1-4m_\chi^2/s}}{1-2m_\chi^2/s}$$

$$\sigma_{\rm ann} v = a + bv^2 + \mathcal{O}(v^4)$$

$$\langle \sigma_{\rm ann} v \rangle = a + 6bx^{-1} + \mathcal{O}(x^{-2}), \quad x \equiv m_\chi / T$$

Center-of-mass frame:

$$\chi(q_1) + \chi(q_2) \to \phi^{(*)}(q) \to X(p_1) + \bar{X}(p_2)$$

$$s = q^2 = (q_1+q_2)^2 = 2m_\chi^2 + 2q_1\cdot q_2 = (p_1+p_2)^2 = 2m_X^2 + 2p_1\cdot p_2$$

$$q_1^0 = q_2^0 = p_1^0 = p_2^0 = \frac{\sqrt{s}}{2}, \quad q_1\cdot q_2 = \frac{s}{2} - m_\chi^2, \quad p_1\cdot p_2 = \frac{s}{2} - m_X^2$$

$$|\mathbf{q}_1| = |\mathbf{q}_2| = \frac{\sqrt{s}}{2}\beta_\chi, \quad |\mathbf{p}_1| = |\mathbf{p}_2| = \frac{\sqrt{s}}{2}\beta_X, \quad \beta_\chi \equiv \sqrt{1-4m_\chi^2/s}, \quad \beta_X \equiv \sqrt{1-4m_X^2/s}$$

$$q_2\cdot p_2 = q_1\cdot p_1 = q_1^0 p_1^0 - |\mathbf{q}_1||\mathbf{p}_1|\cos\theta = \frac{s}{4}(1-\beta_\chi\beta_X\cos\theta)$$

$$q_2\cdot p_1 = q_1\cdot p_2 = q_1^0 p_1^0 + |\mathbf{q}_1||\mathbf{p}_1|\cos\theta = \frac{s}{4}(1+\beta_\chi\beta_X\cos\theta)$$

$$q\cdot q_1 = q\cdot q_2 = q\cdot p_1 = q\cdot p_2 = \frac{s}{2}$$

$$t = (q_1-p_1)^2 = m_\chi^2 + m_X^2 - 2q_1\cdot p_1 = m_\chi^2 + m_X^2 - \frac{s}{2}(1-\beta_\chi\beta_X\cos\theta)$$

$$u = (q_1-p_2)^2 = m_\chi^2 + m_X^2 - 2q_1\cdot p_2 = m_\chi^2 + m_X^2 - \frac{s}{2}(1+\beta_\chi\beta_X\cos\theta)$$

$$\frac{d\sigma_{\rm ann}}{d\Omega} = n_{\rm id}\frac{1}{2p_1^0 2p_2^0 |\mathbf{v}_1 - \mathbf{v}_2|}\frac{|\mathbf{p}_1|}{(2\pi)^2 4E_{\rm CM}}\frac{1}{n_{\rm spin}}\sum_{\rm spins}|\mathcal{M}|^2 = n_{\rm id}\frac{\beta_X}{64\pi^2 s\beta_\chi}\frac{1}{n_{\rm spin}}\sum_{\rm spins}|\mathcal{M}|^2$$

$$n_{\rm id} = \begin{cases} 1, & X \neq \bar{X} \\ \frac{1}{2}, & X = \bar{X} \end{cases} \qquad n_{\rm spin} = \begin{cases} 1, & \text{spin-0 } \chi \\ 4, & \text{spin-}\frac{1}{2}\ \chi \\ 9, & \text{spin-1 } \chi \end{cases}$$

$$\sigma_{\rm ann} = n_{\rm id}\int d\Omega \frac{\beta_X}{64\pi^2 s\beta_\chi}\frac{1}{n_{\rm spin}}\sum_{\rm spins}|\mathcal{M}|^2 = n_{\rm id}\frac{\beta_X}{32\pi s\beta_\chi}\int_0^\pi \sin\theta d\theta \frac{1}{n_{\rm spin}}\sum_{\rm spins}|\mathcal{M}|^2$$

$$\chi(q_1) + \chi(q_2) \to \phi^{(*)}(q) \to \gamma(p_1) + X(p_2)$$

$$s = q^2 = (q_1+q_2)^2 = 2m_\chi^2 + 2q_1\cdot q_2 = (p_1+p_2)^2 = m_X^2 + 2p_1\cdot p_2$$

$$q_1^0 = q_2^0 = \frac{\sqrt{s}}{2}, \quad p_1^0 = \frac{s-m_X^2}{2\sqrt{s}}, \quad p_2^0 = \frac{s+m_X^2}{2\sqrt{s}}, \quad q_1\cdot q_2 = \frac{s}{2} - m_\chi^2, \quad p_1\cdot p_2 = \frac{s-m_X^2}{2}$$

$$|\mathbf{q}_1| = |\mathbf{q}_2| = \frac{\sqrt{s}}{2}\beta_\chi, \quad |\mathbf{p}_1| = |\mathbf{p}_2| = \frac{s-m_X^2}{2\sqrt{s}}, \quad \beta_\chi \equiv \sqrt{1-4m_\chi^2/s}$$

$$q_1\cdot p_1 = q_1^0 p_1^0 - |\mathbf{q}_1||\mathbf{p}_1|\cos\theta = \frac{s-m_X^2}{4}(1-\beta_\chi\cos\theta)$$

$$q_2\cdot p_2 = q_2^0 p_2^0 - |\mathbf{q}_1||\mathbf{p}_1|\cos\theta = \frac{1}{4}[s(1-\beta_\chi\cos\theta) + m_X^2(1+\beta_\chi\cos\theta)]$$

$$q_2\cdot p_1 = q_2^0 p_1^0 + |\mathbf{q}_1||\mathbf{p}_1|\cos\theta = \frac{s-m_X^2}{4}(1+\beta_\chi\cos\theta)$$

$$q_1\cdot p_2 = q_1^0 p_2^0 + |\mathbf{q}_1||\mathbf{p}_1|\cos\theta = \frac{1}{4}[s(1+\beta_\chi\cos\theta) + m_X^2(1-\beta_\chi\cos\theta)]$$

$$q\cdot q_1 = q\cdot q_2 = \frac{s}{2}, \quad q\cdot p_1 = \frac{s-m_X^2}{2}, \quad q\cdot p_2 = \frac{s+m_X^2}{2}$$

$$\sigma_{\rm ann} = n_{\rm id}\frac{1-m_X^2/s}{32\pi s\beta_\chi}\int_0^\pi \sin\theta d\theta \frac{1}{n_{\rm spin}}\sum_{\rm spins}|\mathcal{M}|^2$$

(1) $\mathcal{L}_{\rm M1}$, CP-even scalar ϕ, Majarana fermion χ

$$\chi(q_1)+\chi(q_2)\to\gamma(p_1)+\gamma(p_2)$$

$$i\mathcal{M}=ig_\chi\bar{v}(q_2)u(q_1)\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\frac{4ik_{\rm AA}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$=\frac{4ig_\chi k_{\rm AA}}{\Lambda(s-m_\phi^2+im_\phi\Gamma_\phi)}\bar{v}(q_2)u(q_1)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$(i\mathcal{M})^*=-\frac{4ig_\chi k_{\rm AA}}{\Lambda(s-m_\phi^2-im_\phi\Gamma_\phi)}\bar{u}(q_1)v(q_2)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)\varepsilon_\rho(p_1)\varepsilon_\sigma(p_2)$$

$$\frac{1}{4}\sum_{\rm spins}|\mathcal{M}|^2=\frac{4g_\chi^2k_{\rm AA}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)$$
$$\times\sum_{\rm spins}\bar{v}(q_2)u(q_1)\bar{u}(q_1)v(q_2)\varepsilon_\mu^*(p_1)\varepsilon_\rho(p_1)\varepsilon_\nu^*(p_2)\varepsilon_\sigma(p_2)$$

$$=\frac{4g_\chi^2k_{\rm AA}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}{\rm Tr}[(\not{q}_2-m_\chi)(\not{q}_1+m_\chi)](g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)(-g_{\mu\rho})(-g_{\nu\sigma})$$

$$=\frac{4s^2k_{\rm AA}^2g_\chi^2(s-4m_\chi^2)}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\rm ann}(\chi\chi\to\gamma\gamma)=\frac{k_{\rm AA}^2g_\chi^2s(s-4m_\chi^2)}{8\pi\Lambda^2\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a(\chi\chi\to\gamma\gamma)=0$$

$$b(\chi\chi\to\gamma\gamma)=\frac{k_{\rm AA}^2g_\chi^2m_\chi^4}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\chi(q_1)+\chi(q_2)\to Z(p_1)+Z(p_2)$$

$$i\mathcal{M}=ig_\chi\bar{v}(q_2)u(q_1)\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\frac{4ik_{\rm ZZ}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$\frac{1}{4}\sum_{\rm spins}|\mathcal{M}|^2=\frac{4g_\chi^2k_{\rm ZZ}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}{\rm Tr}[(\not{q}_2-m_\chi)(\not{q}_1+m_\chi)]$$
$$\times(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)\left(-g_{\mu\rho}+\frac{p_{1\mu}p_{1\rho}}{m_Z^2}\right)\left(-g_{\nu\sigma}+\frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$=\frac{4g_\chi^2k_{\rm ZZ}^2(s-4m_\chi^2)(s^2-4sm_Z^2+6m_Z^4)}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\rm ann}(\chi\chi\to ZZ)=\frac{k_{\rm ZZ}^2g_\chi^2\beta_Z(s-4m_\chi^2)(s^2-4sm_Z^2+6m_Z^4)}{8\pi\Lambda^2s\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=0$$

$$b=\frac{k_{\rm ZZ}^2g_\chi^2\rho_Z(8m_\chi^4-8m_\chi^2m_Z^2+3m_Z^4)}{8\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]},\quad \rho_Z\equiv\sqrt{1-m_Z^2/m_\chi^2}$$

$\chi(q_1)+\chi(q_2)\to\gamma(p_1)+Z(p_2)$

$i\mathcal{M}=ig_\chi\bar{v}(q_2)u(q_1)\dfrac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\dfrac{2ik_{\mathrm{AZ}}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{g_\chi^2k_{\mathrm{AZ}}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}\mathrm{Tr}[(\not{q}_2-m_\chi)(\not{q}_1+m_\chi)]$$

$$\times(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)(-g_{\mu\rho})\left(-g_{\nu\sigma}+\frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$=\frac{k_{\mathrm{AZ}}^2g_\chi^2(s-4m_\chi^2)(s-m_Z^2)^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to\gamma Z)=\frac{k_{\mathrm{AZ}}^2g_\chi^2(s-4m_\chi^2)(s-m_Z^2)^3}{16\pi\Lambda^2s^2\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$a=0$

$$b=\frac{k_{\mathrm{AZ}}^2g_\chi^2(4m_\chi^2-m_Z^2)^3}{128\pi\Lambda^2m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$\chi(q_1)+\chi(q_2)\to W^+(p_1)+W^-(p_2)$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{4g_\chi^2k_2^2(s-4m_\chi^2)(s^2-4sm_W^2+6m_W^4)}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to W^+W^-)=\frac{k_2^2g_\chi^2\beta_W(s-4m_\chi^2)(s^2-4sm_W^2+6m_W^4)}{4\pi\Lambda^2s\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$a=0$

$$b=\frac{k_2^2g_\chi^2\rho_W(8m_\chi^4-8m_\chi^2m_W^2+3m_W^4)}{4\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]},\quad \rho_W\equiv\sqrt{1-m_W^2/m_\chi^2}$$

$\chi(q_1)+\chi(q_2)\to g(p_1)+g(p_2)$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{4s^2k_3^2g_\chi^2(s-4m_\chi^2)}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to gg)=\frac{k_3^2g_\chi^2s(s-4m_\chi^2)}{\pi\Lambda^2\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$a=0$

$$b=\frac{8k_3^2g_\chi^2m_\chi^4}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$\chi(q_1)+\chi(q_2)\to\phi(p_1)+\phi(p_2)$

$$i\mathcal{M}_t=\bar{v}(q_2)ig_\chi\frac{i(\not{q}_1-\not{p}_1+m_\chi)}{t-m_\chi^2}ig_\chi u(q_1)=-ig_\chi^2\bar{v}(q_2)\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}u(q_1),\quad i\mathcal{M}_u=-ig_\chi^2\bar{v}(q_2)\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}u(q_1)$$

$$i\mathcal{M}=i\mathcal{M}_t+i\mathcal{M}_u=-ig_\chi^2\bar{v}(q_2)\left(\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}+\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}\right)u(q_1)$$

$$(i\mathcal{M})^*=ig_\chi^2\bar{u}(q_1)\left(\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}+\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}\right)v(q_2)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{g_\chi^4}{4}\mathrm{Tr}\left[(\not{q}_2-m_\chi)\left(\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}+\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}\right)(\not{q}_1+m_\chi)\left(\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}+\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}\right)\right]$$

$\sigma_{\text{ann}}(\chi\chi\to\phi\phi)=\text{complex expression}$

$a=0$

$$b=\frac{g_\chi^4m_\chi^2\rho_\phi(9m_\chi^4-8m_\chi^2m_\phi^2+2m_\phi^4)}{24\pi(2m_\chi^2-m_\phi^2)^4},\quad \rho_\phi\equiv\sqrt{1-m_\phi^2/m_\chi^2}$$

(2) $\mathcal{L}_{\rm M2}$, CP-odd scalar ϕ, Majarana fermion χ

$$\chi(q_1)+\chi(q_2)\to\gamma(p_1)+\gamma(p_2)$$

$$i\mathcal{M}=-g_\chi\bar{v}(q_2)\gamma_5 u(q_1)\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\frac{4ik_{\rm AA}}{\Lambda}\varepsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$=\frac{4g_\chi k_{\rm AA}}{\Lambda(s-m_\phi^2+im_\phi\Gamma_\phi)}\bar{v}(q_2)\gamma_5 u(q_1)\varepsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$(i\mathcal{M})^*=-\frac{4g_\chi k_{\rm AA}}{\Lambda(s-m_\phi^2-im_\phi\Gamma_\phi)}\bar{u}(q_1)\gamma_5 v(q_2)\varepsilon^{\rho\sigma\gamma\delta}p_{1\gamma}p_{2\delta}\varepsilon_\rho(p_1)\varepsilon_\sigma(p_2)$$

$$\frac{1}{4}\sum_{\rm spins}|\mathcal{M}|^2=-\frac{4g_\chi^2k_{\rm AA}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}{\rm Tr}[(\not{q}_2-m_\chi)\gamma_5(\not{q}_1+m_\chi)\gamma_5]\varepsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\varepsilon^{\rho\sigma\gamma\delta}p_{1\gamma}p_{2\delta}(-g_{\mu\rho})(-g_{\nu\sigma})$$

$$=\frac{4s^3k_{\rm AA}^2g_\chi^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\rm ann}(\chi\chi\to\gamma\gamma)=\frac{s^2k_{\rm AA}^2g_\chi^2}{8\pi\Lambda^2\beta_\chi[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{4k_{\rm AA}^2g_\chi^2m_\chi^4}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_{\rm AA}^2g_\chi^2m_\chi^4(m_\phi^4+m_\phi^2\Gamma_\phi^2-16m_\chi^4)}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to Z(p_1)+Z(p_2)$$

$$\frac{1}{4}\sum_{\rm spins}|\mathcal{M}|^2=-\frac{4g_\chi^2k_{\rm ZZ}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}{\rm Tr}[(\not{q}_2-m_\chi)\gamma_5(\not{q}_1+m_\chi)\gamma_5]$$

$$\times\varepsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\varepsilon^{\rho\sigma\gamma\delta}p_{1\gamma}p_{2\delta}\left(-g_{\mu\rho}+\frac{p_{1\mu}p_{1\rho}}{m_Z^2}\right)\left(-g_{\nu\sigma}+\frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$=\frac{4s^2g_\chi^2k_{\rm ZZ}^2(s-4m_Z^2)}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\rm ann}(\chi\chi\to ZZ)=\frac{k_{\rm ZZ}^2g_\chi^2\beta_Z s(s-4m_Z^2)}{8\pi\Lambda^2\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{4k_{\rm ZZ}^2g_\chi^2m_\chi^4\rho_Z^3}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_{\rm ZZ}^2g_\chi^2m_\chi^2\rho_Z[2m_\chi^2(m_\phi^4+m_\phi^2\Gamma_\phi^2-16m_\chi^4)+m_Z^2(m_\phi^4-24m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2+80m_\chi^4)]}{2\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to\gamma(p_1)+Z(p_2)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=-\frac{g_\chi^2k_{\text{AZ}}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}\text{Tr}[(\not{q}_2-m_\chi)\gamma_5(\not{q}_1+m_\chi)\gamma_5]$$

$$\times\varepsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\varepsilon^{\rho\sigma\gamma\delta}p_{1\gamma}p_{2\delta}(-g_{\mu\rho})\left(-g_{\nu\sigma}+\frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$=\frac{sk_{\text{AZ}}^2g_\chi^2(s-m_Z^2)^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to\gamma Z)=\frac{k_{\text{AZ}}^2g_\chi^2(s-m_Z^2)^3}{16\pi\Lambda^2s\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{k_{\text{AZ}}^2g_\chi^2(4m_\chi^2-m_Z^2)^3}{32\pi\Lambda^2m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_{\text{AZ}}^2g_\chi^2(4m_\chi^2-m_Z^2)^2[2m_\chi^2(m_\phi^4+\Gamma_\phi^2m_\phi^2-16m_\chi^4)+m_Z^2(m_\phi^4-12m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2+32m_\chi^4)]}{64\pi\Lambda^2m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to W^+(p_1)+W^-(p_2)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{4s^2g_\chi^2k_2^2(s-4m_W^2)}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to W^+W^-)=\frac{k_2^2g_\chi^2\beta_Ws(s-4m_W^2)}{4\pi\Lambda^2\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{8k_2^2g_\chi^2m_\chi^4\rho_W^3}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_2^2g_\chi^2m_\chi^2\rho_W[2m_\chi^2(m_\phi^4+m_\phi^2\Gamma_\phi^2-16m_\chi^4)+m_W^2(m_\phi^4-24m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2+80m_\chi^4)]}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to g(p_1)+g(p_2)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{4s^3k_3^2g_\chi^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to gg)=\frac{k_3^2g_\chi^2s^2}{\pi\Lambda^2\beta_\chi[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{32k_3^2g_\chi^2m_\chi^4}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{8k_3^2g_\chi^2m_\chi^4(m_\phi^4+m_\phi^2\Gamma_\phi^2-16m_\chi^4)}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to\phi(p_1)+\phi(p_2)$$

$$i\mathcal{M}_t=\overline{v}(q_2)(-g_\chi\gamma_5)\frac{i(\not{q}_1-\not{p}_1+m_\chi)}{t-m_\chi^2}(-g_\chi\gamma_5)u(q_1)=ig_\chi^2\overline{v}(q_2)\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}u(q_1),\quad i\mathcal{M}_u=ig_\chi^2\overline{v}(q_2)\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}u(q_1)$$

$$i\mathcal{M}=i\mathcal{M}_t+i\mathcal{M}_u=ig_\chi^2\overline{v}(q_2)\gamma_5\left(\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}+\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}\right)\gamma_5u(q_1)$$

$$(i\mathcal{M})^*=-ig_\chi^2\overline{u}(q_1)\gamma_5\left(\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}+\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}\right)\gamma_5v(q_2)$$

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{g_\chi^4}{4}\text{Tr}\left[(\not{q}_2-m_\chi)\gamma_5\left(\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}+\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}\right)\gamma_5(\not{q}_1+m_\chi)\gamma_5\left(\frac{\not{q}_1-\not{p}_1+m_\chi}{t-m_\chi^2}+\frac{\not{q}_1-\not{p}_2+m_\chi}{u-m_\chi^2}\right)\gamma_5\right]$$

$$\sigma_{\text{ann}}(\chi\chi\to\phi\phi)=\text{complex expression}$$

$$a=0$$

$$b=\frac{g_\chi^4m_\chi^6\rho_\phi^5}{24\pi(2m_\chi^2-m_\phi^2)^4}$$

(3) $\mathcal{L}_\mathrm{S}$, CP-even scalar ϕ, Real scalar χ

$$\chi(q_1)+\chi(q_2)\to\gamma(p_1)+\gamma(p_2)$$

$$i\mathcal{M}=ig_\chi\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\frac{4ik_{\mathrm{AA}}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$=\frac{4ig_\chi k_{\mathrm{AA}}}{\Lambda(s-m_\phi^2+im_\phi\Gamma_\phi)}(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$(i\mathcal{M})^*=-\frac{4ig_\chi k_{\mathrm{AA}}}{\Lambda(s-m_\phi^2-im_\phi\Gamma_\phi)}(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)\varepsilon_\rho(p_1)\varepsilon_\sigma(p_2)$$

$$\sum_{\text{spins}}|\mathcal{M}|^2=\frac{16g_\chi^2k_{\mathrm{AA}}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)\sum_{\text{spins}}\varepsilon_\mu^*(p_1)\varepsilon_\rho(p_1)\varepsilon_\nu^*(p_2)\varepsilon_\sigma(p_2)$$

$$=\frac{16g_\chi^2k_{\mathrm{AA}}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)(-g_{\mu\rho})(-g_{\nu\sigma})$$

$$=\frac{8s^2k_{\mathrm{AA}}^2g_\chi^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\mathrm{ann}}(\chi\chi\to\gamma\gamma)=\frac{k_{\mathrm{AA}}^2g_\chi^2s}{4\pi\Lambda^2\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{2k_{\mathrm{AA}}^2g_\chi^2m_\chi^2}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{4k_{\mathrm{AA}}^2g_\chi^2m_\chi^4(m_\phi^2-4m_\chi^2)}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to Z(p_1)+Z(p_2)$$

$$i\mathcal{M}=ig_\chi\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\frac{4ik_{ZZ}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$\sum_{\text{spins}}|\mathcal{M}|^2=\frac{16g_\chi^2k_{ZZ}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)\left(-g_{\mu\rho}+\frac{p_{1\mu}p_{1\rho}}{m_Z^2}\right)\left(-g_{\nu\sigma}+\frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$=\frac{8g_\chi^2k_{ZZ}^2(s^2-4sm_Z^2+6m_Z^4)}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\mathrm{ann}}(\chi\chi\to ZZ)=\frac{k_{ZZ}^2g_\chi^2\beta_Z(s^2-4sm_Z^2+6m_Z^4)}{4\pi\Lambda^2s\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{k_{ZZ}^2g_\chi^2\rho_Z(8m_\chi^4-8m_\chi^2m_Z^2+3m_Z^4)}{4\pi\Lambda^2m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_{ZZ}^2g_\chi^2}{32\pi\Lambda^2m_\chi^4\rho_Z[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}[128m_\chi^8(m_\phi^2-4m_\chi^2)+8m_\chi^4m_Z^2(3m_\phi^4-56m_\phi^2m_\chi^2+3m_\phi^2\Gamma_\phi^2+176m_\chi^4)$$
$$-4m_\chi^2m_Z^4(9m_\phi^4-116m_\phi^2m_\chi^2+9m_\phi^2\Gamma_\phi^2+320m_\chi^4)+3m_Z^6(5m_\phi^4-56m_\phi^2m_\chi^2+5m_\phi^2\Gamma_\phi^2+144m_\chi^4)]$$

$$\chi(q_1)+\chi(q_2)\to\gamma(p_1)+Z(p_2)$$

$$i\mathcal{M}=ig_\chi\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\frac{2ik_{\rm AZ}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$\sum_{\rm spins}|\mathcal{M}|^2=\frac{4g_\chi^2k_{\rm AZ}^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)(-g_{\mu\rho})\left(-g_{\nu\sigma}+\frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$=\frac{2k_{\rm AZ}^2g_\chi^2(s-m_Z^2)^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\rm ann}(\chi\chi\to\gamma Z)=\frac{k_{\rm AZ}^2g_\chi^2(s-m_Z^2)^3}{8\pi\Lambda^2s^2\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{k_{\rm AZ}^2g_\chi^2(4m_\chi^2-m_Z^2)^3}{64\pi\Lambda^2m_\chi^4[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_{\rm AZ}^2g_\chi^2(4m_\chi^2-m_Z^2)^2[32m_\chi^4(m_\phi^2-4m_\chi^2)+m_Z^2(3m_\phi^4-32m_\phi^2m_\chi^2+3m_\phi^2\Gamma_\phi^2+80m_\chi^4)]}{256\pi\Lambda^2m_\chi^4[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to W^+(p_1)+W^-(p_2)$$

$$\sum_{\rm spins}|\mathcal{M}|^2=\frac{8g_\chi^2k_2^2(s^2-4sm_W^2+6m_W^4)}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\rm ann}(\chi\chi\to W^+W^-)=\frac{k_2^2g_\chi^2\beta_W(s^2-4sm_W^2+6m_W^4)}{2\pi\Lambda^2s\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{k_2^2g_\chi^2\rho_W(8m_\chi^4-8m_\chi^2m_W^2+3m_W^4)}{2\pi\Lambda^2m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_2^2g_\chi^2}{16\pi\Lambda^2m_\chi^4\rho_W[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}[128m_\chi^8(m_\phi^2-4m_\chi^2)+8m_\chi^4m_W^2(3m_\phi^4-56m_\phi^2m_\chi^2+3m_\phi^2\Gamma_\phi^2+176m_\chi^4)$$
$$-4m_\chi^2m_W^4(9m_\phi^4-116m_\phi^2m_\chi^2+9m_\phi^2\Gamma_\phi^2+320m_\chi^4)+3m_W^6(5m_\phi^4-56m_\phi^2m_\chi^2+5m_\phi^2\Gamma_\phi^2+144m_\chi^4)]$$

$$\chi(q_1)+\chi(q_2)\to g(p_1)+g(p_2)$$

$$\sum_{\rm spins}|\mathcal{M}|^2=\frac{8k_3^2g_\chi^2s^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\rm ann}(\chi\chi\to gg)=\frac{2k_3^2g_\chi^2s}{\pi\Lambda^2\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{16k_3^2g_\chi^2m_\chi^2}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{32k_3^2g_\chi^2m_\chi^4(m_\phi^2-4m_\chi^2)}{\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to\phi(p_1)+\phi(p_2)$$

$$i\mathcal{M}_t=ig_\chi\frac{i}{t-m_\chi^2}ig_\chi=-i\frac{g_\chi^2}{t-m_\chi^2},\quad i\mathcal{M}_u=-i\frac{g_\chi^2}{u-m_\chi^2}$$

$$i\mathcal{M}=i\mathcal{M}_t+i\mathcal{M}_u=-ig_\chi^2\left(\frac{1}{t-m_\chi^2}+\frac{1}{u-m_\chi^2}\right)$$

$$(i\mathcal{M})^*=ig_\chi^2\left(\frac{1}{t-m_\chi^2}+\frac{1}{u-m_\chi^2}\right)$$

$$|\mathcal{M}|^2=g_\chi^4\left(\frac{1}{t-m_\chi^2}+\frac{1}{u-m_\chi^2}\right)^2$$

$$\sigma_{\rm ann}(\chi\chi\to\phi\phi)=\text{complex expression}$$

$$a=\frac{g_\chi^4\rho_\phi}{16\pi m_\chi^2(2m_\chi^2-m_\phi^2)^2}$$

$$b=\frac{g_\chi^4(-80m_\chi^6+148m_\chi^4m_\phi^2-80m_\chi^2m_\phi^4+15m_\phi^6)}{384\pi m_\chi^4\rho_\phi(2m_\chi^2-m_\phi^2)^4}$$

(4) $\mathcal{L}_{\text{V}}$, CP-even scalar ϕ, Real vector χ

$$\chi(q_1)+\chi(q_2)\to\gamma(p_1)+\gamma(p_2)$$

$$i\mathcal{M}=ig_\chi g^{\alpha\beta}\varepsilon_\alpha(q_1)\varepsilon_\beta(q_2)\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\frac{4ik_{\text{AA}}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$=\frac{4ig_\chi k_{\text{AA}}}{\Lambda(s-m_\phi^2+im_\phi\Gamma_\phi)}g^{\alpha\beta}(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\alpha(q_1)\varepsilon_\beta(q_2)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$(i\mathcal{M})^*=-\frac{4ig_\chi k_{\text{AA}}}{\Lambda(s-m_\phi^2-im_\phi\Gamma_\phi)}g^{\gamma\delta}(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)\varepsilon_\gamma^*(q_1)\varepsilon_\delta^*(q_2)\varepsilon_\rho(p_1)\varepsilon_\sigma(p_2)$$

$$\frac{1}{9}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{16g_\chi^2k_{\text{AA}}^2}{9\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}g^{\alpha\beta}g^{\gamma\delta}(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)$$
$$\times\sum_{\text{spins}}\varepsilon_\alpha(q_1)\varepsilon_\gamma^*(q_1)\varepsilon_\beta(q_2)\varepsilon_\delta^*(q_2)\varepsilon_\mu^*(p_1)\varepsilon_\rho(p_1)\varepsilon_\nu^*(p_2)\varepsilon_\sigma(p_2)$$

$$=\frac{16g_\chi^2k_{\text{AA}}^2}{9\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}g^{\alpha\beta}g^{\gamma\delta}\left(-g_{\alpha\gamma}+\frac{q_{1\alpha}q_{1\gamma}}{m_\chi^2}\right)\left(-g_{\beta\delta}+\frac{q_{2\beta}q_{2\delta}}{m_\chi^2}\right)$$
$$\times(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)(-g_{\mu\rho})(-g_{\nu\sigma})$$

$$=\frac{2s^2k_{\text{AA}}^2g_\chi^2(s^2-4sm_\chi^2+12m_\chi^4)}{9\Lambda^2m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to\gamma\gamma)=\frac{k_{\text{AA}}^2g_\chi^2s(s^2-4sm_\chi^2+12m_\chi^4)}{144\pi\Lambda^2\beta_\chi m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{2k_{\text{AA}}^2g_\chi^2m_\chi^2}{3\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{2k_{\text{AA}}^2g_\chi^2m_\chi^2(m_\phi^4-2m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2-8m_\chi^4)}{9\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to Z(p_1)+Z(p_2)$$

$$i\mathcal{M}=ig_\chi g^{\alpha\beta}\varepsilon_\alpha(q_1)\varepsilon_\beta(q_2)\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\frac{4ik_{\text{ZZ}}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$\frac{1}{9}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{16g_\chi^2k_{\text{ZZ}}^2}{9\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}g^{\alpha\beta}g^{\gamma\delta}\left(-g_{\alpha\gamma}+\frac{q_{1\alpha}q_{1\gamma}}{m_\chi^2}\right)\left(-g_{\beta\delta}+\frac{q_{2\beta}q_{2\delta}}{m_\chi^2}\right)$$
$$\times(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)\left(-g_{\mu\rho}+\frac{p_{1\mu}p_{1\rho}}{m_Z^2}\right)\left(-g_{\nu\sigma}+\frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$=\frac{2g_\chi^2k_{\text{ZZ}}^2(s^2-4sm_\chi^2+12m_\chi^4)(s^2-4sm_Z^2+6m_Z^4)}{9\Lambda^2m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to ZZ)=\frac{k_{\text{ZZ}}^2g_\chi^2\beta_Z(s^2-4sm_\chi^2+12m_\chi^4)(s^2-4sm_Z^2+6m_Z^4)}{144\pi\Lambda^2s\beta_\chi m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{k_{\text{ZZ}}^2g_\chi^2\rho_Z(8m_\chi^4-8m_\chi^2m_Z^2+3m_Z^4)}{12\pi\Lambda^2m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_{\text{ZZ}}^2g_\chi^2}{288\pi\Lambda^2m_\chi^4\rho_Z[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}[64m_\chi^6(m_\phi^4-2m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2-8m_\chi^4)-8m_\chi^4m_Z^2(7m_\phi^4+40m_\phi^2m_\chi^2+7m_\phi^2\Gamma_\phi^2-272m_\chi^4)$$
$$-4m_\chi^2m_Z^4(5m_\phi^4-172m_\phi^2m_\chi^2+5m_\phi^2\Gamma_\phi^2+608m_\chi^4)+3m_Z^6(7m_\phi^4-104m_\phi^2m_\chi^2+7m_\phi^2\Gamma_\phi^2+304m_\chi^4)]$$

$$\chi(q_1)+\chi(q_2)\to\gamma(p_1)+Z(p_2)$$

$$i\mathcal{M}=ig_\chi g^{\alpha\beta}\varepsilon_\alpha(q_1)\varepsilon_\beta(q_2)\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}\left(-\frac{2ik_{\mathrm{AZ}}}{\Lambda}\right)(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)\varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)$$

$$\frac{1}{9}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{4g_\chi^2k_{\mathrm{AZ}}^2}{9\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}g^{\alpha\beta}g^{\gamma\delta}\left(-g_{\alpha\gamma}+\frac{q_{1\alpha}q_{1\gamma}}{m_\chi^2}\right)\left(-g_{\beta\delta}+\frac{q_{2\beta}q_{2\delta}}{m_\chi^2}\right)$$

$$\times(g^{\mu\nu}p_1\cdot p_2-p_2^\mu p_1^\nu)(g^{\rho\sigma}p_1\cdot p_2-p_2^\rho p_1^\sigma)(-g_{\mu\rho})\left(-g_{\nu\sigma}+\frac{p_{2\nu}p_{2\sigma}}{m_Z^2}\right)$$

$$=\frac{k_{\mathrm{AZ}}^2g_\chi^2(s^2-4sm_\chi^2+12m_\chi^4)(s-m_Z^2)^2}{18\Lambda^2m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to\gamma Z)=\frac{k_{\mathrm{AZ}}^2g_\chi^2(s^2-4sm_\chi^2+12m_\chi^4)(s-m_Z^2)^3}{288\pi\Lambda^2s^2\beta_\chi m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{k_{\mathrm{AZ}}^2g_\chi^2(4m_\chi^2-m_Z^2)^3}{192\pi\Lambda^2m_\chi^4[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_{\mathrm{AZ}}^2g_\chi^2(4m_\chi^2-m_Z^2)^2[16m_\chi^2(m_\phi^4-2m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2-8m_\chi^4)+m_Z^2(5m_\phi^4-64m_\phi^2m_\chi^2+5m_\phi^2\Gamma_\phi^2+176m_\chi^4)]}{2304\pi\Lambda^2m_\chi^4[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to W^+(p_1)+W^-(p_2)$$

$$\frac{1}{9}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{2g_\chi^2k_2^2(s^2-4sm_\chi^2+12m_\chi^4)(s^2-4sm_W^2+6m_W^4)}{9\Lambda^2m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to W^+W^-)=\frac{k_2^2g_\chi^2\beta_W(s^2-4sm_\chi^2+12m_\chi^4)(s^2-4sm_W^2+6m_W^4)}{72\pi\Lambda^2s\beta_\chi m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{k_2^2g_\chi^2\rho_W(8m_\chi^4-8m_\chi^2m_W^2+3m_W^4)}{6\pi\Lambda^2m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{k_2^2g_\chi^2}{144\pi\Lambda^2m_\chi^4\rho_W[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}[64m_\chi^6(m_\phi^4-2m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2-8m_\chi^4)-8m_\chi^4m_W^2(7m_\phi^4+40m_\phi^2m_\chi^2+7m_\phi^2\Gamma_\phi^2-272m_\chi^4)$$

$$-4m_\chi^2m_W^4(5m_\phi^4-172m_\phi^2m_\chi^2+5m_\phi^2\Gamma_\phi^2+608m_\chi^4)+3m_W^6(7m_\phi^4-104m_\phi^2m_\chi^2+7m_\phi^2\Gamma_\phi^2+304m_\chi^4)]$$

$$\chi(q_1)+\chi(q_2)\to g(p_1)+g(p_2)$$

$$\frac{1}{9}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{2s^2k_3^2g_\chi^2(s^2-4sm_\chi^2+12m_\chi^4)}{9\Lambda^2m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$\sigma_{\text{ann}}(\chi\chi\to gg)=\frac{k_3^2g_\chi^2s(s^2-4sm_\chi^2+12m_\chi^4)}{18\pi\Lambda^2\beta_\chi m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a=\frac{16k_3^2g_\chi^2m_\chi^2}{3\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b=\frac{16k_3^2g_\chi^2m_\chi^2(m_\phi^4-2m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2-8m_\chi^4)}{9\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}$$

$$\chi(q_1)+\chi(q_2)\to\phi(p_1)+\phi(p_2)$$

$$i\mathcal{M}_t=\varepsilon_\mu(q_1)ig_\chi g^{\mu\rho}\frac{-i}{t-m_\chi^2}\left[g_{\rho\sigma}-\frac{(q_1-p_1)_\rho(q_1-p_1)_\sigma}{m_\chi^2}\right]ig_\chi g^{\sigma\nu}\varepsilon_\nu(q_2)=ig_\chi^2 g^{\mu\rho}g^{\sigma\nu}\varepsilon_\mu(q_1)\varepsilon_\nu(q_2)\frac{1}{t-m_\chi^2}\left[g_{\rho\sigma}-\frac{(q_1-p_1)_\rho(q_1-p_1)_\sigma}{m_\chi^2}\right]$$

$$i\mathcal{M}_u=ig_\chi^2 g^{\mu\rho}g^{\sigma\nu}\varepsilon_\mu(q_1)\varepsilon_\nu(q_2)\frac{1}{u-m_\chi^2}\left[g_{\rho\sigma}-\frac{(q_1-p_2)_\rho(q_1-p_2)_\sigma}{m_\chi^2}\right]$$

$$i\mathcal{M}=i\mathcal{M}_t+i\mathcal{M}_u=ig_\chi^2 g^{\mu\rho}g^{\sigma\nu}\varepsilon_\mu(q_1)\varepsilon_\nu(q_2)\left\{\frac{1}{t-m_\chi^2}\left[g_{\rho\sigma}-\frac{(q_1-p_1)_\rho(q_1-p_1)_\sigma}{m_\chi^2}\right]+\frac{1}{u-m_\chi^2}\left[g_{\rho\sigma}-\frac{(q_1-p_2)_\rho(q_1-p_2)_\sigma}{m_\chi^2}\right]\right\}$$

$$(i\mathcal{M})^*=-ig_\chi^2 g^{\alpha\gamma}g^{\delta\beta}\varepsilon_\alpha^*(q_1)\varepsilon_\beta^*(q_2)\left\{\frac{1}{t-m_\chi^2}\left[g_{\gamma\delta}-\frac{(q_1-p_1)_\gamma(q_1-p_1)_\delta}{m_\chi^2}\right]+\frac{1}{u-m_\chi^2}\left[g_{\gamma\delta}-\frac{(q_1-p_2)_\gamma(q_1-p_2)_\delta}{m_\chi^2}\right]\right\}$$

$$\frac{1}{9}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{g_\chi^4}{9}g^{\mu\rho}g^{\sigma\nu}g^{\alpha\gamma}g^{\delta\beta}\left(-g_{\mu\alpha}+\frac{q_{1\mu}q_{1\alpha}}{m_\chi^2}\right)\left(-g_{\nu\beta}+\frac{q_{2\nu}q_{2\beta}}{m_\chi^2}\right)$$
$$\times\left\{\frac{1}{t-m_\chi^2}\left[g_{\rho\sigma}-\frac{(q_1-p_1)_\rho(q_1-p_1)_\sigma}{m_\chi^2}\right]+\frac{1}{u-m_\chi^2}\left[g_{\rho\sigma}-\frac{(q_1-p_2)_\rho(q_1-p_2)_\sigma}{m_\chi^2}\right]\right\}$$
$$\times\left\{\frac{1}{t-m_\chi^2}\left[g_{\gamma\delta}-\frac{(q_1-p_1)_\gamma(q_1-p_1)_\delta}{m_\chi^2}\right]+\frac{1}{u-m_\chi^2}\left[g_{\gamma\delta}-\frac{(q_1-p_2)_\gamma(q_1-p_2)_\delta}{m_\chi^2}\right]\right\}$$

$$\sigma_{\text{ann}}(\chi\chi\to\phi\phi)=\text{complex expression}$$

$$a=\frac{g_\chi^4\rho_\phi(6m_\chi^4-4m_\chi^2m_\phi^2+m_\phi^4)}{144\pi m_\chi^6(2m_\chi^2-m_\phi^2)^2}$$

$$b=\frac{g_\chi^4(-224m_\chi^{10}+616m_\chi^8m_\phi^2-656m_\chi^6m_\phi^4+362m_\chi^4m_\phi^6-100m_\chi^2m_\phi^8+11m_\phi^{10})}{3456\pi m_\chi^8\rho_\phi(2m_\chi^2-m_\phi^2)^4}$$

Direct detection

Majarana fermion χ

$$\mathcal{L}_{S,N} = \sum_{N=p,n} G_{S,N} \bar{\chi}\chi \bar{N}N$$

$$\sigma_{\chi N}^{\rm SI} = \frac{4}{\pi}\mu_{\chi N}^2 G_{S,N}^2, \quad \mu_{\chi N} \equiv \frac{m_\chi m_N}{m_\chi + m_N}$$

$$\mathcal{L}_{S,q} = \sum_q G_{S,q} \bar{\chi}\chi \bar{q}q$$

$$\langle N | m_q \bar{q}q | N \rangle = f_q^N M_N \quad \Rightarrow \quad \langle N | \bar{q}q | N \rangle = M_N \frac{f_q^N}{m_q}$$

$$G_{S,N} = m_N \sum_q \frac{G_{S,q} f_q^N}{m_q} = m_N \left(\sum_{q=u,d,s} \frac{G_{S,q}}{m_q} f_q^N + \sum_{q=c,b,t} \frac{G_{S,q}}{m_q} f_Q^N \right), \quad f_Q^N = \frac{2}{27}\left(1 - \sum_{q=u,d,s} f_q^N\right)$$

hep-ph/0001005:

$$f_u^p = 0.020 \pm 0.004, \quad f_d^p = 0.026 \pm 0.005, \quad f_u^n = 0.014 \pm 0.003, \quad f_d^n = 0.036 \pm 0.008, \quad f_s^p = f_s^n = 0.118 \pm 0.062$$

$$\Rightarrow \quad f_Q^p = 0.0619, \quad f_Q^n = 0.0616$$

$$m_N \langle N | N \rangle = \langle N | \sum_{q=u,d,s} m_q \bar{q}q - \frac{9}{8\pi}\alpha_s G_{\mu\nu}^a G^{a\mu\nu} | N \rangle$$

[Ref: Shifman, Vainshtein & Zakharov, Phys. Lett. B78, 443 (1978)]

$$\langle N | G_{\mu\nu}^a G^{a\mu\nu} | N \rangle = -\frac{8\pi}{9\alpha_s} \langle N | m_N - \sum_{q=u,d,s} m_q \bar{q}q | N \rangle = -\frac{8\pi}{9\alpha_s} m_N \left(1 - \sum_{q=u,d,s} f_q^N\right)$$

(1) $\mathcal{L}_{\rm M1}$, CP-even scalar ϕ, Majarana fermion χ

$$i\frac{k_3}{\Lambda} G_{\mu\nu}^a G^{a\mu\nu} \frac{i}{q^2 - m_\phi^2} \frac{1}{2} i g_\chi \bar{\chi}\chi \quad \xrightarrow{q^2 \to 0} \quad i\frac{k_3 g_\chi}{2\Lambda m_\phi^2} G_{\mu\nu}^a G^{a\mu\nu} \bar{\chi}\chi$$

$$\mathcal{L}_{\chi\chi GG} = \frac{k_3 g_\chi}{2\Lambda m_\phi^2} G_{\mu\nu}^a G^{a\mu\nu} \bar{\chi}\chi$$

$$G_{S,N} = -\frac{8\pi}{9\alpha_s} m_N \left(1 - \sum_{q=u,d,s} f_q^N\right) \frac{k_3 g_\chi}{2\Lambda m_\phi^2} = -\frac{4\pi k_3 g_\chi m_N}{9\alpha_s \Lambda m_\phi^2} \left(1 - \sum_{q=u,d,s} f_q^N\right)$$

$$\sum_{q=u,d,s} f_q^n \simeq \sum_{q=u,d,s} f_q^p \quad \Rightarrow \quad G_{S,n} = G_{S,p}$$

$$\sigma_{\chi N}^{\rm SI} = \frac{4}{\pi}\mu_{\chi N}^2 G_{S,N}^2 = \frac{64\pi k_3^2 g_\chi^2 m_N^2 \mu_{\chi N}^2}{81\alpha_s^2 \Lambda^2 m_\phi^4} \left(1 - \sum_{q=u,d,s} f_q^N\right)^2$$

(3) CP-even scalar ϕ, real scalar χ

$$\mathcal{L}_{S,N} = \sum_{N=p,n} G_{S,N}\chi^2 \bar{N}N, \quad \sigma_{\chi N}^{\mathrm{SI}} = \frac{\mu_{\chi N}^2}{\pi m_\chi^2} G_{S,N}^2$$

$$i\frac{k_3}{\Lambda} G_{\mu\nu}^a G^{a\mu\nu} \frac{i}{q^2 - m_\phi^2} \frac{1}{2} i g_\chi \chi^2 \quad \xrightarrow{q^2 \to 0} \quad i\frac{k_3 g_\chi}{2\Lambda m_\phi^2} G_{\mu\nu}^a G^{a\mu\nu} \chi^2$$

$$\mathcal{L}_{\chi\chi GG} = \frac{k_3 g_\chi}{2\Lambda m_\phi^2} G_{\mu\nu}^a G^{a\mu\nu} \chi^2$$

$$G_{S,N} = -\frac{4\pi k_3 g_\chi m_N}{9\alpha_s \Lambda m_\phi^2} \left(1 - \sum_{q=u,d,s} f_q^N \right)$$

$$\sigma_{\chi N}^{\mathrm{SI}} = \frac{\mu_{\chi N}^2}{\pi m_\chi^2} G_{S,N}^2 = \frac{16\pi k_3^2 g_\chi^2 m_N^2 \mu_{\chi N}^2}{81\alpha_s^2 \Lambda^2 m_\phi^4 m_\chi^2} \left(1 - \sum_{q=u,d,s} f_q^N \right)^2$$

(4) CP-even scalar ϕ, real vector χ

$$\mathcal{L}_{S,N} = \sum_{N=p,n} G_{S,N}\chi^\mu \chi_\mu \bar{N}N, \quad \sigma_{\chi N}^{\mathrm{SI}} = \frac{\mu_{\chi N}^2}{\pi m_\chi^2} G_{S,N}^2$$

$$i\frac{k_3}{\Lambda} G_{\mu\nu}^a G^{a\mu\nu} \frac{i}{q^2 - m_\phi^2} \frac{1}{2} i g_\chi \chi^\mu \chi_\mu \quad \xrightarrow{q^2 \to 0} \quad i\frac{k_3 g_\chi}{2\Lambda m_\phi^2} G_{\mu\nu}^a G^{a\mu\nu} \chi^\mu \chi_\mu$$

$$\mathcal{L}_{\chi\chi GG} = \frac{k_3 g_\chi}{2\Lambda m_\phi^2} G_{\mu\nu}^a G^{a\mu\nu} \chi^\mu \chi_\mu$$

$$G_{S,N} = -\frac{4\pi k_3 g_\chi m_N}{9\alpha_s \Lambda m_\phi^2} \left(1 - \sum_{q=u,d,s} f_q^N \right)$$

$$\sigma_{\chi N}^{\mathrm{SI}} = \frac{\mu_{\chi N}^2}{\pi m_\chi^2} G_{S,N}^2 = \frac{16\pi k_3^2 g_\chi^2 m_N^2 \mu_{\chi N}^2}{81\alpha_s^2 \Lambda^2 m_\phi^4 m_\chi^2} \left(1 - \sum_{q=u,d,s} f_q^N \right)^2$$

Including interactions with fermions

CP-even scalar ϕ

$$\mathcal{L}_{0^+} \supset \sum_f k_f \frac{m_f}{\Lambda} \phi \bar{f} f$$

$\phi(q) \to f(p_1) + \bar{f}(p_2)$ decay width

$$i\mathcal{M} = ik_f \frac{m_f}{\Lambda} \bar{u}(p_1) v(p_2), \quad (i\mathcal{M})^* = -ik_f \frac{m_f}{\Lambda} \bar{v}(p_2) u(p_1)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{k_f^2 m_f^2}{\Lambda^2} \text{Tr}[(\not{p}_1 + m_f)(\not{p}_2 - m_f)] = \frac{4k_f^2 m_f^2}{\Lambda^2}(p_1 \cdot p_2 - m_f^2) = \frac{4k_f^2 m_f^2}{\Lambda^2}\left[\frac{1}{2}(m_\phi^2 - 2m_f^2) - m_f^2\right]$$

$$= \frac{2k_f^2 m_f^2}{\Lambda^2}(m_\phi^2 - 4m_f^2) = \frac{2k_f^2 m_f^2 m_\phi^2 \eta_f^2}{\Lambda^2}$$

$$\Gamma(\phi \to f\bar{f}) = \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{8\pi} \frac{1}{m_\phi^2} \frac{m_\phi}{2} \eta_f \frac{2k_f^2 m_f^2 m_\phi^2 \eta_f^2}{\Lambda^2} = \frac{\eta_f^3 k_f^2 m_f^2 m_\phi}{8\pi\Lambda^2}$$

$$\eta_f \equiv \sqrt{1 - 4m_f^2 / m_\phi^2}$$

(1) Majarana fermion χ

$\chi(q_1) + \chi(q_2) \to f(p_1) + \bar{f}(p_2)$ annihilation

$$i\mathcal{M} = ig_\chi \bar{v}(q_2) u(q_1) \frac{i}{s - m_\phi^2 + im_\phi\Gamma_\phi} ik_f \frac{m_f}{\Lambda} \bar{u}(p_1) v(p_2) = -\frac{ig_\chi k_f m_f}{\Lambda(s - m_\phi^2 + im_\phi\Gamma_\phi)} \bar{v}(q_2) u(q_1) \bar{u}(p_1) v(p_2)$$

$$(i\mathcal{M})^* = \frac{ig_\chi k_f m_f}{\Lambda(s - m_\phi^2 - im_\phi\Gamma_\phi)} \bar{u}(q_1) v(q_2) \bar{v}(p_2) u(p_1)$$

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{g_\chi^2 k_f^2 m_f^2}{4\Lambda^2[(s - m_\phi^2)^2 + m_\phi^2\Gamma_\phi^2]} \text{Tr}[(\not{q}_2 - m_\chi)(\not{q}_1 + m_\chi)] \text{Tr}[(\not{p}_1 + m_f)(\not{p}_2 - m_f)]$$

$$\sigma_{\text{ann}}(\chi\chi \to f\bar{f}) = \frac{\beta_f k_f^2 m_f^2 g_\chi^2 (s - 4m_f^2)(s - 4m_\chi^2)}{16\pi\Lambda^2 s \beta_\chi [(s - m_\phi^2)^2 + m_\phi^2\Gamma_\phi^2]}$$

$$a(\chi\chi \to f\bar{f}) = 0$$

$$b(\chi\chi \to f\bar{f}) = \frac{k_f^2 g_\chi^2 m_\chi^2 m_f^2 \rho_f^3}{8\pi\Lambda^2[(m_\phi^2 - 4m_\chi^2)^2 + m_\phi^2\Gamma_\phi^2]}$$

$\chi N \to \chi N$ scattering

$$\sigma_{\chi N}^{\text{SI}} = \frac{4}{\pi} \mu_{\chi N}^2 G_{S,N}^2$$

$$ik_q \frac{m_q}{\Lambda} \bar{q} q \frac{i}{q^2 - m_\phi^2} \frac{1}{2} ig_\chi \bar{\chi}\chi \xrightarrow{q^2 \to 0} i\frac{k_q g_\chi m_q}{2\Lambda m_\phi^2} \bar{q} q \bar{\chi}\chi \quad \Rightarrow \quad G_{S,q} = \frac{k_q g_\chi m_q}{2\Lambda m_\phi^2}$$

$$G_{S,N} = m_N \sum_q \frac{G_{S,q} f_q^N}{m_q} = \frac{g_\chi m_N}{2\Lambda m_\phi^2} \sum_q k_q f_q^N = \frac{g_\chi m_N}{2\Lambda m_\phi^2}\left(\sum_{q=u,d,s} k_q f_q^N + f_Q^N \sum_{q=c,b,t} k_q\right)$$

(3) Real scalar χ

$\chi(q_1)+\chi(q_2)\to f(p_1)+\bar{f}(p_2)$ annihilation

$$i\mathcal{M}=ig_\chi\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}ik_f\frac{m_f}{\Lambda}\bar{u}(p_1)v(p_2)=-\frac{ig_\chi k_f m_f}{\Lambda(s-m_\phi^2+im_\phi\Gamma_\phi)}\bar{u}(p_1)v(p_2)$$

$$(i\mathcal{M})^*=\frac{ig_\chi k_f m_f}{\Lambda(s-m_\phi^2-im_\phi\Gamma_\phi)}\bar{v}(p_2)u(p_1)$$

$$\sum_{\text{spins}}|\mathcal{M}|^2=\frac{g_\chi^2 k_f^2 m_f^2}{\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}\mathrm{Tr}[(\not{p}_1+m_f)(\not{p}_2-m_f)]$$

$$\sigma_{\text{ann}}(\chi\chi\to f\bar{f})=\frac{\beta_f k_f^2 m_f^2 g_\chi^2(s-4m_f^2)}{8\pi\Lambda^2 s\beta_\chi[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a(\chi\chi\to f\bar{f})=\frac{k_f^2 g_\chi^2 m_f^2\rho_f^3}{4\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b(\chi\chi\to f\bar{f})=\frac{k_f^2 g_\chi^2 m_f^2\rho_f}{32\pi\Lambda^2 m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}[-2m_\chi^2(m_\phi^4-16m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2+48m_\chi^4)+m_f^2(5m_\phi^4-56m_\phi^2m_\chi^2+5m_\phi^2\Gamma_\phi^2+144m_\chi^4)]$$

$\chi N\to\chi N$ scattering

$$\sigma_{\chi N}^{\text{SI}}=\frac{\mu_{\chi N}^2}{\pi m_\chi^2}G_{S,N}^2$$

$$ik_q\frac{m_q}{\Lambda}\bar{q}q\frac{i}{q^2-m_\phi^2}\frac{1}{2}ig_\chi\chi^2 \xrightarrow{q^2\to 0} i\frac{k_q g_\chi m_q}{2\Lambda m_\phi^2}\bar{q}q\chi^2 \quad\Rightarrow\quad G_{S,q}=\frac{k_q g_\chi m_q}{2\Lambda m_\phi^2}$$

$$G_{S,N}=\frac{g_\chi m_N}{2\Lambda m_\phi^2}\sum_q k_q f_q^N$$

(4) Real vector χ

$\chi(q_1)+\chi(q_2)\to f(p_1)+\bar{f}(p_2)$ annihilation

$$i\mathcal{M}=ig_\chi g^{\alpha\beta}\varepsilon_\alpha(q_1)\varepsilon_\beta(q_2)\frac{i}{s-m_\phi^2+im_\phi\Gamma_\phi}ik_f\frac{m_f}{\Lambda}\bar{u}(p_1)v(p_2)=-\frac{ig_\chi k_f m_f}{\Lambda(s-m_\phi^2+im_\phi\Gamma_\phi)}g^{\alpha\beta}\varepsilon_\alpha(q_1)\varepsilon_\beta(q_2)\bar{u}(p_1)v(p_2)$$

$$(i\mathcal{M})^*=\frac{ig_\chi k_f m_f}{\Lambda(s-m_\phi^2-im_\phi\Gamma_\phi)}g^{\gamma\delta}\varepsilon_\gamma^*(q_1)\varepsilon_\delta^*(q_2)\bar{v}(p_2)u(p_1)$$

$$\frac{1}{9}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{g_\chi^2 k_f^2 m_f^2}{9\Lambda^2[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}g^{\alpha\beta}g^{\gamma\delta}\left(-g_{\alpha\gamma}+\frac{q_{1\alpha}q_{1\gamma}}{m_\chi^2}\right)\left(-g_{\beta\delta}+\frac{q_{2\beta}q_{2\delta}}{m_\chi^2}\right)\mathrm{Tr}[(\not{p}_1+m_f)(\not{p}_2-m_f)]$$

$$\sigma_{\text{ann}}(\chi\chi\to f\bar{f})=\frac{\beta_f k_f^2 m_f^2 g_\chi^2(s-4m_f^2)(s^2-4sm_\chi^2+12m_\chi^4)}{288\pi\Lambda^2 s\beta_\chi m_\chi^4[(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$a(\chi\chi\to f\bar{f})=\frac{k_f^2 g_\chi^2 m_f^2\rho_f^3}{12\pi\Lambda^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]}$$

$$b(\chi\chi\to f\bar{f})=\frac{k_f^2 g_\chi^2 m_f^2\rho_f}{288\pi\Lambda^2 m_\chi^2[(m_\phi^2-4m_\chi^2)^2+m_\phi^2\Gamma_\phi^2]^2}[2m_\chi^2(m_\phi^4+16m_\phi^2m_\chi^2+m_\phi^2\Gamma_\phi^2-80m_\chi^4)+m_f^2(7m_\phi^4-104m_\phi^2m_\chi^2+7m_\phi^2\Gamma_\phi^2+304m_\chi^4)]$$

$\chi N\to\chi N$ scattering

$$\sigma_{\chi N}^{\text{SI}}=\frac{\mu_{\chi N}^2}{\pi m_\chi^2}G_{S,N}^2$$

$$ik_q\frac{m_q}{\Lambda}\bar{q}q\frac{i}{q^2-m_\phi^2}\frac{1}{2}ig_\chi\chi^\mu\chi_\mu \xrightarrow{q^2\to 0} i\frac{k_q g_\chi m_q}{2\Lambda m_\phi^2}\bar{q}q\chi^\mu\chi_\mu \quad\Rightarrow\quad G_{S,q}=\frac{k_q g_\chi m_q}{2\Lambda m_\phi^2}$$

$$G_{S,N}=\frac{g_\chi m_N}{2\Lambda m_\phi^2}\sum_q k_q f_q^N$$

CP-odd scalar ϕ

$$\mathcal{L}_{0^-} \supset \sum_f k_f \frac{m_f}{\Lambda} \phi \bar{f} i \gamma_5 f$$

$\phi(q) \to f(p_1) + \bar{f}(p_2)$ decay width

$$i\mathcal{M} = -k_f \frac{m_f}{\Lambda} \bar{u}(p_1)\gamma_5 v(p_2), \quad (i\mathcal{M})^* = k_f \frac{m_f}{\Lambda} \bar{v}(p_2)\gamma_5 u(p_1)$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 = -\frac{k_f^2 m_f^2}{\Lambda^2} \mathrm{Tr}[(\not{p}_1 + m_f)\gamma_5(\not{p}_2 - m_f)\gamma_5] = \frac{4k_f^2 m_f^2}{\Lambda^2}(p_1 \cdot p_2 + m_f^2)$$

$$= \frac{4k_f^2 m_f^2}{\Lambda^2}\left[\frac{1}{2}(m_\phi^2 - 2m_f^2) + m_f^2\right] = \frac{2k_f^2 m_f^2 m_\phi^2}{\Lambda^2}$$

$$\Gamma(\phi \to f\bar{f}) = \frac{1}{8\pi} \frac{|\mathbf{p}_1|}{m_\phi^2} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{8\pi} \frac{1}{m_\phi^2} \frac{m_\phi}{2} \eta_f \frac{2k_f^2 m_f^2 m_\phi^2}{\Lambda^2} = \frac{\eta_f k_f^2 m_f^2 m_\phi}{8\pi\Lambda^2}$$

(2) Majarana fermion χ

$\chi(q_1) + \chi(q_2) \to f(p_1) + \bar{f}(p_2)$ annihilation

$$i\mathcal{M} = -g_\chi \bar{v}(q_2)\gamma_5 u(q_1) \frac{i}{s - m_\phi^2 + i m_\phi \Gamma_\phi} \left(-k_f \frac{m_f}{\Lambda}\right) \bar{u}(p_1)\gamma_5 v(p_2)$$

$$= \frac{i g_\chi k_f m_f}{\Lambda(s - m_\phi^2 + i m_\phi \Gamma_\phi)} \bar{v}(q_2)\gamma_5 u(q_1) \bar{u}(p_1)\gamma_5 v(p_2)$$

$$(i\mathcal{M})^* = -\frac{i g_\chi k_f m_f}{\Lambda(s - m_\phi^2 - i m_\phi \Gamma_\phi)} \bar{u}(q_1)\gamma_5 v(q_2) \bar{v}(p_2)\gamma_5 u(p_1)$$

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{g_\chi^2 k_f^2 m_f^2}{4\Lambda^2[(s - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2]} \mathrm{Tr}[(\not{q}_2 - m_\chi)\gamma_5(\not{q}_1 + m_\chi)\gamma_5] \mathrm{Tr}[(\not{p}_1 + m_f)\gamma_5(\not{p}_2 - m_f)\gamma_5]$$

$$\sigma_{\text{ann}}(\chi\chi \to f\bar{f}) = \frac{s \beta_f k_f^2 m_f^2 g_\chi^2}{16\pi\Lambda^2 \beta_\chi [(s - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2]}$$

$$a(\chi\chi \to f\bar{f}) = \frac{k_f^2 g_\chi^2 m_\chi^2 m_f^2 \rho_f}{2\pi\Lambda^2[(m_\phi^2 - 4m_\chi^2)^2 + m_\phi^2 \Gamma_\phi^2]}$$

$$b(\chi\chi \to f\bar{f}) = \frac{k_f^2 g_\chi^2 m_f^2 [16 m_\chi^4 (m_\phi^2 - 4m_\chi^2) + m_f^2 (m_\phi^4 - 24 m_\phi^2 m_\chi^2 + m_\phi^2 \Gamma_\phi^2 + 80 m_\chi^4)]}{16\pi\Lambda^2 \rho_f [(m_\phi^2 - 4m_\chi^2)^2 + m_\phi^2 \Gamma_\phi^2]^2}$$