银心 GeV 伽马射线超出: 暗物质与 τ 轻子相互作用

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1 湮灭过程运动学

银心 GeV 伽马射线超出 [1, 2, 3] 可以用暗物质湮灭到 $\tau^+\tau^-$ 来解释, 对应的暗物质粒子质量和湮灭截面 (对自共轭暗物质粒子而言) 分别为 [2]

$$m_\chi = 9.43\ (^{+0.63}_{-0.52}\ \text{stat.})\ (\pm 1.2\ \text{sys.})\ \text{GeV}, \tag{1}$$

$$\langle \sigma_{\text{anni}} v \rangle = (0.51 \pm 0.24) \times 10^{-26}\ \text{cm}^3\ \text{s}^{-1}. \tag{2}$$

在质心系中, 湮灭过程 $\chi(p_1) + \bar{\chi}(p_2) \to \tau^-(k_1) + \tau^+(k_2)$ 的各个运动学变量有如下关系.

$$p_1^0 = p_2^0 = k_1^0 = k_2^0 = \frac{\sqrt{s}}{2}, \quad p_1 \cdot p_2 = \frac{s}{2} - m_\chi^2, \quad k_1 \cdot k_2 = \frac{s}{2} - m_\tau^2, \tag{3}$$

$$|\mathbf{p}_1| = |\mathbf{p}_2| = \sqrt{\frac{s}{4} - m_\chi^2} = \frac{\sqrt{s}}{2}\beta_\chi, \quad \beta_\chi \equiv \sqrt{1 - 4m_\chi^2/s}, \tag{4}$$

$$|\mathbf{k}_1| = |\mathbf{k}_2| = \sqrt{\frac{s}{4} - m_\tau^2} = \frac{\sqrt{s}}{2}\beta_\tau, \quad \beta_\tau \equiv \sqrt{1 - 4m_\tau^2/s}, \tag{5}$$

$$p_2 \cdot k_2 = p_1 \cdot k_1 = p_1^0 k_1^0 - |\mathbf{p}_1||\mathbf{k}_1|\cos\theta = \frac{s}{4}(1 - \beta_\chi \beta_\tau \cos\theta), \tag{6}$$

$$p_2 \cdot k_1 = p_1 \cdot k_2 = p_1^0 k_1^0 + |\mathbf{p}_1||\mathbf{k}_1|\cos\theta = \frac{s}{4}(1 + \beta_\chi \beta_\tau \cos\theta), \tag{7}$$

$$q = p_1 + p_2 = k_1 + k_2 \quad \Rightarrow \quad q \cdot p_1 = q \cdot p_2 = q \cdot k_1 = q \cdot k_2 = \frac{s}{2}, \tag{8}$$

$$t = (p_1 - k_1)^2 = m_\chi^2 + m_\tau^2 - 2p_1 \cdot k_1 = m_\chi^2 + m_\tau^2 - \frac{s}{2}(1 - \beta_\chi \beta_\tau \cos\theta), \tag{9}$$

$$u = (p_1 - k_2)^2 = m_\chi^2 + m_\tau^2 - 2p_1 \cdot k_2 = m_\chi^2 + m_\tau^2 - \frac{s}{2}(1 + \beta_\chi \beta_\tau \cos\theta). \tag{10}$$

在低速极限下, $s \to 4m_\chi^2$, $\beta_\chi \to 0$, 则

$$t \to m_\tau^2 - m_\chi^2, \quad u \to m_\tau^2 - m_\chi^2, \tag{11}$$

$$p_2 \cdot k_2 = p_1 \cdot k_1 \to m_\chi^2, \quad p_2 \cdot k_1 = p_1 \cdot k_2 \to m_\chi^2, \tag{12}$$

$$p_1 \cdot p_2 = m_\chi^2, \quad k_1 \cdot k_2 \to 2m_\chi^2 - m_\tau^2. \tag{13}$$

微分截面可表达为

$$\frac{d\sigma_{\text{anni}}}{d\Omega} = \frac{1}{2p_1^0 2p_2^0 |\mathbf{v}_1 - \mathbf{v}_2|} \frac{|\mathbf{k}_1|}{(2\pi)^2 4E_{\text{CM}}} \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{2\beta_\chi} \frac{\beta_\tau}{32\pi^2 s} \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2. \tag{14}$$

2 t-channel 湮灭模型 (tau portal)

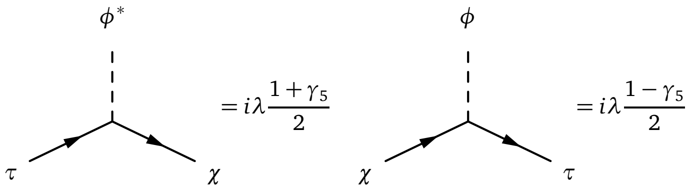
2.1 Dirac fermionic DM, scalar mediator

假设暗物质粒子是 Dirac 费米子, 而且是标准模型规范单态. 参考文献 [5, 6, 7, 8], 考虑暗物质粒子 (χ) 通过 t-channel 交换标量粒子 ϕ 湮灭到 $\tau^+\tau^-$, 相互作用拉氏量为

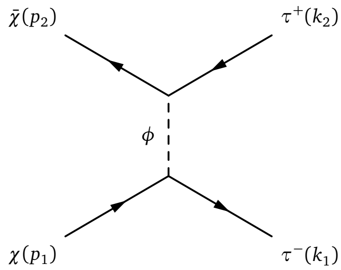
$$\mathcal{L}_\phi = \lambda(\phi^*\bar{\chi}_L\tau_R + \phi\bar{\tau}_R\chi_L) = \frac{\lambda}{2}[\phi^*\bar{\chi}(1+\gamma_5)\tau + \phi\bar{\tau}(1-\gamma_5)\chi]. \tag{15}$$

标量粒子 ϕ 是 $SU(2)_L$ 单态, τ 轻子数为 $+1$, 电荷为 -1, 弱超荷为 -1.

费曼规则如下.



湮灭过程 $\chi(p_1)+\bar{\chi}(p_2)\to\tau^-(k_1)+\tau^+(k_2)$ 的费曼图为



不变振幅

$$\begin{aligned}
i\mathcal{M} &= i\frac{\lambda}{2}\bar{u}(k_1)(1-\gamma_5)u(p_1)\frac{i}{(p_1-k_1)^2-m_\phi^2}i\frac{\lambda}{2}\bar{v}(p_2)(1+\gamma_5)v(k_2) \\
&= -\frac{i\lambda^2}{4(t-m_\phi^2)}\bar{u}(k_1)(1-\gamma_5)u(p_1)\bar{v}(p_2)(1+\gamma_5)v(k_2), \qquad (16)\\
(i\mathcal{M})^* &= \frac{i\lambda^2}{4(t-m_\phi^2)}\bar{u}(p_1)(1+\gamma_5)u(k_1)\bar{v}(k_2)(1-\gamma_5)v(p_2), \qquad (17)
\end{aligned}$$

则

$$\begin{aligned}
\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 &= \sum_{\text{spins}}\frac{\lambda^4}{64(t-m_\phi^2)^2}\bar{u}(k_1)(1-\gamma_5)u(p_1)\bar{u}(p_1)(1+\gamma_5)u(k_1)\bar{v}(p_2)(1+\gamma_5)v(k_2)\bar{v}(k_2)(1-\gamma_5)v(p_2) \\
&= \frac{\lambda^4}{64(t-m_\phi^2)^2}\mathrm{Tr}[(\not{k}_1+m_\tau)(1-\gamma_5)(\not{p}_1+m_\chi)(1+\gamma_5)]\mathrm{Tr}[(\not{p}_2-m_\chi)(1+\gamma_5)(\not{k}_2-m_\tau)(1-\gamma_5)] \\
&= \frac{\lambda^4}{(t-m_\phi^2)^2}(p_1\cdot k_1)(p_2\cdot k_2) = \frac{\lambda^4}{(t-m_\phi^2)^2}\frac{s}{4}(1-\beta_\chi\beta_\tau\cos\theta)\frac{s}{4}(1-\beta_\chi\beta_\tau\cos\theta) \\
&= \frac{\lambda^4}{16}\frac{s^2(1-\beta_\chi\beta_\tau\cos\theta)^2}{(t-m_\phi^2)^2}\xrightarrow{s\to4m_\chi^2,\ \beta_\chi\to0}\frac{\lambda^4m_\chi^4}{(m_\phi^2+m_\chi^2-m_\tau^2)^2}. \qquad (18)
\end{aligned}$$

在低速近似下,

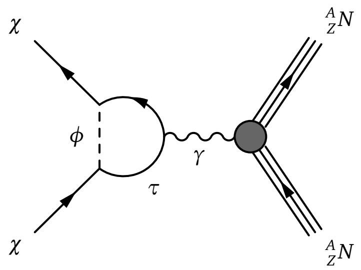
$$\sigma_{\text{anni}}v \simeq 4\pi\frac{\beta_\tau}{32\pi^2 4m_\chi^2}\frac{\lambda^4m_\chi^4}{(m_\phi^2+m_\chi^2-m_\tau^2)^2} = \frac{\lambda^4m_\chi^2\beta_\tau}{32\pi(m_\phi^2+m_\chi^2-m_\tau^2)^2}, \tag{19}$$

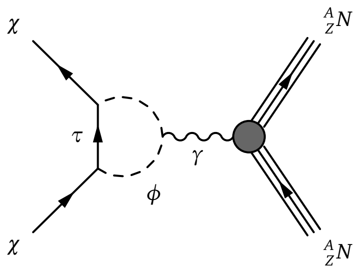
其中 $\beta_\tau = \sqrt{1 - m_\tau^2/m_\chi^2}$. 当 $m_\tau \ll m_\chi \ll m_\phi$ 时, 有

$$\frac{1}{2}\left\langle \sigma_{\rm anni} v \right\rangle \simeq \frac{\lambda^4 m_\chi^2}{64\pi m_\phi^4} = 5\times 10^{-27}\ {\rm cm^3\ s^{-1}} \left(\frac{m_\chi}{10\ {\rm GeV}}\right)^2 \left(\frac{\lambda}{m_\phi/185\ {\rm GeV}}\right)^4. \tag{20}$$

考虑到 Dirac 暗物质由正反粒子组成, 将湮灭截面乘上一个 $1/2$ 因子, 才能与假设暗物质粒子自共轭所得出的湮灭截面相比较.

暗物质粒子可以通过单圈过程与原子核发生相互作用:





暗物质粒子与核子的 SI 散射截面可表达为 [6]

$$\sigma_{\chi N} = \frac{Z^2 e^2 c_1^2 \mu_{\chi N}^2}{\pi A^2}, \tag{21}$$

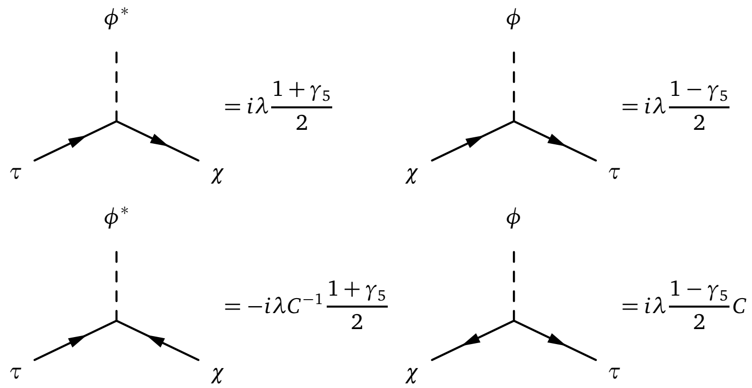
其中约化质量 $\mu_{\chi N} \equiv (m_\chi m_N)/(m_\chi + m_N)$, 而

$$c_1 = -\frac{\lambda^2 e}{64\pi^2 m_\phi^2}\left[\frac{1}{2} + \frac{2}{3}\ln\left(\frac{m_\tau^2}{m_\phi^2}\right)\right]. \tag{22}$$

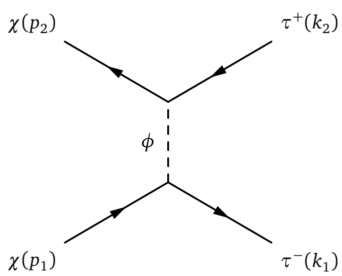
对于 LUX 实验, $Z = 54$, $A = 129$. 取 $m_\chi = 10$ GeV, $m_\phi = 185$ GeV, $\lambda = 1$, 可得 $\sigma_{\chi N} = 9.2\times 10^{-45}\ {\rm cm}^2$.

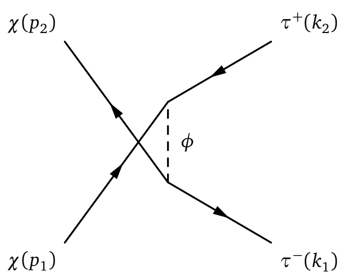
2.2 Majorana fermionic DM, scalar mediator

假设暗物质粒子是 Majorana 费米子. 相互作用拉氏量与上一小节相同. 费曼规则如下 (参考文献 [4]).



湮灭过程 $\chi(p_1) + \chi(p_2) \to \tau^-(k_1) + \tau^+(k_2)$ 的费曼图为





注意到

$$
\begin{aligned}
C^{\mathrm{T}} &= -C, \quad C^{-1}\gamma_\mu C = -\gamma_\mu^{\mathrm{T}}, \quad C^{-1}\gamma_5 C = \gamma_5, \\
u(p_i) &= C\bar{v}(p_i)^{\mathrm{T}}, \quad u^{\mathrm{T}}(p_i) = \bar{v}(p_i)C^{\mathrm{T}} = -\bar{v}(p_i)C,
\end{aligned} \tag{23}
$$

两幅费曼图的不变振幅分别为

$$
\begin{aligned}
i\mathcal{M}_1 &= i\frac{\lambda}{2}\bar{u}(k_1)(1-\gamma_5)u(p_1)\frac{i}{(p_1-k_1)^2-m_\phi^2}i\frac{\lambda}{2}\bar{v}(p_2)(1+\gamma_5)v(k_2) \\
&= -\frac{i\lambda^2}{4(t-m_\phi^2)}\bar{u}(k_1)(1-\gamma_5)u(p_1)\bar{v}(p_2)(1+\gamma_5)v(k_2),
\end{aligned} \tag{24}
$$

$$
(i\mathcal{M}_1)^* = \frac{i\lambda^2}{4(t-m_\phi^2)}\bar{u}(p_1)(1+\gamma_5)u(k_1)\bar{v}(k_2)(1-\gamma_5)v(p_2), \tag{25}
$$

$$
\begin{aligned}
i\mathcal{M}_2 &= i\frac{\lambda}{2}\bar{u}(k_1)(1-\gamma_5)C\bar{v}(p_2)^{\mathrm{T}}\frac{i}{(p_1-k_2)^2-m_\phi^2}(-i)\frac{\lambda}{2}u(p_1)^{\mathrm{T}}C^{-1}(1+\gamma_5)v(k_2) \\
&= \frac{i\lambda^2}{4(u-m_\phi^2)}\bar{u}(k_1)(1-\gamma_5)C\bar{v}(p_2)^{\mathrm{T}}u(p_1)^{\mathrm{T}}C^{-1}(1+\gamma_5)v(k_2) \\
&= -\frac{i\lambda^2}{4(u-m_\phi^2)}\bar{u}(k_1)(1-\gamma_5)u(p_2)\bar{v}(p_1)(1+\gamma_5)v(k_2),
\end{aligned} \tag{26}
$$

$$
(i\mathcal{M}_2)^* = \frac{i\lambda^2}{4(u-m_\phi^2)}\bar{u}(p_2)(1+\gamma_5)u(k_1)\bar{v}(k_2)(1-\gamma_5)v(p_1), \tag{27}
$$

则

$$
\begin{aligned}
&\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_1|^2 \\
=& \sum_{\text{spins}}\frac{\lambda^4}{64(t-m_\phi^2)^2}\bar{u}(k_1)(1-\gamma_5)u(p_1)\bar{u}(p_1)(1+\gamma_5)u(k_1)\bar{v}(p_2)(1+\gamma_5)v(k_2)\bar{v}(k_2)(1-\gamma_5)v(p_2) \\
=& \frac{\lambda^4}{64(t-m_\phi^2)^2}\mathrm{Tr}[(\not{k}_1+m_\tau)(1-\gamma_5)(\not{p}_1+m_\chi)(1+\gamma_5)]\mathrm{Tr}[(\not{p}_2-m_\chi)(1+\gamma_5)(\not{k}_2-m_\tau)(1-\gamma_5)] \\
=& \frac{\lambda^4(p_1\cdot k_1)(p_2\cdot k_2)}{(t-m_\phi^2)^2},
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
&\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_2|^2 \\
=& \sum_{\text{spins}}\frac{\lambda^4}{64(u-m_\phi^2)^2}\bar{u}(k_1)(1-\gamma_5)u(p_2)\bar{u}(p_2)(1+\gamma_5)u(k_1)\bar{v}(p_1)(1+\gamma_5)v(k_2)\bar{v}(k_2)(1-\gamma_5)v(p_1) \\
=& \frac{\lambda^4}{64(u-m_\phi^2)^2}\mathrm{Tr}[(\not{k}_1+m_\tau)(1-\gamma_5)(\not{p}_2+m_\chi)(1+\gamma_5)]\mathrm{Tr}[(\not{p}_1-m_\chi)(1+\gamma_5)(\not{k}_2-m_\tau)(1-\gamma_5)] \\
=& \frac{\lambda^4(p_2\cdot k_1)(p_1\cdot k_2)}{(u-m_\phi^2)^2},
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
&-\frac{1}{4}\sum_{\text{spins}}(\mathcal{M}_1^*\mathcal{M}_2+\text{h.c.}) \\
=& \sum_{\text{spins}}\frac{\lambda^4}{64(t-m_\phi^2)(u-m_\phi^2)}[\bar{u}(p_1)(1+\gamma_5)u(k_1)\bar{v}(k_2)(1-\gamma_5)v(p_2) \\
&\qquad\times\bar{u}(k_1)(1-\gamma_5)C\bar{v}(p_2)^{\mathrm{T}}u(p_1)^{\mathrm{T}}C^{-1}(1+\gamma_5)v(k_2)+\text{h.c.}] \\
=& \sum_{\text{spins}}\frac{\lambda^4}{64(t-m_\phi^2)(u-m_\phi^2)}[u(k_1)^{\mathrm{T}}(1+\gamma_5)^{\mathrm{T}}\bar{u}(p_1)^{\mathrm{T}}u(p_1)^{\mathrm{T}}C^{-1}(1+\gamma_5)v(k_2)
\end{aligned}
$$

$$
\begin{aligned}
&\qquad\qquad\qquad\qquad \times\bar{v}(k_2)(1-\gamma_5)v(p_2)\bar{v}(p_2)C^{\mathrm{T}}(1-\gamma_5)^{\mathrm{T}}\bar{u}(k_1)^{\mathrm{T}}+\mathrm{h.c.}]\\
&= \frac{\lambda^4}{64(t-m_\phi^2)(u-m_\phi^2)}\mathrm{Tr}[C^{\mathrm{T}}(1-\gamma_5)^{\mathrm{T}}(\not{k}_1+m_\tau)^{\mathrm{T}}(1+\gamma_5)^{\mathrm{T}}(\not{p}_1+m_\chi)^{\mathrm{T}}C^{-1}\\
&\qquad\qquad\qquad\qquad \times(1+\gamma_5)(\not{k}_2-m_\tau)(1-\gamma_5)(\not{p}_2-m_\chi)]+\mathrm{h.c.}\\
&= \frac{\lambda^4}{64(t-m_\phi^2)(u-m_\phi^2)}\mathrm{Tr}[-(1-\gamma_5)(-\not{k}_1+m_\tau)(1+\gamma_5)(-\not{p}_1+m_\chi)\\
&\qquad\qquad\qquad\qquad \times(1+\gamma_5)(\not{k}_2-m_\tau)(1-\gamma_5)(\not{p}_2-m_\chi)]+\mathrm{h.c.}\\
&= -\frac{\lambda^4 m_\chi^2(k_1\cdot k_2)}{2(t-m_\phi^2)(u-m_\phi^2)}+\mathrm{h.c.} = -\frac{\lambda^4 m_\chi^2(k_1\cdot k_2)}{(t-m_\phi^2)(u-m_\phi^2)}.
\end{aligned}
\tag{30}
$$

从而,

$$
\begin{aligned}
\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 &= \frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_1-\mathcal{M}_2|^2 = \frac{\lambda^4(p_1\cdot k_1)(p_2\cdot k_2)}{(t-m_\phi^2)^2}+\frac{\lambda^4(p_2\cdot k_1)(p_1\cdot k_2)}{(u-m_\phi^2)^2}-\frac{\lambda^4 m_\chi^2(k_1\cdot k_2)}{(t-m_\phi^2)(u-m_\phi^2)}\\
&\xrightarrow{s\to 4m_\chi^2,\ \beta_\chi\to 0} \frac{\lambda^4 m_\chi^4}{(m_\phi^2+m_\chi^2-m_\tau^2)^2}+\frac{\lambda^4 m_\chi^4}{(m_\phi^2+m_\chi^2-m_\tau^2)^2}-\frac{\lambda^4 m_\chi^2(2m_\chi^2-m_\tau^2)}{(m_\phi^2+m_\chi^2-m_\tau^2)^2}\\
&= \frac{\lambda^4 m_\tau^2 m_\chi^2}{(m_\chi^2+m_\phi^2-m_\tau^2)^2}.
\end{aligned}
\tag{31}
$$

在低速近似下,

$$
\sigma_{\mathrm{anni}}v \simeq 4\pi\frac{\beta_\tau}{32\pi^2 4m_\chi^2}\frac{\lambda^4 m_\tau^2 m_\chi^2}{(m_\chi^2+m_\phi^2-m_\tau^2)^2} = \frac{\lambda^4 m_\tau^2\beta_\tau}{32\pi(m_\chi^2+m_\phi^2-m_\tau^2)^2}.
\tag{32}
$$

当 $m_\tau \ll m_\chi \ll m_\phi$ 时, 有

$$
\langle\sigma_{\mathrm{anni}}v\rangle \simeq \frac{\lambda^4 m_\tau^2}{32\pi m_\phi^4} = 5\times 10^{-27}\ \mathrm{cm^3\ s^{-1}}\left(\frac{\lambda}{m_\phi/93\ \mathrm{GeV}}\right)^4.
\tag{33}
$$

暗物质粒子与原子核的散射通过 anapole moment 发生, 截面很小, 当前直接探测实验对它不灵敏 [6, 7].

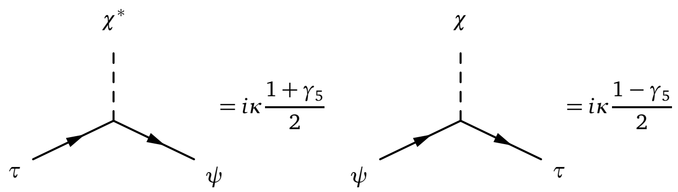
2.3 Complex scalar DM, fermionic mediator

假设暗物质粒子是复标量粒子, 而且是标准模型规范单态. 参考文献 [6, 7], 考虑暗物质粒子 (χ) 通过 t-channel 交换费米子 ψ 湮灭到 $\tau^+\tau^-$, 相互作用拉氏量为

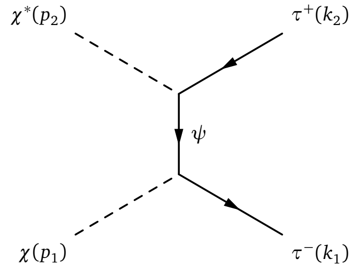
$$
\mathcal{L}_\psi = \kappa(\chi^*\bar{\psi}_L\tau_R+\chi\bar{\tau}_R\psi_L) = \frac{\kappa}{2}[\chi^*\bar{\psi}(1+\gamma_5)\tau+\chi\bar{\tau}(1-\gamma_5)\psi].
\tag{34}
$$

Dirac 费米子 ψ 是 $SU(2)_L$ 单态, τ 轻子数为 $+1$, 电荷为 -1, 弱超荷为 -1.

费曼规则如下.



湮灭过程 $\chi(p_1)+\chi^*(p_2)\to\tau^-(k_1)+\tau^+(k_2)$ 的费曼图为



不变振幅

$$
\begin{aligned}
i\mathcal{M} &= \bar{u}(k_1) i\frac{\kappa}{2}(1-\gamma_5)\frac{i(\not{k}_1 - \not{p}_1 + m_\psi)}{(k_1-p_1)^2 - m_\psi^2} i\frac{\kappa}{2}(1+\gamma_5)v(k_2) \\
&= -\frac{i\kappa^2}{4(t-m_\psi^2)}\bar{u}(k_1)(1-\gamma_5)(\not{k}_1 - \not{p}_1 + m_\psi)(1+\gamma_5)v(k_2), \qquad (35)\\
(i\mathcal{M})^* &= \frac{i\kappa^2}{4(t-m_\psi^2)}\bar{v}(k_2)(1-\gamma_5)(\not{k}_1 - \not{p}_1 + m_\psi)(1+\gamma_5)u(k_1), \qquad (36)
\end{aligned}
$$

则

$$
\begin{aligned}
\sum_{\text{spins}} |\mathcal{M}|^2 &= \sum_{\text{spins}} \frac{\kappa^4}{16(t-m_\psi^2)^2}\bar{u}(k_1)(1-\gamma_5)(\not{k}_1 - \not{p}_1 + m_\psi)(1+\gamma_5)v(k_2)\bar{v}(k_2)(1-\gamma_5)(\not{k}_1 - \not{p}_1 + m_\psi)(1+\gamma_5)u(k_1) \\
&= \frac{\kappa^4}{16(t-m_\psi^2)^2}\mathrm{Tr}[(\not{k}_1 + m_\tau)(1-\gamma_5)(\not{k}_1 - \not{p}_1 + m_\psi)(1+\gamma_5)(\not{k}_2 - m_\tau)(1-\gamma_5)(\not{k}_1 - \not{p}_1 + m_\psi)(1+\gamma_5)] \\
&= \frac{2\kappa^4}{(t-m_\psi^2)^2}[-2m_\tau^2(k_2\cdot p_1) + (m_\tau^2 - m_\chi^2)(k_1\cdot k_2) + 2(k_1\cdot p_1)(k_2\cdot p_1)] \\
&= \frac{\kappa^4}{4(t-m_\psi^2)^2}\{s^2(1-\beta_\chi^2\beta_\tau^2\cos^2\theta) - 4m_\chi^2 s - 4m_\tau^2 s\beta_\chi\beta_\tau\cos\theta + 8m_\tau^2(m_\chi^2 - m_\tau^2)\} \\
&\xrightarrow{s\to 4m_\chi^2,\ \beta_\chi\to 0} \frac{2\kappa^4 m_\tau^2 m_\chi^2\beta_\tau^2}{(m_\psi^2 + m_\chi^2 - m_\tau^2)^2}. \qquad (37)
\end{aligned}
$$

在低速近似下,

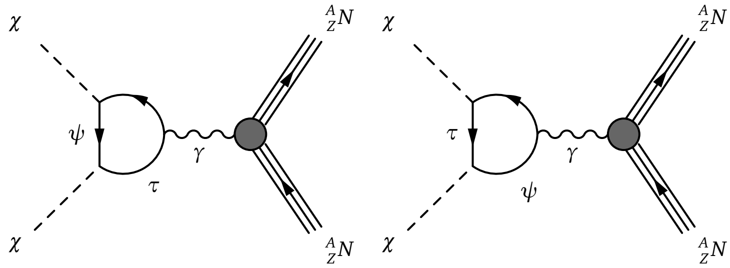
$$
\sigma_{\text{anni}}v \simeq 4\pi\frac{\beta_\tau}{32\pi^2 4m_\chi^2}\frac{2\kappa^4 m_\tau^2 m_\chi^2\beta_\tau^2}{(m_\psi^2 + m_\chi^2 - m_\tau^2)^2} = \frac{\kappa^4 m_\tau^2\beta_\tau^3}{16\pi(m_\psi^2 + m_\chi^2 - m_\tau^2)^2}, \qquad (38)
$$

当 $m_\tau \ll m_\chi \ll m_\psi$ 时, 有

$$
\frac{1}{2}\left\langle\sigma_{\text{anni}}v\right\rangle \simeq \frac{\kappa^4 m_\tau^2}{32\pi m_\psi^4} = 5\times 10^{-27}\ \text{cm}^3\text{s}^{-1}\left(\frac{\kappa}{m_\psi/93\ \text{GeV}}\right)^4. \qquad (39)
$$

考虑到复标量暗物质由正反粒子组成, 将湮灭截面乘上一个 $1/2$ 因子, 才能与假设暗物质粒子自共轭所得出的湮灭截面相比较.

暗物质粒子可以通过单圈过程与原子核发生相互作用:



暗物质粒子与核子的 SI 散射截面可表达为 [6]

$$\sigma_{\chi N} = \frac{Z^2 e^2 C^2(m_\tau, m_\psi)\mu_{\chi N}^2}{8\pi A^2}, \tag{40}$$

其中

$$\begin{aligned} C(m_\tau, m_\psi) &= -\frac{\kappa^2 e}{16\pi^2}\left[\frac{m_\psi^4 - 6m_\psi^2 m_\tau^2 + m_\tau^4}{\left(m_\psi^2 - m_\tau^2\right)^3} + \frac{2(m_\psi^2 + m_\tau^2)(m_\psi^4 - 5m_\psi^2 m_\tau^2 + m_\tau^4)}{3\left(m_\psi^2 - m_\tau^2\right)^4}\ln\left(\frac{m_\tau^2}{m_\psi^2}\right)\right] \\ &\simeq -\frac{\kappa^2 e}{16\pi^2 m_\psi^2}\left[1 + \frac{2}{3}\ln\left(\frac{m_\tau^2}{m_\psi^2}\right)\right]. \end{aligned} \tag{41}$$

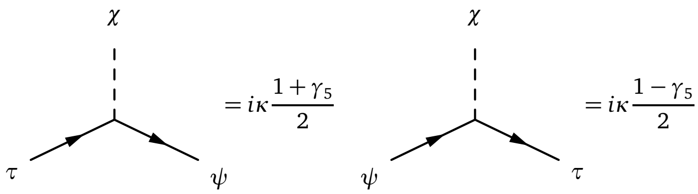
对于 LUX 实验, $Z = 54$, $A = 129$. 取 $m_\chi = 10$ GeV, $m_\psi = 93$ GeV, $\kappa = 1$, 可得 $\sigma_{\chi N} \simeq 1.6 \times 10^{-43}$ cm^2.

2.4 Real scalar DM, fermionic mediator

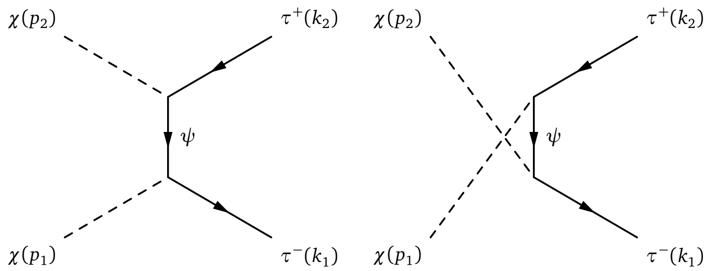
假设暗物质粒子是实标量粒子, mediator 性质与上一小节一样, 相互作用拉氏量与略有不同:

$$\mathcal{L}_\psi = \kappa\chi(\bar{\psi}_L \tau_R + \bar{\tau}_R \psi_L) = \frac{\kappa}{2}\chi[\bar{\psi}(1+\gamma_5)\tau + \bar{\tau}(1-\gamma_5)\psi]. \tag{42}$$

费曼规则如下.



湮灭过程 $\chi(p_1) + \chi(p_2) \to \tau^-(k_1) + \tau^+(k_2)$ 的费曼图为



两幅费曼图的不变振幅分别为

$$\begin{aligned} i\mathcal{M}_1 &= \bar{u}(k_1) i\frac{\kappa}{2}(1-\gamma_5)\frac{i(\not{k}_1 - \not{p}_1 + m_\psi)}{\left(k_1 - p_1\right)^2 - m_\psi^2} i\frac{\kappa}{2}(1+\gamma_5)v(k_2) \\ &= -\frac{i\kappa^2}{4(t - m_\psi^2)}\bar{u}(k_1)(1-\gamma_5)(\not{k}_1 - \not{p}_1 + m_\psi)(1+\gamma_5)v(k_2), \end{aligned} \tag{43}$$

$$(i\mathcal{M}_1)^* = \frac{i\kappa^2}{4(t - m_\psi^2)}\bar{v}(k_2)(1-\gamma_5)(\not{k}_1 - \not{p}_1 + m_\psi)(1+\gamma_5)u(k_1), \tag{44}$$

$$\begin{aligned} i\mathcal{M}_2 &= \bar{u}(k_1) i\frac{\kappa}{2}(1-\gamma_5)\frac{i(\not{k}_1 - \not{p}_2 + m_\psi)}{\left(k_1 - p_2\right)^2 - m_\psi^2} i\frac{\kappa}{2}(1+\gamma_5)v(k_2) \\ &= -\frac{i\kappa^2}{4(u - m_\psi^2)}\bar{u}(k_1)(1-\gamma_5)(\not{k}_1 - \not{p}_2 + m_\psi)(1+\gamma_5)v(k_2), \end{aligned} \tag{45}$$

$$(i\mathcal{M}_2)^* = \frac{i\kappa^2}{4(u - m_\psi^2)}\bar{v}(k_2)(1-\gamma_5)(\not{k}_1 - \not{p}_2 + m_\psi)(1+\gamma_5)u(k_1), \tag{46}$$

则

$$
\begin{aligned}
&\sum_{\text{spins}} |\mathcal{M}_1|^2 \\
=& \sum_{\text{spins}} \frac{\kappa^4}{16(t-m_\psi^2)^2} \bar{u}(k_1)(1-\gamma_5)(\not{k}_1-\not{p}_1+m_\psi)(1+\gamma_5)v(k_2)\bar{v}(k_2)(1-\gamma_5)(\not{k}_1-\not{p}_1+m_\psi)(1+\gamma_5)u(k_1) \\
=& \frac{\kappa^4}{16(t-m_\psi^2)^2} \mathrm{Tr}[(\not{k}_1+m_\tau)(1-\gamma_5)(\not{k}_1-\not{p}_1+m_\psi)(1+\gamma_5)(\not{k}_2-m_\tau)(1-\gamma_5)(\not{k}_1-\not{p}_1+m_\psi)(1+\gamma_5)] \\
=& \frac{2\kappa^4}{(t-m_\psi^2)^2}[-2m_\tau^2(k_2\cdot p_1)+(m_\tau^2-m_\chi^2)(k_1\cdot k_2)+2(k_1\cdot p_1)(k_2\cdot p_1)], \qquad (47)\\
&\sum_{\text{spins}} |\mathcal{M}_2|^2 \\
=& \sum_{\text{spins}} \frac{\kappa^4}{16(u-m_\psi^2)^2} \bar{u}(k_1)(1-\gamma_5)(\not{k}_1-\not{p}_2+m_\psi)(1+\gamma_5)v(k_2)\bar{v}(k_2)(1-\gamma_5)(\not{k}_1-\not{p}_2+m_\psi)(1+\gamma_5)u(k_1) \\
=& \frac{\kappa^4}{16(u-m_\psi^2)^2} \mathrm{Tr}[(\not{k}_1+m_\tau)(1-\gamma_5)(\not{k}_1-\not{p}_2+m_\psi)(1+\gamma_5)(\not{k}_2-m_\tau)(1-\gamma_5)(\not{k}_1-\not{p}_2+m_\psi)(1+\gamma_5)] \\
=& \frac{2\kappa^4}{(u-m_\psi^2)^2}[-2m_\tau^2(k_2\cdot p_2)+(m_\tau^2-m_\chi^2)(k_1\cdot k_2)+2(k_1\cdot p_2)(k_2\cdot p_2)], \qquad (48)\\
&\sum_{\text{spins}} (\mathcal{M}_1^*\mathcal{M}_2+\text{h.c.}) \\
=& \sum_{\text{spins}} \frac{\kappa^4}{16(t-m_\psi^2)(u-m_\psi^2)} [\bar{v}(k_2)(1-\gamma_5)(\not{k}_1-\not{p}_1+m_\psi)(1+\gamma_5)u(k_1)\bar{u}(k_1)(1-\gamma_5) \\
&\qquad\times(\not{k}_1-\not{p}_2+m_\psi)(1+\gamma_5)v(k_2)+\text{h.c.}] \\
=& \frac{\kappa^4}{16(t-m_\psi^2)(u-m_\psi^2)} \mathrm{Tr}[(\not{k}_2-m_\tau)(1-\gamma_5)(\not{k}_1-\not{p}_1+m_\psi)(1+\gamma_5)(\not{k}_1+m_\tau)(1-\gamma_5) \\
&\qquad\times(\not{k}_1-\not{p}_2+m_\psi)(1+\gamma_5)]+\text{h.c.} \\
=& \frac{4\kappa^4}{(t-m_\psi^2)(u-m_\psi^2)}[(k_1\cdot p_2)(k_2\cdot p_1)+(k_1\cdot p_1)(k_2\cdot p_2)-(k_1\cdot k_2)(p_1\cdot p_2)-m_\tau^2(k_2\cdot p_1+k_2\cdot p_2-k_1\cdot k_2)]. \\
& \qquad (49)
\end{aligned}
$$

从而,

$$
\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2=\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_1+\mathcal{M}_2|^2 \xrightarrow{s\to 4m_\chi^2,\ \beta_\chi\to 0} \frac{8\kappa^4 m_\tau^2 m_\chi^2\beta_\tau^2}{(m_\psi^2+m_\chi^2-m_\tau^2)^2}. \qquad (50)
$$

在低速近似下,

$$
\sigma_{\text{anni}}v \simeq 4\pi\frac{\beta_\tau}{32\pi^2 4m_\chi^2}\frac{8\kappa^4 m_\tau^2 m_\chi^2\beta_\tau^2}{(m_\psi^2+m_\chi^2-m_\tau^2)^2}=\frac{\kappa^4 m_\tau^2\beta_\tau^3}{4\pi(m_\psi^2+m_\chi^2-m_\tau^2)^2}. \qquad (51)
$$

当 $m_\tau \ll m_\chi \ll m_\psi$ 时, 有

$$
\langle\sigma_{\text{anni}}v\rangle \simeq \frac{\kappa^4 m_\tau^2}{4\pi m_\psi^4} = 5\times 10^{-27}\ \text{cm}^3\text{s}^{-1}\left(\frac{\kappa}{m_\psi/156\ \text{GeV}}\right)^4. \qquad (52)
$$

暗物质粒子与原子核的散射通过双圈过程发生, 截面非常小, 当前直接探测实验对它不灵敏 [7].

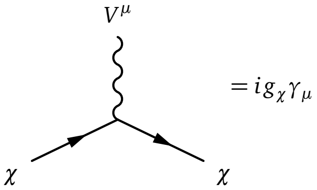
3 s-channel 湮灭模型

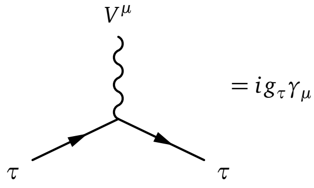
参考文献 [9], 考虑暗物质粒子通过 s-channel 湮灭到 $\tau^+\tau^-$. 假设中介粒子分别为矢量粒子 V, 轴矢量粒子 U 和赝标量粒子 S, 而暗物质粒子是 Dirac 费米子.

设矢量粒子 V 中介的相互作用拉氏量为

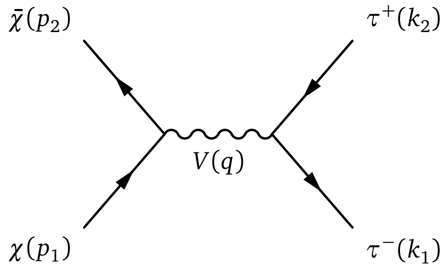
$$\mathcal{L}_V = (g_\chi \bar{\chi}\gamma_\mu \chi + g_\tau \bar{\tau}\gamma_\mu \tau)V^\mu. \tag{53}$$

相应费曼规则如下.





湮灭过程 $\chi(p_1) + \bar{\chi}(p_2) \to \tau^-(k_1) + \tau^+(k_2)$ 的费曼图为



不变振幅

$$\begin{aligned} i\mathcal{M} &= ig_\chi \bar{v}(p_2)\gamma_\mu u(p_1)\frac{-i(g^{\mu\nu} - q^\mu q^\nu/m_V^2)}{q^2 - m_V^2} ig_\tau \bar{u}(k_1)\gamma_\nu v(k_2) \\ &= ig_\chi g_\tau \frac{g^{\mu\nu} - q^\mu q^\nu/m_V^2}{s - m_V^2}\bar{v}(p_2)\gamma_\mu u(p_1)\bar{u}(k_1)\gamma_\nu v(k_2), \end{aligned} \tag{54}$$

$$(i\mathcal{M})^* = -ig_\chi g_\tau \frac{g^{\rho\sigma} - q^\rho q^\sigma/m_V^2}{s - m_V^2}\bar{u}(p_1)\gamma_\rho v(p_2)\bar{v}(k_2)\gamma_\sigma u(k_1), \tag{55}$$

则

$$\begin{aligned} \frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 &= \sum_{\text{spins}}\frac{(g_\chi g_\tau)^2}{4(s-m_V^2)^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_V^2}\right)\left(g^{\rho\sigma} - \frac{q^\rho q^\sigma}{m_V^2}\right)\bar{v}(p_2)\gamma_\mu u(p_1)\bar{u}(p_1)\gamma_\rho v(p_2)\bar{u}(k_1)\gamma_\nu v(k_2)\bar{v}(k_2)\gamma_\sigma u(k_1) \\ &= \frac{(g_\chi g_\tau)^2}{4(s-m_V^2)^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_V^2}\right)\left(g^{\rho\sigma} - \frac{q^\rho q^\sigma}{m_V^2}\right)\mathrm{Tr}[(\not{p}_2 - m_\chi)\gamma_\mu(\not{p}_1 + m_\chi)\gamma_\rho]\mathrm{Tr}[(\not{k}_1 + m_\tau)\gamma_\nu(\not{k}_2 - m_\tau)\gamma_\sigma] \\ &= \frac{(g_\chi g_\tau)^2}{(s-m_V^2)^2}s[s(1+\beta_\chi^2\beta_\tau^2\cos^2\theta) + 4m_\chi^2 + 4m_\tau^2] \xrightarrow{s\to 4m_\chi^2,\ \beta_\chi\to 0} \frac{16(g_\chi g_\tau)^2 m_\chi^2(2m_\chi^2 + m_\tau^2)}{(m_V^2 - 4m_\chi^2)^2}. \end{aligned} \tag{56}$$

在低速近似下,

$$\sigma_{\text{anni}}v \simeq 4\pi\frac{\beta_\tau}{32\pi^2 4m_\chi^2}\frac{16(g_\chi g_\tau)^2 m_\chi^2(2m_\chi^2 + m_\tau^2)}{(m_V^2 - 4m_\chi^2)^2} = \frac{(g_\chi g_\tau)^2(2m_\chi^2 + m_\tau^2)\beta_\tau}{2\pi(m_V^2 - 4m_\chi^2)^2}. \tag{57}$$

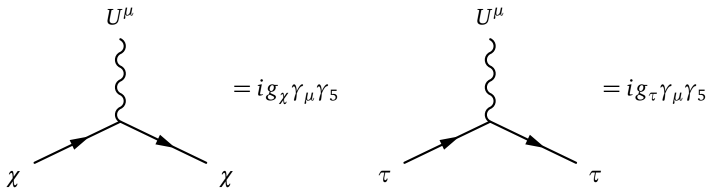
当 $m_\tau \ll m_\chi \ll m_\phi$ 时, 有

$$\frac{1}{2}\langle\sigma_{\text{anni}}v\rangle \simeq \frac{(g_\chi g_\tau)^2 m_\chi^2}{2\pi m_V^4} = 5\times 10^{-27}\ \text{cm}^3\ \text{s}^{-1}\left(\frac{m_\chi}{10\ \text{GeV}}\right)^2\left(\frac{\sqrt{g_\chi g_\tau}}{m_V/439\ \text{GeV}}\right)^4. \tag{58}$$

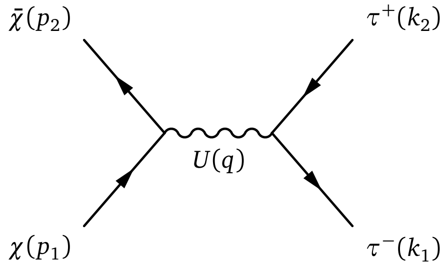
设轴矢量粒子 U 中介的相互作用拉氏量为

$$\mathcal{L}_U = (g_\chi \bar{\chi}\gamma_\mu\gamma_5 \chi + g_\tau \bar{\tau}\gamma_\mu\gamma_5 \tau)U^\mu. \tag{59}$$

相应费曼规则如下.



湮灭过程 $\chi(p_1) + \bar{\chi}(p_2) \to \tau^-(k_1) + \tau^+(k_2)$ 的费曼图为



不变振幅

$$\begin{aligned} i\mathcal{M} &= ig_\chi \bar{v}(p_2)\gamma_\mu\gamma_5 u(p_1) \frac{-i(g^{\mu\nu} - q^\mu q^\nu/m_U^2)}{q^2 - m_U^2} ig_\tau \bar{u}(k_1)\gamma_\nu\gamma_5 v(k_2) \\ &= ig_\chi g_\tau \frac{g^{\mu\nu} - q^\mu q^\nu/m_U^2}{s - m_U^2} \bar{v}(p_2)\gamma_\mu\gamma_5 u(p_1)\bar{u}(k_1)\gamma_\nu\gamma_5 v(k_2), \end{aligned} \tag{60}$$

$$(i\mathcal{M})^* = -ig_\chi g_\tau \frac{g^{\rho\sigma} - q^\rho q^\sigma/m_U^2}{s - m_U^2} \bar{u}(p_1)\gamma_\rho\gamma_5 v(p_2)\bar{v}(k_2)\gamma_\sigma\gamma_5 u(k_1), \tag{61}$$

则

$$\begin{aligned} &\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 \\ =& \sum_{\text{spins}} \frac{(g_\chi g_\tau)^2}{4(s - m_U^2)^2} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_U^2}\right)\left(g^{\rho\sigma} - \frac{q^\rho q^\sigma}{m_U^2}\right) \bar{v}(p_2)\gamma_\mu\gamma_5 u(p_1)\bar{u}(p_1)\gamma_\rho\gamma_5 v(p_2)\bar{u}(k_1)\gamma_\nu\gamma_5 v(k_2)\bar{v}(k_2)\gamma_\sigma\gamma_5 u(k_1) \\ =& \frac{(g_\chi g_\tau)^2}{4(s - m_U^2)^2} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_U^2}\right)\left(g^{\rho\sigma} - \frac{q^\rho q^\sigma}{m_U^2}\right) \mathrm{Tr}[(\not{p}_2 - m_\chi)\gamma_\mu\gamma_5(\not{p}_1 + m_\chi)\gamma_\rho\gamma_5]\mathrm{Tr}[(\not{k}_1 + m_\tau)\gamma_\nu\gamma_5(\not{k}_2 - m_\tau)\gamma_\sigma\gamma_5] \\ =& \frac{(g_\chi g_\tau)^2}{(s - m_U^2)^2} \left[s^2(1 + \beta_\chi^2\beta_\tau^2\cos^2\theta) - 4m_\chi^2 s - 4m_\tau^2 s + 32m_\chi^2 m_\tau^2 - 32m_\tau^2 m_\chi^2 s/m_U^2 + 16m_\tau^2 m_\chi^2 s^2/m_U^4\right] \\ =& \frac{(g_\chi g_\tau)^2}{(s - m_U^2)^2} \left[s^2(1 + \beta_\chi^2\beta_\tau^2\cos^2\theta) - 4m_\chi^2 s + 16m_\tau^2 m_\chi^2 \left(2 - \frac{s}{4m_\chi^2} - 2\frac{s}{m_U^2} + \frac{s^2}{m_U^4}\right)\right] \\ &\xrightarrow{s\to 4m_\chi^2,\ \beta_\chi\to 0} \frac{16(g_\chi g_\tau)^2 m_\tau^2 m_\chi^2}{(m_U^2 - 4m_\chi^2)^2}\left(1 - \frac{4m_\chi^2}{m_U^2}\right)^2 = \frac{16(g_\chi g_\tau)^2 m_\tau^2 m_\chi^2}{m_U^4}. \end{aligned} \tag{62}$$

在低速近似下,

$$\sigma_{\text{anni}} v \simeq 4\pi \frac{\beta_\tau}{32\pi^2 4m_\chi^2} \frac{16(g_\chi g_\tau)^2 m_\tau^2 m_\chi^2}{m_U^4} = \frac{(g_\chi g_\tau)^2 m_\tau^2 \beta_\tau}{2\pi m_U^4}. \tag{63}$$

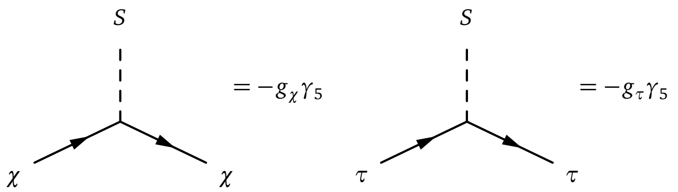
当 $m_\tau \ll m_\chi \ll m_\phi$ 时, 有

$$\frac{1}{2}\left\langle \sigma_{\text{anni}} v \right\rangle \simeq \frac{(g_\chi g_\tau)^2 m_\tau^2}{4\pi m_U^4} = 5 \times 10^{-27}\ \text{cm}^3\ \text{s}^{-1} \left(\frac{\sqrt{g_\chi g_\tau}}{m_U/156\ \text{GeV}}\right)^4. \tag{64}$$

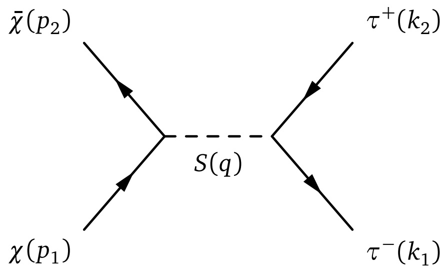
设赝标量粒子 S 中介的相互作用拉氏量为

$$\mathcal{L}_S = (g_\chi \bar{\chi} i\gamma_5 \chi + g_\tau \bar{\tau} i\gamma_5 \tau) S. \tag{65}$$

相应费曼规则如下.



湮灭过程 $\chi(p_1)+\bar{\chi}(p_2)\to\tau^-(k_1)+\tau^+(k_2)$ 的费曼图为



不变振幅

$$
\begin{aligned}
i\mathcal{M} &= (-g_\chi)\bar{v}(p_2)\gamma_5 u(p_1)\frac{i}{q^2-m_S^2}(-g_\tau)\bar{u}(k_1)\gamma_5 v(k_2) \\
&= \frac{ig_\chi g_\tau}{s-m_S^2}\bar{v}(p_2)\gamma_5 u(p_1)\bar{u}(k_1)\gamma_5 v(k_2), \qquad (66)\\
(i\mathcal{M})^* &= -\frac{ig_\chi g_\tau}{s-m_S^2}\bar{u}(p_1)\gamma_5 v(p_2)\bar{v}(k_2)\gamma_5 u(k_1), \qquad (67)
\end{aligned}
$$

则

$$
\begin{aligned}
\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 &= \sum_{\text{spins}}\frac{(g_\chi g_\tau)^2}{4(s-m_S^2)^2}\bar{v}(p_2)\gamma_5 u(p_1)\bar{u}(p_1)\gamma_5 v(p_2)\bar{u}(k_1)\gamma_5 v(k_2)\bar{v}(k_2)\gamma_5 u(k_1) \\
&= \frac{(g_\chi g_\tau)^2}{4(s-m_S^2)^2}\mathrm{Tr}[(\not{p}_2-m_\chi)\gamma_5(\not{p}_1+m_\chi)\gamma_5]\mathrm{Tr}[(\not{k}_1+m_\tau)\gamma_5(\not{k}_2-m_\tau)\gamma_5] \\
&= \frac{(g_\chi g_\tau)^2 s^2}{(s-m_S^2)^2}\xrightarrow{s\to 4m_\chi^2,\ \beta_\chi\to 0}\frac{16(g_\chi g_\tau)^2 m_\chi^4}{(m_S^2-4m_\chi^2)^2}. \qquad (68)
\end{aligned}
$$

在低速近似下,

$$
\sigma_{\text{anni}}v\simeq 4\pi\frac{\beta_\tau}{32\pi^2 4m_\chi^2}\frac{16(g_\chi g_\tau)^2 m_\chi^4}{(m_S^2-4m_\chi^2)^2}=\frac{(g_\chi g_\tau)^2 m_\chi^2\beta_\tau}{2\pi(m_S^2-4m_\chi^2)^2}. \qquad (69)
$$

当 $m_\tau\ll m_\chi\ll m_\phi$ 时, 有

$$
\frac{1}{2}\langle\sigma_{\text{anni}}v\rangle\simeq\frac{(g_\chi g_\tau)^2 m_\chi^2}{4\pi m_S^4}=5\times 10^{-27}\ \mathrm{cm}^3\ \mathrm{s}^{-1}\left(\frac{m_\chi}{10\ \mathrm{GeV}}\right)^2\left(\frac{\sqrt{g_\chi g_\tau}}{m_S/369\ \mathrm{GeV}}\right)^4. \qquad (70)
$$

A 标量 QED 费曼规则

一个电荷为 Q 的复标量场对应的标量 QED 拉氏量为

$$
\begin{aligned}
\mathcal{L} &= (D^\mu\phi)^\dagger D_\mu\phi-m^2\phi^\dagger\phi \\
&= (\partial^\mu\phi)^\dagger\partial_\mu\phi-m^2\phi^\dagger\phi-iQeA_\mu(\partial^\mu\phi^\dagger)\phi+iQeA_\mu\phi^\dagger\partial^\mu\phi+Q^2e^2A^\mu A_\mu\phi^\dagger\phi, \qquad (71)
\end{aligned}
$$

其中协变导数有如下形式

$$
D_\mu\phi=\partial_\mu\phi-iQeA_\mu\phi,\quad (D^\mu\phi)^\dagger=\partial^\mu\phi^\dagger+iQeA^\mu\phi^\dagger. \qquad (72)
$$

下面导出此理论的费曼规则.

依照 Peskin & Schroeder [10] 的惯例, 标量场可展开为

$$\phi(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\mathbf{p}}}}(a_{\mathbf{p}}e^{-ip\cdot x}+b_{\mathbf{p}}^\dagger e^{ip\cdot x}),\quad \phi^\dagger(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{\mathbf{p}}}}(b_{\mathbf{p}}e^{-ip\cdot x}+a_{\mathbf{p}}^\dagger e^{ip\cdot x}),\tag{73}$$

其中产生湮灭算符的非零对易关系为

$$[a_{\mathbf{p}},a_{\mathbf{q}}^\dagger]=(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{q}),\quad [b_{\mathbf{p}},b_{\mathbf{q}}^\dagger]=(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{q}).\tag{74}$$

设入射正粒子态为 $|p\rangle=\sqrt{2E_{\mathbf{p}}}a_{\mathbf{p}}^\dagger|0\rangle$, 出射正粒子态为 $\langle q|=\langle 0|a_{\mathbf{q}}\sqrt{2E_{\mathbf{q}}}$, 则

$$\langle 0|\phi(x)|p\rangle=e^{-ip\cdot x},\quad \langle 0|\phi^\dagger(x)|p\rangle=0,\quad \langle q|\phi(x)|0\rangle=0,\quad \langle q|\phi^\dagger(x)|0\rangle=e^{iq\cdot x},\tag{75}$$

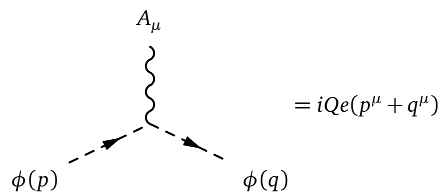
而

$$\langle q|:\phi^\dagger(x)\partial^\mu\phi(x):|p\rangle = e^{iq\cdot x}\partial^\mu e^{-ip\cdot x}=-ip^\mu e^{-i(p-q)\cdot x},\tag{76}$$
$$\langle q|:[\partial^\mu\phi^\dagger(x)]\phi(x):|p\rangle = (\partial^\mu e^{iq\cdot x})e^{-ip\cdot x}=+iq^\mu e^{-i(p-q)\cdot x}.\tag{77}$$

于是, $A\phi\phi$ 顶点的费曼规则可按如下方式提取:

$$\begin{aligned}\mathcal{O}_{A\phi\phi} &= iQeA_\mu\phi^\dagger\partial^\mu\phi-iQeA_\mu(\partial^\mu\phi^\dagger)\phi\to iQeA_\mu\langle q|:[\phi^\dagger\partial^\mu\phi-(\partial^\mu\phi^\dagger)\phi]:|p\rangle\\ &\to i^2QeA_\mu[-ip^\mu-(+iq^\mu)]=iQe(p^\mu+q^\mu),\end{aligned}\tag{78}$$

即



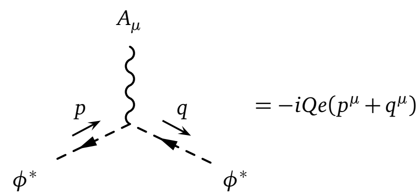
若设入射反粒子态为 $|p\rangle=\sqrt{2E_{\mathbf{p}}}b_{\mathbf{p}}^\dagger|0\rangle$, 出射反粒子态为 $\langle q|=\langle 0|b_{\mathbf{q}}\sqrt{2E_{\mathbf{q}}}$, 则

$$\langle 0|\phi(x)|p\rangle=0,\quad \langle 0|\phi^\dagger(x)|p\rangle=e^{-ip\cdot x},\quad \langle q|\phi(x)|0\rangle=e^{iq\cdot x},\quad \langle q|\phi^\dagger(x)|0\rangle=0,\tag{79}$$

而

$$\langle q|:\phi^\dagger(x)\partial^\mu\phi(x):|p\rangle = (\partial^\mu e^{iq\cdot x})e^{-ip\cdot x}=iq^\mu e^{-i(p-q)\cdot x},\tag{80}$$
$$\langle q|:[\partial^\mu\phi^\dagger(x)]\phi(x):|p\rangle = e^{iq\cdot x}\partial^\mu e^{-ip\cdot x}=-ip^\mu e^{-i(p-q)\cdot x}.\tag{81}$$

因此, 该按如下方式应用费曼规则.

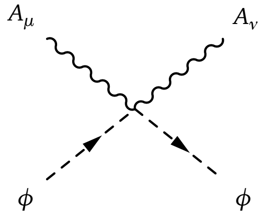


即是说, 对于反粒子, 动量方向与虚线上的方向相反, 而且表达式中动量应换成其相反数.

对于 $AA\phi\phi$ 顶点, 费曼规则可按如下方式提取:

$$\mathcal{O}_{AA\phi\phi}=Q^2e^2A^\mu A_\mu\phi^\dagger\phi=Q^2e^2g^{\mu\nu}A_\mu A_\nu\phi^\dagger\phi\to 2iQ^2e^2g^{\mu\nu},\tag{82}$$

即



$$= 2iQ^2e^2g^{\mu\nu}$$

费曼规则的另一种处理方式可参见 Srednicki [11] §61.

参考文献

[1] L. Goodenough and D. Hooper, "Possible Evidence For Dark Matter Annihilation In The Inner Milky Way From The Fermi Gamma Ray Space Telescope," arXiv:0910.2998 [hep-ph].

[2] K. N. Abazajian, N. Canac, S. Horiuchi and M. Kaplinghat, "Astrophysical and Dark Matter Interpretations of Extended Gamma Ray Emission from the Galactic Center," arXiv:1402.4090 [astro-ph.HE].

[3] T. Daylan, D. P. Finkbeiner, D. Hooper, T. Linden, S. K. N. Portillo, N. L. Rodd and T. R. Slatyer, "The Characterization of the Gamma-Ray Signal from the Central Milky Way: A Compelling Case for Annihilating Dark Matter," arXiv:1402.6703 [astro-ph.HE].

[4] H. E. Haber and G. L. Kane, "The Search for Supersymmetry: Probing Physics Beyond the Standard Model," Phys. Rept. **117**, 75 (1985).

[5] P. Agrawal, S. Blanchet, Z. Chacko and C. Kilic, "Flavored Dark Matter, and Its Implications for Direct Detection and Colliders," Phys. Rev. D **86**, 055002 (2012) [arXiv:1109.3516 [hep-ph]].

[6] Y. Bai and J. Berger, "Lepton Portal Dark Matter," arXiv:1402.6696 [hep-ph].

[7] S. Chang, R. Edezhath, J. Hutchinson and M. Luty, "Leptophilic Effective WIMPs," arXiv:1402.7358 [hep-ph].

[8] P. Agrawal, B. Batell, D. Hooper and T. Lin, "Flavored Dark Matter and the Galactic Center Gamma-Ray Excess," arXiv:1404.1373 [hep-ph].

[9] E. Izaguirre, G. Krnjaic and B. Shuve, "The Galactic Center Excess from the Bottom Up," arXiv:1404.2018 [hep-ph].

[10] M. E. Peskin and D. V. Schroeder, "An Introduction to quantum field theory," Reading, USA: Addison-Wesley (1995) 842 p.

[11] M. Srednicki, "Quantum field theory," Cambridge, UK: Univ. Pr. (2007) 641 p.