$$\frac{p^i\sigma^i}{|\mathbf{p}|}\xi_\lambda(p)=\lambda\xi_\lambda(p),\quad \lambda=\pm$$

$$\xi_+(p)=\frac{1}{\sqrt{2|\mathbf{p}|(|\mathbf{p}|+p_z)}}\begin{pmatrix}|\mathbf{p}|+p_z\\ p_x+ip_y\end{pmatrix},\quad \xi_-(p)=\frac{1}{\sqrt{2|\mathbf{p}|(|\mathbf{p}|+p_z)}}\begin{pmatrix}-p_x+ip_y\\ |\mathbf{p}|+p_z\end{pmatrix}$$

$$p_z=-|\mathbf{p}|:\quad \xi_+(p)=\begin{pmatrix}0\\1\end{pmatrix},\quad \xi_-(p)=\begin{pmatrix}-1\\0\end{pmatrix}$$

$$u(p,\lambda)=\begin{pmatrix}\omega_{-\lambda}(p)\xi_\lambda(p)\\ \omega_\lambda(p)\xi_\lambda(p)\end{pmatrix},\quad v(p,\lambda)=\begin{pmatrix}-\lambda\omega_\lambda(p)\xi_{-\lambda}(p)\\ \lambda\omega_{-\lambda}(p)\xi_{-\lambda}(p)\end{pmatrix},\quad \omega_\lambda(p)=\sqrt{E_p+\lambda|\mathbf{p}|},\quad E_p=\sqrt{|\mathbf{p}|^2+m^2}$$

$$\not{p}u(p,\lambda)=mu(p,\lambda),\quad \not{p}v(p,\lambda)=-mv(p,\lambda)$$

$$E_p\gg m,\quad \omega_\lambda(p)\to\sqrt{(1+\lambda)|\mathbf{p}|},\quad \omega_+(p)\to\sqrt{2|\mathbf{p}|},\quad \omega_-(p)\to 0$$

$$u(p,-)\to\sqrt{2|\mathbf{p}|}\begin{pmatrix}\xi_-(p)\\0\end{pmatrix},\quad u(p,+)\to\sqrt{2|\mathbf{p}|}\begin{pmatrix}0\\ \xi_+(p)\end{pmatrix}$$

$$v(p,-)\to-\sqrt{2|\mathbf{p}|}\begin{pmatrix}0\\ \xi_+(p)\end{pmatrix},\quad v(p,+)\to-\sqrt{2|\mathbf{p}|}\begin{pmatrix}\xi_-(p)\\0\end{pmatrix}$$

$$P_Lu(p,-)\to u(p,-),\quad P_Ru(p,+)\to u(p,+),\quad P_Ru(p,-)\to 0,\quad P_Lu(p,+)\to 0$$

$$P_Rv(p,-)\to v(p,-),\quad P_Lv(p,+)\to v(p,+),\quad P_Lv(p,-)\to 0,\quad P_Rv(p,+)\to 0$$

Expansion: $E_p\simeq|\mathbf{p}|+\frac{m^2}{2|\mathbf{p}|},\quad \omega_+(p)\simeq\sqrt{2|\mathbf{p}|}+\frac{m^2}{4\sqrt{2}|\mathbf{p}|^{3/2}}=\sqrt{2|\mathbf{p}|}\left(1+\frac{m^2}{8|\mathbf{p}|^2}\right),\quad \omega_-(p)\simeq\frac{m}{\sqrt{2|\mathbf{p}|}}$

$$u(p,-)\simeq\begin{pmatrix}\sqrt{2|\mathbf{p}|}\xi_-(p)\\ \frac{m}{\sqrt{2|\mathbf{p}|}}\xi_-(p)\end{pmatrix},\quad u(p,+)\simeq\begin{pmatrix}\frac{m}{\sqrt{2|\mathbf{p}|}}\xi_+(p)\\ \sqrt{2|\mathbf{p}|}\xi_+(p)\end{pmatrix}$$

$$v(p,-)\simeq\begin{pmatrix}\frac{m}{\sqrt{2|\mathbf{p}|}}\xi_+(p)\\ -\sqrt{2|\mathbf{p}|}\xi_+(p)\end{pmatrix},\quad v(p,+)\simeq\begin{pmatrix}-\sqrt{2|\mathbf{p}|}\xi_-(p)\\ \frac{m}{\sqrt{2|\mathbf{p}|}}\xi_-(p)\end{pmatrix}$$

Fermion-antifermion pair: $P=(-)^{L+1},\quad C=(-)^{L+S},\quad CP=(-)^{S+1}$

$L=1,\quad S=1\quad\Rightarrow\quad P=+,\quad C=+,\quad CP=+$

$L=0,\quad S=0\quad\Rightarrow\quad P=-,\quad C=+,\quad CP=-$

$$q(q_1)+\bar{q}(q_2)\to g^*(q)\to t(k_1)+\bar{t}(k_2)+h(p),\quad g(q_1)+g(q_2)\to g^*(q)\to t(k_1)+\bar{t}(k_2)+h(p),\quad s\text{-channel}$$

$$i\mathcal{M}\propto \bar{u}(k_1)\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i(\not{k}_1+\not{p}+m_t)}{(k_1+p)^2-m_t^2}\gamma^\mu v(k_2)+\bar{u}(k_1)\gamma^\mu\frac{i(-\not{k}_2-\not{p}+m_t)}{(-k_2-p)^2-m_t^2}\left(-i\frac{y_t}{\sqrt{2}}\right)v(k_2)$$

$$\propto \bar{u}(k_1)\left[\frac{\not{k}_1+\not{p}+m_t}{m_h^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{k}_2-\not{p}+m_t}{m_h^2+2k_2\cdot p}\right]v(k_2)=\bar{u}(k_1)\left[\frac{\not{p}+2m_t}{m_h^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}+2m_t}{m_h^2+2k_2\cdot p}\right]v(k_2)$$

$$\mathcal{M}_{\text{unlike}}\propto \bar{u}(k_1)P_R\left[\frac{\not{p}+2m_t}{m_h^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}+2m_t}{m_h^2+2k_2\cdot p}\right]P_L v(k_2)=\bar{u}(k_1)\left[\frac{\not{p}P_L+2m_tP_R}{m_h^2+2k_1\cdot p}\gamma^\mu P_L+P_R\gamma^\mu\frac{-P_R\not{p}+2m_tP_L}{m_h^2+2k_2\cdot p}\right]v(k_2)$$

$$=\frac{2m_t(2m_h^2+2k_1\cdot p+2k_2\cdot p)}{(m_h^2+2k_1\cdot p)(m_h^2+2k_2\cdot p)}\bar{u}_L(k_1)\gamma^\mu v_L(k_2)\simeq\frac{4m_t(q\cdot p)}{(m_h^2+2k_1\cdot p)(m_h^2+2k_2\cdot p)}\bar{u}_L(k_1,-)\gamma^\mu v_L(k_2,+)+\mathcal{O}(m_t^2)$$

$$\mathcal{M}_{\text{like}}\propto \bar{u}(k_1)P_L\left[\frac{\not{p}+2m_t}{m_h^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}+2m_t}{m_h^2+2k_2\cdot p}\right]P_L v(k_2)=\bar{u}(k_1)\left[\frac{\not{p}P_R+2m_tP_L}{m_h^2+2k_1\cdot p}\gamma^\mu P_L+P_L\gamma^\mu\frac{-P_R\not{p}+2m_tP_L}{m_h^2+2k_2\cdot p}\right]v(k_2)$$

$$=\bar{u}_R(k_1)\left[\frac{\not{p}}{m_h^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}}{m_h^2+2k_2\cdot p}\right]v_L(k_2)\simeq\bar{u}_R(k_1,+)\left[\frac{\not{p}}{m_h^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}}{m_h^2+2k_2\cdot p}\right]v_L(k_2,+)+\mathcal{O}(m_t)$$

$$q(q_1)+\bar{q}(q_2)\to g^*(q)\to t(k_1)+\bar{t}(k_2)+A(p),\quad g(q_1)+g(q_2)\to g^*(q)\to t(k_1)+\bar{t}(k_2)+A(p),\quad s\text{-channel}$$

$$i\mathcal{M}\propto \bar{u}(k_1)\frac{y_t}{\sqrt{2}}\gamma_5\frac{i(\not{k}_1+\not{p}+m_t)}{(k_1+p)^2-m_t^2}\gamma^\mu v(k_2)+\bar{u}(k_1)\gamma^\mu\frac{i(-\not{k}_2-\not{p}+m_t)}{(-k_2-p)^2-m_t^2}\frac{y_t}{\sqrt{2}}\gamma_5 v(k_2)$$

$$\propto \bar{u}(k_1)\left[\gamma_5\frac{\not{k}_1+\not{p}+m_t}{m_A^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{k}_2-\not{p}+m_t}{m_A^2+2k_2\cdot p}\gamma_5\right]v(k_2)=\bar{u}(k_1)\left[\gamma_5\frac{\not{p}}{m_A^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}}{m_A^2+2k_2\cdot p}\gamma_5\right]v(k_2)$$

$$\mathcal{M}_{\text{unlike}}\propto \bar{u}(k_1)P_R\left[\gamma_5\frac{\not{p}}{m_A^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}}{m_A^2+2k_2\cdot p}\gamma_5\right]P_L v(k_2)=\bar{u}(k_1)\left[P_R\frac{\not{p}}{m_A^2+2k_1\cdot p}\gamma^\mu P_L-P_R\gamma^\mu\frac{-\not{p}}{m_A^2+2k_2\cdot p}P_L\right]v(k_2)=0$$

$$\mathcal{M}_{\text{like}}\propto \bar{u}(k_1)P_L\left[\gamma_5\frac{\not{p}}{m_A^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}}{m_A^2+2k_2\cdot p}\gamma_5\right]P_L v(k_2)=-\bar{u}(k_1)\left[P_L\frac{\not{p}}{m_A^2+2k_1\cdot p}\gamma^\mu P_L+P_L\gamma^\mu\frac{-\not{p}}{m_A^2+2k_2\cdot p}P_L\right]v(k_2)$$

$$=-\bar{u}_R(k_1)\left[\frac{\not{p}}{m_h^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}}{m_h^2+2k_2\cdot p}\right]v_L(k_2)\simeq-\bar{u}_R(k_1,+)\left[\frac{\not{p}}{m_h^2+2k_1\cdot p}\gamma^\mu+\gamma^\mu\frac{-\not{p}}{m_h^2+2k_2\cdot p}\right]v_L(k_2,+)+\mathcal{O}(m_t)$$

$g(q_1)+g(q_2)\to t(k_1)+\bar{t}(k_2)+h(p),\quad t\text{-channel}$

$$
\begin{aligned}
i\mathcal{M} &\propto \bar{u}(k_1)\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i(\not{k}_1+\not{p}+m_t)}{(k_1+p)^2-m_t^2}\gamma^\mu\frac{i(\not{q}_2-\not{k}_2+m_t)}{(q_2-k_2)^2-m_t^2}\gamma^\nu v(k_2)+\bar{u}(k_1)\gamma^\mu\frac{i(\not{k}_1-\not{q}_1+m_t)}{(k_1-q_1)^2-m_t^2}\gamma^\nu\frac{i(-\not{k}_2-\not{p}+m_t)}{(-k_2-p)^2-m_t^2}\left(-i\frac{y_t}{\sqrt{2}}\right)v(k_2)\\
&\quad+\bar{u}(k_1)\gamma^\mu\frac{i(\not{k}_1-\not{q}_1+m_t)}{(k_1-q_1)^2-m_t^2}\left(-i\frac{y_t}{\sqrt{2}}\right)\frac{i(\not{q}_2-\not{k}_2+m_t)}{(q_2-k_2)^2-m_t^2}\gamma^\nu v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}\\
&\propto \bar{u}(k_1)\left[\frac{(\not{p}+2m_t)\gamma^\mu(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma^\nu(-\not{p}+2m_t)}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{\text{unlike}} &\propto \bar{u}(k_1)P_R\left[\frac{(\not{p}+2m_t)\gamma^\mu(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma^\nu(-\not{p}+2m_t)}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]P_L v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}\\
&=\bar{u}(k_1)\left[\frac{(\not{p}P_L+2m_tP_R)\gamma^\mu(P_L\not{q}_2-P_L\not{k}_2+m_tP_R)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{\gamma^\mu(\not{k}_1P_R-\not{q}_1P_R+m_tP_L)\gamma^\nu(-P_R\not{p}+2m_tP_L)}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1P_R-\not{q}_1P_R+m_tP_L)(P_L\not{q}_2-P_L\not{k}_2+m_tP_R)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}\\
&=\bar{u}_L(k_1)\left[\frac{m_t[\not{p}\gamma^\mu\gamma^\nu+2\gamma^\mu(\not{q}_2-\not{k}_2)\gamma^\nu]}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{m_t[2\gamma^\mu(\not{k}_1-\not{q}_1)\gamma^\nu-\gamma^\mu\gamma^\nu\not{p}]}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{m_t\gamma^\mu(\not{k}_1-\not{q}_1+\not{q}_2-\not{k}_2)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v_L(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}\\
&\propto m_t\bar{u}_L(k_1,-)\otimes v_L(k_2,+)+\mathcal{O}(m_t^2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{\text{like}} &\propto \bar{u}(k_1)P_L\left[\frac{(\not{p}+2m_t)\gamma^\mu(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma^\nu(-\not{p}+2m_t)}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]P_L v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}\\
&=\bar{u}(k_1)\left[\frac{(\not{p}P_R+2m_tP_L)\gamma^\mu(P_L\not{q}_2-P_L\not{k}_2+m_tP_R)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{\gamma^\mu(\not{k}_1P_L-\not{q}_1P_L+m_tP_R)\gamma^\nu(-P_R\not{p}+2m_tP_L)}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1P_L-\not{q}_1P_L+m_tP_R)(P_L\not{q}_2-P_L\not{k}_2+m_tP_R)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}\\
&=\bar{u}_R(k_1)\left[\frac{\not{p}\gamma^\mu(\not{q}_2-\not{k}_2)\gamma^\nu+2m_t^2\gamma^\mu\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{-\gamma^\mu(\not{k}_1-\not{q}_1)\gamma^\nu\not{p}+2m_t^2\gamma^\mu\gamma^\nu}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1)(\not{q}_2-\not{k}_2)\gamma^\nu+m_t^2\gamma^\mu\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v_L(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}\\
&\propto \bar{u}_R(k_1,+)\otimes v_L(k_2,+)+\mathcal{O}(m_t)
\end{aligned}
$$

$g(q_1)+g(q_2)\to t(k_1)+\overline{t}(k_2)+A(p)$, $\quad t$-channel

$$i\mathcal{M}\propto \bar{u}(k_1)\frac{y_t}{\sqrt{2}}\gamma_5\frac{i(\not{k}_1+\not{p}+m_t)}{(k_1+p)^2-m_t^2}\gamma^\mu\frac{i(\not{q}_2-\not{k}_2+m_t)}{(q_2-k_2)^2-m_t^2}\gamma^\nu v(k_2)+\bar{u}(k_1)\gamma^\mu\frac{i(\not{k}_1-\not{q}_1+m_t)}{(k_1-q_1)^2-m_t^2}\gamma^\nu\frac{i(-\not{k}_2-\not{p}+m_t)}{(-k_2-p)^2-m_t^2}\frac{y_t}{\sqrt{2}}\gamma_5 v(k_2)$$

$$+\bar{u}(k_1)\gamma^\mu\frac{i(\not{k}_1-\not{q}_1+m_t)}{(k_1-q_1)^2-m_t^2}\frac{y_t}{\sqrt{2}}\gamma_5\frac{i(\not{q}_2-\not{k}_2+m_t)}{(q_2-k_2)^2-m_t^2}\gamma^\nu v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}$$

$$\propto \bar{u}(k_1)\left[\frac{\gamma_5\not{p}\gamma^\mu(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{-\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma^\nu\not{p}\gamma_5}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma_5(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}$$

$$\mathcal{M}_{\text{unlike}}\propto \bar{u}(k_1)P_R\left[\frac{\gamma_5\not{p}\gamma^\mu(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{-\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma^\nu\not{p}\gamma_5}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma_5(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]P_L v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}$$

$$=\bar{u}(k_1)\left[\frac{\not{p}P_L\gamma^\mu(P_L\not{q}_2-P_L\not{k}_2+m_tP_R)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{\gamma^\mu(\not{k}_1P_R-\not{q}_1P_R+m_tP_L)\gamma^\nu P_R\not{p}}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1P_R-\not{q}_1P_R+m_tP_L)\gamma_5(P_L\not{q}_2-P_L\not{k}_2+m_tP_R)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}$$

$$=\bar{u}_L(k_1)\left[\frac{m_t\not{p}\gamma^\mu\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{m_t\gamma^\mu\gamma^\nu\not{p}}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{-m_t\gamma^\mu\not{p}\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v_L(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}$$

$$\propto m_t\bar{u}_L(k_1,-)\otimes v_L(k_2,+)+\mathcal{O}(m_t^2)$$

$$\mathcal{M}_{\text{like}}\propto \bar{u}(k_1)P_L\left[\frac{\gamma_5\not{p}\gamma^\mu(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{-\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma^\nu\not{p}\gamma_5}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1+m_t)\gamma_5(\not{q}_2-\not{k}_2+m_t)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]P_L v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}$$

$$=\bar{u}(k_1)\left[\frac{-\not{p}P_R\gamma^\mu(P_L\not{q}_2-P_L\not{k}_2+m_tP_R)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{\gamma^\mu(\not{k}_1P_L-\not{q}_1P_L+m_tP_R)\gamma^\nu P_R\not{p}}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1P_L-\not{q}_1P_L+m_tP_R)\gamma_5(P_L\not{q}_2-P_L\not{k}_2+m_tP_R)\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}$$

$$=\bar{u}_R(k_1)\left[\frac{-\not{p}\gamma^\mu(\not{q}_2-\not{k}_2)\gamma^\nu}{-2k_2\cdot q_2(m_h^2+2k_1\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1)\gamma^\nu\not{p}}{-2k_1\cdot q_1(m_h^2+2k_2\cdot p)}+\frac{\gamma^\mu(\not{k}_1-\not{q}_1)(-\not{q}_2+\not{k}_2)\gamma^\nu+m_t^2\gamma^\mu\gamma^\nu}{4(k_1\cdot q_1)(k_2\cdot q_2)}\right]v_L(k_2)+\{q_1\leftrightarrow q_2,\mu\leftrightarrow\nu\}$$

$$\propto \bar{u}_R(k_1,+)\otimes v_L(k_2,+)+\mathcal{O}(m_t)$$

Top pair production

$$q(q_1,\sigma_1)+\bar{q}(q_2,\sigma_2)\to g^*(q,\lambda)\to t(k_1,\lambda_1)+\bar{t}(k_2,\lambda_2)$$

$$-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}=\sum_{\lambda=\pm1,0}\varepsilon^*_\mu(q,\lambda)\varepsilon_\nu(q,\lambda)$$

$$\mathcal{M}_{\sigma_1\sigma_2\lambda_1\lambda_2}\propto\sum_{\lambda=\pm1,0}J'^{\lambda}_{\sigma_1\sigma_2}J^{\lambda}_{\lambda_1\lambda_2}$$

$$J^{\lambda}_{\lambda_1\lambda_2}=\varepsilon_\mu(q,\lambda)\bar{u}(k_1,\lambda_1)\gamma^\mu v(k_2,\lambda_2)$$

$$q=(\sqrt{s},\mathbf{0}),\quad k_1=(k_1^0,|\mathbf{k}_1|\sin\theta\cos\phi,|\mathbf{k}_1|\sin\theta\sin\phi,|\mathbf{k}_1|\cos\theta),\quad k_2=(k_1^0,-\mathbf{k}_1),\quad k_1^0=\frac{1}{2}\sqrt{s}$$

$$\varepsilon_\mu(q,+1)=\frac{1}{\sqrt{2}}(0,-1,-i,0),\quad \varepsilon_\mu(q,-1)=\frac{1}{\sqrt{2}}(0,1,-i,0),\quad \varepsilon_\mu(q,0)=(0,0,0,1)$$

$$\xi_+(k_1)=\frac{1}{\sqrt{2(1+\cos\theta)}}\begin{pmatrix}1+\cos\theta\\ e^{i\phi}\sin\theta\end{pmatrix}=\begin{pmatrix}c_{\theta/2}\\ e^{i\phi}s_{\theta/2}\end{pmatrix},\quad \xi_-(k_1)=\frac{1}{\sqrt{2(1+\cos\theta)}}\begin{pmatrix}-e^{-i\phi}\sin\theta\\ 1+\cos\theta\end{pmatrix}=\begin{pmatrix}-e^{-i\phi}s_{\theta/2}\\ c_{\theta/2}\end{pmatrix}$$

$$\xi_+(k_2)=\frac{1}{\sqrt{2(1-\cos\theta)}}\begin{pmatrix}1-\cos\theta\\ -e^{i\phi}\sin\theta\end{pmatrix}=\begin{pmatrix}s_{\theta/2}\\ -e^{i\phi}c_{\theta/2}\end{pmatrix},\quad \xi_-(k_2)=\frac{1}{\sqrt{2(1-\cos\theta)}}\begin{pmatrix}e^{-i\phi}\sin\theta\\ 1-\cos\theta\end{pmatrix}=\begin{pmatrix}e^{-i\phi}c_{\theta/2}\\ s_{\theta/2}\end{pmatrix}$$

$$J^{\lambda}_{\lambda_1\lambda_2}=\varepsilon_\mu(q,\lambda)\begin{pmatrix}\omega_{-\lambda_1}(p)\xi^\dagger_{\lambda_1}(k_1) & \omega_{\lambda_1}(p)\xi^\dagger_{\lambda_1}(k_1)\end{pmatrix}\begin{pmatrix}\bar{\sigma}^\mu & \\ & \sigma^\mu\end{pmatrix}\begin{pmatrix}-\lambda_2\omega_{\lambda_2}(k_2)\xi_{-\lambda_2}(k_2)\\ \lambda_2\omega_{-\lambda_2}(k_2)\xi_{-\lambda_2}(k_2)\end{pmatrix}$$

$$=-\lambda_2\omega_{-\lambda_1}(p)\omega_{\lambda_2}(k_2)\varepsilon_\mu(q,\lambda)\xi^\dagger_{\lambda_1}(k_1)\bar{\sigma}^\mu\xi_{-\lambda_2}(k_2)+\lambda_2\omega_{\lambda_1}(p)\omega_{-\lambda_2}(k_2)\varepsilon_\mu(q,\lambda)\xi^\dagger_{\lambda_1}(k_1)\sigma^\mu\xi_{-\lambda_2}(k_2)$$

$$J^+_{++}=\sqrt{2}m_t\sin\theta,\quad J^0_{++}=-2m_te^{-i\phi}\cos\theta,\quad J^-_{++}=-\sqrt{2}m_te^{-2i\phi}\sin\theta$$

$$J^+_{+-}=\sqrt{2}k_1^0e^{i\phi}(1+\cos\theta),\quad J^0_{+-}=2k_1^0\sin\theta,\quad J^-_{+-}=\sqrt{2}k_1^0e^{-i\phi}(1-\cos\theta)$$

$$J^\pm_{--}=(J^\mp_{++})^*,\quad J^0_{--}=-(J^0_{++})^*,\quad J^\pm_{-+}=-(J^\mp_{+-})^*,\quad J^0_{-+}=(J^0_{+-})^*$$

$$J^{\lambda}_{\lambda_1\lambda_2}=a^{\lambda}_{\lambda_1\lambda_2}\,\mathrm{e}^{i\phi[\lambda-(\lambda_1+\lambda_2)/2]}\,d^1_{\lambda,(\lambda_1-\lambda_2)/2}(\theta)$$

$$d^1_{-1,-1}(\theta)=d^1_{+1,+1}(\theta)=\frac{1}{2}(1+\cos\theta)=c^2_{\theta/2},\quad d^1_{-1,+1}(\theta)=d^1_{+1,-1}(\theta)=\frac{1}{2}(1-\cos\theta)=s^2_{\theta/2}$$

$$d^1_{0,0}(\theta)=\cos\theta,\quad d^1_{0,-1}(\theta)=d^1_{+1,0}(\theta)=-\frac{1}{\sqrt{2}}\sin\theta,\quad d^1_{0,+1}(\theta)=d^1_{-1,0}(\theta)=\frac{1}{\sqrt{2}}\sin\theta$$

$$a^\lambda_{--}=2m_t=-a^\lambda_{++},\quad a^\lambda_{-+}=-2\sqrt{2}k_1^0=a^\lambda_{+-}$$

$$J'^{\lambda}_{\sigma_1\sigma_2} = \overline{v}(q_2,\sigma_2)\gamma^{\mu}u(q_1,\sigma_1)\varepsilon^{*}_{\mu}(q,\lambda)$$

$$q_1 = (|\,\mathbf{q}_1\,|,0,0,|\,\mathbf{q}_1\,|), \quad q_2 = (|\,\mathbf{q}_1\,|,0,0,-|\,\mathbf{q}_1\,|), \quad |\,\mathbf{q}_1\,| = k_1^0$$

$$\xi_+(q_1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \xi_-(q_1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \xi_+(q_2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \xi_-(q_2) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$J'^{+}_{+-} = -2\sqrt{2}k_1^0, \quad J'^{-}_{-+} = 2\sqrt{2}k_1^0, \quad 0 \text{ for others}$$

$$\mathcal{M}_{+-\lambda_1\lambda_2} \propto J'^{+}_{+-}J^{+}_{\lambda_1\lambda_2} = -2\sqrt{2}k_1^0 J^{+}_{\lambda_1\lambda_2} = -2\sqrt{2}k_1^0 a^{+}_{\lambda_1\lambda_2}\,\mathrm{e}^{i\phi[1-(\lambda_1+\lambda_2)/2]}\,d^1_{+1,(\lambda_1-\lambda_2)/2}(\theta)$$

$$\mathcal{M}_{-+\lambda_1\lambda_2} \propto J'^{-}_{-+}J^{-}_{\lambda_1\lambda_2} = 2\sqrt{2}k_1^0 J^{-}_{\lambda_1\lambda_2} = 2\sqrt{2}k_1^0 a^{-}_{\lambda_1\lambda_2}\,\mathrm{e}^{i\phi[-1-(\lambda_1+\lambda_2)/2]}\,d^1_{-1,(\lambda_1-\lambda_2)/2}(\theta)$$

$$\mathcal{M}_{+-+-} \propto d^1_{+1,+1}(\theta) = \frac{1}{2}(1+\cos\theta), \quad \mathcal{M}_{+--+} \propto d^1_{+1,-1}(\theta) = \frac{1}{2}(1-\cos\theta)$$

$$\mathcal{M}_{+-++} \propto d^1_{+1,0}(\theta) = -\frac{1}{\sqrt{2}}\sin\theta, \quad \mathcal{M}_{+---} \propto d^1_{+1,0}(\theta) = -\frac{1}{\sqrt{2}}\sin\theta$$

$$\mathcal{M}_{-++-} \propto d^1_{-1,+1}(\theta) = \frac{1}{2}(1-\cos\theta), \quad \mathcal{M}_{-+-+} \propto d^1_{-1,-1}(\theta) = \frac{1}{2}(1+\cos\theta)$$

$$\mathcal{M}_{-+++} \propto d^1_{-1,0}(\theta) = \frac{1}{\sqrt{2}}\sin\theta, \quad \mathcal{M}_{-+--} \propto d^1_{-1,0}(\theta) = \frac{1}{\sqrt{2}}\sin\theta$$

$$\sigma(t_+\overline{t}_-) \propto [(1+\cos\theta)^2 + (1-\cos\theta)^2]\sin\theta = 2(1+\cos^2\theta)\sin\theta$$

$$\sigma(t_+\overline{t}_+) \propto \sin^3\theta$$

Ref: Dicus & Hong-Jian He, hep-ph/0409131

$$\mathcal{M}_{\sigma_1\sigma_2\lambda_1\lambda_2}(\theta,\phi) = 16\pi e^{i\sigma\phi}\sum_j (2j+1) d^j_{\lambda\sigma}(\theta) a^j_{\lambda\sigma}, \quad a^j_{\lambda\sigma} = \frac{1}{32\pi} e^{-i\sigma\phi}\int_0^\pi d\theta \sin\theta\, d^j_{\lambda\sigma}(\theta)\mathcal{M}(\theta,\phi)$$

$$\sigma = (\sigma_1 - \sigma_2)s_{\rm in}, \quad \lambda = (\lambda_1 - \lambda_2)s_{\rm out}, \quad \int_0^\pi d\theta \sin\theta\, d^j_{\lambda\sigma}(\theta) d^{j'}_{\lambda\sigma}(\theta) = \frac{2}{2j+1}\delta_{jj'}, \quad d^j_{00}(\theta) = P_j(\theta)$$

Polarization vectors

$$\varepsilon^\mu_\pm(k) = \frac{1}{\sqrt{2}}[\mp\varepsilon^\mu_{(1)}(k) - i\varepsilon^\mu_{(2)}(k)], \quad \varepsilon^\mu_{(1)}(k) = \frac{1}{|\mathbf{k}|k_{\rm T}}(0, k^1k^3, k^2k^3, -k^2_{\rm T}), \quad \varepsilon^\mu_{(2)}(k) = \frac{1}{k_{\rm T}}(0, -k^2, k^1, 0)$$

e^+e^- annihilation into 2 photons

$$e^-(q_1,\sigma_1) + e^+(q_2,\sigma_2) \to \gamma(k_1,\lambda_1) + \gamma(k_2,\lambda_2)$$

$$k_1^\mu = |\mathbf{k}_1|(1, s_\theta c_\phi, s_\theta s_\phi, c_\theta), \quad \varepsilon^\mu_{(1)}(k_1) = (0, c_\theta c_\phi, c_\theta s_\phi, -s_\theta), \quad \varepsilon^\mu_{(2)}(k_1) = (0, -s_\phi, c_\phi, 0)$$

$$\varepsilon^\mu_\pm(k_1) = \frac{1}{\sqrt{2}}(0, \mp c_\theta c_\phi + is_\phi, \mp c_\theta s_\phi - ic_\phi, \pm s_\theta)$$

$$k_2^\mu = |\mathbf{k}_1|(1, -s_\theta c_\phi, -s_\theta s_\phi, -c_\theta), \quad \varepsilon^\mu_{(1)}(k_2) = (0, c_\theta c_\phi, c_\theta s_\phi, -s_\theta), \quad \varepsilon^\mu_{(2)}(k_2) = (0, s_\phi, -c_\phi, 0)$$

$$\varepsilon^\mu_\pm(k_2) = \frac{1}{\sqrt{2}}(0, \mp c_\theta c_\phi - is_\phi, \mp c_\theta s_\phi + ic_\phi, \pm s_\theta)$$

$$\begin{aligned} i\mathcal{M}_{\sigma_1\sigma_2\lambda_1\lambda_2} &= \bar{v}_{\sigma_2}(q_2)(-ie\gamma_\mu)\varepsilon^{*\mu}_{\lambda_2}(k_2)\frac{i(\not{q}_1 - \not{k}_1 + m_e)}{(q_1-k_1)^2 - m_e^2}(-ie\gamma_\nu)\varepsilon^{*\nu}_{\lambda_1}(k_1)u_{\sigma_1}(q_1) \\ &\quad + \bar{v}_{\sigma_2}(q_2)(-ie\gamma_\nu)\varepsilon^{*\nu}_{\lambda_1}(k_1)\frac{i(\not{q}_1 - \not{k}_2 + m_e)}{(q_1-k_2)^2 - m_e^2}(-ie\gamma_\mu)\varepsilon^{*\mu}_{\lambda_2}(k_2)u_{\sigma_1}(q_1) \\ &= -ie^2\varepsilon^{*\mu}_{\lambda_2}(k_2)\varepsilon^{*\nu}_{\lambda_1}(k_1)\bar{v}_{\sigma_2}(q_2)\left[\frac{\gamma_\mu(\not{q}_1 - \not{k}_1 + m_e)\gamma_\nu}{-2q_1\cdot k_1} + \frac{\gamma_\nu(\not{q}_1 - \not{k}_2 + m_e)\gamma_\mu}{-2q_1\cdot k_2}\right]u_{\sigma_1}(q_1) \end{aligned}$$

$$\mathcal{M}_{\sigma_1\sigma_2\lambda_1\lambda_2} = -e^2\varepsilon^{*\mu}_{\lambda_2}(k_2)\varepsilon^{*\nu}_{\lambda_1}(k_1)\bar{v}_{\sigma_2}(q_2)\left[\frac{\gamma_\mu\not{k}_1\gamma_\nu - 2\gamma_\mu q_{1\nu}}{2q_1\cdot k_1} + \frac{\gamma_\nu\not{k}_2\gamma_\mu - 2\gamma_\nu q_{1\mu}}{2q_1\cdot k_2}\right]u_{\sigma_1}(q_1)$$

$$d^2_{+2,+1}(\theta) = -2s_{\theta/2}c^3_{\theta/2} = -\frac{s_\theta(1+c_\theta)}{2} = -d^2_{-2,-1}(\theta), \quad d^2_{-2,+1}(\theta) = 2s^3_{\theta/2}c_{\theta/2} = \frac{s_\theta(1-c_\theta)}{2} = -d^2_{+2,-1}(\theta)$$

$$d^2_{+2,0}(\theta) = d^2_{-2,0}(\theta) = \sqrt{6}s^2_{\theta/2}c^2_{\theta/2} = \frac{\sqrt{6}}{4}s^2_\theta, \quad d^0_{0,0}(\theta) = 1$$

$$\mathcal{M}_{+-+-} = -\frac{2e^2\beta_e e^{i\phi}s_\theta(1+c_\theta)}{1-\beta_e^2c_\theta^2} = 4e^2\beta_e\frac{e^{i\phi}d^2_{+2,+1}(\theta)}{1-\beta_e^2c_\theta^2}, \quad \mathcal{M}_{+--+} = \frac{2e^2\beta_e e^{i\phi}s_\theta(1-c_\theta)}{1-\beta_e^2c_\theta^2} = 4e^2\beta_e\frac{e^{i\phi}d^2_{-2,+1}(\theta)}{1-\beta_e^2c_\theta^2}$$

$$\mathcal{M}_{-+-+} = -\frac{2e^2\beta_e e^{-i\phi}s_\theta(1+c_\theta)}{1-\beta_e^2c_\theta^2} = -4e^2\beta_e\frac{e^{-i\phi}d^2_{-2,-1}(\theta)}{1-\beta_e^2c_\theta^2}, \quad \mathcal{M}_{-++-} = \frac{2e^2\beta_e e^{-i\phi}s_\theta(1-c_\theta)}{1-\beta_e^2c_\theta^2} = -4e^2\beta_e\frac{e^{-i\phi}d^2_{+2,-1}(\theta)}{1-\beta_e^2c_\theta^2}$$

$$\mathcal{M}_{+-++} = \mathcal{M}_{+---} = \mathcal{M}_{-+++} = \mathcal{M}_{-+--} = 0 \quad \text{[Angular momentum is not conserved]}$$

$$\mathcal{M}_{+++-} = \frac{4e^2m_e\beta_e s_\theta^2}{\sqrt{s}(1-\beta_e^2c_\theta^2)} = \frac{16e^2m_e\beta_e}{\sqrt{6}\sqrt{s}}\frac{d^2_{+2,0}(\theta)}{1-\beta_e^2c_\theta^2}, \quad \mathcal{M}_{++-+} = \frac{4e^2m_e\beta_e s_\theta^2}{\sqrt{s}(1-\beta_e^2c_\theta^2)} = \frac{16e^2m_e\beta_e}{\sqrt{6}\sqrt{s}}\frac{d^2_{-2,0}(\theta)}{1-\beta_e^2c_\theta^2}$$

$$\mathcal{M}_{---+} = -\mathcal{M}_{+++-} = -\frac{16e^2m_e\beta_e}{\sqrt{6}\sqrt{s}}\frac{d^2_{-2,0}(\theta)}{1-\beta_e^2c_\theta^2}, \quad \mathcal{M}_{--+-} = -\mathcal{M}_{++-+} = -\frac{16e^2m_e\beta_e}{\sqrt{6}\sqrt{s}}\frac{d^2_{+2,0}(\theta)}{1-\beta_e^2c_\theta^2}$$

$$\mathcal{M}_{++++} = -\frac{4e^2m_e(1+\beta_e)}{\sqrt{s}(1-\beta_e^2c_\theta^2)} = -\frac{4e^2m_e(1+\beta_e)}{\sqrt{s}}\frac{d^0_{0,0}(\theta)}{1-\beta_e^2c_\theta^2}, \quad \mathcal{M}_{++--} = \frac{4e^2m_e(1-\beta_e)}{\sqrt{s}(1-\beta_e^2c_\theta^2)} = \frac{4e^2m_e(1-\beta_e)}{\sqrt{s}}\frac{d^0_{0,0}(\theta)}{1-\beta_e^2c_\theta^2}$$

$$\mathcal{M}_{----} = -\mathcal{M}_{++++} = \frac{4e^2m_e(1+\beta_e)}{\sqrt{s}}\frac{d^0_{0,0}(\theta)}{1-\beta_e^2c_\theta^2}, \quad \mathcal{M}_{--++} = -\mathcal{M}_{++--} = -\frac{4e^2m_e(1-\beta_e)}{\sqrt{s}}\frac{d^0_{0,0}(\theta)}{1-\beta_e^2c_\theta^2}$$

$$\mathcal{M}_{\sigma_1\sigma_2\lambda_1\lambda_2} = \tilde{a}^j_{\sigma_1\sigma_2\lambda_1\lambda_2}\frac{e^{i\sigma\phi}d^j_{\lambda\sigma}(\theta)}{1-\beta_e^2c_\theta^2}, \quad \sigma = \frac{1}{2}(\sigma_1-\sigma_2), \quad \lambda = \lambda_1-\lambda_2, \quad j = 0,2$$

$$\tilde{a}^j_{\sigma_1\sigma_2\lambda_1\lambda_2} \in \mathbf{R}, \quad \tilde{a}^j_{-\sigma_1,-\sigma_2,-\lambda_1,-\lambda_2} = -\tilde{a}^j_{\sigma_1\sigma_2\lambda_1\lambda_2}$$

CLs hypothesis test

Ref: Junk, NIMA 434, 435 (1999); Read, CERN-2000-005, p.81

n independent counting search analyses

i-th channel: estimated signal s_i, estimated background b_i, observed candidate number d_i

Test statistic (Likelihood ratio) $X = \prod_{i=1}^{n} X_i, \quad X_i = \frac{e^{-(s_i+b_i)}(s_i+b_i)^{d_i}}{d_i!} \bigg/ \frac{e^{-b_i} b_i^{d_i}}{d_i!} = \frac{\mathcal{P}(d_i; s_i+b_i)}{\mathcal{P}(d_i; b_i)}$

Poisson distribution $\mathcal{P}(k;\lambda) = \frac{e^{-\lambda}\lambda^k}{k!}$

Confidence level (CL) for exclusing the possibility of simultaneous presence of new particle production and background (the $s+b$ hypothesis) $\mathrm{CL}_{s+b} = P_{s+b}(X \le X_{\mathrm{obs}})$

$$P_{s+b}(X \le X_{\mathrm{obs}}) = \sum_{X(\{d_i'\}) \le X(\{d_i\})} \prod_{i=1}^{n} \frac{e^{-(s_i+b_i)}(s_i+b_i)^{d_i'}}{d_i'!}$$

Confidence level for background alone $\mathrm{CL}_b = P_b(X \le X_{\mathrm{obs}})$

Modified Frequentist confidence level $\mathrm{CL}_s = \frac{\mathrm{CL}_{s+b}}{\mathrm{CL}_b}$

One-sided definition for confidence level $\mathrm{CL} = 1 - \mathrm{CL}_s$

$\mathrm{CL}_s = 0.32 \quad \to \quad 68\%\ \mathrm{CL}, \quad \mathrm{CL}_s = 0.05 \quad \to \quad 95\%\ \mathrm{CL}$

$\left[\begin{array}{l}\text{Two-sided definition for confidence level } \mathrm{CL} = 1 - 2\mathrm{CL}_s \\ \mathrm{CL}_s = 0.16 \quad \to \quad 68\%\ \mathrm{CL}, \quad \mathrm{CL}_s = 0.025 \quad \to \quad 95\%\ \mathrm{CL}\end{array}\right]$

$$\ln X_i = d_i \ln(s_i+b_i) - (s_i+b_i) - \ln(d_i!) - [d_i \ln b_i - b_i - \ln(d_i!)] = d_i \ln\left(1 + \frac{s_i}{b_i}\right) - s_i$$

$$Q(\{d_i\}) = \ln X = \sum_i \ln X_i = \sum_i \left[d_i \ln\left(1 + \frac{s_i}{b_i}\right) - s_i \right]$$

Pseudo-experiments:

1) n_{MC} pseudo-experiments generate n_{MC} sets of $\{\tilde{b}_i\}$ according to normal distribution $\tilde{b}_i \sim \mathcal{N}(b_i, \sigma_{b_i})$

2) For each set of $\{\tilde{b}_i\}$, randomly draw pseudo-data $\{d_i^{(b)}\}$ and $\{d_i^{(s+b)}\}$ according to Poisson distributions $d_i^{(b)} \sim \mathcal{P}(\tilde{b}_i)$ and $d_i^{(s+b)} \sim \mathcal{P}(s_i + \tilde{b}_i)$

3) Calculate n_{MC} sets of $Q_b = Q(\{d_i^{(b)}\})$ and $Q_{s+b} = Q(\{d_i^{(s+b)}\})$, which will approximately obey normal distributions

$P(Q_b \le Q_x) = x \quad \Rightarrow \quad Q_x$

$x_{\mathrm{med}} = 0.5, \quad x_{\pm 1\sigma} = 0.5 \pm 0.34, \quad x_{\pm 2\sigma} = 0.5 \pm 0.475$

$$\mathrm{CL}_s(x) = \frac{\mathrm{CL}_{s+b}(x)}{\mathrm{CL}_b(x)} = \frac{P(Q_{s+b} \le Q_x)}{P(Q_b \le Q_x)} = \frac{P(Q_{s+b} \le Q_x)}{x}$$

Discriminating two signal scenarios

Likelihood for signal s: $\mathcal{L}(s)=\prod_i \frac{e^{-(s_i+b_i)}(s_i+b_i)^{d_i}}{d_i!}=\mathcal{P}(d_i;s_i+b_i)$

Test statistic for a benchmark signal and an alternative signal $Q=\ln\frac{\mathcal{L}(s^{\text{alt}})}{\mathcal{L}(s^{\text{bm}})}$

$$\ln\frac{\mathcal{P}(d_i;s_i^{\text{alt}}+b_i)}{\mathcal{P}(d_i;s_i^{\text{bm}}+b_i)}=d_i\ln(s_i^{\text{alt}}+b_i)-(s_i^{\text{alt}}+b_i)-\ln(d_i!)-[d_i\ln(s_i^{\text{bm}}+b_i)-(s_i^{\text{bm}}+b_i)-\ln(d_i!)]=d_i\ln\frac{s_i^{\text{alt}}+b_i}{s_i^{\text{bm}}+b_i}+s_i^{\text{bm}}-s_i^{\text{alt}}$$

$$Q(\{d_i\})=\sum_i\left[d_i\ln\frac{s_i^{\text{alt}}+b_i}{s_i^{\text{bm}}+b_i}+s_i^{\text{bm}}-s_i^{\text{alt}}\right]$$

Pseudo-experiments:

1) n_{MC} pseudo-experiments generate n_{MC} sets of $\{\tilde{b}_i\}$ according to normal distributions $\tilde{b}_i\sim\mathcal{N}(b_i,\sigma_{b_i})$

2) For each set of $\{\tilde{b}_i\}$, randomly draw pseudo-data $\{d_i^{\text{bm}}\}$ and $\{d_i^{\text{alt}}\}$ according to Poisson distributions $d_i^{\text{bm}}\sim\mathcal{P}(s_i^{\text{bm}}+\tilde{b}_i)$ and $d_i^{\text{alt}}\sim\mathcal{P}(s_i^{\text{alt}}+\tilde{b}_i)$

3) Calculate n_{MC} sets of $Q_{\text{bm}}=Q(\{d_i^{\text{bm}}\})$ and $Q_{\text{alt}}=Q(\{d_i^{\text{alt}}\})$, which will approximately obey normal distributions

$$P(Q_{\text{bm}}\le Q_x)=x \quad\Rightarrow\quad Q_x$$

$$\text{CL}_s(x)=\frac{P(Q_{\text{alt}}\le Q_x)}{P(Q_{\text{bm}}\le Q_x)}=\frac{P(Q_{\text{alt}}\le Q_x)}{x}$$