The theoretical basis for high-energy physics in particular is quantum field theories and the Standard Model of particle physics, which are not typically available at the undergraduate level. The existence of quark-gluon plasma in fact was predicted theoretically by one such field theory, quantum chromodynamics, before it was observed experimentally.

Theoretical predictions in general are very important to the advancement of experimental high-energy physics because detections are so sensitive, may not happen frequently, and often must be deduced from indirect evidence; experimentalists generally must look for things that have been predicted theoretically. Although the Standard Model has enjoyed numerous experimental successes, it still does not describe all phenomena, and is inconsistent with the also-celebrated theory of gravitation, Einstein’s theory of General Relativity. There is therefore reason, not only to study the Standard Model in detail, but also to study alternative predictive methods for the behavior of matter at high energies and densities. Hydrodynamics is one method for modeling the physical properties of such matter, but there certainly are (and will continue to be) many theories describing all sorts of aspects of these experiments. As an aspiring mathematical physicist, I hope to contribute to the theoretical backing of high-energy or plasma physics.

Recent experiments at CERN and at Brookhaven’s Relativistic Heavy Ion Collider (RHIC) have realized a state of matter with such high energy nuclear particles such as protons and neutrons cannot form from their constituent particles, quarks and gluons. The quarks and gluons that at almost all energies are bound tightly in atomic and subatomic particles are in this case ‘de-confined’ and free to roam about. This state is called quark-gluon plasma. Quark-gluon plasmas are of fantastic experimental and theoretical interest, as quark-gluon plasma is thought to have been the state of matter very shortly after the Big Bang, and is also suspected to exist at the core of neutron stars.

Quark-gluon plasmas remain exceptionally difficult to study experimentally - the plasmas exist for femtoseconds before the energy density decreases enough for protons and neutrons to re-form. The experimental observation of elliptic flow in quark-gluon plasmas was therefore a major success and sparked significant theoretical interest. Elliptic flow is a phenomenon in which the wide dimension of an elliptical density profile changes as time evolves, and is a characteristic of strongly-coupled quantum fluids with anomalously low viscosity. In fact, experimental data suggests that quark-gluon plasma may have the lowest viscosity of any fluid yet observed. Particularly in light of an absolute lower bound on fluid viscosity proposed recently through string-theoretic methods, there is much interest in finally constraining the viscosity of quark-gluon plasma. Hydrodynamics is one promising method for the theoretical study of viscosity in quark-gluon plasmas.

The observation of elliptic flow in quark-gluon plasma allows analogies to be drawn with other systems that exhibit elliptic flow, which include several strongly-coupled cold atomic fluids. One such system is the unitary Fermi gas, which is an ultra-cold collection of fermions (often Lithium) in the limit (called unitarity) where the Fermi gas exhibits the strongest coupling. This system has analogous material properties to quark-gluon plasmas, but is not relativistic. My current contribution to the study of quark-gluon plasmas is to study the material properties of unitary Fermi gases through hydrodynamic simulations. The advantage of studying this system is that it is experimentally-verifiable: elliptic flow in Fermi gases is observed on the scale of hundreds of microseconds, which is about 12 orders of magnitude longer of a time scale than the one on which the expansion of quark-gluon plasmas can be observed. Developing simulations of quark-gluon plasmas out of simulations of Fermi gases that have been verified through experiment puts the non-relativistic limit on a solid experimental foundation. An extension of these methods, however, requires a relativistic formulation of hydrodynamics. It is important to note as well that, though these fluids are of interest for their anomalously low viscosity, if we would like to constrain a value for this viscosity we must use viscous hydrodynamic methods.

Relativistic hydrodynamics is a developing field with important applications in astrophysics, magneto-hydrodynamics, and high-energy condensed matter (as we were discussing). For viscous flows, the first-order diffusion relations typically used to model viscous non-relativistic flows, the Navier-Stokes equations, violate causality. Several techniques for re-working this model exist in the literature (generally some sort of second-order diffusion), but the equations for relativistic viscous hydrodynamics remain very complicated and numerical solutions are prohibitively computationally-intensive in most cases. Advances in hydrodynamic modeling of quark-gluon plasmas and other high-energy matter will require improvements in the computational efficiency of relativistic hydrodynamic methods. This problem certainly has not been solved and is an issue I would like to contribute to in my undergraduate thesis.

Moving forward in my physics and mathematics education into graduate school, a wider diversity of methods will become available to me for the study of high-energy matter and plasmas. The theoretical basis for high-energy physics in particular is quantum field theories and the Standard Model of particle physics, which are not typically available at the undergraduate level. The existence of quark-gluon plasma in fact was predicted theoretically by one such field theory, quantum chromodynamics, before it was observed experimentally.

Theoretical predictions in general are very important to the advancement of experimental high-energy physics because detections are so sensitive, may not happen frequently, and often must be deduced from indirect evidence; experimentalists generally must look for things that have been predicted theoretically. Although the Standard Model has enjoyed numerous experimental successes, it still does not describe all phenomena, and is inconsistent with the also-celebrated theory of gravitation, Einstein’s theory of General Relativity. There is therefore reason, not only to study the Standard Model in detail, but also to study alternative predictive methods for the behavior of matter at high energies and densities. Hydrodynamics is one method for modeling the physical properties of such matter, but there certainly are (and will continue to be) many theories describing all sorts of aspects of these experiments. As an aspiring mathematical physicist, I hope to contribute to the theoretical backing of high-energy or plasma physics.

**References**

Nearly Perfect Fluidity: From Cold Atomics Gases to Hot Quark Gluon Plasmas

“Observation of a strongly interacting degenerate Fermi gas of atoms”

“Universal Quantum Viscosity in a Unitary Fermi Gas”

Courses in quantum field theory are typically not available to undergraduates, but as I enter graduate school a theoretical backing for particle physics in quantum field theory becomes accessible. I hope that my background in theoretical mathematics will aid me at this point in…

Quark-gluon plasma, especially near the phase transition for its condensation (re-confinement) into a hadron gas, is a strongly-coupled quantum fluid with anomalously low shear viscosity. Other fluids with these properties include strongly-coupled Bose fluids and Fermi gases in the unitarity limit (i.e. at the Feshbach resonance, when the scattering length goes to infinity). My present contribution to the study of elliptic flow and the material properties of quark-gluon plasmas is to study the hydrodynamics of the unitary Fermi gas. One advantage of developing a hydrodynamic model for this system is that the model is experimentally-verifiable. Elliptic flow in this strongly-coupled quantum system is observed on the scale of hundreds of microseconds, and though its experimental realization is relatively recent there are several sets of experimental data (e.g. “Observation of a strongly interacting degenerate Fermi gas of atoms”, “Universal Quantum Viscosity in a Unitary Fermi Gas”, and others). The approximately 12 orders of magnitude longer time scale of the expansion of the Fermi gas relative to the lifetime of the quark-gluon plasma makes this system much more experimentally-verifiable. Therefore we can develop hydrodynamic models of the Fermi gas at unitarity, verify our methods with experiment, and then use these models to develop hydrodynamic models of quark-gluon plasmas. It is important to note that Fermi gases are non-relativistic. An extension of these methods into the study of quark-gluon plasmas requires a fully relativistic formulation of hydrodynamics.

For ideal hydrodynamics (zero fluid viscosity), fluid dynamics equations in both non-relativistic and relativistic formulations are conservation equations for the fluid’s energy-momentum tensor (“New Developments in Relativistic Viscous Hydrodynamics”). However, “New Developments in Relativistic Viscous Hydrodynamics” also shows that the relativistic formulation of the Navier-Stokes equations (typically used to describe fluids with non-zero viscosity) tend to break down for fluids without analytic solutions whose dynamics must be solved numerically (which are a lot of them). Of course, if one would like to constrain the viscosity of quark-gluon plasma by theoretical means, assuming zero viscosity will not cut it and the hydrodynamic model must include viscous effects.

“A fluid is a material that can be described by the laws of fluid dynamics. These laws imply that the response of a fluid to slowly varying external perturbation is completely governed by conservation laws”.

Shear viscosity is friction force per unit area, and is related to the momentum transport of individual molecules.

“A perfect fluid dissipates the smallest possible amount of energy, and satisfies the laws of fluid dynamics in the largest possible domain.”

Quark-gluon plasma is a state of matter in which quarks and gluons are not confined to nuclear particles. Quark-gluon plasma has only been realized experimentally within the last few years (at CERN and at Brookhaven’s Relativistic Heavy Ion Collider (RHIC)), and remains exceptionally difficult to study experimentally because the plasma exists only for femtoseconds before the matter cools enough for neutrons and protons to re-form. There is strong interest in understanding the physical and material properties of quark-gluon plasma, especially its viscosity. String theory has recently proposed an absolute lower bound on fluid viscosity, and experiments at CERN and RHIC suggest that quark-gluon plasma is possibly the fluid with the lowest observed shear viscosity, which makes it of theoretical importance. Furthermore, as QGP is thought to have existed at the beginning of the universe, and is also thought to exist at the core of neutron stars, understanding its physical properties is central to theories about these objects and scenarios.

Quark-gluon plasma and the surrounding phenomena fall under the theoretical domain of quantum chromodynamics, however the study of the material properties of the quark-gluon plasma phase also seems to belong inside condensed-matter physics. There is potential in studying the behavior of high-energy, high-density matter (and specifically quark-gluon plasma) through condensed-matter physics, and more specifically through methods of hydrodynamics.

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