HW4-JiawenTong

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```
In [1]: %matplotlib inline
        import numpy as np
        from math import *
        import matplotlib.pyplot as plt
```

0.1 Problem 1

** Please see proof of Problem 1 (a), (b) on the main writeup report.**

0.2 Problem 2

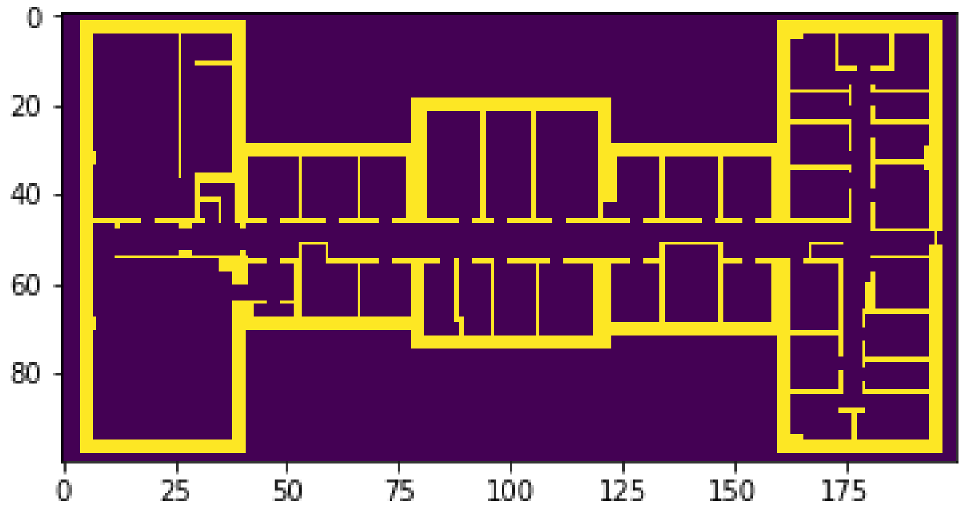
0.2.1 (a)

```
In [2]: # Pressure in the region S
        def src_f(t):
            p0 = 10
            w = 100*pi
            return p0*sin(w*t)

        # Determine if (x=j, y=k) is in S
        def is_in_S(j, k):
            if j>=57 and j<=60 and k>=15 and k<=18:
                return True
            else:
                return False
```

```
In [3]: # Read in the map of Pierce Hall
        M, N = 100, 200
        pierce_map = np.zeros((M, N))
        lines = open('pierce.txt').readlines()
        for i, line in enumerate(lines):
            pierce_map[i, :] = line.replace('\n', '').split(' ')
```

```
In [4]: plt.imshow(pierce_map)
```

```
Out[4]: <matplotlib.image.AxesImage at 0x110bc8908>
```




```
In [5]: # Computer the big matrix for the pressure field
        h = 36.6
        c = 3.43e4
        dt = h/(2*c)
        snaps = 2000
        nu = c*c*dt*dt/(h*h)
```

0.2.2 Can directly load the pressure field np matrix using

`large_P = np.load('./pierce/pierce_t2000.npy')`

It takes ~ 11 mins to run the following cell on my Mac...

```
In [ ]: P = np.zeros((M, N, snaps+1)) # The Pressure Field

        # Initial condition t = 0
        P[:, :, 0] = 0

        # Initial condition t = 1 (t=0 is already satisfied)
        for j in range(M):
            for k in range(N):
                if is_in_S(j, k):
                    P[j, k, 1] = src_f(dt)

        import time
        start_time = time.time()

        # Iteration <<< Reference: Lecture 15 Code Example Heat.py
        for i in range(2, snaps+1):
            for j in range(M):
```

```
            for k in range(N):
                if pierce_map[j, k] == 1: # (j, k) is wall, skip
                    continue

                if is_in_S(j, k): # (j,k) IN Region S
                    P_jk = src_f((i-1)*dt)
                    s1 = 2*src_f((i-1)*dt) - src_f((i-2)*dt)
                else: # (j,k) NOT IN Region S
                    P_jk = P[j, k, i-1]
                    s1 = 2*P[j, k, i-1] - P[j, k, i-2]

                s2 = -4 * P_jk

                orth_neighbors = np.array([(j-1, k), (j+1, k), (j, k-1), (j, k+1)])
                for jn, kn in orth_neighbors:
                    if jn<0 or jn>=M or kn<0 or kn>=N: # (jn, kn) is NOT in map, s2 += 0,
                        continue
                    else:
                        if pierce_map[jn, kn] == 1: # Is Wall
                            s2 += P_jk
                        elif is_in_S(jn, kn): # NOT WALL & In Region S
                            s2 += src_f((i-1)*dt)
                        else: # NOT WALL & NOT IN Region S
                            s2 += P[jn, kn, i-1]

                P[j, k, i] =  s1 + nu*s2 # Update P

    print("--- Duration = %s seconds ---" % (time.time() - start_time))

    np.save('./pierce/pierce_t2000', P)
```

```
In [6]: large_P = np.load('./pierce/pierce_t2000.npy')
```

0.2.3 (b)

```
In [7]: # Provided code for custom plot
        import sys

        # Returns a scaled value of a function in the range 0 to 1, truncating it if necessary
        def fscale(v,vmin,vsca):
            vs=(v-vmin)*vsca
            if vs<0: vs=0
            if vs>1: vs=1
            return vs

        # Returns a red->white->blue color scheme
        def palette1(v):
            v*=2
```

```
    if(v>1):
        v=2-v
        return (v,v,min(1,0.5+3*v))
    else:
        return (min(1,0.5+3*v),v,v)

# Outputs a 2D image from a field using the palette1 color scheme
# fn: the filename to save to
# p: the 2D field to plot
# wa: the matrix describing walls to overlay
# (vmin,vmax): the field range
# ups: the upsampling factor to use (so each field point is convert into an ups by ups
def plot1(fn,p,wa,vmin,vmax,ups):

    # Check matrix dimensions are the same
    (m,n)=p.shape
    if (m,n)!=wa.shape:
        print("Matrix dimension mismatch")

    # Set up output array and scaling constant
    o=np.zeros((m*ups,n*ups,3))
    vsca=1.0/(vmax-vmin)

    # Assemble the output array
    for i in range(m):
        iu=i*ups
        for j in range(n):
            ju=j*ups
            if wa[i,j]==1:
                o[iu:iu+ups,ju:ju+ups,0]=0
                o[iu:iu+ups,ju:ju+ups,1]=0
                o[iu:iu+ups,ju:ju+ups,2]=0
            else:
                (re,gr,bl)=palette1(fscale(p[i,j],vmin,vsca))
                o[iu:iu+ups,ju:ju+ups,0]=re
                o[iu:iu+ups,ju:ju+ups,1]=gr
                o[iu:iu+ups,ju:ju+ups,2]=bl

    # Save the image
    plt.imsave(fn,o)
    plt.imshow(o)

# Returns a black->red->yellow color scheme (matching Gnuplot's PM3D scheme)
def palette2(v):
    if v>0.5:
        vs=0
    else:
        vs=sin(2*pi*v)
```

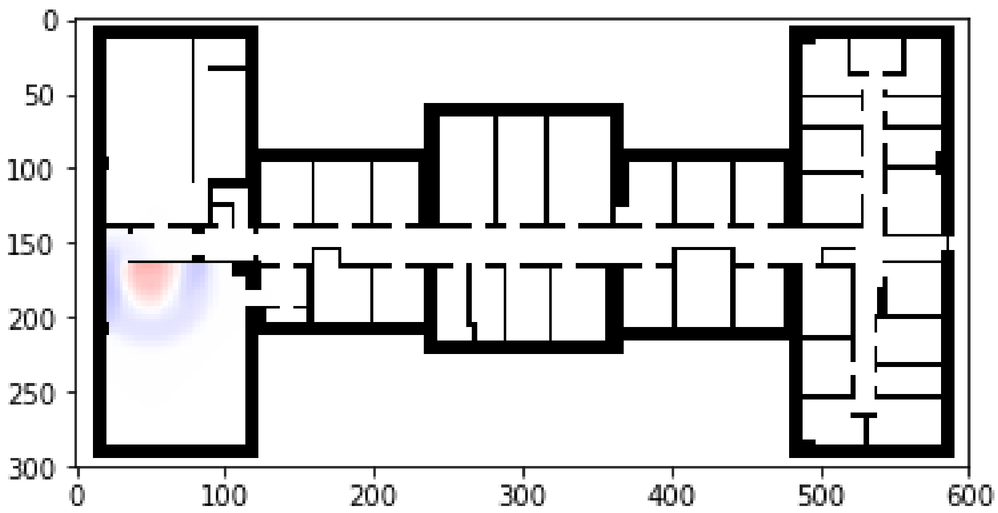
```
        return (sqrt(v),v*v*v,vs)

    # Outputs a 2D image from a field using the palette2 color scheme
    # fn: the filename to save to
    # p: the 2D field to plot
    # wa: the matrix describing walls to overlay
    # (vmin,vmax): the field range
    # ups: the upsampling factor to use (so each field point is convert into an ups
    #      by ups square)
    def plot2(fn,p,wa,vmin,vmax,ups):

        # Check matrix dimensions are the same
        (m,n)=p.shape
        if (m,n)!=wa.shape:
            print("Matrix dimension mismatch")

        # Set up output array and scaling constant
        o=np.zeros((m*ups,n*ups,3))
        vsca=1.0/(vmax-vmin)

        # Assemble the output array
        for i in range(m):
            iu=i*ups
            for j in range(n):
                ju=j*ups
                if wa[i,j]==1:
                    o[iu:iu+ups,ju:ju+ups,0]=1
                    o[iu:iu+ups,ju:ju+ups,1]=1
                    o[iu:iu+ups,ju:ju+ups,2]=1
                else:
                    (re,gr,bl)=palette2(fscale(p[i,j],vmin,vsca))
                    o[iu:iu+ups,ju:ju+ups,0]=re
                    o[iu:iu+ups,ju:ju+ups,1]=gr
                    o[iu:iu+ups,ju:ju+ups,2]=bl

        # Save the image
        plt.imsave(fn,o)
        plt.imshow(o)
```

In [8]:
```
P_last = large_P[:, :, -1]
p_min = np.min(P_last)
p_max = np.max(P_last)
print('The magitude of the P in the last iteration is {} ~ {}'.format(p_min, p_max))
```

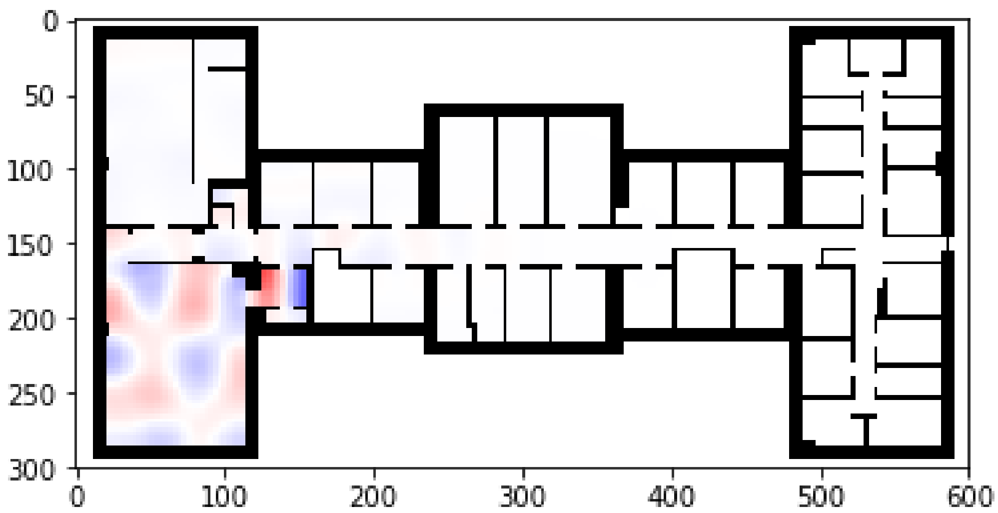
```
The magitude of the P in the last iteration is -43.34730364254081 ~ 41.53652109925552
```

In [9]:
```
ts = [0.015, 0.105, 0.505, 1.005]
```

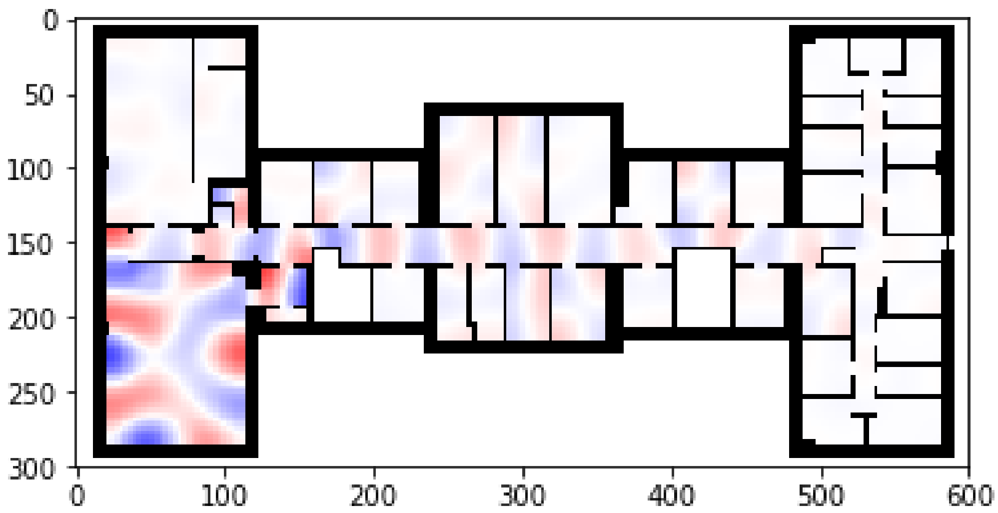
```
# Load in the wall matrix
wall = np.loadtxt("pierce.txt",dtype=np.int8)

# Plot t = 0.015
plot1("./pierce/t0.png", large_P[:, :, int(ts[0]/dt)], wall, -45, 45, 3)
```



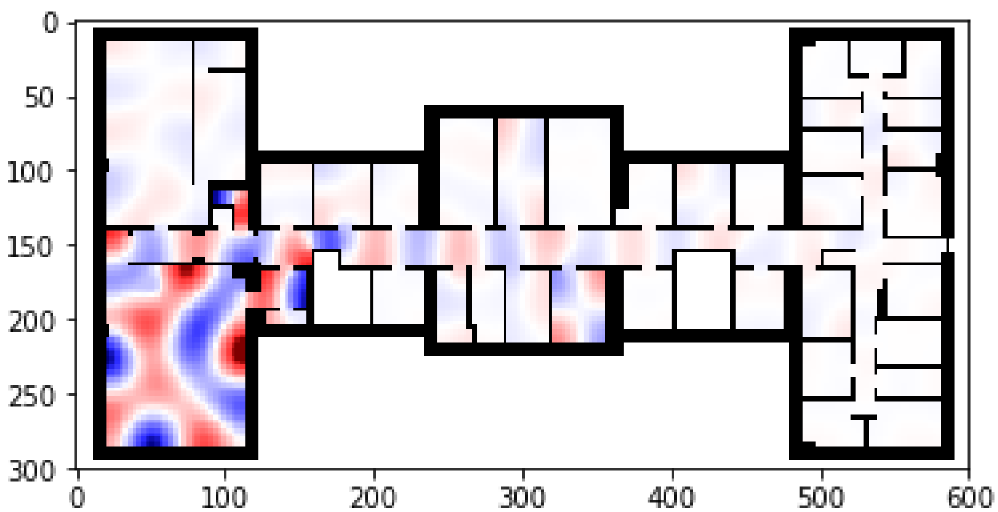

```
In [10]: # Plot t = 0.105
         plot1("./pierce/t1.png", large_P[:, :, int(ts[1]/dt)], wall, -45, 45, 3)
```




```
In [11]: # Plot t = 0.505
         plot1("./pierce/t2.png", large_P[:, :, int(ts[2]/dt)], wall, -45, 45, 3)
```




```
In [12]: # Plot t = 1.005
         plot1("./pierce/t3.png", large_P[:, :, int(ts[3]/dt)], wall, -45, 45, 3)
```



0.2.4 (c)

```
In [13]: P_C_t = large_P[35, 73, :]
         P_G_t = large_P[61, 109, :]
         P_M_t = large_P[91, 188, :]
         t_C, t_G, t_M = -1, -1, -1
```

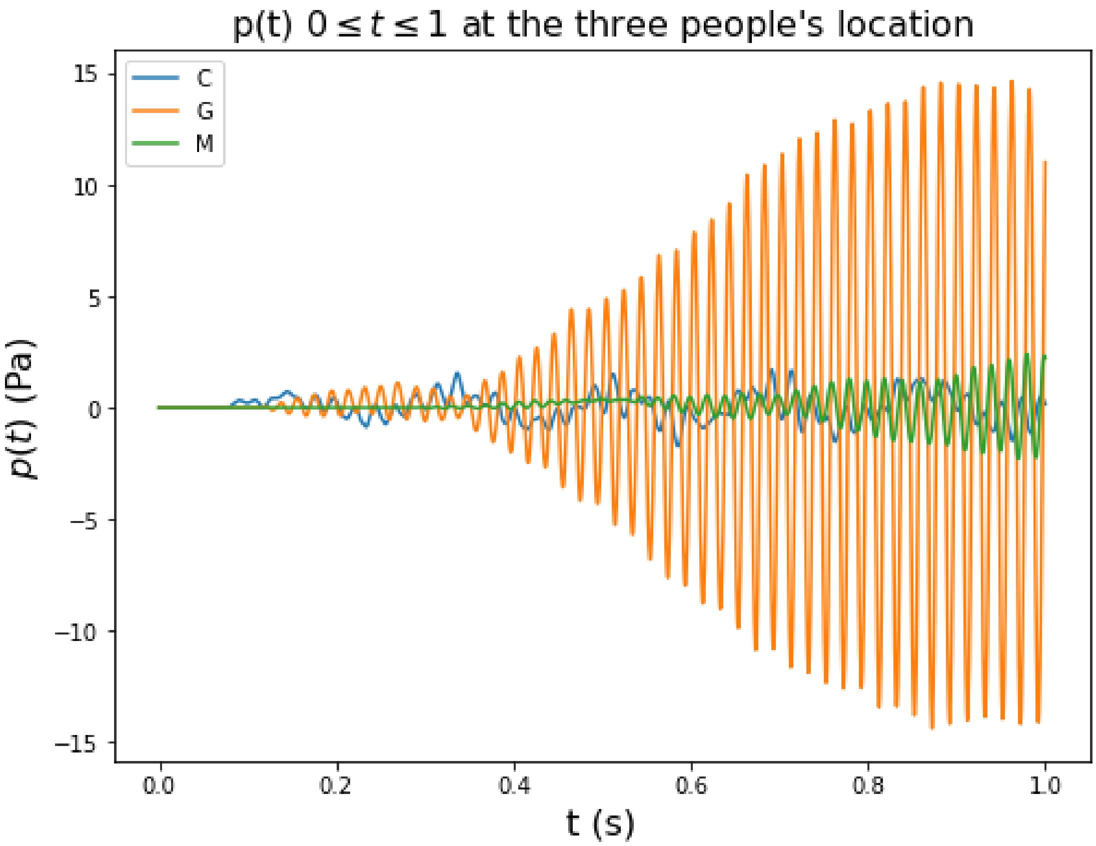
```
n_time_step = large_P.shape[-1]
for n in range(n_time_step):
    if t_C != -1 and t_G != -1 and t_M != -1:
        break
    if abs(P_C_t[n]) > 1e-3 and t_C == -1:
        t_C = n * dt
    if abs(P_G_t[n]) > 1e-3 and t_G == -1:
        t_G = n * dt
    if abs(P_M_t[n]) > 1e-3 and t_M == -1:
        t_M = n * dt

print('Time (sec) when the three people first hear the sound\n')
print('C: {} \nG: {} \nM: {}'.format(t_C, t_G, t_M))
```

```
Time (sec) when the three people first hear the sound

C: 0.07362682215743441
G: 0.1088396501457726
M: 0.2310174927113703
```

0.2.5 (d)

In [14]:
```
# Plot p(t) at the three people's location for 0 <= t <= 1
fig, ax = plt.subplots(1, 1, figsize=(8, 6))
t_steps = int(1/dt) + 1
t_lin = np.linspace(0, 1, t_steps)
ax.plot(t_lin, P_C_t[:t_steps], label='C')
ax.plot(t_lin, P_G_t[:t_steps], label='G')
ax.plot(t_lin, P_M_t[:t_steps], label='M')
ax.set_xlabel('t (s)', fontsize=16)
ax.set_ylabel('$p(t)$ (Pa)', fontsize=16)
ax.set_title('p(t) $0 \leq t \leq 1$ at the three people\'s location', fontsize=16)
ax.legend()
```

Out[14]:
```
<matplotlib.legend.Legend at 0x126bfcdd8>
```



0.3 Problem 3

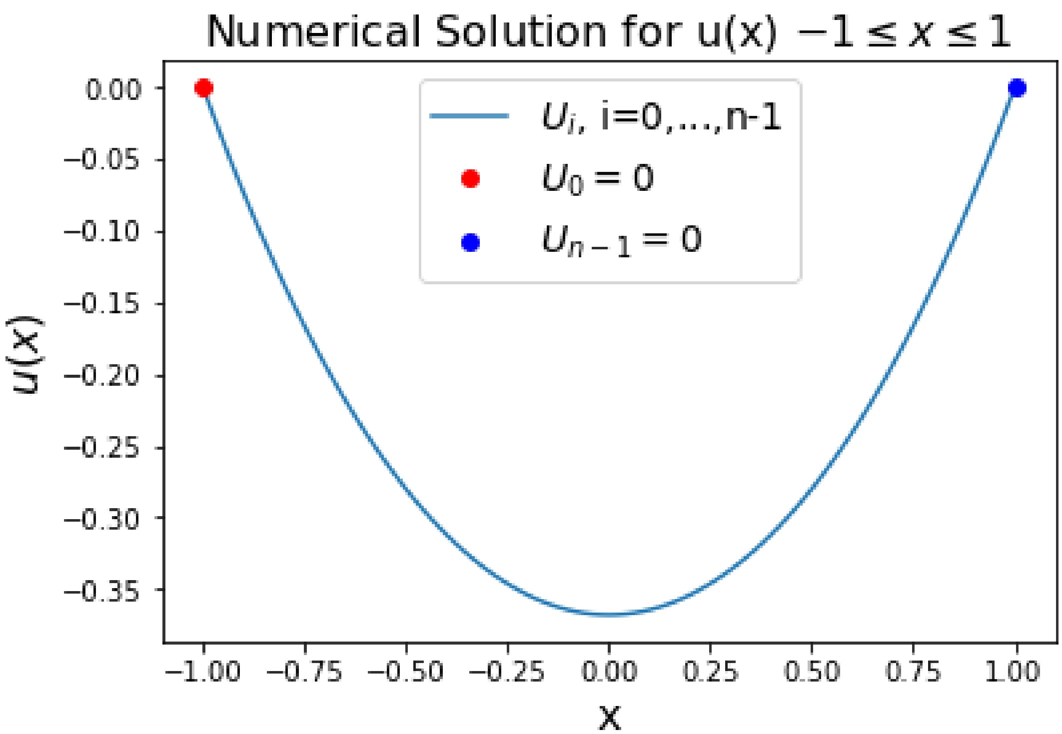
0.3.1 (b)

In [15]:
```
n = 101
h = 2/(n-1)
b = 1/(h*h)

def get_F(U): # F(U)
    F = np.zeros((n-2,))
    F[0] = (U[1]-2*U[0])*b - exp(U[0])
    F[n-3] = (-2*U[n-3]+U[n-4])*b - exp(U[n-3])
    for i in range(1, n-3):
        F[i] = (U[i+1] - 2*U[i] + U[i-1])*b - exp(U[i])
    return F

def get_J(U): # Jacobian of F(U)
    A = np.zeros((n-2, n-2))
    A[0:n-3, 1:n-2] = b*np.eye(n-3)
    B = np.zeros((n-2, n-2))
```

```
    B[1:n-2, 0:n-3] = b*np.eye(n-3)
    J = np.add(-2*b*np.eye(n-2), A, B)

    for i in range(n-2):
        J[i,i] += -exp(U[i])
    return J

# Newton Roots Finding: F(U) = 0
U0 = np.zeros((n-2,))
F = get_F(U0)
J = get_J(U0)
delta_U = np.linalg.solve(J, -1*F)
U = U0
relative_step = 100
while relative_step > 1e-10:
    U = np.add(delta_U, U)
    F = get_F(U)
    J = get_J(U)
    delta_U = np.linalg.solve(J, -1*F)
    relative_step = np.linalg.norm(delta_U)/np.linalg.norm(U)
```

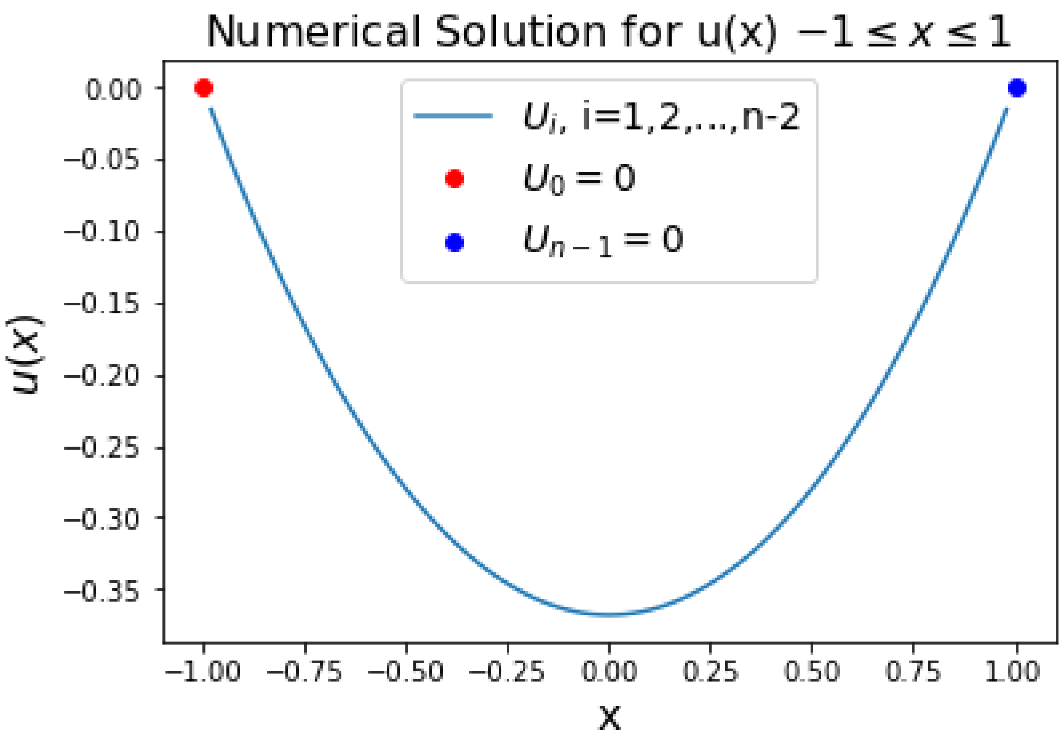
In [16]:
```
# Pad the approximate solution U with U0 & Un-1
U_all = np.zeros((n,))
U_all[0] = 0
U_all[-1] = 0
U_all[1:-1] = np.copy(U)

# Plot U
plt.plot(np.linspace(-1, 1, n), U_all, '-', label='$U_i$, i=0,...,n-1')

plt.plot(-1, 0, 'ro', label='$U_0 = 0$')
plt.plot(1, 0, 'bo', label='$U_{n-1} = 0$')

plt.legend(fontsize=14)
plt.title('Numerical Solution for u(x) $-1 \leq x \leq 1$', fontsize=16)
plt.xlabel('x', fontsize=16)
plt.ylabel('$u(x)$', fontsize=16)
```

Out[16]:
```
<matplotlib.text.Text at 0x126e92320>
```



In [17]:
```
# Plot the approximate solution U
plt.plot(np.linspace(-1, 1, n)[1:-1], U, '-', label='$U_i$, i=1,2,...,n-2')

# Pad with U0 & Un-1
plt.plot(-1, 0, 'ro', label='$U_0 = 0$')
plt.plot(1, 0, 'bo', label='$U_{n-1} = 0$')

plt.legend(fontsize=14)
plt.title('Numerical Solution for u(x) $-1 \leq x \leq 1$', fontsize=16)
plt.xlabel('x', fontsize=16)
plt.ylabel('$u(x)$', fontsize=16)
```

Out[17]: <matplotlib.text.Text at 0x125e7ffd0>




```
In [18]: print('Approximation to u(0) is {}, index={}'.format(U[int(1/h-1)], int(1/h-1)))
```

```
Approximation to u(0) is -0.36804818818603935, index=49
```

```
In [ ]:
```