LAB 1 PROSPECTUS

1. Introduction

The objective of this game is to locate a series of charged particles that have been hidden in the xy plane by analyzing the trajectories of moving particles subjected to the electrostatic forces generated by the hidden charges. The player enters a set of initial conditions for the moving particle — initial velocity, initial angle, and initial position — and examines its trajectory. This can be repeated at the player's discretion. Once the player comes up with an answer, they enter it into the console. The program compares the player's answer with the true location of the particles and provides feedback accordingly. This process repeats until the player has successfully located the particles.

This prospectus is intended to explicate the program behind the game. We will begin with a brief overview of the physics used in developing the game, and the rest of the paper will be devoted to a summary of the game program and mechanics.

2. Game physics

If Q is one of the aforementioned hidden charges, the electrostatic force generated by this charge is given by

$$\mathbf{F}(r) = \frac{kQ}{r}\hat{\mathbf{r}}$$

where r is the distance from Q to a moving particle, $\hat{r}$ is the unit vector pointing in the direction of r, and k is the coupling constant. The magnitude of this force is inversely proportional to the magnitude of the distance $\mathbf{r}$.

This force function is additive, so the total force generated by multiple charges is the vector sum of the individual electrostatic forces.

The second equation we will be using is the potential function corresponding to $\mathbf{F}(\mathbf{r})$, which we denote by $\phi(r)$.

$$\phi(r) = kQ\ln(\tfrac{r_0}{r})$$

It can be shown that $\nabla\mathbf{F}(r) = \phi(r)$. These functions will govern the movement of particles in the xy plane. The motion of such particles can be modeled using Newton's second law and techniques of differential equations. Further details will be discussed in the next section.

3. Implementation

In this section, we will present and explain the structure of the program behind the game. The three hidden charges are defined in the first few lines, including their charges and their positions in the xy plane. These values are stored in a 3×3 array, with each row devoted to a single charge.

3.1. **The Reveal Potential Function.** Per its name, the purpose of this function is to calculate the electrostatic potential over a region of the xy plane. The graph of the potential lines is then shown to the player upon winning the game. By inspecting the plot, one can determine the locations of the particles.

The function works by individually calculating the potential at each point in a two-dimensional matrix whose points are indexed by the variables m and n. Two nested for loops traverse this matrix and call the calculate potential function at each point. The total potential created by the three charges is then returned and added to the plot.

3.2. **The Plot Trajectory Function.** This function is designed to graph the motion of particles placed in the force field generated by the three hidden particles. The x and y components of motion are separately determined by solving two differential equations which, via Newton's second law, relate the electrostatic force on the moving particles with their acceleration.

The first and second derivatives of the position functions are stored in a four-dimensional array called "derivatives." Using the initial conditions (velocity, angle, position) specified by the player, the x and y position functions are determined with respect to t over the interval $[0, 50]$ and plotted on a graph which is then shown to the player.

The intention is that the player plot multiple position functions corresponding to a variety of initial conditions so as to figure out the locations of the hidden charges.

3.3. **The Solve It Function.** This function is designed to assess the player's guesses for the charge locations. The player is presented with four prompts corresponding to the four quadrants of the xy plane, and they guess whether or there exist one or multiple positive or negative charges in each plane. The four answer choices for each quadrant are (1) positive, (2) negative, (3) neither, (4) both. If the player guesses correctly, the function returns a congratulatory message. Otherwise, it asks the player to try again.

Once the player enters a number, it is compared with a boolean expression corresponding to their answer. The player will earn a point

if and only if the boolean expression is true. At the end, the points are added. Their sum must equal four for the player to win.

4. Game play and game solution

A typical game would involve the player plotting multiple trajectory functions and analyzing how the particle's motion changes depending on its position in the xy plane. Based on this, the player can approximate the location of the hidden charges by considering the interaction of charged particles of differing signs (i.e. positive charges and negative charges attract, whereas like charges repel). For instance, if multiple positive particles suddenly deviate from a straight path when they approach a certain point in the xy plane, one might infer that a hidden particle with a positive charge exists at that point. Likewise, if multiple positive particles converge on to a point in the xy plane, one might infer that a hidden particle with a negative charge exists at the point. If the player then guesses correctly, a graph of the potential function will be shown.