Probability

Lectures by Steven Miller

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Introduction

While probability began with a study of games, it has grown to become a discipline with numerous applications throughout mathematics and the sciences. Drawing on gaming examples for motivation, this course will present axiomatic and mathematical aspects of probability. Included will be discussions of random variables, expectation, independence, laws of large numbers, and the Central Limit Theorem. Many interesting and important applications will also be presented, including some from coding theory, number theory and nuclear physics.

These lectures are mainly based on the books The Probability Lifesaver by Steven Miller published by Princeton University Press.

These notes were live-TeXed, though I edited for typos and added diagrams requiring the TikZ package separately. I used the editor TeXstudio.

I am responsible for all faults in this document, mathematical or otherwise; any merits of the material here should be credited to the lecturer, not to me.

Please email any corrections or suggestions to jaafar_zhang@163.com.

Acknowledgments

Thank you to all of my friends who will send me suggestions and corrections. My notes will be much improved due to your help.

I would like to especially thank Professor Miller who puts this course in website.

Lecture 1

Introduction

Lecture 2

Combinatorics, Birthday Problem, Pentium Bug, QWERTY, Programming, Babylonian Mathematics

Lecture 3

Set Theory, Probability Wish List, Coding

Lecture 4

Axioms of Probability, Consequences, Sniffing out Formulas

Lecture 5

Supplemental: Infinities, Generating Functions, Differentiating Identities

Lecture 6

Factorial Function, Binomial Coefficients, Poker Hands, Pascal's Triangle Mod 2

Lecture 7

Probability and Mathematical Modeling I

Lecture 8

Probability and Mathematical Modeling II

Lecture 9

Card Shuffling

Lecture 10

Trump Splits, Conditional Probability, Bayes' Theorem

Lecture 11

Independence, Derangements, Inclusion-Exclusion, Induction

Lecture 12

Basics of pdfs and Random Variables

Lecture 13

Review cont/discrete RV, expectation, moments, Cauchy, Taylor

Lecture 14

Joint pdfs, linearity of expectation

Lecture 15

Introduction to Statistics and Modeling

Lecture 16

Simpson's paradox, Ace of Hearts method

Lecture 17

Linearity of expectation, variances and covariances, power of linearity of expectation, bernoulli and binomial, convolution

Lecture 18

Marriage Problem, Two Envelope Problem, Buffon's Needle

Lecture 19

Differentiating Identiteis (Gaussian, Exponential, Geometric, Negative Binomial)

Lecture 20

Sums of Uniform Random Variables, Sums of Gaussian Random Variables, Cauchy Distribution, Gregory-Leibnitz Formula

Lecture 21

Pythagoras, Gamma Function, Chi-Square Distribution, Surface Area

Lecture 22

Markov and Chebyshev's inequalities, Divide and Conquer vs Newton's Method

Lecture 23

Poisson Random Variables, Exponential Function, Stirling's Formula, Dyadic Decomposition, CLT to Stirling

Lecture 24

CLT for random walk of fair coin tosses, intro to generating fns via sums Poisson rvs

Lecture 25

Generating Functions and Moment Generating Functions

Lecture 26

M & M Game: Memoryless Processes, Geometric Series, Double Recurrences, Hypergeometric

Lecture 27

Generating Functions III: Properties of MGF, Poisson and Normal Example, Poisson to CLT

Lecture 28

Method of Least Squares

Lecture 29

Proof of the CLT

Lecture 30

German Tank Problem

Lecture 31

Coding Examples