Deep Learning	**Fall 2019**

Lecture 2: Recurrent neural network

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Definition 2.1 (State space model). *Given observation sequence $x^1, ..., x^s$. Identify hidden activities h with the state of a dynamical system. Discrete time evolution of hidden state space sequence*

$$h^t = F\left(h^{t-1}, x^t, \theta\right), \quad h^0 = 0, \quad t = 1, ..., s \tag{2.1}$$

1. *Markov property: hidden state at time t depends on input of time t as well as previous hidden state*
2. *Time-invariance: state evolution function F is independent of time t*

How should F be chosen?

Definition 2.2 (Recurrent Neural Network). *Linear dynamical system w/ elementwise non-linearity*

$$\begin{gathered}\overline{F}(h, x, \theta) = Wh + Ux + b, \quad \theta = (U, W, b, ...) \\ F = \sigma \circ \overline{F}, \quad \sigma \in \{\log istic, \ \tanh, \ \operatorname{Re} LU, ...\}\end{gathered} \tag{2.2}$$

Optionally produce outputs via

$$y = H(h, \theta), \quad H(h, \theta) \triangleq \sigma(Vh + c), \quad \theta = (..., V, c) \tag{2.3}$$

Unfolding of recurrency. Recurrent networks: feeding back activities (with time delays). Unfold computational graph over time (also called unrolling)

Lossy memorization: what does a recurrent network (RNN) do?

1. hidden state can be thought of as a noisy memory or a noisy data summary.
2. learn to memorize relevant aspects of partial observation sequence:

$$\left(x^1, \cdots, x^{t-1}\right) \mapsto h^t \tag{2.4}$$

3. more powerful than just memorizing fixed-length context.

Feedforward vs. Recurrent networks:

1. for any fixed length s, the unrolled recurrent network corresponds to a feedforward network with s hidden layers
2. however, inputs are processed in sequence and (optionally) outputs are produced in sequence
3. main difference: sharing of parameters between layers – same function F and H at all layers / time steps.

Backpropagation through time:

1. backpropagation is straightforward: propagete derivatives backwards through time
2. parameter sharing leads to sum over t, when dealing with derivatives of weights
3. define shortcut $\dot{\sigma}_i^t \triangleq \sigma'\left(\bar{F}_i\left(h^{t-1}, x^t\right)\right)$, then

$$\begin{aligned}\frac{\partial \mathcal{R}}{\partial w_{ij}} &= \sum_{t=1}^{s} \frac{\partial \mathcal{R}}{\partial h_i^t} \cdot \frac{\partial h_i^t}{\partial w_{ij}} = \sum_{t=1}^{s} \frac{\partial \mathcal{R}}{\partial h_i^t} \cdot \dot{\sigma}_i^t \cdot h_j^{t-1} \\ \frac{\partial \mathcal{R}}{\partial u_{ik}} &= \sum_{t=1}^{s} \frac{\partial \mathcal{R}}{\partial h_i^t} \cdot \frac{\partial h_i^t}{\partial u_{ij}} = \sum_{t=1}^{s} \frac{\partial \mathcal{R}}{\partial h_i^t} \cdot \dot{\sigma}_i^t \cdot x_k^t\end{aligned} \tag{2.5}$$

RNN gradients: RNN where output is produced in last step: $y = y^s$. Remember backpropagation in MLPs:

$$\nabla_x \mathcal{R} = J_{F^1} \cdots J_{F^L} \nabla_y \mathcal{R} \tag{2.6}$$

Shared weights: $F^t = F$, yet evaluated at different points

$$\nabla_{x^t} \mathcal{R} = \left[\prod_{r=t+1}^{s} W^T S\left(h^r\right)\right] \cdot \underbrace{J_H \cdot \nabla_y \mathcal{R}}_{\triangleq z} \tag{2.7}$$

where $S\left(h^r\right) = diag\left(\dot{\sigma}_1^r, ..., \dot{\sigma}_n^r\right)$, which is $\leq I$ for $\sigma \in \{logistic,\ tanh,\ ReLU\}$.

Exploding and/or vanishing gradients: spectral norm of matrix which is the largest singular value

$$\|A\|_2 = \max_{x:\|x\|=1} \|Ax\| = \sigma_{\max}(A) \tag{2.8}$$

Note that $\|AB\|_2 \leqslant \|A\|_2 \cdot \|B\|_2$, hence with $S(\cdot) \leqslant I$

$$\left\|\prod_{s=t+1}^{s} W^T S\left(h^t\right)\right\|_2 \leqslant \left\|\prod_{s=t+1}^{s} W^T\right\|_2 \leqslant \|W\|_2^{s-t} = \left[\sigma_{\max}(W)\right]^{s-t} \tag{2.9}$$

If $\sigma_{\max}(W) < 1$, gradients are vanishing, i.e.

$$\|\nabla_{x^t} R\| \leqslant \sigma_{\max}(W)^{s-t} \cdot \|z\| \overset{(s-t)\to\infty}{\to} 0 \tag{2.10}$$

Conversely, if $\sigma_{\max}(W) > 1$ gradients may explode. (depends on gradient direction [1]).

Bi-directional recurrent networks: hidden state evolution does not always have to follow direction of time (or causal direction).

Define reverse order sequence

$$g^t = G\left(x^t, g^{t+1}, \theta\right) \tag{2.11}$$

as model w/ separate parameters.

Now we can interweave hidden state sequences. Backpropagation is also bi-directional.

Deep recurrent networks: hierchical hidden state:

$$\begin{aligned} h^{t,1} &= F^1\left(h^{t-1,1}, x^t, \theta\right) \\ h^{t,l} &= F^l\left(h^{t-1,l}, x^t, \theta\right) \quad l = 1, ..., L\end{aligned} \tag{2.12}$$

Output connected to last hidden layer

$$y^t = H\left(h^{t,L}, \theta\right) \tag{2.13}$$

Can be combined with bi-directionality (how?).

Differentiable memory: long-term dependencies

1. sometimes: important to model long-term dependencies $\Rightarrow$ network needs to memorize features from the distant past
2. recurrent networks: hidden state needs to preserve memory
3. conflicts with short-term fluctuations and vanishing gradients
4. conclusion: difficult to learn long-term dependencies with standard recurrent network
5. popular remedy: gated units

LSTM: overrall architecture, Long-Short-Term-Memory: unit for memory management

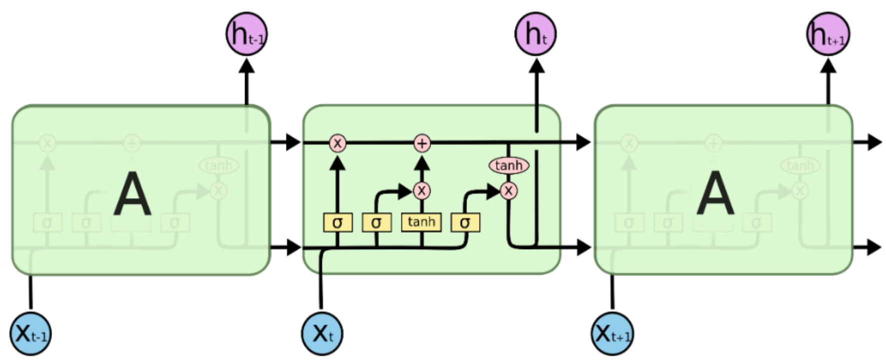


Figure 2.1: The repeating module in an LSTM contains four interacting layers

where

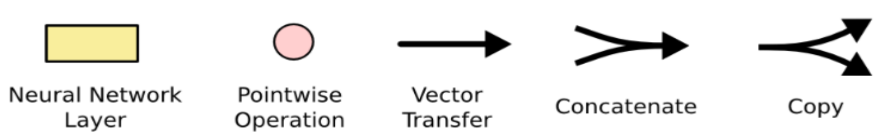


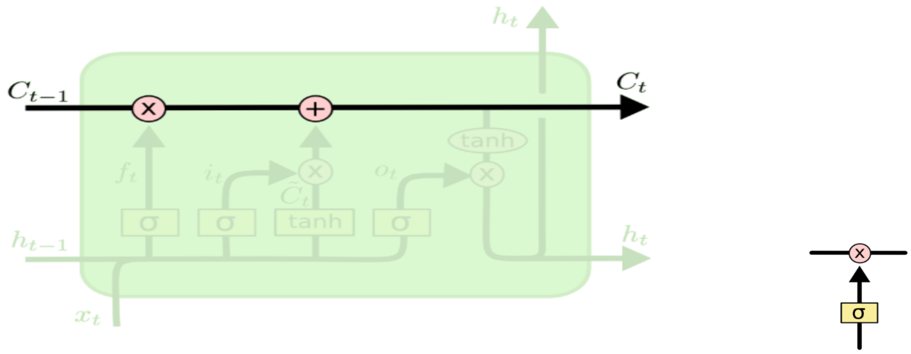
Figure 2.2: from http://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTM: flow of information

1. information propagates along the chain like on a conveyor belt
2. information can flow unchanged and is only selectively changed (vector addition) by σ-gates

where

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right) \tag{2.14}$$



(a) (b)

Figure 2.3

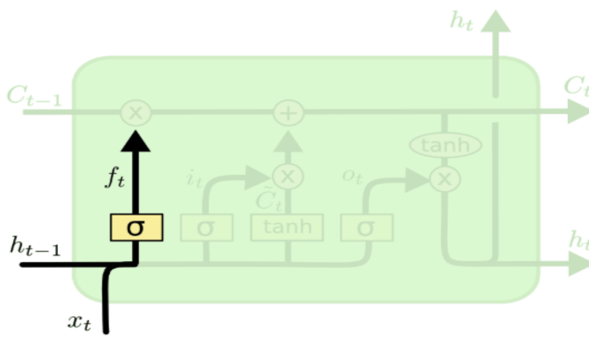


Figure 2.4: LSTM: forget gate

1. keeping or forgetting of stored content?

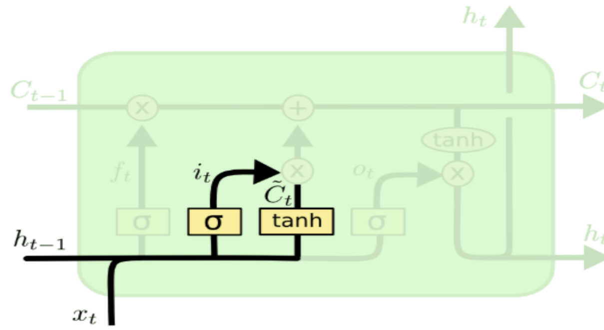


Figure 2.5: LSTM: input → memory value

where

$$
\begin{aligned}
i_t &= \sigma\left(W_i \cdot [h_{t-1}, x_t] + b_i\right) \\
\widetilde{C}_t &= \tanh\left(W_C \cdot [h_{t-1}, x_t] + b_C\right)
\end{aligned}
\tag{2.15}
$$

1. preparing new input information to be added to the memory

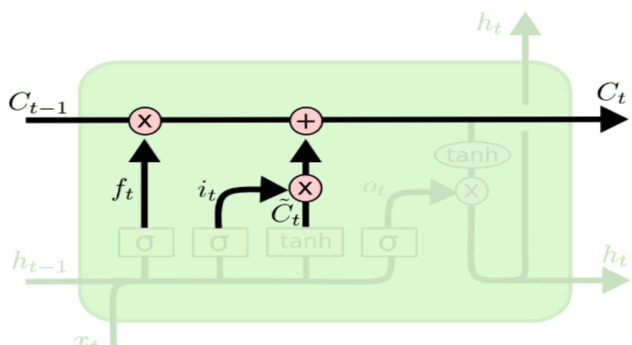


Figure 2.6: LSTM: updating memory

where

$$
C_t = f_t * C_{t-1} + i_t * \widetilde{C}_t \tag{2.16}
$$

1. combining stored and new information

where

$$
\begin{aligned}
o_t &= \sigma\left(W_o\left[h_{t-1}, x_t\right] + b_o\right) \\
h_t &= o_t * \tanh\left(C_t\right)
\end{aligned}
\tag{2.17}
$$

1. computing output selectively

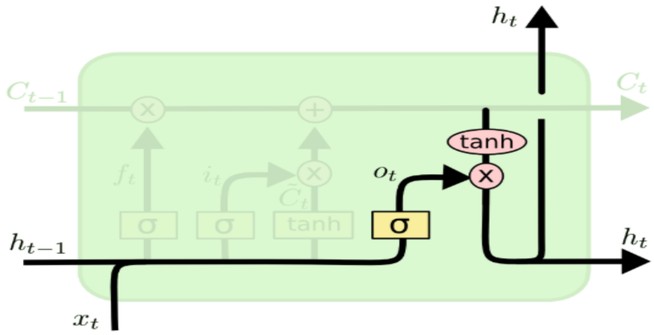


Figure 2.7: LSTM: output gate

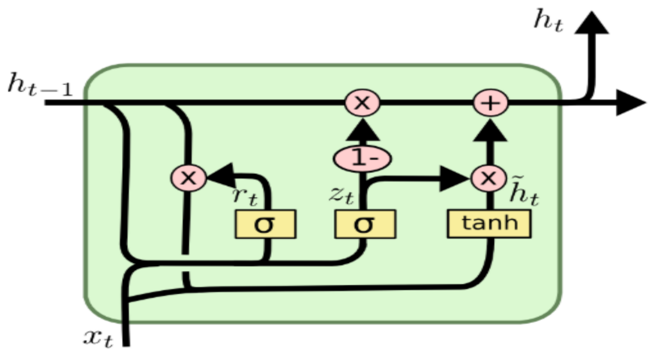


Figure 2.8: LSTM: gate memory units

where

$$\begin{aligned} z_t &= \sigma\left(W_z \cdot [h_{t-1}, x_t]\right) \\ r_t &= \sigma\left(W_r \cdot [h_{t-1}, x_t]\right) \\ \widetilde{h}_t &= \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right) \\ h_t &= (1 - z_t) * h_{t-1} + z_t * \widetilde{h}_t \end{aligned} \tag{2.18}$$

1. memory state = output. modification to logic [2]
2. convex combination of old and new information

Gated memory units:

1. GRUs and LSTMs can learn active memory strategies: what to memorize, overwrite and recall when
2. successful use cases:
   (a) handwriting recognition
   (b) speech recognition (also: Google)
   (c) machine translation
   (d) image captioning

3. notoriously difficult to understand what units learn... Resource-hungry. Slow in learning.

Language modeling:

Model	Test Perplexity	Number of Params [billions]
Sigmoid-RNN-2048 (Ji et al., 2015a)	68.3	4.1
Interpolated KN 5-gram, 1.1B n-grams (Chelba et al., 2013)	67.6	1.76
Sparse Non-Negative Matrix LM (Shazeer et al., 2015)	52.9	33
RNN-1024 + MaxEnt 9-gram features (Chelba et al., 2013)	51.3	20
LSTM-512-512	54.1	0.82
LSTM-1024-512	48.2	0.82
LSTM-2048-512	43.7	0.83
LSTM-8192-2048 (No Dropout)	37.9	3.3
LSTM-8192-2048 (50% Dropout)	32.2	3.3
2-Layer LSTM-8192-1024 (BIG LSTM)	30.6	1.8
BIG LSTM+CNN Inputs	**30.0**	**1.04**
BIG LSTM+CNN Inputs + CNN Softmax	39.8	**0.29**
BIG LSTM+CNN Inputs + CNN Softmax + 128-dim correction	35.8	**0.39**
BIG LSTM+CNN Inputs + Char LSTM predictions	47.9	**0.23**

Figure 2.9: Best results of single models on the 1B word benchmark [3]

1. evaluation on corpus w/ 1B words
2. number of parameters can be in the 100Ms or even Bs!
3. ensembles can reduce perplexity to ~ 23 (best result 06/2016)

Sequence to sequence learning:

1. important use of of memory units: sequence to sequence learning. Seminal paper [4]
2. encoder-decoder architecture

   Encode sequence (e.g. sentence) into vector, decode sequence (e.g. translate) from vector(w/ autoregressive output feedback)

RNN encoder/decoder: How to make this work? [4]

1. deep LSTMs (multiple layers, e.g. 4)
2. different RNNs for encoding and decoding
3. teacher forcing (maximum likelihood) during training
4. beam search for decoding at test time
5. reverse order of source sequence
6. ensemble-ing
7. $\Rightarrow$ state-of-the art results on WMT benchmarks at the time. Today: use of attention-based models.

Reading List

[1] R. Pascanu, T. Mikolov and Y. Bengio, "On the difficulty of training Recurrent Neural Networks," *ArXiv*, 2013.

[2] K.Cho, B. Merrienboer, C. Gulcehre, F. Bougares, H. Schwenk and Y. Bengio, "Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation," *Conference on Empirical Methods in Natural Language Processing (EMNLP 2014)* , 2014.

[3] R. Jozefowicz, O. Vinyals, M. Schuster, N. Shazeer and Y. Wu, "Exploring the Limits of Language Modeling," *CoRR*, 2016.

[4] I. Sutskever, O. Vinyals and Q. Le, "Sequence to sequence learning with neural networks," *NIPS'14 Proceedings of the 27th International Conference on Neural Information Processing Systems*, 2014, Vol. 2014, pp. 3104-3112 .