

Org Mode ConTeXt Export Demo

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1 Problem 1

Problem Evaluate numerically the integral

$$I = \int_0^{\pi/2} \ln(\sin x) dx$$

and compare with the exact value $I = -\frac{\pi}{2} \ln 2$

Solution ...

Note that $\ln(\sin x) = \ln\left(x \frac{\sin x}{x}\right) = \ln x + \ln \frac{\sin x}{x}$

Therefore we just need to find $\int_0^{\pi/2} \ln x dx$ analytically and evaluate $\int_0^{\pi/2} \ln \frac{\sin x}{x} dx$ numerically.

$\int_0^{\pi/2} \ln x dx$ can be integrated by parts.

Let $u = \ln x, v' = dx, u' = \frac{1}{x}, v = x$. Therefore,

$$\begin{aligned} \int \ln x dx &= \int uv' dx = uv - \int vu' dx \\ uv - \int vu' dx &= x \ln x - \int x \frac{1}{x} dx = x \ln x - x \end{aligned}$$

And so, $\int_0^{\pi/2} \ln x dx = x \ln x - x \Big|_0^{\pi/2}$

$$\begin{aligned} \lim_{x \rightarrow 0} x \ln x &= \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x}} \\ &= \lim_{x \rightarrow 0} -\frac{\frac{1}{x}}{\frac{1}{x^2}} \\ &= \lim_{x \rightarrow 0} -x \\ &= 0 \end{aligned}$$

And, $\int_0^{\pi/2} \ln x dx = \frac{\pi i}{2} (\ln x - 1) \approx -0.861$

```
from sympy import log, pi, integrate
from sympy.abc import x
print((pi/2 * (log(pi/2) - 1)).evalf())
print(integrate(log(x), (x, 0, pi/2)).evalf())

-0.861451872082119
-0.861451872082119
```

So we have a value for part of our problem. The other part is to evaluate the integral

$$\int_0^{\pi/2} \ln \frac{\sin x}{x} dx$$

We can do this with Simpson's rule. Recall that

$$S = \frac{H}{9} \sum_{i=0}^{n-1} f(x_0 + iH) + 4f\left(x_0 + \left(i + \frac{1}{2}\right)H\right) + f(x_0 + (i+1)H)$$

```
from math import pi, sin, log
num_points = 10000
xmin = 0
xmax = pi/2
def f(x):
    if x == 0:
        return 0
    return log(sin(x)/x)
H = (xmax - xmin) / num_points
def s(x0, i, H, f):
    return H/6 * (
        f(x0 + i * H)
        + 4 * f(x0 + (i + 1/2) * H)
        + f(x0 + (i + 1) * H)
    )
print(sum(s(xmin, i, H, f) for i in range(num_points)))

-0.22734117306968255
```

1.1 Sample Subheading

- Item

Some prose

- Item 2
Some more prose

Here's some prose in the middle

1. Numbered item
 - A subitem
 - ★ A subsubitem
2. Another numbered item

Here's some bold *here's some italics* ~~Here's some strikethrough~~ here's some
code here's some verbatim here's some underline!? *Here's some underlined italics*

```
local function s(a)
  if control[s] then
    foo(s)
  end
end

beginfig(1)
  draw fullcircle scaled 3cm ;
endfig ;
end ;
```

Here's a quote from Donald Knuth:

The best programs are written so that computing machines can perform them quickly and so that human beings can understand them clearly. A programmer is ideally an essayist who works with traditional aesthetic and literary forms as well as mathematical concepts, to communicate the way that an algorithm works and to convince a reader that the results will be correct.