Goat Discovery Bot

a) Logically, we can model in the following way:
Firstly, there must be a sheep in A or B or C, and it can only be in one place.

$$In\ A \vee In\ B \vee In\ C = True$$
$$In\ A \wedge In\ B = False$$
$$In\ A \wedge In\ C = False$$
$$In\ B \wedge In\ C = False$$

Every time we perform a search, suppose in location A, we get the value of $In\ A$ and add it into our knowledge base.

Probabilistically, we have:

$$P(In\ A), P(In\ B), P(In\ C)$$

They might be same or different regrading of the extra information we have. If there is no extra information about location A, B or C (For example terrain), then initially we have:

$$P(In\ A), P(In\ B), P(In\ C) = \frac{1}{3}$$

If we have searched a location (suppose A) and found nothing, then we update the probability:

$$P(In\ B|\ not\ In\ A) = P(In\ C|\ not\ In\ A) = \frac{1}{2}$$

However, if we have extra information (for example, if the sheep in not in A, it has a lower probability in B), the situation might be different.

b) Logically, if we do not know the exact value of $In\ A$, $In\ B$ and $In\ C$, there is no best choice. Three choice and equally good.

c) Probabilistically, if there is no extra information provided, and $P(In\ A), P(In\ B), P(In\ C) = \frac{1}{3}$, there is no best choice. Also, if $P(In\ B|not\ In\ A) = P(In\ C|not\ In\ A)$, there is no best choice for the second step.

d) Logically, we have:

$$In\ B = False$$

So that:

$$In\ A \vee In\ C = True$$
$$In\ A \wedge In\ C = False$$

e) Probabilistically, we have:

$$P(In\ B) = 0$$

What is interesting is that we can model the behavior of $MoreHelpfulBot$:

$$P(CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ In\ A) = \frac{1}{2}$$
$$P(CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ In\ B) = 0$$
$$P(CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ In\ C) = 1$$

f) Logically, $Re-Select\ A$ and $Re-Select\ C$ are equally good choice, and $Re-Select\ B$ is obviously bad choice, so $Goat\ Discovery\ Bot$ will only choose between the former two.

g) We have:

$$P(PCBMHBot\ say\ not\ in\ B)$$
$$= P(CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ In\ A) \times P(Pick\ A\ and\ In\ A)$$
$$+ P(CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ InB) \times P(Pick\ A\ and\ In\ B)$$
$$+ P(CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ InC) \times P(Pick\ A\ and\ In\ C)$$
$$= \frac{1}{2} \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3} = \frac{1}{2}$$

So:

$$P(Pick\ A\ and\ In\ A|CBMHBot\ say\ not\ in\ B)$$
$$= \frac{P(CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ In\ A) \times P(Pick\ A\ and\ In\ A)}{P((CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ In\ A)} = \frac{1}{3}$$

$$P(Pick\ A\ and\ In\ C|CBMHBot\ say\ not\ in\ B)$$
$$= \frac{P(CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ In\ C) \times P(Pick\ A\ and\ In\ C)}{P((CBMHBot\ say\ not\ in\ B|Pick\ A\ and\ In\ C)} = \frac{2}{3}$$

In this situation, $Re - Select\ C$ is the best choice.

h) Logically, there is no different between stick with location A and change. With no extra information gained, the $Goat\ Discovery\ Bot$ are more likely to stick with location A.

i) Probabilistically,

$$P(Pick\ A\ and\ In\ A|CBMHBot\ say\ not\ in\ B) < P(Pick\ A\ and\ In\ C|CBMHBot\ say\ not\ in\ B)$$

The $Goat\ Discovery\ Bot$ will change and select location C.

j) $ProbabilisticGoatDiscoveryBot$ is much better for two reasons

Firstly, if there are extra information of location A, B and C, $ProbabilisticGoatDiscoveryBot$ can model the information as we discuss in answer(a).

Secondly, if we have a $CBMHBot$, $ProbabilisticGoatDiscoveryBot$ is able to make use of the information provided by it and makes better choice.