Environmental Impacts on the Great Barrier Reef

Math 42 Final

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Data Collection

The Australian Institute of Marine Science (AIMS) provided comprehensive data regarding the great barrier reef.

Their data collection method consisted of towing a diver behind a boat (manta-tow).

We chose to analyze the following variables: coral coverage, Crown of Thorns starfish population, temperature, pH level

Problem Background

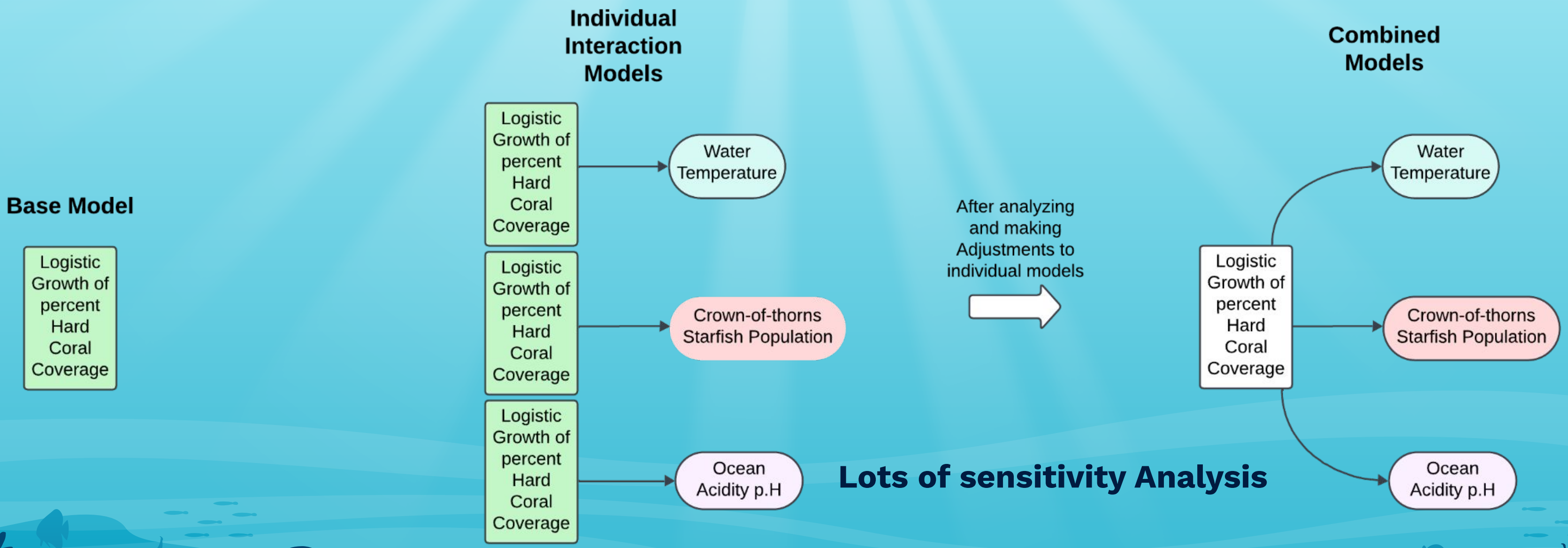
Questions to address:

- How do environmental impacts affect a base coral model individually?
- What is the effect of a combined model on hard coral coverage over time?
- How significant are the environmental impacts on hard coral coverage?
- What are the benefits of identifying negative impacts on hard coral coverage?

Methods Used

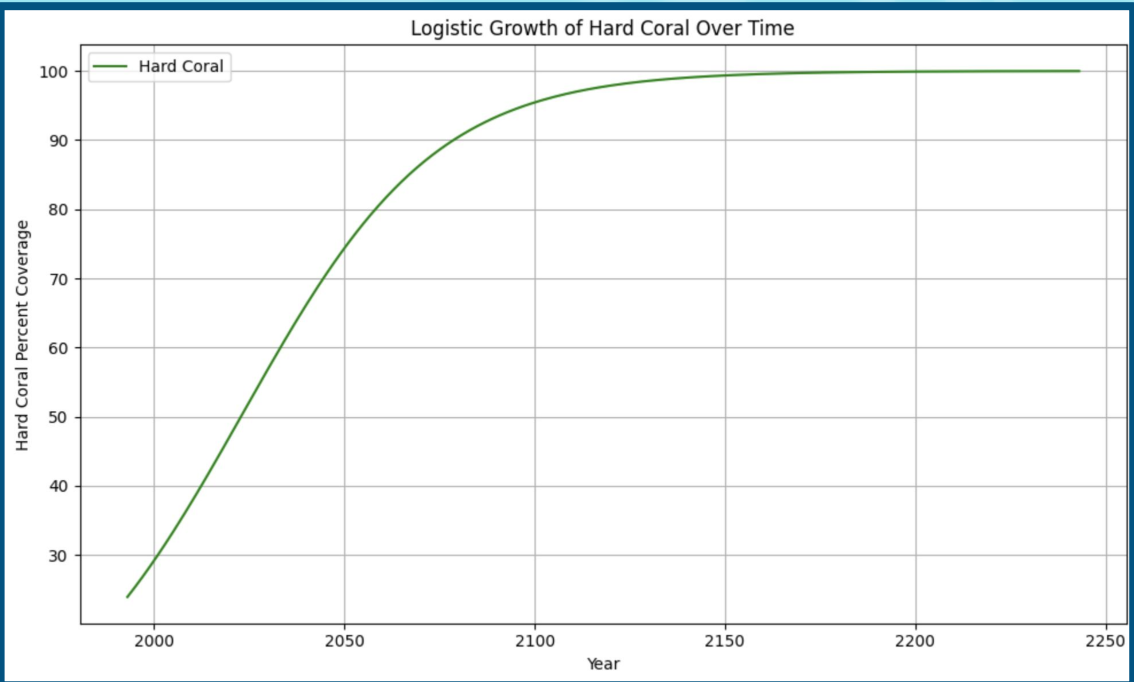
- Deterministic modeling
- Discrete modeling
- Stochastic simulations (Extrinsic Stochasticity)
- Non-constant coefficients (Growth Rate as an exponential function of pH & Temperature)
- Logistic growth
- Logarithmic decline

Methods Used



Lots of sensitivity Analysis

Model 0: Base Logistic Coral Growth



In order to establish a basis for our overall model, we chose to graph the coral population using the logistic growth model.

$$x(n) = x(n - 1) + r * x(n-1) * (1 - x(n-1) / K)$$

Where r is the growth rate and K is the carrying capacity. Since our data had coral coverage as a percent, these variables were adjusted accordingly.

$r = 0.039$, $K = 100$ (100%),
$x(n)$ = coral coverage array

Assumptions: Coral growth is not impacted by anything else, coral has a constant growth rate, can cover 100% of area, & assume logistic growth. Initial year: 1993 and Initial Hard Coral Coverage: 23.92

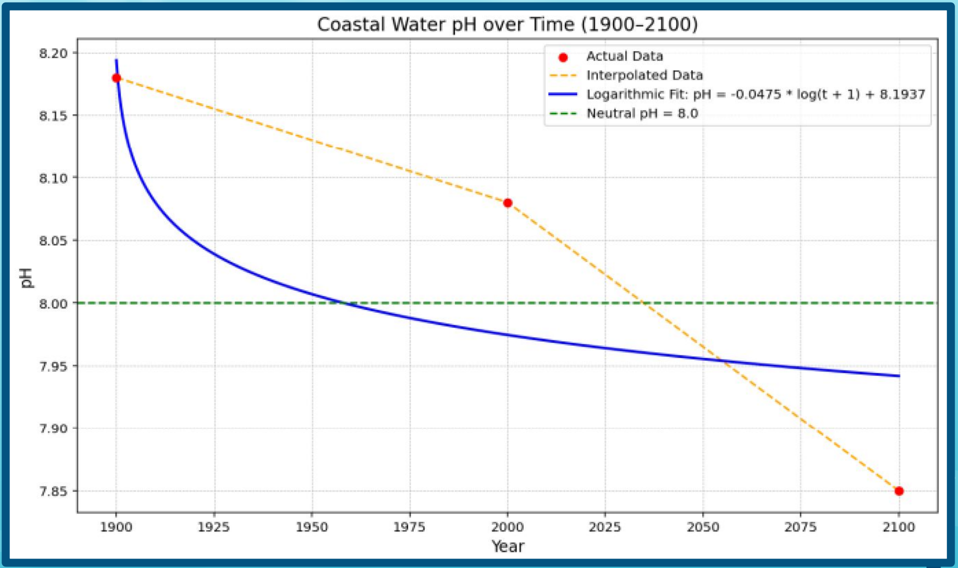
Model 1: Coral Growth with pH Affects

Assumptions:

- pH should be modeled logarithmically.

pH equation:

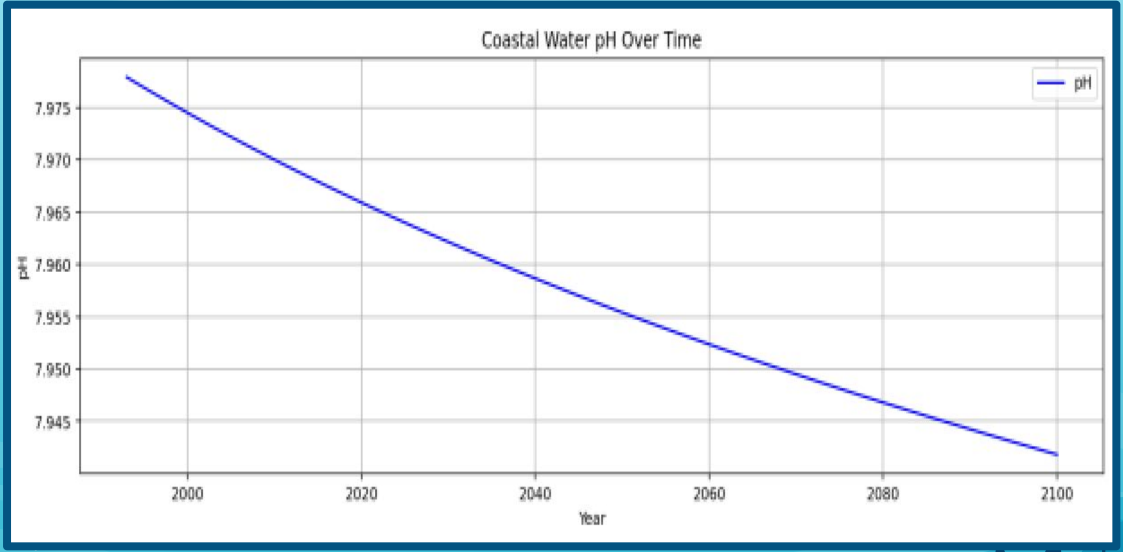
pH(t) = -0.0475*log(t+1) + 8.1937



Model 1: Coral Growth with pH Affects

Assumptions:

- pH should be modeled logarithmically.
- Considering pH from 1993 and onward only

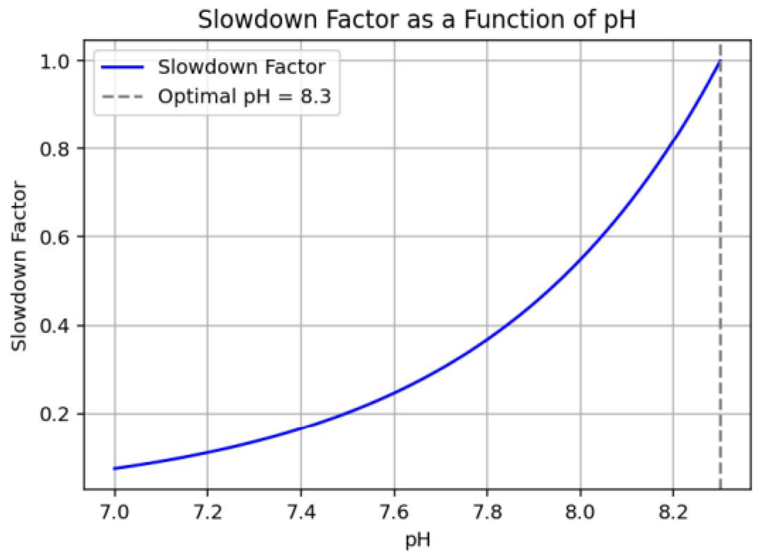


Model 1: Coral Growth with pH Affects

Assumptions:

- 8.3 is the optimal pH for coral growth, anything below affects coral growth. If >= 8.3 growth rate not affected.
- Using exponential decay to calculate slowdown factor due to sensitivity
- Sensitivity Factor: -2

$$e^{\left(-2\cdot\left(8.3-pH(t)\right)\right)}$$



Coral growth slows down more as pH decreases. Acidity makes it harder for them to form their skeletons.

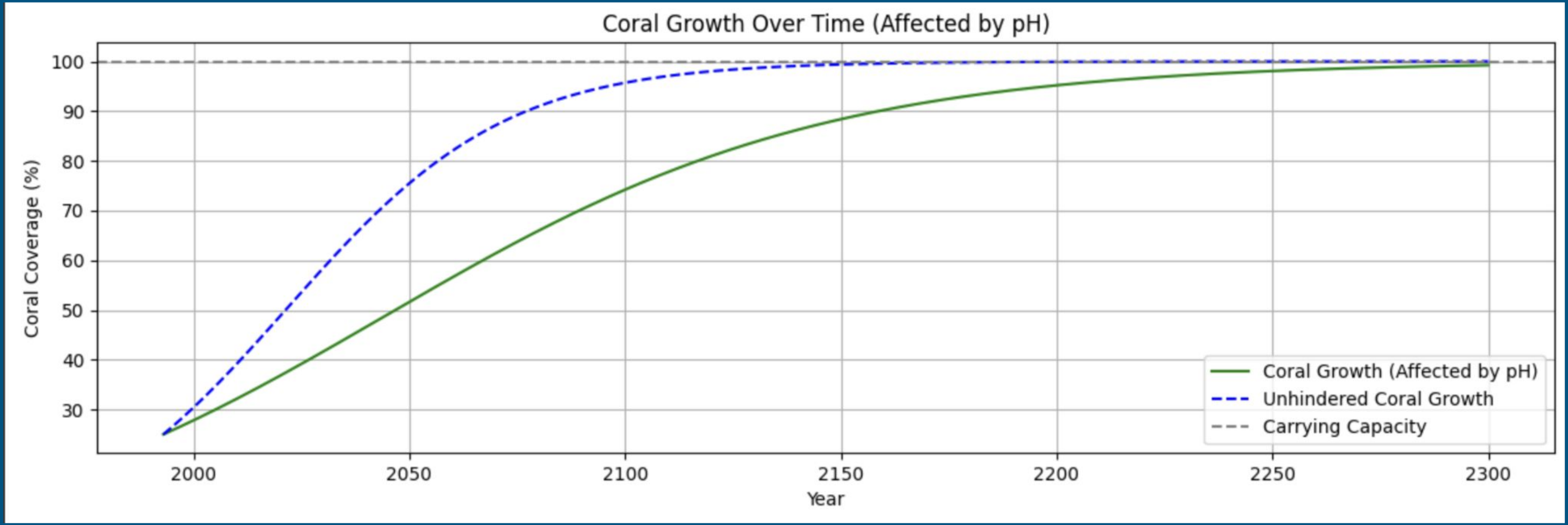
Model 1: Coral Growth with pH Affects

- Apply slowdown factor to unhindered growth rate every year for updated growth rate

$$r = 0.039 \cdot e^{\left(-2 \cdot \left(8.3 - pH(t)\right)\right)}$$

Model 1: Coral Growth with pH Affects

Combined model with base logistic growth



Model 2: Coral Growth with Temperature Affects

Increased temperatures can cause coral bleaching, increasing the risk of coral death (NOAA).

Coral bleaching is present when sustained temperatures of 29 degrees celsius or more occur (Great Barrier Reef Foundation).

Model 2: Coral Growth with Temperature Affects

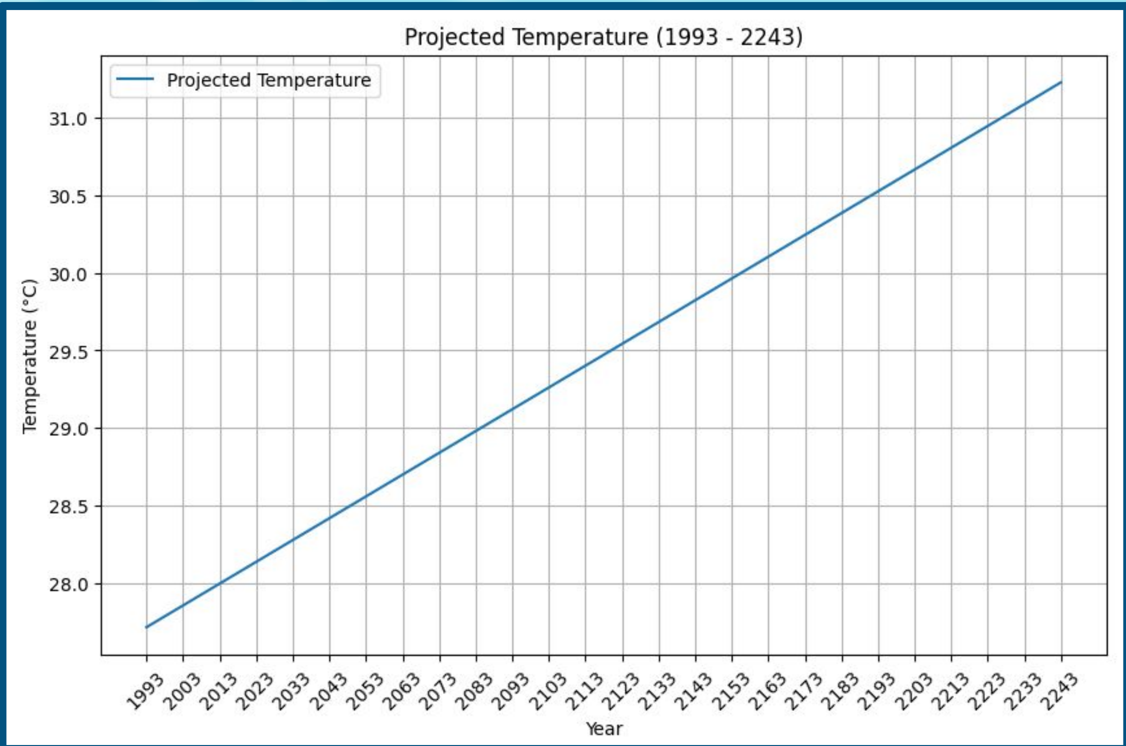
Assumptions:

- Temperature is only relevant during the warmest months
- Temperature increase is a constant 0.014 C per year
- Starting temperature of 27.71 C in 1993

Model:

$$P = 27.71 + dt$$

- P = projected temperature
- d = mean yearly difference
- t = years after 1993



Model 2: Coral Growth with Temperature Affects

Model:

- Uses logistic base model for coral growth
- Uses increasing temperature stochasticity over time
- Exponential coral cover decay when temperatures are above 29 C
- No effect below 29 C
- The growth rate is an exponential function of temperature that becomes negative as temperature grows large

Coral growth when temperature ≤ 29

$$r = .039$$

Coral growth when temperature > 29

$$r = .039\left(2 - e^{1.5\left(temperature - 29\right)}\right)$$

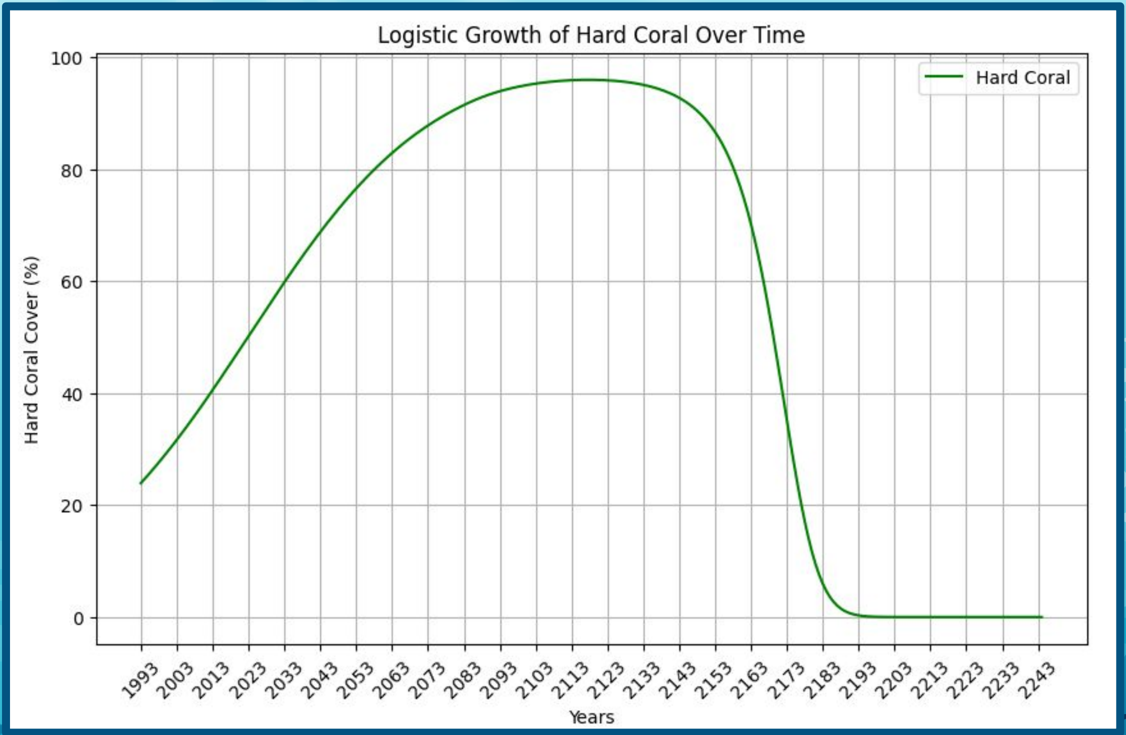
Model 2: Coral Growth with Temperature Affects

Assumptions:

- Temperature is only relevant during the warmest months
- Bleaching events occurs above 29 C
- Bleaching effect on coral cover increases exponentially as temperature increases

Model:

- Base logistic coral model with the growth rate as exponential function of temperature



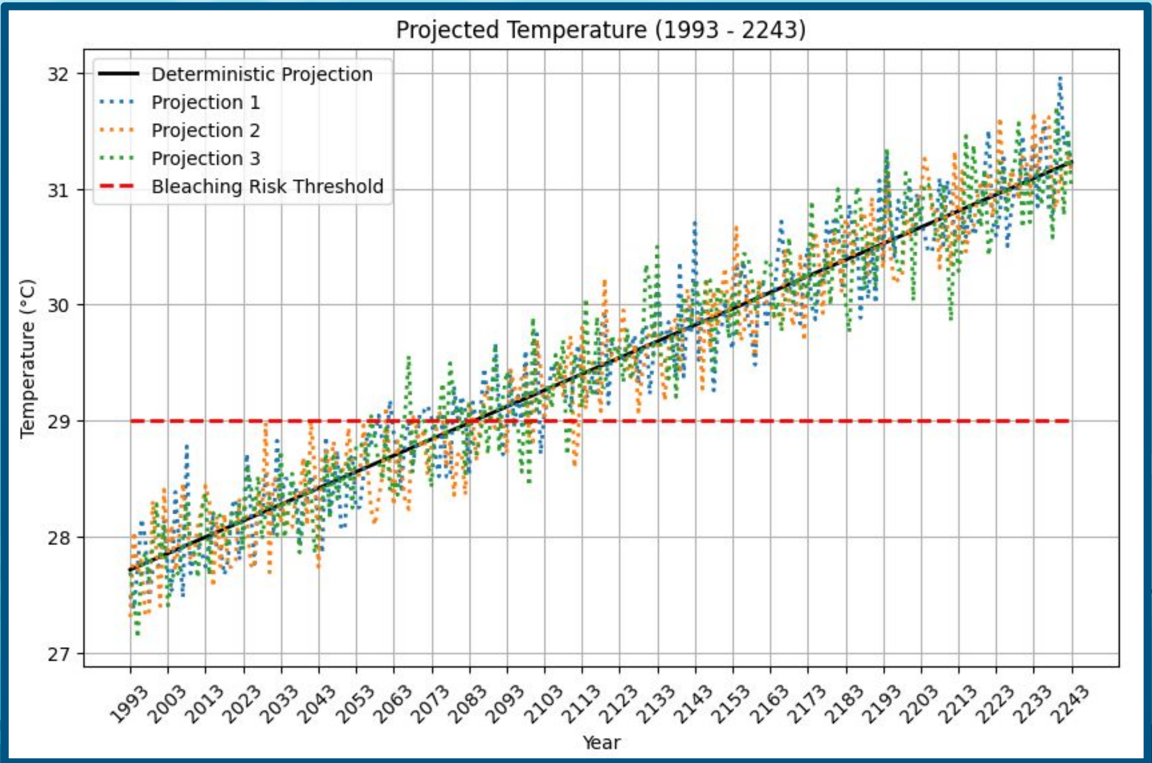
Model 2: Coral Growth with Temperature Affects

Assumptions:

- We assume a standard deviation of 0.31 degrees
- Stochasticity is normally distributed N(0, 0.31)

Model:

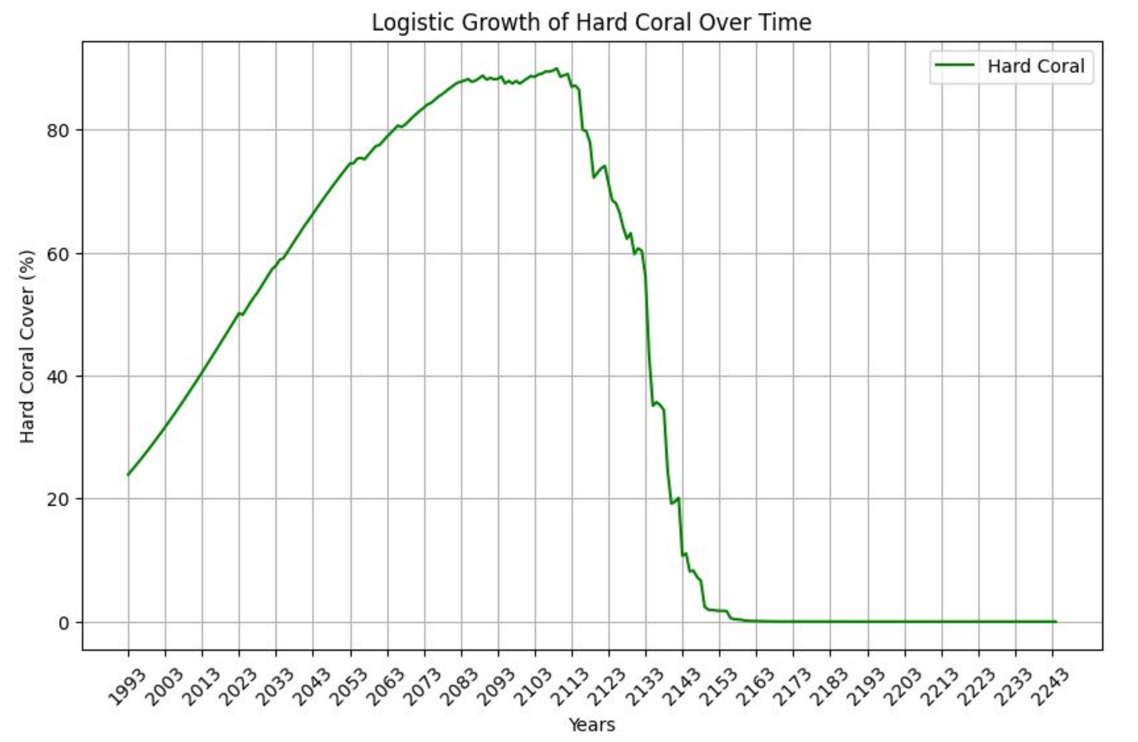
- This model adds stochasticity for random temperature variations
- The stochasticity is normally distributed and based on the standard deviation



Model 2: Coral Growth with Temperature Affects

Model:

- Builds on the logistic coral model with bleaching penalty by making the penalty have stochastic variation



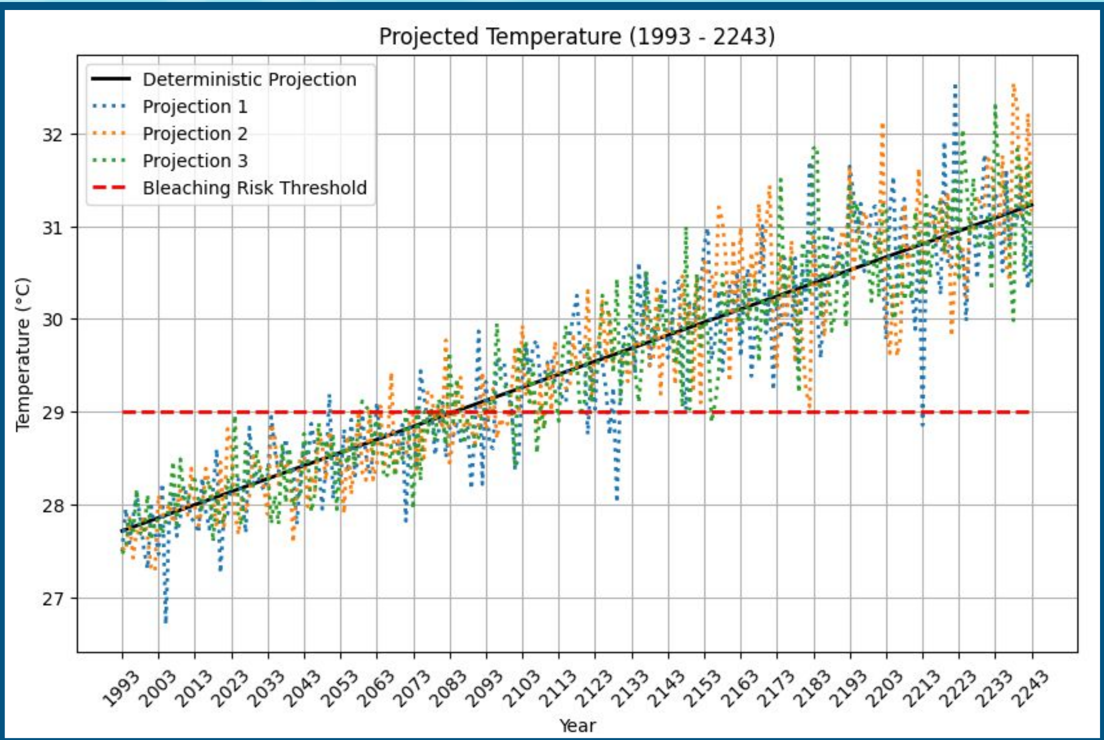
Model 2: Coral Growth with Temperature Affects

Assumptions:

- We assume that standard deviation increases by 0.001 degrees per year
- Stochasticity is normally distributed
  $N(0, 0.31 + 0.001(\Delta year))$

Model:

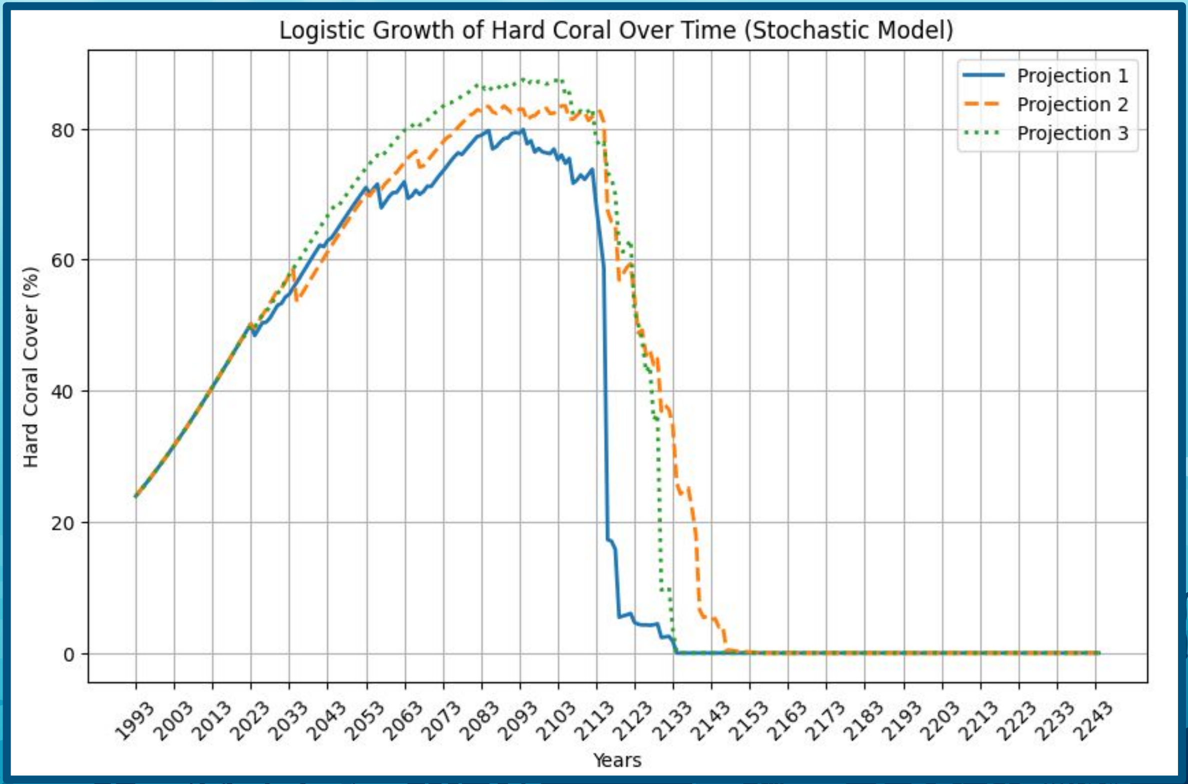
- This updates the temperature stochasticity to have standard deviation that increases over time



Model 2: Coral Growth with Temperature Affects

Model:

- Updates the logistic coral model with bleaching penalty to account for the stochasticity with changing standard deviation



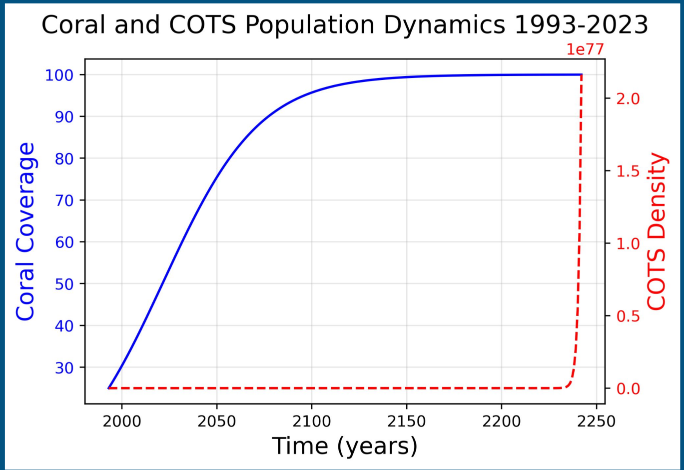
Model 3: Crown of Thorns Starfish (COTS) Growth Predator-Prey Model

Assumptions:

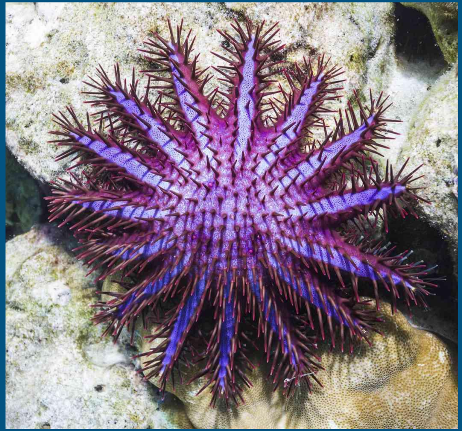
- Each starfish couple produces 2 babies
- Starfish do not die
- Discrete Modeling

Variables:

- Starfish growth: r = 2
- N = years
- f(n) number of starfish at time n



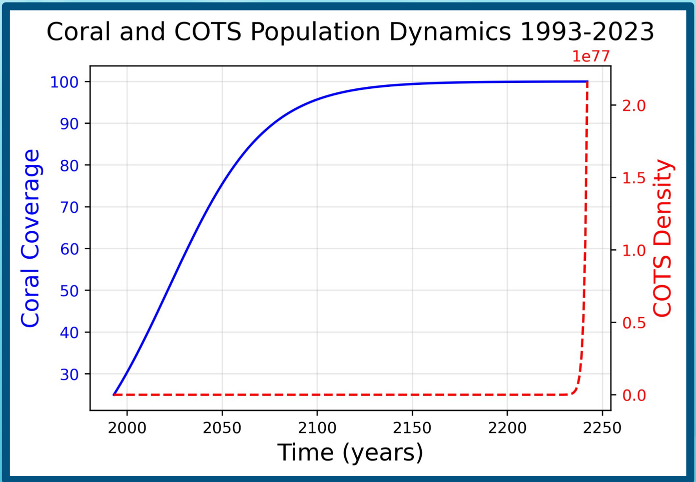
$$f(n) = 2f(n-1)$$



Model 3: Analysis

Analysis

- In this first model, we assumed the starfish grew exponentially with no restriction, and does not die.
- Model does not show any interaction between species
- Starfish population goes to infinite
- Coral Reef approaches 100% sea floor coverage



$$f(n) = 2f(n-1)$$

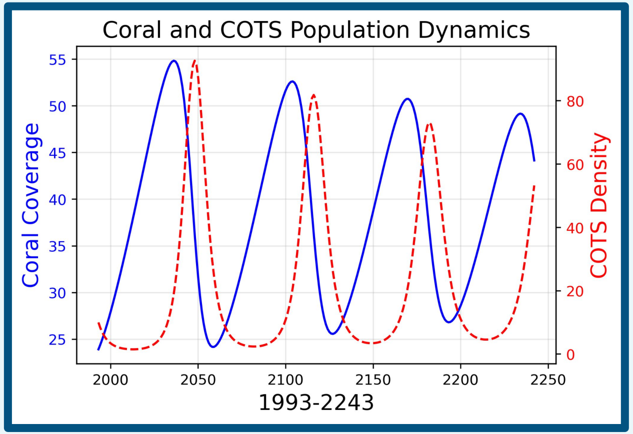
Model 3: Crown of Thorns Starfish (COTS) Growth Predator-Prey Model

Assumptions:

- Starfish grows **proportionally** to Coral Coverage
- Starfish die naturally
- Coral grows logistically
- Coral coverage reduced **proportionally** by starfish density

Variables:

- $f(n)$ models starfish population $f(0) = 10$
- Starfish proportion growth factor: $g = 0.12$
- Starfish death rate: $d_2 = 0.45$
- $c(n)$ models coral population $c(0) = 23.92\%$
- Coral logistic growth rate: $r = 0.039$
- Coral reduced proportal factor: $d_1 = .001$



Model:

$$c(n) = c(n-1)\left(1 + r\left(1 - \frac{c(n-1)}{k}\right) - f(n)d_1\right)$$

$$f(n) = f(n-1)\left(1 + gc(n) - d_2\right)$$

Model 3: Fixed Point Analysis

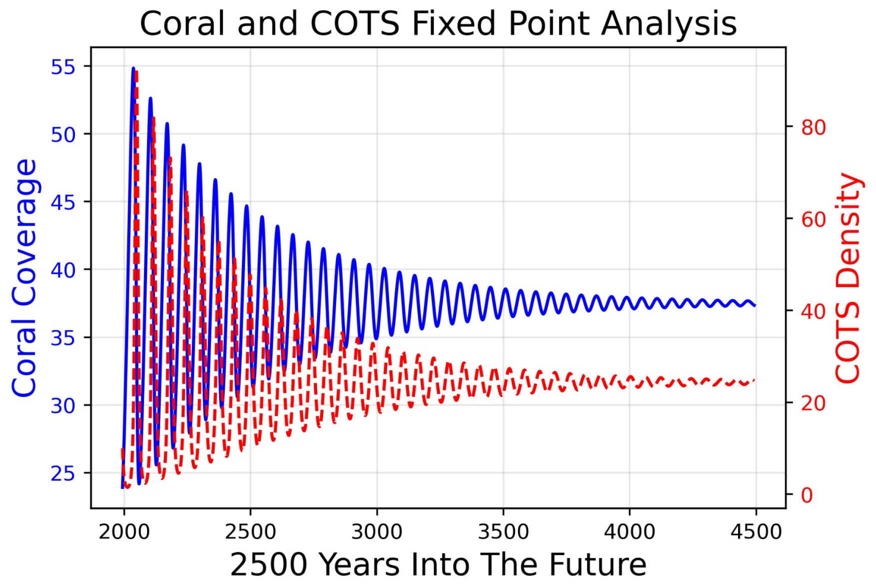
Coral Coverage Fixed Point:

$$c_* = \frac{d_2}{e} \qquad c_* = \frac{starfish\ death}{starfish\ growth} \qquad c_* = \frac{0.45}{0.012} = 37.5$$

Starfish Density Fixed Point:

$$f_* = \frac{r(1 - \frac{d_2}{ek})}{d_1} \qquad f_* = \frac{coral\ growth\ (1 - \frac{starfish\ death}{starfish\ growth * carrying\ capacity})}{coral\ death}$$

$$f_* = \frac{.039(1 - \frac{.45}{.012*100})}{.001} = 24.375$$



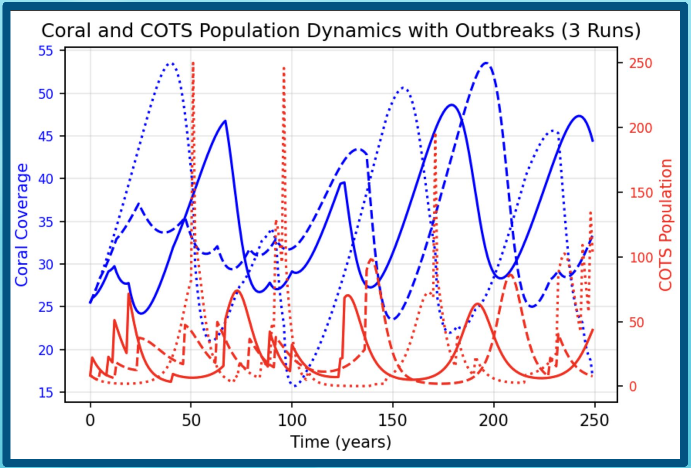
Model 3: Crown of Thorns Starfish (COTS) Stochastic Growth

Assumptions (Includes assumptions from model 3.1)

- Starfish grows **proportionally** to Coral Coverage with stochastic outbreaks
- Divers will kill portion of starfish population

Variables (Includes variables from model 3.1)

- Outbreak threshold (Divers intervene): b = 150
- Outbreak chance: c = .05 yearly
- Outbreak mean: 3
- Outbreak Standard deviation: 0.1
- Divers kill: N(.5, .1)



Combined Model

All variables integrated into one model

- Coral Coverage (%)
- pH
- Temperature
  - Stochastic
- Crown-of-thorns Starfish (COTS)
  - Stochastic

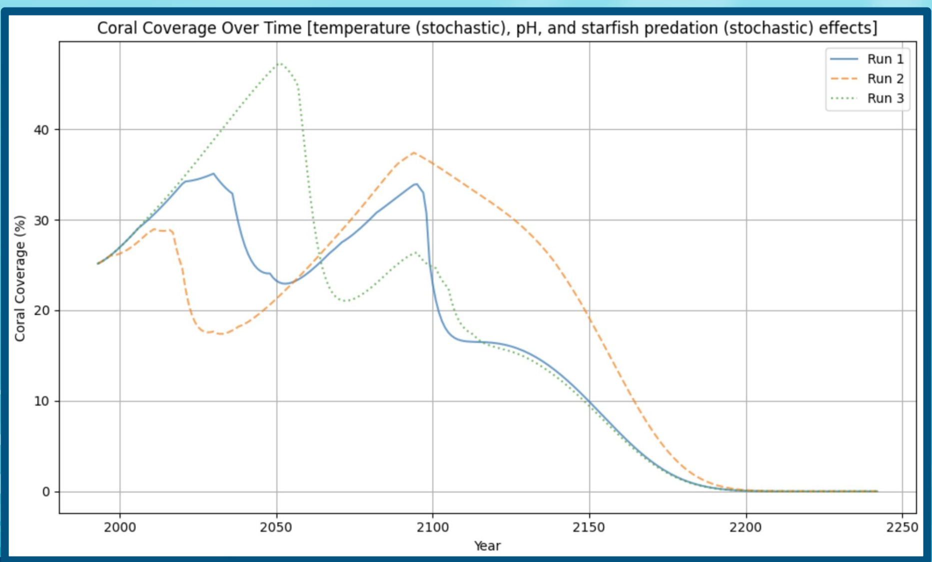
Combined Model

Coral Combined Model

- Coral logistic growth
- Predator Prey Model with Starfish
- Growth rate is an exponential function of pH and Temperature
  - pH slows down growth
  - High Temperatures send population into decline

Assumption: pH and temperature don't directly affect starfish density

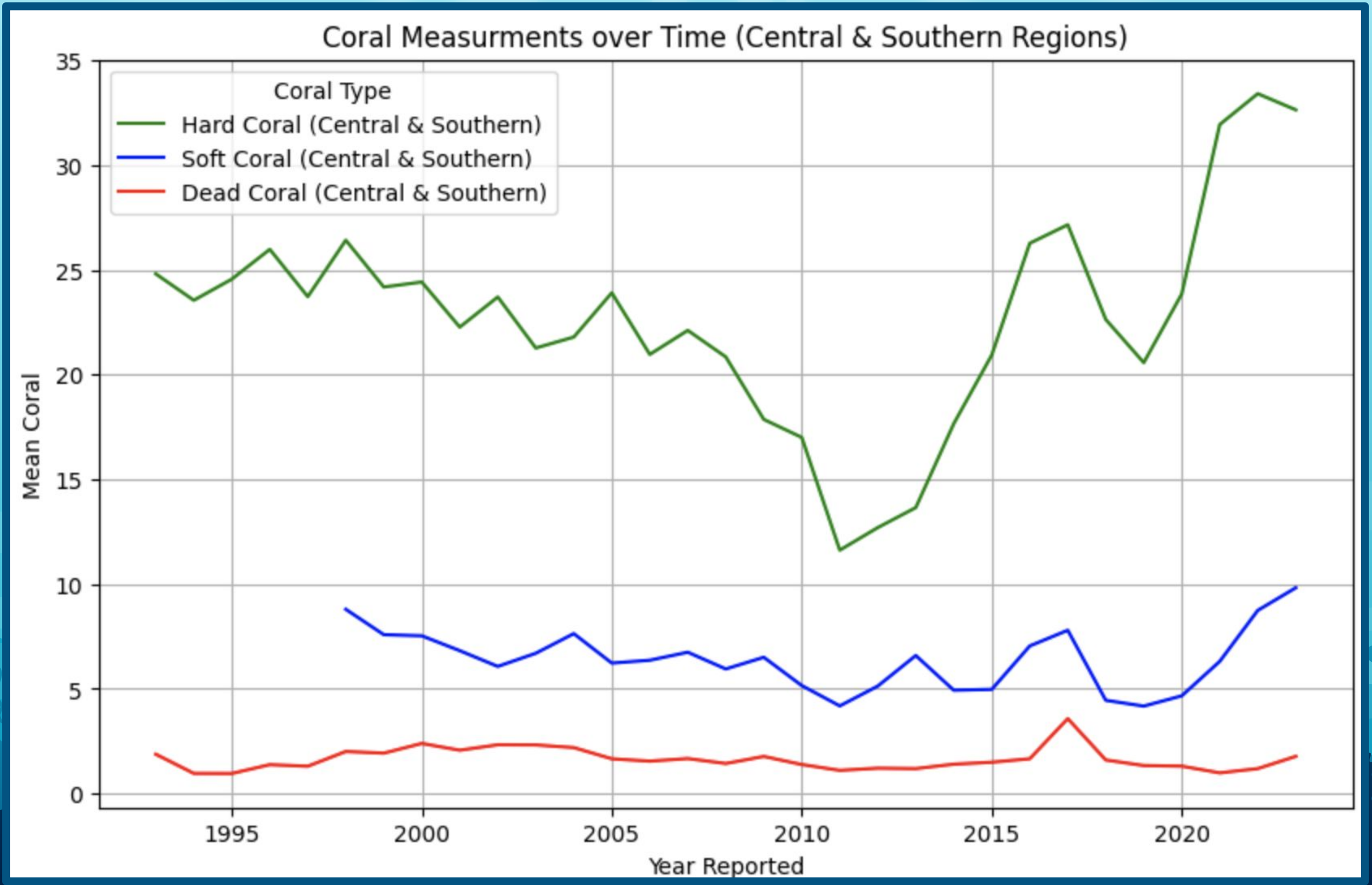
Combined Model



Model

- Model includes all variables of interest
- At lower temperatures, the predator prey relationship is dominantant
- As time and temperature increase, the effect of temperature dominates model and kills the coral

Real-World Model (Raw Data)



Conclusion

We have identified ocean acidification (p.H), temperature increases, and crown-of-thorns starfish as significantly harmful to the great barrier reef.

However, temperature stands out as the most likely to to wipe out the Great Barrier Reef.

Thank you!

Appendix: Combined model

Coral and COTS Population Dynamics with Cases

Coral Dynamics ($x(n)$)

$$x(n) = \begin{cases} x(n-1)\left[1 + 0.039\left(1 - \frac{x(n-1)}{100}\right) - 0.001 \cdot y(n-1)\right], & \text{if pH} \geq 8.3 \text{ and temp} < 29 \\ x(n-1)\left[1 + e^{-2(8.3-\text{pH})} \cdot 0.039\left(1 - \frac{x(n-1)}{100}\right) - 0.001 \cdot y(n-1)\right], & \text{if pH} < 8.3 \text{ and temp} < 29 \\ x(n-1)\left[1 + e^{-2(8.3-\text{pH})} \cdot 0.039\left(1 - \frac{x(n-1)}{100}\right) \cdot \left(2 - e^{1.5(\text{temp}-29)}\right) - 0.001 \cdot y(n-1)\right], & \text{if pH} < 8.3 \text{ and temp} \geq 29 \\ x(n-1)\left[1 + 0.039\left(1 - \frac{x(n-1)}{100}\right) \cdot \left(2 - e^{1.5(\text{temp}-29)}\right) - 0.001 \cdot y(n-1)\right], & \text{if pH} \geq 8.3 \text{ and temp} \geq 29 \end{cases}$$

COTS Dynamics ($y(n)$)

$$y(n) = \begin{cases} y(n-1)\left[1 + 0.012 \cdot x(n-1) - 0.45\right], & \text{if } y(n-1) \leq 125 \\ y(n-1) \cdot \mathcal{N}(0.5, 0.1)\left[1 + 0.012 \cdot x(n-1) - 0.45\right], & \text{if } y(n-1) > 125 \end{cases}$$