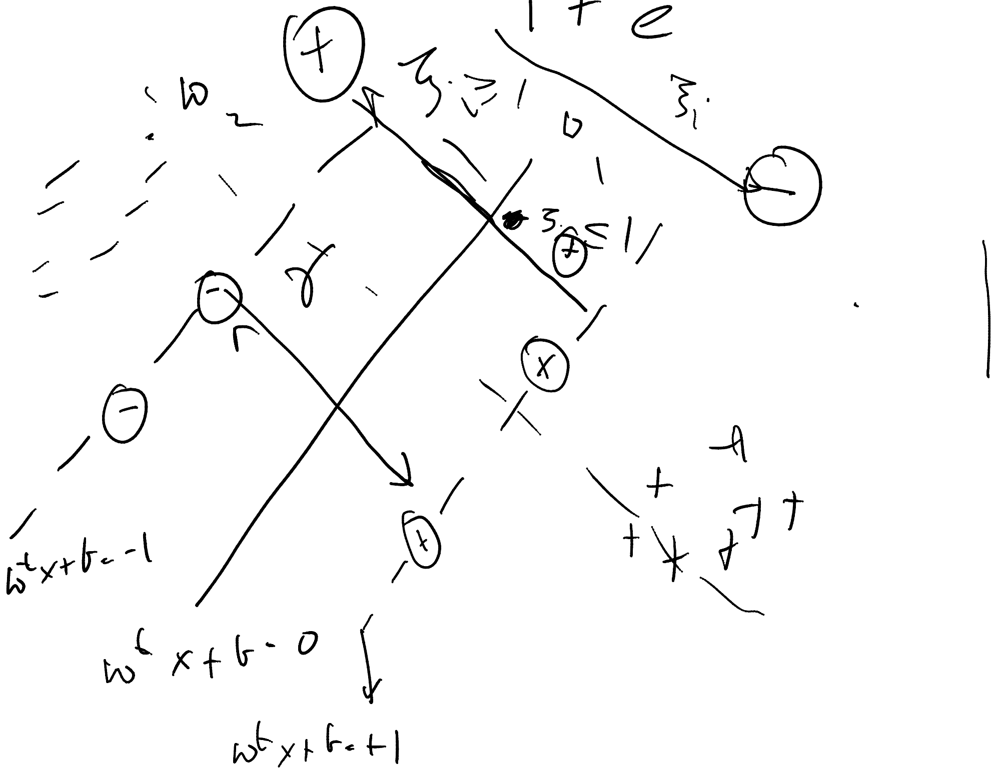
Lec 5: Friday, 16th August - Linear SVMs

16 August 2013
12:37

MAX MARGIN IS GOOD

$$\underset{w}{\operatorname{ARGMAX}} \prod_{i=1}^{n} p(y_i \mid x_i, w)\, p(w)$$

$$p(y_i = +1 \mid x_i, w) = \frac{1}{1 + e^{-w^t x_i}}$$



$$\boxed{\gamma = \frac{2}{\sqrt{w^t w}}}$$

$$\frac{2}{\sqrt{w^t w}}$$

$$\underset{w}{\text{ARGMAX}} \left[\prod_{i=1}^{n} \frac{1}{1+e^{-y_i w^t x_i}}\right] \cancel{*\, e^{\sqrt{w^t w}}}$$

$$p(y_i \mid x_i, w)\; p(w)$$

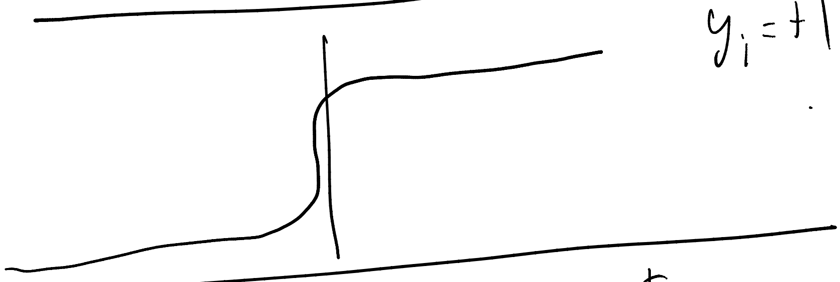
$$p(w) \propto e^{2/\sqrt{w^t w}}$$

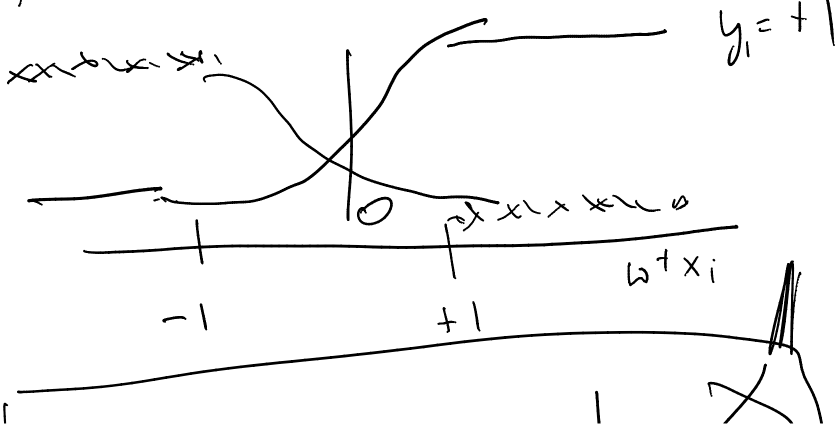
$$\propto e^{-\frac{1}{2}\sqrt{w^t w}}$$

$$\propto e^{-1/2\, w^t w}$$

$$p(w) \propto e$$

$$\boxed{p(y_1 \mid x_1, w)}$$





$$\boxed{p(y_i \mid x_i, w) = \frac{1}{1 + e^{-y_i w^t x_i}}}$$

$$\boxed{p(w) \propto e^{-\frac{1}{2} w^t w}}$$

$$p(y_i = +1 \mid x_i, w) + p(y_i = -1 \mid x_i, w) = 1$$

$$\underset{w}{\text{ARGMAX}} \; p(w) \prod_{i=1}^{p} p(y_i \mid x_i, w)$$

$$\equiv \underset{w}{\text{ARGMAX}} \; \underbrace{\log p(w)}_{\text{REG}} + \underbrace{\sum_{i=1}^{n} \log (p(y_i \mid x_i, w))}_{\text{LOSS}}$$

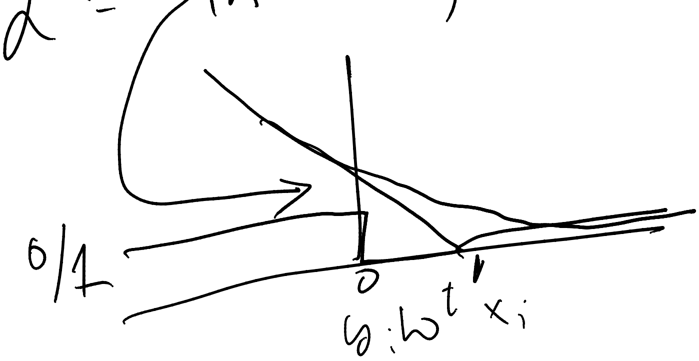
$$\underset{w,v}{\text{ARGMIN}} \; \text{REG}(w, v) \; \lambda \sum_{i=1}^{n} \mathcal{L}(y_i, x_i, \theta)$$

$$p(x_i, y_i)$$

$$\underset{w}{\text{ARGMIN}} \; \frac{1}{2} w^t w + C \sum_{i=1}^{p} \mathcal{L}(y_i, w^t x_i)$$

$$\mathcal{L} = \frac{1}{1+e^{-y_i w^t x_i}}, \frac{1}{1+e^{1-y_i w^t x_i}}$$

$$\mathcal{L} = \text{MAX}\left(0, 1 - y_i w^t x_i\right)$$



$$\text{MAX}(A, B)$$

$$\equiv$$

$$\text{MIN} \ \xi$$

$$\xi \geq A$$

$$\xi \geq B$$

$$\boxed{\begin{aligned} &\min_{w,\xi} \ \frac{1}{2} w^t w + C \sum_{i=1}^{n} \xi_i \\ &\begin{cases} y_i w^t x_i \geq 1 - \xi_i & \forall i \quad 1 \leq i \leq n \\ \xi_i \geq 0 \end{cases} \end{aligned}}$$

$$L_w = \frac{1}{2} w^t w + C \sum_{i=1}^{N} \mathrm{MAX}(0, 1 - y_i w^t x_i)$$

$$\nabla_w L_w = w + C \sum_{i \in S} -y_i x_i$$

$$w_{t+1} = w_t - \eta_t \nabla_w L_w$$