Lec 8: Friday, 18th October - Cutting Plane Optimization for Linear SVMs

18 October 2013
11:01

Original Primal

$$\min_{w} \frac{1}{2} w^t w + C \sum_{i=1}^{n} \max(0, 1 - y_i w^t x_i)$$

$$w = [w_0, b]$$

$$V_i = 1 - y_i w^t x_i$$

$$\min_{w} \frac{1}{2} w^t w + C \sum_{i=1}^{n} \max(0, V_i)$$

$$\sum_{i=1}^{N} \text{MAX}(0, V_i)$$

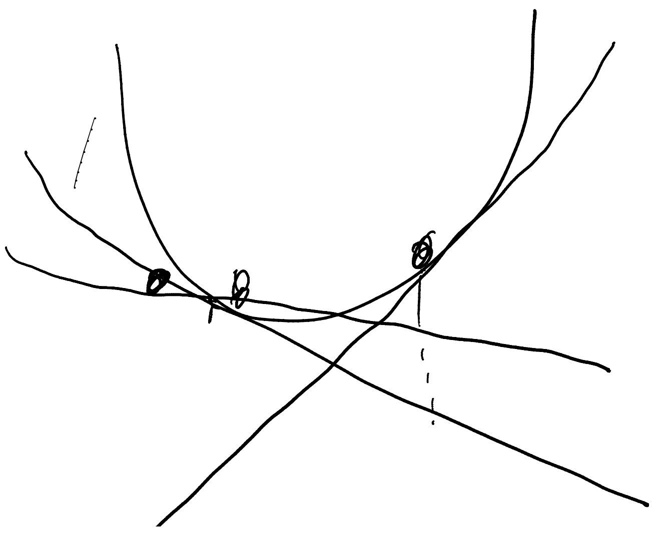
$$= \sum^{N} \text{MAX} \; S_i V_i$$

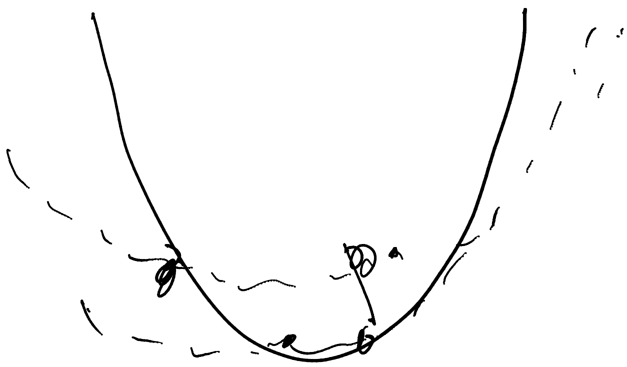
$$i=1 \quad \delta_i \in \{0,1\}$$

$$= \max_{\delta \in \{0,1\}^n} \left[\sum_{i=1}^{n} \delta_i v_i \right]$$

$$\equiv$$

$$\boxed{\begin{aligned} &\min_{w} \ \tfrac{1}{2} w^t w + C\xi \\ &\xi \geq \sum_{i=1}^{n} \delta_i v_i \quad \forall \delta \in \{0,1\}^n \end{aligned}}$$





$$\min_{w,\xi} \frac{1}{2} w^t w + C \xi$$

$$\xi \geq \sum_{i=1}^{n} \delta_i V_i \quad \forall \delta \in 2^n$$

$$\xi \geq \max_{\delta \in 2^n} \sum_{i=1}^{n} \delta_i V_i$$

Cutting plane Algo

1) $K_0 = \phi$

2) till convergence

$w_t, \xi_t \leftarrow$ solve opt (K_t)

$$\min_{w,\xi} \frac{1}{2} w^t w + \xi$$

$$\xi \geq \sum \delta_i V_i$$
$$\delta \in K$$

SMO, liblinear → SVM

3) $\delta_t \longleftarrow \text{max violater}(w_t, \xi_t)$

$$\delta_t = \max_{\delta \in 2^n} \sum_{i=1}^{n} \delta_i V_i$$

$$= \sum_{i=1}^{n} \max_{\delta_i \in [0,1]} \delta_i V_i$$

$$\boxed{O(n)}$$

$\delta_t = [\, v_i > 0 \,]$ $\quad v_i = 1 - y_i w^t x_i$

4) $K_t = K_{t-1} \cup \delta_t$

end

<u>stopping criterion</u> :

ε :

In step 3 : If $\dfrac{\sum \delta_{ti} v_i}{< \sum \delta_{t-1,i} v_i + \varepsilon}$

stop.

If $\delta_t \in K_{t-1}$

t-1 step

w^*_{t-1}, ξ^*_{t-1}

$$w^*, \xi^* = \arg\min \frac{1}{2} w^t w + C\xi$$
$$\xi \geq \sum \delta_i V_i \quad \delta \in 2^n$$

$$K_{t-1} \subseteq 2^n$$

$$w^*_{t-1}, \xi^*_{t-1} = \arg\min \frac{1}{2} w^t w + C\xi$$
$$\xi \geq \sum \delta_i V_i \quad \delta \in K_{t-1}$$

If $\delta_t \in K_{t-1} \Leftrightarrow$

then $\xi \geq \sum \delta_i V_i$, $\delta \in 2^n$

$$\delta_t = \operatorname*{argmax}_{\delta \in 2^n} \sum \delta_i V_i$$

$$\xi \geq \sum_{\delta_t} \delta_i V_i$$

Stop : if $\delta_t \in K_{t-1}$

$$\xi \geq \sum_i \delta_{ti} V_i$$

Stop : $\xi \geq \sum_i \delta_{ti} V_i - \varepsilon$

$$\text{Value}(K_{t-1}) \leq \text{Value}(K_t)$$

$$\text{Value}(K_T) \leq \text{Obj value}$$

$$\leq \text{Value}(K_T) + \varepsilon$$

$\underline{v^*}(w^*, \xi^*) = \arg\min_{w, \xi} \frac{1}{2} w^t w + C\xi$

$\xi \geq \max_{\delta} \sum \delta_i v_i$

$\underline{w, \xi}$: $\xi \geq \max_{\delta} \sum \delta_i v_i$

$\underline{\text{Thm}}$: for given ε,

algo $\max\left(\frac{2}{\varepsilon}, \frac{8CR^2}{\varepsilon^2}\right)$

iterations

$C \leftarrow \frac{1}{2} w^t w + C\xi$

$R = \max_i \|x_i\|$

$\rightarrow \quad w = 0, \xi^i \geq n$

$\Rightarrow Cn$

$Cn - \max\left(\frac{2}{\varepsilon}, \frac{8CR^2}{}\right)$

$$\frac{cn}{\Delta} = \max\left(\frac{2}{\varepsilon}, \frac{O(R^2)}{\varepsilon^2}\right)$$

$$w_t, \xi_t = \arg\min_{w, \xi} \frac{1}{2} w^t w + C\xi$$

$$\xi \geq \sum_{\delta \in K_t} d_i V_i$$

Svm light

$$O\left(\frac{dn\,|K|}{\varepsilon^2}\right)$$