Lec 7: Tuesday, 15 October 2013

15 October 2013
12:19

C-SVM PRIMAL

$\sim \text{MAX}(0, 1 - y_i(w^t x_i + b))$

$$\frac{1}{2} w^t w + C \sum_{i=1} \xi_i$$

$\rightarrow \xi_i \geq 0, \quad y_i(w^t x_i + b) \geq 1 - \xi_i$

$$w^{t+1} = w^t - \eta^t \nabla_w P(x)$$

$$\eta^t = \frac{\eta_0}{\lambda t}$$

$$\nabla_w P = \lambda w - \delta_i y_i x_i$$

$O(D)$ - 1 ITER

$O(nD)$ - 1 PASS

---

1. MONOTONIC DECREASE

2. FIXED STEP

3. NO STOPPING CRITERION

4. RANDOM PT SELECTION

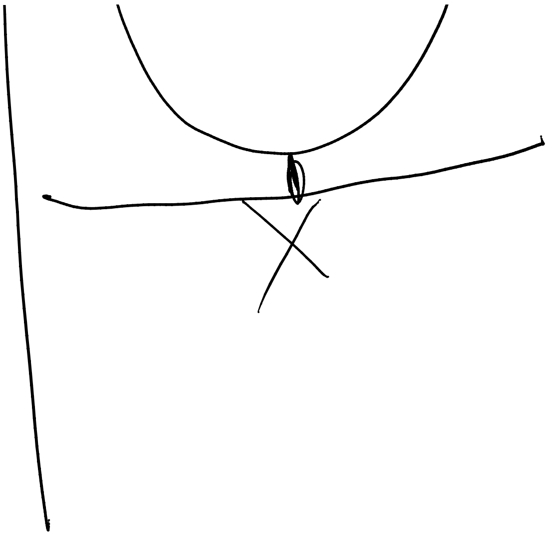
DUAL OPTIMIZATION

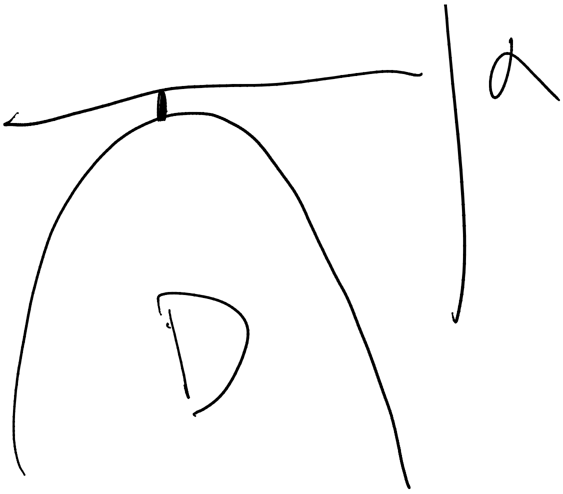
$$\min_{x} \; f_0(x)$$

$$f_i(x) \leq 0 \quad \forall\, 1 \leq i \leq m$$

$$h_i(x) = 0$$

$\|(P)$





$$\mathcal{L}(x, \lambda, \mu) = f_0(x) + \sum_{i=1}^{M} \lambda_i f_i(x) + \sum \mu_i h_i(x)$$

$$D = \underset{\alpha}{\mathrm{MAX}} \ \underset{x}{\mathrm{MIN}} \ \mathcal{L}(x, \lambda, \mu)$$

$$\alpha \geq 0$$

$P \geq D$ (WEAK DUALITY)

$P = D$ (STRONG DUALITY)

CONVEX

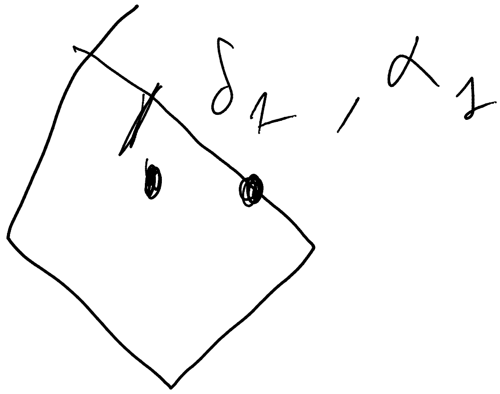
STRONG DUALITY $\Rightarrow x^*, \lambda^*, \mu^*$

① PRIMAL FEAS $f_i(x) \leq 0, h_i(x) = 0$

② DUAL FEAS. $\lambda^* \geq 0$

③ STATIONARITY $\nabla_x \mathcal{L}(x, \lambda, \mu) = 0$
$|_{x^*, \lambda^*, \mu^*}$

④ C. SLACKNESS
$\lambda_i^* f_i(x^*) = 0$



$P = \frac{1}{2} w^t w + C \sum \xi_i$
$[y_i(w^t x_i + b) \geq 1 - \xi_i]$

$$\xi_i \geq 0$$

$$\boxed{\begin{aligned} D = \underset{\alpha}{\mathrm{MAX}}\ & 1^t\alpha - \tfrac{1}{2}\alpha^t Y^t X^t X Y \alpha \\ & \underline{0 \leq \alpha \leq C},\ \underline{1^t Y\alpha = 0} \end{aligned}}$$

α - N - VECTOR

$$\alpha_i^* \left[y_i (w^{*t} x_i + b) - 1 + \xi_i^* \right] = 0$$

$$Y - N \times N \quad \begin{bmatrix} y_1 & & & & \\ & y_2 & & & \\ & & y_3 & & \\ & & & \ddots & \\ & & & & y_N \end{bmatrix}$$

X - $D \times N$ - MATRIX

$$f(x_i) = w^t x_i + b$$

$$w^* = X Y \alpha^* \to w = \sum_i \alpha_i x_i y_i$$

$$\boxed{D = \max_{\alpha} \; 1^t \alpha - \tfrac{1}{2} \alpha^t H \alpha \qquad 0 \le \alpha \le C, \; 1^t Y \alpha = 0}$$

$w \quad \xi \quad b$	N
$D, F N + 1$	$2N+1$
$2N$	

① ✓

$$y_i f(x_i) > 1$$

$$\Rightarrow \left[y_i f(x_i) - 1 + \xi_i \right] > 0$$

$$\Rightarrow \quad \alpha_i = 0$$

② X

$$y_i f(x_i) < 1$$

$$\xi_i = \max(0, 1 - y_i f(x_i)) > 0$$

$$\beta_i^* \xi_i^* = 0, \quad \alpha \geq 0$$

$$\boxed{C = \alpha_i} + \beta_i$$

③ $y_i f(x_i) = 1$

LIBLINEA

$$D = \max \; 1^t \alpha - \tfrac{1}{2} \alpha^t y^t X^t X y \alpha$$

$$0 \leq \alpha \leq C$$

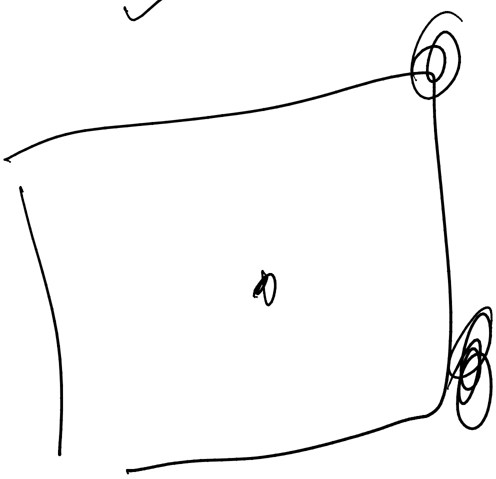
$$G = \underset{n \times 1}{1} - \underbrace{y^t X^t X y}_{H =} \alpha \qquad \alpha = H^{-1} 1$$

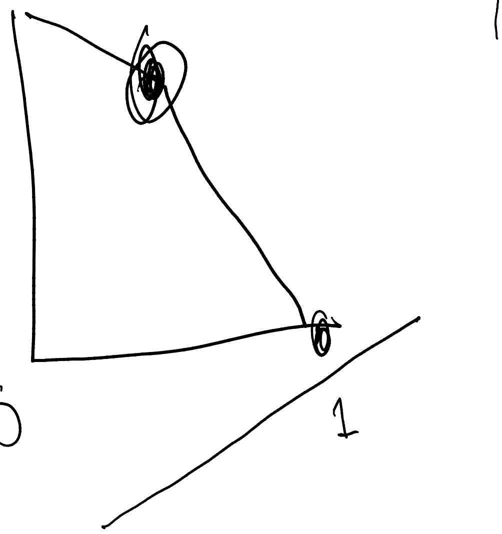
$H\alpha = 1$

X $\quad w^t z = \begin{bmatrix} x \\ 10^6 \end{bmatrix} = w^t x + w_0$

$[w \quad w_0]$

$\frac{1}{2} w^t w \equiv \frac{1}{2} w^t w + \frac{1}{2} w_0^2$





$D(\alpha) = 1^t \alpha - \frac{1}{2} \alpha^t H \alpha$

$\alpha \geq 0$

$\alpha + s e_i$

$$\max_s \; 1^t\alpha + s - \tfrac{1}{2}(\alpha+se_i)^t H (\alpha+se_i)$$

$$0 \leq \alpha_i + s \leq C$$

$$D(\alpha+se_i) = -\frac{1}{2}s^2 H_{ii} - s\,\underline{\alpha^t h_i} + s + D(\alpha)$$

$$O(N^2)$$

$$\underbrace{(1 - \alpha^t h_i)}\, s \qquad (1 - y_i w^t x_i)$$

$$y_i \sum_{j=1}^{n} \alpha_j y_j x_i^t x_j$$

$$w(\alpha) = \sum \alpha_j y_j x_j = XY\alpha$$

$$\boxed{w(\alpha + se_i) = w(\alpha) + s\, x_i y_i}$$

$$\boxed{w(\alpha + s e_i) = w(\alpha) + s\, x_i y_i}$$