Lec 4: Tuesday, 13th August

13 August 2013
13:53

$$P(Y \mid X, \theta) = \frac{P(X, Y \mid \theta)}{P(X \mid \theta)}$$

$$= \frac{\prod_{i=1}^{n} P(x_i, y_i \mid \theta)}{P(X \mid \theta)}$$

$$= \prod_{i=1}^{n} \frac{p(y_i \mid x_i, \theta)\, p(x_i \mid \theta)}{P(X \mid \theta)}$$

$$= \prod_{i=1}^{n} p(y_i \mid x_i, \theta) \left[\frac{\prod_{i=1}^{n} p(x_i \mid \theta)}{P(X \mid \theta)} \right]$$

$$= \prod_{i=1}^{n} p(y_i \mid x_i; \theta) \checkmark$$

$$p(Y \mid X, w) \neq \prod_{i=1}^{n} p(y_i \mid x_i, w)$$

$$L = \frac{P(X, Y \mid w)}{P(X \mid w)} \neq \frac{\prod_{i=1}^{n} P(x_i, y_i \mid w)}{P(X \mid w)}$$

$$\theta_{MAP} = \underset{\theta}{\mathrm{ARGMAX}}$$

$$p(\theta) \prod_{i=1}^{N} p(x_i \mid y_i, \theta)\, p(y_i \mid \theta)$$

$$p(\theta_x)\, p(\theta_y) \prod_{i=1}^{N} p(x_i \mid y_i, \theta_x)\, p(y_i \mid \theta_y)$$

$$\left[\underset{\theta_x}{\mathrm{ARGMAX}}\ p(\theta_x) \prod_{i=1}^{N} p(x_i \mid y_i, \theta_x)\right]$$

$$\times \left[\underset{\theta_y}{\mathrm{ARGMA}}\ p(\theta_y) \prod_{i=1}^{N} p(y_i \mid \theta_y)\right]$$

$$p(y_i = 1 \mid \theta_y) = \hat{\pi}$$

$$p(y_i = -1 \mid \theta) = 1 - \bar{\pi}$$

$$p(\pi) \propto \mathrm{CONST}$$

$$\underset{\bar{\pi}}{\mathrm{ARGMAX}} \prod_{i=1}^{N} \bar{\pi}^{\frac{(1+y_i)}{2}} (1-\bar{\pi})^{\left(\frac{1-y_i}{2}\right)}$$

$$\bar{\pi} = N_+ / N$$

ARGMAX

$\theta \quad x$

$$p(x \mid y = \pm 1, \mu, \sigma)$$

$$\propto e^{-\frac{1}{2\sigma^2} \|x - \mu^{\pm}\|_2^2}$$

$$\underset{\mu^+, \mu^-}{\text{ARGMAX}} \prod_{i=1}^{N} e^{-\frac{1}{2\sigma^2} \|x - \mu^{\pm}\|_2^2}$$

$$(\quad) \times (\quad)$$

$\mu_+ \qquad \mu_-$

$$\prod_{i=1}^{N_+} e^{-\frac{1}{2\sigma^2} \|x_i - \mu^+\|_2^2}$$

$$-\frac{1}{2} \sum_{i=1}^{N_+} \|(x_i - \mu^+\|)^2$$

$$\Rightarrow \sum_{i=1}^{N_+} x_i - \mu^+$$

$$\Rightarrow \boxed{\mu_+ = \frac{1}{N_+} \sum_{i=1}^{N_d} x_i}$$

$\mu_+, \quad \mu_-, \quad \sigma^2, \quad \pi, \quad \Sigma$

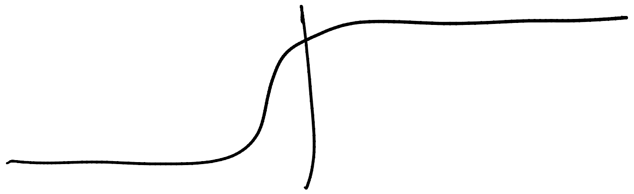
$D, \quad D, \quad 1, \quad 1, \quad D^2$

$$p(x \mid y) = e^{-\|x-y\|^2}$$

$$\propto e^{-\frac{1}{2}(x-\mu)^t \Sigma^{-1} (x-\mu)}$$

μ^+, μ^-, Σ

D^2



w, b

$D, 1$

$$\boxed{\begin{aligned} w &= \Sigma^{-1}(\mu_+ - \mu_-) \\ \hat{w} &= \hat{\Sigma}^{-1}(\hat{\mu}_+ - \hat{\mu}_-) \end{aligned}}$$

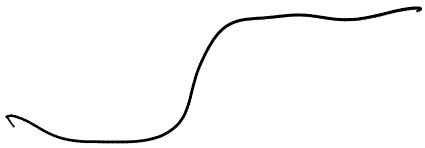
$$\theta^*_{MAP} = \underset{\theta}{\mathrm{ARGMAX}}\; p(\theta) \prod_{i=1}^{n} p(y_i \mid x_i, \theta)\, p(x_i \mid \theta)$$

$$\theta = \{ w, w_\perp \}$$

$$= p(w)\, p(w_t) \prod_{i=1}^{n} p(y_i \mid x_i, w)\, p(x_i \mid w_t)$$

$$\left(p(w) \prod_{i=1}^{n} p(y_i \mid x_i, w) \right) * \left(p(w_t) \prod p(x_i \mid w_t) \right)$$

?



$$p(y_i = 1 \mid x_i, w) = \frac{e^{w^t x + b}}{1 + e^{w^t x + b}}$$

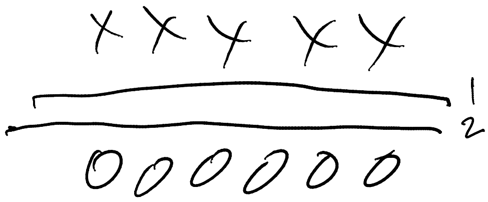
$$p(w) = e^{-\lambda/2\, w^t w}$$

$$\boxed{w_{MAP} = \underset{w}{\text{ARGMAX}}\ \lambda/2\, w^t w + \prod_{i=1}^{n} \log\left(1 + e^{-y_i(w^t x_i + b)}\right)}$$

SVMs

① OUTLIERS

②



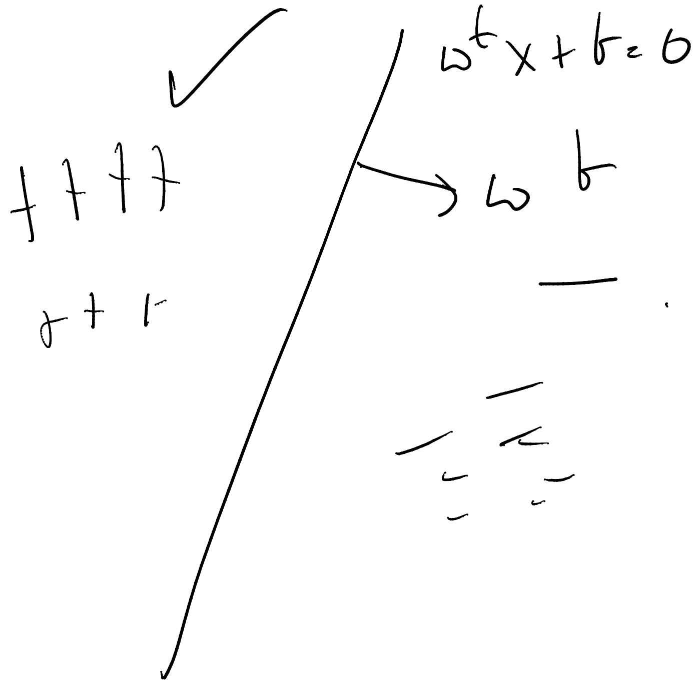
$f(x_i) = w^t x_i + b$

LR

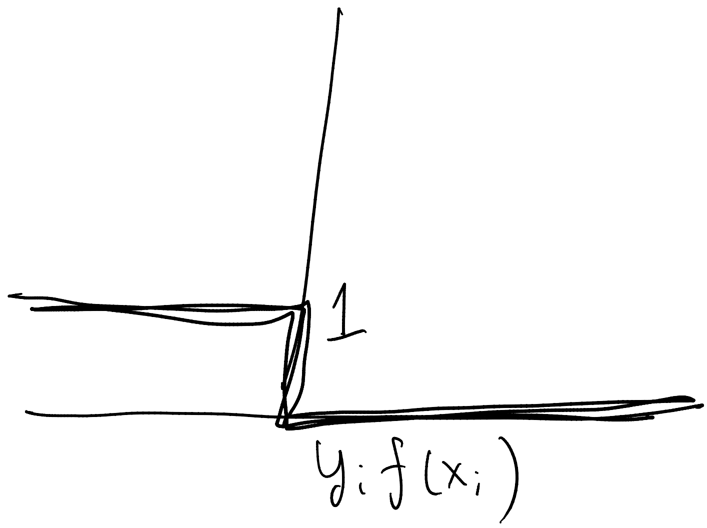
$$\min_{w,b} \ \tfrac{1}{2} w^t w + \boxed{C \sum_{i=1}^{n} \log\left(1 + e^{1-y_i f(x_i)}\right)}$$

SVM

$$\min_{w,b} \ \boxed{\tfrac{1}{2} w^t w + C \sum_{i=1}^{n} \max\left(0, 1-y_i f(x_i)\right)}$$

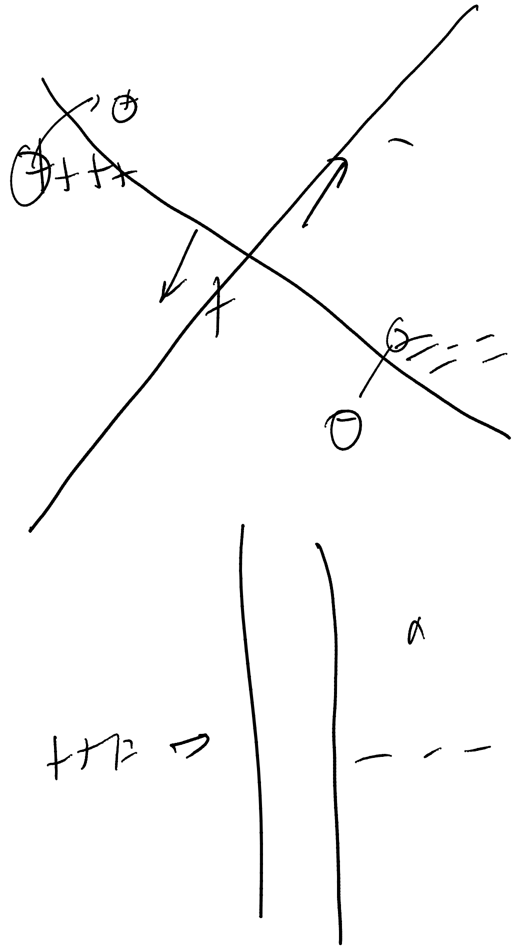


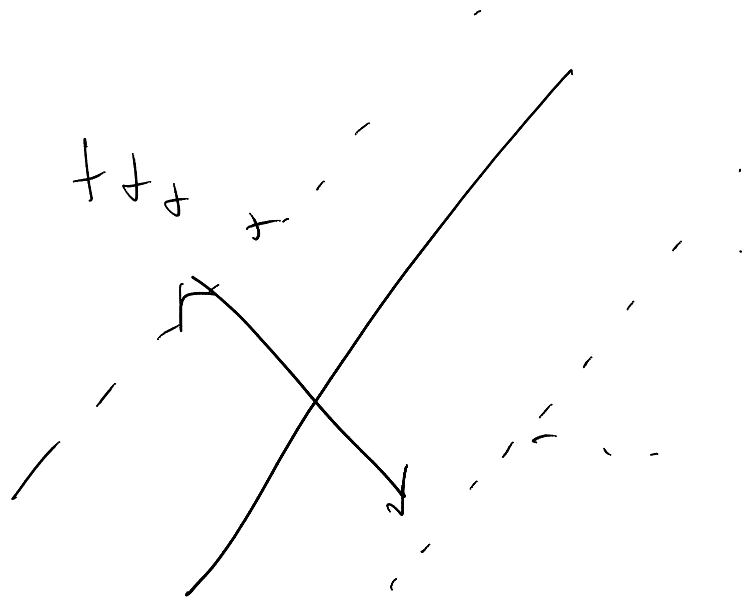
① $\min_{w} \ \# MIS(w)$



$$\mathrm{SIGN}(f(x_i))$$

$$\boxed{\min_{w,r} \sum_{i=1}^{n} \mathcal{L}(y_i, f(x_i))}$$





MAX MARGIN + MTN $\boxed{\mathcal{L}}$