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MUST DO BEFORE STARTING EXAM

(a) WRITE AND MARK YOUR NAME AND ID ON THE SCANTRON (THREE POINTS OFF IF NOT)

(b) WRITE YOUR NAME ON BOTH SIDES OF THE CHEAT SHEET. (THREE POINTS OFF IF NOT)

(c) DO NOT DETACH ANY PAGES FROM THIS EXAM. EXAM MUST STAY STAPLED DURING THE WHOLE EXAM. (3 POINTS OFF IF YOU DO)

(d) Write here your UCLA ID: –
NO ID NO EXAM. ID MUST BE ON YOUR DESK AT ALL TIMES. AT BEGINNING OF EXAM and THROUGHOUT THE EXAM.

(e) Write here your LAST NAME (Please, PRINT): – FIRST NAME: – – – – – – – – – – –

Other important Instructions–Read. Points lost for not following directions.

- Closed books, closed notes. Material covered is up to last day of lecture before the exam.
- Only scientific calculator allowed for computations. You may not use your phone or any other electronic device as calculator. **Graphics calculators are not allowed.** No exceptions.
- Phones and other electronic devices must be disconnected before you enter the classroom and not turn on again until you are out of the room. While in the classroom, they must be in your backpack and your backpack on the floor. Phones in pockets will lead to big loss of points in the exam. It is not worth the risk.
- Answer for multiple choice questions will be marked in scantron AND the exam. Work will not be read in multiple choice;Failure to mark your name, ID or some answers will result in point deduction from the exam grade.
- Left handed students must sit in a seat for left-handed students. The professor will tell students where to sit. Please, let the professor know that you are left handed once seated and she indicate where to move.
- ID must be ready to show BEFORE and at all times during the exam. NO ID, no exam.
- This midterm must show your individual work. Talking to others during the midterm, not adhering to the above, sharing information or breaking any other aspect of the student code of conduct at UCLA will not be tolerated and will be referred to the Dean of Students office. You can not exchange papers or information. All your things must be on the floor. You may not use the empty seats next to you to put things. Put the tables down. Honor code applies.
- Cheat sheet can have only formulas and definitions, no solved problems, no examples of any kind, no proofs, no numerical examples, no intermediate steps and no drawings or graphs of any kind.YOUR NAME MUST BE ON CHEAT SHEET AT ALL TIMES. Be ready to show your cheat sheet when the instructor requests it. The cheat sheet must be written all in English. Cheat sheets that do not comply will result in lower grade in the exam. You may have two sides of an 11 by 8 sheet.

- You may not speak to each other in the exam room. Wait until you are out of the room.

NOTE: This practice exam is only an indication of format of the exam. All your exam will be in multiple choice format. A practice midterm is to test yourself. Be aware that the questions in the midterm will be different from the ones in the practice midterm. So studying for the test by studying only the practice midterm is a bad idea. With this midterm, you can test yourself after studying. Look at the answers only when you have done it by yourself. It will give you an indication of how prepared are you to answer new questions.

MULTIPLE CHOICE QUESTIONS. ONLY ONE ANSWER IS CORRECT. CHOICE MUST BE MARKED ON THE SCANTRON, AND ALSO HERE ON THE EXAM. ONLY THE SCANTRON WILL BE GRADED. NO MARKS ON SCANTRON OR MORE THAN ONE MARK WILL RESULT IN 0 POINTS FOR THE QUESTION NOT MARKED, EVEN IF IT IS MARKED ON THE EXAM. You may use the space near the question for scratch work, but scratch work will not be read.

Question 1. Police report that 78% of drivers stopped on suspicion of drunk driving are given a breath test, 36% a blood test, and 22% both tests. What is the probability that a randomly selected DWI suspect is given a blood test or a breath test, but not both?

(a) 0.7

(b) 0.3

(c) 0.08

(d) 0.22

(e) 0.12

Solution 1. $P(Bb^c \cup B^c b) = (0.78 - 0.22) + (0.36 - 0.22) = 0.56 + 0.14 = 0.7$

Question 2. Let $S = (0, 1]$ and define $A_i = ((1/i), 1], \; i = 1, 2, ...$ The union of these events is

(a) S, the whole sample space.

(b) The interval $[0, 0.5]$.

(c) The set with only 1 in it.

(d) The empty set.

(e) The interval $[0, 10]$.

Solution 2. The union is the sample space, $\bigcup_{i=1}^{\infty} A_i = (0, 1] \quad = S$

Question 3. The Center for Disease Control says that about 30% of high school students smoke tobacco (down from a high of 38% in 1997). Suppose you randomly select high school students to survey them on their attitude towards scenes of smoking in the movies. What is the probability that the first smoker is the 6th person you choose?

(a) 0.000729

(b) 0.0504

(c) 0.000504

(d) 0.5

(e) 0.002

Solution 3. $P(X = 6) = (0.7^5)(0.3) = 0.0504$

Question 4. A series of 3 jobs arrive at a computing centre with 3 processors. Assume that each of the jobs is equally likely to go through any of the processors. Find the probability that all processors are occupied. Circle one, work will not be read.

(a) 5/9

(b) 1/3

(c) 4/7

(d) 1/10

(e) 2/9

Solution 4. 2/9

Why?

For all 3 processors to be busy when 3 jobs arrive, there must be one job in each processor. Thus the probability is $(3!)/3^3 = 2/9$

Question 5. Of 25 laptops available in a business, 10 are used by the accounting personnel (A), 5 are used by the management (M), and 15 are not used by anybody(N). Which of the following statements is true?

(a) The number of laptops used by management or accounting but not used by both is 5

(b) The event $MA^c \cup AM^c$ has 10 laptops in it.

(c) MA^c contains 5 laptops.

(d) The number of laptops used by at least one of the groups is 5.

(e) MA is the empty set.

Solution 5.

Event	Write event symbolically	Number of laptops
Used by management and accounting	MA=M	5
Only used by management	MA^c	Empty set, none
Used by management or accounting but not used by both	$MA^c \cup AM^c = MA^c$	5
Used by at least one of the groups	$M \cup A$	10

Question 6. A certain firm produces resistors and markets them as 10-ohm resistors. However, the actual ohms of resistance produced by the resistors may vary. Research has established that 10 percent of the values are below 8 ohms and 20 percent are above 10 ohms. If three resistors, randomly selected, are used in a system, what is the probability that the three resistors have values larger than 10?

(a) 0.9

(b) 0.008

(c) 0.8

(d) 0.002

(e) 0.6

Solution 6. $0.2^3 = 0.008$

Question 7. Daily sales records for a car dealership show that it will sell 0, 1, 2, or 3 cars, with probabilities as listed

Number of cars (X)	0	1	2	3
Probability (P(X))	0.5	0.3	0.15	0.05

The expected value of X^3 is

(a) 0.75+1

(b) 2.85

(c) 5

(d) 3

(e) 0

Solution 7. 1*0.3+8*0.15+27*0.05 [1] 2.85

Question 8. 7 people (A,B, C,D,E, F, G) are seated in a row at random. The probability that persons F and G sit next to each other is (choose one):

(a) 0.25

(b) 0.75

(c) 0.071

(d) 0.2857

(e) 9

Solution 8. There are $7! = 5040$ seating arrangements. And there are $2!6! = 1440$ seating arrangements where F and G sit together. Thus, probability is $1440/5040 = 0.2857$

Question 9. Consider the following table published by a newspaper in a small town of 1000 adults. The table classifies individuals in that town according to their political affiliation and their sex.

	Republican	Independent	Democrat
Male	20%	30%	50%
Female	40%	19%	41%

The number 20% means

(a) 20% of Males are republican

(b) 20% of Republicans are male

(c) 20% of people in the town are republican and male

(d) 20% of people in the town are republican and female

(e) 20% of Republican are independent.

Solution 9. These are conditional probabilities, the 20% means that 20% of males are republican.

Question 10. Suppose a foreman must select one worker from a pool of four available workers (numbered 1, 2, 3, 4) for a special job. He selects the worker by mixing the four names and randomly selecting one. Let A denote the event that worker 1 or 2 is selected, let B denote the event that worker 1 or 3 is selected, and let C denote the event that worker 1 is selected. One of the following is true; which one?

(a) A and B are not independent

(b) A and C are independent

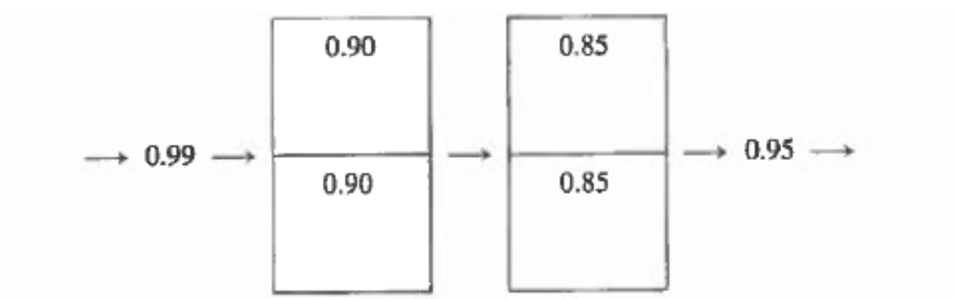
(c) A and C are not independent

(d) The probability of event A is 1/4

(e) The probability of event C is 1/2

Solution 10. Randomly selected means that the probability of selecting each of the workers is 1/4 .
P(A)=P(1 or 2) = 1/4 + 1/4 = 1/2
P(B)=P(1 or 3) = 1/4 +1/4 = 1/2
P(C) = 1/4; P(AB)=P(1)= 1/4
Are A and B independent? Yes, 1/4 = P(AB)=P(A)P(B) = 1/4
A and C are not independent: P(AC) = P(1)=1/4 ≠ P(A)P(C)= 1/2 x 1/4 = 1/8

Question 11. The following system in Figure 11 operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that the devices are independent. Calculate the reliability of the system (i.e., the probability that the system works).



(a) 0.91

(b) 0.5504

(c) 0.99

(d) 0.87

(e) 0.15

Solution 11. Reliability= $0.99(1 - 0.1^2)(1 - 0.15^2)(0.95) = 0.91$

Question 12. An insurance company believes that people can be divided into two classes: those who are accident prone (A) and those who are not (B). Their statistics show that if a person is accident-prone s/he will have an accident (C) at some time within a fixed 1-year period with probability 0.4, whereas if a person is non-accident prone this probability decreases to 0.2. We know that 30 percent of the population is accident prone. The joint probability that a randomly chosen customer is not accident prone and has an accident within 1 year is (choose 1).

(a) 0.14

(b) 0.06

(c) 0.08

(d) 0.12

(e) 0.87

Solution 12. $P(A^cC) = P(C \mid A^c)P(A^c) = 0.2(0.7) = 0.14$

Question 13. Of 100 students who completed an introductory statistics course, 20 were business majors. Further, 10 students received A's in the course, and 3 of these were business majors. What is the probability that a business major did not get an A ?

(a) 0.85

(b) 0.91

(c) 0.67

(d) 0.188

(e) 0.51

Solution 13.

	B	B^c	Total
A	3	7	10
A^c	17	73	90
Total	20	80	100

$P(A^c \mid B) = \dfrac{17}{20} = 0.85$

Question 14. An incoming lot of silicon wafers is to be inspected for defectives by an engineer in a microchip manufacturing plant. Suppose that, in a tray containing twenty wafers, four are defective. Two wafers are to be selected randomly for inspection. The probability that at least one is defective is (mark your answer here and in the scantron.):

(a) 0.500

(b) 0.1011

(c) 0.6316

(d) 0.3684

(e) 0.441

Solution 14. Answer: 0.3684.

P(at least one defective)= 1- P(none defective) = 1 - (16/20)(15/19) =0.3684 Alternatively, we could find this using the hypergeometric.

Question 15. A box with 15 VLSI (Very Large Scale Integrated) chips contains five defective ones. If a random sample of three chips is drawn, what is the probability that more than 1 are defective? Mark your answer here and in the scantron.

(a) -0.2615

(b) 0.2197

(c) 0.0219

(d) 0.2417

(e) 0.2793

Solution 15. X = number of defectives in a sample of 3 $P(X > 1) = \dfrac{\binom{5}{2}\binom{10}{1}}{\binom{15}{3}} + \dfrac{\binom{5}{3}\binom{10}{0}}{\binom{15}{3}} = 0.241758$

Question 16. A town has 1000 people. These people can be classified by their income level and their education, with the following information:

- 100 have ≤ 16 years of schooling and make $> \$50,000$

- 300 have > 16 years of schooling and make $> \$50,000$
- 400 have ≤ 16 years of schooling and make $\leq \$50,000$.

The probability that those that have ≤ 16 years of schooling make $> \$50,000$ is (mark your answer here and in the scantron)

(a) 0.41

(b) 0.87

(c) 0.5

(d) 0.2

(e) 0.62

Solution 16. Answer: 0.2 $P(Income > 5000 \mid education \leq 16) = 100/500 = 0.2$

Question 17. Suppose that you and a friend are matching balanced coins (i.e., each coin has probability 1/2 of landing head). Each of you tosses a coin. If the upper faces match, you win 3.00 dollar; if they do not match, you lose 3.00 dollar (your friend wins 3.00 dollar). Let X be the random sample representing your winnings. Your expected gains and standard deviation of the gains are, respectively, (mark your answer here and in the scantron)

(a) μ= 0.5; σ= 0.2

(b) μ=0; $\sigma = 3$

(c) μ=3; σ=0.5

(d) μ=0; σ=0.5

(e) $\mu = 1.5; \sigma = 0.1$

Solution 17. E(X)=3(1/2)-3(1.2)=0,
Var(X) = 9(1/2)+9(1/2)= 9,
sd= $\sqrt{9} = 3$,
$\mu = 0, \sigma = 3$

Question 18. Let A and B be two events. Which of the following statements is true? Mark your answer here and in the scantron.

(a) $P(B \cap A^c) = P(B) - P(A \cap B)$..

(b) If A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B) - P(AB)$

(c) If A and B are independent, $P(A \cup B) = P(A) + P(B) - P(A \mid B)$

(d) $P(A \cup B) = P(B)P(A)$ if A and B are independent.

(e) None of the above.

Solution 18. $P(B \cap A^c) = P(B) - P(A \cap B)$..

Question 19. The Center for Disease Control says that about 30% of high school students smoke tobacco (down from a high of 38% in 1997). Suppose you randomly select 10 high school students to survey them on their attitude towards scenes of smoking in the movies. What is the probability that there are 2 smokers among the 10 people you choose?

(a) 0.000729

(b) 0.0504

(c) 0.000504

(d) 0.2334

(e) 0.002

Solution 19. Binomial.

$P(X = 2) = 45(0.3^2)(0.7^8) = 0.2334$

Question 20. The number of failures per day in a certain plant has a Poisson distribution with expected value 4. Present maintenance facilities can repair 3 machines per day, otherwise a contractor is called out. On any given day, what is the probability of having machines repaired by a contractor?

(a) 0.75

(b) 0.43335

(c) 0.5665

(d) 0.101

(e) 0.96

Solution 20. Answer: 0.57 Why ?

$$P(X > 3) = 1 - P(X \leq 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)] = 1 - 0.43 = 0.57$$

Question 21. The number of people arriving to an emergency room can be modeled by a Poisson process with a rate parameter λ of five per hour.

What is the probability that at least 4 people arrive during a particular hour?

(a) 0.265

(b) 0.735

(c) 5

(d) 0.8

(e) 0.1

Solution 21. 0.735 Why? $P(X \geq 4) = 1 - P(X < 3) = 1 - .265 = .735$

Question 22. The number of people arriving to an emergency room can be modeled by a Poisson process with a rate parameter λ of five per hour. How many people do you expect to arrive during a 45 minute period?

(a) 45

(b) 3.75

(c) 5

(d) 0.75

(e) 0.1

Solution 22. Answer: 3.75 Why?

Arrivals occur at the rate of 5 per hour, so for a 45 minute period the rate is (5)(.75) = 3.75, which is also the expected number of arrivals in a 45 minute period.

Question 23. Let X have Binomial distribution with parameters p=0.6 and n=20. One of the following is true. Which? (Mark your answer here and in the scantron). Note: The expected value of a binomial random variable has formula $\mu = np$, and the variance of a binomial random variable is $\sigma^2 = np(1 - p)$.

(a) E(3X + 5) =41

(b) Var(3X + 5)= 35

(c) E(3X + 5) =17

(d) Var(5X) = 5

(e) Var(5X+2)= 7

Solution 23. If p=0.6 and n=20, E(X)= 20(0.6)= 12 and Var(X) = 20(0.6)(0.4) = 4.8 Answer: $E(3X + 5) = 3E(X) + 5 = 3(12) + 5 = 41$

Question 24. Daily sales records for a car dealership show that it will sell 0, 1, 2, or 3 cars, with probabilities as listed

Number of cars (X)	0	1	2	3
Probability (P(X))	0.5	0.3	0.15	0.05

The expected value of X^3 is

(a) Impossible to compute without more information

(b) 0.75+1

(c) 2.85

(d) 5

(e) 3

(f) 0

Solution 24. $E(X^3) = 1(0.3) + 8(0.15) + 27(0.05) = 2.85$

Question 25. Suppose that the moment generating function of a random variable X is given by

$$M_x(t) = e^{3(e^t-1)}.$$

The Probability that X is 0 is

(a) 0.3467

(b) 0.1456

(c) 0.0497

(d) 0.0001

(e) 1.237

Solution 25. $X \sim$ Poisson ($\lambda = 3$)

$$P(X = 0) = \frac{3^0 e^{-3}}{0!} = 0.0497$$

Question 26. Let X denote a random variable that has the following probability mass function.

X	-1	0	$+1$
P(X)	0.2	0.5	0.3

What is the expected value of X^2 ? (mark your answer here and in the scantron)

(a) 0.1

(b) 0.5

(c) 0.3

(d) 0.4

(e) 0.01

Solution 26. Answer: 0.5 $E(X^2) = 1(0.2) + 0(0.5) + 1(0.3) = 0.5$

Question 27. Suppose that you and a friend are matching balanced coins (i.e., each coin has probability 1/2 of landing head). Each of you tosses a coin. If the upper faces match, you win 3.00 dollar; if they do not match, you lose 3.00 dollar (your friend wins 3.00 dollar). Let X be the random sample representing your winnings. Your expected gains and standard deviation of the gains are, respectively, (mark your answer here and in the scantron)

(a) μ= 0.5; σ= 0.2

(b) μ=0; $\sigma = 3$

(c) μ=3; σ=0.5

(d) μ=0; σ=0.5

(e) $\mu = 1.5; \sigma = 0.1$

Solution 27. E(X)=3(1/2)-3(1.2)=0,
Var(X) = 9(1/2)+9(1/2)= 9,
sd= $\sqrt{9} = 3$,
$\mu = 0, \sigma = 3$

Question 28. A recruiting firm finds that 30 percent of the applicants for a certain industrial job have received advanced training in computer programming. Applicants are interviewed sequentially and selected at random from the pool. Suppose that the first applicant with advanced training is offered the position, and the applicant accepts. If each interview costs \$30, find the expected value and variance of the total cost of interviewing until the job is filled. Mark your answer here and in the scantron.

(a) $\mu = 100; \sigma^2 = 7000$

(b) $\mu = 3.33; \sigma^2 = 7.777$

(c) $\mu = 50; \sigma^2 = 9$

(d) $\mu = 200; \sigma^2 = 1000$

(e) $\mu = 150; \sigma^2 = 500$

Solution 28. Geometric r.v.
$E(30X) = 30(1/0.3) = 100$
$Var(X) = (1 - 0.3)/0.3^2 = 7.777778$
$VAR(30X) = ((1 - 0.3)/0.3^2)900 = 7000$

Question 29. A coin which lands heads with probability p is tossed repeatedly. Assuming independence of the tosses, the probability that the fifth head appears on the 12th toss is

(a) $\binom{9}{5}p^5(1-p)^4$

(b) $(1-p)^6 p$

(c) $\sum_{k=0}^{5} \binom{8}{k} p^k (1-p)^{8-k} . \binom{5}{k} p^k (1-p)^{5-k}$

(d) $\binom{11}{4} p^4 (1-p)^7 p$

(e) 2/x

Solution 29. $\binom{11}{4} p^4 (1-p)^7 . p$