COMP4418 Assignment 3

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Social Choice Theory

Consider the school choice problem in which students have strict preferences over schools and schools have strict priorities over students. Students are allowed to express certain schools as unacceptable (a student would rather be unmatched than be matched to an unacceptable school). We are interested in finding a matching that satisfies justified envy-freeness such that the matching has as many pairs as possible. Reminder from Week 4.
Justified envy-freeness: there exists no agent i who prefers another school s over her match and s admits j a lower priority agent than i.

[1.1] Is the Student Proposing Deferred Acceptance algorithm (described in Week 4) a suitable approach to solving this problem? If yes, prove it in one or two sentences (or by referring to the appropriate theorem). If not, then explain why not in one or two sentences.

- Yes, the deferred acceptance algorithm is a suitable approach to solving this school choice problem, because according to the deferred acceptance algorithm, for each school c ∈ C, let Sc be the set of students who are matched to c or who now apply to c. Each time school c selects a maximum of q(c) acceptable students depending on the school preference from among Sc and rejects the rest, thus if s admits j a lower priority agent than i, and this has two situations, one is j is currently match with school s, as a result, schools s will reject j and match with i, and the second situation is i is rejected by school s and j appies to school s, and obviously that school s will reject j. As a result, there exists no agent i who prefers another school s over her match and s admits j a lower priority agent than i., which satisfies justified envy-freeness.

The instance will be provided to your program via three predicates. The predicates schoolRank/3 and studentRank/3 indicate the preference of school over students and students over school respectively.
For instance schoolRank(randwick,jane,1) means that Randwick School would love to have Jane as student, as she is ranked first in their list. If a student find a school unacceptable, then there is no studentRank statement for that student for that school (e.g., the situation of Jane in the example below who is not ranking Maroubra School because it's unacceptable to her). The predicate quota/2 indicates the number of students that can be accepted by some school. The solution should be output using a predicate match/2 taking a school s and a student p arguments to indicate that p is

matched with s, for instance match(randwick,jane). Consider the following instance
schoolRank (randwick , jane ,1).
schoolRank (randwick , kurt ,2).
schoolRank (randwick , lisa ,3).
schoolRank (randwick , maud ,4).
schoolRank (maroubra , jane ,1).
schoolRank (maroubra , maud ,2).
schoolRank (maroubra , lisa ,3).
schoolRank (maroubra , kurt ,4).
studentRank (jane , maroubra ,1).
studentRank (kurt , randwick ,1).
studentRank (kurt , maroubra ,2).
studentRank (lisa , randwick ,1).
studentRank (lisa , maroubra ,2).
studentRank (maud , randwick ,1).
quota (randwick ,2).
quota (maroubra ,2).

[1.2] Find a justified-envy-free matching of largest cardinality and return it using the predicate match/2.

- `match(randwick,kurt) match(randwick,lisa) match(maroubra,jane)`

[1.3] Design an ASP program that identifies a largest justified-envy-free matching on any input instance.

- The code is shown in matching.lp.

Answer Set Programming

The problem is based on the Crazy Frog Puzzle and is inspired by the Logic and Constraint programming contest LP/CP2021 https://github.com/alviano/lpcp-contest-2021/tree/ main/problem-1. You have the control of a little frog, capable of very long jumps, and today is your lucky day! The little frog just woke up in an S × S land, with few obstacles and a lot of insects. Time to eat them all! The frog can jump as far as you want within the land, but only in the four cardinal directions (don’t ask why) and you cannot land on any obstacle or already visited places (you will see why). Don’t leave any insect alive! Input format An input file contains a predicate instance size/1 indicating the size of the land. The coordinates of the place where you woke up in a predicate start/2. And a predicate obstacle/2 where each instance indicates the presence of an obstacle at the coordinates. The size S is guaranteed to be between 4 and 60. Output format Your

program should compute a model containing a predicate jump/3 indicating for each time step (starting at 1) where the frog lands. Example Consider the following instance.
size (4).
obstacle (2 ,1).
obstacle (4 ,2).
obstacle (1 ,3).
obstacle (2 ,4).
start (3 ,3).

One possible solution would contain the predicate instances
Figure 1: Crazy Frog Puzzle instance 0 and sequence of positions corresponding to a solution. The initial state is given in the top left picture, and then reading left to right then up to down, we have a sequence of states until all insects have been eaten without visiting the same place twice.

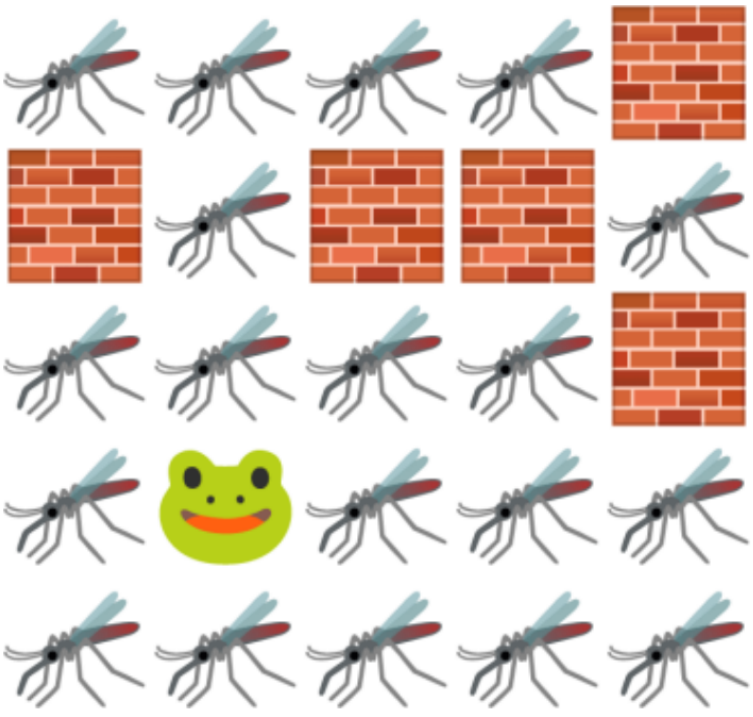
jump (1 ,3 ,4)
jump (2 ,4 ,4)
jump (3 ,1 ,4)
jump (4 ,1 ,2)
jump (5 ,1 ,1)
jump (6 ,3 ,1)

jump (7 ,4 ,1)
jump (8 ,4 ,3)
jump (9 ,2 ,3)
jump (10 ,2 ,2)
jump (11 ,3 ,2)
The starting state of the instance as well as the successive steps represented by the solution are depicted in Figure 1.

[2.1] Given a size S, a list of k obstacles, and assuming the frog starts in location (i, j), is it possible to compute the number of jumps needed so that all non-obstacle location are visited exactly once? If yes, explain how and why. If not, then provide a concrete counter example with two solutions having a distinct number of jumps.

- Yes, it is possible. To compute the number of jumps needed so that all non-obstacle locations are visited exactly once, we can first think that if there's no obstacle, then it's clear that we need **((S * S) - 1)** jumps to visit all non-obstacle location exactly once, and now if we have k obstacles, since we cannot land on any obstacle, so there are only ((S * S) - 1 - k) locations we can visit, also we know that each non-obstacle location can be only visited once, therefore if we are given a size S, a list of k obstacles, to visit all non-obstacle locations exactly once, we need **((S * S) - 1 - k)** jumps.

Figure 2: Crazy Frog Puzzle instance 1.



[2.2] Consider the starting instance in Figure 2. Provide the ASP code describing this instance using the input format given above.

- size (5).
- obstacle (5, 1).
- obstacle (1, 2).
- obstacle (3, 2).
- obstacle (4, 2).
- obstacle (5, 3).
- start (2, 4).

[2.3] Consider the instance in Figure 2. Identify a solution, if any, and provide the list of jumps required in the output format described above. If there are no solutions, then state it and explain how you can prove/detect the absence of any solution. You can identify the solution by any reasonable means you want (any reasonable means = non-cheating, non-collaborative, non-looking it up online, etc.), including human reasoning and manual computation, computer reasoning using ASP, running an appropriate algorithm in the programming language of your choice. If you find a solution, list it and then provide a very short sentence in English describing your approach. If you find that no solution exists, then explain your approach in details.

- jump(1,2,3)
- jump(2,4,3)
- jump(3,4,5)
- jump(4,2,5)
- jump(5,3,5)
- jump(6,5,5)
- jump(7,1,5)
- jump(8,1,3)
- jump(9,1,4)
- jump(10,5,4)
- jump(11,5,2)
- jump(12,2,2)
- jump(13,2,1)
- jump(14,3,1)
- jump(15,3,3)
- jump(16,3,4)
- jump(17,4,4)
- jump(18,4,1)
- jump(19,1,1)

I used ASP code to generate the answer, firstly we get the valid locations and count the number of the valid positions, which also means the number of jumps we need totally((S * S) - 1 - k), after that we use the generator to generate ((S * S) - 1 - k)=19 jumps, and use several integrity constraints(Each valid position should only be visited once / Next jump should have the same X or Y with the previous jump) to filter and get our final answer as shown above.

[2.4] Give an ASP program identifying solutions to any input instance and computing answer sets such that the projection of a model on the jump/3 predicate constitute a solution list of jumps.

- The code is shown in frogs.lp.