'Colour Space': A Contrived Problem Domain for Investigating Adaptive Models

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Abstract-Adaptive models are complex systems that are especially difficult to investigate. In particular, when researching such systems it easy to blur the line between model and domain, given that the former, the later, and both are vital adaptive system research pursuits. When investigating a model, one may become 'caught-up' on model performance resulting in a vicious cycle of increasing the complexity of benchmark problem instances, and loosing site of the characteristic model behaviours that one is seeking. This work proposes a simple 'colour space' domain from which trivial problem instances may be derived to investigate complex systems. Example optimization and pattern-recognition problem instances are described, and the paper is rounded off with a discussion of the potential limitations of using the domain to investigate adaptive models.

Keywords- Colour Domain, Color, Test Problem, Domain, Problem Suite, Artificial Immune System, Adaptive Model, Colour Space, Color Space

I. INTRODUCTION

The Self-Organizing Map (SOM) is a competitive and unsupervised learning algorithm that is renowned for its feature extraction and dimensionality reduction capabilities (see [4] for an excellent treatment of SOM). In his doctorial work, Honkela [5] (page 14-18) used a colour domain (red-green-blue or RGB), and a dataset of known colours in that domain. The learning and ordering of the domain of RGB colours based on the dataset was used as an intuitive example of the topological preserving properties of the algorithm. Although Honkela may not have been the first to employ this example domain to the SOM, it has become a canonical example domain by the simplicity and suitability of the colour domain to qualitatively and quantifiably (numerical measures) demonstrate the properties of the SOM, this work seeks to define a 'colour space' domain from which a suite of test problem instances bay be drawn to investigate adaptive systems. This work is biased toward the investigation of immunologically inspired adaptive systems, and this endeavour is intended to provide a framework and test suite suitable for investigating the models proposed by Brownlee in [2] and [3]. As such, the domain formalism and name are biased by the 'shape space' formalism [1] employed in theoretical immunology.

Section II outlines the design goals for a problem domain, defines the terminology for a domain that meets these design goals, and lists measures that may be used within the domain. Section III specifies a few illustrative example domains and problem instances suitable for general optimization and pattern recognition based investigations. Finally, section IV summarises some potential limitations and criticisms of using the domain.

II. A CONTRIVED DOMAIN: 'COLOUR SPACE'

To avoid confusion, the domain does *not* provide a framework for investigating the so-called colour space theory, which refers to the study of colour models used in display and printer systems. Nor is the intention of the defined test domain for the study of colour theory, the investigation of colour segregation, colour recognition, palette compression, or colour clustering (or any other practical colour-based application domain).

Given what the intentions are not, this section outlines a series of general goals for designing a test domain for adaptive systems, and follows it up by defining a simple 'colour space' domain that meets these goals.

A.Domain Goals

The ambition is to contrive a domain from which trivial problem instances can be drawn to preliminary assess specific properties of adaptive models. These models may be engaged in processes that equate to pattern recognition and optimization, and may involved the evaluation of broad characteristics such as learning, memory, and adaptation. See Table 1 for a list of the general design goals (perhaps requirements) for a problem domain suitable for this purpose.

Ref#	Goal			
G1	The domain must provide a numerical and or combinatorial basis			
	that may be manipulated meaningfully by operators			
G2	The response surface must be correlated such that localised regions			
	in the response surface have a gradient			
G3	The domain must be suitable for optimization tasks			
G4	The domain must be suitable for pattern recognition tasks			
G5	The domain must be parameterised such that the relative complexity may be meaningfully adjusted			
G6	The domain must be easily and meaningfully visualised both online			
	(dynamically) and offline (end of run)			

¹ There are many applet and tutorial web sites on the internet that use this example to demonstrate the SOM, the listing of which is not necessary

G7	The domain must be easy to understand, easy to implement, and		
	easy to analyse		
G8	The properties of the domain (eg the response surface or visualisation) should facilitate rather than mask model behaviours being investigated		

Table 1 - List of goals for a 'colour space' domain

B. Domain Definition

This section proposes a 'colour space' domain formalism that attempts to meet the list of goals proposed in Table 1. The formalism consists of a series of terms to describe the characteristics of a colour space domain. The characteristics identified include the environment; the cardinality, the representation, and the mapping function (see Table 2 for a summary).

Term	Name	Summary
E	Environment (Colour Space)	The search space (colour space) which defines the scope of feasible coordinates defined by its dimensionality $(1-n)$, a boundary (limits of each dimension), and cardinality (C) .
R	Representation	An encoding used and manipulated by adaptive models. May be binary, integer, real, and may or may not directly represent a coordinate in the colour space (<i>E</i>).
M	Mapping Function	Converts a given representation (<i>R</i>) into a coordinate in the colour space (<i>E</i>). If the representation used matches the coordinate system of the environment, than the mapping function returns an unchanged coordinate.
С	Cardinality (Palette)	The number of discrete points in the domain, and ideally (given display capabilities) the number of colours available for visualisation.
I	Intention (Task)	The goal of a system within the domain, a task or action it is to perform or achieve. This is a cover-all term for something to do in a defined colour-space.

Table 2 - Summary of terms for the colour space domain formalism

The environment (E) is an n-dimensional hypercube (for visualisation purposes perhaps limited to 1-3 dimensions), where each dimension represents a different colour axis. The intention is that a distinct coordinate in a given environment (colour space) represent a distinct colour. For example, a monochrome (one-colour) environment would be implemented as a one-dimensional space (line) perhaps a gradient of white to black. An RGB colour space would be implemented as a three-dimensional (cube) environment.

An environment is a volume of feasible discrete points, and the cardinality (C) defines the number of points in that volume. The granularity of each dimension in the environment also determines the number of colours, thus cardinality may be thought of as the palette of the environment. Further, the palette analogy may be exploited further in defining standard cardinalities, such as those common palettes that are used in computer display systems. These palettes are defined in terms of the number of bits to store a set of possible colours. For example, a monochrome cardinality has an 8-bit palette with 256 (2^8) distinct colours, a 16-bit palette has 65,536 (2^{16}) distinct colours. A 24-bit RGB (three-dimensional) cardinality would be a cube of 16,777,216 distinct colours (2^{8x3}) .

The numerical coordinate system for a given environment may be integer (for example 0-255 for 8-

bit), or floating point (0-1). This coordinate-based representation (*R*) may be used directly by an adaptive system. Alternatively a symbolic or 'lower-level' representation may be used (such as bit-strings), which require a mapping function (*M*) (binary-coded decimal or gray code in the case of bit-strings) to transform representation into a given environments coordinate system.

Thus, a domain may be defined given (E, C, R, and M). An adaptive system needs something to do in this space, a task or intention (*I*). Intentions may include the optimization of a randomly initialised system to a set of pre-selected points, or the adaptation of a system to a defined region within the environment.

C.Domain Measures

One may measure the Euclidean distance between points in the colour space as a quantitative measure of difference. Distances measures may not be limited to Euclidean distance; Manhattan distance (and many others, see [4] pages 17-29) may be used in the colour space. Distance measures may be augmented with coordinate radii such that a point of interest in the space may represent a collective of points in its vicinity. Such regions may be defined as hyper-cubes or spheres, and may have a distance-based falloff such as linear, Gaussian or exponential.

These distances and coordinate neighbourhoods are also qualitatively meaningful. Following a line (for example in a monochrome or RGB) in colour space shows a transition through intermediate colour coordinates (colours). These intermediate colours are quantifiably distinct (numerical coordinates), and qualitatively different (assuming the colour transition can be detected by the human eye). This correlation is also useful for visualisation: agents representing coordinates in the colour space may be implemented, which may permit qualitative measures and observations of system behaviour.

III. ILLUSTRATIVE EXAMPLES

This section proposes and discusses a number of example *domains* (E, R, M, C) and *tasks* (I) that adaptive systems that may be employed or extended to investigate adaptive models.

A.Domains

Given that visualisation is an important design goal with regard to qualitative observations, domains of one, two, and three dimensions are given as examples. The examples are mealy illustrative and by no means encompass the possibility space for implement a colour space domain.

- **1-D**: This is a line colour space. It may be implemented as a transition from black to white (greyscale), or from black or white to a given colour (monochrome), although obviously the transition may be between any two colours (gradient).
- **2-D**: This is a rectangular colour space such as a square. One may implement the domain as a gradient along or horizontal to an axis. Alternatively, one may

implement two different gradients (four colours) perpendicular to each other with regard to the domains axis.

3-D: The obvious choice for the dimensionality is to implement red, green, and blue (RGB) on each axis, thus giving the full the range of colours accustomed to computer displays.

B.Optimization

One may define an optimization domain, by preselecting a coordinate in the space (that is withheld from the adaptive system) and defining an objective function (cost function) as a distance measure to the given coordinate. For example if a space was defined as an 8-bit one-dimensional domain (0-255), a coordinate was pre-selected at 127, and an Euclidean distance measure was employed as a cost function, then the shape of the response surface would be a triangle with the apex at the selected coordinate.

There are many extensions to this simple linear optimization task. An example extension would be to expand an objective coordinate to a set of *S* coordinates. Two varied objective tasks that may result: the *parallel* and *sequential* optimization of *S*.

Parallel: Coordinates of the set *S* are exposed to the system in parallel, thus the solution is the ability of a system to satisfy all of *S*.

Sequential: Coordinates of the set *S* are exposed to the system sequentially (ordered, random, or otherwise). Thus, the objective function changes through time.

C.Pattern Recognition

Although in the strictest definition, the above optimization example may also be considered definition, one may consider classification as a pattern recognition task. A geometry can be defined within the colour space and class labels can be assigned to the resulting concave regions. One may use the gradient (correlated) colour as defined by the colour space to represent coordinates of a given concave region, or induce an additional class-colour mapping function to clearly differentiate regions. Given that the class boundaries are withheld from an adaptive system addressing the task, the former represents an 'adaptive system perspective' of the domain, where as the latter represents a 'task perspective' or *a priori* perspective of the domain.

Given the same 8-bit one-dimensional (0-255) example used for optimization, one may define a region (sub-line) that extends from the coordinate 49 to 99. Given an *a priori* sample of coordinates from within this space, the task of an adaptive system may be to identify future coordinates within this space given an ongoing sample of random coordinates from the whole domain. This is a trivial example of the one-class classification problem. Again, there are many obvious extensions to this task, such as the expansion of the region to a set of regions (*R*) and the use of more complex convex geometries.

IV. CRITICISMS AND LIMITATIONS

When one formulates a domain for the purposes of investigating experimental questions (hypothesis) it is critical to consider the limitations of such a platform. The following are some of these concerns.

Triviality: The domain and resultant problem definitions are primitive. The problem instances are likely easily solved rapidly by standard techniques, and even by rudimentary approaches such as enumeration and random search.

Transferability: The results achieved on one or a set of derived problem instances are very likely to be not transferable to problem domains of interest. Conclusions regarding model performance are limited to the instances on which they are testes, and perhaps related trivial domains.

Difficulty: Although superficiality they appear trivial, there is no innate sense of the absolute or relative difficulty of the derived problem definitions.

Visibility: For high-cardinality domains (such as those at or above 24-bit or 32-bit RGB) the human eye cannot detect colour differences between coordinates within close proximity.

Novelty: The colour space domain and derived problem instances are not novel, they are generalisations and simplifications of existent test domains and benchmark problems.

With brevity in mind, it is prudent not to rebut these criticisms, but to acknowledge them as limitations (designed or otherwise) of the domain and framework that it provides. The domain and resultant problem instances are intended to be trivial (easy to solve) such that the attention remain on the model under study. The performance of a given model is not intended to be transferable, what is expected to be transferable are the general functional behaviours that the domain assists in isolating and evaluating.

The relative difficulty of problem instances may be determined by baseline strategies. Comparable relative and absolute difficulty may be defined using the tools of probability, statistical mechanics, and additional mathematical apparatus. The (suitable) visual distinctiveness of close-proximity coordinates in highcardinality spaces may be exaggerated where appropriate. For example, a dynamically normalised colour scale may be allocated to coordinates of interest. Finally, novelty is not claimed, rather the domain is a reformulation of classical optimization and pattern recognition benchmark domains.

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