3.6

(a) $P(X\geq 200) = 1 - P(X<200) = 1 - \int_0^{200} f(x)dx$

$= 1 - \int_0^{200} \frac{20000}{(x+100)^3}dx = 1 - \left(-\frac{1}{2}\times\frac{20000}{(x+100)^2}\right)\Bigg|_0^{200}$

$= 1 + \frac{10000}{(x+100)^2}\Bigg|_0^{200} = 1 + \frac{10000}{300^2} - \frac{10000}{100^2}$

$= \frac{10000}{90000} = \frac{1}{9}$ #

A: $\frac{1}{9}$

(b) $P(80 \leq X \leq 120) = \int_{80}^{120} f(x)dx$

$= \int_{80}^{120} \frac{20000}{(x+100)^3}dx = -\frac{1}{2}\times\frac{20000}{(x+100)^2}\Bigg|_{80}^{120}$

$= -\frac{10000}{(x+100)^2}\Bigg|_{80}^{120} = -\left(\frac{10000}{220^2} - \frac{10000}{180^2}\right)$

$= 0.102$ #

A: 0.102

3.15 $f(0) = \frac{\binom{2}{0}\binom{5}{3}}{\binom{7}{3}} = \frac{\frac{2!}{2!0!}\times\frac{5!}{3!2!}}{\frac{7!}{3!4!}} = \frac{1\times\frac{5\times4}{2\times1}}{\frac{7\times6\times5}{3\times2\times1}} = \frac{5\times2}{7\times5} = \frac{2}{7}$

$f(1) = \frac{\binom{2}{1}\binom{5}{2}}{\binom{7}{3}} = \frac{\frac{2!}{1!1!}\frac{5!}{2!3!}}{\frac{7!}{3!4!}} = \frac{\frac{2}{1}\times\frac{5\times4}{2\times1}}{\frac{7\times6\times5}{3\times2\times1}} = \frac{5\times4}{7\times5} = \frac{4}{7}$

$f(2) = \frac{\binom{2}{2}\binom{5}{1}}{\binom{7}{3}} = \frac{\frac{2!}{2!}\frac{5!}{1!4!}}{\frac{7!}{3!4!}} = \frac{1\times5}{\frac{7\times6\times5}{3\times2\times1}} = \frac{5}{7\times5} = \frac{1}{7}$

$F(0) = f(0) = \frac{2}{7}$ $F(1) = f(0) + f(1) = \frac{2}{7} + \frac{4}{7} = \frac{6}{7}$

$F(2) = f(0) + f(1) + f(2) = \frac{2}{7} + \frac{4}{7} + \frac{1}{7} = 1$

(a) $P(X=1) = F(1) - F(0) = \frac{6}{7} - \frac{2}{7} = \frac{4}{7}$ #

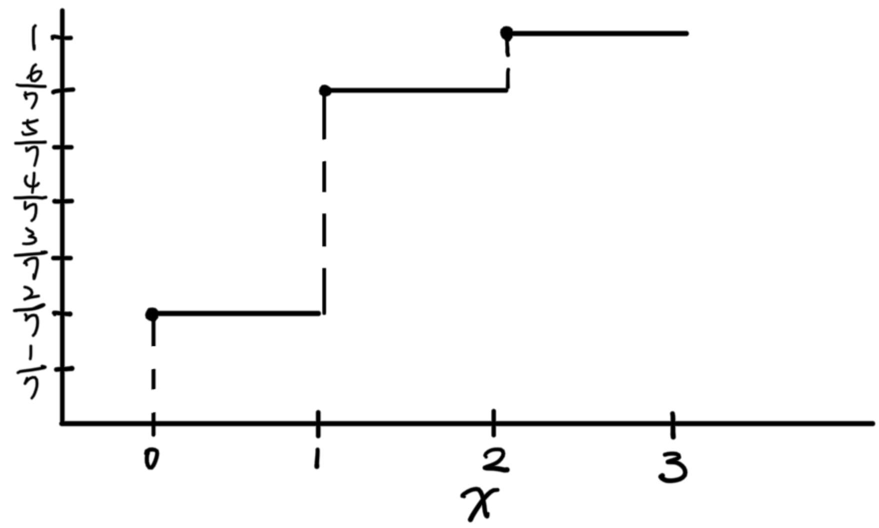
A: $\frac{4}{7}$

(b) $P(0<X\le 2) = P(X\le 2) - P(X\le 0) = F(2) - F(0)$

$= 1 - \frac{2}{7} = \frac{5}{7}$ #

A: $\frac{5}{7}$

3.16



3.24 Let X be a random variable whose values x are the number of comic book are selected.

$$f(x) = \begin{cases} \dfrac{\binom{5}{x}\binom{5}{4-x}}{\binom{10}{4}}, & x = 0,1,2,3,4, \\ 0, & \text{elsewhere} \end{cases}$$

#

3.30

(a) $\int_{-\infty}^{\infty} k(3-x^2)\,dx = 1 \Rightarrow \int_{-1}^{1} k(3-x^2)\,dx = 1$

$\int_{-1}^{1} k(3-x^2)dx = k\int_{-1}^{1}(3-x^2)dx = k\left[3x - \frac{x^3}{3}\right]_{-1}^{1}$

$= k\left[(3-\frac{1}{3}) - (-3+\frac{1}{3})\right] = \frac{16}{3}\times k = 1 \Rightarrow k = \frac{3}{16}$ #

A: $\frac{3}{16}$

(b) $P(X<\frac{1}{2})=P(-1\leq X\leq \frac{1}{2})=\int_{-1}^{\frac{1}{2}}\frac{3}{16}(3-X^2)\,dX$

$=\frac{3}{16}\left[3X-\frac{X^3}{3}\right]_{-1}^{\frac{1}{2}}=\frac{3}{16}\left[(\frac{3}{2}-\frac{1}{24})-(-3+\frac{1}{3})\right]$

$=\frac{3}{16}\left(\frac{36}{24}-\frac{1}{24}+3-\frac{1}{3}\right)=\frac{3}{16}\times\left(\frac{35}{24}+\frac{8}{3}\right)$

$=\frac{3}{16}\times\frac{99}{24}=\frac{99}{128}$ #

A: $\frac{99}{128}$

(c) $P(|X|>0.8)=1-P(-0.8<X<0.8)=1-\int_{-0.8}^{0.8}\frac{3}{16}(3-X^2)\,dX$

$=1-\frac{3}{16}\left[3X-\frac{X^3}{3}\right]_{-0.8}^{0.8}$

$=1-\frac{3}{16}\left[(\frac{12}{5}-\frac{64}{375})-(-\frac{12}{5}-\frac{64}{375})\right]$

$=1-\frac{3}{16}\times\frac{1672}{375}=1-\frac{209}{250}=\frac{41}{250}$ #

A: $\frac{41}{250}$

3.40

(a) $\int_{-\infty}^{\infty}f(x,y)\,dy=\int_{0}^{1}\frac{2}{3}(x+2y)\,dy=\frac{2}{3}\int_{0}^{1}(x+2y)\,dy$

$=\frac{2}{3}\left[xy+y^2\right]_0^1=\frac{2}{3}\left[(x+1^2)-(0+0)\right]=\frac{2}{3}(x+1)$ #

A: $\frac{2}{3}(x+1)$

(b) $\int_{-\infty}^{\infty}f(x,y)\,dx=\int_{0}^{1}\frac{2}{3}(x+2y)\,dx=\frac{2}{3}\int_{0}^{1}(x+2y)\,dx$

$=\frac{2}{3}\left[\frac{x^2}{2}+2xy\right]_0^1=\frac{2}{3}\left[(\frac{1}{2}+2y)-(0+0)\right]=\frac{2}{3}(\frac{1}{2}+2y)$

$=\frac{1}{3}(4y+1)=\frac{4y+1}{3}$ #

A: $\frac{4y+1}{3}$

(c) $P(X<0.5) = P(0 \le X \le 0.5 \mid 0 \le Y \le 1)$

$$= \int_0^1 \int_0^{0.5} \frac{2}{3}(x+2y)\,dx\,dy = \frac{2}{3}\int_0^1\int_0^{0.5}(x+2y)\,dx\,dy$$

$$= \frac{2}{3}\int_0^1 \left(\frac{x^2}{2}+2xy\right)\Big|_0^{0.5} dy = \frac{2}{3}\int_0^1\left(\frac{1}{8}+y\right)dy$$

$$= \frac{2}{3}\left(\frac{y}{8}+\frac{y^2}{2}\right)\Big|_0^1 = \frac{2}{3}\left(\frac{1}{8}+\frac{1}{2}\right) = \frac{2}{3}\times\frac{5}{8} = \frac{5}{12} \text{ \#}$$

A: $\frac{5}{12}$

3.50

(a) Let marginal distribution of X 為 $g(x)$

$g(2) = \sum_y f(2,y) = 0.1+0.2+0.1 = 0.4$

$g(4) = \sum_y f(4,y) = 0.15+0.3+0.15 = 0.6$

x	2	4
$g(x)$	0.4	0.6

\#

(b) Let marginal distribution of Y 為 $h(y)$

$h(1) = \sum_x f(x,1) = 0.10+0.15 = 0.25$

$h(3) = \sum_x f(x,3) = 0.20+0.30 = 0.50$

$h(5) = \sum_x f(x,5) = 0.10+0.15 = 0.25$

y	1	3	5
$h(y)$	0.25	0.50	0.25

\#