7.12

$$y_2 = \frac{x_1}{(x_1+x_2)} = \frac{x_1}{y_1} \Rightarrow x_1 = y_1 y_2$$

$$y_1 = x_1 + x_2 = y_1 y_2 + x_2 \Rightarrow x_2 = y_1 - y_1 y_2 = y_1(1-y_2)$$

for $y_1 > 0$ 和 $0 < y_2 < 1$

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix} = \begin{vmatrix} y_2 & y_1 \\ 1-y_2 & -y_1 \end{vmatrix} = -y_1 y_2 - (y_1 - y_1 y_2) = -y_1$$

$g(y_1, y_2) = f(y_1 y_2, y_1(1-y_2))|J| = y_1 e^{-y_1}$ for $y_1 > 0$ 和 $0 < y_2 < 1$

$$g(y_1) = \int_0^1 y_1 e^{-y_1} dy_2 = y_1 e^{-y_1} \quad , y_1 > 0$$

$$g(y_2) = \int_0^\infty y_1 e^{-y_1} dy_1 = \Gamma(2) = 1 \ , \ 0 < y_2 < 1$$

$\because g(y_1, y_2) = g(y_1) g(y_2)$

$\therefore$ the random variables Y_1 and Y_2 independent #

7.14

$$y = x^2 \Rightarrow x_1 = \sqrt{y}, \ x_2 = -\sqrt{y}$$

$$\Rightarrow J_1 = \frac{dx_1}{dy} = \frac{d(\sqrt{y})}{dy} = \frac{1}{2\sqrt{y}}$$

$$\Rightarrow J_2 = \frac{dx_2}{dy} = \frac{d(-\sqrt{y})}{dy} = -\frac{1}{2\sqrt{y}}$$

$$g(y) = f(x_1)|J_1| + f(x_2)|J_2| = \frac{1+\sqrt{y}}{2} \times \frac{1}{2\sqrt{y}} + \frac{1-\sqrt{y}}{2} \times \frac{1}{2\sqrt{y}}$$

$$= \frac{1+\sqrt{y}+1-\sqrt{y}}{4\sqrt{y}} = \frac{2}{4\sqrt{y}} = \frac{1}{2\sqrt{y}}$$

$$g(y) = \begin{cases} \frac{1}{2\sqrt{y}} & , 0 < y < 1 \\ 0 & , \text{elsewhere} \end{cases}$$ #

7.18

$$M_X(t)=E(e^{tX})=\sum_{x=1}^{\infty}e^{tx}pq^{x-1}=pe^t\sum_{x=1}^{\infty}e^{tx}q^{x-1}=pe^t\sum_{x=0}^{\infty}e^{tx}q^{x}$$

$$=pe^t\sum_{x=0}^{\infty}(e^tq)^x=pe^t\frac{1}{1-qe^t}=\frac{pe^t}{1-qe^t}$$

$$\text{mean}=\mu_1=M_X'(0)=\left.\frac{(1-qe^t)pe^t-(-qe^t)pe^t}{(1-qe^t)^2}\right|_{t=0}=\left.\frac{pe^t}{(1-qe^t)^2}\right|_{t=0}$$

$$=\frac{p}{(1-q)^2}=\frac{p}{p^2}=\frac{1}{p}\ \#$$

$$E(X^2)=\mu_2'=M_x''(0)=\frac{d}{dt}\left(\frac{pe^t}{(1-qe^t)^2}\right)\Big|_{t=0}$$

$$=\left.\frac{(1-qe^t)^2pe^t-2(1-qe^t)(-qe^t)pe^t}{(1-qe^t)^4}\right|_{t=0}=\left.\frac{pe^t-pq^2e^t}{(1-qe^t)^4}\right|_{t=0}$$

$$=\frac{p-pq^2}{(1-q)^4}=\frac{p(1-q^2)}{p^4}=\frac{p(1+q)(1-q)}{p^3(1-q)}=\frac{1+q}{p^2}=\frac{1+1-p}{p^2}=\frac{2-p}{p^2}$$

$$\text{variance}=\sigma^2=\mu_2'-(\mu_1')^2=\frac{2-p}{p^2}-\left(\frac{1}{p}\right)^2=\frac{2-p-1}{p^2}=\frac{1-p}{p^2}=\frac{q}{p^2}\ \#$$

7.22

$$M_X(t)=(1-2t)^{-\frac{v}{2}}$$

$$\text{mean}=\mu_x=\mu_1'=\frac{d}{dt}(1-2t)^{-\frac{v}{2}}=\left(-\frac{v}{2}\cdot(1-2t)^{-\frac{v}{2}-1}(-2)\right)\Big|_{t=0}$$

$$=-\frac{v}{2}\times1\times(-2)=v\ \#$$

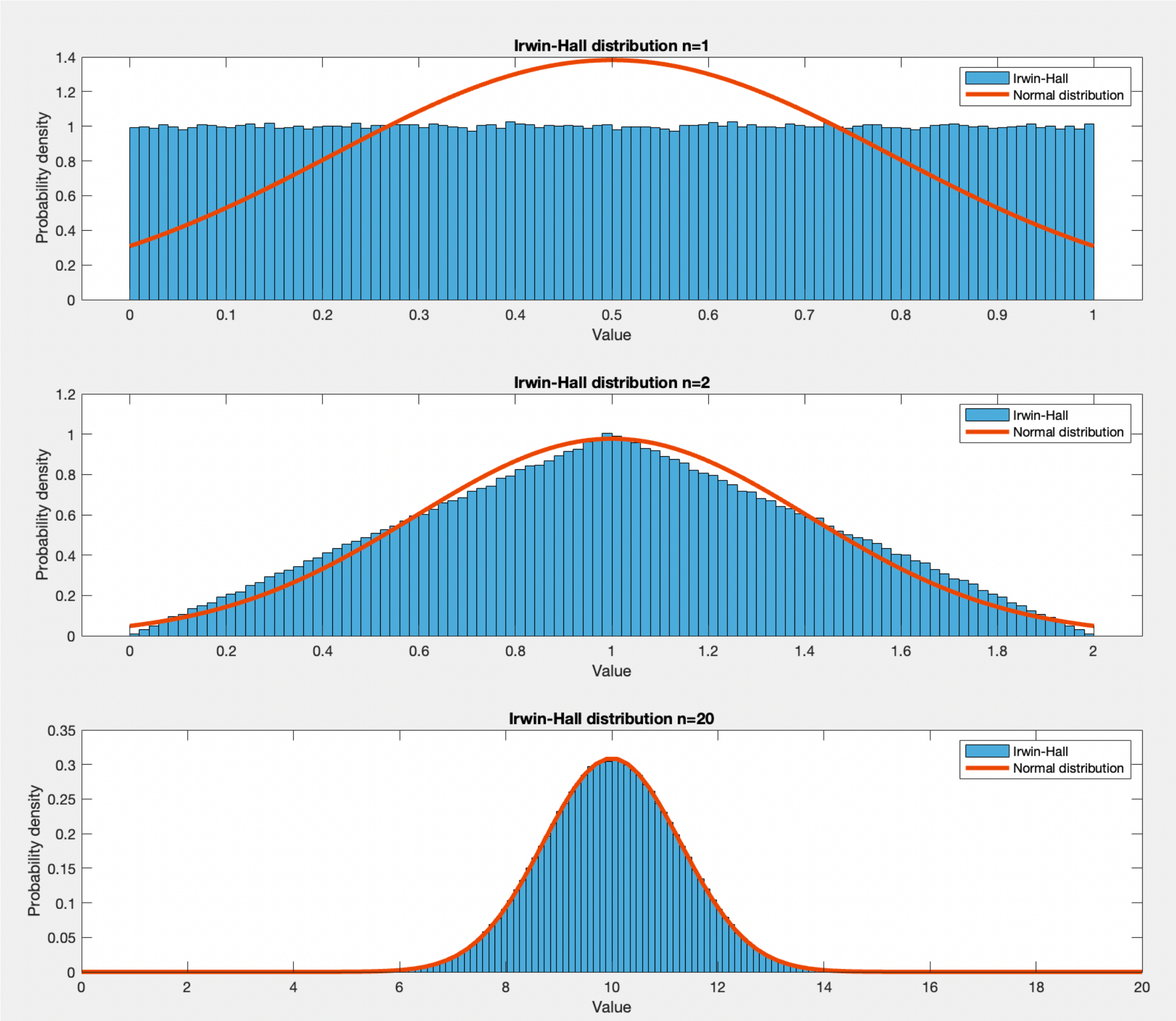
$$\text{variance}=\sigma_x^2=\mu_2'-(\mu_1')^2=\frac{d}{dt}\left[-\frac{v(1-2t)^{-\frac{v}{2}-1}(-2)}{2}\right]\Big|_{t=0}-v^2$$

$$=\frac{d}{dt}\left[v(1-2t)^{-\frac{v}{2}-1}\right]\Big|_{t=0}-v^2$$

$$=\left(v\cdot\left(-\frac{v}{2}-1\right)(1-2t)^{-\frac{v}{2}-2}\times(-2)\right)\Big|_{t=0}-v^2$$

$$=\left((v^2+2v)(1-2t)^{-\frac{v}{2}-2}\right)\Big|_{t=0}-v^2=v^2+2v-v^2=2v\ \#$$

Matlab 1.(b)



在 n=1 時，用 Irwin-Hall distribution to approximate a normal distribution，可以發現之間的 error 非常明顯、非常大，這也是三者中 error 最大的。
在 n=2 時，用 Irwin-Hall distribution to approximate a normal distribution，可以發現之間的 error 有比 n=1 時小不少，但是誤差還是可以直接看得出來。
在 n=20 時，用 Irwin-Hall distribution to approximate a normal distribution，可以發現之間的 error 和 n=1 與 n=2 相比明顯變小許多，這也是三者中 error 最小的。