4.24

	$X=0$	$X=1$	$X=2$	$X=3$
$Y=0$	0	$\frac{\binom{3}{1}\binom{2}{0}\binom{3}{3}}{\binom{8}{4}}=\frac{3}{70}$	$\frac{\binom{3}{2}\binom{2}{0}\binom{3}{2}}{\binom{8}{4}}=\frac{9}{70}$	$\frac{\binom{3}{3}\binom{2}{0}\binom{3}{1}}{\binom{8}{4}}=\frac{3}{70}$
$Y=1$	$\frac{\binom{3}{0}\binom{2}{1}\binom{3}{3}}{\binom{8}{4}}=\frac{2}{70}$	$\frac{\binom{3}{1}\binom{2}{1}\binom{3}{2}}{\binom{8}{4}}=\frac{18}{70}$	$\frac{\binom{3}{2}\binom{2}{1}\binom{3}{1}}{\binom{8}{4}}=\frac{18}{70}$	$\frac{\binom{3}{3}\binom{2}{1}\binom{3}{0}}{\binom{8}{4}}=\frac{2}{70}$
$Y=2$	$\frac{\binom{3}{0}\binom{2}{2}\binom{3}{2}}{\binom{8}{4}}=\frac{3}{70}$	$\frac{\binom{3}{1}\binom{2}{2}\binom{3}{1}}{\binom{8}{4}}=\frac{9}{70}$	$\frac{\binom{3}{2}\binom{2}{2}\binom{3}{0}}{\binom{8}{4}}=\frac{3}{70}$	0

$$\binom{8}{4}=\frac{8!}{4!4!}=\frac{8\times7\times6\times5}{4\times3\times2\times1}=70$$

(a) $$E(X^2Y-2XY)=\sum_x\sum_y(x^2y-2xy)f(x,y)$$

$$=\sum_{x=0}^{3}\sum_{y=0}^{2}xy(x-2)f(x,y)$$

$$=(0)(0)(0-2)f(0,0)+(0)(1)(0-2)f(0,1)$$
$$+(0)(2)(0-2)f(0,2)+(1)(0)(1-2)f(1,0)$$
$$+(1)(1)(1-2)f(1,1)+(1)(2)(1-2)f(1,2)$$
$$+(2)(0)(2-2)f(2,0)+(2)(1)(2-2)f(2,1)$$
$$+(2)(2)(2-2)f(2,2)+(3)(0)(3-2)f(3,0)$$
$$+(3)(1)(3-2)f(3,1)+(3)(2)(3-2)f(3,2)$$
$$=-1\times\frac{18}{70}+(-2)\times\frac{9}{70}+3\times\frac{2}{70}+6\times0=\frac{-30}{70}$$
$$=-\frac{3}{7}\ \#$$

A: $-\frac{3}{7}$

(b) $$\mu_X-\mu_Y=\sum_x\sum_y xf(x,y)-\sum_x\sum_y yf(x,y)$$
$$=\sum_3\sum_2(x-y)f(x,y)=(0-0)f(0,0)+(0-1)f(0,1)$$
$$+(0-2)f(0,2)+(1-0)f(1,0)+(1-1)f(1,1)+(1-2)f(1,2)$$
$$+(2-0)f(2,0)+(2-1)f(2,1)+(2-2)f(2,2)+(3-0)f(3,0)$$
$$+(3-1)f(3,1)+(3-2)f(3,2)$$
$$=-\frac{2}{70}-\frac{6}{70}+\frac{3}{70}+\left(-\frac{9}{70}\right)+\frac{18}{70}+\frac{18}{70}+\frac{9}{70}+\frac{4}{70}=\frac{35}{70}=\frac{1}{2}\ \#$$

A: $\frac{1}{2}$

4.44

$\sum_{y=0}^{2} f(0,y) = 0 + \frac{2}{70} + \frac{3}{70} = \frac{5}{70}$

$\sum_{y=0}^{2} f(1,y) = \frac{3}{70} + \frac{18}{70} + \frac{9}{70} = \frac{30}{70}$

$\sum_{y=0}^{2} f(2,y) = \frac{9}{70} + \frac{18}{70} + \frac{3}{70} = \frac{30}{70}$

$\sum_{y=0}^{2} f(3,y) = \frac{3}{70} + \frac{2}{70} + 0 = \frac{5}{70}$

$\mu_x = 0 \times \frac{5}{70} + 1 \times \frac{30}{70} + 2 \times \frac{30}{70} + 3 \times \frac{5}{70} = \frac{105}{70} = \frac{3}{2}$

$\sum_{x=0}^{3} f(x,0) = 0 + \frac{3}{70} + \frac{9}{70} + \frac{3}{70} = \frac{15}{70}$

$\sum_{x=0}^{3} f(x,1) = \frac{2}{70} + \frac{18}{70} + \frac{18}{70} + \frac{2}{70} = \frac{40}{70}$

$\sum_{x=0}^{3} f(x,2) = \frac{3}{70} + \frac{9}{70} + \frac{3}{70} + 0 = \frac{15}{70}$

$\mu_y = 0 \times \frac{15}{70} + 1 \times \frac{40}{70} + 2 \times \frac{15}{70} = \frac{70}{70} = 1$

$E(XY) = \sum_x \sum_y xy f(x,y) = \sum_{x=0}^{3} \sum_{y=0}^{2} xy f(x,y)$

$= 0(0)f(0,0) + 0(1)f(0,1) + 0(2)f(0,2) + 1(0)f(1,0) + 1(1)f(1,1) + 1(2)f(1,2)$

$+ 2(0)f(2,0) + 2(1)f(2,1) + 2(2)f(2,2) + 3(0)f(3,0) + 3(1)f(3,1) + 3(2)f(3,2)$

$= 1 \times \frac{18}{70} + 2 \times \frac{9}{70} + 2 \times \frac{18}{70} + 4 \times \frac{3}{70} + 3 \times \frac{2}{70} + 6 \times 0 = \frac{90}{70} = \frac{9}{7}$

$\sigma_{XY} = E(XY) - \mu_X \mu_Y = \frac{9}{7} - \frac{3}{2} \times 1 = \frac{18}{14} - \frac{21}{14} = -\frac{3}{14}$ #

A: $-\frac{3}{14}$

4.60

$E(X) = 2(0.15 + 0.25 + 0.15) + 4(0.1 + 0.25 + 0.1)$

$= 2 \times 0.55 + 4 \times 0.45 = 1.1 + 1.8 = 2.9$

$E(Y) = 1(0.15 + 0.10) + 3(0.25 + 0.25) + 5(0.15 + 0.10)$

$= 0.25 + 1.5 + 1.25 = 3$

(a) $E(2X - 3Y) = 2E(X) - 3E(Y) = 2 \times 2.9 - 3 \times 3 = -3.2$ #

A: -3.2

(b) $E(XY) = E(X)E(Y) = 2.9 \times 3 = 8.7$ #

A: 8.7

4.78

$$\mu=\int_0^1 x\left[30x^2(1-x)^2\right]dx=\int_0^1\left[30x^3(1-2x+x^2)\right]dx$$

$$=\int_0^1 30x^5-60x^4+30x^3\,dx=5x^6-12x^5+\frac{15}{2}x^4\Big|_0^1=\left(5-12+\frac{15}{2}\right)-0=\frac{1}{2}$$

$$E(X^2)=\int_0^1 x^2\left[30x^2(1-x)^2\right]dx=\int_0^1\left[30x^4(1-2x+x^2)\right]dx$$

$$=\int_0^1 30x^6-60x^5+30x^4\,dx=\frac{30}{7}x^7-10x^6+6x^5\Big|_0^1$$

$$=\left(\frac{30}{7}-10+6\right)-0=\frac{2}{7}\qquad \sigma^2=E(X^2)-\mu^2=\frac{2}{7}-\left(\frac{1}{2}\right)^2=\frac{8-7}{28}=\frac{1}{28}$$

$$\sigma=\sqrt{\frac{1}{28}}=\frac{\sqrt{28}}{28}=\frac{2\sqrt{7}}{28}=\frac{\sqrt{7}}{14}\qquad \mu-2\sigma=\frac{1}{2}-2\times\frac{\sqrt{7}}{14}=\frac{7-2\sqrt{7}}{14}$$

$$\mu+2\sigma=\frac{1}{2}+2\times\frac{\sqrt{7}}{14}=\frac{7+2\sqrt{7}}{14}$$

$$P\left(\frac{7-2\sqrt{7}}{14}<X<\frac{7+2\sqrt{7}}{14}\right)=\int_{\frac{7-2\sqrt{7}}{14}}^{\frac{7+2\sqrt{7}}{14}}30x^2(1-x)^2dx$$

$$=\int_{\frac{7-2\sqrt{7}}{14}}^{\frac{7+2\sqrt{7}}{14}}30x^2(1-2x+x^2)dx=\int_{\frac{7-2\sqrt{7}}{14}}^{\frac{7+2\sqrt{7}}{14}}30x^4-60x^3+30x^2dx$$

$$=6x^5-15x^4+6x^3\Big|_{\frac{7-2\sqrt{7}}{14}}^{\frac{7+2\sqrt{7}}{14}}=\left[6\left(\frac{7+2\sqrt{7}}{14}\right)^5-15\left(\frac{7+2\sqrt{7}}{14}\right)^4+6\left(\frac{7+2\sqrt{7}}{14}\right)^3\right]$$

$$-\left[6\left(\frac{7-2\sqrt{7}}{14}\right)^5-15\left(\frac{7-2\sqrt{7}}{14}\right)^4+6\left(\frac{7-2\sqrt{7}}{14}\right)^3\right]=0.96998_{\#}$$

根據 Chebyshev's theorem：$P(\mu-k\sigma<X<\mu+k\sigma)\geq 1-\frac{1}{k^2}$

$$P(\mu-2\sigma<X<\mu+2\sigma)=P\left(\frac{7-2\sqrt{7}}{14}<X<\frac{7+2\sqrt{7}}{14}\right)\geq 1-\frac{1}{k^2}=1-\frac{1}{2^2}=0.75$$

$\Rightarrow 0.96998\geq 0.75\qquad\Rightarrow$ 符合 Chebyshev's theorem $_{\#}$

4.98 Let $f(x,y)$ be the joint probability function.
Let $g(x)$ be the marginal density function of X.
Let $h(x)$ be the marginal density function of Y.

(a)
$g(0)=f(0,0)+f(0,1)+f(0,2)=0.12+0.04+0.04=0.20$ #
$g(1)=f(1,0)+f(1,1)+f(1,2)=0.08+0.19+0.05=0.32$ #
$g(2)=f(2,0)+f(2,1)+f(2,2)=0.06+0.12+0.30=0.48$ #
$h(0)=f(0,0)+f(1,0)+f(2,0)=0.12+0.08+0.06=0.26$ #
$h(1)=f(0,1)+f(1,1)+f(2,1)=0.04+0.19+0.12=0.35$ #
$h(2)=f(0,2)+f(1,2)+f(2,2)=0.04+0.05+0.30=0.39$ #

$$f(x|2)=\frac{f(x,2)}{h(2)}$$

$$f(0|2)=\frac{f(0,2)}{h(2)}=\frac{0.04}{0.39}=\frac{4}{39}\ \#$$

$$f(1|2)=\frac{f(1,2)}{h(2)}=\frac{0.05}{0.39}=\frac{5}{39}\ \#$$

$$f(2|2)=\frac{f(2,2)}{h(2)}=\frac{0.30}{0.39}=\frac{30}{39}\ \#$$

(b)
$$E(X)=\sum_x x\,g(x)=\sum_{x=0}^{2} x\,g(x)=0\times0.20+1\times0.32+2\times0.48=1.28\ \#$$

$$Var(X)=\sum_x\left[x-E(X)\right]^2\cdot g(x)=\sum_{x=0}^{2}\left[x-E(X)\right]^2\cdot g(x)$$
$$=(0-1.28)^2\times0.20+(1-1.28)^2\times0.32+(2-1.28)^2\times0.48$$
$$=0.3277+0.0251+0.2488=0.6016\ \#$$

A: $E(X)=1.28$, $Var(X)=0.6016$

(c)
$$E(X|Y=2)=\sum_x x f(x|2)=\sum_{x=0}^{2} x f(x|2)=0\times\frac{4}{39}+1\times\frac{5}{39}+2\times\frac{30}{39}=\frac{65}{39}=\frac{5}{3}\ \#$$

$$Var(X|Y=2)=\sum_x\left[x-E(X|Y=2)\right]^2\cdot f(x|2)=\sum_{x=0}^{2}\left(x-\frac{5}{3}\right)^2\cdot f(x|2)$$
$$=\left(0-\frac{5}{3}\right)^2\times\frac{4}{39}+\left(1-\frac{5}{3}\right)^2\times\frac{5}{39}+\left(2-\frac{5}{3}\right)^2\times\frac{30}{39}$$
$$=\frac{25\times4+4\times5+1\times30}{9\times39}=\frac{150}{351}=\frac{50}{117}\ \#$$

A: $E(X|Y=2)=\frac{5}{3}$, $Var(X|Y=2)=\frac{50}{117}$