1. i)

   the sequence of states entered: (q_1, q_1, q_1, q_1). Output string: "000".

   ii)

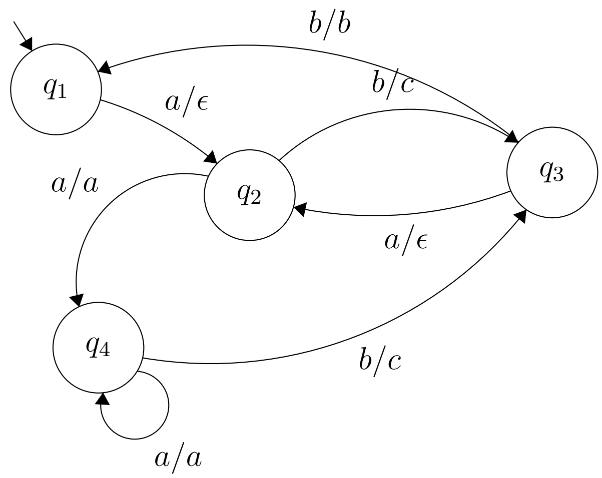
   the sequence of states entered: $(q_1, q_1, q_2, q_1, q_2)$. Output string: "0101".

   iii)

   the sequence of states entered: $(q_1, q_3, q_2, q_3, q_2)$. Output string: "1111".

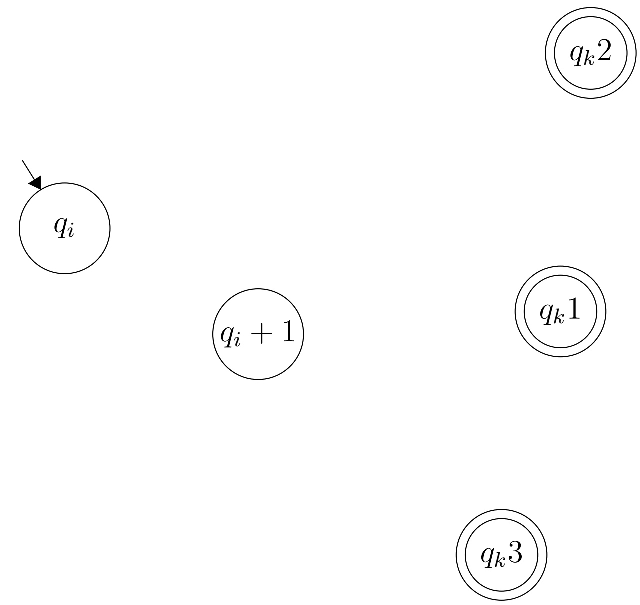
   iv)

   the sequence of states entered: (q_1). Output string: ϵ

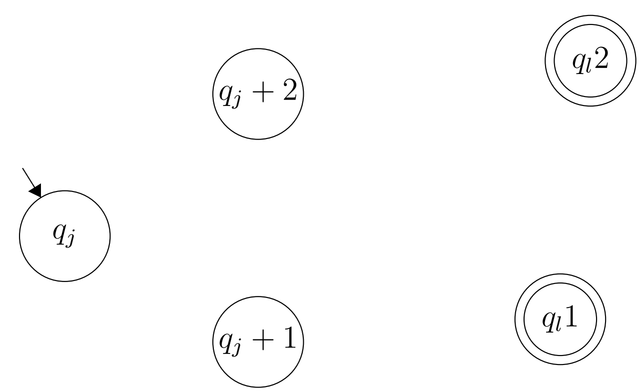
   

   The problem with the FST above is that if the program ends in state q2 or q4. The output string will be lacking one 'a' symbol. To fix this we could define the state machine to affix an a to the output string if the input string terminates in either of these states.

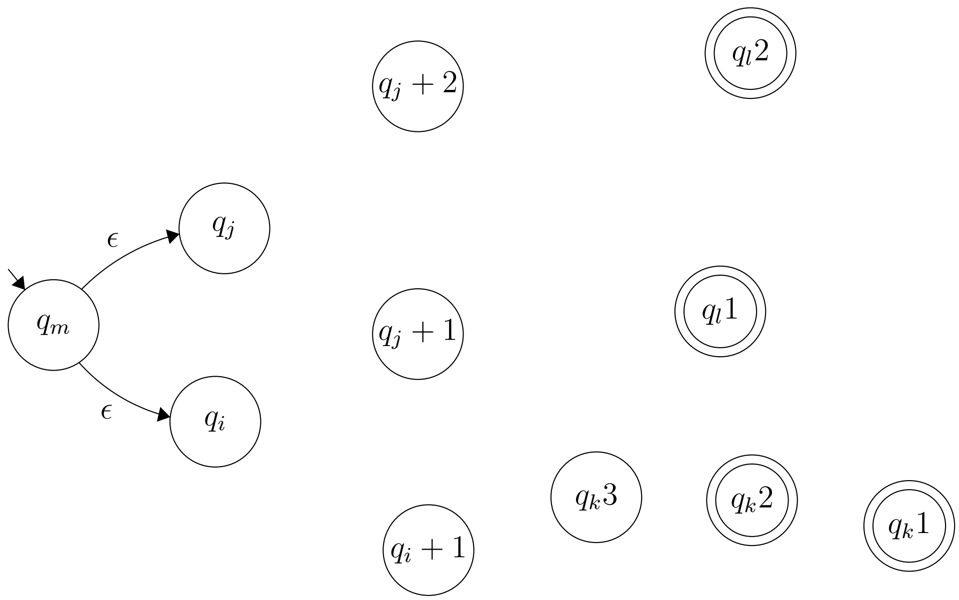
2. below is a shadow-abstract figure of a FST A1



Below is a shadow-abstraction figure of a FST A2



To show that the class of regular languages is closed under DROP-OUT() operation we simply combine the two machines known to accept a word w if the word is valid under the DROP-OUT() operation for their respective languages, A1 and A2. which are both of the same class which accepts any word w if w is legal as defined by the definition for DROP-OUT(A) in question 2 of homework 2. In order to combine them we construct a new machine with a start state which allows free transition to the start states of every machine which accepts any language in the class of A. the accept states for the new machine A_m is the union of all of the accept states of every machine in the class of A.



A_m accepts (w| w is a string of elements such that there is at least one element of the string is not the empty string and with the condition that if exactly one symbol from the string is removed, the new string is an element of the language A).

Let A_1 = (Q_1, Σ, δ_1, q_i, F_1) recognize A_1

Let A_2 = (Q_2, Σ, δ_2, q_j, F_2) recognizes A_2

Construct A_m = (Q, Σ, δ, q_0, F) to recognize A^*

1. Q = $\{q_0\} \cup Q_1 \cup Q_2$

the states A_m are the same as union of the states of every machine in the class A of machines in addition to a new start state.

2. the state q_0 is the new sart state.

3. F = $\{F_1 \cup F_2\}$

the accept states are the same as the accept states of the individual machines.

4. Define δ so that for any q $\in$ Q and any a $\in$ Σ_ϵ

δ (q, a) =

δ_1 (q,a) $q \in Q_1$ δ_2 (q,a) $q \in Q_2$ δ $\{q_i \cup q_j\}$ q = q_0 and a = ϵ

---

3. the NFA $D_k, = (Q_1, \Sigma, \delta_1, q_1, F_1)$.

   D_k accepts (w| w is a string composed of elements of Σ and such that the element k places from the right most end of the string is the character 'a').

Where

$Q_1 = \{q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

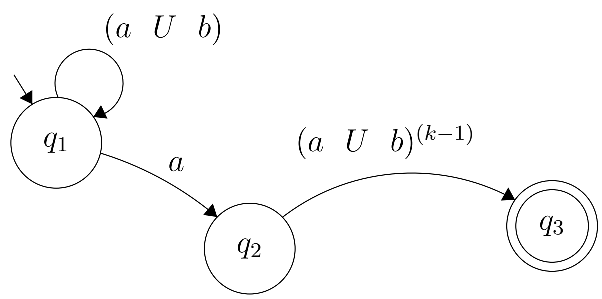
$\delta_1 = \{\ (q_1, a) \rightarrow q_2$

$(q_1, (a \cup b)) \rightarrow q_1$

$(q_2, (a \cup b)^{k-1}) \rightarrow q_3$

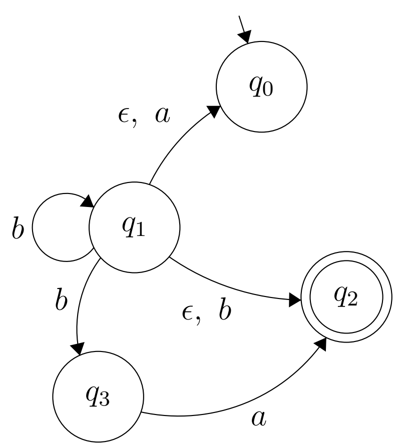
$F_1 = \{q_3\}$ ———————————————

below is a diagram of NFA D_k



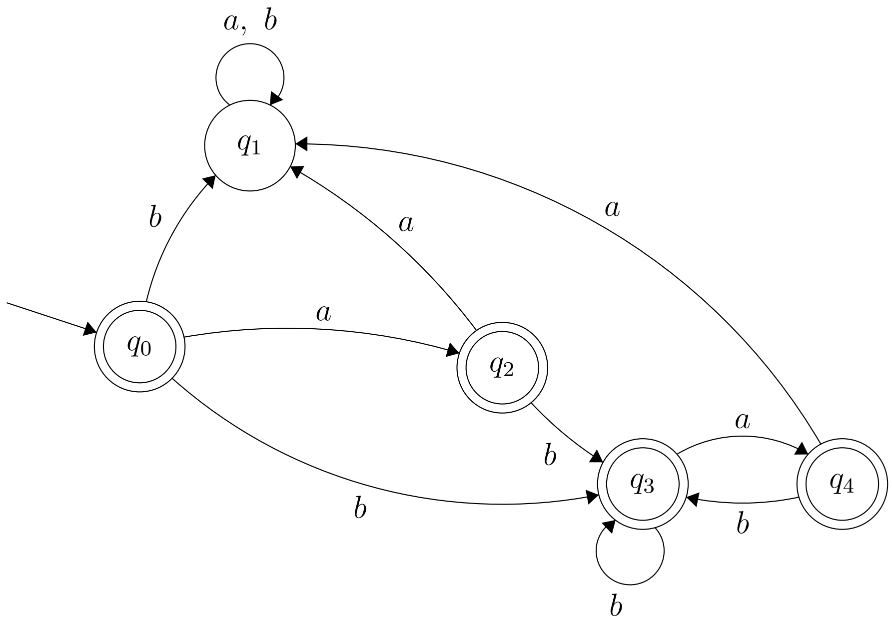
———————————————

4. a) NFA



———————————————-

b AND c) DFA



a, b
q_1
b
a
a
a
q_0
q_2
b
a
q_3
q_4
b
b
b