

Probability Practice

Jason Antal

2024-08-18

QUESTION: Part A. Visitors to your website are asked to answer a single survey question before they get access to the content on the page. Among all of the users, there are two categories: Random Clicker (RC), and Truthful Clicker (TC). There are two possible answers to the survey: yes and no. Random clickers would click either one with equal probability. You are also giving the information that the expected fraction of random clickers is 0.3. After a trial period, you get the following survey results: 65% said Yes and 35% said No. What fraction of people who are truthful clickers answered yes? Hint: use the rule of total probability.

```
knitr::opts_chunk$set(echo = TRUE)

p_rc <- 0.3 # Probability of being a Random Clicker
p_yes <- 0.65 # Overall probability of answering Yes
p_yes_rc <- 0.5 # Random Clickers have 50% chance of answering Yes

p_yes_tc <- (p_yes - p_yes_rc * p_rc) / (1 - p_rc)

#ANSWER:
cat("The percent of Truthful Clickers who answered Yes is:", round(p_yes_tc,4)*100, "%.", '\n',"As a fr

## The percent of Truthful Clickers who answered Yes is: 71.43 %.
## As a fraction, this means roughly 7/10 of truthful clickers answered yes.
```

QUESTION: Part B. Imagine a medical test for a disease with the following two attributes:

The sensitivity is about 0.993. That is, if someone has the disease, there is a probability of 0.993 that they test positive. The specificity is about 0.9999. This means that if someone doesn't have the disease, there is a probability of 0.9999 that they test negative. In the general population, incidence of the disease is reasonably rare: about 0.0025% of all people have the disease.

Suppose someone tests positive. What is the probability that they have the disease?

```
sensitivity <- 0.993 # P(Positive | Disease)
specificity <- 0.9999 # P(Negative | No Disease)
prevalence <- 0.000025 # P(Disease)

# Calculate P(Positive)
p_positive <- sensitivity * prevalence + (1 - specificity) * (1 - prevalence)

# Calculate P(Disease | Positive)
p_disease_given_positive <- (sensitivity * prevalence) / p_positive

# ANSWER
cat('The probability that someone who tests positive for the disease actually has the disease is', round(p_disease_given_positive,4)*100, "%.", '\n')
```

The probability that someone who tests positive for the disease actually has the disease is 19.89 %.