MATH4853_PDE_PS1_Q3A

October 14, 2020

1 DT8248 Introduction to PDEs Problem Set 1

1.1 Question 3a

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 1 \qquad (1)$$

$$u(x,0) = x^2 \qquad x > 0$$

1.1.1 Solve using Parametric form of Method of Characteristics

$$\frac{dx}{dt} = 1 \qquad (1) \qquad (2)$$
$$\frac{dy}{dt} = 1 \qquad (2) \qquad (3)$$
$$\frac{du}{dt} = 1 - u \qquad (3) \qquad (4)$$
$$(5)$$

We need to parameterize the initial conditions. Let s be the parameter along the data curve. Let (s,0) be a typical point along the data curve. s > 0. The initial conditions tell us that:

$$u = s^2, \qquad at \quad (s,0), \quad s > 0$$

At t = 0:

$$x = s, \quad y = 0, \quad u = s^2$$

1.1.2 Solving Equation (1)

$$\frac{dx}{dt} = 1 \qquad (6)$$
$$\therefore \int dx = \int dt \qquad (7)$$
$$\therefore x = t + A \qquad (8)$$

When t = 0, x = s.

$$\therefore s = 0 + A \tag{9}$$
$$\therefore A = s \tag{10}$$
$$\therefore x = t + s \tag{11}$$

1.1.3 Solving Equation (2)

$$\frac{dy}{dt} = 1 \tag{12}$$
$$\therefore \int dy = \int dt \tag{13}$$
$$\therefore y = t + B \tag{14}$$

When t = 0, y = 0.

$$\therefore 0 = 0 + B \tag{15}$$
$$\therefore B = 0 \tag{16}$$
$$\therefore y = t \tag{17}$$

1.1.4 Solving Equation 3

$$\frac{du}{dt} = 1 - u \tag{18}$$
$$\therefore \int \frac{1}{1-u} du = \int dt \tag{19}$$
$$\therefore \int \frac{1}{u-1} du = -\int dt \tag{20}$$
$$\therefore \ln |(u-1)| = -t + C \tag{21}$$
$$\therefore |(u-1)| = e^{-t+C} \tag{22}$$
$$\therefore |(u-1)| = e^C e^{-t} \tag{23}$$
$$\therefore u - 1 = \pm e^C e^{-t} \tag{24}$$
$$\therefore u = De^{-t} + 1 \tag{25}$$

When t=0, s^2

$$\therefore s^2 = De^{-0} + 1 \tag{26}$$
$$\therefore s^2 = D + 1 \tag{27}$$
$$\therefore D = s^2 - 1 \tag{28}$$
$$\therefore u = (s^2 - 1)e^{-t} + 1 \tag{29}$$

We need to find t and s in terms of x and y. We know $y = t$.

$$\therefore x - y = s \tag{30}$$

$$\therefore u(x, y) = [(x - y)^2 - 1]e^{-y} + 1 \tag{31}$$

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