

MATH4853_PDE_PS1_Q3A

October 15, 2020

1 DT8248 Introduction to PDEs Problem Set 1

1.1 Question 3a

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 1 \quad (1)$$

$$u(x, 0) = x^2 \quad x > 0$$

1.1.1 Solve using Parametric form of Method of Characteristics

$$\frac{dx}{dt} = 1 \quad (1) \quad (2)$$

$$\frac{dy}{dt} = 1 \quad (2) \quad (3)$$

$$\frac{du}{dt} = 1 - u \quad (3) \quad (4)$$

$$(5)$$

We need to parameterize the initial conditions. Let s be the parameter along the data curve. Let $(s, 0)$ be a typical point along the data curve. $s > 0$. The initial conditions tell us that:

$$u = s^2, \quad \text{at } (s, 0), \quad s > 0$$

At $t = 0$:

$$x = s, \quad y = 0, \quad u = s^2$$

1.1.2 Solving Equation (1)

$$\frac{dx}{dt} = 1 \quad (6)$$

$$\therefore \int dx = \int dt \quad (7)$$

$$\therefore x = t + A \quad (8)$$

When $t = 0$, $x = s$.

$$\therefore s = 0 + A \quad (9)$$

$$\therefore A = s \quad (10)$$

$$\therefore x = t + s \quad (11)$$

1.1.3 Solving Equation (2)

$$\frac{dy}{dt} = 1 \quad (12)$$

$$\therefore \int dy = \int dt \quad (13)$$

$$\therefore y = t + B \quad (14)$$

When $t = 0$, $y = 0$.

$$\therefore 0 = 0 + B \quad (15)$$

$$\therefore B = 0 \quad (16)$$

$$\therefore y = t \quad (17)$$

1.1.4 Solving Equation 3

$$\frac{du}{dt} = 1 - u \quad (18)$$

$$\therefore \int \frac{1}{1-u} du = \int dt \quad (19)$$

$$\therefore \int \frac{1}{u-1} du = - \int dt \quad (20)$$

$$\therefore \ln |(u-1)| = -t + C \quad (21)$$

$$\therefore |(u-1)| = e^{-t+C} \quad (22)$$

$$\therefore |(u-1)| = e^C e^{-t} \quad (23)$$

$$\therefore u - 1 = \pm e^C e^{-t} \quad (24)$$

$$\therefore u = D e^{-t} + 1 \quad (25)$$

When $t=0$, s^2

$$\therefore s^2 = D e^{-0} + 1 \quad (26)$$

$$\therefore s^2 = D + 1 \quad (27)$$

$$\therefore D = s^2 - 1 \quad (28)$$

$$\therefore u = (s^2 - 1)e^{-t} + 1 \quad (29)$$

We need to find t and s in terms of x and y . We know $y = t$.

$$\therefore x - y = s \quad (30)$$

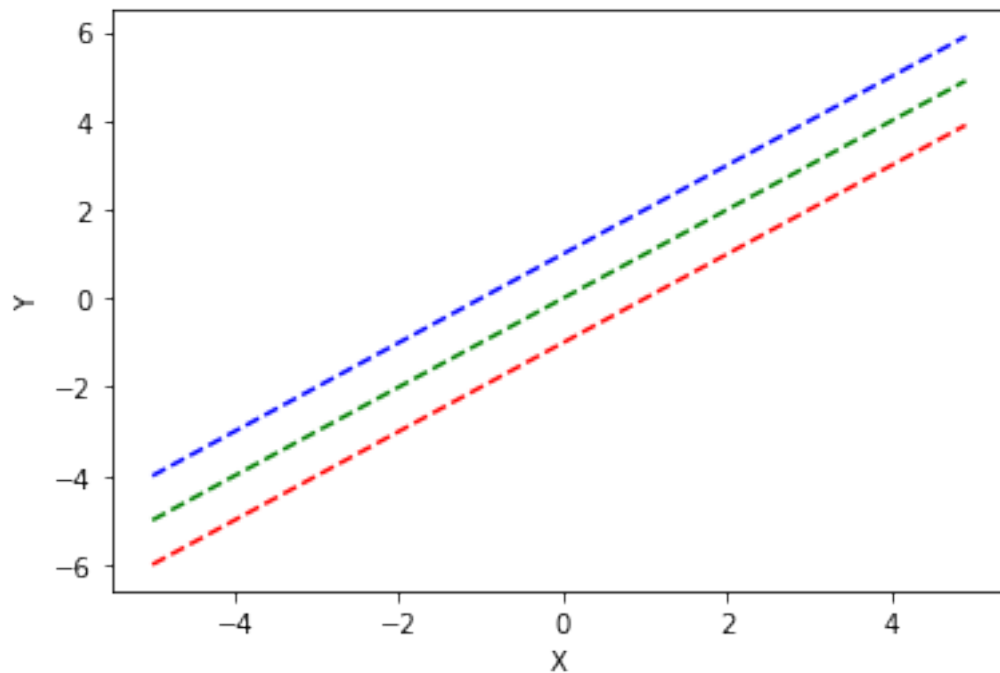
$$\therefore u(x, y) = [(x - y)^2 - 1]e^{-y} + 1 \quad (31)$$

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[2]: import numpy as np
import math
import matplotlib.pyplot as plt #Matplotlib for python's basic plotting
import plotly
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[4]: X = np.arange(-5, 5, 0.1)

Y0 = X - 1
Y1 = X + 0
Y2 = X + 1

plt.plot(X, Y0, 'r--', X, Y1, 'g--', X, Y2, 'b--')
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```



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