MATH4853_PDE_PS1_Q3B

October 15, 2020

1 DT8248 Introduction to PDEs Problem Set 1

1.1 Question 3b

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 1 \tag{1}$$

$$u(x, x) = x^2 \qquad x > 0$$

1.1.1 Solve using Parametric form of Method of Characteristics

$$\frac{dx}{dt} = 1 \qquad (1) \tag{2}$$

$$\frac{dy}{dt} = 1 \qquad (2) \tag{3}$$

$$\frac{du}{dt} = 1 - u \qquad (3) \tag{4}$$

$$\tag{5}$$

We need to parameterize the initial conditions. Let s be the parameter along the data curve. Let (s,s) be a typical point along the data curve. s > 0. The initial conditions tell us that:

$$u = s^2, \qquad at \quad (s, s), \quad s > 0$$

At t = 0:

$$x = s, \quad y = s, \quad u = s^2$$

1.1.2 Solving Equation (1)

$$\frac{dx}{dt} = 1 \tag{6}$$

$$\therefore \int dx = \int dt \tag{7}$$

$$\therefore x = t + A \tag{8}$$

When t = 0, x = s.

$$\therefore s = 0 + A \tag{9}$$
$$\therefore A = s \tag{10}$$
$$\therefore x = t + s \tag{11}$$

1.1.3 Solving Equation (2)

$$\frac{dy}{dt} = 1 \tag{12}$$
$$\therefore \int dy = \int dt \tag{13}$$
$$\therefore y = t + B \tag{14}$$

When t = 0, y = s.

$$\therefore s = 0 + B \tag{15}$$
$$\therefore B = s \tag{16}$$
$$\therefore y = t + s \tag{17}$$

1.1.4 Solving Equation 3

$$\frac{du}{dt} = 1 - u \tag{18}$$
$$\therefore \quad \int \frac{1}{1-u} du = \int dt \tag{19}$$
$$\therefore \quad \int \frac{1}{u-1} du = -\int dt \tag{20}$$
$$\therefore \quad \ln |(u-1)| = -t + C \tag{21}$$
$$\therefore \quad |(u-1)| = e^{-t+C} \tag{22}$$
$$\therefore \quad |(u-1)| = e^{C} e^{-t} \tag{23}$$
$$\therefore \quad u - 1 = \pm e^{C} e^{-t} \tag{24}$$
$$\therefore \quad u = De^{-t} + 1 \tag{25}$$

When t=0, s^2

$$\therefore s^2 = De^{-0} + 1 \tag{26}$$
$$\therefore s^2 = D + 1 \tag{27}$$
$$\therefore D = s^2 - 1 \tag{28}$$
$$\therefore u = (s^2 - 1)e^{-t} + 1 \tag{29}$$

We need to find t and s in terms of x and y.
We know $y = t + s$, and $x = t + s$. So $x = y$, therefore $s = 0$ and

$$\therefore t = \frac{x+y}{2} \tag{30}$$

$$\therefore u(x,y) = -e^{-\frac{x+y}{2}} + 1 \tag{31}$$

[2]:

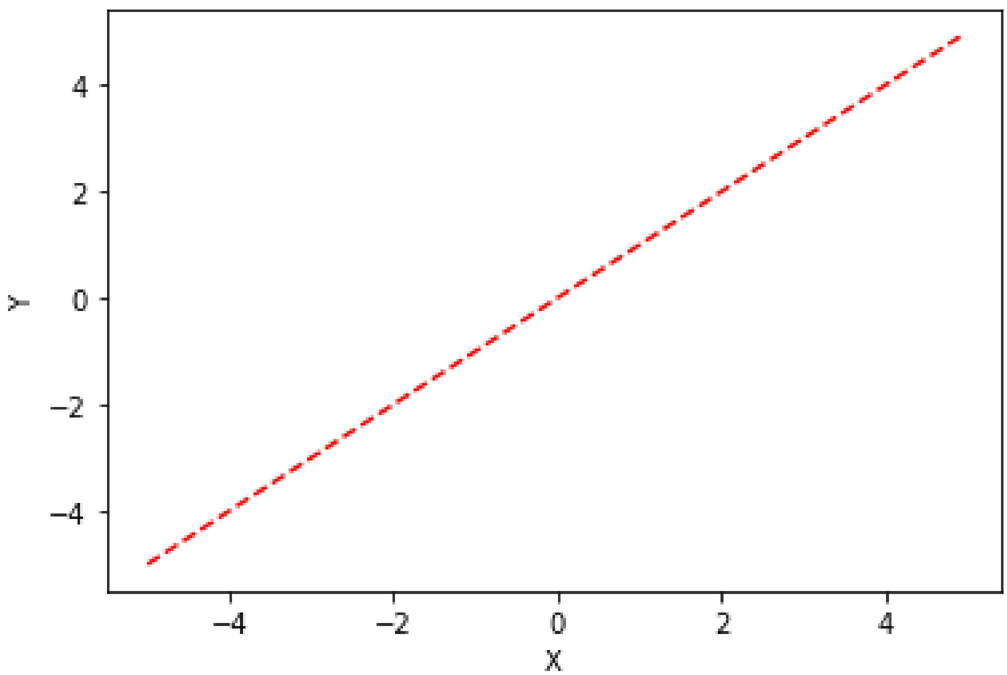
```
import numpy as np
import math
import matplotlib.pyplot as plt #Matplotlib for pythons basic plotting
import plotly
```

[3]:

```
X = np.arange(-5, 5, 0.1)

Y0 = X +0


plt.plot(X, Y0,'r--')
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```



[ ]: