# MATH4853_PDE_PS2_Q3B

October 29, 2020

### 0.1 Problem Sheet 2: Q3b

Find, in parametric form, the solution to the PDE and hence obtain a relationship between $u, x, y$ in:

$$uu_x + u_y = 1$$

Where $u = \frac{s}{2}$ when $x = s = y$ for $0 \leq s \leq 1$

$$\frac{dx}{dt} = u \quad (1) \qquad (1)$$
$$\frac{dy}{dt} = 1 \quad (2) \qquad (2)$$
$$\frac{du}{dt} = 1 \quad (3) \qquad (3)$$
$$(4)$$

Solving Equation 2

$$\frac{dy}{dt} = 1 \qquad (5)$$
$$\therefore \int dy = \int dt \qquad (6)$$
$$\therefore y = t + A \qquad (7)$$

When t = 0, y = s.

$$\therefore s = 0 + A \qquad (8)$$
$$\therefore A = s \qquad (9)$$
$$\therefore y = t + s \quad (4) \qquad (10)$$

Solving Equation 3

$$\frac{du}{dt} = 1 \tag{11}$$

$$\therefore \int du = \int dt \tag{12}$$

$$\therefore u = t + B \tag{13}$$

When t = 0, $u = \frac{s}{2}$.

$$\therefore \frac{s}{2} = 0 + B \tag{14}$$

$$\therefore B = \frac{s}{2} \tag{15}$$

$$\therefore u = t + \frac{s}{2} \qquad (5) \tag{16}$$

Solving Equation 1 using (5)

$$\frac{dx}{dt} = u \tag{17}$$

$$\frac{dx}{dt} = t + \frac{s}{2} \tag{18}$$

$$\therefore \int dx = \int t + \frac{s}{2} dt \tag{19}$$

$$\therefore x = t^2 + \frac{s}{2} t + C \tag{20}$$

When t = 0, $x = s$.

$$\therefore s = 0 + 0 + C \tag{21}$$

$$\therefore C = s \tag{22}$$

$$\therefore x = t^2 + \frac{s}{2} t + s \qquad (6) \tag{23}$$

Need to eliminate $s$ and $t$ from (4), (5), and (6)

$$x = t^2 + \frac{s}{2} t + s \tag{24}$$

$$\therefore x = t(t + \frac{s}{2}) + s \tag{25}$$

$$\therefore x = tu + s \qquad (7)\, as\, u = t + \frac{s}{2} \tag{26}$$

$$\tag{27}$$

From (5): $s = 2u - 2t$ (8) From (7) using (8):

$$x = tu + 2u - 2t \tag{28}$$
$$\therefore x = t(u-2) + 2u \tag{29}$$
$$\therefore t(u-2) = x - 2u \tag{30}$$
$$\therefore t = \frac{x-2u}{u-2}(9) \tag{31}$$
$$\tag{32}$$

From (4):

$$y = t + s \tag{33}$$
$$\therefore s = y - t \tag{34}$$
$$\therefore s = y - \frac{x-2u}{u-2} \qquad (10) \tag{35}$$
$$\tag{36}$$

Sub (9) and (10) in (7):

$$x = tu + s \tag{37}$$
$$\therefore x = u\frac{x-2u}{u-2} + y - \frac{x-2u}{u-2} \tag{38}$$
$$\therefore (x-y)(u-2) = ux - 2u^2 - x + 2u \tag{39}$$
$$\therefore xu - 2x - yu + 2y = ux - 2u^2 - x + 2u \tag{40}$$
$$\therefore 2u^2 - u(y+2) = x - 2y \tag{41}$$
$$\tag{42}$$

Doesn't seem much point simplifying further (could use quadratic formula).

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