Chapter 24

24.1-2

- if there is a path from s to v then:
  there must be a shortest-path between s and v, so we can assume the path has k edges, so the path is: $s \to v_0 \to v_1 \ldots v_{k-3} \to v$
  from ***Lemma 24.2*** we know $v.d = \delta(s.v)$
  so $v.d = w(s, v_0) + w(v_0, v_1) + \ldots + w(v_{k-3}, s) < \infty$

- if **Bellman-Ford** terminates with $v.d < \infty$ then:
  we get that the vertice v is relaxed, we can easily get that if a vertice can be relaxed, so:
  $$v.d \leq \delta(s'.u) + w(s', v)$$
  so s' can be reached from s, and s' has edges with v, so there is a path between s and v.

24.1-3

Algorithm 1 Bellman-Ford(G,w,s)

1: INITIALIZE-SINGLE-SOURCE(G,S)
2: copy each vertice's d to a array $COPY[|G.V|]$
3: **for** $i = 1$ to $|G.v - 1|$ **do**
4:     **for** each edge $\in G.v$ **do**
5:         RELAX(u, v, w)
6:     **end for**
7:     compare the latest vertice's d to the old one in $COPY$ array, if nothing changed, we make it and break.
8: **end for**
9: **for** each edge $(u, v) \in G.E$ **do**
10:     **if** $v.d > u.d + w(u, v)$ **then return** FALSE
11:     **end if**
12: **end for**

24.1-4

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we only need to change $return\ FALSE$ to $v.d = -\infty$ in line 7
because if vertice v iS the vertice we want to find, and we assume the vertice u is also in the path including negative-weight cycles we know before each $REALX(u, v)$, v.d will be larger than $u.d+w(u.v)$ so only vertices like v can match the $line6$'s condition.

24.2-2

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because the last vertice's adjance list is empty, it doesn't matter if the last loop does or not, because no more RELAX func will be called

24.3-2

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in proof, we get $\delta(s,y) \leq \delta(s.u)$, but when there are negative-weight path, $\delta(s,y) \leq \delta(s.u)$ will not be true always. it can easily get when there is a negative-wight edge from u to y

24.3-4

- first, $s.d = 0 s.\pi = NIL$
- for any vertice, if its π is NIL, its d should be $+\infty$

Algorithm 2 Check-Bellman-Ford(G,w,s)

```
for  each vertice v ∈ G.v do
    for  each edge u ∈ G.adj[v] do
        if v.d > u.d + w(u.v) then return False
        end if
    end for
end for
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24.3-7

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the num of edges $edges = \sum_{(u,v)\in E} w(u,v) - 1$
the num of vertices $vertices = edges - 1$

24.5-2

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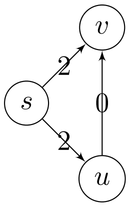
let vertice s, u, v be the three vertices of a triangle,and the weight of $(u,v), (s,u), (s,v)$ are 2,2,0



24.5-5

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the result we want get is like this

$$v.\pi = v_1, v_1.\pi = v_2, \dots, v_n.\pi = v$$

to simplify, we can let $v.\pi = u, u.\pi = v$ so the graphy in $24.5 - 2$ can match it, there is a shortest path to v is $s \to u \to v$, and also a shortest path to u is $s \to v \to u$, so let $v.\pi = u, u.\pi = v$, we make it.

24.2

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assume that box $a(a_1, a_2, \ldots, a_d)$ nests within box $b(b_1, b_2, \ldots, b_d)$, and box $c(c_1, c_2, \ldots, c_d)$ nests within box c so there is permution π on $\{1, 2, \ldots, d\}$,such that:

$$a_{\pi(1)} < b_1, a_{\pi(2)} < b_2, \ldots, a_{\pi(d)} < b_d$$

$$b_{\pi(1)} < c_1, b_{\pi(2)} < c_2, \ldots, b_{\pi(d)} < c_d$$

let the above two sets be two func f and g,we can easily get a new func $h = f(g)$ beause both are bijection. we can get

$$a_{\pi(1)} < c_1, a_{\pi(2)} < c_2, \ldots, a_{\pi(d)} < c_d$$

so nesting relation is transitive.

b

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sort both two dimension from small to large, then let $booleana = x_i > y_i foriinrange(1, d+1)$ after the a be initialized, and if its value changes, then one can not nests within another total time is $O(d \log d)$

c

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we can first sort all n box's dimension. which is $O(nd \log d)$, we let each box be a vertice in a graphand then for each pair vertices u, v in $G.V$ first we should find if there is path from u to v, if it is, that menas u can be nests within v else to check if one box can nest within another,if u can nest within v so add a arrow from u to v. at last find the longest path in G.
the running time is $O(nd(\log d + n))$

24.3

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I refer to the answer on the website，the main idea is tranforming the formula

$$R[i_1, i_2] \cdot R[i_2, i_3] \ldots R[i_k, i-1] > 1$$

to

$$-\ln(R[i_1, i_2]) - \ln(R[i_2, i_3]) \ldots - \ln(R[i_k, i_1]) < 0$$

so we kan use Bellman Ford algorithm to detect if exists. when BF returns FALSE, it exists

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when BF return FALSE, which vertice gose wrong?
the vertice v is the one whose d will be relaxed in $|V|_{th}$ loop in the code, but unfortunately we only have $|V| - 1$ loop, so we can change Bellman Ford algorithm let it has $|2 \times V| - 1$ loop but after $|V| - 1$ loop

we will check and we will save the vertice that cause FALSE and not return，and let its predecessor be u, all the vertices we save are the vertices on the nagetive-weight cycles. so we can get the cycle by these vertices and their predecessor