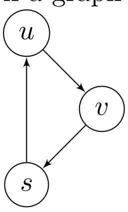
7.1

a

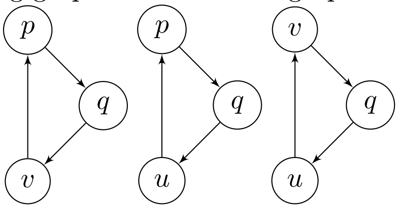
suppose D is not strong, so there are two vertices m, n, s.t there is a m n path ,but there isn't a $n - m$ path, so we choose another vertex v(neight m or n). if we remove v,so D-v is not strong because there is not a n m path. So D is strong.

b

if a graph C od order 3 is strong, so it must be a circle like this.



so if the graph D's 4 vertices is u, v, p, q, so if we remove u,v,p each time D-u,D-v,D-p is still a strong graph.so these three graphs maybe



we know the sum of indegree and outdegree of vertex q is 3, is can be 2 in-edge and 1 out-edge of 1 in-edge and 2 out-edge, but each triangle that q lies on will choose two edges that one incident to q and another incident from q and every triangle chooses differently. it's impossible because vertex q can provide at most two different combinations but there are three triangles.

7.2

if G is Eulerian, we can find a Eulerian circuit and we can assign a direction on tcircuit, and assign each edge a direction, so itis an Eulerian orientation.

7.4

if digraph D is strong

then for eac pair vertices u,v there is a u-v path and a v-u path. if we reverse the direction iof every arc of D, so u-v path will be v-u path in $\vec{D}$, and v-u path will be a u-v path in $\vec{D}$, so for each pair u,v vertices in $\vec{D}$, there are both u-v path and v-u path

if digraph $\vec{D}$ is strong it's same as we prove above, bacause it's symmetrical

7.5

1. if D is strong

for any two sets A and B. then for a vertex in A, a vertex in B, because there is a u-v path, so there is an arc from A to B. because there is a v-u path, so there is an arr from B to A

2 prove D is strong

D's underlying graph is connected, otherwise D will has morn than two components and obviously impossible.

for each pair vertices u,v in V firstly, we let A be u and let v in B. for any vertices incident to u in B, we add them to A and keep finding the vertices that incident to them. if we find v, stop. for any vertices incident from u in B, we add them to B and keep finding the vertices that incident from them. if we find v, stop.

we must can find v because (fail to prove it)

7.9

if toutnament T is transitive:

because there is Hamiltonain path in T $v_1 \to v_2 ... \to v_n$, so if the outdegree of v_i is k then v_{i-1} is at least k+1 because T is transitive ervery vertex incident from v_i is incident from v_{i-1} and v_i is also incident from v_{i-1} so for each pair vertices v_i, v_j the degree v_i is larger than v_j is i<j

if every two vertices of T has distinct outdegrees.

we just prove that T don't contain Hamiltonain circle use Pigeonhole principle, if T has a Hamiltonain circle then n vertices there outdegree is from 1 to n-1,must has at least two vertices has same outdegree.

so T contains no Hamiltonain circle. so T is transitive

7.10

the shortest u-v path is folowingf, and the vertives from v_1 to v will be incident to u otherwise the $\vec{d}$ will be smaller than k. the number of vertices from v_1 to v is k-1 so id u >= k-1 $u \to v_1 \to v_2 ... v$

7.13

min ($\vec{d}$(u,v),$\vec{d}$(v,u)) is 1, and another must be larger than 1, because there is only one between arc(u,v) and arc(v,u)

7.14

a

if the Tournaments of order 3 contains a Hamiltonain circle. every team can be the champion.

b

first we know in a Trounaments $\sum indrgeee$ and $\sum outdegree$ equals,so if a Trounaments with even order, each vertex has same indegree m,and outdegree n ,because m+n is odd so ,m!=n so $\sum indrgeee$!= $\sum outdegree$. so it impossible fo all vertices has same outdegree and same indegree.

7.15

from Throrem 7.10 we know T has a Hamiltonain circle of length n;

from Throrem 7.11 we know there exists a T-v is a strong Tournament;

from Throrem 7.10 we know T-v has a Hamiltonain circle of length n-1;

....

from Throrem 7.11 we know there exists a tournament T_3 of order 4 is a strong Tournament;

from Throrem 7.10 we know T_3 has a Hamiltonain circle of length 3;
so we get it.