Electric Charge, Force and Field, Continued

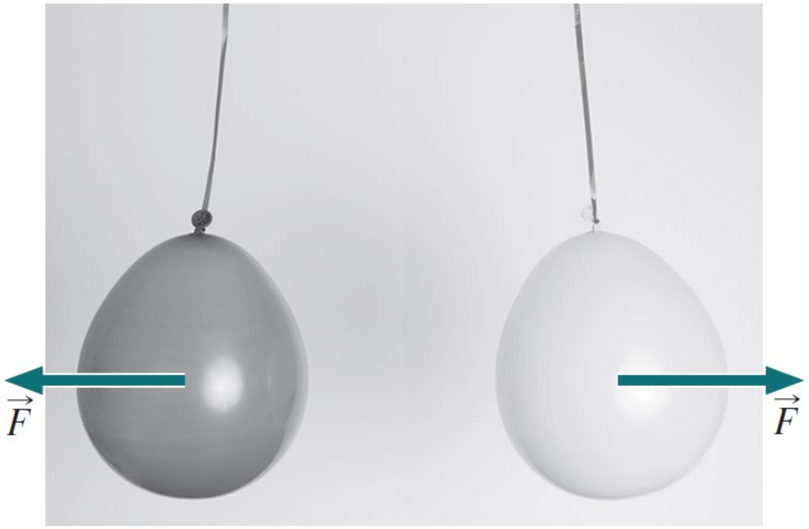
2025-03-03

Announcements/reminders

- Chapter 20 HW is due Friday March 7
- Update: MT1 scores will be posted by tomorrow, graded exams returned on Wednesday
- Today:
  - Continue on Electric Charge, Force and Field – Chapter 20

From last time: Coulomb’s Law

- Like charges repel, and opposite charges attract.
- According to **Coulomb’s law**, when charges are small compared to their separation (point-like charges), the force between the charges:
    - Is directed along the line joining the charges.
    - Is proportional to the product of the charges.
    - Is inversely proportional to the square of their separation.



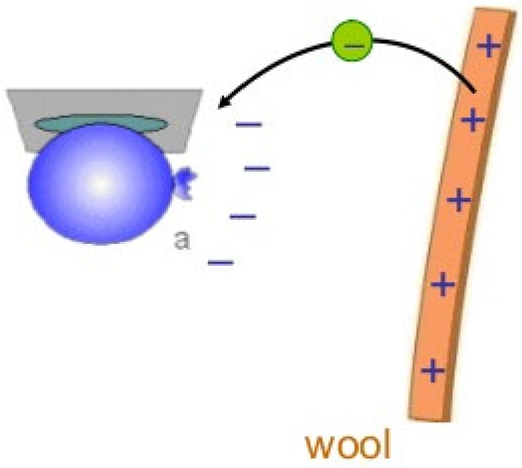
Static Electricity

From phys.org

Static electricity

Sometimes electrons can get transferred from one material to another

If you rub a balloon against a wool sweater, electrons get transferred from the wool to the balloon – the wool becomes positively charged, the balloon becomes negatively charged



Charge transfer and the triboelectric series

There is a hierarchy of which materials most readily give up electrons and which tend to gain electrons when put in contact

…again, the charge which flows is the negative charge, electrons. Positive nuclei do not have the mobility to flow from one material to another

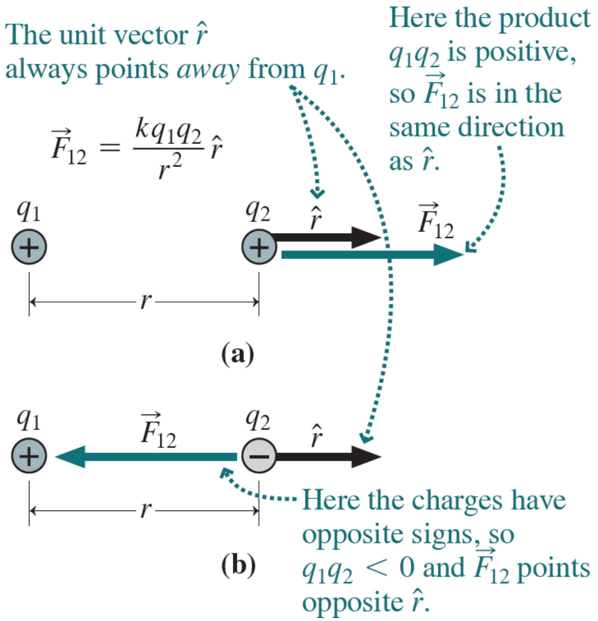
Most positively charged
+
Polyurethane foam
Hair, oily skin
Nylon, dry skin
Glass
Acrylic, Lucite
Leather
Rabbit's fur
Quartz
Cat's fur
0
Wood (*Small negative charge*)
Rubber balloon
Nickel, Copper
Brass, Silver
Gold, Platinum
Synthetic rubber
Polyester
Styrene and polystyrene
Plastic wrap
Polyethylene (like Scotch tape)
Polypropylene
Vinyl (PVC)
Teflon
Silicone rubber
–
Most negatively charged

Coulomb's Law (continued)

- In mathematical terms, the electric force that a charge q_1 exerts on q_2 is given by Coulomb's law:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$$

Where k is roughly $9.0\times10^9\,\mathrm{N\,m^2/C^2}$, and $\hat{r}$ is a unit vector pointing from q_1 to q_2.



Useful numbers

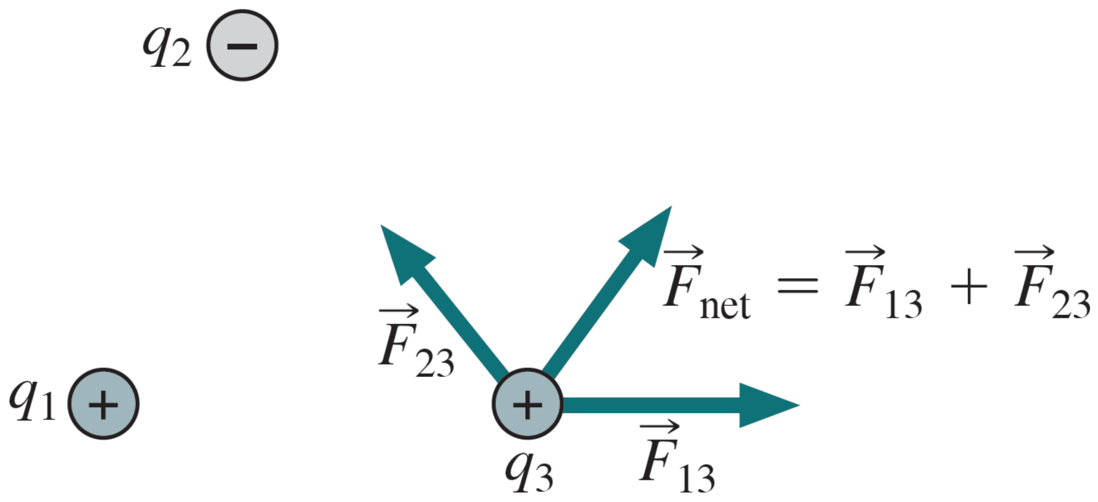
- **Proton (p)**
  - Charge= $+1.6\text{x}10^{-19}$C=$+1e$
  - Mass=m_p=$1.66\text{x}10^{-27}$kg
- **Electron (e)**
  - Charge= $-1.6\text{x}10^{-19}$C=$-1e$
  - mass=m_e=$9.11\text{x}10^{-31}$kg
- **Neutron (n)**
  - Charge= neutral =$0e$
  - Mass=m_n=$1.67\text{x}10^{-27}$kg

The total electric charge on an object is always some multiple of e

Electrons cannot be cut in half!

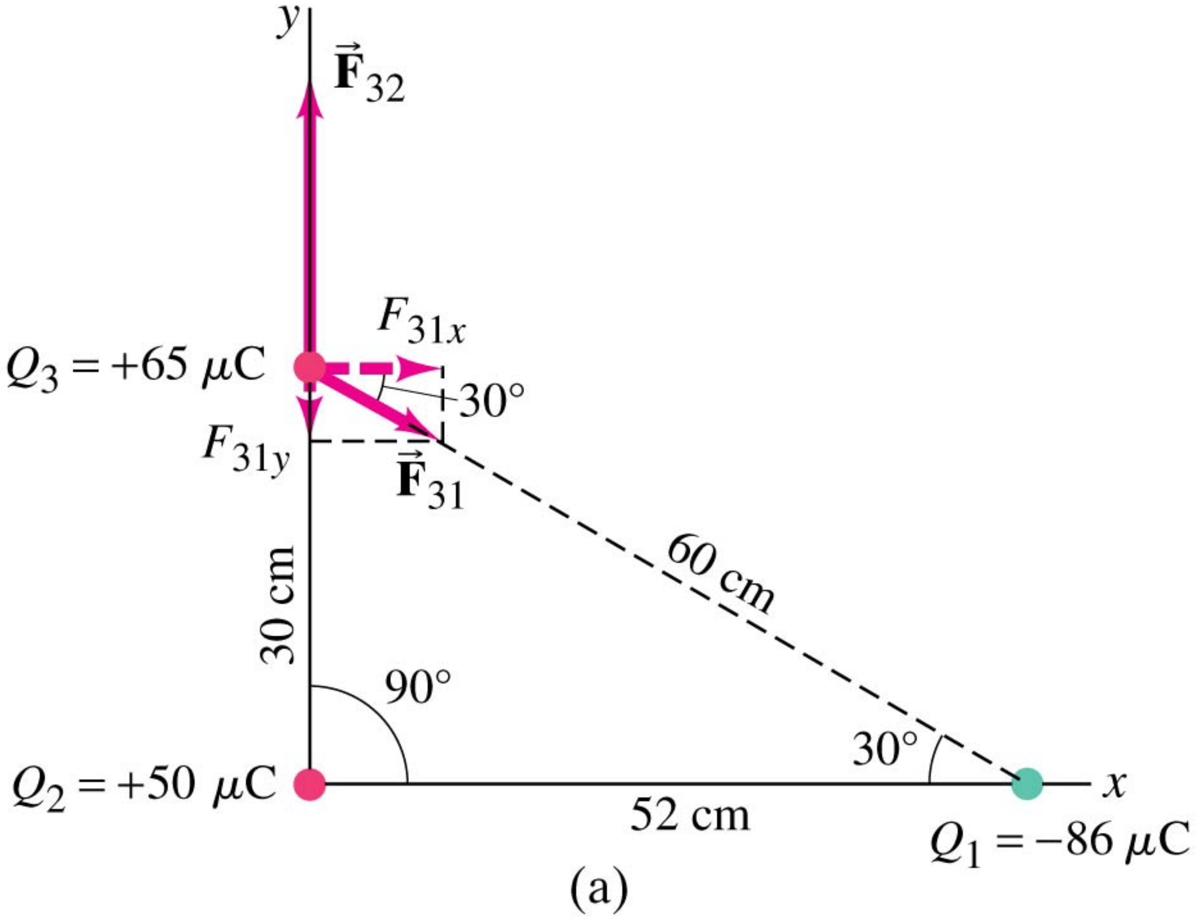
Point Charges and the Superposition Principle

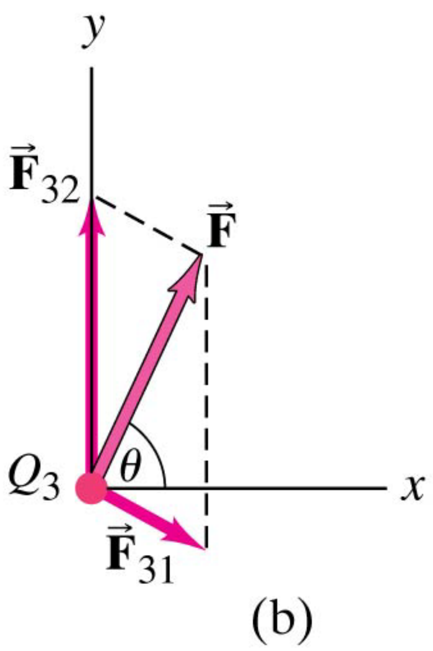
- In the figure below, how can we calculate the electric force experienced by q_3 when it is simultaneously in the presence of both q_1 and q_2?
  - Experiment shows that we can still calculate the force that q_1 exerts on q_3 and the force that q_2 exerts on q_3 using Coulomb's law.
  - The net force acting on q_3 is found to be the vector sum of these two forces.



Example

Calculate the net force on Q_3 due to Q_1 and Q_2





See solution on board

Point Charges and the Superposition Principle (continued)

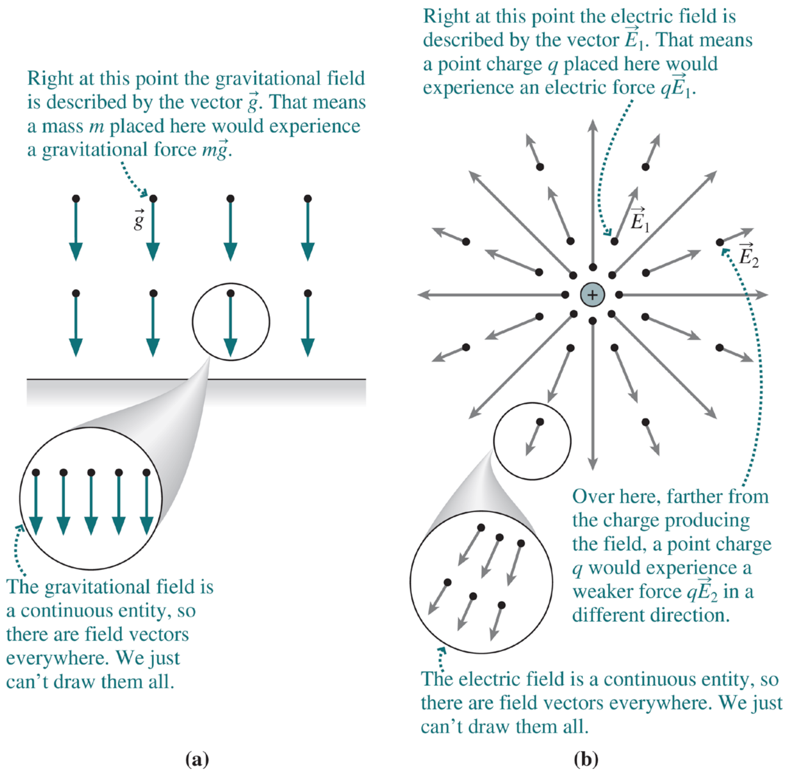
- In general, the fact that electric forces act independently and add vectorially is known as the **superposition principle**. To determine the force that any collection of charge (called a **charge distribution**) exerts on a given charge q, we proceed as follows:
  - Divide the charge distribution into simpler parts (perhaps point charges), whose effect is already understood.
  - Find the force that each part exerts on q.
  - Add the forces vectorially to find the net force that the charge distribution exerts on q.

The Electric Field

- The electric field at a point in space is the force per unit charge that a charge *q* placed at that point would experience:

$$\vec{E} = \frac{\vec{F}}{q}$$

- The force on a charge *q* in an electric field is $\vec{F} = q\vec{E}$.
- The electric field is analogous to the gravitational field, which gives the force per unit mass.



The Electric Field (continued)

- We can determine the electric field at any point with a small **test charge**—a large test charge might cause nearby charges to move and thus alter the field we want to measure!

$$\vec{E} = \frac{\vec{F}}{q}$$

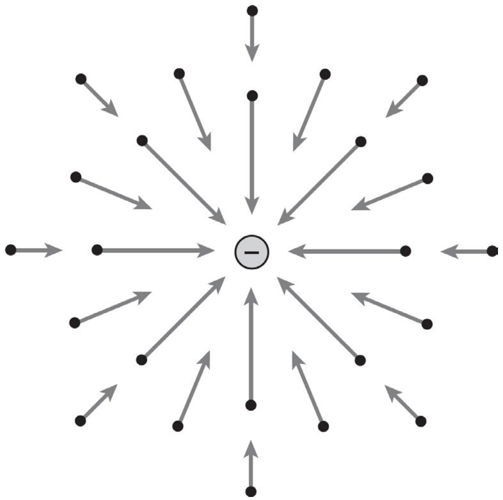
- The magnitude of the electric field, which is known as the **field strength**, is measured in N/C.
- The force on a positive test charge is parallel to the electric field.
- The force on a negative test charge is opposite to the electric field.

The Field of a Point Charge

- The field of a point charge is radial, outward for a positive charge and inward for a negative charge.

$$\vec{E}_{\text{point charge}} = \frac{kq}{r^2}\hat{r}$$

- Note that in the figure below, $\hat{r}$ points radially outward, but $q\hat{r}$ points radially inward—since the charge is negative.



Fields of Charge Distributions

- The superposition principle we talked about for Coulomb's law works for field calculations too:

$$\vec{E} = \sum \vec{E}_i = \sum \frac{kq_i}{r_i^2}\hat{r}_i$$

In this equation, r_i represents the distance from the given source point (where each q_i is located) to the **field point** (the fixed point where the electric field is calculated). The

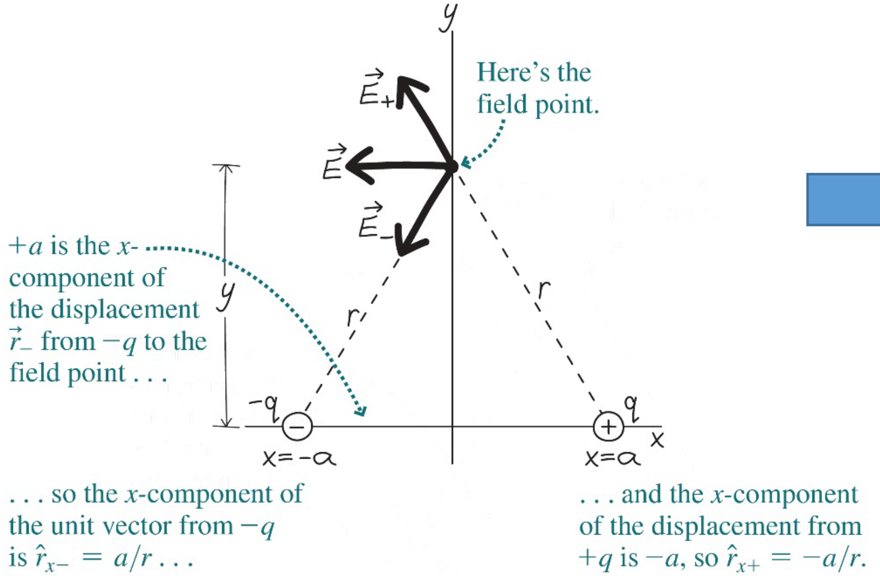
vector $\hat{r}_i$ always points from the corresponding source point to the field point.

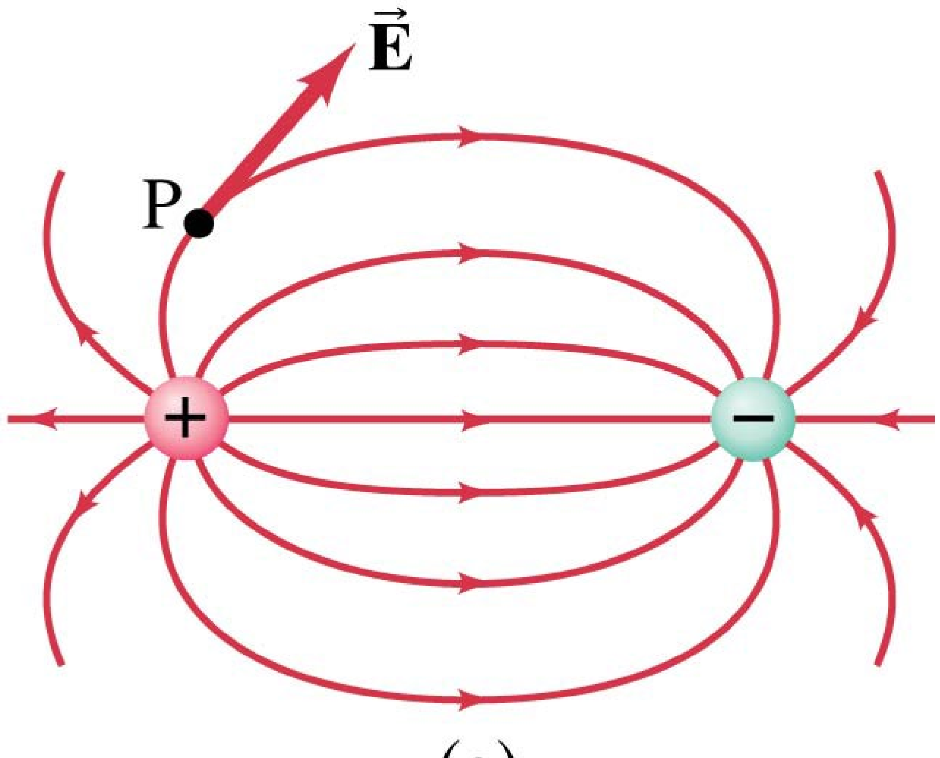
EXAMPLE: Let's examine the electric field of two charges of equal magnitude but opposite sign… +q and -q

Example: electric dipole

An **electric dipole** consists of two point charges of equal magnitude but opposite signs, held a short distance apart:

The dipole is electrically neutral, but the separation of its charges results in an electric field.





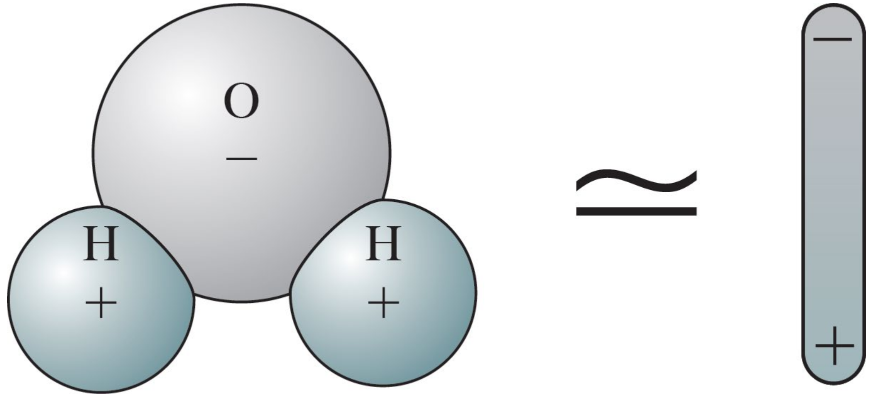
(a)

See E-field simulation

The Electric Dipole

Many charge distributions, especially molecules, behave like electric dipoles.

- When you are far enough from a dipole, its electric field is:
  - Proportional to its **electric dipole moment:** $p = qd$.
  - Inversely proportional to the **cube** of the distance to the dipole.
  - Note: we will revisit the dipole approximation soon, when we talk about electric potential



E-field simulator

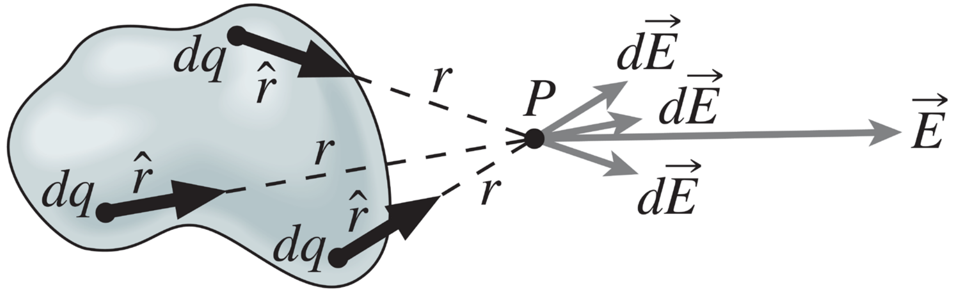
Experiment with different charge configurations at:

https://phet.colorado.edu/sims/html/charges-and-fields/latest/charges-and-fields_en.html

Continuous Charge Distributions

- Charge ultimately resides on individual particles, but it's often convenient to consider it distributed continuously on a line, over an area, or throughout space:
  - The electric field of a charge distribution follows by summing—that is, integrating—the fields of individual charge elements *dq*, each treated as a point charge:

$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$



Charge distribution

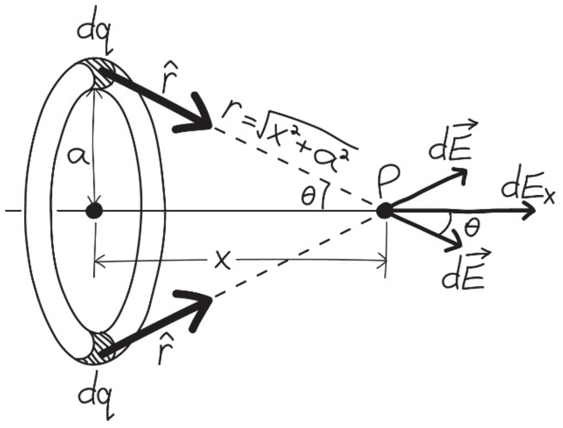
Evaluating the Field: A Charged Ring

- A ring of radius *a* carries an evenly distributed charge *Q*. Find the electric field at any point on its axis:
  - Divide the ring into charge elements *dq*.
  - By symmetry, the components of the electric field parallel to the plane of the ring cancel.
  - Using $\hat{r}_x = \cos\theta = x/r,$ we obtain:

$$E = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{kxdq}{r^3} = \int_{\text{ring}} \frac{kxdq}{(x^2+a^2)^{3/2}} = \frac{kx}{(x^2+a^2)^{3/2}} \int_{\text{ring}} dq,$$

which reduces to

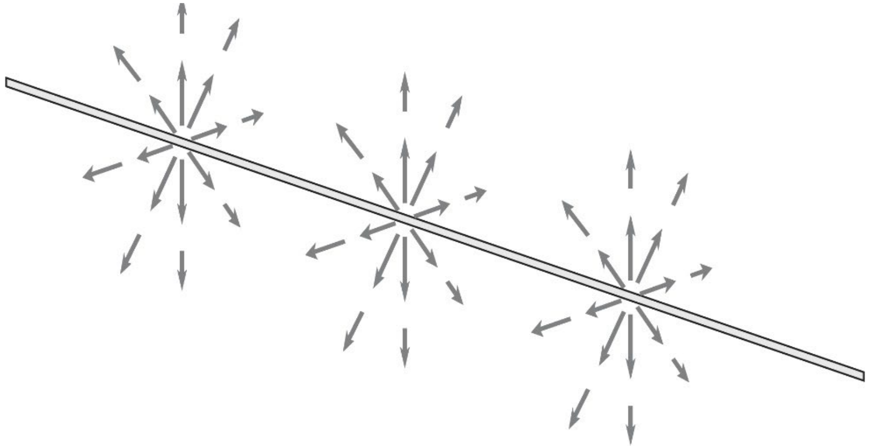
$$E = \frac{kQx}{(x^2+a^2)^{3/2}}.$$

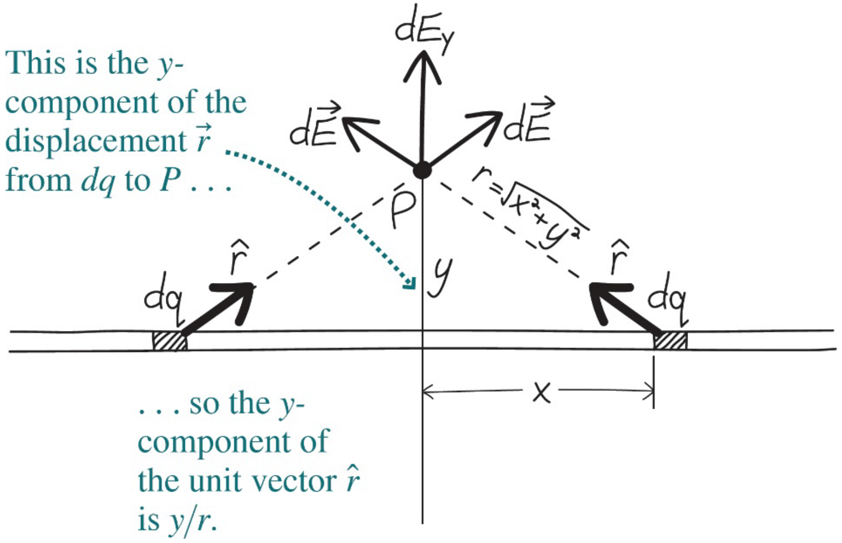


Infinite Line of Charge (1 of 2)

- Find the electric field of a long wire (treat as infinite) that coincides with the *x*-axis and carries a uniform line charge density λ (units: C/m):
  - By symmetry, the field is directed radially from the wire.
  - To find the field at a point on the *y*-axis, we set

$$dq = \lambda dx \text{ and } r = \sqrt{x^2 + y^2}.$$

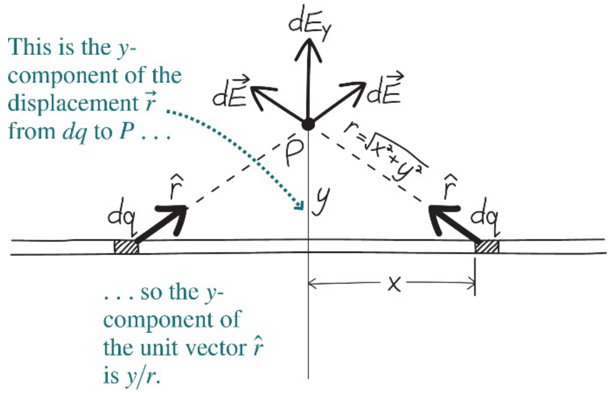




Infinite Line of Charge (2 of 2)

- The integral becomes:

$$E = E_y = \int_{-\infty}^{+\infty} \frac{k\lambda dx}{r^2} \cdot \frac{y}{r} = k\lambda y \int_{-\infty}^{+\infty} \frac{dx}{(x^2 + y^2)^{3/2}}$$



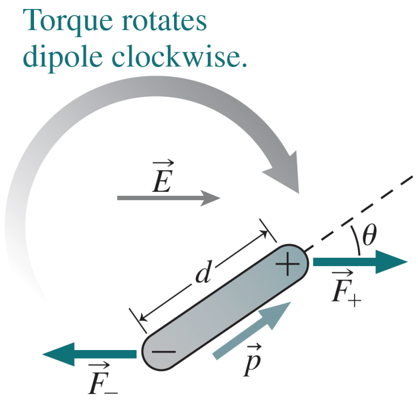
- This integral can be evaluated by standard methods to give:

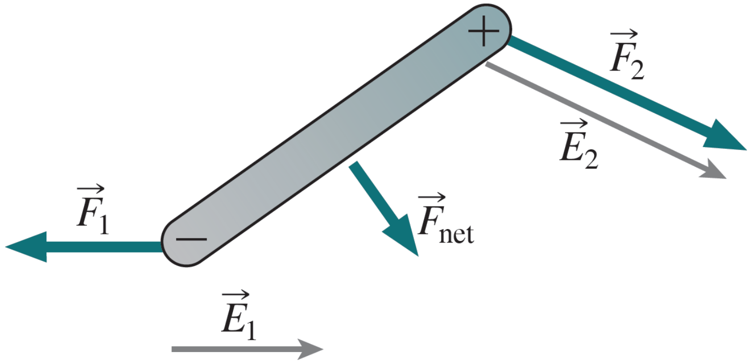
$$E = \frac{2k\lambda}{y}$$

- Note that *y* represents the radial distance from the line, which is usually called *r* (not the same as the *r* used in our analysis).
- Thus, the field of an infinite line decreases at a slower rate than that of a point charge.

Matter in Electric Fields

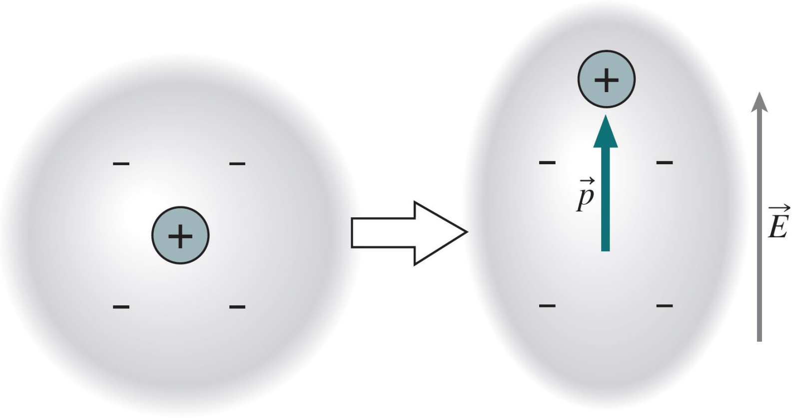
- For a point charge q in an electric field $\vec{E}$,
$$\vec{a} = q\vec{E}/m.$$
- A dipole in an electric field experiences a torque that tends to align the dipole moment with the field: $\vec{\tau} = \vec{p} \times \vec{E}$.
- If the field is not uniform, the dipole also experiences a net force.
- A dipole in an electric field has a potential energy $U = -\vec{p} \cdot \vec{E}$.

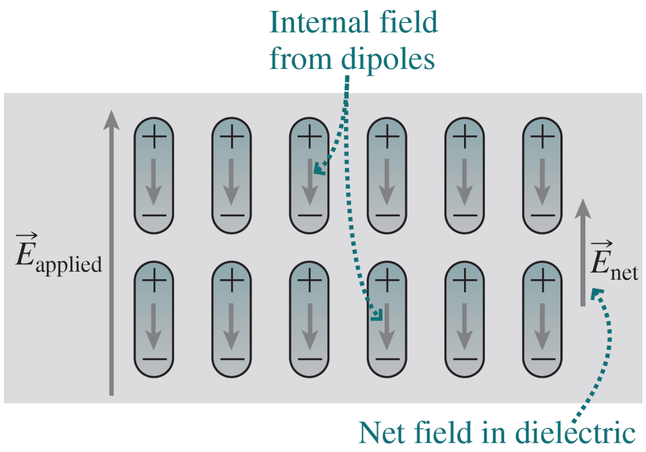




Conductors, Insulators, and Dielectrics

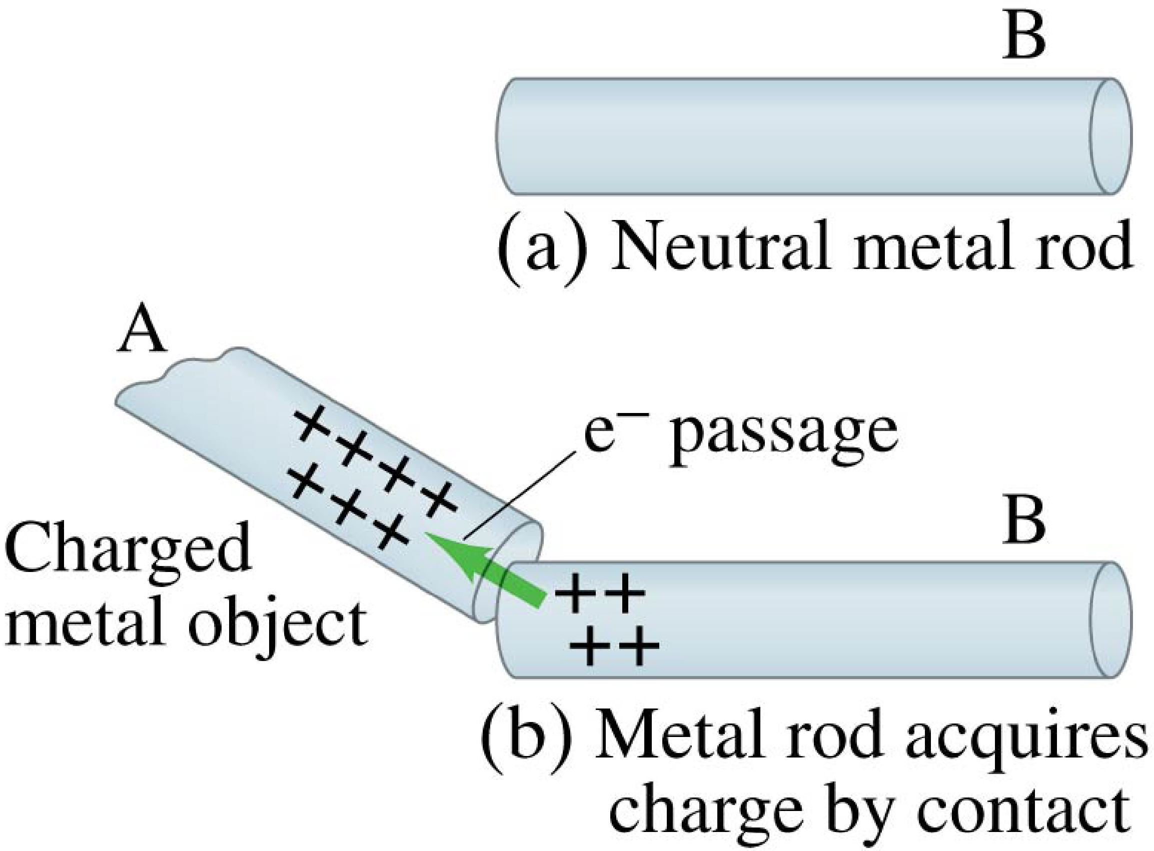
- Materials in which charge is free to move are **conductors**.
- Materials in which charge is not free to move are **insulators**:
  - Insulators generally contain molecular dipoles, which experience torques and forces in electric fields:
    - Such materials are called **dielectrics**.
  - Even if molecules are not intrinsically dipoles, they acquire **induced dipole moments** as a result of electric forces stretching the molecule.
  - Alignment of molecular dipoles reduces an externally applied field.



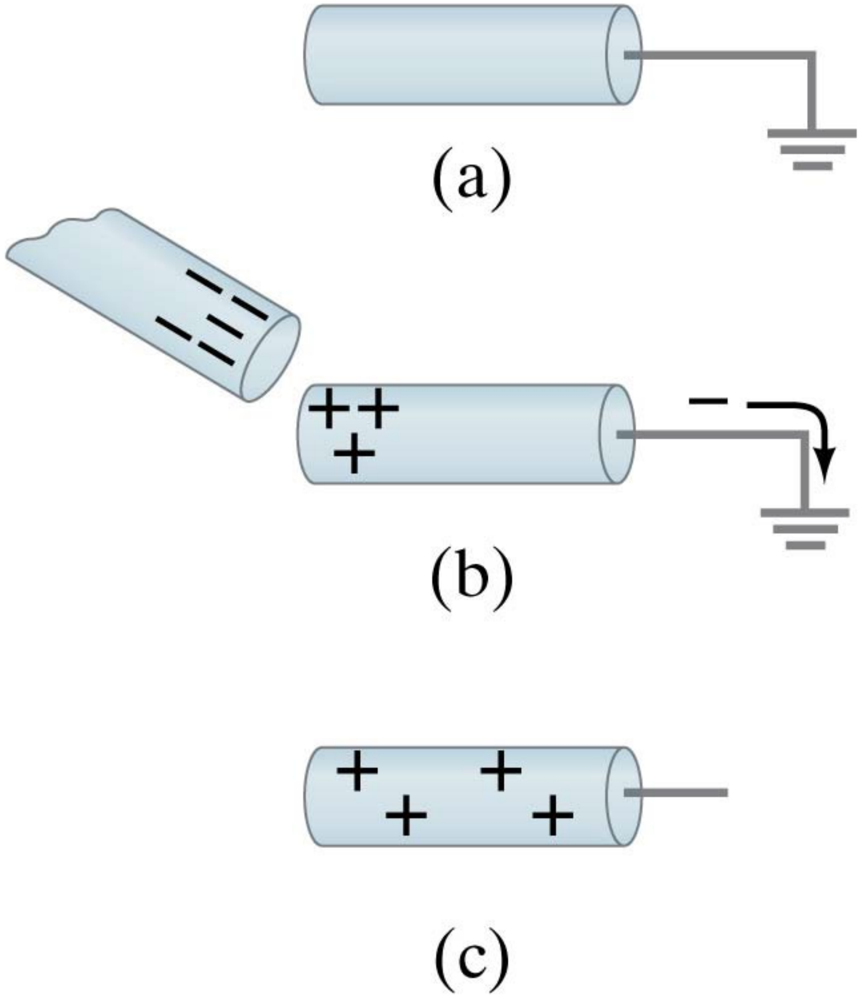


Charging via conduction

Metal objects can be charged via conduction



… or by <u>induction</u>

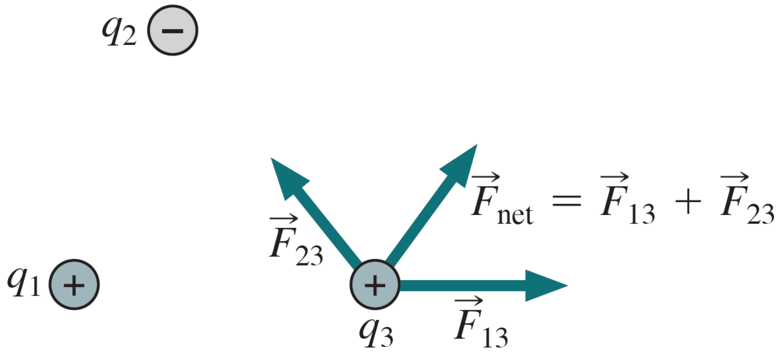


Summary

- **Electric charge** is a fundamental property of matter:
  - Charge comes in two varieties, positive and negative.
  - Charge is conserved.
  - The force between two charges is given by Coulomb's law:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$$

- The electric force obeys the **superposition principle,** meaning the forces due to individual charges sum vectorially.



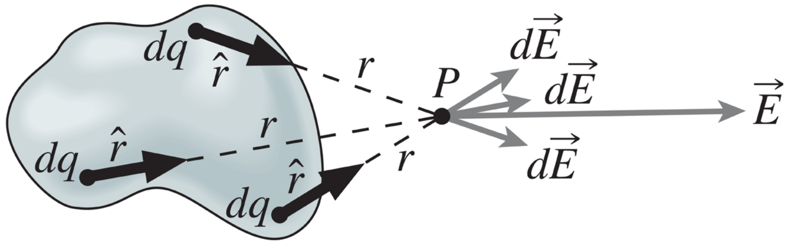
Summary (continued)

- The **electric field** describes the force per unit charge at a given point:

$$\vec{E} = \vec{F}/q$$

  - The fields of discrete charge distributions are calculated by summation.
  - The fields of continuous charge distributions are calculated by integration.

- A point charge experiences a force $\vec{F} = q\vec{E}$ in an electric field.
- A dipole in an electric field experiences a torque. If the field is not uniform, the dipole also experiences a net force.



Charge distribution