Data Driven Reduced Order Modeling

Aryn Harmon[1] and Jason Turner[2]
University of Illinois at Urbana-Champaign, Champaign, Illinois, U.S.A.
[1] arynh2@illinois.edu, [2] jasonet2@illinois.edu

Abstract

Reduced Order Modeling (ROM) can be applied to high-dimensional problems to reduce the computational cost of finding solutions. A high fidelity model was created for several partial differential equations of physical significance. Using data from these models, accurate reduced order models were constructed.

1 Main Objectives

1. Make online computation possible in computationally challenging problems.
2. Speed up analysis of problems with many degrees of freedom.
3. Produce an understanding of physical processes which are currently obscured by complexity.

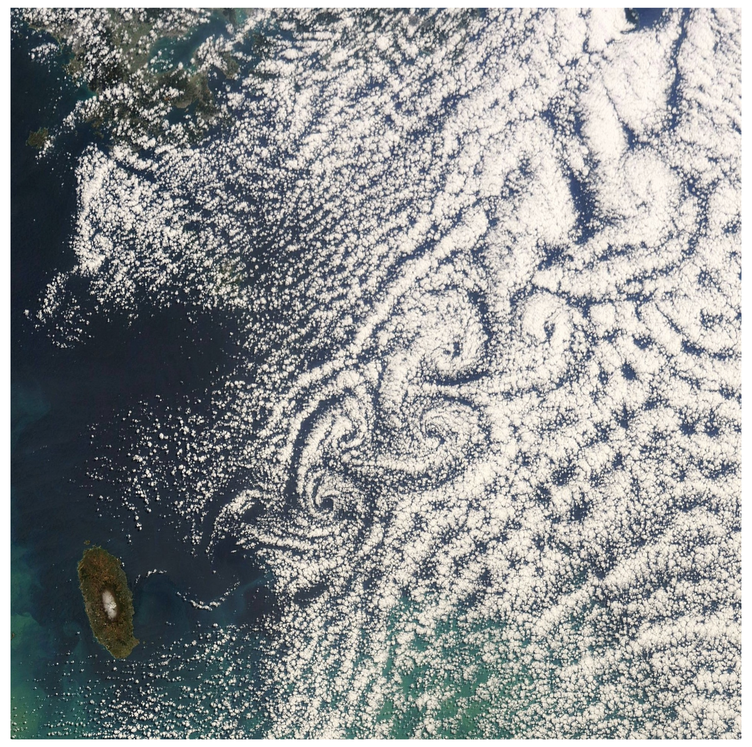


Figure 1: Atmospheric von Karman Vortex Street [1]

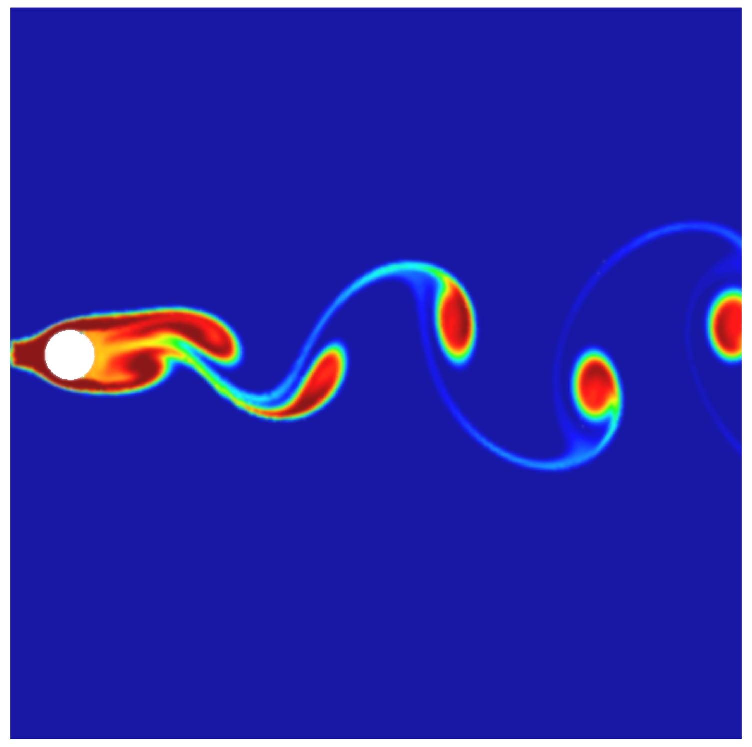


Figure 2: Simulated von Karman Vortex Street [2]

2 Introduction

- Linear Convection and Diffusion Equation describes movement and diffusion of a wave in one dimension.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$
$$\vec{u}^{\,n+1} = \mathcal{L}(\vec{u}^{\,n}) = \mathbf{B}\vec{u}^{\,n}$$

- Viscous Burger's Equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$
$$\vec{u}^{\,n+1} = \mathcal{L}(\vec{u}^{\,n}) + \mathcal{N}(\vec{u}^{\,n})$$

- Black-Scholes Model prices European options.

$$\frac{\partial w}{\partial t} = rw - rx\frac{\partial w}{\partial x} - \frac{1}{2}v^2x^2\frac{\partial^2 w}{\partial x^2}$$

3 Methods

3.1 Full Order/High Fidelity Model

3.1.1 Linear Convection and Diffusion Equation

Finite Difference Scheme:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} + \mathcal{O}(\Delta x^2)$$
$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} + \mathcal{O}(\Delta x^2)$$
$$\vdots$$
$$\frac{\partial \vec{u}_i}{\partial t} = \left(\frac{\nu}{\Delta x^2} - \frac{c}{2\Delta x}\right)u_{i+1} - \left(\frac{2}{\Delta x^2}\right)u_i + \left(\frac{\nu}{\Delta x^2} + \frac{c}{2\Delta x}\right)u_{i-1}$$

3.1.2 Viscous Burger's Equation

Newton's Method:

$$\mathbf{J}\left(u^{n+1} - u^n\right) - \mathbf{R}\left(u^n\right) = 0$$

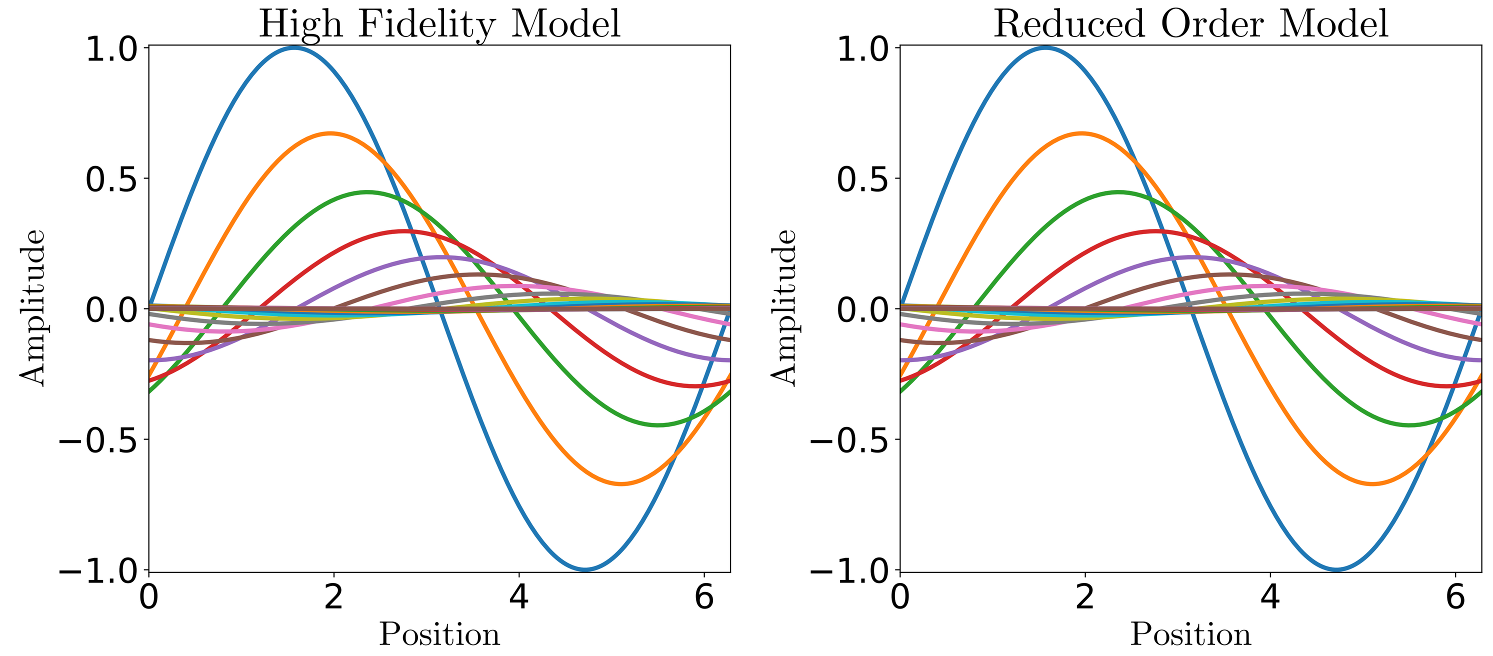
3.2 Reduced Order Model

The goal of ROM is to lower the rank of the matrices involved in evolving the system; this can be done with **singular value decomposition**.

$$\min\{||\mathbf{X} - \tilde{\mathbf{X}}||_F\} \ni \operatorname{rank}(\tilde{\mathbf{X}}) \ll \operatorname{rank}(\mathbf{X})$$
$$\tilde{\mathbf{X}} = U\Sigma V^T$$

The number of modes we use to create the model is dependent on how many singular values are used. In the case of linear convection and diffusion, we used five modes to model with nearly no error.

4 Results



Preliminary timing tests show a speed-up of about 50%.

5 Forthcoming Research

- Model Black-Scholes Equation
- Merton Model

$$\frac{\partial u}{\partial \tau} = \frac{1}{2}\sigma_s^2 s^2 \frac{\partial^2 u}{\partial s^2} + (r - \mu\xi)s\frac{\partial u}{\partial s} - (r+\mu)u + \mu\int_0^\infty u(sy,\tau)p(y)dy$$

- Discrete empirical interpolation method

References

[1] Environmental Visualization Laboratory at the National Oceanic and Atmospheric Administration. *Karman vortex formation in clouds.*

[2] Dan Ruggirello. *Reynolds 100.* Dec 2003.

6 Acknowledgements

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