Data Driven Reduced Order Modeling

Aryn Harmon[1] and Jason Turner[2]

University of Illinois at Urbana-Champaign, Champaign, Illinois, U.S.A.

[1] arynh2@illinois.edu, [2] jasonet2@illinois.edu

Abstract

Model Order Reduction (MOR) can be applied to many-dimensional problems to reduce the computational cost of finding solutions. We created a high fidelity model for the linear convection and diffusion equation using finite difference methods. Using data from this model, we successfully constructed an accurate lower order model using proper orthogonal decomposition.

1 Main Objectives

1. Make online computation possible in computationally challenging problems.
2. Speed up analysis of problems with many degrees of freedom.
3. Produce an understanding of physical processes which are currently obscured by complexity.

2 Introduction

- Linear Convection and Diffusion Equation describes movement and diffusion of a wave in one dimension.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial x^2}$$
$$\vec{u}^{\,n+1} = \mathcal{L}(\vec{u}^{\,n}) = \mathbf{B}\vec{u}^{\,n}$$

- Black-Scholes Model prices European options.

$$\frac{\partial w}{\partial t} = rw - rx\frac{\partial w}{\partial x} - \frac{1}{2}v^2x^2\frac{\partial^2 w}{\partial x^2}$$

- Viscous Burger's Equation (short description, mention non-linearity)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$
$$\mathbf{J}\left(u^{n+1} - u^n\right) - \mathbf{R}\left(u^n\right) = 0 \implies u^{n+1} = \mathcal{L}(\vec{u}^{\,n}) + \mathcal{N}(\vec{u}^{\,n})$$

- MOR uses symmetries in problems to find answers faster.

3 Methods

3.1 Full Order Model

Finite Difference Scheme:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} + \mathcal{O}(\Delta x^2)$$
$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} + \mathcal{O}(\Delta x^2)$$
$$\vdots$$
$$\frac{\partial \vec{u}_i}{\partial t} = \left(\frac{1}{\mathrm{Re}\Delta x^2} - \frac{c}{2\Delta x}\right)u_{i+1} - \frac{2}{\Delta x^2}u_i + \left(\frac{1}{\mathrm{Re}\Delta x^2} + \frac{c}{2\Delta x}\right)u_{i-1}$$

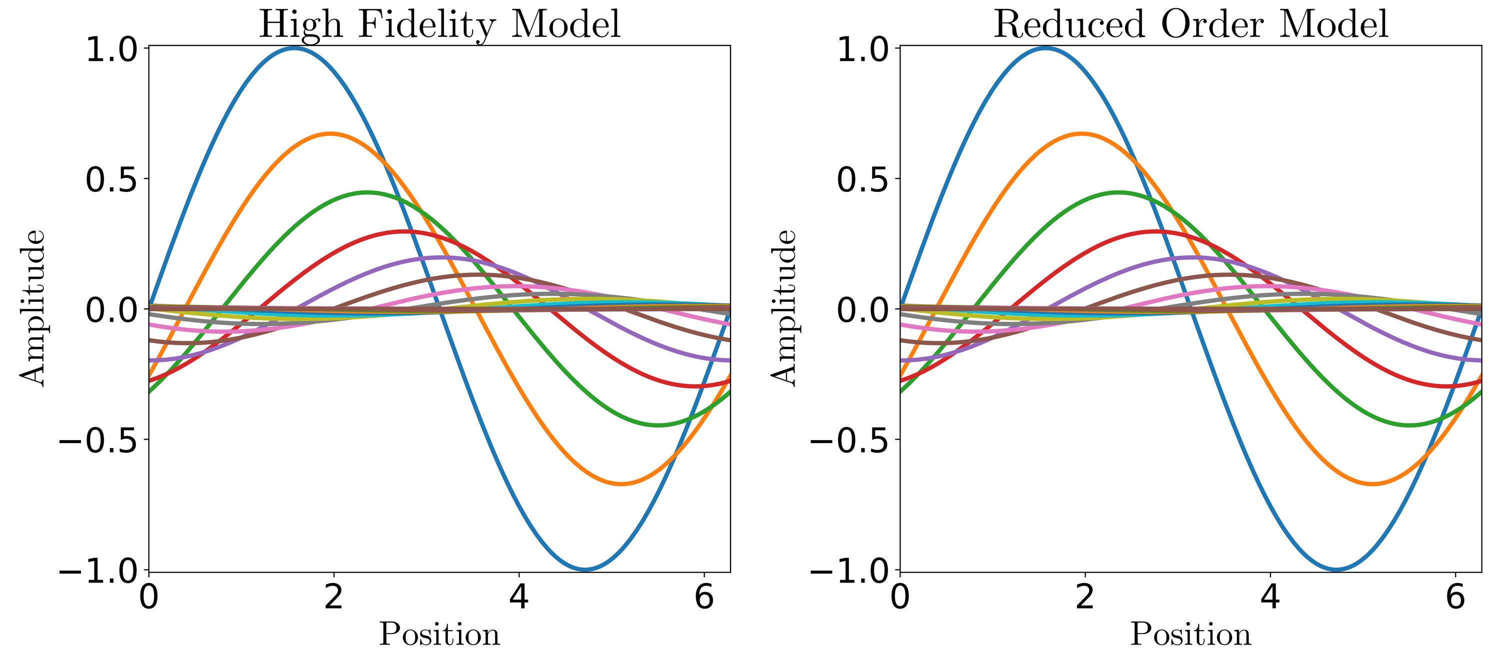
3.2 Reduced Order Model

The goal of MOR is to lower the rank needed to evolve a system to solve it; this can be done with **singular value decomposition**.

$$\min\{||\mathbf{X} - u \cdot v||_F\} \implies A = U\Sigma V^T$$

The number of modes we use to create the model is dependent on how many singular values are used. In the case of linear convection and diffusion, we used three modes to model with nearly no error.

4 Results



5 Conclusions

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6 Forthcoming Research

7 Acknowledgements