**Section 1.6**

4) a) P: kangaroos live in Australia

Q: Kangaroos are marsupials

P ∧ Q

Concludes Q. This is Simplification.

b) P: it is hotter than 100 degrees today

Q: the pollution is dangerous

P ∨ Q

⌐ P

Concludes Q. This is Disjunctive syllogism.

c) P: Linda is an excellent swimmer

Q: Linda can work as a lifeguard

P

P → Q

Concludes Q. This is Modus Ponens.

d) P: Steve will work at a computer company this summer

Q: Steve will be a beach bum

P

Concludes P ∨ Q. This is Addition.

e) P: I work all night on this homework

Q: I can answer all the exercises

R: I will understand the material

P → Q

Q → R

Concludes P → R. This is hypothetical syllogism.

10) a) P: I play hockey

Q: I am sore the next day

R: I use the whirlpool

P → Q

Q → R

⌐ R

This can be simplified using hypothetical syllogism to:

P → R

⌐ R

Then using modus tollens, you conclude:

⌐ P. I did not play hockey.

b) P: I work on x

Q: it is sunny on x

R: it is partly sunny on x

P(x) → (Q(x) ∨ R(x))

P(Monday) ∨ P(Friday)

⌐ Q(Tuesday)

⌐ R(Friday)

This can be simplified using disjunctive syllogism to:

P(Friday) → Q(Friday)

P(Monday) → (Q(Monday) ∨ R(Monday))

Concluding that if I worked on monday it was sunny or partly sunny or if I worked on Friday it was sunny.

c) P: x have six legs

Q: x are insects

R: x are dragonflies

S: x are spiders

Q(x) → P(x)

R(x) → Q(x)

S(x) → ⌐ P(x)

Can be simplified using hypothetical syllogism to:

R(x) → P(x)

S(x) → ⌐ P(x)

Contrapositive of Q(x) → P(x) gives you:

S(x) → ⌐ Q(x)

Concluding that spiders do not have 6 legs, so they are not insects.

d) P: x has an internet account

Q: x is a student

x = students

Y = homer

Z = maggie

∀x(P(x))

Ǝy(⌐ P(x))

Ǝz(P(x))

By using Modus Tollens you can determine that:

Q(Y) → ⌐ P(x) concluding that Homer is not a student.

16) a) x = students

P: x is enrolled in the university

Q: x has lived in a dormitory

Y = mia

∀x(P(x) → Q(x))

⌐ Q(y)

By using modus tollens you can determine that:

⌐ P(x), concluding that Mia is not enrolled in the university.

The argument is correct.

b) P: is a convertible

Q: is fun to drive

Ǝx(P(x) → Q(x))

⌐ P(issac)

Although you cannot conclude:

⌐ Q(issac)

This would be an inverse error.

**Section 1.7**

16) x,y,z ɛ ƶ

x+y+z = odd

Then at least one of x,y,z is odd

X = 2m + 1

Y = 2n

Z =2L

(2m+1) + (2n) + (2L) = 2m + 2n + 2L +1 = 2(m + n + L) + 1

Where (m + n + L) = p, any integer.

We know that any 2\*p is even and (2\*p) + 1 is odd.

X = 2m+1 meaning odd integer

Y = 2n meaning even integer

Z = 2L meaning even integer

So at least one x, y, or z is odd in x+y+z = odd integer.

18) m,n ɛ ƶ

Mn = even integer

M = 2p

N = q

(2p)(q) = even integer.

Where pq = t, a product of two integers.

2t = even integer, because any integer multiplied by 2 results in an even number.

M = 2p, an even integer

N = q, an odd integer

So at least 1 M or N is even, and the product of the two is even.

20) a) n ɛ ƶ

3n+2 = even

Then, n is even.

P: 3n+2 is even

Q: n is even

P → Q

Contraposition: ⌐Q → ⌐P.

We know n is odd.

Trying to prove that 3n+2 is odd because of contraposition.

Any odd integer, n, multiplied by an odd number, 3, is odd. Call this T.

T+2, any odd number T, plus 2 will be odd. Proving when n is off, 3n+2 2 is odd.

b) proving by contradiction means that we want to prove that ⌐P is false.

Let 3n+2 be odd. And n be even. Want to prove that this is false.

n=2p

3(2p)+2 = 6p+2. Any integer multiplied by an even number is even.

So, 6p = t, an even integer.

T+2 = even number, meaning that 3n+2 = odd is false when n is even.

28) n is a positive integer, then n is even if and only if 7n+4 is even.

(if n is positive) → ( (n is even) ↔ (7n+4 is even) )

N = 2p

7(2p) + 4 = 2(7p + 2) = even number because it is a multiple of 2.

N = 2p+1

7(2p+1) + 4 = 14p+11 = odd because multiple of 2 plus 1.

So if n is even, then 7n+4 is even.

If n is odd, then 7n+4 is odd.

Proving that if n is a positive integer, n is even if and only if 7n+4 is even.

**Section 2.1**

46) a) There is at least one real number, x, where (x^3) = -1.

This is true, x = -1. (-1^3) = -1

b) There is at least one integer, x, where (x+1) > x.

This is true. 3+1 > 3. 4 > 3

c) For all integers, x, where (x-1) is within the set of integers.

This is true. -1-1 = -2 is an integer.

d) There is at least one integer, x, where x^2 is within the set of integers.

This is true. (2^2) = 4. 4 is an integer.

48) a) (x^3) >= 1 is all integers greater than or equal to 1.

So, {1,2,3,... }

b) (x^2) = 2 is no integer. sqrt(2) != integer.

So, {∅}.

c) x < (x^2) is all integers > 1.

So, {...,-3,-2,-1,2,3,4,...}

**Section 2.2**

2) A = set of sophomores

B = set of students in discrete mathematics

a) S ⊆ (A ∩ B)

b) S ⊆ (A-B)

c) S ⊆ (A ∪ B)

d) S ⊆ (⌐A ∪ ⌐B)