Section 5.1

8)

2(-7)^n = (1 - (-7)^(n+1))/4

P(1) : 2 - 2\*7 = -12

P(1): (1 - (-7)^(2))/4 = -12

N = k

2(-7)^k = (1-(-7)^(k+1))/4

2(-7)^k + 2(-7)^(k+1) = (1-(-7)^(k+2))/4

(1-(-7)^(k+1))/4 + 2(-7)^(k+1) = (1-(-7)^(k+2))/4

(1-(-7)^(k+1) + 8(-7)^(k+1))/4

(-(-7)^(k+1) (-1+8))/4

(1-(-7)^(k+1) (-7))/4

(1-(-7)^(k+2))/4

P(k) proves P(k+1)

P(k+1) holds.

10)

1/(n\*(n+1))

N(1) = 1/(1\*2) = ½

N(2) = ½ + 1/(2\*3) = ½ + ⅙ = ⅔

N(3) = ½ + ⅙ + 1/12 = ¾

n/n+1

N / (n+1) = 1/(n\*(n+1)

P(1) : ½ = ½

N = k

1/(k\*(k+1)) + 1/(k+1)((k+1)+1) = (k+1)/(k+2)

k/(k+1) + 1/(k+1)(k+2)

k(k+2) + 1 / (k+1)(k+2)

k^2 + 2k +1 / (k+1)(k+2)

(k+1)/(k+2) = (k+1)/(k+1)+1

P(k) proves P(k+1) P(k+1) holds true.

12)

(-½)^n = 2^(n+1) + (-1)^n / 3\*2^n

P(1) :1 - ½ = 1/2

P(1) : 3/6 = ½

N = k

(-½)^k + (-½)^k+1 = 2^(k+2) + (-1)^(k+1) / 3\*2^(k+1)

2^(k+1) + (-1)^k / 3\*2^k + (-½)^k+1

2^(k+1) + (-1)^k / 3\*2^k + (-1^k+1) / 2^(k+1)

2^(k+2) + 2(-1)^k + 3(-1^k+1) / 3\*2^k+1

2^(k+2) + (2-3)(-1)^k / 3(2^K+1)

2^(k+2) + (-1)^k+1 / 3\*2^k+1

P(k) proves P(k+1)

p(k+1) holds.

14)

k2^k = (n-1)2^(n+1) + 2

P(1) : 2 = 2

K = n

n\*2(^n) = (n-1)2^(n+1) + 2

n\*2^n + (n+1)\*2^(n+1) = (n)2^(n+2) + 2

(n-1)2^(n+1) + 2 + (n+1)\*2^(n+1)

(2^(n+1))(n-1+n+1) + 2

(2^(n+1))(2n) + 2

n2^(n+2) + 2

P(n) proves P(n+1)

P(n+1) holds.

18)

P(2) : 2! < 2^2

P(2) : 2 < 4

N = k

P(k) : k! < 2^k k>1

Have P(k) proves P(k+1)

Where P(k+1) = (K+1)! < 2^(K+1)

(K+1)! < (K+1)k^k

k(k^k) + k^k

K^(K+1) + k^k

(k+1)^(K+1)

(K+1)! < (k+1)^(k+1)

P(k) proves P(k+1)

P(k+1) holds

You are setting up an inequality to check if n!<2^n. Then we setup the inequality for n+1, and see if that is also true. We manipulate the inequality until we get the same inequality for n+1.

32)

n^3 + 2n

P(1) : 1 + 2 = 3

P(1) : 3/3 = 1

N = k , k>0

M = k

k^3 + 2k = 3m

(k+1)^3 + 2(k+1) = 3m

(k+1)(k^2 + 2k + 1) + 2k +2

k^3 + 3k^2 + 3k + 1

k^3 +2k + 3k^2 + k +1

k^3 + 2k + 3(k^2 + ⅓k + ⅓)

Since k^3 + 2k is a multiple of 3, and (k^2 + ⅓k + ⅓) is being multiplied by 3, this is a multiple of 3 as well, proving P(k+1) = 3m.

P(k) proves P(k+1)

P(K+1) holds.

Section 5.2

4)

A postage of n cents can be formed using just 4 cents stamps and 7 cent stamps.

P(18) : 18 = 7 + 7 + 4.

P(19) : 19 = 7 + 4 + 4 + 4.

P(20) : 20 = 4 + 4 + 4 + 4 + 4.

P(21) : 21 = 7 + 7 + 7.

P(18), P(19), P(20), P(21) are true.

So, P(k>=18) is true.

Prove P(k+1) is true, by combining 4 + 7 cents to get k+1.

k>=21

We know P(18) is true, so that is k-3.

K + 1 = k-3 + 4

P(n) is true for all n>=18 because we proved that P(n>=18) is true by using mathematical induction.

12)

P(1) : 2^0 = 1

P(2) : 2^1 = 2

P(3) : 2^0 + 2^1 = 3

P(4): 2^2 = 4

N = k

(K+1)/2 = (sum of powers of 2) for even numbers

K+1 = 2(sum of powers of 2)

Multiplying by 2 increases the powers by 1.

K+1 still equals the sum of various powers of 2.

When k+1 is even, the proposition is true.

K = sum of powers of 2, except 2^0

Adding 2^0 gives you odd numbers,

So when k+1 is odd, the proposition is true.

Therefore, all positive integers are true for the proposition.