1a. Distance traveled while running was 2.9 km and 9 min total time to do so.

$$V_{avg.} = \frac{\text{displacement}}{\text{time}} = \frac{2.9\text{ km}}{9\text{ min}} = \boxed{5.37\ \frac{m}{s}}$$

(we generally prefer meters per second)

b. Distance traveled while walking is the same while running, but 30 min to do so:

$$V_{avg.} = \frac{\text{displacement}}{\text{time}} = \frac{-2.9\text{km}}{30\text{ min}} = \boxed{-1.61\ \frac{m}{s}}$$

The reason 2.9 km is negative is because the runner traveled in the opposite direction, as we generally define our reference frame to have left be negative and right positive.

c. Average velocity for the whole trip

0 (he ends up where he started)

$$v_{avg} = \frac{\text{displacement}}{\text{total time}} = \boxed{0\ \frac{m}{s}}$$

(30 + 9) min

d. Average speed for the whole trip:

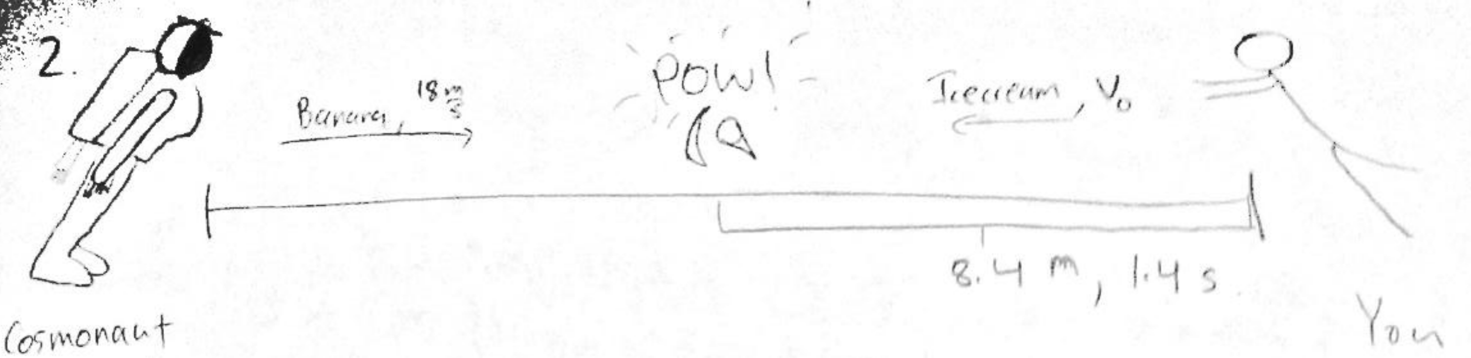
$$s_{avg} = \frac{\text{distance}}{\text{time}}$$

Distance can never be negative, and so we have to split the distance into two parts: the walking part and the running part.

$$s_{avg} = \frac{\text{total distance}}{\text{total time}} = \frac{|\text{displacement}_{,\text{walking}}| + |\text{displacement, running}|}{\text{total time}}$$

$$= \frac{|-2.9\text{km}| + |2.9\text{ km}|}{30\text{ min} + 9\text{ min}} = \frac{5.8\text{ km}}{39\text{ min}} = \boxed{2.48\ \frac{m}{s}}$$

2.



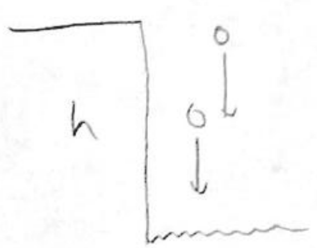
They ask how fast – that's a speed

a) $V_0 = \dfrac{\text{displacement}}{\text{time}} = \dfrac{-8.4\,m}{1.4\,s} = -6.0\,\frac{m}{s} \rightarrow \boxed{6.0\,\frac{m}{s}}$

b) Distance from Andrei = Your distance from collision point + his distance from collision point

$= |-8.4\,m| + |18\,\frac{m}{s} \cdot 1.4\,s| = \boxed{33.6\,m}$

3. Now we need to start using our more sophisticated equations because we have an acceleration. Specifically, gravity acts in this problem.



Let h be the height of the cliff, v_1 and v_2 the initial velocities of the first and second rocks, respectively, and Δt be the time between which they are thrown. Also, I choose my reference frame so that downward is POSITIVE!

$v_1 = 0$ because rock is dropped

initial y position

$$v_1(t) = gt + v_1 \xrightarrow{\text{Integration}} y_1(t) = \tfrac{1}{2}gt^2 + y_1$$

$$v_2(t) = g(t-\Delta t) + v_2 \xrightarrow{\text{Integration}} y_2(t) = \tfrac{1}{2}g(t-\Delta t)^2 + v_2(t-\Delta t) + y_2$$

Since both particles start and end at the place, at the time we are interested in, $y_1(t) - y_1 = y_2(t) - y_2 = h$. So we have:

$$h = \tfrac{1}{2}gt^2 \qquad h = \tfrac{1}{2}g(t-\Delta t)^2 + v_2(t-\Delta t)$$

If we plug in our given values, we have:

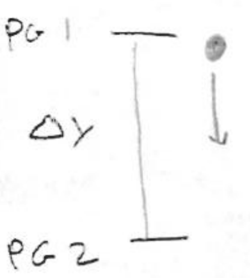
$$h = \frac{1}{2} g t^2 \longrightarrow h = \frac{1}{2}(9.81)t^2$$

$$h = \frac{1}{2} g (t - \Delta t)^2 + v_2 (t - \Delta t) \longrightarrow h = \frac{1}{2}(9.81)(t - 1.8)^2 + 47(t - 1.8)$$

Now, this is just a simple problem in solving for h, which we can do using standard algebra techniques. Doing so, we have:

$$\boxed{h = 26.9 \text{ m}}$$

4. Let's draw a picture to solve this problem!



a) Because there is no initial velocity, our velocity is given by:

$$v(t) = gt \longrightarrow \frac{dy}{dt} = gt$$

Integrating, we have:

$y(t) = \frac{1}{2} g t^2 + y_0$; since $\Delta y = y(t) - y_0$, we have.

$$\Delta y = \frac{1}{2} g t^2 \longrightarrow \boxed{g = \frac{2\Delta y}{t^2}}$$

And we prove the result. Realize that $\Delta t = t - t_0$, but I'm assuming $t_0 = 0$, so our result agrees with that in the problem.

b) For this set up, $\Delta y = 1.0$ m, so:

$$\Delta y = \frac{1}{2} g t^2 \longrightarrow t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(1.0 \text{ m})}{9.81 \frac{m}{s^2}}} = \boxed{0.45 \text{ s}}$$

c) The height of the first photogate:

$1.00\,m - 0.50\,cm = 1.00\,m - 0.005\,m =$

$0.995\,m$, whereas the perceived height is

$1.00\,m$.

actual Δy The actual time travelled, as measured by photogates:

$\Delta y = \frac{1}{2}gt^2 \longrightarrow t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(0.995\,m)}{9.81\,\frac{m}{s^2}}}$

The calculated value of g:

perceived Δy

total time

$\Delta y = \frac{1}{2}g_{exp}t^2 \longrightarrow g_{exp} = \frac{2\Delta y}{t^2} = \frac{2(1.00\,m)}{\left(\sqrt{\frac{2(0.995\,m)}{9.81\,\frac{m}{s^2}}}\right)^2}$

$= \boxed{9.86\,\frac{m}{s^2}}$

$\%\text{ difference} = \frac{|g_{exp} - g_{real}|}{g_{real}} \cdot 100\% = 100\% \cdot \frac{|9.86\,\frac{m}{s^2} - 9.81\,\frac{m}{s^2}|}{9.81\,\frac{m}{s^2}}$

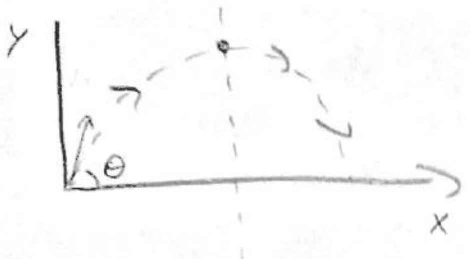
$= \boxed{0.51\%\text{ difference}}$

5. There are many ways to do this problem. I will do it without calculus. We know, from our kinematics equations, that (provided the starting position is (0,0)).

$x(t) = v_0 \cos\theta\, t$

$y(t) = v_0 \sin\theta\, t - \frac{1}{2}gt^2$

We have discussed that the shape of this parametrization is a parabola, which makes sense based on what we see in real life!



We see here that the y-position of the ball is 0 at two times! 0, and some t_f we don't know how to determine without numbers!

$$0 = v_0 \sin\theta \, t_f - \frac{1}{2} g t_f^2 \longrightarrow t_f = \frac{2 v_0 \sin\theta}{g}$$

Because of the symmetrical nature of the parabola, the apex, which has the maximum height, is reached at time $\frac{t_f}{2}$.

$$h_{max} = v_0 \sin\theta \left(\frac{t_f}{2}\right) - \frac{1}{2} g \left(\frac{t_f}{2}\right)^2$$

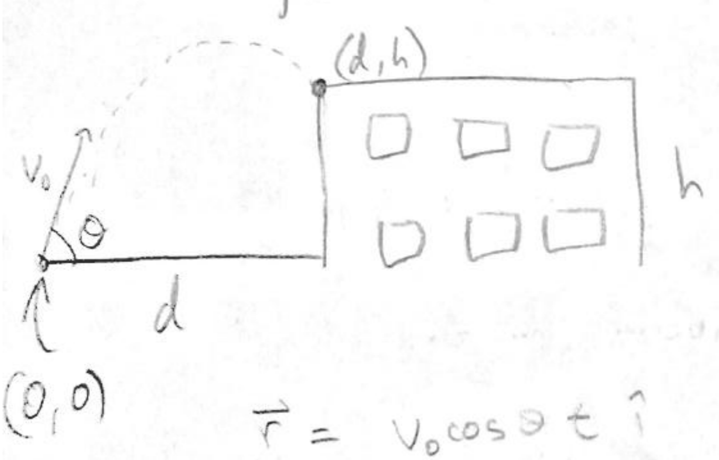
$$= v_0 \sin\theta \left(\frac{2 v_0 \sin\theta}{2g}\right) - \frac{1}{2} g \left(\frac{2 v_0 \sin\theta}{2g}\right)^2$$

$$= \frac{v_0^2 \sin^2\theta}{g} - \frac{v_0^2 \sin^2\theta}{2g}$$

$$= \boxed{\frac{v_0^2 \sin^2\theta}{2g}}$$

6. This problem may seem easy at first, but it is not so straightforward as it seems.

a) Trying to use the kinematics equations at the point which the ball is thrown can be tedious. Instead, we use a clever technique — we pretend as if the ball were launched from the ground to the roof. Then it becomes easier to employ our kinematics equations.



$\theta = 51°$

$d = 25$ m

$t = 1.61$ s

$$\vec{r} = V_0 \cos\theta\, t\, \hat{i} + \left(V_0 \sin\theta\, t - \tfrac{1}{2} g t^2\right)\hat{j}.$$

We know after 1.61 seconds, the ball has travelled horizontally 25 m. Using the x-component part of our position vector, we have:

$$x(t) = V_0 \cos\theta\, t \longrightarrow V_0 = \frac{x(t)}{\cos\theta\, t} = \frac{25\text{ m}}{\cos(51°)\, 1.61\text{ s}}$$

$$= 24.67\ \frac{\text{m}}{\text{s}}$$

Therefore the position vector describing our reversed ground-to-roof equation is as follows:

$$\vec{r}(t) = 24.67 \cos 51°\, t\, \hat{i} + \left(24.67 \sin 51°\, t - \tfrac{1}{2} 9.81 t^2\right)\hat{j}$$

Therefore, we can find h, using the y-component of our position vector. After 1.61 seconds, the ball is at the top of the roof, at height h:

$$h = 24.67 \sin 51^\circ (1.61) - \frac{1}{2}(9.81)(1.61)^2$$
$$= \boxed{18.16 \text{ m}}$$

b) We know that by taking the derivative of our position vector, we have our velocity vector:

$$\vec{r}(t) = v_0 \cos\theta \, t \, \hat{i} + (v_0 \sin\theta \, t - \tfrac{1}{2} g t^2)\hat{j}$$
$$\vec{v}(t) = v_0 \cos\theta \, \hat{i} + (v_0 \sin\theta - gt)\hat{j}$$

So at 1.61 seconds, the velocity vector is given by

$$\vec{v}(t) = 24.67 \cos(51^\circ)\hat{i} + \left(24.67(\sin 51^\circ) - (9.81)(1.61)\right)\hat{j}$$

And its magnitude:

$$|\vec{v}(t)| = \sqrt{[24.67 \cos(51^\circ)]^2 + [24.67 (\sin 51^\circ) - (9.81)(1.61)]^2}$$
$$= \boxed{15.89 \ \frac{m}{s}}$$

Realize that V_0 is NOT the velocity with which the ball is thrown in the problem! V_0 is the launch velocity we introduced so that we could "reverse" the problem and get a velocity vector for the top of the roof.

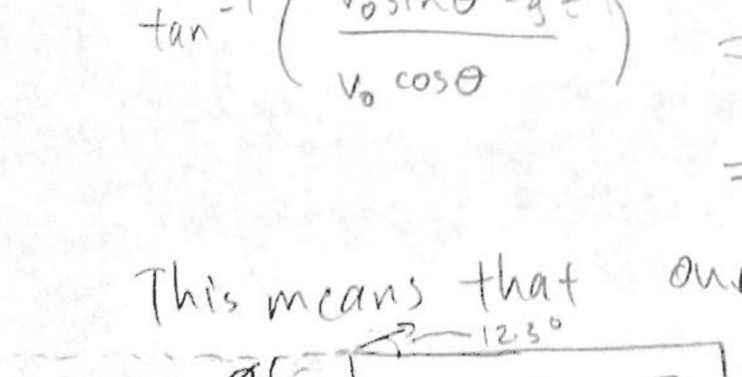
Infact, thevelocity vector in which the ball is lauched at the top of the roof in the problem is <u>different</u> from the velocity vector in our solution, but their magnitudes are the same. This is why we need to be careful with clever solutions, as we will see in this next part as we get the angle (which I call ϕ).

The angle of a velocity vector is given by $\tan^{-1}\left(\frac{v_y}{v_x}\right)$ where v_y and v_x are the y and x components of the velocity vector.

$$\tan^{-1}\left(\frac{V_0\sin\theta - gt}{V_0\cos\theta}\right) = \tan^{-1}\left(\frac{24.67\sin 51^\circ - 9.81\cdot 1.61}{24.67\cos 51^\circ}\right)$$

$$= 12.3^\circ$$

This means that our picture looks like this:



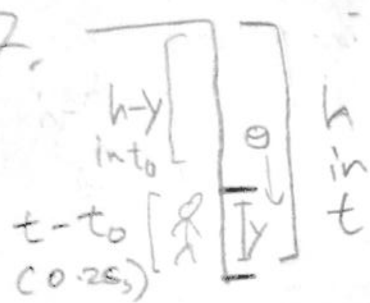
So as the ball reaches the top of the roof, at hasn't reached its apex yet.

we can see from this new picture based on the new information that we have that ϕ, the angle at which the ball is launched from the top of the roof in the real problem is also 12.3°, but below the horizontal. So our answer for part b is:

$$\boxed{15.89 \ \frac{m}{s} \text{ at } 12.3° \text{ degrees below the horizontal}}$$

This also inherently contains the answer for part c.

7.



Let h be the height from the ledge to the bottom of the window, and y the height of the window. Let be t be the total time and t_0 the time needed to get to the top of the window. This means $t-t_0$ is the time needed to pass the window. We are interested in finding h-y.

When the flowerpot gets to the window it has velocity gt_0.

$$h-y = \frac{1}{2}gt_0^2$$

$$y = \frac{1}{2}g(t-t_0)^2 + \underbrace{v_0}_{gt_0}(t-t_0)$$

$$= \frac{1}{2}g(t-t_0)^2 + gt_0(t-t_0)$$

We know that $t - t_0 = 0.25\,s$ and $x = 3.5\,m$

$$3.5\,m = \frac{1}{2} g (0.25)^2 + 9.81 \cdot t_0 (0.25)$$

$$t_0 = 1.30$$

$$h - x = \frac{1}{2} g t_0^2$$

$$= \frac{1}{2} g (1.30)^2$$

$$= \boxed{8.32\,m}$$

The solutions to this problem set were written by me, Arun Kannan. If you notice any problems with the solutions or have any questions, please contact me at 2015akannan@tjhsst.edu.