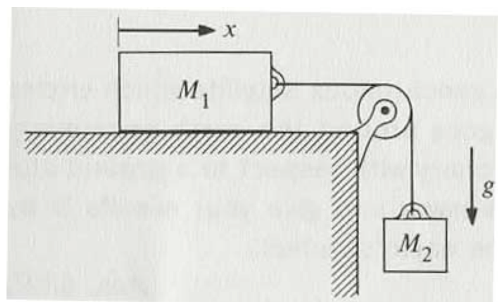
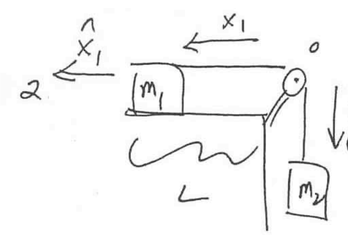
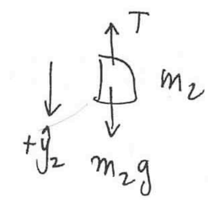
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group

Problem Set 2 Solutions

Problem 2: The two blocks shown in the figure are connected by a string of negligible mass. If the system is released from rest, find how far the block of mass m_1 slides in time t. Neglect friction.

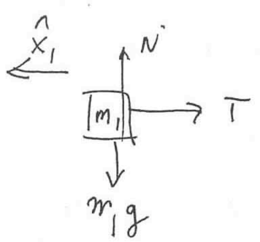






$\vec{F}_2$	$= m_2 \vec{a}_2$
$\hat{y}_2: m_2 g - T$	$= m_2 a_{2,y}$

$$\Rightarrow m_2 g - T = m_2 a_{2,y} \quad (1)$$



$\vec{F}_1 =$	$m_1 \vec{a}_1$
$-T =$	$m_1 a_{x,1}$ (2)

constraint $a \equiv a_{2,y} = -a_{x,1}$

eq (1) $\Rightarrow$ $m_2 g - T = m_2 a$ $\Rightarrow$ $m_2 g - m_1 a = m_2 a$

eq (2) $\Rightarrow$ $T = m_1 a$ $\Rightarrow$ $a = \dfrac{m_2 g}{m_1 + m_2}$

$$x_1 = x_{1,0} + \frac{1}{2} a_{x,1} t^2$$

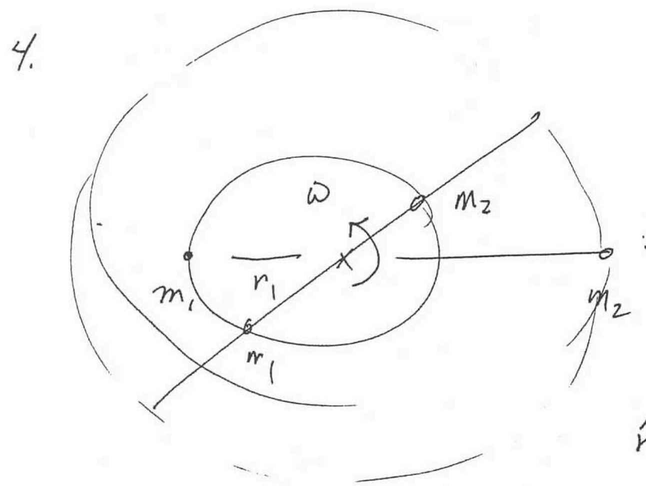
$$x_1 = L - \frac{1}{2} a t^2 = L - \frac{1}{2}\left(\frac{m_2 g}{m_1 + m_2}\right) t^2$$

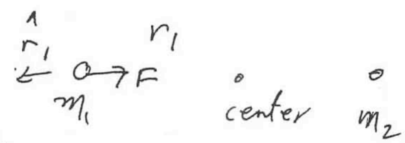
Problem 4: Two particles of mass m_1 and m_2 undergo uniform circular motion about each other at a separation R under the influence of an attractive force of magnitude F. The angular velocity is ω radians per second.

a) Show that $R = (F/\omega^2)(1/m_1 + 1/m_2)$.

b) Explain why you can think of this problem is equivalent to a single body of mass μ where $1/\mu = (1/m_1 + 1/m_2)$ undergoing circular motion of radius R due to the influence of a central attractive force of magnitude F.

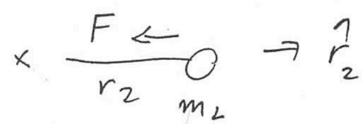
4.





	$\vec{F}$	$= m_1 \vec{a}_1$
$\hat{r}_1$:	$-F$	$= -m_1 r_1 \omega^2$

$$F = m_1 r_1 \omega^2 \quad (1)$$



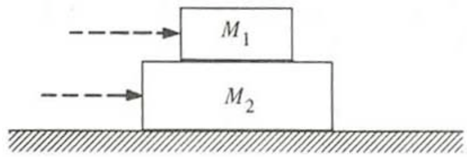
	$\vec{F}$	$= m_2 \vec{a}_2$
$\hat{r}_2$:	$-F$	$= -m_2 r_2 \omega^2$

$$\Rightarrow \quad F = m_2 r_2 \omega^2$$

$$L = r_1 + r_2 = \frac{F}{m_1 \omega^2} + \frac{F}{m_2 \omega^2} = \frac{F}{\omega^2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

or $$F = \frac{\omega^2 L}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}.$$

Problem 7: Consider two textbooks that are resting one on top of the other. The lower book has $m_2 = 0.8\ \text{kg}$ m and is resting on a nearly frictionless surface. The upper book has mass $m_2 = 2.0\ \text{kg}$. Suppose the coefficient of static friction is given by $\mu_s = 0.1$.

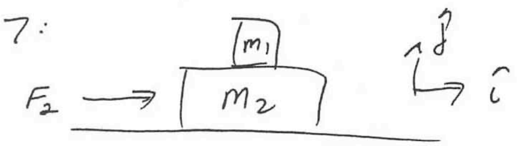


a) What is the maximum force which the upper book can be pushed horizontally so that the two books move together without slipping? Identify all action-reaction pairs of forces in this problem.

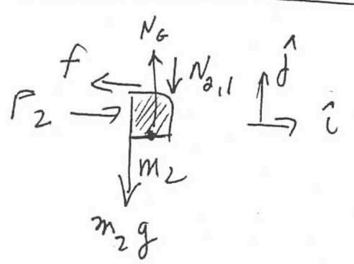
b) What is the maximum force which the lower book can be pushed horizontally so that the two books move together without slipping? Identify all action-reaction pairs of forces in this problem.

c) Explain why one of your forces in parts a) and b) is larger than the other.

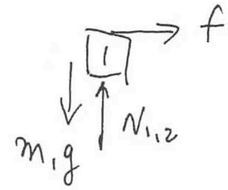
7:



$a_1 = a_2$



	$\vec{F}$	$m_2 \vec{a}_2$	
$\hat{i}$:	$F_2 - f =$	$m_2 a_2$	(1)
$\hat{j}$:	$N_G - N_{2,1} - m_2 g$	$= 0$	(2)



	$\vec{F}$	$m_1 \vec{a}_1$	
$\hat{i}$:	$f =$	$m_1 a_1$	(3)
$\hat{j}$:	$N_{1,2} - m_1 g$	$= 0$	(4)

Just slipping condition $f = \mu N_{1,2}$

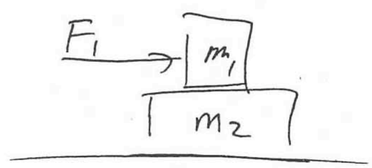
eq (4) $\Rightarrow N_{1,2} = m_1 g \Rightarrow f = \mu m_1 g$

eq (3) $f = m_1 a_1 \Rightarrow \mu m_1 g = m_1 a_1 \Rightarrow \boxed{\mu g = a_1}$

$= \mu m_1 g + m_2 \mu g$

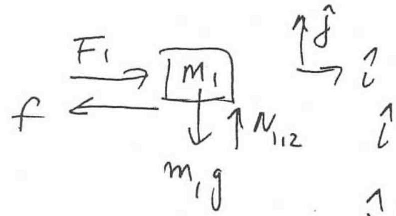
$$\boxed{F_2 = \mu g (m_1 + m_2)}$$

parts 5)

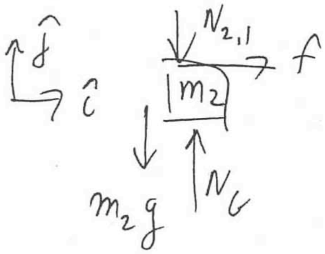


$a_1 = a_2$

$f = \mu N_{1,2}$ just slipping



$\vec{F}$	$= m_1 \vec{a}_1$	
$\hat{i}: F_1 - f$	$= m_1 a_1$	(5)
$\hat{j}: N_{1,2} - m_1 g$	$= 0$	(6)



$\vec{F}$	$= m_2 \vec{a}_2$	(7
$\hat{i}: f$	$= m_2 a_2$	(7)
$\hat{j}: N_G - N_{2,1} - m_2 g$	$= 0$	(8)

eq (6) $\Rightarrow N_{1,2} = m_1 g \Rightarrow f = \mu m_1 g$

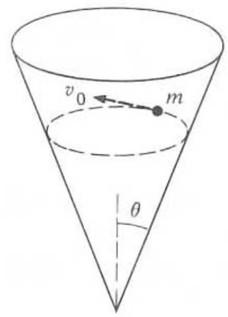
eq (7) $\Rightarrow \mu m_1 g = m_2 a_2 \Rightarrow \boxed{a_2 = \mu \frac{m_1}{m_2} g}$

eq (1) $\Rightarrow F_1 = f + m_1 a_1 \qquad a_1 = a_2$

$= \mu m_1 g + m_1 \mu \frac{m_1}{m_2} g$

$$\boxed{F_1 = \mu g m_1 \left(1 + \frac{m_1}{m_2}\right)}$$

Problem KK 2.9: A body of mass m is moving in a horizontal circle of radius r with a constant speed v_0 on the inside wall of a cone. Assume the wall of the cone is frictionless. The wall of the cone makes an angle θ with the vertical.



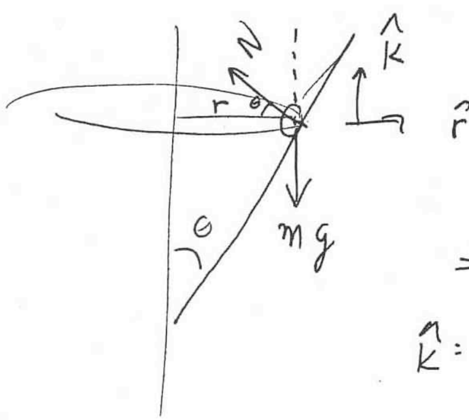
a) Draw a free body force diagram showing all the forces acting on the mass.

b) Find the radius of the circular orbit in terms of v_0, θ, and g.

c) How long will the mass take to go around the circle?

d) Now assume there is a coefficient of static friction μ_s. Find the maximum speed the mass can move on the inside of a cone and still move in a circular orbit of radius r.

9.



$\vec{F}$	$m\vec{a}$
$\hat{r}: -N\cos\theta =$	$-\dfrac{m v_0^2}{r}$

$$\Rightarrow \quad N\cos\theta = \frac{m v_0^2}{r} \quad (1)$$

$$\hat{k}: \quad N\sin\theta - mg = 0$$

$$N\sin\theta = mg \quad (2)$$

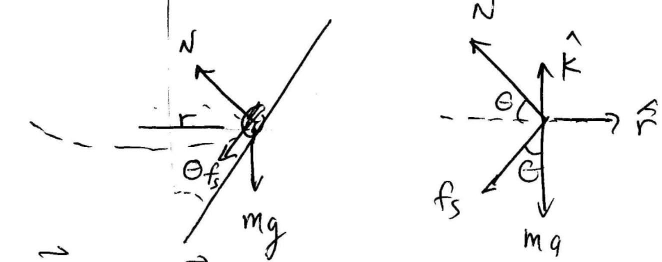
$$\frac{N\sin\theta}{N\cos\theta} = \frac{mg}{\frac{m v_0^2}{r}} \Rightarrow \tan\theta = \frac{rg}{v_0^2} \Rightarrow$$

$$\boxed{r = \frac{v_0^2 \tan\theta}{g}}$$

The object will take a time $T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi v_0 \tan\theta}{g}$ to complete an orbit.

d) Now assume there is a coefficient of static friction μ_s. Find the maximum speed the mass can move on the inside of a cone and still move in a circular orbit of radius r. We must assume that the object can roll so that the only friction is static friction. If the object rolls very rapidly it will tend to slide up the inclined plane, so static friction must point down the plane.

Let's assume the object can roll so there is only static friction acting on it.



$$\vec{F} = m\vec{a}$$

$$\hat{r}: -(N\cos\theta + f_s\sin\theta) = -m\frac{v_0^2}{r} \qquad (1)$$

$$\hat{k}: \quad N\sin\theta - f_s\cos\theta - mg = 0 \qquad (2)$$

when $v_0 = v_m$, object just starts to slip upwards, then

$$(f_s)_{max} = \mu_s N \qquad (3)$$

Eq (1) and (2) become

$$N\cos\theta + \mu_s N\sin\theta = m\frac{(v_m)^2}{r}$$

$$N\sin\theta - \mu_s N\cos\theta = mg$$

divide these two equations yielding

$$\frac{\cos\theta + \mu_s\sin\theta}{\sin\theta - \mu_s\cos\theta} = \frac{(v_m^2)}{rg}$$

$$v_{max} = \sqrt{rg\left(\frac{\cos\theta + \mu_s\sin\theta}{\sin\theta - \mu_s\cos\theta}\right)}$$

Problem 2.10

The earth is spinning about its axis with a period of 23 hours 56 min and 4 sec. The equatorial radius of the earth is $6.38 \times 10^{6}\ \mathrm{m}$. The latitude of Cambridge, Mass is $42^{o}\ 22'$.

a) Find the velocity of a person at MIT as they undergo circular motion about the earth's axis of rotation.

b) Find the person's centripetal acceleration.

c) The rotation of the Earth is slowing down. In 1977, the Earth took $1.01\,\mathrm{s}$ longer to complete 365 rotations than in 1990. What was the average angular deceleration of the Earth in the time interval from 1900 to 1977?

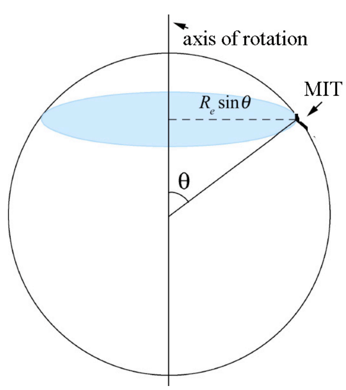
d) Find the radius of the orbit of a synchronous satellite which circles the earth. (A synchronous satellite goes around the earth once every rotation of the earth, so that its position appears stationary with respect to a ground station).

Solution:

a) The rotational period of the earth is given by

$$T_{\text{earth},2004} = (23\ \mathrm{hr})(3600\ \mathrm{s}\cdot\mathrm{hr}^{-1}) + (56\ \mathrm{min})(60\ \mathrm{s}\cdot\mathrm{min}^{-1}) + 4\,\mathrm{s} = 86164\ \mathrm{s}. \qquad (1.1)$$

Note that this period is less than $24\,\mathrm{hr}$. Twenty-four hours is one *solar* day (noon to noon), while the above period is one *sidereal* day; sidereal means with respect to the "fixed" stars, and you should be able to see why the two are different.



A person at MIT undergoes circular motion about the axis of the earth. The radius of the orbit is given by $R = R_e \sin\theta$ where θ is the angle between MIT and the axis of rotation. Since the latitude λ is measured form the equator, $\sin\theta = \sin(\pi/2 - \lambda) = \cos\lambda$ (the angle

θ is sometimes called the "colatitude"). Hence $\theta = \pi / 2 - \lambda$, and the radius of the orbit of a person at MIT is

$$R = R_e \cos\lambda = (6.38\times10^6\,\mathrm{m})(\cos 42.36) = 4.71\times10^6\,\mathrm{m} \tag{1.2}$$

Since the motion is uniform, during one period of rotation the person travels a distance

$$s = 2\pi R = vT \tag{1.3}$$

where v is the magnitude of the velocity and $s = 2\pi R$ is the circumference.

The magnitude of the velocity (speed) is then given by

$$v = \frac{2\pi R}{T} \tag{1.4}$$

So for a person at MIT, the magnitude of the velocity is

$$v = \frac{2\pi R}{T} = \frac{2\pi R_e \cos\lambda}{T} = \frac{(2\pi)(4.714\times10^6\,\mathrm{m})}{(86164\,\mathrm{s})} = 3.44\times10^2\,\mathrm{m}\cdot\mathrm{s}^{-1}. \tag{1.5}$$

b) The centripetal acceleration is given by

$$\left|a_r\right| = \frac{v^2}{R} = \frac{v^2}{R_e\cos\lambda} = \frac{(3.44\times10^2\,\mathrm{m}\cdot\mathrm{s}^{-1})^2}{(4.71\times10^6\,\mathrm{m})} = 2.51\times10^{-2}\,\mathrm{m}\cdot\mathrm{s}^{-2}. \tag{1.6}$$

c) The period of the Earth's rotation in 1900 was $T_{\mathrm{earth},1900} = T_{\mathrm{earth},1977} - 1.01\,\mathrm{s}/365$ so $\Delta T = T_{\mathrm{earth},1977} - T_{\mathrm{earth},1900} = 1.01\,\mathrm{s}/365$.

When an object undergoes uniform circular motion, the angular velocity is constant and is equal to the velocity divided by the radius of the orbit,

$$\left|\omega\right| = \frac{v}{R} = \frac{2\pi R}{T}\frac{1}{R} = \frac{2\pi}{T}. \tag{1.7}$$

The change in angular velocity is then

$$\Delta\omega = \omega_{1977} - \omega_{1900} = \frac{2\pi}{T_{1977}} - \frac{2\pi}{T_{1900}} = 2\pi \frac{(T_{1900} - T_{1977})}{T_{1900}\, T_{1977}} \simeq -\frac{2\pi\, \Delta T}{T_{1977}^2} \tag{1.8}$$
$$= -2.34 \times 10^{-12} \text{ rad} \cdot \text{s}^{-1}.$$

Note: that we approximated the denominator in Eq. (1.8) as

$$T_{1900}\, T_{1977} = (T_{1977} - \Delta T) T_{1977} \simeq T_{1977}^2 \tag{1.9}$$

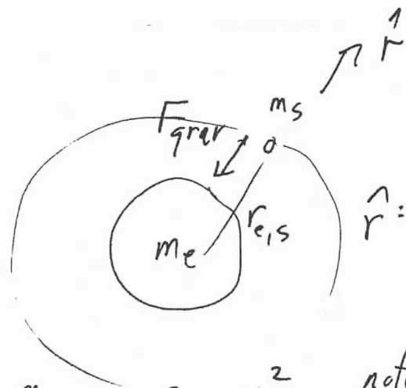
since the term $\Delta T\ T_{1977} << T_{1977}^2$ and hence can be ignored. This slowing down occurred over a period of 80 years for which we shall use the average period of 86163.5 s. So the time interval is

$$\Delta t = (80 \text{ yr})(365.25 \text{ day} \cdot \text{yr}^{-1})(86163.5 \text{ s} \cdot \text{day}^{-1}) = 2.52 \times 10^9 \text{ s}. \tag{1.10}$$

The average angular acceleration is then

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{(-2.34 \times 10^{-12} \text{ rad} \cdot \text{s}^{-1})}{(2.52 \times 10^9 \text{ s})} = -9.29 \times 10^{-22} \text{ rad} \cdot \text{s}^{-2}. \tag{1.11}$$

10.



	$\vec{F}$	$= m\vec{a}$
$\hat{r}$:	$\dfrac{-Gm_s m_e}{r_{e,s}^2}$	$= -m_s r_{e,s}\,\omega^2$

$$\Rightarrow (1)\ \frac{Gm_e}{r_{e,s}^2} = r_{e,s}\,\omega^2, \quad \text{note: } \omega = \frac{2\pi}{T} \Rightarrow$$

also $\dfrac{Gm_e m}{R_e^2} = mg \Rightarrow \dfrac{Gm_e}{R_e^2} = g \Rightarrow Gm_e = gR_e^2$

$$\Rightarrow \text{eq (1)} \quad \underline{Gm_e} = r_{e,s}^3\,\omega^2 \Rightarrow gR_e^2 = R_{e,s}^3\left(\frac{2\pi}{T}\right)^2$$

$$R_{e,s}^3 = \frac{gR_e^2 T^2}{4\pi^2}$$

$$R_{e,s} = \left(\frac{gR_e^2T^2}{4\pi^2}\right)^{1/3} = \left(\frac{\left(9.8\frac{m}{s^2}\right)\left(6.38\times10^6 m\right)^2\left(24hr\right)^2\left(\frac{3.6\times10^3 s}{1hr}\right)^2}{4\pi^2}\right)^{1/3}$$

$$= 6.19\times10^7 m \qquad \text{note:}$$

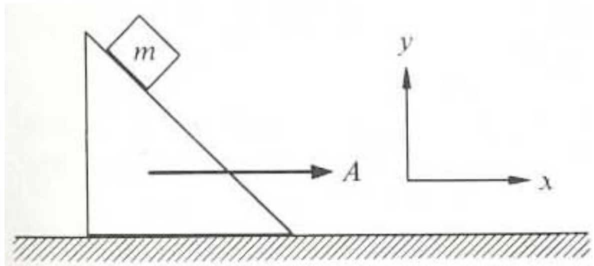
$R_{e,s} - R_e = 6.19\times10^7 m - 6.38\times10^6 m$ height above surface

also $6.6\,R_e = (6.6)(6.38\times10^6 m) = 4.2\times10^7 m$

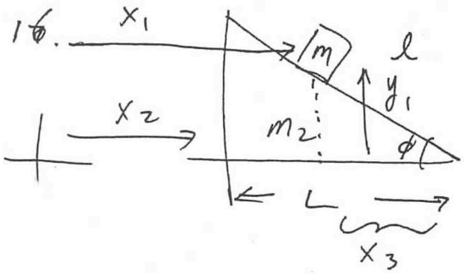
so $R_{e,s} \simeq 6.6\,R_e$

Problem 2.16

A 45^o wedge is pushed along a table with constant acceleration A. A block of mass m slides without friction down the wedge. Find its acceleration. (Gravity is directed down.)



16.



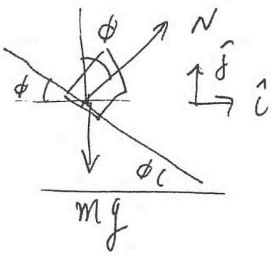
$$L = x_1 - x_2 + y_1 \cot\phi$$

$$y_1 = x_3 \tan\phi$$

$$\ddot L = 0 \Rightarrow \boxed{\ddot x_1 - \ddot x_2 + \ddot y_1 \cot\phi = 0}$$

$$A = \ddot x_2 \Rightarrow \ddot x_1 = A - \ddot y_1 \cot\phi \quad (1)$$

$$\boxed{\ddot y_1 = \frac{A - \ddot x_1}{\cot\phi}}$$



$$\vec F = m_1 \vec a_1$$

$$\hat j: \quad N\cos\phi - mg = m_1 \ddot y_1 \quad (2)$$

$$\hat i: \quad N\sin\phi = m \ddot x_1 \quad (3)$$

$$\frac{N\sin\phi}{N\cos\phi} = \frac{m\ddot x_1}{mg + m_1\ddot y_1}$$

$$\tan\phi = \frac{\ddot x_1}{g + \ddot y_1} = \frac{\ddot x_1}{g + \frac{A - \ddot x_1}{\cot\phi}} = \frac{\ddot x_1}{g + (A - \ddot x_1)\tan\phi}$$

$$\Rightarrow (\tan\phi)(g + (A - \ddot x_1)\tan\phi) = \ddot x_1$$

$$g \tan\phi + (A - \ddot{x}_1)\tan^2\phi = \ddot{x}_1$$

$$g \tan\phi + A \tan^2\phi = \ddot{x}_1 (1 + \tan^2\phi)$$

$$\ddot{x}_1 = \frac{g \tan\phi + A \tan^2\phi}{1 + \tan^2\phi}$$

$$\phi = 45^\circ, \quad \tan\phi = 1$$

$$\ddot{x}_1 = \frac{g + A}{2}$$

$$\ddot{y}_1 = \frac{A - \ddot{x}_1}{\cot\phi} = A - \ddot{x}_1 = A - \left(\frac{g+A}{2}\right)$$

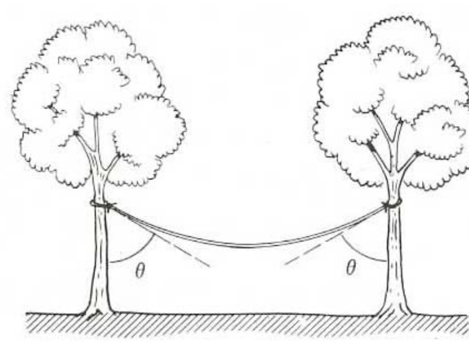
$$\ddot{y}_1 = \frac{A}{2} - \frac{g}{2}$$

$$a_1 = (\ddot{x}_1^2 + \ddot{y}_1^2)^{1/2} = \left(\left(\frac{g+A}{2}\right)^2 + \left(\frac{A-g}{2}\right)^2\right)^{1/2}$$

$$a_1 = \left(\frac{g^2}{2} + \frac{A^2}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}(g^2 + A^2)^{1/2}$$

Problem 2.22

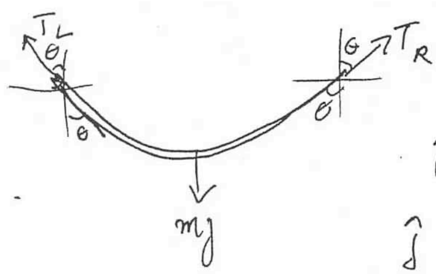
Suppose a rope of mass m hangs between two trees. The ends of the rope are at the same height and they make an angle θ with the trees.



a) What is the tension at the ends of the rope where it is connected to the trees?

b) What is the tension in the rope at a point midway between the trees?

22:



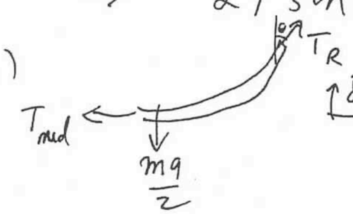
$\hat{j}$ $\hat{i}$ $\vec{F} = m\vec{a}$

$\hat{i}: T_R\cos\theta - T_L\cos\theta = 0 \Rightarrow T_R = T_L$

$\hat{j}: T_R\sin\theta + T_L\sin\theta - mg = 0$

$\Rightarrow 2T\sin\theta = mg \Rightarrow \boxed{T = \frac{mg}{2\sin\theta}}$

b)



$\hat{j}$ $\hat{i}$

$\vec{F} = ma$

$T_R\cos\theta - T_{mid} = 0 \Rightarrow T_{mid} = T\cos\theta$

$\boxed{T_{mid} = \frac{mg\cos\theta}{2\sin\theta}}$