Pset 3 - Problem 34

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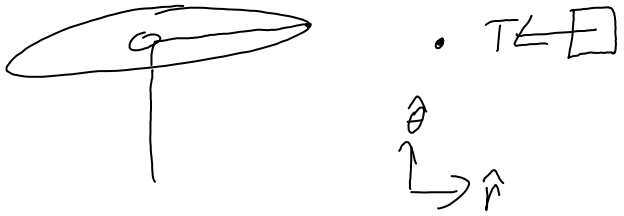
8:25 PM

a) 

b) $\frac{d\vec{r}}{dt} = \frac{d}{dt}(r\hat{r}) = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

$\frac{d^2\vec{r}}{dt^2} = \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}(-\dot{\theta}\hat{r})$

$= (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} - r\dot{\theta}^2\hat{r}$

$= (2v\omega + r\dot{\omega})\hat{\theta} - r\omega^2\hat{r}$

Since $\vec{F} = m\vec{a}$ is only radially inwards,

$$\boxed{2v\omega + r\dot{\omega} = 0}$$

c) $r\frac{d\omega}{dt} = -2v\omega$, so $\frac{d\omega}{\omega} = -\frac{2v}{r}dt$

then $\omega(t) = Ce^{-2v/r}$

since $\omega(0) = \omega_0 = Ce^{-2v/r_0}$, so

$$\boxed{\omega(t) = \omega_0 e^{2v(1/r - 1/r_0)}}$$

d) $\vec{F} = m\vec{a} = -mr\omega^2\hat{r}$

e) $\|\vec{L}\| = \|\vec{r} \times m\vec{v}\| = \vec{r}m(\omega r\hat{\theta} + \dot{r}\hat{r})$

since $\vec{r} \perp \hat{\theta}$, and $\vec{r} \parallel \hat{r}$,

$\|\vec{L}\| = rm\omega r = m\omega r^2$

$$\frac{d}{dt}(m\omega r^2) = m\dot{\omega} r^2 + 2m\omega r\dot{r}$$
$$= mr(\dot{\omega} r + 2\omega \dot{r})$$
$$= mr(\dot{\omega} r + 2\omega v) = 0 \quad \text{by} \quad b)$$