MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group
Physics 8.012

Problem Set 9 Solutions

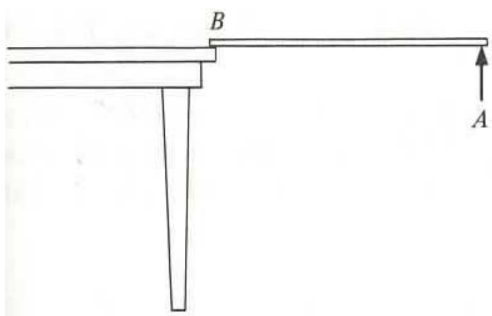
Readings: (KK) Kleppner, Daniel and Kolenkow, Robert, An Introduction to Mechanics, McGraw Hill, Inc., New York, 1973, Chapter 6.

Problems:

Week Nine Chapter 6: 14, 18, 24, 29, 30, 37, 41

Problem 14:

A uniform stick of mass m and length l is suspended horizontally with end B at the edge of a table and the other end A is held by hand. Point A is suddenly released. At the instant after release:

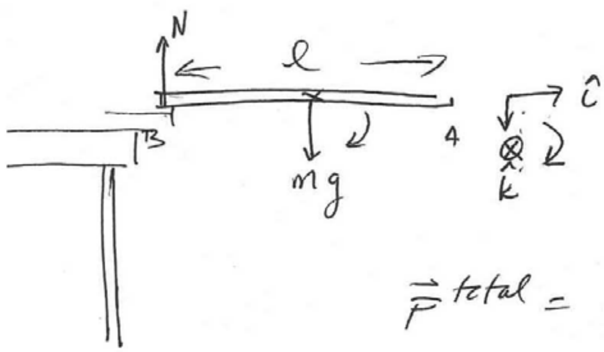


a) What is the torque about the end B on the table?

b) What is the angular acceleration about the end B on the table?

c) What is the vertical acceleration of the center of mass?

d) What is the vertical component of the hinge force at B? Does the hinge force have a horizontal component at the instant after release?



Since the pivot B is free, the only pivot force is the vertical normal force.

$$\vec{F}^{total} = m^{total}\vec{a}_{cm}$$

$$\hat{j}: \quad -N + mg = ma \qquad (1)$$

$$\vec{\tau}_B = I_B\,\vec{\alpha}$$

$$\vec{r}_{B,cm} \times m\vec{g} = \frac{1}{3} m l^2 \alpha \quad \hat{k}$$

$$\hat{k}: \quad +\frac{l}{2} mg = \frac{1}{3} m l^2 \alpha \qquad (2)$$

$$\alpha = \frac{3}{2}\frac{g}{l} \qquad (2a)$$

The angular acceleration and vertical accelerations are related by

$$a = \frac{l}{2}\alpha \qquad (3)$$

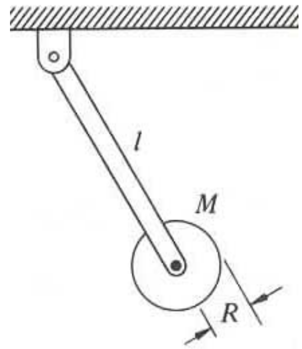
so

$$a = \frac{3}{4} g$$

From eq (1) $\quad N = mg - ma = \frac{1}{4} mg$

Problem 18:

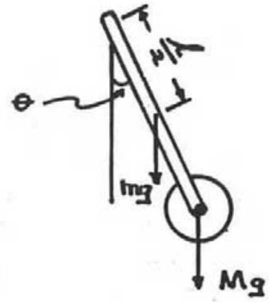
A physical pendulum consists of a disc of radius R and mass m_d fixed at the end of a rod of mass m_r and length l.



a) Find the period of the pendulum.

b) How does the period change if the disk is mounted to the rod by a frictionless bearing so that it is perfectly free to spin?

$$T = I\ddot{\theta}$$



$$-mg\frac{\ell}{2}\theta - Mg\ell\theta = I\ddot{\theta}$$

$$\ddot{\theta} + \frac{mg\frac{\ell}{2} + Mg\ell}{I}\theta = 0$$

$$I = \frac{1}{3}m\ell^2 + \frac{1}{2}MR^2 + M\ell^2 \qquad T = 2\pi\sqrt{\frac{\left(\frac{m}{3}+M\right)\ell^2 + \frac{M}{2}R^2}{\left(\frac{m}{2}+M\right)g\ell}}$$

If disk is on a free bearing it does not contribute to the rotational motion. Hence

$$I' = \frac{1}{3}m\ell^2 + M\ell^2 \qquad T' = 2\pi\sqrt{\frac{\left(\frac{m}{3}+M\right)\ell^2}{\left(\frac{m}{2}+M\right)g\ell}}$$

Energy Method: $E = \left(mg\frac{\ell}{2} + Mg\ell\right)(1-\cos\theta)$

$$+ \frac{1}{2}\left[\frac{1}{3}m\ell^2 + \left(\frac{1}{2}MR^2 + M\ell^2\right)\right]\dot{\theta}^2$$

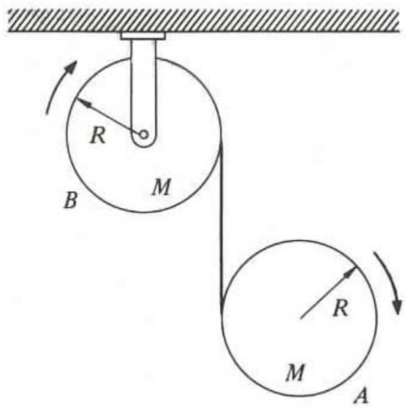
Since $E = \text{const.}$, $\frac{dE}{dt} = 0$

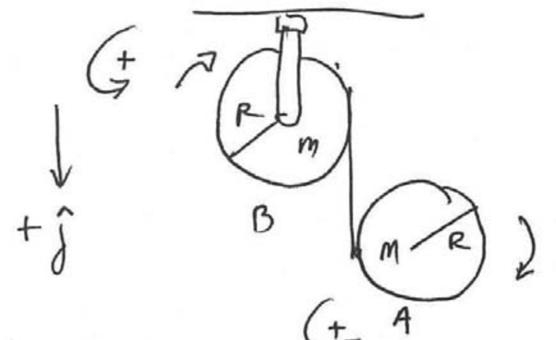
$$\Rightarrow 0 = \left(mg\frac{\ell}{2} + Mg\ell\right)\sin\theta + \left(\frac{1}{3}m\ell^2 + \frac{1}{2}MR^2 + M\ell^2\right)\ddot{\theta}$$

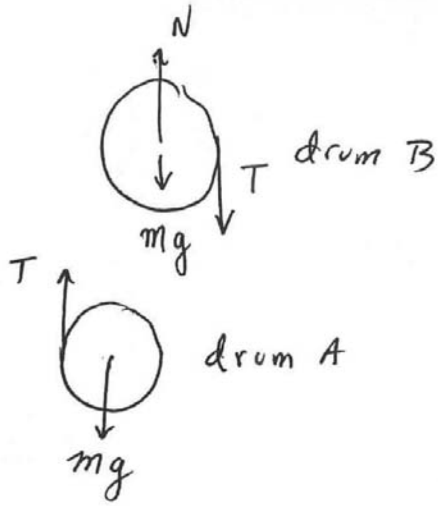
as above. If disk is free, the term $\frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2$ should not be included in E.

Problem 24:

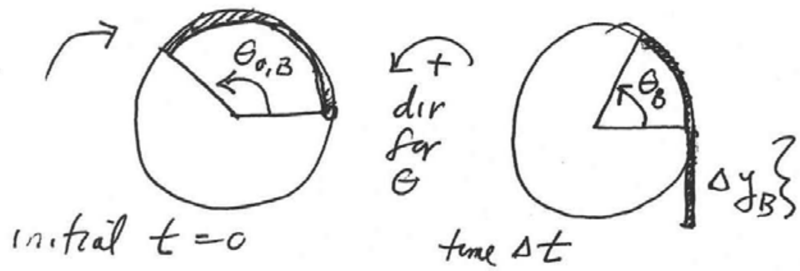
A drum A of mass m and radius R is suspended from a drum B also of mass m and radius R, which is free to rotate about its axis. The suspension is in the form of a massless metal tape wound around the outside of each drum, and free to unwind. Gravity is directed downwards. Both drums are initially at rest. Find the initial acceleration of drum A, assuming that it moves straight down.







The difficult part about this problem is to correctly identify the constraint condition. Consider drum B



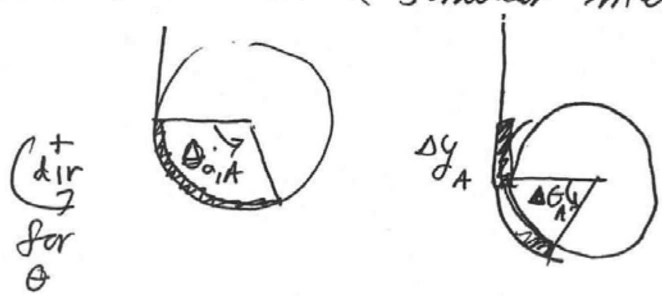
$$\frac{d\theta}{dt} < 0$$

As the drum rotates in time Δt

$$\Delta y_B = R\,\Delta\theta_B = R(\theta_{0,B} - \theta_B(\Delta t))$$

so $$\frac{\Delta y}{\Delta t} = -R\frac{d\theta_B}{dt} \Rightarrow \frac{dy_B}{dt} = -R\frac{d\theta_B}{dt}$$

drum A: in a similar manner



$$\frac{dy_A}{dt} = -R\frac{d\theta_A}{dt}$$

Thus $\Delta y^{total} = \Delta y_A + \Delta y_B$

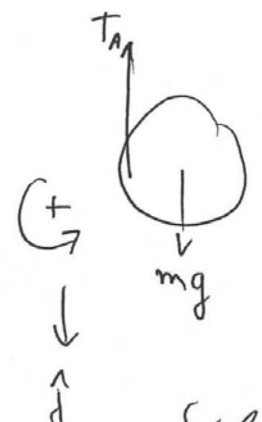
or $$\frac{dy}{dt} = -R\frac{d\theta_A}{dt} - R\frac{d\theta_B}{dt}$$

$$\frac{d^2y}{dt^2} = -R\alpha_A - R\alpha_B \qquad (0)$$

with $\alpha_A < 0$ and $\alpha_B < 0$

The torque about the center of mass on drum A is

$$\tau_{cm,A} = I_{cm,A}\,\alpha_A$$



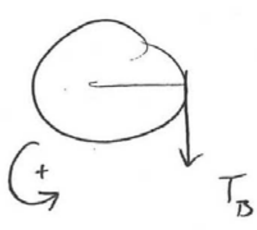
$$-RT_A = \frac{1}{2} m R^2 \alpha_A \qquad (1)$$

the force equation is

$$\vec{F}^{total}_A = m^{total}_A \vec{a}$$

$$mg - T_A = m a \qquad (2)$$

The torque about the center of mass on drum B is



$$-RT_B = \frac{1}{2} m R^2 \alpha_B \qquad (3)$$

$$T_A = T_B \qquad (4)$$

From eq (1) $\quad \alpha_A = \dfrac{-RT}{\frac{1}{2}mR^2} = \dfrac{-2T}{mR}$

From eq (3) $\quad \alpha_B = \dfrac{-2T}{mR}$

Thus from the constraint condition

$$a = -R\alpha_A - R\alpha_B$$

$$a = +\frac{2T}{m} + \frac{2T}{m} = \frac{4T}{m} \Rightarrow T = \frac{ma}{4}$$

From eq (2)

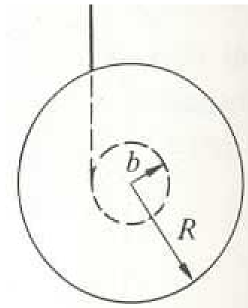
$$mg - T = ma$$

$$mg - \frac{ma}{4} = ma$$

$$mg = \frac{5}{4}ma \Rightarrow a = \frac{4}{5}g$$

Problem 29:

A Yo-Yo of mass m has an axle of radius b and a spool of radius R. It's moment of inertia can be taken to be $I = (1/2)mR^2$ and the thickness of the string can be neglected. The Yo-Yo is released from rest.



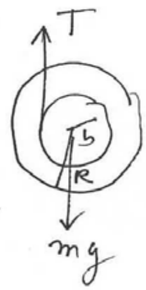
a) What is the tension in the cord as the Yo-Yo descends and as it ascends?

b) The center of the Yo-Yo descends a distance h before the string is fully unwound. Use conservation of energy to find the angular velocity of the Yo-Yo when it reaches its lowest point.

c) What happens to the Yo-Yo at the bottom of the string?

d) Assuming it reverses direction with uniform angular velocity, find the average force on the string while the Yo-Yo turns around.

a)



descending

$\downarrow \hat{j}$ $\quad \otimes \hat{k}$ $\quad$ (+)

$$\begin{array}{l|l} \vec{F} = & m\vec{a} \\ \hline \hat{j}: \; mg - T & = m a \end{array} \qquad (1)$$

$$I_{cm} = \frac{1}{2} m R^2$$

$$\begin{array}{l|l} \vec{\tau}_{cm} = & I_{cm}\vec{\alpha} \\ \hline \hat{k}: \; bT & = \frac{1}{2} m R^2 \alpha \end{array} \qquad (2) \Rightarrow \alpha = \frac{bT}{\frac{1}{2} m R^2} \quad (2a)$$

constraint condition

$$a = b\alpha \qquad (3) \Rightarrow a = \frac{b^2 T}{\frac{1}{2} m R^2} \qquad (3a)$$

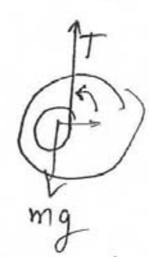
Substituting eq's (2a) and (3a) into eq (1) yields:

$$mg - T = \frac{m b^2 T}{\frac{1}{2} m R^2}$$

Solve for T

$$T = \frac{mg}{1 + \frac{2b^2}{R^2}}$$

ascending:



(+) $\otimes \hat{k}$ $\quad \downarrow \hat{j}$

$$\vec{F} = m\vec{a}$$

$\hat{j}$: $mg - T = ma$ (1)

$$\vec{\tau}_{cm} = I_{cm}\vec{\alpha}$$

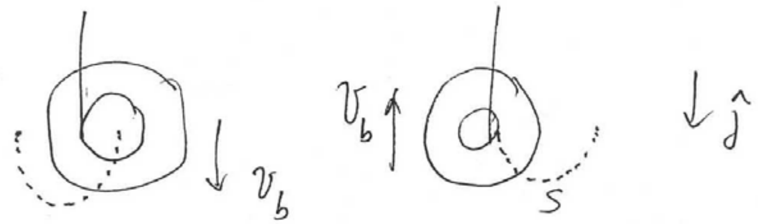
$\hat{k}$: $-bT = \frac{1}{2} m R^2 \alpha$ (4)

$a = -b\alpha$ (5)

note minus sign in constraint eq(4) with eq (5) yields same equations as descending case.

So the tension in the string is the same for ascending as descending

b)



The center of mass rotates through a half circle

$$s = \pi b$$

By conservation of energy

$$E_0 = mgh = E_f = \tfrac{1}{2} m v_b^2 + \tfrac{1}{2} I_{cm} \omega_b^2$$

with $b\omega_b = v_b$. So

$$mgh = \tfrac{1}{2} m v_b^2 + \tfrac{1}{2}\left(\tfrac{1}{2} m R^2\right)\frac{v_b^2}{b^2}$$

$$\Rightarrow \quad v_b^2 = \frac{2gh}{\left(1 + \frac{1}{2}\frac{R^2}{b^2}\right)}$$

The $\vec{F}_{ave}\,\Delta t = \vec{P}_f - \vec{P}_0 = -2mv_b\,\hat{j}$

the time to complete the switch satisfies

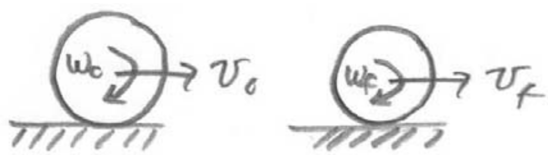
$$v_b\,\Delta t = \pi b \Rightarrow \Delta t = \frac{\pi b}{v_b}$$

so $\vec{F}_{ave} = \dfrac{-2mv_b}{\Delta t}\,\hat{j} = \dfrac{-2mv_b}{\pi b/v_b}\hat{j} = -\dfrac{2m\,v_b^2}{\pi b}\,\hat{j}$

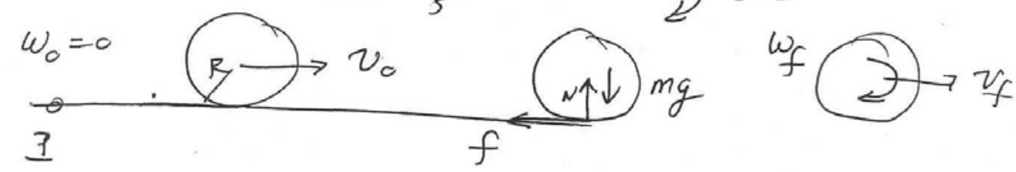
$$\vec{F}_{ave} = \frac{-2m}{\pi b}\,\frac{2gh}{(1+\frac{1}{2}R^2/b^2)}\,\hat{j}$$

Problem 30:

A bowling ball of mass m and radius R is initially thrown down an alley with an initial velocity v_0 and it slides without rolling but due to friction it begins to roll. The moment of inertia of the ball about its center of mass is $I_{cm} = (2/5)mR^2$. What is the velocity of the bowling ball when its just start to roll without slipping.



$I_{cm} = \frac{2}{5} m R^2$ $\quad$ $\otimes \hat{k}$



the torque about the point P is

$$\vec{\tau}_P = \vec{r}_{P,m} \times (\vec{N} + m\vec{g}) = 0$$

since $\vec{N} = -m\vec{g}$. Hence $\vec{\tau}_P = \frac{d\vec{L}_P}{dt} = 0$

so $\vec{L}_P = $ constant.

$$\vec{L}_P = \vec{r}_{P,cm} \times m\vec{v}_{cm} + I_{cm}\vec{\omega}$$

$$\vec{L}_{P,0} = R m v_0 \hat{k}$$

$$\vec{L}_{P,f} = R m v_f \hat{k} + \frac{2}{5} m R^2 \omega_f \hat{k}$$

the rolling without slipping condition is

$$v_f = R\omega_f$$

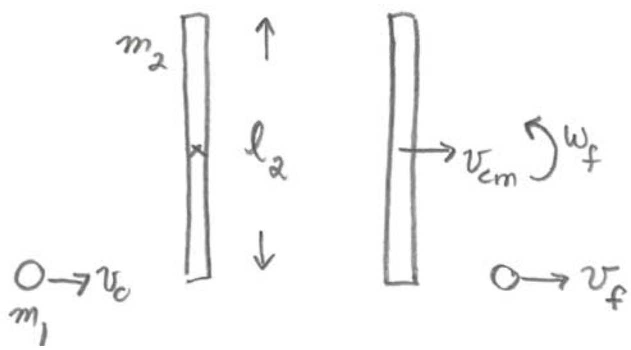
So conservation of angular mom. becomes

$$\hat{k}: \; R m v_0 = R m v_f + \frac{2}{5} m \frac{R^2 v_f}{R} = \frac{7}{5} m R v_f$$

$$\Rightarrow \quad v_f = \frac{5}{7} v_0$$

Problem 37:

A hockey puck of mass m_1 slides along ice with a velocity v_0 and strikes one end of a stick lying on the ice of length l_2 and mass m_2. The center of mass of the stick moves with an unknown magnitude v_{cm}. The stick also rotates about the center of mass with unknown angular velocity ω_f. The puck continues to move in the same straight line as before it hit the stick with velocity v_f. Assume the ice is frictionless and there is no loss of mechanical energy during the collision.



a) Write down the equation for conservation of momentum.

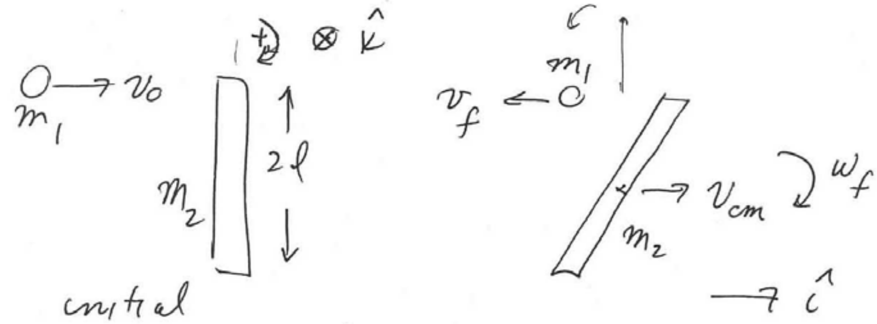
b) Write down the equation for conservation of energy.

c) Is there any external torques acting on the system consisting of the puck and the stick? Write down the equation for conservation of angular momentum about a convenient point.

d) Find the velocity of the center of mass of the stick.

e) Find the velocity of the puck after the collision.

f) Find the angular velocity of the stick after the collision.



Conservation of Energy:

$$(1) \qquad \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_{cm}^2 + \frac{1}{2} I_{cm} \omega_f^2$$

Conservation of Momentum

$$\hat{i}: \qquad m_1 v_0 = - m_1 v_f + m_2 v_{cm} \qquad (2)$$

$$I_{cm} = \frac{1}{12} m_2 (2\ell)^2 = \frac{1}{3} m_2 \ell^2 \qquad (3)$$

Solve eq (2) for v_{cm}

$$\text{eq (2a)}: \qquad v_{cm} = \frac{m_1}{m_2}(v_0 + v_f)$$

Substitute into eq (1)

$$(1a) \quad \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2}(v_0 + v_f) \right)^2 + \frac{1}{2}\left(\frac{1}{3} m_2 \ell^2\right)\omega_f$$

Conservation of Ang momentum about cm:

$$\vec{L}_{0,cm} = \ell\, m_1 v_0 \hat{k}$$
$$\vec{L}_{f,cm} = -\ell\, m_1 v_f + I_{cm} \omega_f \hat{k}$$

$$\vec{L}_{0,cm} = \vec{L}_{f,cm}$$

$$\hat{k}: \quad l\, m_1 v_0 = -l\, m_1 v_f + I_{cm}\, \omega_f$$

$$\Rightarrow \quad \omega_f = \frac{l\, m_1 (v_0 + v_f)}{\frac{1}{3} m_2 l^2} = 3\,\frac{m_1}{m_2}\,(v_0 + v_f)$$

$$\frac{1}{2} I_{cm}\, \omega_f^{\,2} = \frac{1}{2}\, 3\, \frac{l^2 m_1^2 (v_0 + v_f)^2}{m_2\, l^2}$$

$$\frac{1}{2} I_{cm}\, \omega_f^{\,2} = \frac{3}{2}\, \frac{m_1^2}{m_2}\, (v_0 + v_f)^2$$

substitute this into eq (1a)

$$\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 \left(\frac{m_1^2}{m_2^2} (v_0 + v_f)^2 \right) + \frac{3}{2}\, \frac{m_1^2}{m_2} (v_0 + v_f)^2$$

$$\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} \left(4\, \frac{m_1^2}{m_2} (v_0 + v_f)^2 \right)$$

$$\Rightarrow \quad v_0^2 = v_f^2 + 4\,\frac{m_1}{m_2}\,(v_0 + v_f)^2$$

$$v_0^2 = v_f^2 + 4\,\frac{m_1}{m_2}\,(v_0^2 + 2 v_0 v_f + v_f^2)$$

$$0 = v_0^2 \left(-1 + 4\frac{m_1}{m_2}\right) + 8\frac{m_1}{m_2}\, v_0 v_f + v_f^2 \left(1 + 4\frac{m_1}{m_2}\right)$$

$$0 = v_f^2 + \left(\frac{8 m_1/m_2}{1 + 4 m_1/m_2} \right) v_0 v_f + v_0^2\, \frac{\left(-1 + 4\frac{m_1}{m_2}\right)}{\left(1 + 4\frac{m_1}{m_2}\right)}$$

let $\beta = 4 m_1/m_2 \quad \Rightarrow$

$$0 = v_f^2 + \frac{2\beta}{1+\beta} v_0 v_f + v_0^2 \frac{(-1+\beta)}{(1+\beta)}$$

$$v_f = \frac{-\frac{2\beta}{1+\beta} v_0 \pm \left(\frac{4\beta^2 v_0^2}{(1+\beta)^2} - 4v_0^2\left(\frac{-1+\beta}{1+\beta}\right)\right)^{1/2}}{2}$$

$$= -\frac{\beta}{1+\beta} v_0 \pm \frac{v_0}{2}\left(\frac{4\beta^2 - 4(-1+\beta)(1+\beta)}{(1+\beta)^2}\right)^{1/2}$$

$$= \frac{-\beta}{1+\beta} v_0 \pm \frac{(\beta^2 - (-1+\beta^2))^{1/2}}{1+\beta} v_0$$

$$= \frac{-\beta}{1+\beta} v_0 \pm \frac{1}{1+\beta} v_0$$

positive root: $\boxed{v_f = \frac{(-\beta+1)}{(1+\beta)} v_0}$

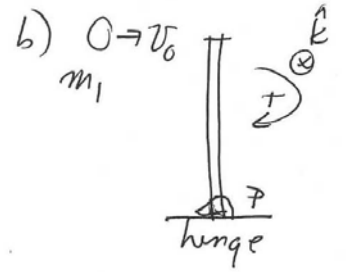
negative root: $v_f = \frac{-\beta-1}{1+\beta} v_0 = -v_0$

This is just the initial state

$$v_f = \frac{\left(-4\frac{m_1}{m_2}+1\right)}{\left(4\frac{m_1}{m_2}+1\right)} v_0$$

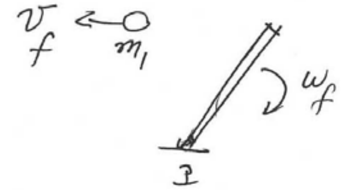
when $m_1 = m_2 \Rightarrow v_f = \frac{3}{5} v_0$

b)



... is not conserved but angular momentum about the pivot point is conserved

$$\vec{L}_{P,0} = m_1 v_0 2l \hat{k}$$



$$\vec{L}_{P,f} = -m_1 v_f 2l \hat{k} + I_P \omega_f \hat{k}$$

$$I_P = \frac{1}{3} m_2 (2l)^2 = \frac{4}{3} m_2 l^2$$

$$(1) \quad m_1 v_0 2l = -m_1 v_f 2l + I_P \omega_f$$

Assume energy is still conserved

$$(2) \quad \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} I_P \omega_f^2$$

Solve eq (1) for $I_P \omega_f = m_1 2l (v_0 + v_f)$ and substitute into eq (2) using

$$\frac{1}{2} I_P \omega_f^2 = \frac{1}{2} \frac{m_1^2 4l^2 (v_0 + v_f)^2}{I_P} = \frac{1}{2} \frac{m_1^2 4l^2 (v_0 + v_f)^2}{\frac{4}{3} m_2 l^2}$$

$$\frac{1}{2} I_P \omega_f^2 = \frac{1}{2} \; 3 \; \frac{m_1^2}{m_2} (v_0 + v_f)^2$$

Eq (2) becomes:

$$\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_f^2 + \frac{1}{2}\, 3\frac{m_1^2}{m_2}(v_0 + v_f)^2$$

$$v_0^2 = v_f^2 + 3\frac{m_1}{m_2}(v_0 + v_f)^2$$

$$v_0^2 = v_f^2 + 3\frac{m_1}{m_2}\left(v_0^2 + 2 v_0 v_f + v_f^2\right)$$

$$\Rightarrow \quad 0 = v_f^2\left(1 + 3\frac{m_1}{m_2}\right) + 3\frac{m_1}{m_2}\, 2 v_0 v_f + v_0^2\left(-1 + 3\frac{m_1}{m_2}\right)$$

Let $\beta = 3 m_1/m_2$ then

$$0 = v_f^2(1+\beta) + 2\beta v_0 v_f + v_0^2(1-\beta)$$

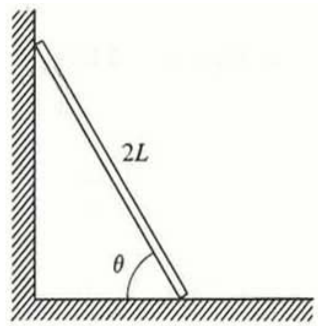
This equation has exactly the same form as part a) so the solution is identical

$$v_f = \frac{1-\beta}{1+\beta} v_0 = \frac{1 - 3\frac{m_1}{m_2}}{1 + 3\frac{m_1}{m_2}} v_0$$

when $m_1 = m_2 \Rightarrow v_f = \frac{1}{2} v_0$

Problem 41:

A plank of length $2l$ leans against a wall. The mass of the plank is m which is uniformly distributed. The plank is initially inclined at an angle θ with respect to the horizontal. It starts to slip downward without friction.

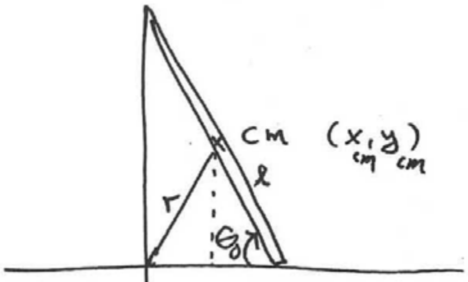


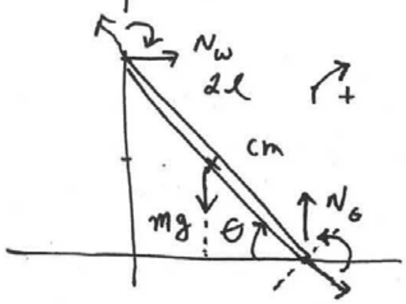
a) Draw a force diagrams showing all the forces acting on the plank. What is the condition that the plank just starts to slip from the wall?

b) Is the mechanical energy of the plank conserved as it slips down the wall?

c) What equations arise from the conditions for static equilibrium for both forces and torque? Think about which point to compute the torque about.

d) Show that the top of the plank loses contact with the wall when it is two-thirds of its initial height against the wall. Hint: only a single variable and its derivatives are needed to describe the motion of the system. Consider the motion of the center of mass of the plank.





$$I_{cm} = \frac{1}{12} m (2\ell)^2 = \frac{1}{3} m \ell^2$$

$$x_{cm} = \ell \cos\theta$$

$$y_{cm} = \ell \sin\theta$$

$\hat{j}$, $\hat{i}$

$$\hat{j}: \quad N_G - mg = m\ddot{y} \qquad (3)$$

$$\hat{i}: \quad N_w = m\ddot{x} \qquad (4)$$

$$\tau_{cm} = I_{cm}\ddot{\theta}$$

$$-N_g \ell \cos\theta + N_w \ell \sin\theta = \frac{1}{3} m \ell^2 \ddot{\theta} \qquad (1)$$

find θ s.t. $N_w = 0$

$$-N_g \ell \cos\theta = \frac{1}{3} m \ell^2 \ddot{\theta} \qquad (1a)$$

$$\text{eq } (4) \Rightarrow \quad 0 = m\ddot{x} \qquad (2)$$

$$x = l\cos\theta$$

$$\dot{x} = -l\sin\theta\,\dot{\theta}$$

$$\ddot{x} = -l\cos\theta\,\dot{\theta}^2 - l\sin\theta\,\ddot{\theta}$$

eq (2) $\Rightarrow$ $0 = -l\cos\theta\,\dot{\theta}^2 - l\sin\theta\,\ddot{\theta}$

$$\ddot{\theta} = \frac{-\cos\theta}{\sin\theta}\,\dot{\theta}^2 \qquad (2a)$$

$$y = l\sin\theta$$

$$\dot{y} = l\cos\theta\,\dot{\theta}$$

$$\ddot{y} = -l\sin\theta\,\dot{\theta}^2 + l\cos\theta\,\ddot{\theta}$$

eq (3) $\Rightarrow$ $N_G - mg = m(-l)(\sin\theta\,\dot{\theta}^2 - \cos\theta\,\ddot{\theta})$

$$N_G = mg - ml(\sin\theta\,\dot{\theta}^2 - \cos\theta\,\ddot{\theta}) \qquad (3a)$$

eq (1) $\quad -N_g\, l\cos\theta = \frac{1}{3} m l^2 \ddot{\theta}$

use eq (2a) in eq (3a)

$$N_G = mg - ml\left(\sin\theta\,\dot{\theta}^2 + \cos\theta\,\frac{\cos\theta}{\sin\theta}\,\dot{\theta}^2\right)$$

$$N_G = mg - ml\left(\frac{1}{\sin\theta}\,\dot{\theta}^2\right)$$

$$\left(-mg+\frac{ml\dot{\theta}^2}{\sin\theta}\right)l\cos\theta = \frac{1}{3}ml^2\ddot{\theta}$$

$$\left(-mg+\frac{ml\dot{\theta}^2}{\sin\theta}\right)l\cos\theta = -\frac{1}{3}ml^2\frac{\cos\theta}{\sin\theta}\dot{\theta}^2$$

$$-mgl\cos\theta = -\frac{1}{3}ml^2\frac{\cos\theta}{\sin\theta}\dot{\theta}^2 - ml^2\frac{\cos\theta}{\sin\theta}\dot{\theta}^2$$

$$(5) \qquad g = +\left(\frac{1}{3}\frac{l}{\sin\theta}+\frac{l}{\sin\theta}\right)\dot{\theta}^2 = \frac{4}{3}\frac{l}{\sin\theta}\dot{\theta}^2$$

$$E_0 = mgl\sin\theta_0$$

$$E_f = \frac{1}{2}mv^2+\frac{1}{2}I_{cm}\omega^2+mgl\sin\theta$$

$$v = l\dot{\theta}$$

$$= \frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}\frac{1}{3}ml^2\dot{\theta}^2+mgl\sin\theta$$

$$mgl\sin\theta_0 = \frac{2}{3}ml^2\dot{\theta}^2+mgl\sin\theta$$

$$mgl\sin\theta_0 - mgl\sin\theta = \frac{2}{3}ml^2\dot{\theta}^2$$

$$g(\sin\theta_0-\sin\theta) = \frac{2}{3}l\dot{\theta}^2$$

eq (5) $\Rightarrow$ $g\frac{\sin\theta}{2} = \frac{2l}{3}\dot{\theta}^2$

$$g(\sin\theta_0 - \sin\theta) = +\frac{g}{2}\sin\theta$$

$$\Rightarrow \quad l\sin\theta_0 = \frac{3}{2}l\sin\theta$$

$$\frac{2}{3}l\sin\theta_0 = l\sin\theta$$

$$\frac{2}{3}h_0 = h_f$$