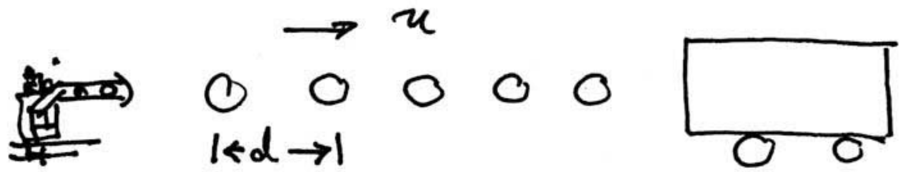
**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**
**Experimental Study Group**
**Physics 8.012**

## Problem 1: Momentum transfer

A baseball pitching machine hurls balls of mass $m_b$ at the back of a cart of mass $m_c$, which moves along the ground. You may ignore friction between the cart and the ground. Also you may ignore any deflection in the path of the ball due to the action of gravity. The pitching machine ejects the balls with speed $u$ at a rate such that the space between the balls is $d$. Let's assume that the balls can be approximated as a continuous stream with density $\lambda = m_b / d$. When a ball hits the cart it sticks briefly and then falls to the ground.




a) Derive the differential equation for the velocity $v$ of the cart at time $t$. Show your momentum flow diagrams at time $t$ and time $t + \Delta t$. Clearly identify your system and label all the mass velocities in your system. Express your answer in terms of the rate that the balls hit the surface $dm / dt$, the speed of the balls $u$, the mass of the cart $m_c$, and the velocity of the cart $v$ and any necessary derivatives.

b) Using conservation of mass, at time $t$, find an expression for the rate that the balls hit the surface, $dm / dt$, as a function of the speed of the cart $v$, the density $\lambda$, and the speed $u$ of the balls.

c) Use your result from part b) in part a) to find a differential equation for the velocity $v$ of the cart at time $t$ only in terms of the speed of the balls $u$, the mass of the cart $m_c$, the mass of the ball $m_b$, the average distance between balls $d$, and the velocity of the cart $v$ and any necessary derivatives.

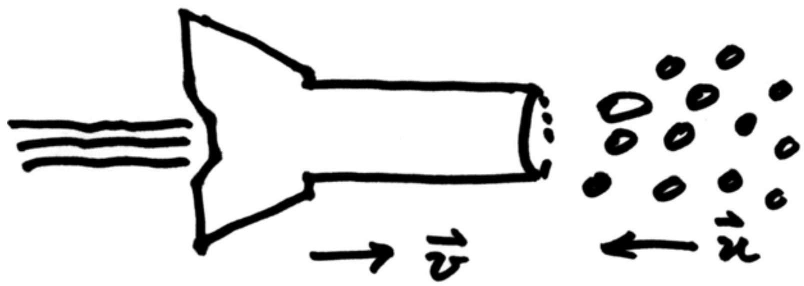
d) What is the limiting value of $v$ as time approaches infinity? You do not need to have integrated your equation in part c) to answer this.

e) At time $t = 0$ the cart is released from rest. Set up an integral expression for the speed of the cart $v$ as a function of time $t$, and integrate this to find the velocity of the cart as a function of time. (The integration is worth 2 points.)

f) At a time $t$ when the cart is moving with speed $v$, what is the average force due to the balls on the cart?

**Problem 2: Momentum Transfer Space Junk**

A spacecraft of cross-sectional area $A$, proceeding along the positive $x$-direction, enters an asteroid storm at time $t = 0$, in which the asteroid mass density is $\rho$ and the average asteroid velocity is $\vec{\mathbf{u}} = -u\,\hat{\mathbf{i}}$ in the negative $x$-direction. As the spacecraft proceeds through the storm, all of the asteroids that hit the spacecraft stick to it.



a) Suppose that at time $t$ the velocity of the spacecraft is $\vec{\mathbf{v}} = v\,\hat{\mathbf{i}}$ in the positive x-direction, and its mass is $m$. Further, suppose that in an interval $\Delta t$, the mass of the spacecraft increases by an amount $\Delta m$. Given that there are no external forces, using conservation of momentum find an equation for the change of the spacecraft velocity $\Delta v$, in terms of $\Delta m$, $u$, and $v$ ?

b) When the spacecraft enters the asteroid storm, the magnitude of its velocity and mass are $v_0$ and $m_0$, respectively. Integrate your differential equation in part a) to find the velocity $v$ of the spacecraft when the mass is $m$.

c) Find an expression for the mass of the asteroids $\Delta m$ that sticks to the spacecraft within the time interval $\Delta t$? (Hint: consider the volume of asteroids swept up by the spacecraft in time $\Delta t$).

d) When the spacecraft enters the asteroid storm, the magnitude of its velocity and mass are $v_0$ and $m_0$, respectively. What is the mass of the spacecraft at time $t$? (Use your results from parts c) and b).)

**Problem 3: Momentum transfer boat and fire hose**

A burning boat of initial mass $m_0$ is initially at rest. A fire fighter stands on the Harvard Bridge and sprays water onto the boat. The water leaves the fire hose with a velocity $u$ at a rate $\alpha$ (measured in $\mathrm{kg \cdot s^{-1}}$). Assume that the motion of the boat and the water jet are horizontal, that gravity does not play any role, and that the river can be treated as a frictionless surface. Also assume that the change in the mass of the boat is only due to the water jet and that all the water from the jet is added to the boat.

a) In a time interval $[t, t+\Delta t]$, an amount of water $\Delta m$ hits the boat. Choose a system. Is the total momentum constant in your system? Write down a differential equation that results from the analysis of the momentum changes inside your system.

b) Integrate the differential equation you found in part a), to find the velocity $v(m)$ as a function of the increasing mass $m$ of the boat, $m_0$, and $u$.