MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group

Physics 8.012, Fall 2010

Problem Set 2 Solutions

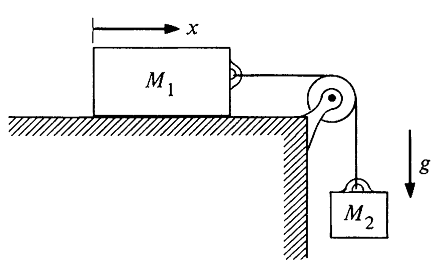
Due: Friday, September 24

Reading: Kleppner and Kolenkow, *An Introduction to Mechanics*, Chapter Two

Problem 1: K&K 2.2

Problem

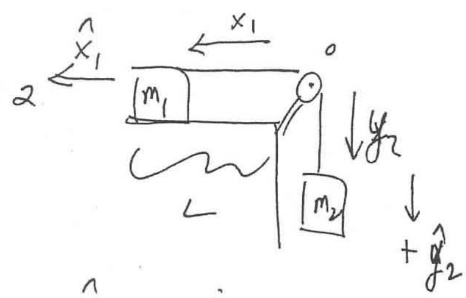
The two blocks shown in the figure are connected by a string of negligible mass. If the system is released from rest, find how far the block of mass m_1 slides in time t. Neglect friction.

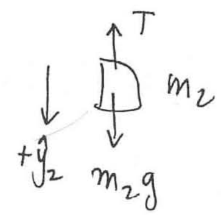


Solution

The system can be treated as a single block of mass $m_1 + m_2$, with a force due to gravity of $m_2 g$. The acceleration is then $\frac{m_2 g}{m_1+m_2}$, so the distance is $\frac{m_2 g}{2(m_1+m_2)} t^2$.

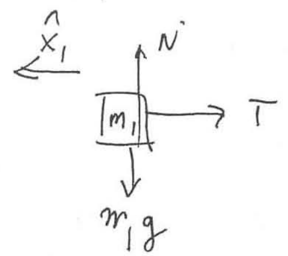
In more detail:





$\vec{F}_2$	$= m_2 \vec{a}_2$
$\hat{y}_2: m_2 g - T$	$= m_2 a_{2,y}$

$$\Rightarrow m_2 g - T = m_2 a_{2,y} \quad (1)$$



$\vec{F}_1$	$= m_1 \vec{a}_1$
$-T$	$= m_1 a_{x,1}$

(2)

constraint $a \equiv a_{2,y} = -a_{x,1}$

eq (1) $\Rightarrow$ $m_2 g - T = m_2 a$ $\Rightarrow$ $m_2 g - m_1 a = m_2 a$

eq (2) $\Rightarrow$ $T = m_1 a$ $\Rightarrow$ $a = \dfrac{m_2 g}{m_1 + m_2}$

$$x_1 = x_{1,0} + \frac{1}{2} a_{x,1} t^2$$

$$x_1 = L - \frac{1}{2} a t^2 = L - \frac{1}{2}\left(\frac{m_2 g}{m_1 + m_2}\right) t^2$$

Problem 2: K&K 2.4

Problem

Two particles of mass m_1 and m_2 undergo uniform circular motion about each other at a separation R under the influence of an attractive force of magnitude F. The angular velocity is ω radians per second.

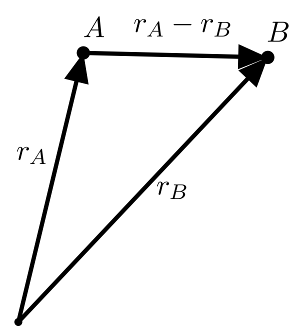
a) Show that $R = \frac{F}{\omega^2} \cdot \left(\frac{1}{m_1} + \frac{1}{m_2}\right)$.

b) Explain why you can think of this problem as equivalent to a single body of mass μ where $\frac{1}{\mu} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)$ undergoing circular motion of radius R due to the influence of a central attractive force of magnitude F.

Solution

a) Let r_1 and r_2 denote the radii of the circles of m_1 and m_2, respectively. Then $R = r_1 + r_2$. Since the movement is uniform and circular, the force must be $\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$. The acceleration of m_1 is then $\frac{v_1^2}{r_1}$, and the acceleration of m_2 is $\frac{v_2^2}{r_2}$. Since the angular velocity is ω, and the circumference is $2\pi r_1$ and $2\pi r_2$, respectively, $v_1 = \frac{2\pi r_1}{\frac{2\pi}{\omega}} = r_1\omega$ and $v_2 = r_2\omega$. Then $F = m_1 r_1 \omega^2 = m_2 r_2 \omega^2$. Since $R = r_1 + r_2$, $F = m_1 r_1 \omega^2 = m_2(R - r_1)\omega^2$. So $F = m_1 r_1 \omega^2 = m_2 R\omega^2 - m_2 r_1 \omega^2$. Dropping the F, $m_1 r_1 = m_2 R - m_2 r_1$, so $(m_1 + m_2) r_1 = m_2 R$. Then $r_1 = \frac{R m_2}{m_1 + m_2}$. Thus, $F = \frac{R m_1 m_2 \omega^2}{m_1 + m_2}$, so $R = \frac{F(m_1+m_2)}{\omega^2 m_1 m_2} = \frac{F}{\omega^2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$.

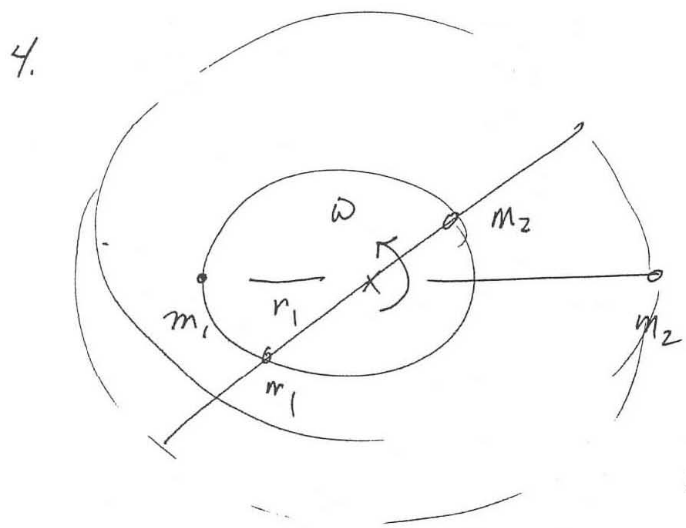
b) Let $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$ as shown below.

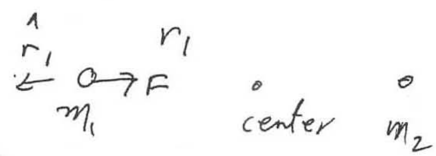


Then $\vec{F}_{AB} = m_B \vec{a}_B$ and $\vec{F}_{BA} = m_A \vec{a}_A = -\vec{F}_{AB}$. $\frac{\vec{F}_{AB}}{m_B} - \frac{\vec{F}_{BA}}{m_A} = \vec{a}_B - \vec{a}_A$, so $\vec{F}_{AB}\left(\frac{1}{m_A} + \frac{1}{m_B}\right) = \frac{\partial^2}{\partial t^2}(\vec{r}_B - \vec{r}_A)$. Then $\vec{F}_{AB}\left(\frac{1}{m_A} + \frac{1}{m_B}\right) = \frac{\partial^2}{\partial t^2}\vec{r}_{AB}$. Define μ such that $\frac{1}{\mu} = \frac{1}{m_A} + \frac{1}{m_B}$, and ω to be $\left|\frac{\partial \hat{r}_{AB}}{\partial t}\right|$. Then $\vec{F}_{AB} = \mu \frac{\partial^2}{\partial t^2}\vec{r}_{AB} = -\mu R \omega^2 \hat{r}_{AB}$, since the vector is rotating with uniform angular momentum. Then $R = \frac{F}{\omega^2}\mu$, which corresponds to the setup in (a).

With diagrams:

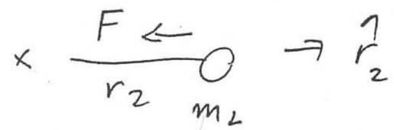
4.





$$\begin{array}{c|c} \vec{F} & m_1 \vec{a}_1 \\ \hline \hat{r}_1: \; -F & -m_1 r_1 \omega^2 \end{array}$$

$$F = m_1 r_1 \omega^2 \quad (1)$$



$$\begin{array}{c|c} \vec{F} & m_2 \vec{a}_2 \\ \hline \hat{r}_2: \; -F & -m_2 r_2 \omega^2 \end{array}$$

$$\Rightarrow \quad F = m_2 r_2 \omega^2$$

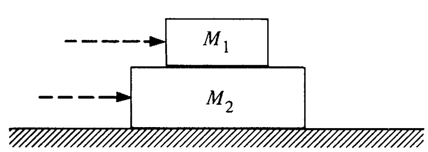
$$L = r_1 + r_2 = \frac{F}{m_1 \omega^2} + \frac{F}{m_2 \omega^2} = \frac{F}{\omega^2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$\text{or} \quad F = \frac{\omega^2 L}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}.$$

Problem 3: K&K 2.7

Problem

Consider two textbooks that are resting one on top of the other. The lower book has $m_2 = 0.8$ kg and is resting on a nearly frictionless surface. The upper book has mass $m_1 = 2.0$ kg. Suppose the coefficient of static friction is given by $\mu_s = 0.1$.



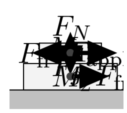
a) What is the maximum force which the upper book can be pushed horizontally so that the two books move together without slipping? Identify all action-reaction pairs of forces in this problem.

b) What is the maximum force which the lower book can be pushed horizontally so that the two books move together without slipping? Identify all action-reaction pairs of forces in this problem.

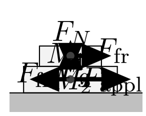
c) Explain why one of your forces in parts a) and b) is larger than the other.

Solution

a) The top block exerts friction on the bottom block, and vice versa. Gravity exerts a force on each block, and the surface below exerts a normal force. The blocks can be treated as a single block with mass $m_1 + m_2$ to which F_{applied} is applied. The acceleration is $\frac{F_{\text{applied}}}{m_1+m_2}$. The maximal force due to friction is $\mu_s F_n = m_1 g \mu_s$. The maximal acceleration of the top block is thus $\frac{\mu_s m_1 g}{m_1} = \mu_s g = 1$ m / s^2. Thus, the maximal applied force is $(1 \text{ m / s}^2)(m_1 + m_2) = 3$ N.



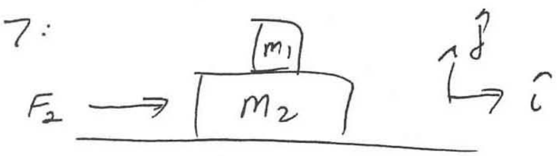
b) The top block exerts friction on the bottom block, and vice versa. Gravity exerts a force on each block, and the surface below exerts a normal force. The blocks can be treated as a single block with mass $m_1 + m_2$ to which F_{applied} is applied. The acceleration is $\frac{F_{\text{applied}}}{m_1+m_2}$. The maximal force due to friction is $\mu_s F_n = m_1 g \mu_s$. The maximal acceleration of the top block is thus $\frac{\mu_s m_1 g}{m_2} = 2.45$ m / s$^2 \approx 2$ m / s^2. Thus, the maximal applied force is $(2 \text{ m / s}^2)(m_1 + m_2) = 7$ N.



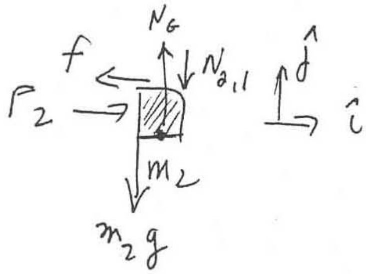
c) The maximal acceleration due to friction is the same in both cases, but the acceleration is of a larger mass in part b), so more force can be applied.

In more detail:

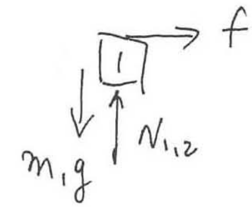
7:



$a_1 = a_2$



	$\vec{F}$	$m_2 \vec{a}_2$	
$\hat{i}$:	$F_2 - f$	$= m_2 a_2$	(1)
$\hat{j}$:	$N_G - N_{2,1} - m_2 g$	$= 0$	(2)



	$\vec{F}$	$m_1 \vec{a}_1$	
$\hat{i}$:	f	$= m_1 a_1$	(3)
$\hat{j}$:	$N_{1,2} - m_1 g$	$= 0$	(4)

Just slipping condition $f = \mu N_{1,2}$

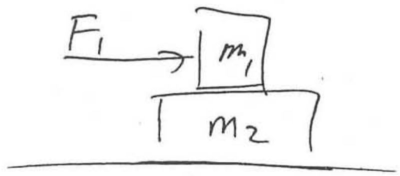
eq (4) $\Rightarrow N_{1,2} = m_1 g \Rightarrow f = \mu m_1 g$

eq (3) $f = m_1 a_1 \Rightarrow \mu m_1 g = m_1 a_1 \Rightarrow \boxed{\mu g = a_1}$

$$= \mu m_1 g + m_2 \mu g$$

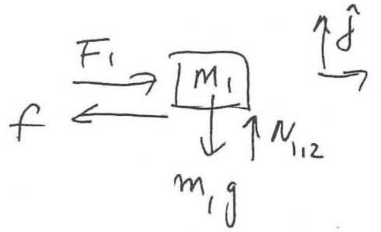
$$\boxed{F_2 = \mu g (m_1 + m_2)}$$

parts 5)

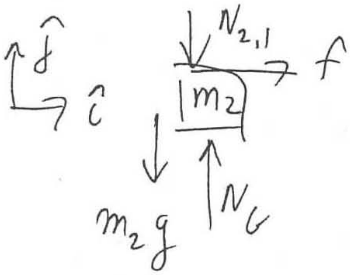


$a_1 = a_2$

$f = \mu N_{1,2}$ just slipping



	$\vec{F}$	$m_1 \vec{a}_1$	
$\hat{i}$:	$F_1 - f$	$= m_1 a_1$	(5)
$\hat{j}$:	$N_{1,2} - m_1 g$	$= 0$	(6)



	$\vec{F}$	$m_2 \vec{a}_2$	(7
$\hat{i}$:	f	$= m_2 a_2$	(7)
$\hat{j}$:	$N_G - N_{2,1} - m_2 g$	$= 0$	(8)

eq (6) $\Rightarrow N_{1,2} = m_1 g \Rightarrow f = \mu m_1 g$

eq (7) $\Rightarrow \mu m_1 g = m_2 a_2 \Rightarrow \boxed{a_2 = \frac{\mu m_1 g}{m_2}}$

eq (1) $\Rightarrow F_1 = f + m_1 a_1$ $\qquad a_1 = a_2$

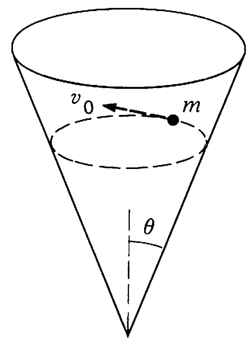
$$= \mu m_1 g + m_1 \mu \frac{m_1}{m_2} g$$

$$\boxed{F_1 = \mu g m_1 \left(1 + \frac{m_1}{m_2}\right)}$$

Problem 4: K&K 2.9

Problem

A body of mass m is moving in a horizontal circle of radius r with a constant speed v_0 on the inside wall of a cone. Assume the wall of the cone is frictionless. The wall of the cone makes an angle θ with the vertical.



a) Draw a free body force diagram showing all the forces acting on the mass.

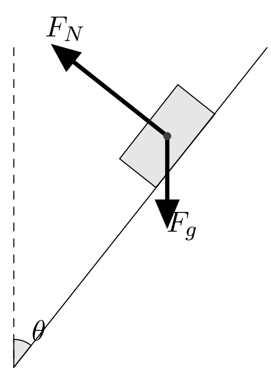
b) What is the speed, v_0, of the body, in terms of r, m, θ, and g?

c) How long will the mass take to go around the circle?

d) Now assume there is a coefficient of static friction μ_s. Find the maximum speed the mass can move on the inside of a cone and still move in a circular orbit of radius r.

Solution

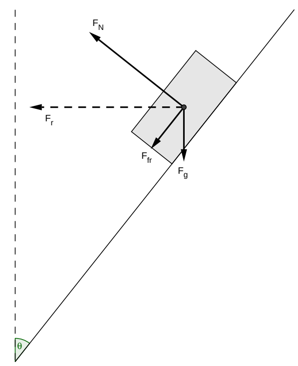
a)



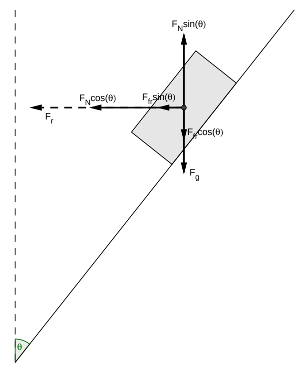
b) For circular motion, $\frac{\partial \vec{r}}{\partial t} = r\frac{\partial \theta}{\partial t}\hat{\theta} = v_0\hat{\theta}$, so $\frac{\partial^2 \vec{r}}{\partial t^2} = -r\left(\frac{\partial \theta}{\partial t}\right)^2\hat{r} = -\frac{v_0^2}{r}\hat{r}$. Since $F_N \sin\theta = mg$, and $\vec{F}_{\text{net}} = -F_N\cos\theta\hat{\imath}$, $\frac{mg}{\sin\theta}\cos\theta = m\frac{v_0^2}{r}$, so $v_0 = \sqrt{gr\cot\theta}$.

c) The circumference is $2\pi r$, and $v_0 = \sqrt{gr\cot\theta}$, so it takes $\frac{2\pi\sqrt{r}}{\sqrt{g\cot\theta}}$.

d) FIX The diagram becomes:



Decomposing the vectors,



The radial acceleration is $\frac{v^2}{r}$, so $\frac{\partial^2 \vec{r}}{\partial t^2} = -\frac{v^2}{r}\hat{i}$. Then $m\frac{v^2}{r} = F_{\text{fr}} \sin\theta + F_g \cos\theta = F_N \mu_s + F_g \cos\theta$. The normal force is $m\frac{v^2}{r}\cos\theta$. Then $m\frac{v^2}{r} = m\frac{v^2}{r}\cos\theta\mu_s + mg\cos\theta$, so $\frac{v^2}{r}(1 - \cos\theta\mu_s) = g\cos\theta$, so $v = \sqrt{\frac{rg\cos\theta}{1-\cos\theta\mu_s}}$. ???

Problem 5: K&K 2.10

Problem

The earth is spinning about its axis with a period of 23 hours 56 min and 4 sec. The equatorial radius of the earth is $6.38 \cdot 10^6$ m. The latitude of Cambridge, Mass is 42° 22'.

a) Find the velocity of a person at MIT as they undergo circular motion about the earth? axis of rotation.

b) Find the person's centripetal acceleration.

c) The rotation of the Earth is slowing down. In 1977, the Earth took 1.01s longer to complete 365 rotations than in 1900. What was the average angular deceleration of the Earth in the time interval from 1900 to 1977?

d) Find the radius of the orbit of a synchronous satellite which circles the earth. (A synchronous satellite goes around the earth once every rotation of the earth, so that its position appears stationary with respect to a ground station).

Note: You may need to use the fact that the force of gravity between two objects of masses m_1 and m_2, separated by a distance d, is $F_g = \frac{Gm_1m_2}{d^2}$, where G is the universal constant of gravitation.

Solution

a) The radius of the circle of rotation is $r_E \cos(42°22') \approx 4710$ km. The circumference is then about 29600 km. The velocity is thus $\frac{29600 \text{ km}}{23 \text{ h } 56 \text{ m } 4 \text{ s}} \approx 343$ m/s.

b) The centripetal acceleration is $\frac{v^2}{r} \approx 0.0251$ m / s^2.

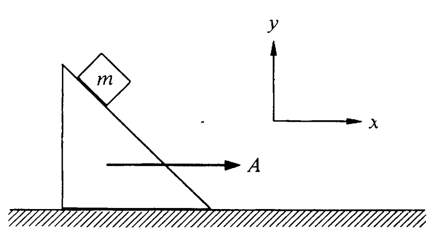
c) The angular velocity is $\frac{2\pi}{t}$, so the angular deceleration is $\frac{\frac{365\cdot 2\pi}{1 \text{ year}} - \frac{365\cdot 2\pi}{1 \text{ year}+1.01 \text{ s}}}{77 \text{ years}} \approx 10^{-21}$ s^{-2}.

d) Since $v = \frac{2\pi r}{t}$, $a = G\frac{m_e}{r^2} = \frac{4\pi^2 r}{t^2}$, so $r = \sqrt[3]{\frac{Gt^2}{4\pi^2}} \approx 42000$ km.

Problem 6: K&K 2.16

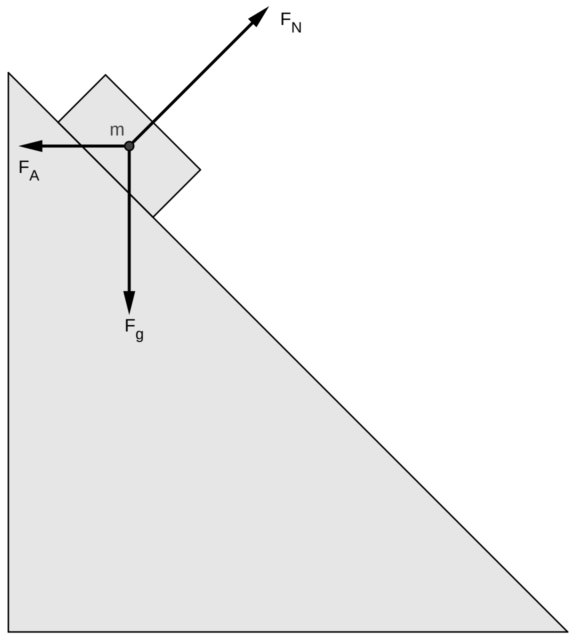
Problem

A 45° wedge is pushed along a table with constant acceleration A. A block of mass m slides without friction down the wedge. Find its acceleration. (Gravity is directed down.)

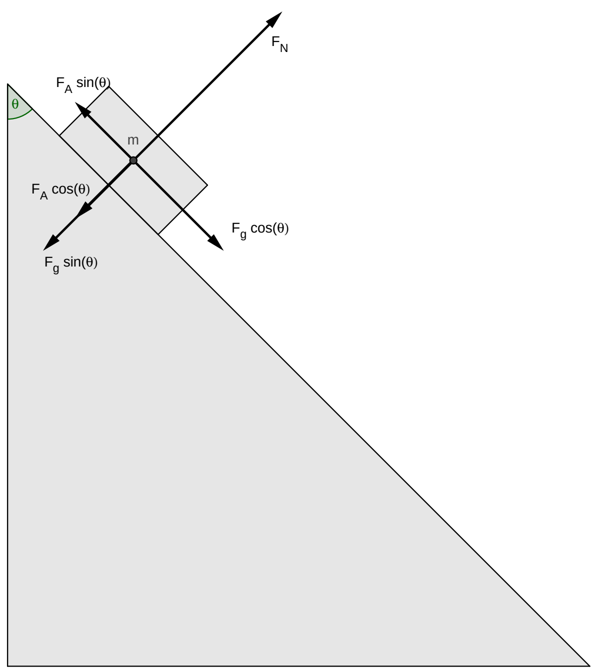


Solution

We begin with a coordinate transformation, substituting the acceleration of the wedge, $\vec{A}$, with a force in the opposite direction, $\vec{F}_A = -m\vec{A}$.



Decomposing the vectors, we get the following.

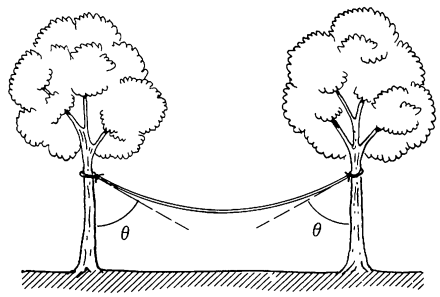


Since $\sin(45°) = \cos(45°) = \frac{1}{\sqrt{2}}$, the magnitude of the net force is $\frac{1}{\sqrt{2}}(F_g - F_A)$ down the wedge. Then the acceleration is $\frac{1}{\sqrt{2}}(g - A)$ down the wedge. Transforming back to the original coordinate system, the net acceleration is $A\hat{\imath} + \frac{1}{\sqrt{2}}(g - A)\left(\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}\right) = \frac{1}{2}(g + A)\hat{\imath} - \frac{1}{2}(g - A)\hat{\jmath}$.

Problem 7: K&K 2.22

Problem

Suppose a rope of mass m hangs between two trees. The ends of the rope are at the same height and they make an angle θ with the trees.



a) What is the tension at the ends of the rope where it is connected to the trees?

b) What is the tension in the rope at a point midway between the trees?

Solution

a) The magnitude of tension at the end of the rope, T, is such that $2T\cos\theta = mg$, so $T = \frac{mg}{2\cos\theta}$.

b) Since there are no external horizontal forces on the rope, and no portion of the rope is accelerating, the horizontal component of tension must be constant throughout the rope. Then $T = \frac{mg}{2}\tan\theta$.