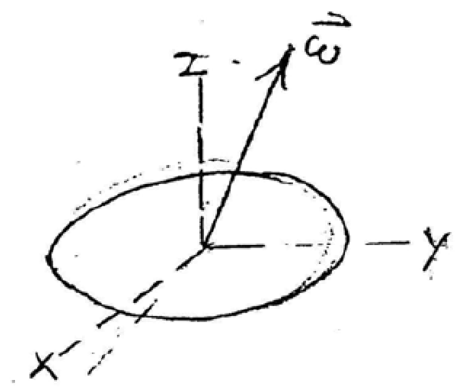
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group

Physics 8.012 **Fall Term 2009**

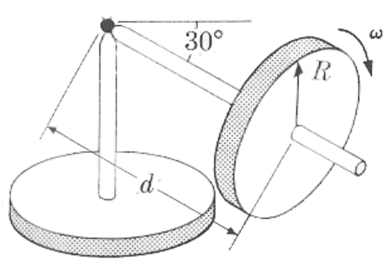
Three Dimensional Rotation Problems

Problem 1 Principal Axes and Angular Momentum

A thin disc of radius R and mass M is instantaneously rotating about its center of mass with angular velocity $\vec{\omega} = \omega_y \hat{j} + \omega_z \hat{k}$. Find the angular momentum about the center of mass of the disc.

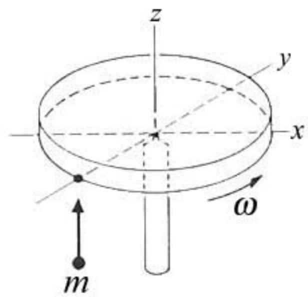


Problem 2 Tilted Gyroscope: A gyroscope consists of a uniform disc of mass radius $M = 1.0\ \text{kg}$ and radius $R = 0.2\ \text{m}$. The disc spins with an angular speed $\omega = 400\ \text{rad}\cdot\text{s}^{-1}$ as shown in the figure below. The gyroscope precesses, with its axle at an angle 30° below the horizontal (see figure). The gyroscope is pivoted about a point $d = 0.3\ \text{m}$ from the center of the disc. What is the direction and magnitude of the precessional angular velocity?



Problem Angular Impulse:

A uniform disc of radius R and mass m, mounted on its center by a universal bearing, rotates originally in a horizontal plane with angular velocity ω shown in the figure below. An object of mass m with speed $v = \omega R / 2$ directed along the *z*-axis collides with the edge of the disc and rebounds with an equal but oppositely directed velocity.



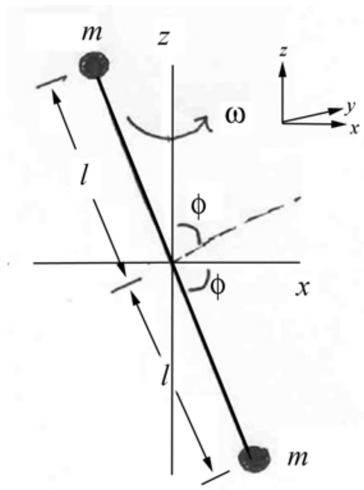
(a) What is the angular momentum of the disc and of the object taken about the center of mass of the disc before the collision?

(b) What angular impulses are imparted to the disc and to the object as a result of the collision?

(c) What is the angular momentum of the disc taken about the center of mass of the disc after the collision?

Problem Skewed Rod

Consider a simple rigid body consisting of two particles of mass m separated by a massless rod of length $2l$. The midpoint of the rod is attached to a vertical axis that rotates at angular velocity $\vec{\omega}$ pointing in the positive z-direction. The perpendicular to the rod is skewed at an angle ϕ with respect to the z-axis. At time $t = 0$ the rod lies in the x-z plane. Find the direction and magnitude of the angular momentum about the midpoint of the rod at that instant.

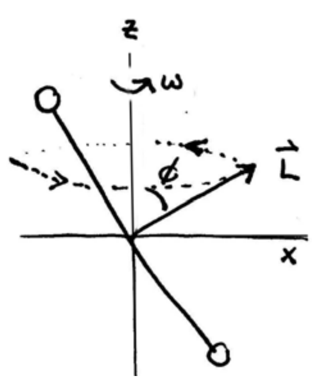


Problem: Principal Axes

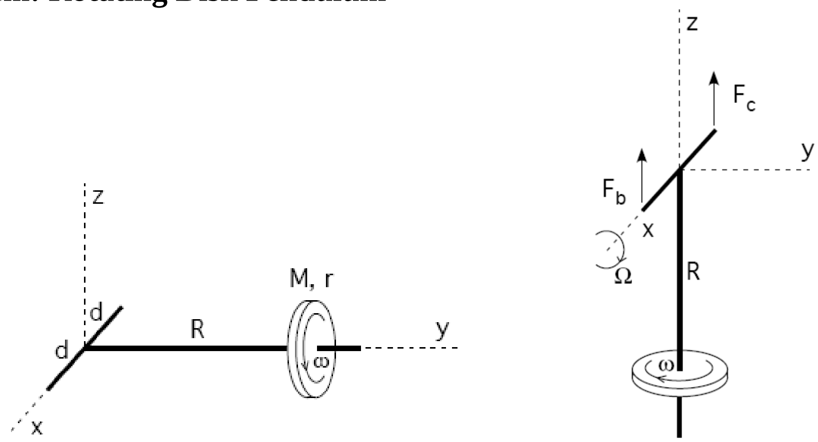
For the rotating skewed rod, what are the principal axes. Find the moment of inertia about those axes. Find the components of the angular velocity about those axes. Find the angular momentum about the center of the skewed rod.

Problem: Torque on Skewed Rod

Based on your result for the angular momentum of the skewed rod, calculate the torque about the center of mass for the rod at time $t = 0$ when it lies in the x-z plane .As the rod rotates, the angular momentum vector precessing at an angular speed ω .



Problem: Rotating Disk Pendulum



A rotating disk pendulum consists of a uniform spinning disk on a shaft attached to a rod pivoted at the origin of the coordinate system. Assume that the rod and shaft are massless. The uniform disk spins on a frictionless bearing. The pendulum is dropped from rest in a horizontal position.

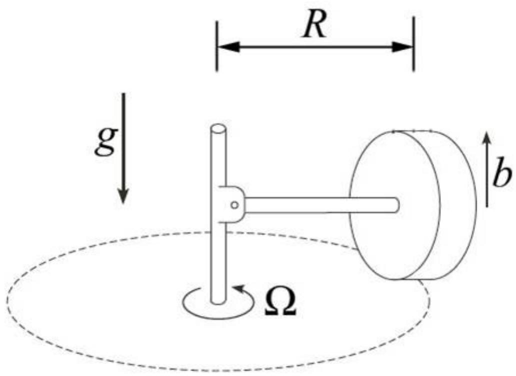
a) Use conservation of energy to find Ω at the bottom of the swing in terms of r, R and g.

b) Find $\vec{L}_0$ about the origin when the pendulum is at the bottom of its swing in terms of Ω, ω, M, r, and R.

c) Find an expression for $F_b - F_c$ in terms of M, r, Ω, ω and d.

Problem: Grain Mill

In a grain mill, grain is ground by a massive wheel that rolls without slipping in a circle on a flat horizontal surface driven by a vertical shaft. The rolling wheel has radius b and is constrained to roll in a horizontal circle of radius R at angular speed Ω about the vertical axis. Because of the stone's angular momentum, the contact force with the surface can be considerably greater than the weight of the wheel. In this problem, the angular speed Ω about the shaft is such that the contact force between the ground and the wheel is equal to twice the weight. The goal of the problem is to find Ω. Assume that the wheel is closely fitted to the axle so that it cannot tip. Neglect friction and the mass of the axle of the wheel. Express your answer in terms of R, b, M, Ω, and g as needed.



a) How is the angular speed ω of the wheel along the axle related to the angular speed Ω about the shaft?

b) What is the horizontal component of the angular momentum vector about the point P in the figure above? Although we have not shown this, for this situation it is correct to compute the horizontal component of the angular momentum by completely ignoring the rotation of the mill wheel about the vertical axis, taking into account only the rotation of the mill wheel about its own axle.

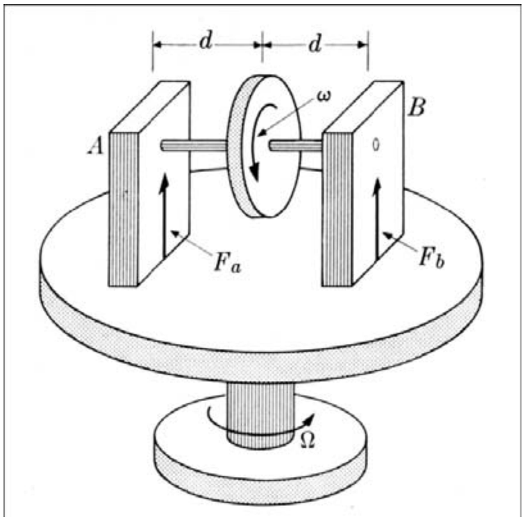
c) Draw a free body force diagram for all the forces acting on the axle–wheel combination.

d) What is the torque about the joint (about the point P in the figure above) due to the forces acting on the axle–wheel combination? Your answer may include any of the given variables R, b, M, Ω and g, and also any forces that you introduced in the force diagram of part (c).

e) Use the torque equation of motion to find the value of Ω that doubles the contact force between the stone and the ground. Your final answer should be expressed in terms of R, b, M and g, as needed.

Gyroscope on Rotating Platform

A gyroscope consists of a axle of negligible mass and a disk of mass m and radius r mounted on a platform that rotates with angular speed Ω as shown in the figure below. The gyroscope is spinning with a spin angular speed ω. Forces F_a and F_b act on the gyroscopic mounts. The goal of this problem is to find the magnitudes of the forces F_a and F_b. You may assume that the moment of inertia of the gyroscope about an axis passing through the center of mass normal to the plane of the disk is given by I_n.



a) Calculate the torque about the center of mass of the gyroscope.

b) Calculate the angular momentum about the center of mass of the gyroscope.

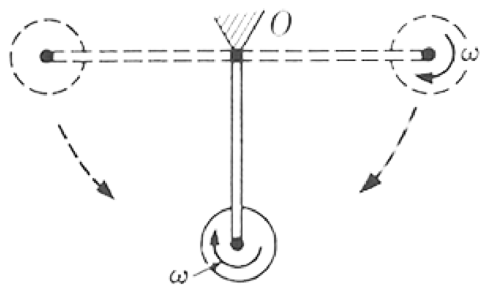
c) Using Newton's Second Law find a relationship between F_a and F_b and the mass m of the gyroscope and g where g is the gravitational constant.

d) Use the torque equation and Newton's Second Law to find expressions for F_a and F_b.

Problem: ***Angular Momentum and Torque***

Consider a pendulum consisting of a thin bar of length L which can rotate about a horizontal axis through its upper end as shown in the figure below. At its other end the bar supports a disc of radius $R << L$ and mass m which spins with a constant angular velocity $\vec{\omega}$ about a horizontal axis through its center. The mass of the bar may be neglected.

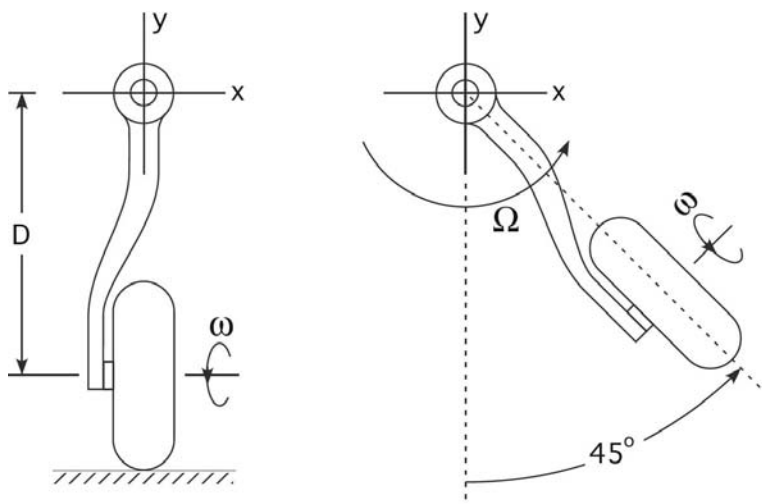
(a) The pendulum is released from its horizontal position. What must be the magnitude and direction of the spin angular velocity of the disc to make the total angular momentum of the pendulum, with respect to the point *O*, be zero at the lowest point of the pendulum? Assume that the pendulum swing is from left to right.



(b) What is the total angular momentum about *O* when it swings from right to left through its lowest point?

Problem Airplane Wheel

The figure shows a landing gear assembly that might be found on a small airplane. After the airplane takes off, the landing gear is retracted into the wing by rotation about the z-axis (which points out of the page in this figure). The wheel continues to spin with a constant angular frequency ω as it is being retracted. When the assembly is at an angle of 45 degrees it is rotating about the z-axis at a constant angular frequency Ω. Neglect all masses except that of the wheel, which has mass M. The wheel has a moment of inertia I_0 about its axle passing through the center of mass and I_d about a diameter.

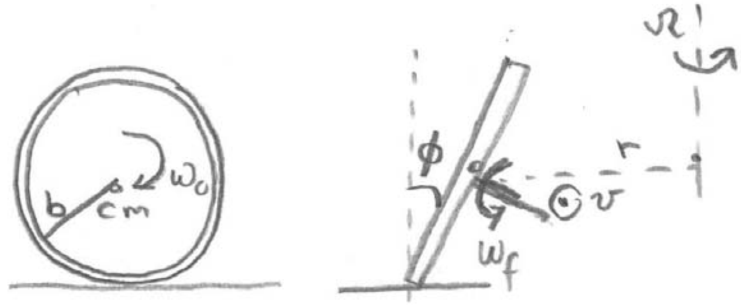


a) What is the total angular momentum $\vec{L}$ of the wheel about the origin when the assembly is at 45 degrees? Express your answer as a vector with components along the x, y, and z directions.

b) Which, if any, of the components you found in a) are changing with time?

c) What is the torque $\vec{\tau}_b$ that must be applied to the landing gear by the bearing at the origin when the assembly is at 45 degrees? Express your answer as a vector with components along the x, y, and z directions. Be sure that your answer would be correct even in the limit $\omega \to 0$.

Problem Tilted Bicycle Wheel: The center of mass of a bicycle wheel is initially at rest and the wheel is spinning with angular velocity ω_0. The wheel starts to skid forward on a level surface until it begins to roll without slipping on a level surface. The moment of inertia of the wheel about the center of mass is $I_{cm} = mb^2$ where b is the radius of the wheel and m is the mass of the wheel.



a) Draw a free body diagram of all the forces acting on the bicycle wheel while it is skidding forward.

b) What is the final angular velocity ω_f of the center of mass of the wheel when it begins to roll without slipping?

c) The wheel hits a bump in the ground causing the wheel to lean over, making a small angle ϕ with respect to a vertical line, and move in a circle of radius $r >> b$ and orbital angular velocity Ω. You may also assume that the final angular velocity $\omega_f >> \Omega$.