MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group

Physics 8.012 **Fall Term 2009**

Problem Set 10 Solutions

Problem 2:

A particle of mass $m = 50\,\text{g}$ moves under an attractive central force of magnitude $F = 4r^3$ dynes. The angular momentum is equal to $1000\,\text{g}\cdot\text{cm}^2/\text{s}$.

a) Find the effective potential energy.

b) Indicate on a sketch of the effective potential the total energy for circular motion.

c) The radius of the particle's orbit varies between r_0 and $2r_0$. Find r_0.

a) $\vec{F} = -4r^3\hat{r}$ $\qquad \underset{\text{center}}{\circ} \qquad \vec{F} \leftarrow \underset{m}{\bigcirc} \quad \rightarrow \hat{r}$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + U(r)$$

$$U(r) - U(0) = -\int_0^r \vec{F}\cdot d\vec{r} = -\int_0^r -4r^3\,dr$$

$$U(r) = \frac{4r^4}{4} = r^4 \quad \text{where } U(0) = 0.$$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + r^4$$

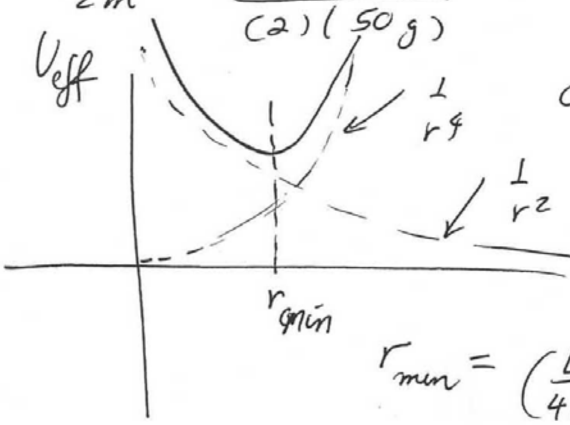
$$L = mr^2\dot{\theta} \Rightarrow \frac{L^2}{mr^4} = \dot{\theta}^2 \qquad \text{so}$$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + r^4 = \frac{1}{2}m\dot{r}^2 + U_{eff}$$

$$U_{eff} = \frac{L^2}{2mr^2} + r^4$$

note: $\frac{L^2}{2m} = \frac{\left(10^3\, g\text{-}\frac{cm^2}{s}\right)^2}{(2)(50\,g)} = 10^4\, \frac{g\text{-}cm^4}{s^2} = 10^4\, \frac{dynes}{cm^2}$

b)



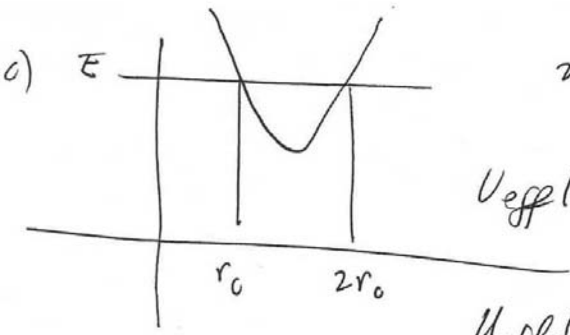
$$0 = \frac{dU_{eff}}{dr} = -\frac{L^2}{m}\frac{1}{r^3} + 4r^3$$

$$r_{min}^6 = \frac{L^2}{4m}$$

$$r_{min} = \left(\frac{L^2}{4m}\right)^{1/6} = \left(\frac{10^4}{2}\frac{dynes}{cm^2}\right)^{1/6}$$

$$r_{min} = 4.1\text{ cm}$$

c)



when $E = V_{eff}(r_0) = V_{eff}(2r_0)$

$$V_{eff}(r_0) = + \frac{L^2}{2mr_0^2} + r_0^4$$

$$U_{eff}(2r_0) = \frac{L^2}{2m\,4r_0^2} + 16r_0^4$$

Setting these equal:

$$\frac{L^2}{2mr_0^2} + r_0^4 = \frac{L^2}{8mr_0^2} + 16r_0^4$$

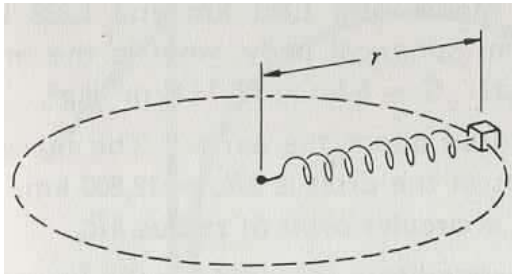
$$\frac{3}{8}\frac{L^2}{mr_0^2} = 15r_0^4 \implies r_0^6 = \frac{1}{40}\frac{L^2}{m}$$

$$r_0^6 = \frac{1}{10} r_{min}^6$$

$$r_0 = \left(\frac{1}{10}\right)^{1/6} r_{min} = 2.8\text{ cm}$$

Problem 5:

A body of mass $2\ kg$ lies on a frictionless table and is attached to one end of massless spring. The other end of the spring is held by a frictionless pivot. The spring produces a force of magnitude $3r$ newtons on the body, where r is the distance in meters from the pivot to the body. The body moves in a circle and has total energy $12\ \mathrm{J}$.

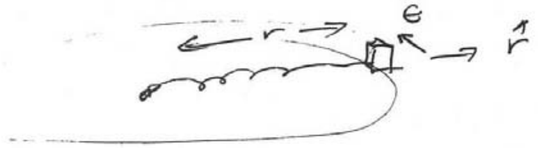


a) Find the radius of the orbit and the velocity of the body.

b) The body is struck by a sudden sharp blow, giving it instantaneous velocity of $1\ \mathrm{m/s}$ radially outward. Show the state of the system before and after the blow on a sketch of the energy diagram.

c) For the new orbit, find the maximum and minimum values of r.

chapter 9 problem 5:



$$\vec{F} = -3r\,\hat{r}$$

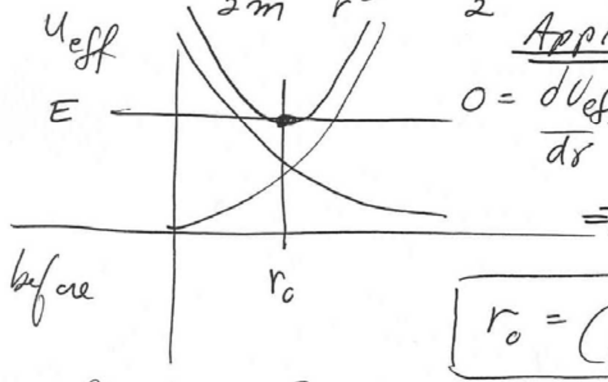
$$U(r) - U(0) = -\int_0^r \vec{F}\cdot d\vec{r}$$

choose $U(0)=0$. Then $U(r) = -\int_0^r -3r\,dr$

$$U(r) = \frac{3r^2}{2} \quad \text{with} \quad U(r)=0.$$

$$E = \frac{1}{2} m\dot{r}^2 + \frac{1}{2} m r^2\dot{\theta}^2 + U(r)$$

$$E = \frac{L^2}{2m}\frac{1}{r^2} + \frac{3r^2}{2} = U_{eff}$$ since $\dot{r}=0$ circular orbit



Approach 1:

$$0 = \frac{dU_{eff}}{dr} = -\frac{L^2}{m}\frac{1}{r_0^3} + 3r_0$$

$$\Rightarrow r_0^4 = \frac{L^2}{3m}$$

$$\boxed{r_0 = \left(\frac{L^2}{3m}\right)^{1/4}}$$

$$U_{eff}(r_0) = \frac{L^2}{2m}\frac{1}{r_0^2} + \frac{3r_0^2}{2} = \frac{L^2}{2m\left(\frac{L^2}{3m}\right)^{1/2}} + \frac{3}{2}\left(\frac{L^2}{3m}\right)^{1/2}$$

$$E = U_{eff}(r_0) = 3\left(\frac{L^2}{3m}\right)^{1/2} \Rightarrow \frac{mE^2}{3} = L^2 \Rightarrow L = \left(\frac{mE^2}{3}\right)^{1/2}$$

Approach 2

For a circular orbit

$$\hat{r}: \ -3r = -mr\dot{\theta}^2 \Rightarrow \dot{\theta}^2 = +\frac{3}{m}$$

Then the energy is

$$E = \frac{1}{2} m r_0^2 \dot{\theta}^2 + \frac{3 r_0^2}{2} = \frac{1}{2} m r_0^2 \frac{3}{m} + \frac{3 r_0^2}{2} = 3 r_0^2$$

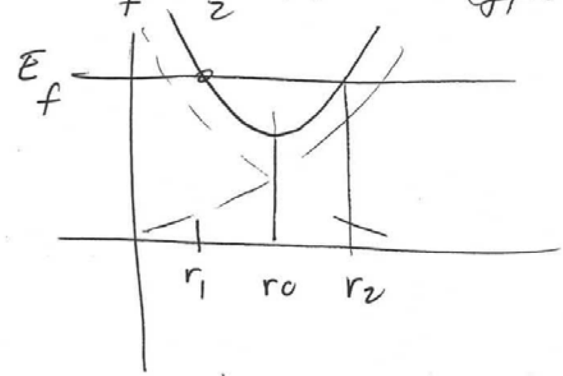
$$r_0 = \left(\frac{E}{3}\right)^{1/2} = \left(\frac{12\ J}{3\ J/m^2}\right)^{1/2} = 4\ m$$

$$v_0 = r_0 \dot{\theta} = \left(\frac{E}{3}\right)^{1/2} \left(\frac{3}{m}\right)^{1/2} = \left(\frac{E}{m}\right)^{1/2}$$

$$v_0 = \left(\frac{12\ J}{2\ kg}\right)^{1/2} = \sqrt{6}\ \frac{m}{s}$$

b) After the collision

$$E_f = \frac{1}{2} m \dot{r}^2 + U_{eff}(r_0) = \frac{1}{2} m \dot{r}^2 + E_0$$



$$E_f = \left(\frac{1}{2}\right)(2kg)\left(\frac{1m}{s}\right)^2 + 12\ J$$

$$E_f = 13\ J$$

c) $U_{eff}(r_1) = U_{eff}(r_2)$

$$\frac{L^2}{2m}\frac{1}{r_1^2} + \frac{3 r_1^2}{2} = \frac{L^2}{2m}\frac{1}{r_2^2} + \frac{3 r_2^2}{2} = E_f$$

Since angular momentum is conserved during the collision (the force is radially outward), we use

$$L^2 = \frac{m E_0^2}{3}$$

Thus we can solve for r_1 and r_2

$$\frac{m E_0^2}{3}\frac{1}{2m}\frac{1}{r^2} + \frac{3r^2}{2} = E_f$$

$$\frac{E_0^2}{6}\frac{1}{r^2} + \frac{3r^2}{2} = E_f$$

$$\frac{3r^4}{2} - E_f r^2 + \frac{E_0^2}{6} = 0$$

$$r^4 - \frac{2}{3}E_f r^2 + \frac{E_0^2}{9} = 0$$

$$r^2 = \frac{\frac{2}{3}E_f \pm \left(\frac{4}{9}E_f^2 - 4\frac{E_0^2}{9}\right)^{1/2}}{2}$$

$$= \frac{1}{3}E_f \pm \frac{1}{3}(E_f^2 - E_0^2)$$

$$r_1 = \frac{1}{3}\left(E_f - (E_f^2 - E_0^2)^{1/2}\right) = \frac{1}{3}\left(13\,J - (13J^2 - 12J^2)^{1/2}\right)$$

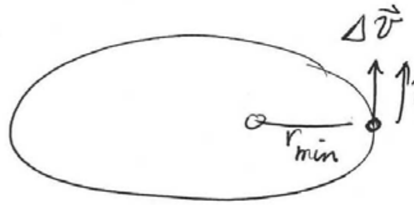
$$r_1 = \frac{8}{3}\,m$$

$$r_2 = \frac{1}{3}\left(E_f + (E_f^2 - E_0^2)^{1/2}\right) = 6\,m$$

Problem 7:

A rocket is in an elliptic orbit around the earth. To put it in escape orbit, its engine is briefly fired, changing the rocket's velocity by $\Delta\vec{\mathbf{v}}$. Where is the orbit, and in what direction, should the firing occur to attain escape with a minimum value of $\Delta\vec{\mathbf{v}}$?

Chapter 9 problem 7



Suppose firing the rocket adds a velocity $\Delta\vec{v}$ then

$$\vec{v}_f = \vec{v}_0 + \Delta\vec{v}$$

$$E_f = \frac{1}{2} m\, \vec{v}_f \cdot \vec{v}_f = \frac{1}{2} m \left(v_0^2 + 2\vec{v}_0 \cdot \Delta\vec{v} + \Delta v^2\right)$$

$$\Delta E = E_f - E_0 = m\, \vec{v}_0 \cdot \Delta\vec{v} + \frac{1}{2} m \Delta v^2$$

In order to maximize ΔE,
v_0 should be maximal (at perigee)
and $\vec{v}_0 \parallel \Delta\vec{v}$

Problem 9:

Halley's comet is in an elliptic orbit about the sun. The eccentricity of the orbit is $\varepsilon = 0.967$ and the period is $T = 76\,y$. The mass of the sun is $m_s = 1.99 \times 10^{30}\,kg$. The mass of Halley's comet is negligible compared to the sun.

a) Using this data, determine the distance of Halley's comet at closest approach to the sun, perihelion, and furthest distance from the sun, aphelion.

b) What is the speed of Halley's comet when it is closest to the sun?

chapter 9 problem 9 $\qquad \varepsilon = .967$

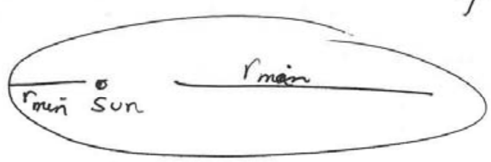
(1) $\quad \varepsilon = \left(1 + \dfrac{2EL^2}{\mu(Gm_1m_2)^2}\right)^{1/2} \quad , \mu = \dfrac{m_1 m_2}{m_1 + m_2}$

(2) $\quad T^2 = \dfrac{\pi^2 A^3}{2(m_1+m_2)G} \quad , A = \text{major axis} = \dfrac{2r_0}{1-\varepsilon^2}$

when $m_1 << m_2 \equiv m_{earth}$ then $\mu \simeq \dfrac{m_1 m_2}{m_2} \simeq m_1$

and $T^2 = \dfrac{\pi^2 A^3}{2 m_2 G} \quad , T = (76\,yrs)\left(3.16 \times 10^7 \dfrac{sec}{yr}\right)$

$T = 2.4 \times 10^9\ sec$



$r_{min} = \dfrac{r_c}{1+\varepsilon} \quad , r_{max} = \dfrac{r_c}{1+\varepsilon} \quad , \quad r_c = \dfrac{L^2}{\mu(Gm_1m_2)}$

eq (2) implies $\quad T^2 \cong \dfrac{\pi^2}{2(m_2)G}\left(\dfrac{2r_c}{1-\varepsilon^2}\right)^3$

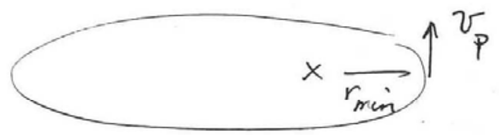
$\Rightarrow \left(\dfrac{2m_2 G T^2 (1-\varepsilon^2)^3}{8\pi^2}\right)^{1/3} = r_0$

$r_c = \left(\dfrac{(2)(2 \times 10^{30}\,kg)(6.67 \times 10^{-11}\,\frac{N\text{-}m^2}{kg^2})(2.4 \times 10^9 s)^2 (1-.967^2)^3}{8\pi^2}\right)$

$r_0 = 1.8 \times 10^{11} m$

$$r_{min} = \frac{r_0}{1+\varepsilon} = \frac{1.8\times10^{11}\,m}{1.967} = 8.9\times10^{10}\,m$$

$$r_{max} = \frac{r_0}{1-\varepsilon} = \frac{1.8\times10^{11}\,m}{(1-.967)} = 5.3\times10^{12}\,m$$



$$L_p = \mu\, r_{min}\, v_p \Rightarrow$$

$$v_p = \frac{L_p}{\mu\, r_{min}} \simeq \frac{L_p}{m_1\, r_{min}}$$

since

$$L_p^2 = \mu\, r_0\, G m_1 m_2 \simeq m_1^2 G r_0 m_2$$

so

$$v_p = \frac{(m_1^2 G r_0 m_2)^{1/2}}{m_1\, r_{min}} = \frac{(G r_0 m_2)^{1/2}}{r_{min}}$$

$$v_p = \frac{\left(\left(6.67\times10^{-11}\,\frac{N\text{-}m^2}{kg^2}\right)(1.8\times10^{11}\,m)(2\times10^{30}\,kg)\right)^{1/2}}{8.9\times10^{10}\,m}$$

$$v_p = 5.5\times10^4\,m/s$$

Problem 10:

A satellite of mass m_s is in a circular orbit about the earth. The radius of the orbit is r_0 and the mass of the earth is m_e .

a) Find the total mechanical energy of the satellite.

b) Now suppose that the satellite moves in the extreme upper atmosphere of the earth where it is retarded by a constant feeble friction force f . The satellite will spiral slowly to the earth. Since the friction force is weak, the change in radius will be very slow. We can therefore assume that at any given instant the satellite is in a circular orbit of average radius r . Find the approximate change in radius per revolution of the satellite, Δr .

c) Find the approximate change in kinetic energy per revolution of the satellite, ΔK .

Chapter 9 problem 10

a. $E = \frac{1}{2} m r^2 \dot{\theta}^2 - \frac{G m_1 m_2}{r}$

For a circular orbit

$$\vec{F} = m \vec{a}$$

$\hat{r}$: $-\frac{G m_1 m_2}{r^2} = -m r \dot{\theta}^2 \Rightarrow \dot{\theta}^2 = \frac{G m_1 m_2}{r^3}$

Then the energy equation becomes

$$E = \frac{1}{2} m r^2 \left(\frac{G m_1 m_2}{r^3} \right) - \frac{G m_1 m_2}{r} = -\frac{1}{2} \frac{G m_1 m_2}{r}$$

note $E = +\frac{1}{2} U$

and $K = \frac{1}{2} m r^2 \dot{\theta}^2 = \frac{1}{2} \frac{G m_1 m_2}{r} = -\frac{1}{2} U$

So $E = -K$

$$\Delta E = \frac{\partial E}{\partial r} \Delta r = \frac{1}{2} \frac{G m_1 m_2}{r^2} \Delta r$$

The work done by friction is

$$\Delta E = \Delta W = -2\pi f r = \int \vec{f} \cdot d\vec{r}$$

If we assume that the radius of the orbit does not significantly change

Then $$\Delta r = \frac{\Delta E}{\frac{G m_1 m_2}{2 r^2}} = \frac{-2\pi f r}{\frac{G m_1 m_2}{r^2}}$$

$$\Delta r = -\frac{2\pi f\, r^3}{G m_1 m_2}$$

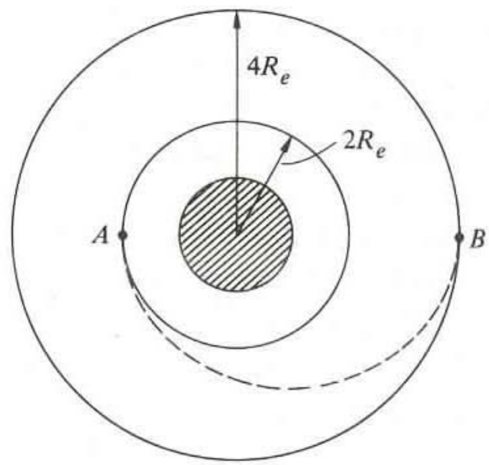
5) Since $\Delta E = -\Delta K$

$$\Delta K = -\Delta E = 2\pi r f$$

so the kinetic energy increases but the potential energy decreases by twice this amount

Problem 12:

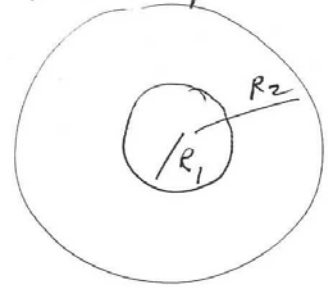
A space vehicle is in a circular orbit about the earth. The mass of the vehicle is $m_s = 3.00 \times 10^3$ kg and the radius of the orbit is $2R_e = 1.28 \times 10^4$ km. It is desired to transfer the vehicle to a circular orbit of radius $4R_e$.



a) What is the minimum energy expenditure required for the transfer?

b) An efficient way to accomplish the transfer is to use a semielliptical orbit from point A from the inner circular orbit at to point B at the outer circular orbit (known as a Hohmann transfer orbit). What velocity changes are required at the points of intersection, A and B?

Chapter 9 problem 12



$R_1 = 2R_e$

$R_2 = 4R_e$

$$E_1 = \frac{1}{2} m_1 v_1^2 - \frac{G m_1 m_2}{2 R_e}$$

From $\vec{F} = m\vec{a}$ for a circular orbit

$$-\frac{G m_1 m_2}{r^2} = -m_1 \frac{v^2}{r}$$

$$\Rightarrow \quad v^2 = \frac{G m_2}{r}$$

$$\Rightarrow \quad E_1 = \frac{1}{2} \frac{m_1 m_2 G}{2 R_e} - \frac{G m_1 m_2}{2 R_e} = -\frac{1}{4} \frac{G m_1 m_2}{R_e}$$

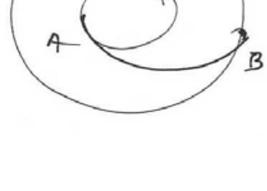
$$E_2 = -\frac{1}{8} \frac{G m_1 m_2}{R_e}$$

$$\Delta E = E_2 - E_1 = -\frac{1}{8} \frac{G m_1 m_2}{R_e} - -\frac{1}{4} \frac{G m_1 m_2}{R_e}$$

$$\Delta E = \frac{1}{8} \frac{G m_1 m_2}{R_e} = \left(\frac{1}{8}\right) \frac{\left(6.67 \times 10^{-11} \frac{N\text{-}m^2}{kg^2}\right)\left(3 \times 10^3 kg\right)\left(5.98 \times 10^{24} kg\right)}{6.37 \times 10^6 m}$$

$$\Delta E = 2.4 \times 10^{10} J$$

b)



After the rockets have moved the satellite into a new trajectory

$$E_A = \frac{1}{2} m_1 v_A^2 - \frac{G m_1 m_2}{2 R_e}$$

$$E_B = \frac{1}{2} m_1 v_B^2 - \frac{G m_1 m_2}{4 R_e}$$

By conservation of angular momentum
E

$$\mu\, 2 R_e v_A = \mu\, 4 R_e v_B$$

$$\Rightarrow \qquad v_A = 2 v_B$$

Thus

$$E_A = \frac{1}{2} m_1 4 v_B^2 - \frac{G m_1 m_2}{2 R_e}$$

$$0 = \Delta E' = E_B - E_A = \frac{1}{2} m_1 v_B^2 - \frac{1}{2} m_1 4 v_B^2 - \frac{G m_1 m_2}{4 R_e} - - \frac{G m_1 m_2}{2 R_e}$$

$$\Delta E' = -\frac{3 m_1 v_B^2}{2} + \frac{1}{4} \frac{G m_1 m_2}{R_e} = 0$$

$$\Rightarrow \qquad v_B^2 = \frac{1}{6} \frac{G m_2}{R_e}$$

hence $\quad v_A^2 = 4 v_B^2 = \frac{2}{3} \frac{G m_2}{R_e}$

For a circular orbit or radius $r = 2R_e$

$$v_{A,0}^2 = \frac{G m_2}{2 R_e} \quad \text{and} \quad v_{B,0}^2 = \frac{G m_2}{4 R_e}$$

$$\Delta v_A = \sqrt{\frac{2}{3}}\left(\frac{G m_2}{R_e}\right)^{1/2} - \left(\frac{G m_2}{2R_e}\right)^{1/2}$$

$$= \left(\sqrt{\frac{2}{3}} - \frac{1}{\sqrt{2}}\right)\left(\frac{G m_2}{R_e}\right)^{1/2} = 8.7 \times 10^2 \, \frac{m}{s}$$

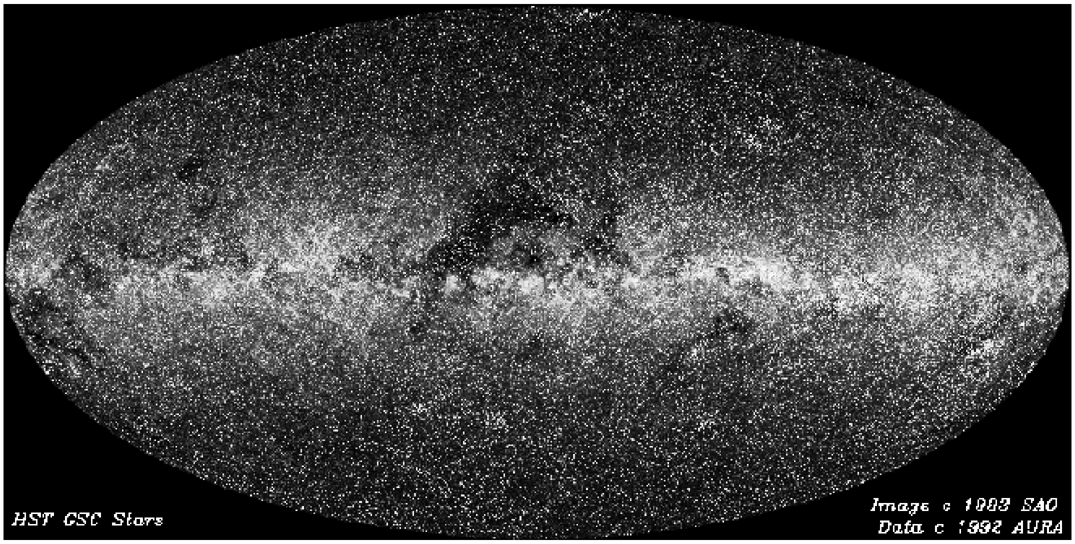
$$\Delta v_B = \frac{1}{\sqrt{6}}\left(\frac{G m_2}{R_e}\right)^{1/2} - \frac{1}{2}\left(\frac{G m_2}{R_e}\right)^{1/2}$$

$$= \left(\frac{1}{\sqrt{6}} - \frac{1}{2}\right)\left(\frac{G m_2}{R_e}\right)^{1/2}$$

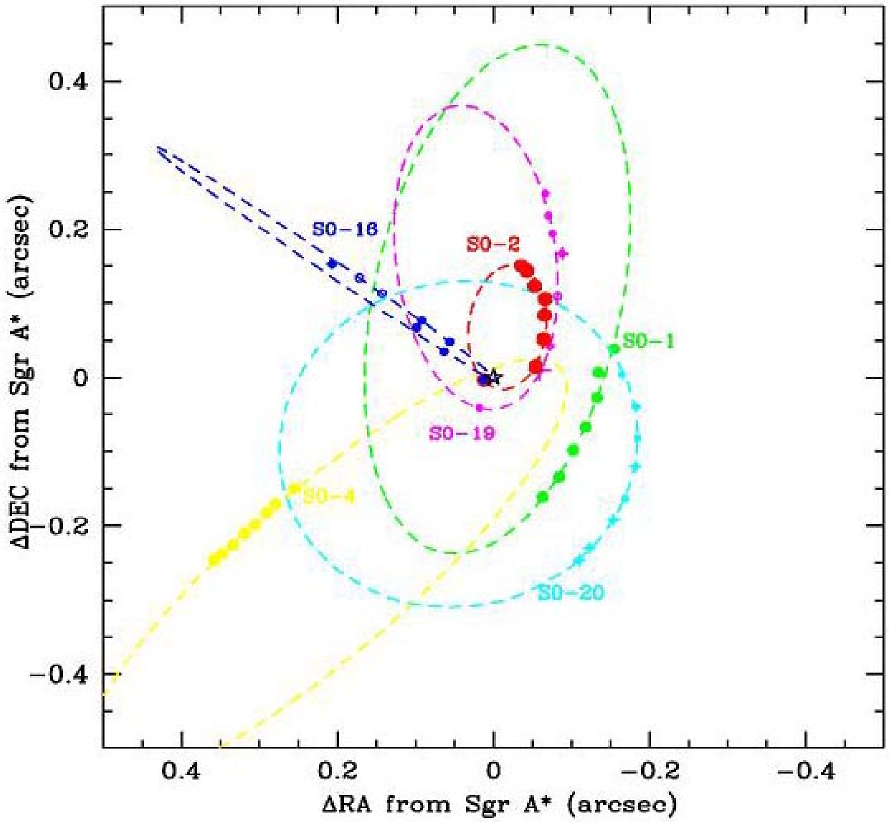
$$= -7.3 \times 10^2 \, m/s$$

Problem : The Motion of SO-2 around the Black Hole at the Galactic Center

The UCLA Galactic Center Group, headed by Dr. Andrea Ghez, reported the following data, (see http://www.astro.ucla.edu/~jlu/gc/ for information about the research group, and http://www.astro.ucla.edu/~jlu/gc/images/2004orbit_animfull_sm.gif for an animation of the orbits about the galactic center), for the orbits of eight stars within $0.8'' \times 0.8''$ of the galactic center.



The orbits of the stars are shown in Figure 1.



A standard astronomical unit is the parsec. One parsec is the distance at which there is one arcsecond = 1/3600 deg angular separation between two objects that are separated by the distance of one astronomical unit, $1AU = 1.50\times10^{11}m$ which is the mean distance between the earth and sun. One astronomical unit is roughly equivalent to eight light minutes, $1AU = 8.3l\min$ One parsec is equal to 3.26 light years, where one light year is the distance that light travels in one earth year, $1pc = 3.26ly = 2.06\times10^{5}AU$ where $1ly = 9.46\times10^{15}m$. The orbital data for the star SO-2, S0-16, and S0-19 are as follows[1]:

Star	Period (yrs)	Eccentricity	Semi-major axis (10^{-3}arc sec)	Periapse (AU)	Apoapse (AU)
S0-2	15.2 (0.68/0.76)	0.8763 (0.0063)	120.7 (4.5)	119.5 (3.9)	1812 (73)
S0-16	29.9 (6.8/13)	0.943 (0.019)	191 (24)	87 (17)	2970 (560)
S0-19	71 (35/11000)	0.889 (0.065)	340 (220)	301 (41)	5100 (3600)

[1] A.M.Ghez, et al., Stellar Orbits Around Galactic Center Black Hole, preprint arXiv:astro-ph/0306130v1, 5 June, 2003.

The period of S0-2 satisfies Kepler's Third Law given by

$$T^2 = \frac{4\pi^2 a^3}{G\left(m_1 + m_2\right)}$$

where m_1 is the mass of S0-2, m_2 is the mass of the black hole, and a is the semi-major axis of the elliptic orbit of S0-2.

The orbit data is given in terms of properties of the elliptic orbit. Consider the ellipse shown in the figure below.

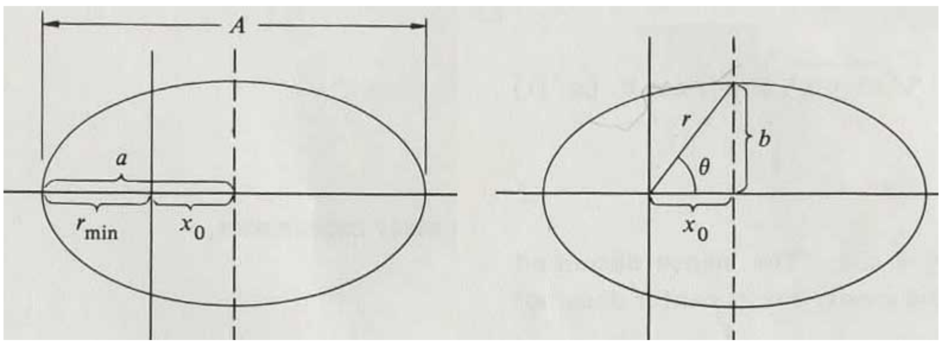


Figure 2: elliptic orbit

In figure 2, let a denote the semi-major axis, b denote the semi-minor axis, and x_0 denote the location of the center of the ellipse from one focal point P.

The orbit equation for the system is given by

$$r = \frac{r_0}{1 - \varepsilon \cos\theta},$$

where r_0 and the eccentricity ε are two constants.

The constant r_0 can be found by considering the lowest energy circular orbit which has radius

$$r_0 = \frac{L^2}{\mu G m_1 m_2},$$

where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is the reduced mass. Note that S0-2 is in a much higher energy orbit. The energy of this circular orbit is

$$E_0 = -\frac{Gm_1m_2}{2r_0}.$$

The eccentricity of the elliptic orbit of S0-2 is then

$$\varepsilon = \left(1 - E/E_0\right)^{1/2} = \left(1 + \frac{2EL^2}{\mu\left(Gm_1m_2\right)^2}\right)^{1/2}$$

The semi major axis a is given by

$$a = \frac{r_p + r_a}{2}$$

where the distance of furthest approach is denoted by r_a, and is called apoapse for the orbit about the galactic center), and the distance of nearest approach is denoted by r_p, and is called periapse for the orbit about the galactic center.

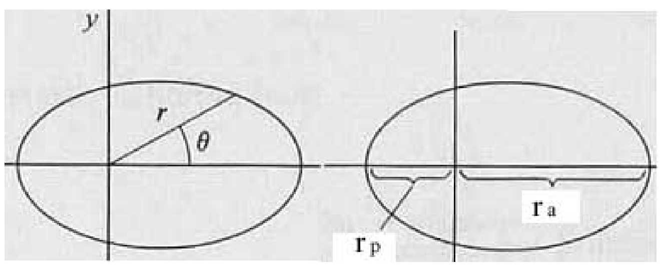


Figure 3: Nearest and furthest approach

Questions:

a) Using the results in the data table for the star S0-2, find the length of the semi-major axis.

the semimajor axis satisfies the condition that

$$2a = r_p + r_a$$

From the data chart

$r_p \equiv r_{periapse} = 119.5\ AU$

$r_a \equiv r_{apoapse} = 1812\ AU$

$$a = \frac{r_p + r_a}{2} = 965.8\ AU$$

$1\ AU = 1.50 \times 10^{11}\ m$ so

$$a = (965.8\ AU)\left(1.50 \times 10^{11}\ \frac{m}{AU}\right) = 1.45 \times 10^{14}\ m$$

b) Using the results in the data table for the star S0-2, find the mass of the black hole that the star S0-2 is orbiting . How many solar masses does this correspond to? Use $G = 6.67 \times 10^{-11} N \cdot m^2 \cdot kg^{-2}$ and the mass of the sun is given by $m_s = 1.99 \times 10^{30} kg$.

$T = 15.2$ yrs from data chart

$1 yr = 3.16 \times 10^7 s$

$$T^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}$$ period

m_2 = mass of black hole

m_1 = mass of S0(2) $<< m_2$

$$T^2 = \frac{4\pi^2 a^3}{G m_2}$$

$$m_2 = \frac{4\pi^2 a^3}{G T^2} = \frac{(4\pi^2)(1.45 \times 10^{14} m)^3}{(6.67 \times 10^{-11} N \cdot m^2 \cdot kg^{-2})(15.2 yr)^2 \left(3.16 \times 10^7 \frac{s}{yr}\right)^2}$$

$m_2 = 7.84 \times 10^{36} kg$

$m_{sun} = 1.99 \times 10^{30} kg$

$$\frac{m_2}{m_{sun}} = \frac{7.84 \times 10^{36} kg}{1.99 \times 10^{30} kg} = 3.94 \times 10^6$$

c) Use the equations for constant energy and angular momentum to find the velocity at periapse and apoapse.

$$\frac{1}{2}\mu v_p^2 - \frac{G m_1 m_2}{r_p} = \frac{1}{2}\mu v_a^2 - \frac{G m_1 m_2}{r_a}$$

$$\mu \simeq m_1$$

$$\Rightarrow \qquad v_p^2 = 2 G m_2 \left(\frac{1}{r_p} - \frac{1}{r_a}\right) + v_a^2$$

$$v_p^2 - v_a^2 = 2 G m_2 \left(\frac{1}{r_p} - \frac{1}{r_a}\right)$$

$$\mu r_p v_p = \mu r_a v_a \quad \Rightarrow v_a = \frac{r_p}{r_a} v_p$$

$$v_p^2 \left(1 - \frac{r_p^2}{r_a^2}\right) = 2 G m_2 \left(\frac{1}{r_p} - \frac{1}{r_a}\right)$$

$$v_p = \left(\frac{2 G m_2 \left(\frac{r_a - r_p}{r_p r_a}\right) r_a^2}{(r_a^2 - r_p^2)}\right)^{1/2}$$

$$= \left(2 G m_2 \frac{(r_a - r_p)\, r_a}{r_p\,(r_a - r_p)(r_a + r_p)}\right)^{1/2}$$

$$= \left(2 G m_2 \frac{r_a}{r_p (r_a + r_p)}\right)^{1/2}$$

$$v_p = \left(2 G m_2 \frac{r_a}{r_p} \frac{1}{2a}\right)^{1/2}$$

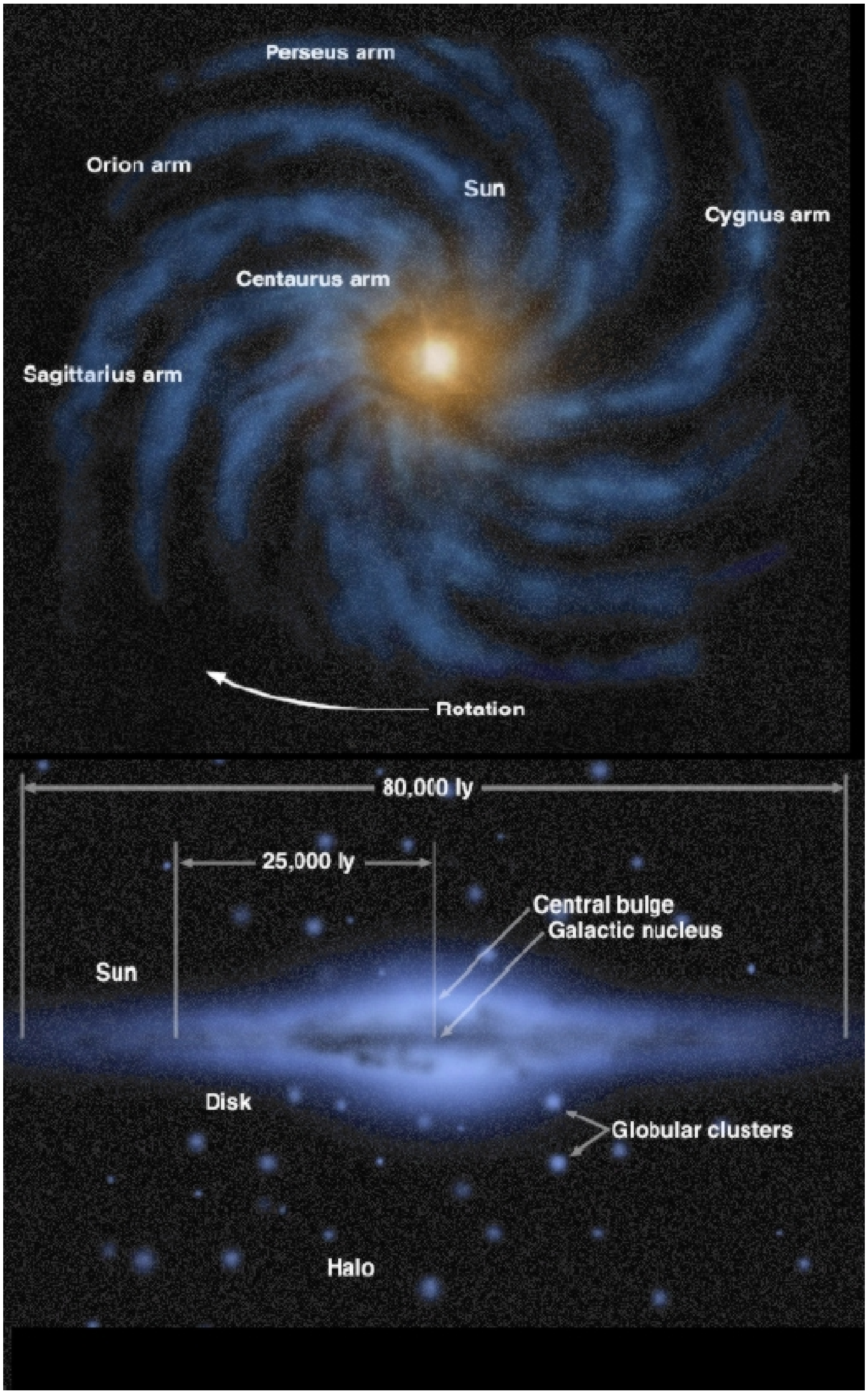
$$v_p = \left(\frac{(2)(6.67\times10^{-11}\,N\cdot m^2\cdot kg^{-2})(7.84\times10^{36}\,kg)}{(2)(1.45\times10^{14}\,m)} \left(\frac{1812}{119.5}\right) \right)^{1/2}$$

$$v_p = 7.4\times10^{6}\ m/s$$

$$v_a = \frac{r_p}{r_a} v_p = \left(\frac{119.5}{1812}\right)\left(7.4\times10^{6}\ m/s\right)$$

$$v_a = 4.9\times10^{5}\ m/s$$

d) Assume that the S0-2 orbit is perpendicular to our line of site. With this assumption, how far away is S0-2 from the earth?



The semi-major axis $a = 1.45 \times 10^{14}$ m and subtends an angle

$$\theta = (120.70 \times 10^{-3} \text{ as})(1 \text{ deg}/3600 \text{ as})(2\pi \text{ rad}/360 \text{ deg}) = 5.85 \times 10^{-7} \text{ rad}.$$

The distance to the earth is then

$$d = a/\theta = 1.45 \times 10^{14} \text{ m}/5.85 \times 10^{-7} \text{ rad} = 2.98 \times 10^{20} \text{ m}$$

One light year equals $1 ly = 9.46 \times 10^{15} m$.

So the galactic center is a distance

$$d = (2.98 \times 10^{20} \text{ m})(1\, ly / 9.46 \times 10^{15} m) = 2.6 \times 10^{4} \ ly$$

The actual distance is about 25,000 ly so this shows that the assumption about the orientation of the orbital plane of S0-2 is reasonably good.