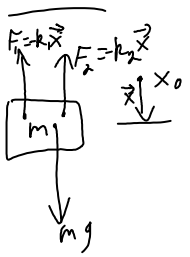
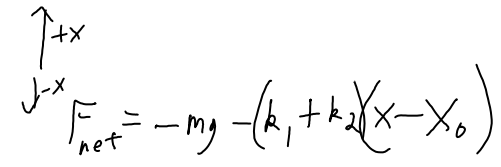
PSet 3 - Problem 31

Wednesday, September 30, 2009
2:07 PM

a)





$$m\ddot{x} = -mg - (k_1+k_2)(x-x_0)$$
$$\ddot{x} = -g - \frac{k_1+k_2}{m}(x-x_0)$$

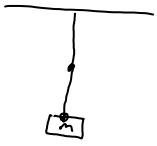
$$0 = -g + \frac{k_1+k_2}{m}x_0$$
$$x_0 = \frac{gm}{k_1+k_2}$$

$$\ddot{x} = -g - \frac{k_1+k_2}{m}x + g$$
$$\ddot{x} = -\frac{k_1+k_2}{m}x$$
$$x = A\cos(\omega t) + B\sin(\omega t)$$

so $\omega = \sqrt{\frac{k_1+k_2}{m}}$

$$x(t) = B\sin\left(\sqrt{\frac{k_1+k_2}{m}}\,t\right)$$
$$\dot{x}(0) = v_0 = B\cos\left(\sqrt{\frac{k_1+k_2}{m}}\,t\right)\sqrt{\frac{k_1+k_2}{m}} = B\sqrt{\frac{k_1+k_2}{m}}$$
$$x(t) = v_0\sqrt{\frac{m}{k_1+k_2}}\sin\left(\sqrt{\frac{k_1+k_2}{m}}\,t\right)$$
$$x\left(\frac{T}{2}\right) = x(0) = 0 \Rightarrow \frac{T}{2}\sqrt{\frac{k_1+k_2}{m}} = \pi$$
$$\Rightarrow T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$$
$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

b)



$$F_s = -kx$$
$$F_1 = -k_1x_1 = F_2 = -k_2x_2$$
$$x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$= F \Big/ \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$$

$$k = \frac{k_1 + k_2}{k_1 k_2}$$

$$T = 2\pi \sqrt{\frac{(k_1 + k_2)/(k_1 k_2)}{m}}$$

$$f = \frac{1}{2\pi \sqrt{\frac{m}{(k_1 + k_2)/(k_1 k_2)}}}$$