MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group
Physics 8.012

Practice Exam 2

Equation Summary

Momentum:

$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$, $\vec{\mathbf{F}}_{ave}\Delta t = \Delta\vec{\mathbf{p}}$, $\vec{\mathbf{F}}_{ext}^{total} = \dfrac{d\vec{\mathbf{p}}^{total}}{dt}$

Impulse: $\vec{\mathbf{I}} \equiv \int_{t=0}^{t=t_f} \vec{\mathbf{F}}(t)dt = \Delta\vec{\mathbf{p}}$

Work-Change in Mechanical Energy:

$W_{nc} = \Delta K^{total} + \Delta U^{total} = \Delta E_{mech}$,

$E_{mech} = K^{total} + U^{total} = K^{orbit} + K^{spin} + U^{total}$

Kinematics Circular Motion:

$$\vec{\mathbf{v}} = \frac{dr}{dt}\hat{\mathbf{r}} + r\frac{d\theta}{dt}\hat{\boldsymbol{\theta}}$$

$$\vec{\mathbf{a}} = \left(\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)\hat{\mathbf{r}} + \left(2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}\right)\hat{\boldsymbol{\theta}}$$

Kinematics:

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t')dt' ,$$

$$x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t')dt'$$

Constant Acceleration:

$$x_1(t) = (x_0)_1 + (v_{x,0})_1 t + \frac{1}{2}(a_x)_1 t^2$$

$$v_{x,1}(t) = (v_{x,0})_1 + (a_x)_1 t$$

Universal Law of Gravity:

$$\vec{\mathbf{F}}_{1,2} = -G\frac{m_1 m_2}{{r_{1,2}}^2}\hat{\mathbf{r}}_{1,2}$$

Surface of earth: $\vec{\mathbf{F}}_{grav} = m_{grav}\vec{\mathbf{g}}$

Coulomb's Law: $\vec{\mathbf{F}}_{1,2} = k_e \dfrac{q_1 q_2}{{r_{1,2}}^2}\hat{\mathbf{r}}_{1,2}$

Contact force: $\vec{\mathbf{F}}_{contact} = \vec{\mathbf{N}} + \vec{\mathbf{f}}$

Static Friction: $0 \le f_s \le f_{s,\max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k\left|\Delta x\right|$, restoring

Center of Mass:

$$\vec{\mathbf{R}}_{cm} = \sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i \Big/ \sum_{i=1}^{i=N} m_i \to \int_{body} dm\vec{\mathbf{r}} \Big/ \int_{body} dm$$

Velocity of Center of Mass:

$$\vec{\mathbf{V}}_{cm} = \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i \Big/ \sum_{i=1}^{i=N} m_i \to \int_{body} dm\vec{\mathbf{v}} \Big/ \int_{body} dm$$

Kinetic Energy:

$$K = \frac{1}{2}mv^2 ;\ \Delta K = \frac{1}{2}m{v_f}^2 - \frac{1}{2}m{v_0}^2$$

Work: $W = \int_{r_0}^{r_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$;

Work- Kinetic Energy: $W^{total} = \Delta K$

Power: $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = dK/dt$

Potential Energy:

$$\Delta U = -W_{conservative} = -\int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}}$$

Potential Energy Functions with Zero Points:

Constant Gravity:

$U(y) = mgy \quad U(y_0 = 0) = 0$.

Inverse Square Gravity:

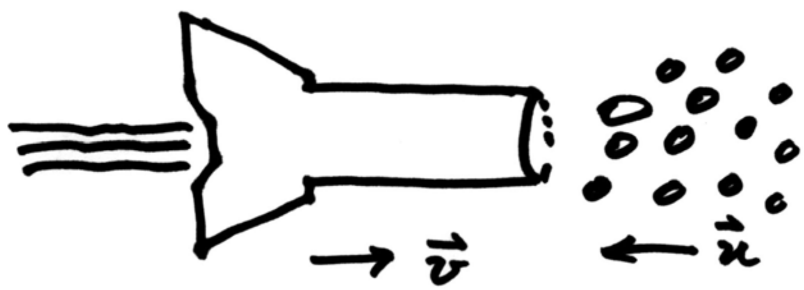
$U_{gravity}(\mathrm{r}) = -\dfrac{Gm_1m_2}{r} \quad U_{gravity}(\mathrm{r}_0 = \infty) \equiv 0$.

Hooke's Law:

$U_{spring}(x) = \dfrac{1}{2}kx^2 \qquad U_{spring}(x = 0) \equiv 0$

Problem 1: Momentum Transfer Space Junk

A spacecraft of cross-sectional area *A*, proceeding along the positive x-direction, enters an asteroid storm at time $t = 0$, in which the asteroid mass density is ρ and the average asteroid velocity is $\vec{\mathbf{u}} = -u\,\hat{\mathbf{i}}$ in the negative *x*-direction. As the spacecraft proceeds through the storm, all of the asteroids that hit the spacecraft stick to it.



a) Suppose that at time t the velocity of the spacecraft is $\vec{\mathbf{v}} = v\,\hat{\mathbf{i}}$ in the positive x-direction, and its mass is m. Further, suppose that in an interval Δt, the mass of the spacecraft increases by an amount Δm. Given that there are no external forces, using conservation of momentum find an equation for the change of the spacecraft velocity Δv, in terms of Δm, u, and v ?

b) When the spacecraft enters the asteroid storm, the magnitude of its velocity and mass are v_0 and m_0, respectively. Integrate your differential equation in part a) to find the velocity v of the spacecraft when the mass is m.

c) Find an expression for the mass of the asteroids Δm that sticks to the spacecraft within the time interval Δt? (Hint: consider the volume of asteroids swept up by the spacecraft in time Δt).

d) When the spacecraft enters the asteroid storm, the magnitude of its velocity and mass are v_0 and m_0, respectively. What is the mass of the spacecraft at time t? (Use your results from parts c) and b).)

Problem 2: Momentum transfer: Boat and fire hose

A burning boat of initial mass m_0 is initially at rest, and slides without negligible resistance on the Charles River. A fireman stands on the Harvard Bridge and sprays water onto the boat. The water leaves the firs hose with a velocity u at a rate α (measured in $\mathrm{kg} \cdot \mathrm{s}^{-1}$). Assume the motion of the boat and the water jet are horizontal, and gravity does not play any role. Also assume that the mass change of the boat is only due to the water jet and all the water from the jet is added to the boat.

a) In a time interval $[t, t+\Delta t]$, an amount of water Δm hits the boat. Choose a system. Is the total momentum constant in your system? Write down a differential equation that results from the analysis of the momentum changes inside your system.

b) Integrate the differential equation you found in part a), to find the velocity $v(m)$ as a function of the increasing mass m of the boat.

c) What is the linear density λ ($\mathrm{kg} \cdot \mathrm{m}^{-1}$) of the water jet? What is the mass flow dm/dt ($\mathrm{kg} \cdot \mathrm{s}^{-1}$) which hits the boat?

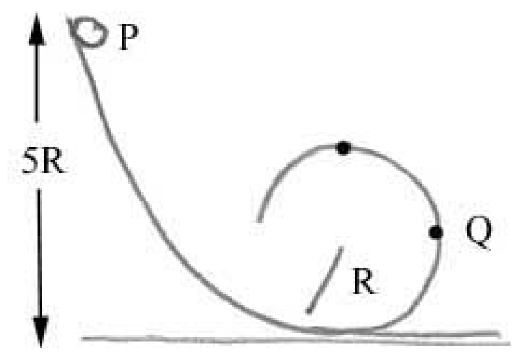
d) Using your results from parts b) and c) , calculate the mass in the boat $m(t)$ as a function of time (express your answer in terms of m_0, u, α, and t.

Problem 3: Energy

The frictionless track shown in the sketch has a final loop that consists of three-quarters of a circle of radius R. A small block of mass m is released from rest at the point P, a distance $5R$ above the ground.

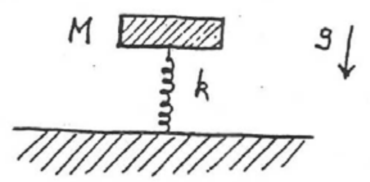
a) What is the force exerted by the track on the block at the point Q, at the right most edge of the track?

b) At what height above the ground should the block be released so that the force exerted by the track at the top of the final loop is equal to the block's weight?



Problem 4: Harmonic Oscillation

A vertical spring, of an unstretched length y_0 and spring constant k, supports a block of mass m. At time $t = 0$, a small bird of mass m_0 lands on the block with negligible incoming velocity. The block now moves downward.



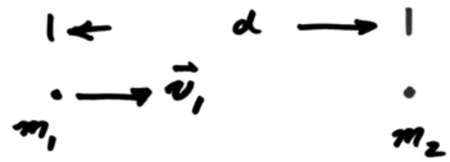
a) If the block is initially a height y_1 above the ground, what is the lowest height it reaches?. You may neglect any friction.

b) At what time is this minimum height reached? Again, neglect friction.

c) If friction were not negligible. The block and bird would oscillate with diminishing amplitude, until they eventually came to rest at time t_f. Calculate the total work done by friction during the time interval $[0, t_f]$.

Problem 5: Center of Mass

Two small particles of mass m_1 and mass m_2 attract each other with a force that varies at the inverse cube of their separation. At time t_0, m_1 has velocity $\vec{\mathbf{v}}_{1,0}$ directed towards m_2, which is at rest a distance d away. At time t_1, the particles collide. How far does m_1 travel in time interval $t_1 - t_0$?



Problem 6: ***Pendulums and Collisions***

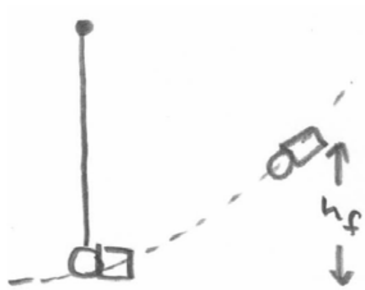
A simple pendulum consists of a bob of mass $m_1 = 0.4\,kg$ that is suspended by a massless string. The bob is pulled out and released from a height $h_0 = 0.2\,m$ as measured from the bottom of the swing and swings downward in a circular orbit. At the bottom of the swing, the bob collides with a block of mass m_2 that is initially at rest on a frictionless table. Assume the pivot point is frictionless.

a) What is the velocity of the bob immediately before the collision at the bottom of the swing

b) Assume the collision is perfectly elastic. The block moves along the table and the bob moves in the opposite direction but with the same speed as the block. What is the mass, m_2, of the block?

c) Suppose the collision is completely inelastic due to some putty that is placed on the block. What is the velocity of the combined system immediately after the collision? (Assume that the putty is massless.)

d) After the completely inelastic collision, the bob and block continue in circular motion. What is the maximum height, h_f, that the combined system rises after the collision?



Problem 7: Small Oscillations

The force on a particle of mass m_1 due to the interaction with a second particle of mass m_2 a distance r away is given by

$$\vec{\mathbf{F}} = \varepsilon(e^{-2(r-r_0)/r_0} - e^{-r/r_0})\hat{\mathbf{r}}$$

where ε and r_0 are positive and $\hat{\mathbf{r}}$ is a unit vector in the direction from the particle of mass m_2 to the particle of mass m_1.

a) What is $U(r)$, the potential energy, when the particles are a distance r a part?

b) Sketch $U(r)$.

c) For what value of r is the force zero?

d) Calculate the angular frequency of small oscillations about the value of r at which the force is zero.