

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Experimental Study Group

Physics 8.012, Fall 2010

Problem Set 1

Due: Friday, September 17

Reading: Chapter One: Kleppner and Kolenkow, *An Introduction to Mechanics*

**Problem 1: Fermi Problem (This problem is hard and should be a challenge.)**

One of the moons of Jupiter, Europa, is reported to have its surface covered by an ocean of water which is 100 km deep. The outermost 8 km are frozen as ice. The radius of Europa is approximately  $1/4$  the radius of the earth. Estimate the pressure at the bottom of Europa's ocean. (Note: there is some speculation that the combination of internal heat and water makes the ocean of Europa the best candidate in the solar system outside the earth for organized life to evolve.)

**Problem 2: Kinematics-One Dimension**

A bus leaves a stop at MIT and accelerates at a constant rate for 5 seconds. During this time the bus traveled 25 meters. Then the bus traveled at a constant speed for 15 seconds. Then the driver noticed a red light 18 meters ahead and slams on the brakes. Assume the bus decelerates at a constant rate and comes to a stop some time later just at the light.

- a) What was the initial acceleration of the bus?
- b) What was the velocity at the bus after 5 seconds?
- c) What was the braking acceleration of the bus? Is it positive or negative?
- d) How long did the bus brake?
- e) What was the distance from the bus stop to the light?
- f) Make a graph of the position vs. time for the entire trip.
- g) Make a graph of the velocity vs. time for the entire trip.
- h) Make a graph of the acceleration vs. time for the entire trip.

### Problem 3: Kinematics-One Dimension

You are running as fast as you can at a constant velocity,  $v_p$ , trying to catch a bus that is at rest at a bus stop. When you are still a distance  $b$  away from the bus stop, the bus starts to accelerate at a constant rate  $a_{\text{bus}}$ .

- What is the minimum velocity that you need to run at in order to just catch the bus?
- Draw graphs showing the motion of the bus and yourself.
- How long did it take to catch the bus?

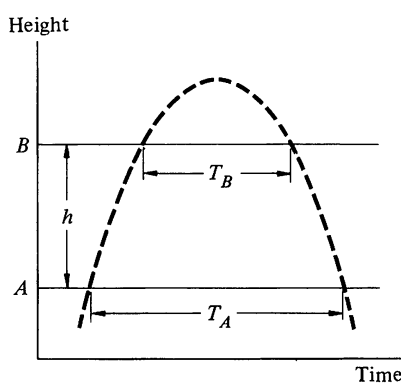
### Problem 4: K&K 1.7

Let  $\hat{\mathbf{a}}$  and  $\hat{\mathbf{b}}$  be unit vectors in the  $xy$  plane making angles  $\theta$  and  $\phi$  with the  $x$  axis, respectively. Show that  $\hat{\mathbf{a}} = \cos \theta \hat{\mathbf{i}} + \sin \theta \hat{\mathbf{j}}$ ,  $\hat{\mathbf{b}} = \cos \phi \hat{\mathbf{i}} + \sin \phi \hat{\mathbf{j}}$ , and using vector algebra prove that  $\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$ .

### Problem 5: K&K 1.12

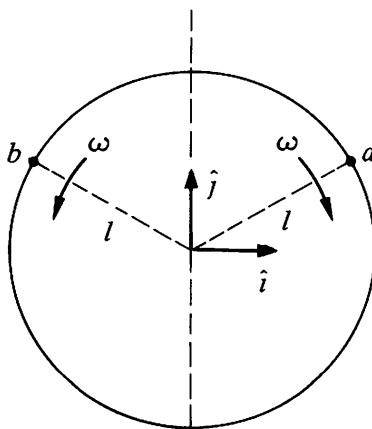
The acceleration of gravity can be measured by projecting a body upward and measuring the time that it takes to pass two given points in both directions. Show that if the time the body takes to pass a horizontal line  $A$  in both directions is  $T_A$ , and the time to go by a second line  $B$  is  $T_B$ , then, assuming that the acceleration is constant, its magnitude is

$$g = \frac{8h}{T_A^2 - T_B^2}$$



### Problem 6: K&K 1.15

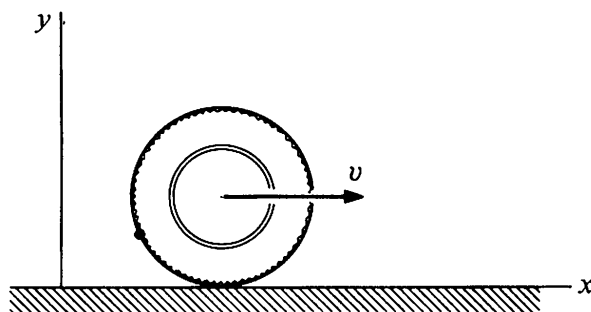
By relative velocity we mean velocity with respect to a specified coordinate system. (The term velocity, alone, is understood to be relative to the observer's coordinate system.)



- A point is observed to have velocity  $\vec{v}_A$  relative to coordinate system  $A$ . What is its velocity relative to coordinate system  $B$ , which is displaced from system  $A$  by distance  $\vec{R}$ ? ( $\vec{R}$  can change in time.)
- Particles  $a$  and  $b$  move in opposite directions around a circle with angular velocity  $\omega$ , as shown. At  $t = 0$  they are both at the point  $\vec{r} = l\hat{j}$ , where  $l$  is the radius of the circle. Find the velocity of  $a$  relative to  $b$ .

### Problem 7: K&K 1.19

A bicycle wheel of radius  $a$  is rolling in a straight line without slipping at a constant horizontal velocity  $V$ . A bead is fixed to a spoke a distance  $b$  from the center of the wheel.



- Find the position and velocity of the bead as a function of time as seen by an observer located at the center of the wheel and moving with the wheel. Make sure you use appropriate unit vectors in your answer.
- What is the position and velocity of the observer at the center of the wheel as seen by an observer fixed to the ground. Assume at  $t = 0$  that the center of the wheel is directly over the observer fixed to the ground. Make sure you use appropriate unit vectors in your answer.
- What is the relation between the angular velocity of the wheel,  $\omega$ , and the horizontal velocity,  $V$ , of the wheel?

- d) Find the position and velocity of the bead as a function of time as seen by the observer fixed to the ground. Make sure you use appropriate unit vectors in your answer.

### Problem 8: K&K 1.20

A particle moves outward along a spiral. Its trajectory is given by  $r = A\theta$ , where  $A$  is a constant,  $A = (1/\pi) \text{ m} \cdot \text{rad}^{-1}$ .  $\theta$  increases in time according to  $\theta = \alpha t^2/2$ , where  $\alpha$  is a constant.

- Sketch the motion, and indicate the approximate velocity and acceleration at a few points.
- Show that the radial acceleration is zero when  $\theta = 1/\sqrt{2} \text{ rad}$ .
- At what angles do the radial and tangential accelerations have equal magnitude?

### Problem 9: K&K 1.21

A person throws a rock from the top of a hill. The hill slopes downward uniformly at angle  $\phi$ . At what angle  $\theta$  from the horizontal should the person throw the rock so that it has the greatest range?

