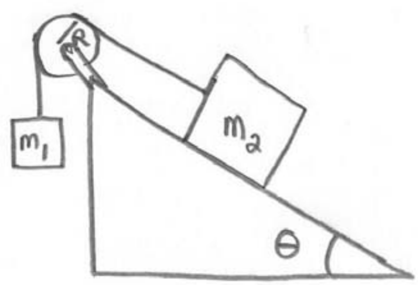
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group

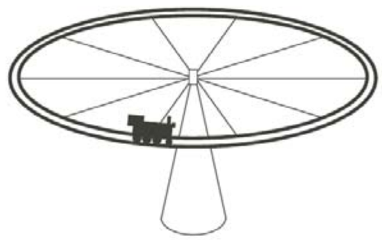
Physics 8.012 **Fall Term 2009**

Exam 3 Practice Problems: Solutions

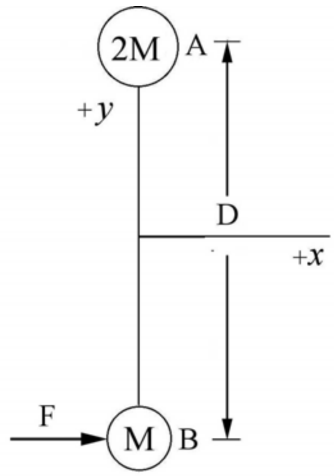
Problem 1 A wheel in the shape of a uniform disk of radius R and mass m_p is mounted on a frictionless horizontal axis. The wheel has moment of inertia about the center of mass $I_\mathrm{cm} = (1/2)m_\mathrm{p}R^2$. A massless cord is wrapped around the wheel and one end of the cord is attached to an object of mass m_2 that can slide up or down a frictionless inclined plane. The other end of the cord is attached to a second object of mass m_1 that hangs over the edge of the inclined plane. The plane is inclined from the horizontal by an angle θ. Once the objects are released from rest, the cord moves without slipping around the disk. Find the accelerations of each object, and the tensions in the string on either side of the pulley.



Problem 2: Toy Locomotive A toy locomotive of mass m_L runs on a horizontal circular track of radius R and total mass m_T. The track forms the rim of an otherwise massless wheel which is free to rotate without friction about a vertical axis. The locomotive is started from rest and accelerated without slipping to a final speed of v relative to the track. What is the locomotive's final speed, v_f, relative to the floor?



Problem 3 Two point-like objects are located at the points A, and B, of respective masses $\mathrm{M_A} = 2\mathrm{M}$, and $\mathrm{M_B} = \mathrm{M}$, as shown in the figure below. The two objects are initially oriented along the y-axis and connected by a rod of negligible mass of length D, forming a rigid body. A force of magnitude $F = |\vec{\mathbf{F}}|$ along the x direction is applied to the object at A at $t = 0$ for a short time interval Δt. Neglect gravity. Give all your answers in terms of M, D, F and Δt as needed.



a) Describe qualitatively in words how the system moves after the force is applied: direction, translation and rotation.

b) How far is the center of mass of the system from the object at point B?

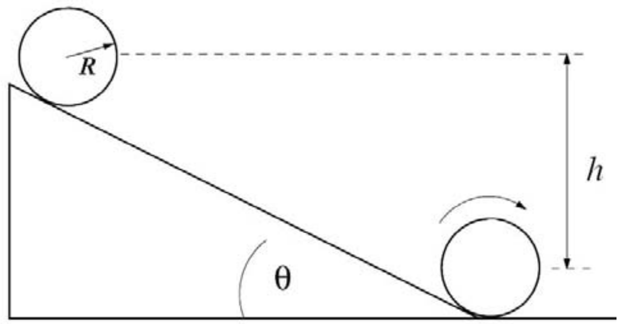
c) What is the direction and magnitude of the linear velocity of the center-of-mass after the collision?

d) What is the magnitude of the angular velocity of the system after the collision?

e) Is it possible to apply another force of magnitude F along the positive x-direction to prevent the system from rotating? Does it matter where the force is applied?

f) Is it possible to apply another force of magnitude F in some direction to prevent the center of mass from translating? Does it matter where the force is applied?

Problem 4 A hollow cylinder of outer radius R and mass M with moment of inertia about the center of mass $I_{\text{cm}} = MR^2$ starts from rest and moves down an incline tilted at an angle θ from the horizontal. The center of mass of the cylinder has dropped a vertical distance h when it reaches the bottom of the incline. Let g denote the gravitational constant. The coefficient of static friction between the cylinder and the surface is μ_s. The cylinder rolls without slipping down the incline. The goal of this problem is to find an expression for the smallest possible value of μ_s such that the cylinder rolls without slipping down the incline plane.



a) Draw a free body force diagram showing all the forces acting on the cylinder.

b) Find an expression for both the angular and linear acceleration of the cylinder in terms of M, R, g, θ and h as needed.

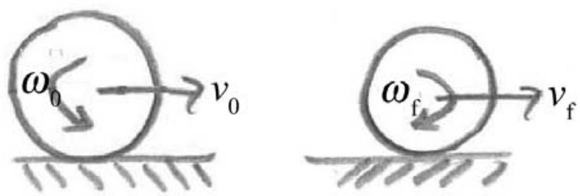
c) What is the minimum value for the coefficient of static friction μ_s such that the cylinder rolls without slipping down the incline plane? Express your answer in terms of M, R, g, θ and h as needed.

d) What is the magnitude of the velocity of the center of mass of the cylinder when it reaches the bottom of the incline? Express your answer in terms of M, R, g, θ and h as needed.

Problem 5 A bowling ball of mass m and radius R is initially thrown down an alley with an initial speed v_0 and backspin with angular speed ω_0, such that $v_0 > R\omega_0$. The moment of inertia of the ball about its center of mass is $I_{\text{cm}} = (2/5)mR^2$. Your goal is to determine the speed v_{f} of the bowling ball when it just starts to roll without slipping.

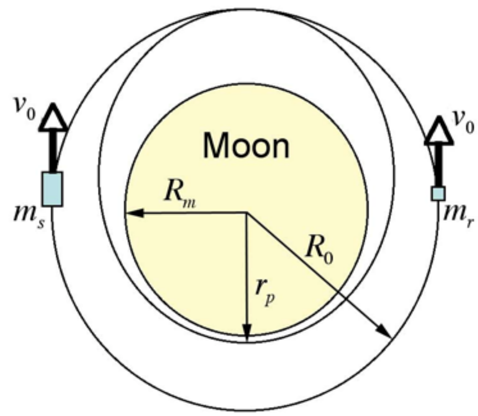
a) Write up your plan for solving this problem. You may find some of the following concepts useful: angular impulse is equal to the change in angular momentum; linear impulse is equal to the change in momentum; Newton's Second Law; torque about the center-of-mass is proportional to the angular acceleration; if the torque about a point is zero, then the angular momentum about that point is constant; etc.

b) What is the speed v_{f} of the bowling ball when it just starts to roll without slipping?



Problem 6 (30 Points)

A lunar mapping satellite of mass m_s is in a circular orbit around the moon, and the orbit has radius $R_0 = 1.5\,R_m$ where R_m is the radius of the moon. A repair robot of mass $m_r < m_s$ is injected into that orbit, but due to a NASA sign error it orbits in the opposite direction. The two collide and stick together in a useless metal mass. The point of this problem is to find whether they create more junk orbiting the moon or crash into the lunar surface. The mass of the moon is denoted by m_m. The universal gravitational constant is denoted by G.



a. What is the initial orbital velocity of the mapping satellite, v_0? Express your answer in terms of R_0, m_m, and G.

b. What is the speed of the space junk (satellite and robot) immediately after the collision? Write it as $f\,v_0$, where you must determine the number f. Express your answer in terms of m_s and m_r.

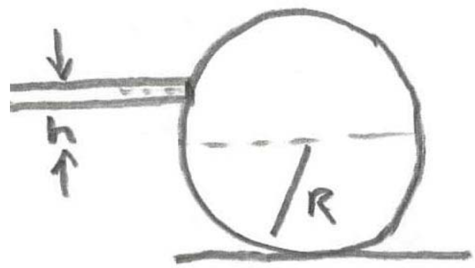
c. After the collision, the orbit of the space junk has changed. Use conservation of energy and angular momentum to solve for f, R_0, and r_p.

Problem 7 A wrench of mass m is pivoted a distance l_{cm} from its center of mass and allowed to swing as a physical pendulum. The period for small-angle-oscillations is T.

a) What is the moment of inertia of the wrench about an axis through the pivot?

b) If the wrench is initially displaced by an angle θ_0 from its equilibrium position, what is the angular speed of the wrench as it passes through the equilibrium position?

Problem 8 A spherical billiard ball of uniform density has mass m and radius R, and moment of inertia about the center of mass $I_{cm} = (2/5)mR^2$. The ball, initially at rest on a table, is given a sharp horizontal impulse by a cue stick that is held an unknown distance h above the centerline (see diagram below). The force applied by the cue to the ball is sufficiently large that you may ignore the friction between the ball and the table during the impulse (as any pool player knows). The ball leaves the cue with a given speed v_0 and an angular velocity ω_0. Because of its initial rotation, the ball eventually acquires a maximum speed of $(9/7)v_0$.

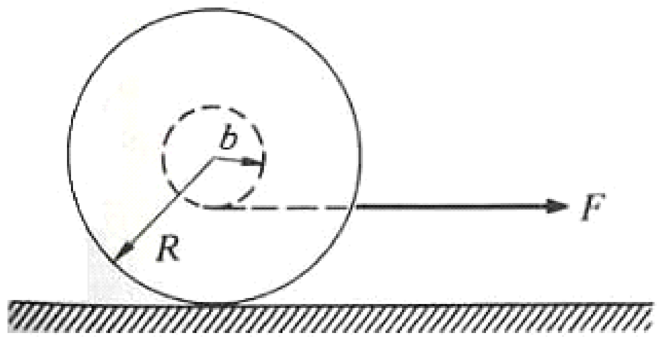


a) Using the fact that the angular impulse on the ball changes the angular momentum, and the linear impulse changes the linear momentum, find an expression for the angular velocity ω_0 of the ball just after the end of the impulse in terms of v_0, R, h and m.

c) Briefly explain why angular momentum is conserved about any point along the line of contact between the ball and the table *after* the impulse.

d) Use conservation of angular momentum about any point along the line of contact between the ball and the table, and your results from part a), to find the ratio h / R.

Problem 9 A Yo-Yo of mass m has an axle of radius b and a spool of radius R. It's moment of inertia about an axis passing through the center of the Yo-Yo can be approximated by $I_0 = (1/2)mR^2$. The Yo-Yo is placed upright on a table and the string is pulled with a horizontal force $\vec{\mathbf{F}}$ to the right as shown in the figure.

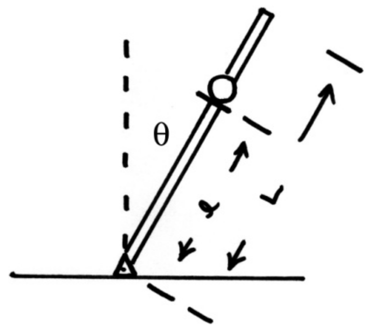


The coefficient of static friction between the Yo-Yo and the table is μ_s.

a) Which way will the Yo-Yo rotate if the string is pulled very gently? If the string is jerked hard, which way will the Yo-Yo rotate?

b) What is the maximum magnitude of the pulling force, $|\vec{\mathbf{F}}|$, for which the Yo-Yo will roll without slipping?

Problem 10 A uniform rod of length d and mass m_r is free to rotate about a pivot at its lower end. A light bead of mass $m_b << m_r$ moves frictionlessly on the rod, but a massless collar fixed on the rod a distance b from the pivot constrains the bead's distance from the pivot to be greater than or equal to b. Initially the rod is at rest, nearly vertical, and the bead is at rest on the collar. The rod is released and falls over.



a) Draw force diagrams for the bead, for the rod, and for the system of bead and rod. Indicate clearly your choice of coordinates and unit vectors.

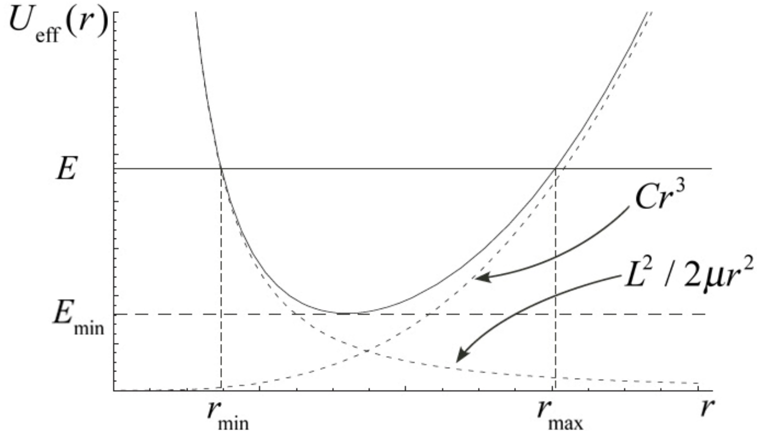
b) What is the total torque on the system of the rod and the bead about the pivot point when the system is at an angle θ with respect to the vertical axis?

c) Using conservation of energy, write down an equation comparing the energy intially with the energy when the system is at an angle θ with respect to the vertical axis.

d) Calculate the the maximum angle, $\theta_{\max}$, that the system makes with the vertical axis when the bead just starts to lose contact with the collar.

e) What is your answer in part d) when $m_b << m_r$?

Problem 11



The effective potential corresponding to a pair of particles interacting through a central force is given by the expression

$$U_{\text{eff}}(r) = \frac{L^2}{2\mu r^2} + Cr^3 \tag{1.1}$$

where L is the angular momentum, μ is the reduced mass and C is a constant. The total energy of the system is E. The relationship between $U_{\text{eff}}(r)$ and E is shown in the figure, along with an indication of the associated maximum and minimum values of r and the minimum allowed energy E_{min}. In what follows, assume that the center of mass of the two particles is at rest.

a) Find an expression for the radial component $f(r)$ of the force between the two particles. Is the force attractive or repulsive?

b) What is the radius r_0 of the circular orbit allowed in this potential? Express your answer as some combination of L, C, and μ.

c) When E has a value larger than E_{min}, find how rapidly the separation between the particles is changing, dr/dt, as the system passes through the point in the orbit where $r = r_0$. Give your answer in terms of some combination of E, E_{min}, L, C, μ and r_0.

d) Does the relative motion between the particles stop when $r = r_{\text{max}}$? If not, what is the total kinetic energy at that point in terms of some combination of E, L, C, μ, r_{max} and r_{min}?