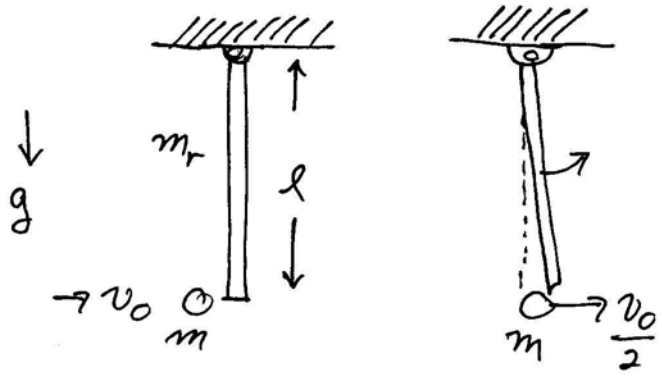
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012 **Fall Term 2009**

Final Exam Practice Problems

Problem 1 An object of mass m and speed v_0 strikes a rigid uniform rod of length l and mass m_r that is hanging by a frictionless pivot from the ceiling. Immediately after striking the rod, the object continues forward but its speed decreases to $v_0/2$. The moment of inertia of the rod about its center of mass is $I_{cm} = (1/12)m_r l^2$. Gravity acts with acceleration g downward.



a) For what value of v_0 will the rod just touch the ceiling on its first swing? You may express your answer in terms of g, m_r, m, and l.

b) For what ratio m_r / m will the collision be elastic?

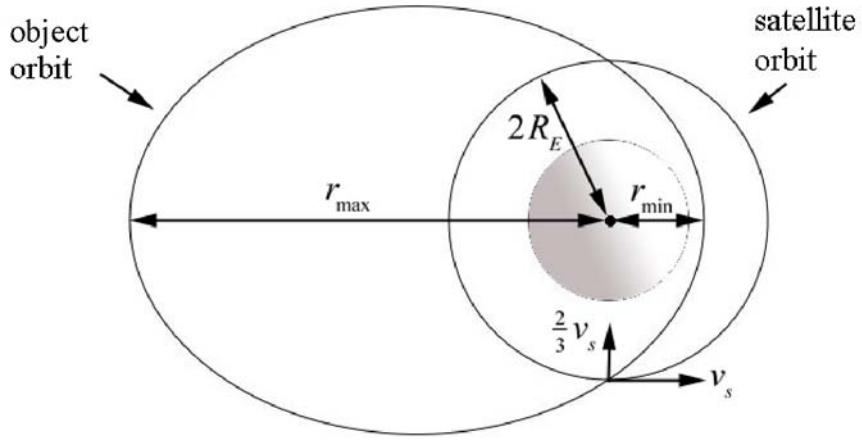
Problem 2 A particle of mass m moves under an attractive central force of magnitude $F = br^3$. The angular momentum is equal to L.

a) Find the effective potential energy and make sketch of effective potential energy as a function of r.

b) Indicate on a sketch of the effective potential the total energy for circular motion.

c) The radius of the particle's orbit varies between r_0 and $2r_0$. Find r_0.

Problem 3 The space shuttle is orbiting the earth with speed v_s in a circular orbit of radius $2R_E$, where R_E is the radius of the Earth. Suppose, in the reference frame of the shuttle, an object of mass m is shot towards the center of Earth at a speed equal to $(2/3)v_s$ (relative to the shuttle). The object moves in an elliptic orbit shown in the figure below. The goal of the problem is to find a quadratic equation whose solutions will give the minimum distance $r_{\min}$ and maximum distance $r_{\max}$ from the center of the Earth attained by the object as it orbits the Earth, expressed in terms of R_E. Let G be the gravitational constant and m_E denote the mass of the Earth.



a) Find an expression for the orbital speed v_s of the shuttle in terms of R_E, m, G, and m_E as needed.

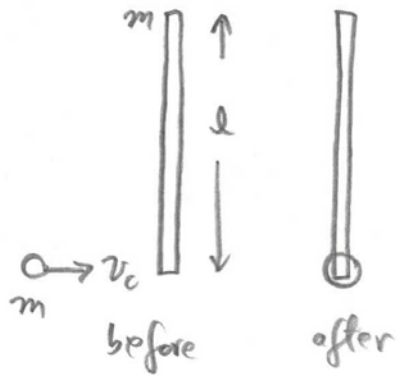
b) Find an expression for the speed v of the object relative to the Earth after it is shot from the shuttle in terms of R_E, m, G, and m_E as needed.

c) What is the magnitude of the angular momentum of the object about the center of the Earth? Express your answer in terms of R_E, m, G, and m_E as needed. Is the angular momentum of the object constant when it is in the elliptic orbit? Explain why or why not?

d) What is the magnitude of the energy (kinetic plus potential) of the object about the center of the Earth? Express your answer in terms of R_E, m, G, and m_E as needed. Is the energy of the object constant when it is in the elliptic orbit? Explain why or why not?

e) What are the minimum distance $r_{\min}$ and maximum distance $r_{\max}$ from the center of the Earth attained by the object, expressed in terms of R_E? Hint: Use conservation of energy and angular momentum to derive a quadratic equation whose two solutions will give these distances.

Problem 4: A long narrow uniform stick of length l and mass m lies motionless on ice (assume the ice provides a frictionless surface). The center of mass of the stick is the same as the geometric center (at the midpoint of the stick). The moment of inertia of the stick about its center of mass is I_{cm}. A puck (with putty on one side) has the same mass m as the stick. The puck slides without spinning on the ice with a speed of v_0 toward the stick, hits one end of the stick, and attaches to it. You may assume that the radius of the puck is much less than the length of the stick so that the moment of inertia of the puck about its center of mass is negligible compared to I_{cm}.



a) How far from the midpoint of the stick is the center of mass of the stick-puck combination after the collision?

b) What is the linear velocity of the stick plus puck after the collision?

c) Is mechanical energy conserved during the collision? Explain your reasoning.

d) What is the angular velocity of the stick plus puck after the collision?

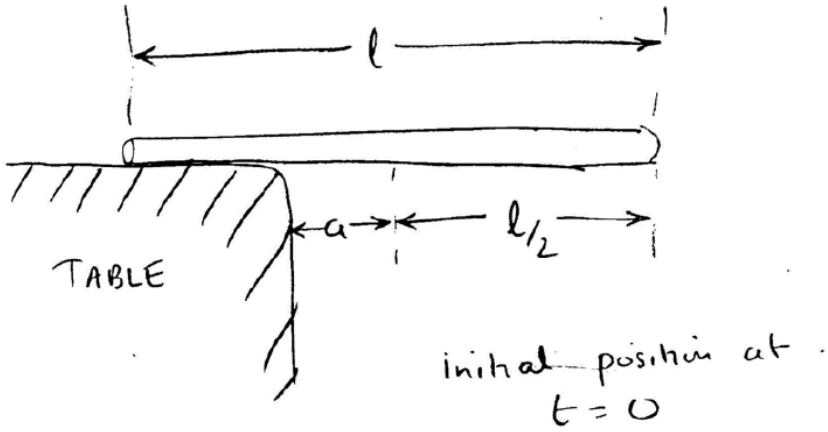
e) How far does the stick's center of mass move during one rotation of the stick?

Problem 5 A uniform rod of mass m and length l is placed on a horizontal table top with its center of mass a distance a from the perpendicular edge as shown in the figure. The rod is released from rest from a horizontal position and begins to rotate about the edge of the table. The coefficient of friction between the rod and the table is μ.

a) Draw a force diagram for the rod when it makes an angle θ with horizontal before it starts slipping, showing the weight of the rod, the normal and frictional forces.

b) Express the coordinates of the center of mass of the rod in terms of the angle θ.

c) Find the maximum angle the rod attains before slipping begins.



Problem 6 Two equal masses M are suspended from a massless and frictionless pulley, as shown. A is a simple weight. B is a uniform cylinder of radius R around which the tape is wrapped. The system is released from rest. Find the acceleration of A.