MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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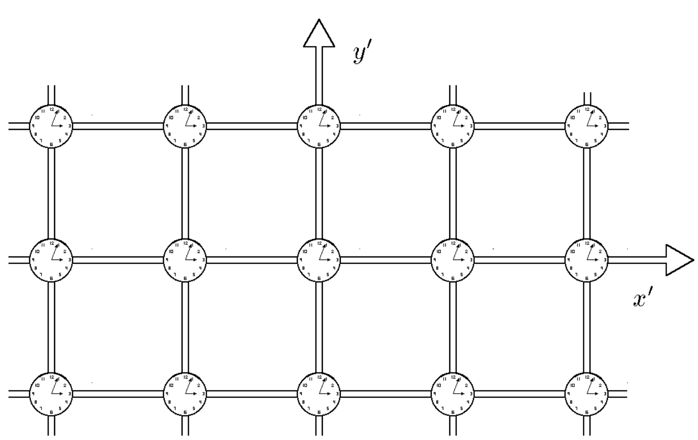
Lecture Notes 2
FURTHER CONSEQUENCES OF SPECIAL RELATIVITY

THE LORENTZ TRANSFORMATION:

The kinematic results of special relativity which were discussed last time—time dilation, Lorentz-Fitzgerald contraction, and the relativity of simultaneity—can all be neatly summarized in a set of equations called the Lorentz transformation. These equations relate the coordinates of an event as seen by one inertial observer to the coordinates of the same event as seen by another inertial observer in relative motion.

The Lorentz transformation can be easily derived from the principles that have already been established. Suppose that the space ship observer, instead of carrying a single SLUG, carries with him an entire network of measuring rods oriented along his x- and y-axes, as shown below. He also has a network of clocks. He determines the spatial coordinates of an event by observing where in this network of measuring rods it occurs, and he determines the time by reading it from a clock located at the site of the event. We will refer to these coordinates as x', y', and t', using the primes to distinguish them from our own coordinate system, which we will continue to call x, y, and t. (To simplify the discussion I am assuming that everything happens in the 2-dimensional plane spanned by the x- and y-axes. The z direction can be reinstated very easily, since its properties are the same as those of the y direction.)

Let us suppose that the moving coordinate system is oriented so that its x'-axis moves to the right along our x-axis, and the clocks are synchronized so that the clock at the origin of each system is set to zero at the time when the two origins cross each other.

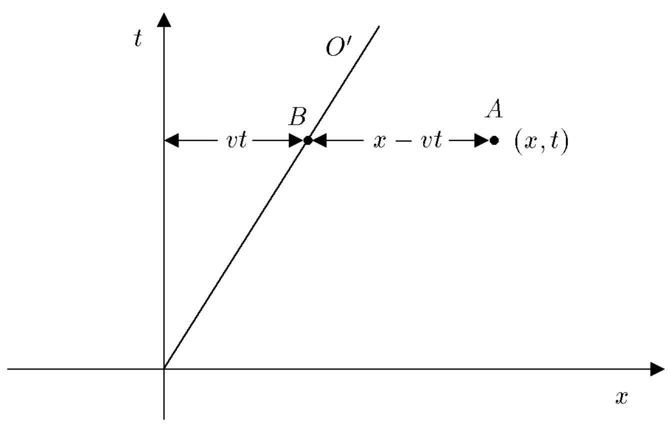


Notice, that since there is no contraction of the measuring rods that are oriented perpendicular to the motion, the y-coordinate of an event has the same value in either frame. This leads to the first transformation equation,

$$y' = y \ . \tag{2.1}$$

If there was a third spatial dimension in the problem, one would similarly conclude that $z' = z$.

Suppose now that an event A occurs in our coordinate system at a spacetime point (x, t), where we will set $y \equiv 0$ for simplicity. We now wish to calculate the coordinates as measured by the moving (primed) system. This event will clearly line up with the measuring rod which constitutes the x'-axis of the moving system, so we can for now ignore the existence of the other measuring rods.



The diagram above shows the trajectory of the origin of the primed coordinate system, which we will call O'. It starts at the origin of our system at $t = 0$, and then moves to the right at speed v. The diagram also shows that the moving measuring rod which connects the event A to O' has length $x - vt$, when measured in our frame. However, since the measuring rod is contracted by a factor

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \ , \tag{2.2}$$

it follows that the length that one would read off from the rod itself must be $\gamma(x - vt)$. Thus,

$$x' = \gamma(x - vt) \ . \tag{2.3}$$

To determine t', we must find the time on the moving clock which coincides with the event A. To do this, consider first the event B which occurs at the same time as event A in our frame, but which is located at the origin O' of the moving system. Since the clock at O' is synchronized with ours at $t = 0$ and then runs slowly by a factor of γ, we know that

$$t'(B) = t/\gamma \ . \tag{2.4}$$

However, the clock at B is trailing the clock at A, and therefore the two clocks will not appear to us to be synchronized. Instead, we have learned that the trailing clock will read a time that is later than the leading clock by an amount $\beta\ell_o/c$, where ℓ_o is rest length of the rod that joins the two clocks. In this case $\ell_o = x' = \gamma(x - vt)$, so

$$\begin{aligned} t'(A) &= t'(B) - \beta\gamma(x - vt)/c \\ &= \left(1 - \beta^2\right)\gamma t - \beta\gamma\left(\frac{x}{c} - \beta t\right) \\ &= \gamma\left(t - \frac{vx}{c^2}\right) \ . \end{aligned} \tag{2.5}$$

This completes the derivation of the Lorentz transformation equations, which can be summarized as follows:

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \ . \end{aligned}} \tag{2.6}$$

We have already verified that there is no distinction between the moving reference frame and ours, so that the moving observer observes the same distortion in our measuring devices that we observe in his. In the formalism of the Lorentz transformation, this fact is verified by inverting the transformation. That is, the above equations can be solved to express the unprimed variables in terms of the primed variables. When this exercise is carried out, it is found that the equations have exactly the same form, except that the sign of the relative velocity v is reversed.

THE LORENTZ-INVARIANT INTERVAL:

So far we have considered only pulses of light that move either parallel or perpendicular to the direction of motion. However, the Lorentz transformation allows us to easily verify that the measured speed of light is the same in ***all*** directions. To see this, consider a spherical light pulse that emanates from the origin. In our system, the wave front moves at the speed of light and therefore satisfies the equation

$$x^2 + y^2 + z^2 = c^2 t^2 \ . \tag{2.7}$$

We need to verify that the same equation holds for the coordinates of the wave front in the primed reference frame. We therefore use the Lorentz transformations to calculate the quantity

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 \ .$$

When we carry out this somewhat complicated but straightforward calculation, we find the following remarkable relation:

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2 \ . \tag{2.8}$$

This quantity,

$$x^2 + y^2 + z^2 - c^2 t^2 \ ,$$

is therefore called the "Lorentz invariant interval" between the event (x, y, z, t) and the origin.

The origin is of course not really a special point, so one can just as well define the Lorentz invariant interval between any two events A and B:

$$\boxed{s^2 \equiv (x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2 - c^2 (t_A - t_B)^2 \ .} \tag{2.9}$$

Although I am calling the Lorentz invariant interval s^2, I obviously do not mean to imply that it is always positive— it can have either sign. I call it s^2 only because it has the units of cm^2. If s^2 is positive, then the two events are said to be spacelike separated. In that case, it can be shown that there exists a frame of reference in which the two events occur at the same time, and the value of s^2 represents the square of the distance between the events in that frame. If s^2 is negative, the two events are said to be timelike separated. In that case there exists a frame of reference in which the two events occur at the same position, and the value of s^2 represents $-c^2$ times the square of the time separation in that frame. Note also that whenever s^2 is negative one can imagine a clock that moves between the two events at a uniform speed— s^2 is then equal to $-c^2$ times the time interval as measured by the clock. This time interval is sometimes called the proper time between the two events.

FOUR-VECTORS:

It is a common and useful practice to put together the four components (x, y, z, t) of an event into a single mathematical entity called a four-vector. One defines

$$\boxed{\begin{aligned} x^0 &\equiv ct \\ x^1 &\equiv x \\ x^2 &\equiv y \\ x^3 &\equiv z \ , \end{aligned}} \tag{2.10}$$

and then one speaks of the four-vector x^μ. It is understood that the index μ takes on the possible values of 0 to 3. (According to standard notation, the symbol $\vec{x}$ is used to denote a three-vector, and is ***not*** used to denote a four-vector.)

The Lorentz transformation can be written in four-vector form as

$$\begin{aligned} x'^0 &= \gamma\left(x^0 - \beta x^1\right) \\ x'^1 &= \gamma\left(x^1 - \beta x^0\right) \\ x'^2 &= x^2 \\ x'^3 &= x^3 \ , \end{aligned} \tag{2.11}$$

which can also be written as the matrix equation

$$\boxed{\begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \ .} \tag{2.12}$$

In fact, one uses the word "four-vector" to refer to any set of four quantities which obey the transformation laws of Eq. (2.12). We will soon see several more examples of four-vectors.

It is also useful to define the Lorentz invariant dot product between two four-vectors x^μ and y^μ:

$$\boxed{x \cdot y \equiv x^1 y^1 + x^2 y^2 + x^3 y^3 - x^0 y^0 \ .} \tag{2.13}$$

The dot product $x \cdot x$ of a vector with itself is usually denoted by x^2. (Note that this usage assumes that the reader remembers that x is a four-vector, and not a number.) The Lorentz invariant separation between two events which occur at x^μ and y^μ is then given simply by

$$s^2 = (x - y)^2 \ . \tag{2.14}$$

(Note that some authors define the dot product to be the negative of the definition given above.)

THE FOUR-VECTOR VELOCITY:

Suppose we are describing the trajectory of a particle moving through space. In Newtonian mechanics, this is usually done by specifying the position as a function of time, $\vec{x}(t)$. The velocity of the particle is then defined by

$$\vec{v} \equiv \frac{d\vec{x}}{dt} \ . \tag{2.15}$$

This definition is also perfectly acceptable in a relativistic setting, and in fact it is precisely the definition of velocity that we have been using so far. The definition has the shortcoming, however, that its transformation properties are complicated. Thus, for situations in which the transformation properties are important, it is useful to introduce a new four-vector quantity to characterize the velocity.

Consider two nearby points along the trajectory of a particle. These are points in spacetime, and may be labeled by two four-vectors x^μ_A and x^μ_B . Since the Lorentz transformations are linear equations, one can easily see that the coordinate differences

$$\Delta x^\mu \equiv x^\mu_B - x^\mu_A \tag{2.16}$$

also transform as a four-vector. Note that if one divides the difference vector Δx^μ by the time interval Δt between the two events, one would have a set of 4 numbers for which the last 3 components would represent the mean velocity vector, as defined by Eq. (2.15). The transformation properties are complicated, however, because the time interval Δt depends on the frame of reference in a complicated way. However, if we divided by a measure of the time interval for which all observers would agree, then the quotient would also be a four-vector. The natural Lorentz invariant definition of the time interval between x^μ_A and x^μ_B is given by the proper time separation between the two events, defined by

$$\Delta x^2 = (x_B - x_A)^2 = -c^2 \Delta\tau^2 \ . \tag{2.17}$$

Thus, the quantity

$$\frac{\Delta x^\mu}{\Delta \tau}$$

transforms as a four-vector. In the limit as $\Delta\tau \to 0$, this quantity becomes an instantaneous measure of velocity, known as the four-velocity u^μ:

$$\boxed{u^\mu \equiv \lim_{\Delta\tau\to 0} \frac{\Delta x^\mu}{\Delta \tau} \equiv \frac{dx^\mu}{d\tau} \ .} \tag{2.18}$$

Since $d\tau$ is the time interval read on the moving clock which is running slowly by a factor of γ, it follows that $d\tau = dt/\gamma$. One can then express the components of the four-velocity in terms of the components of the ordinary velocity:

$$\boxed{\begin{aligned} &u^i = \gamma v^i \ , \qquad \text{for} \quad i = 1, 2, 3 \\ &u^0 = \gamma c \ . \end{aligned}} \tag{2.19}$$

Since $d\tau$ is constructed from the four-vector dx^μ in the numerator, the four components of u^μ are not independent. One can check that they obey the identity

$$\boxed{u^2 \equiv -c^2 \ .} \tag{2.20}$$

THE FOUR-MOMENTUM:

In Newtonian mechanics, the momentum of a particle is defined by

$$\vec{p} \equiv m\vec{v} \quad \text{(Newtonian)}, \tag{2.21}$$

and the total momentum in any collision is conserved.

In a relativistic situation, one can easily show that it is inconsistent to assume that momentum as defined by Eq. (2.21) is conserved. If the total Newtonian momentum is conserved in our frame of reference, one can show that it will not be conserved in the frame of reference of a moving space ship. Since the basic assumption of special relativity is that the laws of physics should be the same in any inertial reference frame, such a situation is intolerable. Thus, we must find a way to modify the definition of momentum so that it can be conserved in all frames of reference simultaneously.

There is a simple solution to this problem— define a four-momentum p^μ by the relation

$$\boxed{p^\mu \equiv m u^\mu \ ,} \tag{2.22}$$

where m is the rest mass of the particle. Since the rest mass is defined in the rest frame of the particle and then ***by definition*** has the same value in any frame, the quantity p^μ is a multiple of a four-vector and hence is also a four-vector.

Since four-vectors transform linearly under the Lorentz transformation, it follows that the sum of the four-momenta can be simultaneously conserved in all reference frames. In more detail, suppose that a particle reaction has two incoming particles and two outgoing particles. In the laboratory frame of reference, suppose the four-momenta of the incoming particles are p_1^μ and p_2^μ, while the four-momenta of the outgoing particles are p_3^μ and p_4^μ. Then conservation of four-momentum, in the laboratory frame, takes the form

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu \ , \tag{2.23}$$

By multiplying each term of this equation by the 4×4 matrix on the right-hand-side of Eq. (2.12), one finds

$$p_1'^{\,\mu} + p_2'^{\,\mu} = p_3'^{\,\mu} + p_4'^{\,\mu} \ . \tag{2.24}$$

Thus, if total four-momentum is conserved in the laboratory frame, then it is also conserved in any frame which is related to the laboratory frame by the matrix multiplication of Eq. (2.12). Thus, total four-momentum would be conserved in any inertial frame of reference.

Note, however, that the Lorentz transformation mixes the 0-component with the others. Thus the desired consistency is achieved only if one assumes that p^0 is also conserved. To get some feeling for what this means, we can expand p^0 in a Taylor series for low velocities. For convenience of interpretation, I will expand cp^0:

$$cp^0 = \gamma m c^2 = mc^2 + \frac{1}{2} m v^2 + \frac{3}{8} m \beta^2 v^2 + \dots \ . \tag{2.25}$$

It is clear that this quantity should be interpreted as the energy of the particle. The first term on the right represents the rest energy of the particle— an energy which the particle possesses even when not in motion. Since this term does not change in Newtonian collisions, it plays no role in the Newtonian limit of the conservation law. The second term on the right is the Newtonian expression for the kinetic energy, and the third term represents a relativistic correction which will be small whenever $\beta \ll 1$.

The first term on the right hand side of Eq. (2.25) is actually one of the most famous results of special relativity, and it is often written by itself: $E = mc^2$. As is well known, this rest energy is in some cases released. When a uranium nucleus undergoes fission, for example, the total rest mass of the decay products is slightly less than the rest mass of the original uranium nucleus. Thus, the difference in rest mass energy must show up in a very high kinetic energy for the decay products, resulting in a tremendous output of heat.

Summarizing the results above, the conserved relativistic three-momentum is given by the first three components of the four-vector p^μ:

$$\vec{p} = \gamma m \vec{v} \ , \tag{2.26}$$

where $\vec{v}$ denotes the ordinary velocity $d\vec{x}/dt$. The conserved relativistic energy is given by cp_0:

$$E = \gamma mc^2 \ . \tag{2.27}$$

E is sometimes called the total energy, composed of the rest energy mc^2 plus the kinetic energy $(\gamma - 1)mc^2$. Using Eqs. (2.20) and (2.22), one sees that $p^2 = -m^2c^2$, where the definition of the Lorentz-invariant dot product (Eq. (2.13)) implies $p^2 = \vec{p}^{\,2} - (E/c)^2$. Thus, one can write

$$E^2 - \vec{p}^{\,2}c^2 = m^2c^4 \ , \tag{2.28}$$

or

$$\boxed{E = \sqrt{\vec{p}^{\,2}c^2 + m^2c^4} \ .} \tag{2.29}$$

To express the velocity of a particle in terms of its energy and/or momentum, one can divide Eq. (2.26) by Eq. (2.27):

$$\boxed{\vec{v} = \frac{\vec{p}c^2}{E} \ .} \tag{2.30}$$

One can then use Eq. (2.28) to eliminate $\vec{p}$, giving

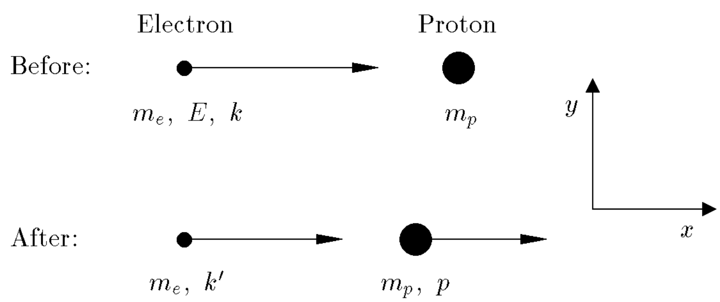
$$|\vec{v}| = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \ . \tag{2.31}$$

Recall that E includes the rest energy and therefore has a minimum value of mc^2, corresponding to $v = 0$. From Eq. (2.31) one can see that $|\vec{v}| \to c$ as $E \to \infty$, so there is no energy which takes a particle beyond the speed of light.

Note that some books summarize Eqs. (2.26) and (2.27) by saying that the particle has a "relativistic mass" given by γm_0, where m_0 is the rest mass. I will, however, follow the practice of most of the newer textbooks, avoiding the concept of relativistic mass. Since "relativistic mass" is just a synonym for "E/c^2", there is no need for it. In these lecture notes the only m that will be used will represent the rest mass, which is related to the momentum and energy by Eqs. (2.26) and (2.27).

RELATIVISTIC CONSERVATION LAWS— AN EXAMPLE:

To illustrate the use of the relativistic conservation laws, let us consider a simple collision problem. An electron, of mass m_e, has energy E and momentum $k\hat{x}$, where $\hat{x}$ denotes a unit vector in the x-direction. Note that E and k are not independent, but must be related by $E = \sqrt{k^2c^2 + m_e^2c^4}$. The electron collides with a proton, of mass m_p, at rest. The final momenta of the electron and proton are also in the x-direction, given by $k'\hat{x}$ and $p\hat{x}$, respectively. Our goal is to find the proton momentum p.



The equation for conservation of the x-component of momentum is given simply by

$$k = k' + p \ , \tag{2.32}$$

which is exactly the same as one would have in the nonrelativistic version of the problem. Conservation of energy takes the more complicated form

$$E + m_pc^2 = \sqrt{k'^2c^2 + m_e^2c^4} + \sqrt{p^2c^2 + m_p^2c^4} \ . \tag{2.33a}$$

For comparison, I will write the nonrelativistic expression for the conservation of energy:

$$\frac{k^2}{2m_e} = \frac{k'^2}{2m_e} + \frac{p^2}{2m_p} \qquad \text{(nonrelativistic)} \ , \tag{2.33b}$$

which can be obtained from Eq. (2.33a) by expanding each of the square roots to lowest order in the squares of the momenta. The nonrelativistic problem can be solved by using Eq. (2.32) to substitute for k' in Eq. (2.33b), giving

$$p = \frac{2m_p}{m_p + m_e}\,k \qquad \text{(nonrelativistic)} \ . \tag{2.34}$$

There is also a solution $p = 0$, corresponding to no scattering at all.

The relativistic problem requires more algebra, but the principles are the same. Again one can use Eq. (2.32) to eliminate k' from the problem. Eq. (2.33a) can then be simplified by moving the second square root on the right-hand-side to the left side, and then squaring the equation. One finds a number of cancelations, leading to

$$(E + m_p c^2)\sqrt{p^2c^2 + m_p^2 c^4} = (E + m_p^2 c^2) m_p c^2 + pkc^2 \ .$$

Squaring again, one finds after a little algebra

$$p = \frac{2m_p c^2 (E + m_p c^2)}{2m_p c^2 E + m_e^2 c^4 + m_p^2 c^4}\, k \ . \tag{2.35}$$

Note that this answer reduces to the nonrelativistic answer when E is dominated by the rest energy $m_e c^2$.

LORENTZ INVARIANCE OF THE LAWS OF PHYSICS:

We have found that the principle of relativity, which requires the speed of light to have the same value when measured by any inertial observer, can be satisfied if a number of peculiar things happen. In particular, moving clocks must run slowly, moving meter sticks must contract in the direction of motion, and pairs of synchronized clocks, moving in unison, must appear unsynchronized to a stationary observer. The amazing thing is that these effects must occur independently of the detailed physics of the measuring devices involved. Our clocks, for example, might keep time by a pendulum moving under gravity, by an oscillating "LC" circuit, by the natural frequency of the spectral line of an atom, or by the decay of an unstable elementary particle. Regardless of which device is used, however, relativity requires the clock to slow by the same amount when the clock is in motion. Nonetheless, I doubt that anyone would even suggest that these devices are conscious of their role as clocks, that they in any way modify their behavior for the purpose of validating special relativity. On the contrary, we believe that these devices are simple physical systems, evolving strictly according to the relevant laws of physics. Apparently, then, relativity implies that the laws of pendulum motion, oscillating circuits, atomic physics, and elementary particle physics all have something in common.

The common principle implied by relativity is a symmetry called Lorentz invariance. To state the principle, one refers to the Lorentz transformation of Eq. (2.6). Suppose the laws of physics— all of the laws of physics— are written in terms of a particular inertial coordinate system (x, y, z, t). Lorentz invariance is the statement that if the laws of physics are re-expressed in terms of new coordinates (x', y', z', t') which are related to the original coordinates by the Lorentz transformation of

Eq. (2.6), then the new equations will be identical to the original equations, except for the presence of the primes.* Stated this way, the property of Lorentz invariance can be tested without using any moving clocks or rulers. One merely has to determine in detail the laws of physics in one particular (x, y, z, t) coordinate system, and then one checks mathematically whether these laws maintain their form when rewritten in terms of Lorentz-transformed coordinates (x', y', z', t').

To complete the logic, we must understand how the property of Lorentz invariance stated above implies that moving clocks and rulers will behave as we stated earlier. So, suppose that all the laws of physics are Lorentz-invariant. Next we need to define a clock: a clock is a device which, when at rest in the (x, y, z, t) coordinate system, emits a sequence of ticks at evenly spaced intervals of the coordinate t. Furthermore, the interval is predictable, in the sense that any two clocks constructed according to the same blueprints will emit ticks at the same interval. Now let us consider what happens if the clock moves along the positive x-axis at speed v, so that its trajectory is given by

$$x_{\text{clock}} = vt \ . \tag{2.36}$$

We can analyze the situation by introducing primed coordinates, related to the original coordinates by the Lorentz transformation of Eq. (2.6). From the top equation, one can see that the trajectory of the clock in the primed coordinates is

$$x'_{\text{clock}} = constant \ . \tag{2.37}$$

Thus, the clock is at rest with respect to the primed coordinates. But Lorentz invariance implies that the laws of physics, when expressed in terms of the primed coordinates, are identical to the laws of physics expressed in terms of the unprimed coordinates. Thus, if the laws of physics caused the clock to emit ticks at evenly spaced, predictable intervals of t when it was at rest in the unprimed coordinates, then the same laws of physics will now cause the clock to emit clicks evenly spaced in t', with the same interval as before. However, given the relationship between the two coordinate systems, this implies that the clock will appear to be running slowly to anyone using the unprimed coordinate system. Thus, Lorentz invariance of the laws of physics implies that moving clocks run slowly.

Similar arguments can be made concerning the synchronization of clocks, and the length of rulers.

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* Note, by the way, that the more familiar symmetries of translation invariance or rotational invariance can be stated in an analogous way. Any statement that the laws of physics maintain their form under a specified transformation of variables is called a symmetry.

ACCELERATING OBSERVERS:

There appears to be a widespread rumor that special relativity describes inertial observers (*i.e.*, observers moving at uniform velocities with respect to an inertial frame), while general relativity is necessary to properly describe an accelerating observer. If you are a victim of this rumor, now is the time to go back to whoever told it to you and straighten him/her out.

We have learned that special relativity predicts that a moving clock runs slower by a factor of $\gamma = 1/\sqrt{1-\beta^2}$, but what should we say about an accelerating clock? After seeing the wondrous implications of special relativity for the behavior of moving clocks, it is tempting to think that general relativity might give us equally powerful insights about the effects of acceleration. A little common sense, however, is all that is needed to dispel this temptation. Consider, for example, a concrete experiment involving the effects of acceleration on a clock. To make the point, let us consider two clocks in particular. The first is a digital wristwatch— for definiteness, let's make it a data-bank-calculator-alarm-chronograph. For a second clock, let's think about an old-fashioned hour-glass. To test the effects of acceleration on these two clocks, we can imagine holding each clock two feet above a concrete floor and then dropping it. (Is there anyone out there who still thinks that general relativity is important to understand the results of this experiment?) I'll admit I haven't actually tried this experiment, but I would guess that the hour-glass would smash to smithereens, but that the data-bank-calculator-alarm-chronograph would probably survive the two foot drop.

In case you haven't gotten the drift, the conclusion is that the effects of acceleration on a clock are complicated, and strongly dependent on the details of the clock mechanism. While nature apparently obeys a symmetry—Lorentz invariance—which determines the effect of uniform motion on a clock, there is no symmetry that determines the effect of acceleration. A similar situation holds with rotational invariance. This symmetry implies that a clock, in the absence of external influences, will run at the same speed no matter how it is oriented in space. If the clock is set rapidly spinning, however, then a detailed analysis of the mechanism would be necessary to predict the clock speed.

Everyone has heard, however, that general relativity has something to do with acceleration, so where does it fit it? General relativity, which we will discuss more later, is a theory of gravity. When Einstein invented special relativity in 1905, he immediately realized that it is inconsistent with Newtonian gravity. Newton's theory implies, for example, that the force on a given object depends on the positions of distant objects at the same time, but relativity tells us that the phrase "at the same time" will have a different meaning to different observers. So Einstein set out to construct a theory of gravity that is consistent with special relativity, and the result was the development in 1915 of general relativity.

One of the key ideas of general relativity is called the principle of equivalence, a principle that is most easily described by considering what is frequently called the Einstein elevator experiment. Consider an elevator, which is initially at rest— the passengers inside experience the gravitational field of the earth, pulling them to the floor of the elevator. Now suppose the elevator cable is cut, and the elevator falls freely, we will assume without friction. Since the elevator and the passengers experience the same acceleration while falling under gravity, Newtonian physics tells us that passengers will appear to float freely inside the elevator, as if gravity were totally absent. The principle of equivalence holds that in the frame of the accelerating elevator, any experiment will produce exactly the same result as it would in empty space, in the absence of any gravitational field. (Of course the principle works only to the extent that the gravitational field is uniform over the volume of the elevator.) Similarly, one can imagine an elevator in empty space, in the absence of gravitational fields, which is being pulled by an accelerating rocket. The passengers would be pressed against the back wall, and to them the effect would be indistinguishable from a uniform gravitational field. The principle of equivalence is a valid principle in Newtonian gravity, and Einstein incorporated it into general relativity. Thus, general relativity describes gravity in terms of acceleration. This idea is used to make inferences about gravity, but it tells us nothing about acceleration that we did not already know.

It is possible to ***define*** an ideal clock, which runs at a rate that is unaffected by acceleration. That is, one can define an ideal clock as one that runs at the same rate as a nonaccelerating clock that is instantaneously moving at the same velocity. A truly ideal clock is impossible to construct, but there is nothing in principle that prevents one from coming arbitrarily close. Since acceleration (unlike uniform velocity) is detectable, it is always possible in principle to design a device to compensate for any effects that acceleration might otherwise produce.

As for accelerating frames of reference, they can be treated by the same ideas that one uses in Newtonian mechanics. Once one knows how to write down the laws of physics in one set of coordinates, it is merely a mathematical exercise to rewrite those laws in terms of arbitrary coordinates. In Newtonian physics, for example, one can transform from inertial to rotating coordinates, and in the process one finds the Coriolis and centrifugal forces— these do not represent any new physics, but merely the rewriting of old physics in the new coordinate system. Similarly, one does not need to know any new physics to transform special relativity equations to an arbitrary coordinate system.