Massachusetts Institute of Technology
Experimental Study Group

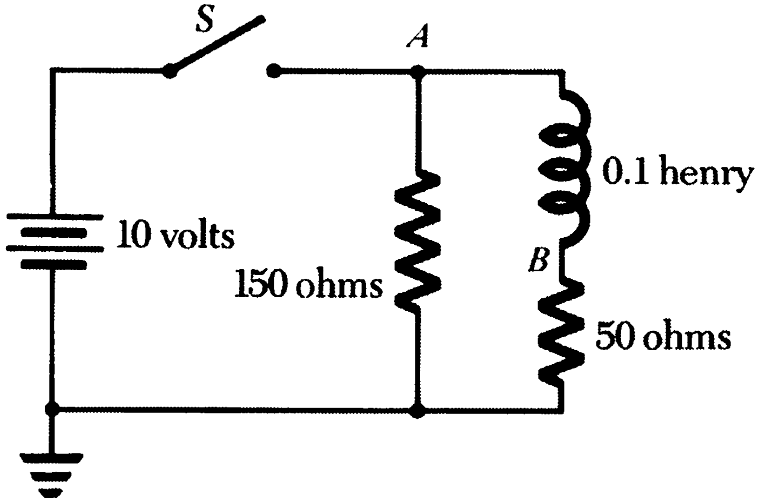
Physics 8.022, Spring 2011

Problem Set 10
RLC circuits, AC circuits

Due: Wednesday, April 27th, 10 pm

Problem 1: Purcell 7.17

In the circuit shown in the diagram the 10-volt battery has negligible internal resistance. The switch S is closed for several seconds, then opened. Make a graph with the potential of point A with respect to ground, just before and then for 10 milliseconds after the opening of switch S. Show also the variation of the potential at point B in the same period of time.



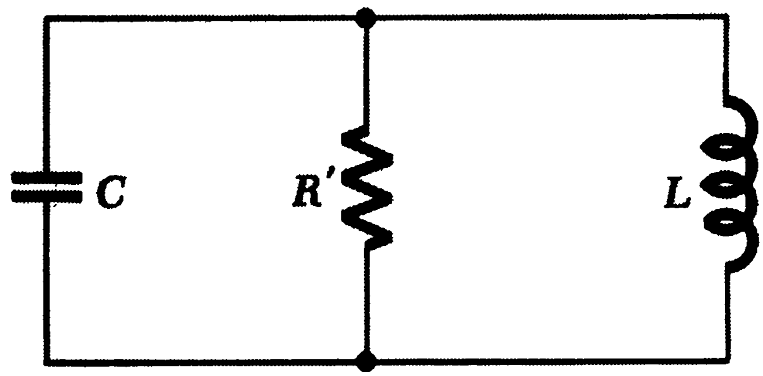
Extra question — By grounding this circuit, we make the switch safer to operate. Describe why a large spark jumps across the switch when it is not grounded, and why the spark does not happen when it is grounded.

Problem 2: Purcell 8.4

In the resonant circuit shown in the figure below, the dissipative element is a resistor R' connected in parallel rather than in series, with the LC combination. Work out the equation analogous to Equation 2 in Purcell,

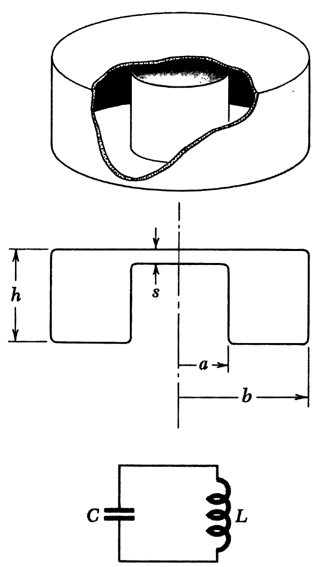
$$\frac{d^2V}{dt^2} + \left(\frac{R}{L}\right)\frac{dV}{dt} + \left(\frac{1}{LC}\right)V = 0,$$

which applies to this circuit. Find also the conditions on the solution analogous to those that hold in the series RLC circuit. If a series RLC and a parallel $R'LC$ circuit have the same L, C, and Q (quality factor), how must R' be related to R?



Problem 3: Purcell 8.7

A resonant cavity of the form illustrated is an essential part of many microwave oscillators. It can be regarded as a simple LC circuit. The inductance is that of a toroid with one turn. Find an expression for the resonant frequency of this circuit and show by a sketch the configuration of the magnetic and electric fields. Hint: the capacitor is composed by the upper and lower disks



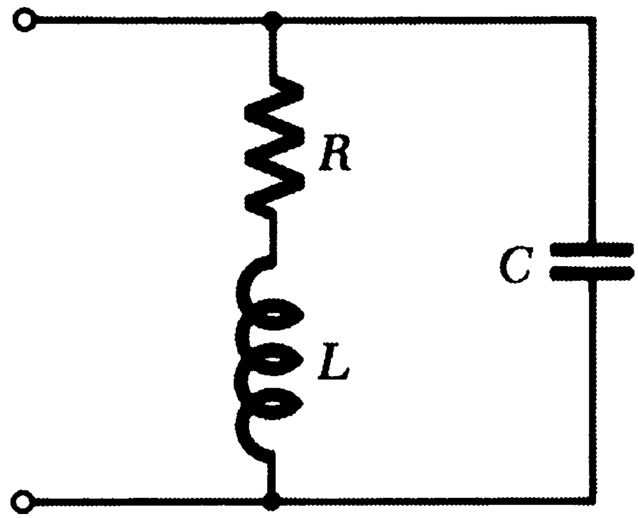
Problem 4: Purcell 8.9

Using the equations 10 and 13 in Purcell (below), express the effect of damping on the frequency of a series RLC circuit.

$$\omega^2 = \frac{1}{LC} - \alpha\frac{R}{L} + \alpha^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

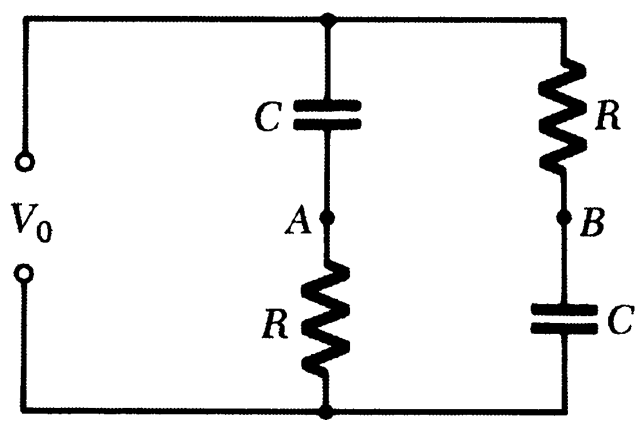
$$Q = \omega\frac{\text{energy stored}}{\text{average power dissipated}}$$

Let $\omega_0 = 1/\sqrt{LC}$ be the frequency of the undamped circuit. Suppose enough resistance is added to bring Q from ∞ down to 1000. By what percentage is the frequency ω thereby shifted from ω_0?

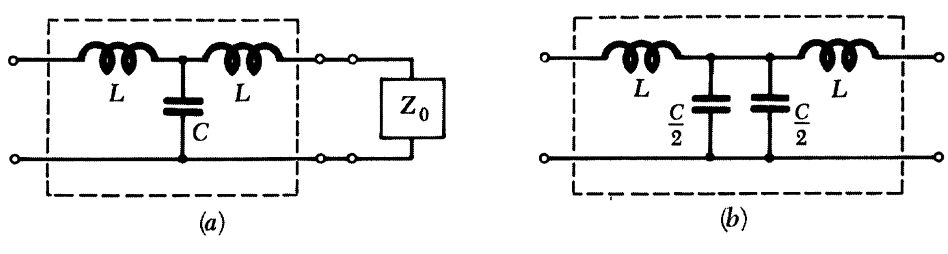


Problem 5: Purcell 8.12

Let $V_{AB} = V_B - V_A$, in this circuit. Show that $|V_{AB}|^2 = V_0^2$ for any frequency ω. Find the frequency for which V_{AB} is 90° out of phase with V_0.



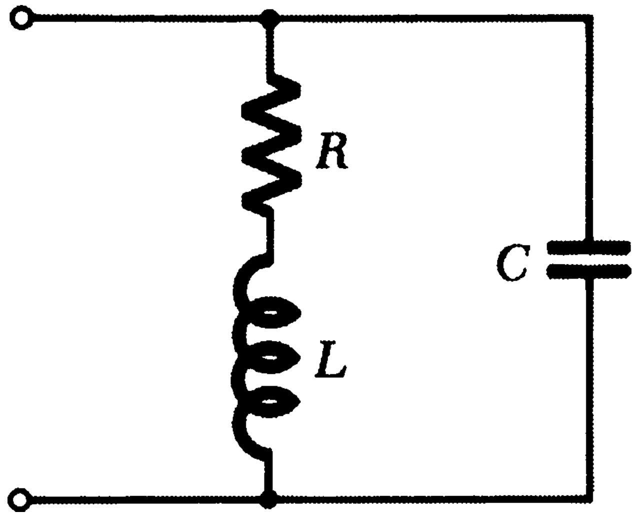
Problem 6: Purcell 8.16



The box (a) with four terminals contains a capacitor C and two inductors of equal inductance L connected as shown. An impedance Z_0 is to be connected to the terminals on the right. For a given frequency ω find the value which Z_0 must have if the resulting impedance across the left terminals is Z_0. You will find that the required Z_0 is a pure resistance R_0 provided $\omega^2 < 2/LC$. What is Z_0 in the special case $\omega = \sqrt{2/LC}$? It helps in understanding that case to note that the contents of the box (a) can be equally well represented by box (b).

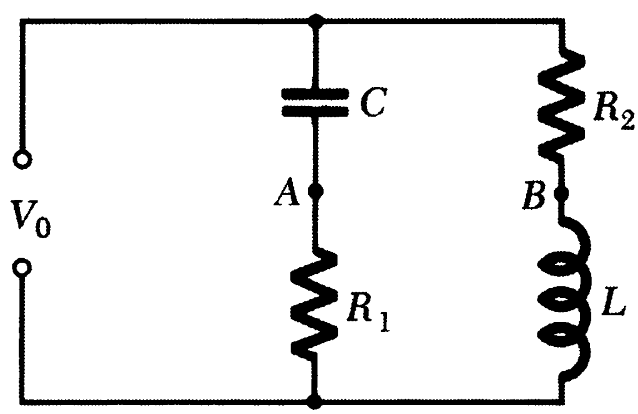
Problem 7: Optional Purcell 8.10

Is it possible to find a frequency at which the impedance at the terminals of the circuit below will be purely real?



Problem 8: Optional Purcell 8.13

Show that, if the condition $R_1 R_2 = L/C$ is satisfied by the components of the circuit below, the difference in voltage between points A and B will be zero at any frequency. Discuss the suitability of this circuit as an AC bridge for measurement of an unknown inductance.



Problem 9: Optional Purcell 8.14

In the laboratory, you find an inductor of unknown inductance L and unknown internal resistance R. Using a DC ohm-meter, an AC volt-meter of high impedance, a 1-microfarad capacitor, and a 1000-Hz signal generator, determine L and R as follows: According to the ohm-meter, R is 35 ohms. You connect the capacitor in series with the inductor and the signal generator. The voltage across both is 10.1 volts. The voltage across the capacitor alone is 15.5 volts. You note also, as a check, that the voltage across the inductor alone is 25.4 volts. How large is L? Is the check consistent?