MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group

Physics 8.022, Spring 2011

Problem Set 10
Maxwell's equations, waves

Due: Wednesday, May 4th 10 am IN CLASS

Problem 1: Purcell 9.1

9.1 If the electric field in free space is $\mathbf{E} = E_0(\hat{\mathbf{x}} + \hat{\mathbf{y}}) \sin (2\pi/\lambda)(z + ct)$, with $E_0 = 2$ statvolts/cm, the magnetic field, not including any static magnetic field, must be what?

Figure 1: Purcell 9.1

Problem 2: Purcell 9.5a

9.5 Here is a particular electromagnetic field in free space:

$$E_x = 0 \qquad E_y = E_0 \sin(kx + \omega t) \qquad E_z = 0$$

$$B_x = 0 \qquad B_y = 0 \qquad B_z = -E_0 \sin(kx + \omega t)$$

(a) Show that this field can satisfy Maxwell's equations if ω and k are related in a certain way.

Figure 2: Purcell 9.5

Problem 3: Discovery of magnetic charge

You discover magnetic charge. The units of magnetic charge density, μ, are chosen such that $\vec{\nabla} \cdot \vec{B} = 4\pi\mu$.
(a) When this magnetic charge is in motion, there is a "magnetic current density" $\vec{L} = \mu\vec{v}$. In analogy to electric charge density and electric current densities, write down the equation of continuity for magnetic charge.
(b) What do Maxwell's equations become with this new charge?

Problem 4: Magnetic field of a moving charge

A charge q moving along the x-axis at constant speed $v \ll c$. When it is at $x = -d$, what is the magnetic field at $(x, y, z) = (0, r, 0)$?

(a) Solve this first using Biot-Savart. (Hint: the current from the moving charge isn't particularly well defined. However, B-S only needs the combination $Idl = (dq/dt)dl = dq(dl/dt) \simeq q_{\text{pt charge}}(dl/dt)$. Sloppy physicist calculus in action!)
(b) Now solve this using displacement current. Look at a circle of radius r centered at the origin and passing through the point $(0, r, 0)$. By symmetry, $\vec{B}$ will be constant on this circle and oriented in the tangential direction. Find a surface which has this circle as a boundary and for which $\int \vec{E} \cdot d\vec{a}$ is simple. Evaluate this flux, apply the "generalized" form of Ampere's law (integral formulation) and you're there.
Note, there's a third way: Lorentz transform from the rest frame electric field (which you used on a previous pset). All three answers should agree, at least in the limit $v \ll c$.

Problem 5: The Director's Challenge — Extra credit!!!

Formulate an interesting problem that relates a topic from 8.022 to your intended major or any other topic about which you are passionate. Give references to help future students to understand the context. Try to give a solution. Any method — theoretical, analytical, numerical, experimental — is acceptable. If you can't give a full solution, outline partial solutions. Enjoy!

Problem 6: Magnetic monopole:experiments

Magnetic monopoles: experiment. [EXTRA CREDIT, 15 bonus points] One way to search for magnetic monpoles is by monitoring the current through a highly conductive (preferably superconducting) loop. Suppose a monopole with magnetic charge s passes through a perfectly conducting circular loop with self-inductance L. The monopole has a constant speed v, perpendicular to the plane of the loop. It approaches from very far away, and then recedes to infinity. Calculate the current I that flows around the loop as a result of the monopole's passage.

(Note: experiments of this type have been running for decades, and have produced a few candidate events, but there has been no unambiguous detection.)

Figure 3: Magnetic monopole