$$\Rightarrow \quad \frac{-e^2}{r^2} - e v_\theta B_z = -m r \omega_f^2$$

$$\Rightarrow \quad \omega_f^2 = \frac{e^2}{mr^3} + \frac{e v_\theta B_z}{mr}$$

$$\omega_f^2 = \omega_0^2 + \frac{e v_\theta B_z}{mr}$$