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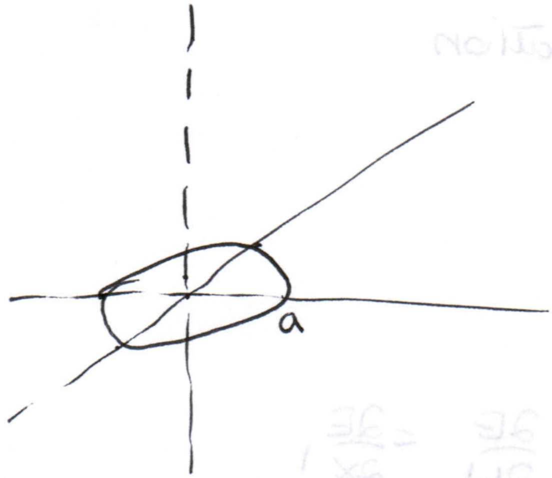

$B_x = 0$

$B_y = 0$

$B_z = B_0 \cos(\omega t - kx)$

$a << \lambda$

a) $\phi_B = \int_S \vec{B} \cdot d\vec{a} \simeq B_0 \cos(\omega t - kx)\pi a^2$

b) $IR = -\frac{d\phi_B}{dt} - L\frac{dI}{dt}$

$-B_0 \omega \sin(\omega t - kx)\pi a^2 = -IR - L\frac{dI}{dt}$

$I(t) = I_0 \sin(\omega t - \phi) \qquad \rightarrow \tilde{I} = I_0 e^{i\omega t} e^{-i\phi}$

$V(t) \equiv \underbrace{B_0 \omega \pi a^2}\, \sin(\omega t - kx)$

$= V_0 \sin(\omega t - kx) \simeq V_0 \sin(\omega t) \rightarrow \tilde{V} = V_0 e^{i\omega t}$

$V_0 \sin(\omega t) = IR + L\frac{dI}{dt}$

Complex

$\tilde{V} = \tilde{I}\underbrace{(R + i\omega L)} \qquad I_0 = \frac{V_0}{|Z_{tot}|} = \frac{V_0}{\sqrt{R^2 + (\omega L)^2}}$

$V_0 e^{i\omega t} = I_0 e^{i\omega t} \boxed{Z_{tot}}\, Z_{tot}\, e^{-i\phi}$

$= I_0 e^{i\omega t} \boxed{|Z_{tot}| e^{i\delta}}\, e^{-i\phi}$

$\downarrow \delta = \phi = \tan^{-1}\frac{\omega L}{R} \rightarrow I = \frac{B_0 \omega \pi a^2}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t - \phi)$

$\doteq \frac{Im(z)}{Re(z)}$

c) $\vec{B}_{center} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} = \frac{\mu_0}{4\pi} I \int_0^{2\pi} \frac{a\, d\theta\, \hat{\theta}\, (-a\hat{r})}{a^3} = \frac{\mu_0 I \hat{z}}{2a} \, \frac{\mu_0}{2a} I_0 \sin(\omega t - \phi)$