$$\frac{\hat{e}}{\hat{v}} = \frac{1}{r^2} = \frac{m \hat{a}}{-e^z \hat{r}} = \frac{m \hat{a}}{-m r \omega_0 r}$$

$$\frac{-e^z \hat{r}}{r^2} = \frac{-m r \omega_0 r}{-m r \omega_0 r}$$

$$\omega_0 = \left(\frac{e^z}{m r^3}\right)^{\frac{1}{3}}$$