Problem 7: ***Electromagnetic Plane Waves.*** Suppose that in the absence of any charges (free space) an electric field exists in the form

$$\vec{\mathbf{E}} = E_0 \sin(kz + \omega t)\,\hat{\mathbf{i}} + E_0 \cos(kz + \omega t)\,\hat{\mathbf{j}}.$$

Show that $\vec{\mathbf{E}}$ satisfies Maxwell's equations provided that a certain magnetic field $\vec{\mathbf{B}}(x, y, z, t)$ also exists, and a relation between ω and k is satisfied.

a) What is the relation between ω and k?

b) What is $\vec{\mathbf{B}}(x, y, z, t)$?

c) Describe what the electric and magnetic fields look like at the origin as a function of time.