# Problem Set 5

1. Solve problem 8 from Pset 4.
2. Reading: the voltaic cell (Purcell 4.9, attached and/or from your favorite Chemistry/Physics textbook)
3. Resistance and resistivity of cake batter (group problem --- Dave Custer, whom we have to thank for suggesting this fun problem, is available to help you with the tools you need at ESG).

   • Choose a cake recipe you like (don't use a cake mix! start from scratch, it is more tasty)
   • Cook the cake by conduction (how?read the attached document for some instructions)
   • Measure the cake batter resistance and estimate the resistivity. What are the uncertainties in your measurement and estimate?
   • Optimize the cake cooking system (shape and size of the container, of the electrodes, recipe proportions,…)
   • Write a report of your findings. Be sure to clarify your goals, procedures, experiment runs, deductions.
      Include plots of the data you collect and illustrations.
   • Bring everything you need to repeat the experiment in class on Monday March 7th 2011.

.

Cake By Conduction
Just Like Ohm

Cook a cake using the heat produced when the cake batter conducts an electric current.
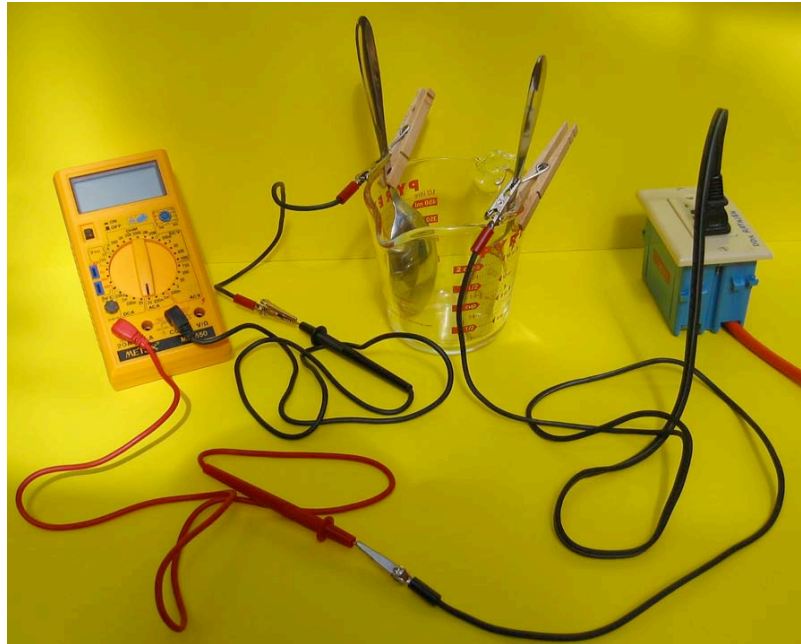
DANGER! READ THESE ITEMS CAREFULLY!
- **THE DEVICE SHOWN ABOVE USES 120 VOLTS AC. WHEN ASSEMBLED AS SHOWN, THERE ARE SEVERAL BARE CONTACTS -- THESE MAY BE LETHAL IF TOUCHED WHILE THE DEVICE IS PLUGGED IN. BE SURE THE DEVICE IS UNPLUGGED BEFORE TOUCHING ANY PART OF IT.**
- **DO NOT PLUG THE DEVICE IN UNTIL YOU HAVE CAREFULLY FINISHED ALL PREPARATION. BE ACUTELY AWARE OF THE LOCATION OF YOUR OWN HANDS AT ALL TIMES!**
- **DO THIS AS A DEMONSTRATION ONLY. KEEP STUDENTS AT A SAFE DISTANCE.**

 The setup shown in the photo consists of the following items connected together in a simple series circuit:
- 2-cup (500 mL) Pyrex measuring cup (or other suitable heat-resistant container) with two large stainless steel serving spoons or soup spoons clipped to opposite sides with wooden spring-loaded clothespins
- AC ammeter (optional) -- meter needs at least a 5 amp scale -- though you likely won't get over 5 amps, a 10 amp scale is an even more comfortable choice -- NOTE: many of the very inexpensive multimeters measure only DC amps, not AC amps

The circuit is plugged into a GFCI (Ground Fault Circuit Interrupter) outlet. The GFCI outlet is a safety feature that reduces (but does not entirely eliminate!) the danger of death or serious injury from electric shock.

The cake mix used in this activity was Betty Crocker Super Moist Milk Chocolate Cake Mix (but other brands and flavors have also been used sucessfully).

The total amount of dry mix in the box was almost 4 cups. Directions on the box called for the addition of 1 1/4 cups of water, 1/3 cup of vegetable oil, and 3 eggs.

I cooked the cake in the 2 cup (500 mL) Pyrex measuring container, and used about a third of the recommended ingredients. Here are the ingredients used:

     1 1/4 cup (about 310 mL) dry mix
     1/2 cup (about 125 mL) water
     2 Tbsp (about 30 mL) canola oil
     1 egg

NOTE: Containers and utensils used for any food activity should be new (or only used previously for food), thoroughly washed (preferably in a dishwasher) before and after each use, and never used for anything else if they are going to be used for food again.

Mix these ingredients **thoroughly**, either in a separate bowl, or in the 2 cup Pyrex measuring container. The cake produced from the ingredients shown will just about fill the 500 ml measuring container after the cake rises.

When the batter is in the Pyrex container with the electrodes in place, and the circuit is set up as shown, plug into the GFCI outlet. **DO NOT TOUCH ANY EXPOSED CONTACTS WHEN THE CIRCUIT IS PLUGGED IN!**

If you want to measure the current as the cake cooks, connect the AC ammeter in SERIES as shown. This is optional. You can then take current readings at regular intervals, e.g. every 30 seconds. With the spoons shown, the current rose to almost 3 amps in one minute, and then steadily decreased to 0.23 amps after 10 minutes and 0.17 after 12 1/2 minutes. The larger the surface area of the electrode used, the larger the current that should be expected, so spoon size is a variable that can be tested.

Depending on the current obtained, a cooking time of 10-15 minutes should be OK. The cooking time used for the array shown was 12 1/2 minutes, since current had leveled off at around 0.2 amps and was very slowly decreasing. It's an arbitrary decision as to when the cake is "done;" pretty much you just judge when nothing much seems to be happening any more. If you aren't using an ammeter, you should probably just cook the cake for 15 minutes. It's better to cook the cake a little too long than not enough.

If you do measure the current at different times as cooking progresses, you can also calculate the corresponding resistances. This can lead to a discussion of the relation of the changes in resistance and current to the cooking process.

DON'T FORGET TO UNPLUG THE SETUP BEFORE TOUCHING THE ELECTRODES OR EXAMINING THE CAKE!

When the cake is "done," **unplug the setup.** Then use the two spoons to remove the cake from the container and place it on a separate plate. Cut it open, and notice where it is done and not done. The portion of the cake between the electrodes should be very much like normal cake in appearance and texture -- and taste! Other parts of the cake may vary from gooey to runny. Don't expect a completely "normal" cake -- but do expect an enjoyable and educational experience!

There are many variables in this activity which may affect current flow, cooking time and quality of the cake. Some have already been noted, but here is a summary:
• brand of cake mix
• water/mix ratio
• size and shape of container, and amount of batter used
• spacing, surface area and configuration of electrodes (e.g., flat spatulas, large or small
    spoons, concave or convex surfaces facing each other, etc.)
These are variables to consider for further experimentation -- a lot of food for thought, so speak! Bon apetit!

Thanks to my friend and colleague Raleigh Ellisen, who first introduced me to this activity many years ago.

Naturally the steady flow of current in a dc circuit requires some source of energy capable of maintaining the electric field that drives the charge carriers. Until now we have avoided the question of the *electromotive force* by studying only parts of entire circuits; we kept the "battery" out of the picture. In Section 4.9 we shall discuss some sources of electromotive force.

ELECTROMOTIVE FORCE AND THE VOLTAIC CELL

4.9   The origin of the electromotive force in a direct-current circuit is some mechanism that transports charge carriers in a direction *opposite* that in which the electric field is trying to move them. A Van de Graaff electrostatic generator (Fig. 4.19) is an example on a large scale. With everything running steadily, we find current in the external resistance flowing in the direction of the electric field **E,** and energy being dissipated there (appearing as heat) at the rate IV_0, or $I^2 R$. Inside the column of the machine, too, there is a downward-directed electric field. Here charge carriers can be moved against the field if they are stuck to a nonconducting belt. They are stuck so tightly that they can't slide backward along the belt in the generally downward electric field. (They can still be removed from the belt by a much stronger field localized at the brush in the terminal. We need not consider here the means for putting charge on and off the belt near the pulleys.) The energy needed to pull the belt is supplied from elsewhere—usually by an electric motor connected to a power line, but it could be a gasoline engine, or even a person turning a crank. This Van de Graaff generator is in effect a battery with an electromotive force, under these conditions, of V_0 volts.

In ordinary batteries it is chemical energy that makes the charge carriers move through a region where the electric field opposes their motion. That is, a *positive* charge carrier may move to a place of *higher* electric potential if by so doing it can engage in a chemical reaction that will yield more energy than it costs to climb the electrical hill.

To see how this works, let us examine one particular voltaic cell. *Voltaic cell* is the generic name for a chemical source of electromotive force. In the experiments of Galvani around 1790 the famous twitching frogs' legs had signaled the chemical production of electric current. It was Volta who proved that the source was not "animal electricity," as Galvani maintained, but the contact of dissimilar metals in the circuit. Volta went on to construct the first battery, a stack of elementary cells, each of which consisted of a zinc disk and a silver disk separated by cardboard moistened with brine. The battery that powers your transistor radio comes in a tidier package, but the principle of operation is the same. Several kinds of voltaic cells are in use, differing in their chemistry but having common features: two elec-

FIGURE 4.19
In the Van de Graaff generator, charge carriers are mechanically transported in a direction opposite that in which the electric field would move them.
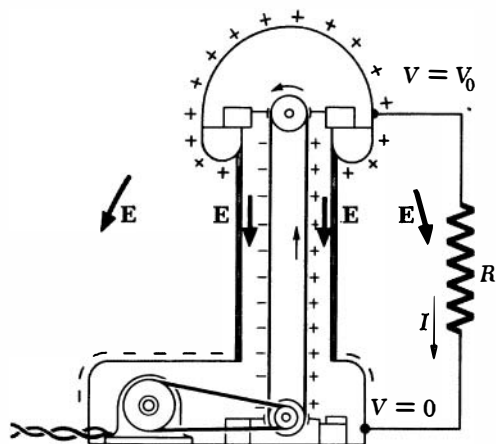
trodes of different material immersed in an ionized fluid, or electrolyte.

As an example, we'll describe the lead–sulfuric acid cell which is the basic element of the automobile battery. This cell has the important property that its operation is readily reversible. With a *storage battery* made of such cells, which can be charged and discharged repeatedly, energy can be stored and recovered electrically.

A fully charged lead–sulfuric acid cell has positive plates which hold lead dioxide, PbO_2, as a porous powder, and negative plates which hold pure lead of a spongy texture. The mechanical framework, or grid, is made of a lead alloy. All the positive plates are connected together and to the positive terminal of the cell. The negative plates, likewise connected, are interleaved with the positive plates, with a small separation. The schematic diagram in Fig. 4.20 shows only a small portion of a positive and a negative plate. The sulfuric acid electrolyte fills the cell, including the interstices of the active material, the porosity of which provides a large surface area for chemical reaction.

The cell will remain indefinitely in this condition if there is no external circuit connecting its terminals. The potential difference between its terminals will be close to 2.1 volts. This open-circuit potential difference is established "automatically" by the chemical interaction of the constituents. This is the *electromotive force* of the cell, for which the symbol $\mathcal{E}$ will be used. Its value depends on the concentration of sulfuric acid in the electrolyte, but not at all on the size, number, or separation of the plates.

Now connect the cell's terminals through an external circuit with resistance R. If R is not too small, the potential difference V between the cell terminals will drop only a little below its open-circuit value $\mathcal{E}$, and a current $I = V/R$ will flow around the circuit (Fig. 4.20b). Electrons flow *into* the positive terminal; other electrons flow *out* of the negative terminal. At each electrode chemical reactions are proceeding, the overall effect of which is to convert lead, lead dioxide, and sulfuric acid into lead sulfate and water. For every molecule of lead sulfate thus made, one charge e is passed around the circuit and an amount of energy $e\mathcal{E}$ is released. Of this energy the amount eV appears as heat in the external resistance R. The difference between $\mathcal{E}$ and V is caused by the resistance of the electrolyte itself, through which the current I must flow inside the cell. If we represent this internal resistance by R_i, the system can be quite well described by the equivalent circuit in Fig. 4.21.

As discharge goes on and the electrolyte becomes more diluted with water, the electromotive force $\mathcal{E}$ decreases somewhat. Normally, the cell is considered discharged when $\mathcal{E}$ has fallen below 1.75 volts. To recharge the cell, current must be forced around the circuit in the opposite direction by connecting a voltage source greater than $\mathcal{E}$ across the cell's terminals. The chemical reactions then run backward
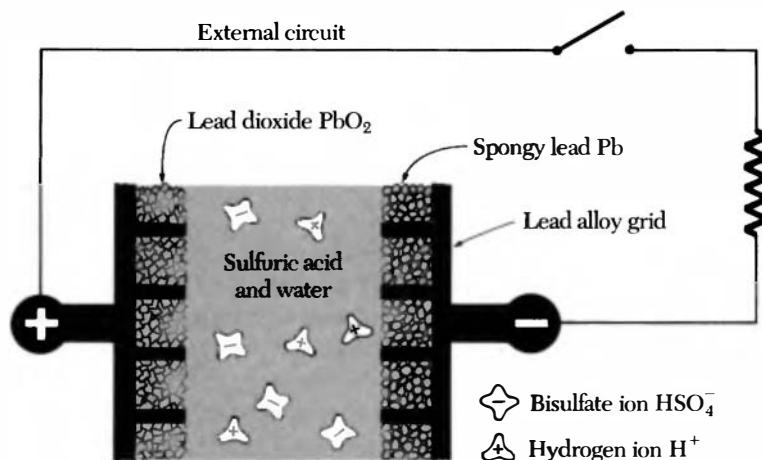
(a) Charged cell.
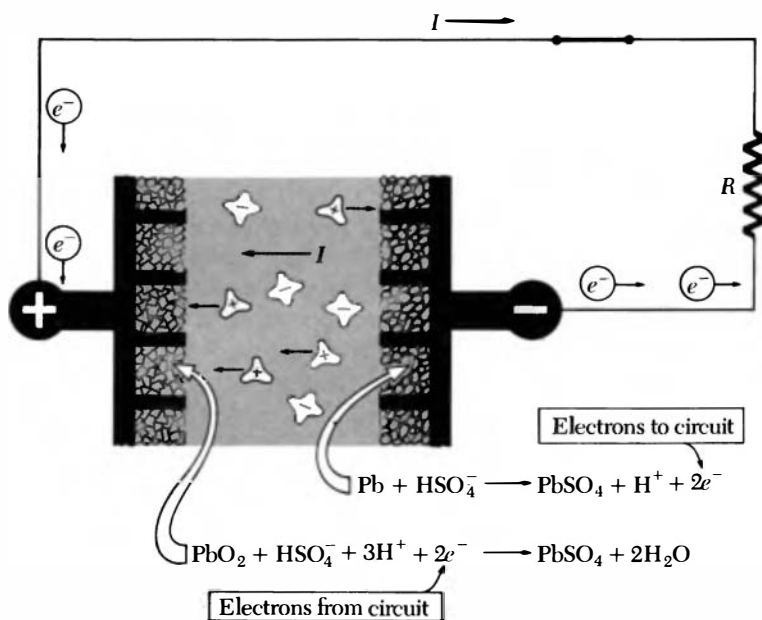
(b) Discharging cell.

FIGURE 4.20
A schematic diagram, not to scale, showing how the lead–sulfuric acid cell works. The electrolyte, sulfuric acid solution, permeates the lead dioxide granules in the positive plate and the spongy lead in the negative plate. The potential difference between the positive and negative terminals is 2.1 volts. With the external circuit closed, chemical reactions proceed at the solid-liquid interfaces in both plates, the depletion of sulfuric acid in the electrolyte, and the transfer of electrons through the external circuit from negative terminal to positive terminal, which constitutes the current I. To recharge the cell, replace the load R by a source with electromotive force greater than 2.1 volts, thus forcing current to flow through the cell in the opposite direction and reversing both reactions.

until all the lead sulfate is turned back into lead dioxide and lead. The investment of energy in charging the cell is somewhat more than the cell will yield on discharge, for the internal resistance R_i causes a power loss $I^2 R_i$ whichever way the current is flowing.

Notice in Fig. 4.20*b* that the current *I* in the electrolyte is produced by a net drift of positive ions toward the positive plate. Evidently the electric field in the electrolyte points toward, not away from, the positive plate. Nevertheless, the line integral of **E** around the whole circuit is zero, as it must be for any electrostatic field. The explanation is this: There are two very steep jumps in potential at the interface of the positive plate and the electrolyte and at the interface of the negative plate and the electrolyte. That is where the ions are moved *against* a strong electric field by forces arising in the chemical reactions. It is this region that corresponds to the belt in a Van de Graaff generator.

Every kind of voltaic cell has its characteristic electromotive force, falling generally in the range of 1 to 3 volts. The energy involved, per molecule, in any chemical reaction is essentially the gain or loss in the transfer of an outer electron from one atom to a different atom. That is never more than a few electron volts. We can be pretty sure that no one is going to invent a voltaic cell with a 12-volt electromotive force. The 12-volt automobile battery consists of six separate lead–sulfuric acid cells connected in series.

NETWORKS WITH VOLTAGE SOURCES

4.10   A network of resistors could contain more than one electromotive force, or voltage source. The circuit in Fig. 4.22 contains two batteries with electromotive force $\mathcal{E}_1$ and $\mathcal{E}_2$, respectively. The positive terminal of each battery is indicated next to the conventional battery symbol. Assume that R_1 includes the internal resistance of one battery, R_2 that of the other. Supposing the resistances given, let us find the currents in this network. Having assigned directions arbitrarily to
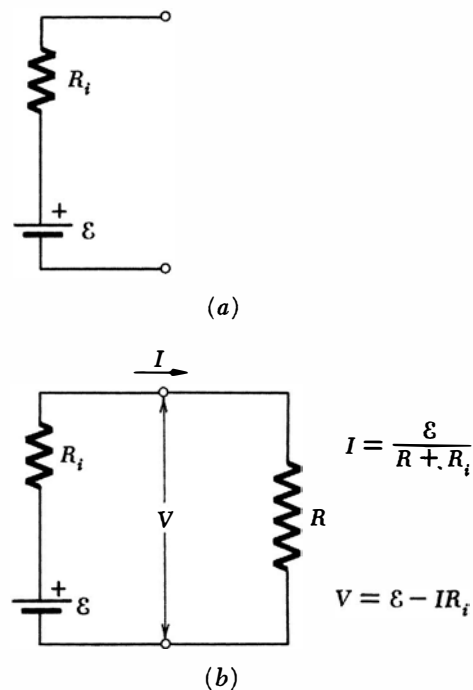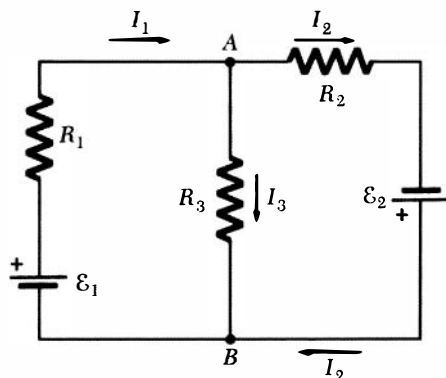

(a)

(b)

FIGURE 4.21

(a) The equivalent circuit for a voltaic cell is simply a resistance R_i in series with an electromotive force $\mathcal{E}$ of fixed value. (b) Calculation of the current in a circuit containing a voltaic cell.

$$I = \frac{\mathcal{E}}{R + R_i}$$

$$V = \mathcal{E} - IR_i$$

FIGURE 4.22

A network with two voltage sources.