Massachusetts Institute of Technology
Experimental Study Group

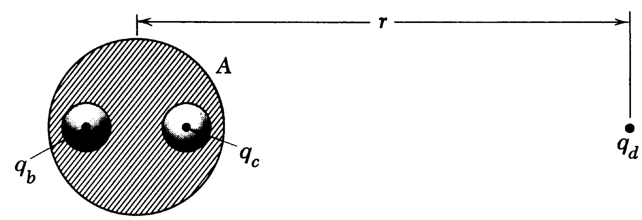
Physics 8.022, Spring 2011

Problem Set 4
Conductors and Capacitors

Due: Sunday, February 27 at 10 pm

Problem 1: Purcell 3.1: Charges in a spherical conductor

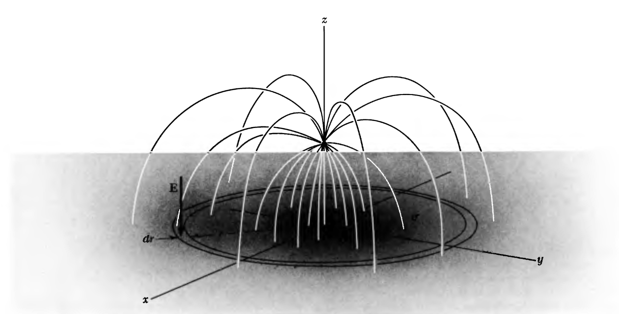
A spherical conductor A contains two spherical cavities. The total charge on the conductor itself is zero. However, there is a point charge q_b at the center of one cavity and q_c at the center of the other. A considerable distance r away is another charge q_d. What force acts on each of the four objects, A, q_b, q_c, q_d? Which answers, if any, are only approximate, and depend on r being relatively large?



Problem 2: Purcell 3.3: Charge over the plane

In the field of a point charge over the plane (Fig. 3.9), if you follow a field line that starts out from the point charge in a horizontal direction, that is, parallel to the plane, where does it meet the surface of the conductor?

Figure 3.9 Some field lines for the charge above the plane. The field strength at the surface, given by the equation below, determines the surface charge density σ.

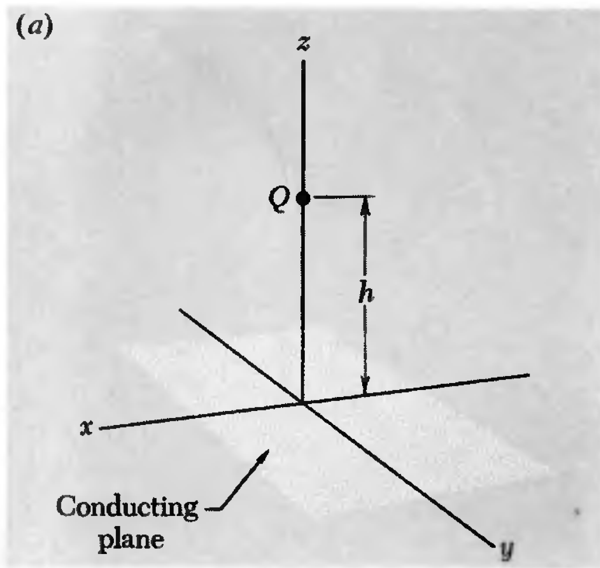


$$E_z = \frac{-2Q}{r^2 + h^2}\cos\theta = \frac{-2Q}{r^2 + h^2} \cdot \frac{h}{(r^2 + h^2)^{1/2}} = \frac{-2Qh}{(r^2 + h^2)^{3/2}}$$

Hint: Sketch the field lines in question. Draw a Gaussian surface that is just barely inside those lines. Close off the bottom. You should have a closed surface that looks like an inverted bowl. What is the total charge contained within this surface? Now compute the total flux through this closed surface: How much goes through the top, curved portion of the inverted bowl? How much goes through the bottom, right next to the conducting plane?

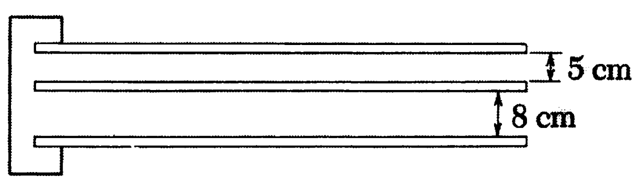
Problem 3: Purcell 3.5: Work pulling a charge away from a conducting plane

A charge Q is located h cm above a conducting plane, just as in Fig. 3.8a. Asked to predict the amount of work that would have to be done to move this charge out to infinite distance from the plane, one student says that it is the same as the work required to separate to infinite distance two charges Q and $-Q$ which are initially $2h$ cm apart, hence $W = Q^2/2h$. Another student calculates the force that acts on the charge as it is being moved and integrates $F\,dx$, but gets a different answer. What did the second student get, and who is right?



Problem 4: Purcell 3.8: Three conducting plates

Three conducting plates are placed parallel to one another as shown. The outer plates are connected by a wire. The inner plate is isolated and carries a charge amounting to 10 esu per square centimeter of plate. In what proportion must this charge divide itself into a surface charge σ_1 on one face of the inner plate and a surface charge σ_2 on the other side of the same plate?



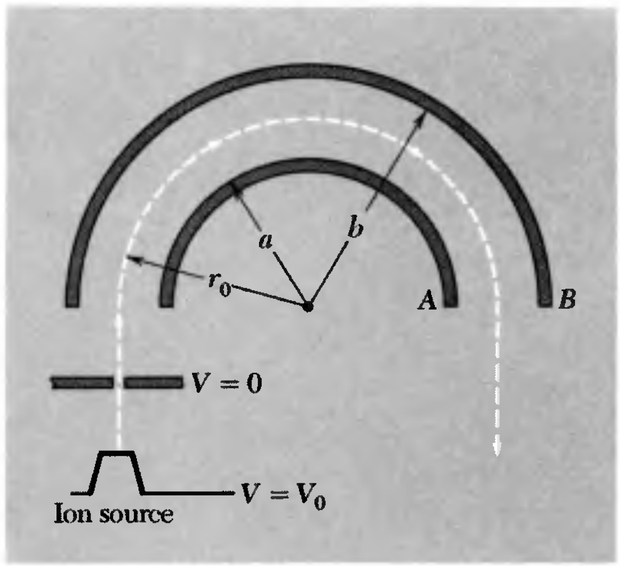
Problem 5: Purcell 3.17: Design a spherical capacitor

We want to design a spherical vacuum capacitor with a given radius a for the outer sphere, which will be able to store the greatest amount of electrical energy subject to the constraint that the electric field strength at the surface of the inner sphere may not exceed E_0. What radius b should be chosen for the inner spherical conductor, and how much energy can be stored?

Ans. $\frac{3}{4}a$; $\frac{27}{512}a^3E_0^2$.

Problem 6: Purcell 3.19: Semicylindrical electrodes

In the apparatus shown, ions are accelerated through a potential difference V_0 and then enter the space between the semicylindrical electrodes A and B. Show that an ion will follow the semicircular path of radius r_0 if the potentials of the outer and inner electrodes are maintained, respectively, at $2V_0 \ln(b/r_0)$ and $2V_0 \ln(a/r_0)$. (The cylindrical electrodes A and B are assumed to be long, in the direction perpendicular to the diagram, compared with the space between them.)



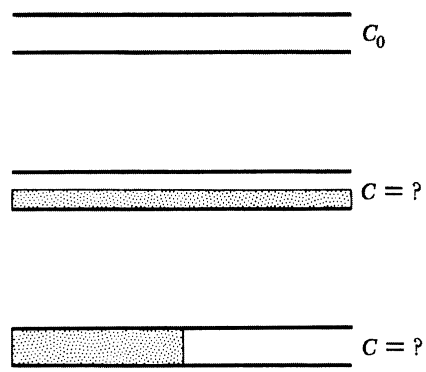
Problem 7: Purcell 10.1: Make a capacitor

(a) You have a supply of polyethylene tape, dielectric constant 2.3, 2.25 inches wide, and 0.001 inch thick; also, a supply of aluminum tape 2 inches wide and 0.0005 inch thick. You want to make a capacitor of about 0.05 microfarad capacitance, in the form of a compact cylindrical roll. Describe how you might do this, estimating the amount of tape of each kind that would be needed, and the overall diameter of the finished capacitor.

(b) (Optional — Group Challenge) Build a Leyden jar, charge it and determine how much charge it stores. Be creative! You can ask Paola and the TAs for hints.

Problem 8: Purcell 10.14: Capacitors with a dielectric

The figure shows three capacitors of the same area and plate separation. Call the capacitance of the vacuum condenser C_0. Each of the others is half-filled with a dielectric, with the same dielectric constant E, but differently disposed, as shown. Find the capacitance of each of these two capacitors. (Neglect edge effects.)



Problem 9: Purcell 10.6: "Fringing" field far away from a parallel-plate capacitor

A parallel-plate capacitor, with a measured capacitance $C = 250\,\text{cm}$, is charged to a potential difference of 6 statvolts. The plates are 1.5 cm apart. We are interested in the field outside the capacitor, the "fringing" field which we usually ignore. In particular, we would like to know the field at a distance from the capacitor large compared with the size of the capacitor itself. This can be found by treating the charge distribution on the capacitor as a dipole. Estimate the electric field strength

(a) At a point 3 meters from the capacitor in the plane of the plates.

(b) At a point the same distance away, in a direction perpendicular to the plates.