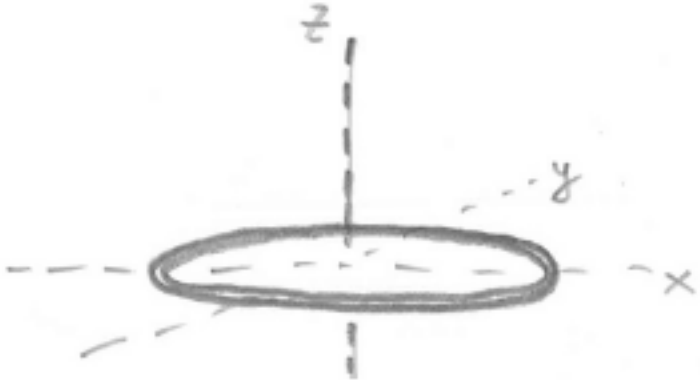
Problem 9: ***Loop Antenna.*** An electromagnetic wave propagating in air has a magnetic field given by

$$B_x = 0 \qquad B_y = 0 \qquad B_z = B_0 \cos(\omega t - kx).$$

It encounters a circular loop antenna of radius a centered at the origin $(x, y, z) = (0,0,0)$ and lying in the x-y plane. The radius of the antenna $a << \lambda$ where λ is the wavelength of the wave. So you can assume that at any time t the magnetic field inside the loop is approximately equal to its value at the center of the loop.



a) What is the magnetic flux, $\Phi_{mag}(t) \equiv \iint_{disk} \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$, through the plane of the loop of the antenna?

The loop has a self-inductance L and a resistance R. Faraday's law for the circuit is

$$IR = -\frac{d\Phi_{mag}}{dt} - L\frac{dI}{dt}.$$

b) Assume a solution for the current of the form $I(t) = I_0 \sin(\omega t - \phi)$ where ω is the angular frequency of the electromagnetic wave, I_0 is the amplitude of the current, and ϕ is a phase shift between the changing magnetic flux and the current.. Find expressions for the constants ϕ and I_0.

c) What is the magnetic field created at the center of the loop by this current $I(t)$?