

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group

Physics 8.022, Spring 2011

Problem Set 2
Gauss's law and electric potential

Due: Sunday, February 13

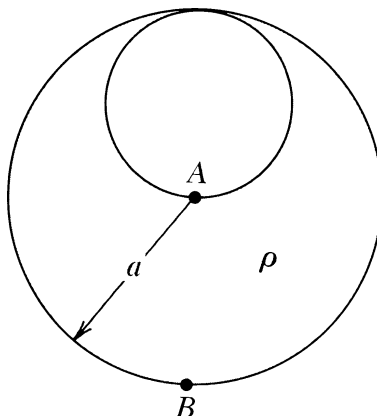
Problem 0: Line integrals; work; units and dimensional analysis (Optional)

A force $\vec{F} = A(y^2\hat{x} + 2x^2\hat{y})$ is acting on a particle which is initially at the origin of the (x, y) coordinate system. We transport the particle on a square path defined by the points $(0, 0)$, $(0, l)$, (l, l) , $(l, 0)$, $(0, 0)$. The constant A is positive.

- (a) Suppose we work in SI units: the coordinates (x, y) are measured in meters, so that the particle moves l meters along each leg of the path; the force is measured in Newtons. What must be the units of A ? Express in terms of kg, m, and s.
- (b) Suppose we work in cgs units: the coordinates (x, y) are measured in centimeters, and the force is measured in dynes. What must be the units of A ? Express in terms of gm, cm, and s.
- (c) How much work does the force do when the particle travels around the path? (Your answer does not depend on the choice of units: express it in terms of the constants A and l , which are assumed to have units built into them.) Is this a conservative force?
- (d) If we place a particle right at the origin, the total force is zero, so it will just stay there. Is this a stable situation? Give any argument you please (mathematical, physical, intuitive) to justify the stability (or instability) of this situation.

Problem 1: Purcell 1.16

The sphere of radius a was filled with positive charge at uniform density ρ . Then a smaller sphere of radius $a/2$ was carved out, as shown in the figure, and left empty. What are the direction and magnitude of the electric field at A ? At B ?



Problem 2: Purcell 1.17

- (a) A point charge q is located at the center of a cube of edge length d . What is the value of $\int \vec{E} \cdot d\vec{a}$ over one face of the cube?
- (b) The charge q is moved to one corner of the cube. What is now the value of the flux of \vec{E} through each of the faces of the cube?

Problem 3: Purcell 1.31

A charged soap bubble experiences an outward electrical force on every bit of its surface. Given the total charge Q on a bubble of radius R , what is the magnitude of the resultant force tending to pull any hemispherical half of the bubble away from the other half? (Should this force divided by $2\pi R$ exceed the surface tension of the soap film interesting behavior might be expected!)

Ans. $Q^2/8R^2$.

Problem 4: Purcell 2.1

The vector function which follows represents a possible electrostatic field:

$$E_x = 6xy \qquad E_y = 3x^2 - 3y^2 \qquad E_z = 0$$

Calculate the line integral of \vec{E} from the point $(0,0,0)$ to the point $(x_1, y_1, 0)$ along the path which runs straight from $(0,0,0)$ to $(x_1, 0, 0)$ and thence to $(x_1, y_1, 0)$. Make a similar calculation for the path which runs along the other two sides of the rectangle, via the point $(0, y_1, 0)$. You ought to get the same answer if the assertion above is true. Now you have the potential function $\phi(x, y, z)$. Take the gradient of this function and see that you get back the components of the given field.

Problem 5: Purcell 2.4

Describe the electric field that goes with the following potential:

$$\begin{aligned} \phi &= x^2 + y^2 + z^2 & \text{for } x^2 + y^2 + z^2 < a^2 \\ \phi &= -a^2 + \frac{2a^3}{(x^2 + y^2 + z^2)^{1/2}} & \text{for } a^2 < x^2 + y^2 + z^2 \end{aligned}$$

Discuss what happens at the boundary ($x^2 + y^2 + z^2 = a^2$).

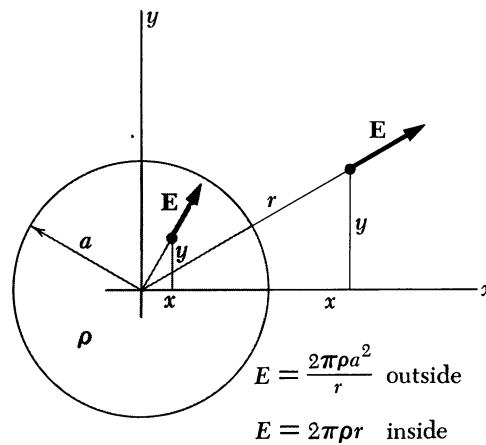
Problem 6: Purcell 2.8

For the cylinder of uniform charge density in Fig. 2.17:

- Show that the expression there given for the field inside the cylinder follows from Gauss's law.
- Find the potential ϕ as a function of r , both inside and outside the cylinder, taking $\phi = 0$ at $r = 0$.

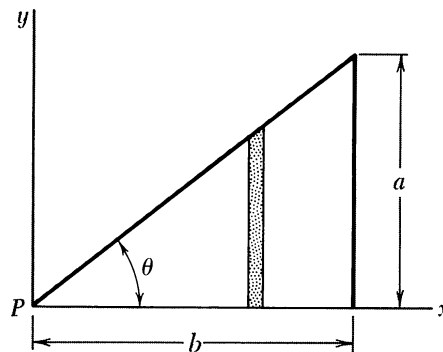
FIGURE 2.17

The field inside and outside a uniform cylindrical distribution of charge.



Problem 7: Purcell 2.12

The right triangle with vertex P at the origin, base b , and altitude a has a uniform density of surface charge σ . Determine the potential at the vertex P . First find the contribution of the vertical strip of width dx at x . Show that the potential at P can be written as $\phi_P = \sigma b \ln[(1 + \sin \theta)/\cos \theta]$.



Problem 8: Purcell 2.30

Consider a charge distribution which has the constant density ρ everywhere inside a cube of edge b and is zero everywhere outside that cube. Letting the electric potential ϕ be zero at infinite distance

from the cube of charge, denote by ϕ_0 the potential at the center of the cube and ϕ_1 the potential at a corner of the cube. Determine the ratio ϕ_0/ϕ_1 . The answer can be found with very little calculation by combining a dimensional argument with superposition. (Think about the potential at the center of a cube with the same charge density and with twice the edge length.)