a) $\frac{\partial E_x}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} \Rightarrow -k^2 E_x = -\frac{\omega^2}{c^2} E_x \Rightarrow$

$\omega = ck$

b) $\vec{B} = -\int (\vec{\nabla} \times \vec{E})\, dt$

$\vec{B} = -\int \frac{\partial}{\partial z} E_x \, dt \, \hat{j} - \int \left(-\frac{\partial E_y}{\partial z}\right) dt \, \hat{i}$

$= -\int k E_0 \cos(kz+\omega t)\, dt \, \hat{j} + \int -E_0 k \sin(kz+\omega t)\, \hat{i}$

$= -\frac{kE_0}{\omega} \sin(kz+\omega t)\, \hat{j} + \frac{E_0 k}{\omega} \cos(kz+\omega t)\, \hat{i}$

$= -\frac{E_0}{c} \sin(kz+\omega t)\, \hat{j} + \frac{E_0}{c} \cos(kz+\omega t)\, \hat{i}$

c) at $z=0$: $\vec{E} = E_0 \sin\omega t \, \hat{i} + E_0 \cos\omega t \, \hat{j}$

$\vec{B} = -\frac{E_0}{c} \sin\omega t \, \hat{j} + \frac{E_0}{c} \cos\omega t \, \hat{i}$