MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

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Lecture Notes 10
PROBLEMS OF THE STANDARD COSMOLOGICAL MODEL

READING ASSIGNMENT: "Inflation and the New Era of High-Precision Cosmology," by Alan Guth, written for the MIT Physics Department annual newsletter, 2002. It is available at

http://web.mit.edu/physics/alumniandfriends/physicsjournal_fall_02_cosmology.pdf.

The data quoted in the article about the nonuniformities of the cosmic microwave background radiation has since been superceded by much better data, but the conclusions remain the same.

INTRODUCTION:

The topics discussed here are also contained in the article assigned above, but that article contains essentially no equations. In these notes I have elaborated on the horizon and flatness problems of the standard cosmological model. The magnetic monopole problem, also discussed in the assigned article, will be discussed in Lecture Notes 12.

THE HORIZON/HOMOGENEITY PROBLEM:

The horizon problem is the difficulty in explaining the large-scale uniformity of the observed universe. This large-scale uniformity is most evident in the microwave background radiation. This radiation appears slightly hotter in one direction than in the opposite direction, by about one part in one thousand— but this nonuniformity can be attributed to our motion through the background radiation. Once this effect is subtracted out, using best-fit parameters for the velocity, it is found that the residual temperature pattern is uniform to about one part in 10^5. However, the universe in the standard cosmological model evolves much too quickly to allow this uniformity to be achieved by the usual processes by which a system approaches thermal equilibrium.

In order to see this, we will not need to know anything about the details of thermal equilibrium processes. We will use only the fact that no information or physical process can propagate faster than light. Thus, no process can propagate beyond the "horizon distance", which is defined as the total distance that a light signal could have traveled since the beginning of the universe. The issue of horizons was introduced into cosmology by W. Rindler in 1956, and the horizon problem is described (without using the words "horizon problem") in two well-known textbooks: S. Weinberg, *Gravitation and Cosmology*, J. Wiley and Sons (1972), and C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, W.H. Freeman & Co. (1973).

In Lecture Notes 5 we found that this horizon distance is given by 3ct for the case of a matter-dominated $k = 0$ universe, and in Lecture Notes 7 we showed that it is 2ct for the case of a radiation-dominated $k = 0$ universe. We also showed in Lecture Notes 7 that the universe became matter-dominated at about 50,000 years after the big bang, and that the cosmic microwave background radiation decoupled from the rest of matter at about 350,000 years after the big bang. Thus, the radiation decoupled well after the universe became matter-dominated, so to a good approximation the horizon distance at this time is given by $3ct \approx 1,000,000$ light years.

For comparison, we would like to calculate the distance between our own galaxy (or, more precisely, the matter which will later become our galaxy) and the site of emission of the cosmic background radiation that we are now receiving. To do this, we can make use of some previous results. In Lecture Notes 7 we learned that the cosmic microwave background radiation was emitted (or more precisely, decoupled) when the temperature was 3000 °K. Since the current temperature is about 2.7 °K, and since $aT = constant$ as the universe expands, it follows that the redshift at the time of emission is given by

$$1 + z = \frac{a(t_0)}{a(t_d)} = \frac{3000°\mathrm{K}}{2.7°\mathrm{K}} \approx 1100 \ , \tag{10.1}$$

where I have also made use of Eq. (3.12) to relate $1 + z$ to the ratio of the scale factors. (I am using the convention that a subscript "0" denotes the present value of a given quantity, and the subscript "d" denotes its value at the time of decoupling.) Knowing $1 + z$ we can find the present distance between our galaxy and the site of emission of the radiation, using the result of Problem 3, Problem Set 2 (2009). You found there that for a matter-dominated $k = 0$ universe, the present physical distance of an object seen at redshift z is given by

$$\ell_p(t_0) = 2cH_0^{-1}\left[1 - \frac{1}{\sqrt{1+z}}\right] \ . \tag{10.2}$$

Using $H_0 = 72$ km-s^{-1}-Mpc^{-1}, one finds that $H_0^{-1} \approx 13.6 \times 10^9$ yr and $\ell_p(t_0) \approx 26 \times 10^9$ light-yr. That is, the region of emission of the cosmic background radiation that we are presently observing is a spherical shell of matter at essentially the present horizon distance. But physical distances vary with time as $a(t)$, so the physical radius of this shell of matter at the time of decoupling t_d is given by

$$\begin{aligned} \ell_p(t_d) &= \frac{a(t_d)}{a(t_0)} \ell_p(t_0) \\ &\approx \frac{1}{1100} \times 26 \times 10^9 \text{ lt-yr} \approx 2.4 \times 10^7 \text{ lt-yr} . \end{aligned} \tag{10.3}$$

Thus, at the time of emission of the cosmic background radiation, the region of emission was a spherical shell with a radius many times larger than the horizon distance. Specifically, the radius was $2.4 \times 10^7 / \ 10^6 \approx 24$ times larger than the horizon distance.

To state the problem most clearly, suppose that one detects the cosmic microwave background in a certain direction in the sky, and suppose that one also detects the radiation from precisely the opposite direction. At the time of emission, the sources of these two signals were separated from each other by about 48 horizon distances. Thus it is absolutely impossible, within the context of this model, for these two sources to have come into thermal equilibrium by any physical process.

This problem is not a genuine inconsistency of the standard model — if the uniformity is assumed in the initial conditions, then the universe will evolve uniformly. The "problem" is that one of the most salient features of the observed universe — its large-scale uniformity — cannot be explained by the standard model; it simply must be assumed as an initial condition. The suggestion then is not that the standard model is wrong, but rather that it is incomplete.

The calculation described above depended on our approximation that the universe was matter-dominated at all relevant times, which is a rather crude approximation. Nonetheless, since 48 is so far from one, we can be confident that this problem will not go away with a more careful calculation.

THE FLATNESS PROBLEM:

A second problem of the standard cosmological model is known as the flatness problem — it refers to the difficulty in understanding why the present value of Ω (the ratio of the mass density ρ to the critical mass density ρ_c) is within an order of magnitude or so of unity. The key fact is that the value $\Omega = 1$ is a point of unstable equilibrium, something like a pencil balancing on its point. The word "equilibrium" implies that if Ω is ever ***exactly*** equal to one, it will remain equal to one forever — that is, a flat ($k = 0$) universe remains a flat universe. However, if Ω is ever slightly larger than one, it will rapidly grow toward infinity; if Ω is ever slightly smaller than one, it will rapidly fall toward zero. Thus, in order for Ω to be anywhere near

1 today, the value of Ω in the early universe must have been extraordinarily close to one.

Like the horizon problem, this problem is not a genuine inconsistency of the standard model. If one is willing to assume that the value of Ω in the early universe was extraordinarily close to one, then the model will describe how the universe evolves to have a value of Ω today within the accepted range. The problem is again the lack of explanatory or predictive power of the model — the extraordinary closeness of Ω to unity in the early universe cannot be explained, but must simply be assumed as an initial condition. The mathematics behind the flatness problem was undoubtedly known to almost anyone who has worked on the big bang theory from the 1920's onward, but apparently the first people to consider it a problem in the sense described below were Robert Dicke and P.J.E. Peebles, who published a discussion in 1979 (in *General Relativity: An Einstein Centenary Survey*, eds: S.W. Hawking & W. Israel (Cambridge University Press, Cambridge)).

To work out the evolution of Ω, we need only recast some relations that we have already derived. The key relation is the first-order equation for the evolution of the scale factor, derived in Lecture Notes 4:

$$H^2 \equiv \left[\frac{1}{a}\left(\frac{da}{dt}\right)\right]^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} \ . \tag{10.4}$$

Recalling that $\rho_c = 3H^2/8\pi G$, one can rewrite this equation to give

$$\rho_c = \rho - \frac{3kc^2}{8\pi G a^2} \ ,$$

which can be manipulated to give

$$\frac{\rho - \rho_c}{\rho} = \frac{3kc^2}{8\pi G a^2 \rho} \ .$$

The left-hand side can then be rewritten by dividing both numerator and denominator by ρ_c, and the right-hand side can be rewritten by multiplying numerator and denominator by T^2. One then has the crucial result

$$\frac{\Omega - 1}{\Omega} = A\frac{T^2}{\rho} \ , \tag{10.5}$$

where

$$A = \frac{3kc^2}{8\pi G a^2 T^2} \ . \tag{10.6}$$

In Lecture Notes 7 we learned that $aT \approx constant$, so therefore $A \approx constant$. The time evolution of Ω is then controlled by Eq. (10.5), provided that one knows how T and ρ evolve.

But the evolution of T and ρ are already understood. For a matter-dominated $k = 0$ universe, we know that $T \propto 1/a$, $\rho \propto 1/a^3$, and $a \propto t^{2/3}$. It follows that

$$\frac{\Omega - 1}{\Omega} \propto a \propto t^{2/3} \quad \text{(matter-dominated).} \tag{10.7}$$

For a radiation-dominated $k = 0$ universe, on the other hand, we have $T \propto 1/a$, $\rho \propto 1/a^4$, and $a \propto t^{1/2}$. This gives

$$\frac{\Omega - 1}{\Omega} \propto a^2 \propto t \quad \text{(radiation dominated).} \tag{10.8}$$

We can now trace the evolution of Ω backward in time. We make the extremely conservative assumption that at the present time

$$0.1 < \Omega_0 < 2 \ , \tag{10.9}$$

from which it follows that

$$\left| \frac{\Omega_0 - 1}{\Omega_0} \right| < 10 \ . \tag{10.10}$$

For mathematical simplicity, we will assume that the universe can be described in terms of a matter-dominated era and a radiation-dominated era, with a sharp transition between the two. The transition occurs at about 50,000 years after the big bang, while we estimate the current age of the universe as 13.7×10^9 years. Using Eq. (10.7), we conclude that the value of Ω at 50,000 years is given by

$$\left(\frac{\Omega - 1}{\Omega} \right)_{t = 50{,}000 \text{ yr}} \approx \left(\frac{50{,}000}{13.7 \times 10^9} \right)^{2/3} \frac{\Omega_0 - 1}{\Omega_0} \approx 2.37 \times 10^{-4} \, \frac{\Omega_0 - 1}{\Omega_0} \ . \tag{10.11}$$

Let us now calculate the value of Ω at 1 second after the big bang. One second is a particularly interesting time, because it is the earliest time for which we have direct evidence that the standard cosmological model seems to be working. The processes which lead to nucleosynthesis begin at about $t = 1$ sec, and the predictions derived from big bang nucleosynthesis calculations are in good agreement with observations.

To find the value of Ω at one second, begin by noting that

$$\frac{1 \text{ sec}}{50{,}000 \text{ yr}} = \frac{1 \text{ sec}}{50{,}000 \text{ yr}} \times \frac{1 \text{ yr}}{3.16 \times 10^7 \text{ sec}} = 6.33 \times 10^{-13} \ .$$

Combining Eqs. (10.8) and (10.11), we find that

$$\begin{aligned}\left(\frac{\Omega-1}{\Omega}\right)_{t=1\text{ sec}} &\approx\; 6.33\times 10^{-13}\left(\frac{\Omega-1}{\Omega}\right)_{t=50{,}000\text{ yr}}\\ &\approx\; 1.50\times 10^{-16}\left(\frac{\Omega_0-1}{\Omega_0}\right)\end{aligned} \tag{10.12}$$

Using Eq. (10.10), we deduce that

$$|\Omega-1|_{t=1\text{ sec}} < 10^{-15}\ . \tag{10.13}$$

(The factor of Ω which appeared in the denominator has been replaced by unity, which is an excellent approximation.) Thus, at one second after the big bang, the value of Ω must have been equal to one to an accuracy of 15 decimal places. The flatness problem is the statement that the standard cosmological model provides no explanation of how the value of Ω came to be tuned so precisely.

As we will see shortly, the horizon and flatness problems provide much of the motivation of the inflationary universe model, which gives a simple resolution to both of them.