a) $$\frac{\partial E_x}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} \Rightarrow -k^2 E_x = -\frac{\omega^2}{c^2} E_x \Rightarrow$$

$$\underline{\omega = ck}$$

b) $$\vec{B} = -\int (\vec{\nabla} \times \vec{E})\, dt$$

$$\vec{B} = -\int \frac{\partial}{\partial z} E_x \, dt\, \hat{j} - \int \left(-\frac{\partial E_y}{\partial z}\right) dt\, \hat{i}$$

$$= -\int k E_0 \cos(kz+\omega t)\, dt\, \hat{j} + \int -E_0 k \sin(kz+\omega t)\, \hat{i}$$

$$= -\frac{k E_0}{\omega} \sin(kz+\omega t)\, \hat{j} + \frac{E_0 k}{\omega} \cos(kz+\omega t)\, \hat{i}$$

$$= -\frac{E_0}{c} \sin(kz+\omega t)\, \hat{j} + \frac{E_0}{c} \cos(kz+\omega t)\, \hat{i}$$

c) at $z = 0$: $$\vec{E} = E_0 \sin \omega t\, \hat{i} + E_0 \cos \omega t\, \hat{j}$$

$$\vec{B} = -\frac{E_0}{c} \sin \omega t\, \hat{j} + \frac{E_0}{c} \cos \omega t\, \hat{i}$$