MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Experimental Study Group

Physics 8.022, Spring 2011

Problem Set 6
Resistances and Circuits

Due: Sunday, March 13 at 10 pm

## Problem 1: Purcell 4.4: Resistance of Atlantic Ocean

The first telegraphic messages crossed the Atlantic Ocean in 1858, by a cable 3000 km long laid between Newfoundland and Ireland. The conductor in this cable consisted of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheath.

(a) Calculate the resistance of the conductor. Use $3 \cdot 10^{-8} \, \Omega \, \mathrm{m}$ for the resistivity of copper, which was of somewhat dubious purity.

(b) A return path for the current was provided by the ocean itself. Given that the resistivity of seawater is about $0.25 \, \Omega \, \mathrm{m}$, see if you can show that the resistance of the ocean return would have been much smaller than that of the cable.

Problem 2: Sea Water

An oceanographer is studying how the ion concentration in sea water depends on depth. She does this by lowering into the water (until completely submerged) a pair of concentric metallic cylinders (see figure) at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius r_a, outer radius r_b, and length L much larger than r_b. The scientist applies a potential difference ΔV between the inner and outer surfaces, producing an outward radial current I. Let ρ represent the resistivity of the water.
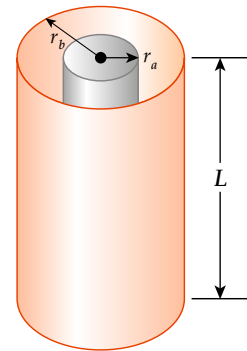
(a) Find the resistance of the water between the cylinders in terms of L, ρ, r_a, and r_b.

(b) Express the resistivity of the water in terms of the measured quantities L, r_a, r_b, ΔV, and I.

Problem 3: Non-uniform Conductivity

A cylindrical glass rod is heated with a torch until it conducts enough current to cause a light bulb to glow. The rod has a length $L = 2$ cm, a diameter $d = 0.5$ cm, and its ends, plated with material of infinite conductivity, are connected to the rest of the circuit. When red hot, the rod's conductivity varies with position x measured from the center of the rod as $\sigma(x) = \sigma_0 L^4 / x^4$, with $\sigma_0 = 4 \cdot 10^{-2} (\Omega \cdot \mathrm{m})^{-1}$.

(a) What is the resistance of the glass rod? Express your answer both symbolically and as a value in ohms.

(b) When a voltage ΔV is applied between the two ends, what is the current density $\vec{J}(x)$ and what is the steady-state electric field $\vec{E}(x)$?

(c) In steady-state, what is the volume charge density $\rho(x)$ found within the rod? (Note that you are not asked for the surface charge σ that will accumulate on the surfaces.)

Problem 4: Battery Lifetime

A standard D cell can supply 10 mA at 1.5 V for about 300 hours. An alkaline D cell can do about the same. Assume that the chemistry of the batteries will produce the same amount of energy and will maintain the emf $\epsilon = 1.5$ V regardless of the current drawn from the battery. However the internal resistance of a standard D cell is about 1 Ω and the internal resistance of an alkaline D cell is about 0.1 Ω, so the amount of energy dissipated internally is different. Suppose that you have a multi-speed winch that is 50% efficient, and that you are trying to lift a mass of 60 kg. The winch acts as load with a variable resistance R_L at different speeds.

(a) Suppose the winch is set to super-slow speed. Then the load resistance is much greater than the internal resistance and you can assume that there is no loss of energy to internal resistance. How high can each battery lift the mass before the battery uses all of its energy?

(b) For each battery, what should the resistance of the winch be set at in order to have the battery lift the mass at the fastest rate?

(c) What is the fastest rate in m/sec that each battery can lift the mass?

(d) At this fastest lift rate, what is the maximum height that you can lift the mass before the battery dies?

(e) How long does it take to reach this height?

Problem 5: Purcell 4.16

In the circuit below, if R_0 is given, what value must R_1 have in order that the input resistance between the terminals shall be equal to R_0?
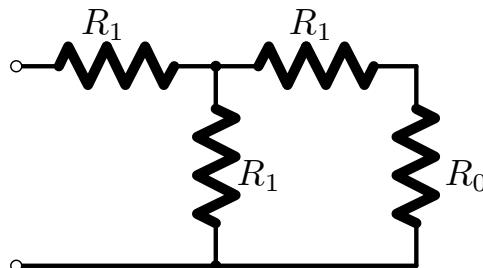
Problem 6: Purcell 4.19: Design an ohmmeter

You own a micro-ammeter that reads $50\,\mu A$ at full scale deflection, and the coil in the meter movement has a resistance of 20 Ω. By adding two resistors, R_1 and R_2, and a 1.5 V battery you can convert this into an ohmmeter. When the two outgoing leads of this ohmmeter are connected together, the meter should register 0 ohms by giving full-scale deflection. When the leads are connected across an unknown resistance R_u, the deflection will indicate the resistance value if the scale is appropriately marked. In particular, we want half-scale deflection to indicate 15 Ω. What values of R_1 and R_2, are required, how should the connections be made, and where on the ohm scale will the marks be (with reference to the old micro-ammeter calibration) for 5 Ω and 50 Ω? Are there several different combinations that can solve this problem?

Problem 7: Composite Conducting Medium

An infinite conducting medium has two regions with conductivity $\sigma_{c,1}$ and $\sigma_{c,2}$, separated by a plane interface. In region 1 a uniform current density $\vec{J_1}$ flows up to the interface, making at an angle θ_1 with the interface. $\vec{J_2}$ flows away from the interface and makes an angle θ_2 with the interface. Assume that the charge density σ is not changing in time, and that the current is steady across the medium.
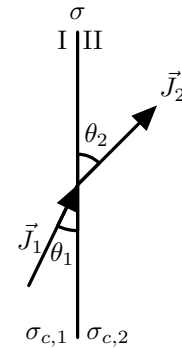
(a) Find the direction and magnitude of the current density $\vec{J_2}$ in region 2.

(b) Find the surface charge density σ on the interface.

Problem 8: Multiple Loop Circuits

Consider the following circuit consisting of a voltage source V_0 , three resistors with resistances R_1, R_2, and R_3, and a capacitor with capacitance C connected together as shown in the Figure below.
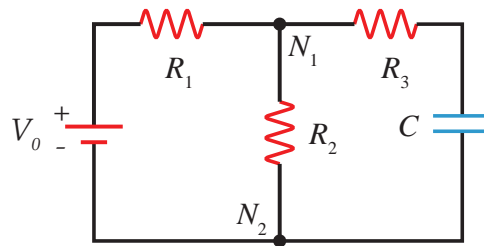
At $t = 0$, the capacitor is uncharged.
HINT: You may want to solve (a) and (b) last.

(a) Find a differential equation for the charge on the capacitor and try to solve it.

(b) What is the time constant for this circuit?

Try to answer the following questions conceptually then check your answer against your solutions.

(c) At $t = 0$, what is the voltage difference across the capacitor?

(d) What is the current that flows in the branch containing the voltage source at $t = 0$.

(e) When a long time has passed after the switch was closed, what is the current that flows in the branch of the circuit that included the capacitor?

(f) When a long time has passed after the switch was closed (based on your result form part e), find the current that flows from the voltage source?