18.S996: Category Theory for Scientists (David Spivak), Problem Set 1

2.1.1.2

The subsets are $\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, and $\{1,2,3\}$.

2.1.2.2

Since each PR cell maps to a unique RG cell, this defines a function $PR \to RG$, but not $RG \to PR$. I would guess that some connections are one-to-one, some are many-to-one, and some are many-to-many.

2.1.2.3

We have $\mathrm{im}(f) = \{y_1, y_2, y_4\}$.

2.1.2.4

In general, $|\mathrm{Hom}_{\mathrm{Set}}(A,B)| = |B|^{|A|}$. In this case, we have $|\mathrm{Hom}_{\mathrm{Set}}(A,B)| = 2^5 = 32$, and $\mathrm{Hom}_{\mathrm{Set}}(B,A) = 5^2 = 25$.

2.1.2.5

The set A is the terminal element, $\{\cdot\}$, and B is the initial element, $\emptyset$.

2.1.2.9

There are $n!$ automorphisms of a set with n elements. Note that $0! = 1$.

2.1.2.11

The sets A and B are not canonically isomorphic (unless I missed something), so there is no single induced function $A \to \{1,2,3,4,5\}$, but the induced function is unique up to a unique automorphism of A. All induced functions send some element of A to 1, some other element to 3, and some other element to 4.

2.1.2.12

Only the initial set ($\{\cdot\}$) satisfies this property; the only solution for x to $\forall y, x^y = x$ (where 0^0 is defined conventionally to be equal to one) is $x = 1$.

2.1.2.13

We have $f(4) = c$ and $s = (1, 4, 9, 16, 25, 36, 49)$.

2.1.2.15

We have $|\{5, 6, 7\} = 3$, $|\mathbb{N}| = \omega$ (or, in your notation, $|\mathbb{N}| \geq \infty$), and $|\{n \in \mathbb{N} \,|n \leq 5\}| = 6$ because there are 6 numbers between 0 and 5 (inclusive on both ends).