

Expected return =  $E(X) = \sum \pi_i x_i$ . Variance =  $\text{var}(x) = E[(x - u)^2]$

Standard Deviation =  $\sqrt{\text{var}(x)}$ . Skewness =  $1/\sigma^3 (E(x - u)^3)$ . Kurtosis =  $1/\sigma^4 (E(x - u)^4)$

Covariance:  $\text{sigma}_{XY} = E(XY) - E(X) * E(Y) = \sum \pi_i x_i y_i - (\sum \pi_i x_i)(\sum \pi_i y_i)$

Correlation coefficient  $\rho = \text{corr}(x, y) = \text{sigma}_{XY} / (\sigma_X * \sigma_Y)$  ( $-1 < \rho < 1$ )

$\text{Var}[aX + bY] = a^2 \text{var}[x] + b^2 \text{var}[y] + 2ab \text{Cov}[x, y]$

$U = u_1 w + u_2 (1 - w)$ .  $\text{Sigma}(2) = w(2) * \text{sigma}_1(2) + (1 - w)(2) * \text{sigma}_2(2) + 2w(1 - w) * \text{sigma}_{12}$

$W = ((1/r) * (u_1 - u_2) + (\text{sigma}_2(2) - \text{sigma}_1(2)) / (\text{sigma}_1(2) + \text{sigma}_2(2) + 2 * \text{sigma}_{12}))$

The minimum variance portfolio:  $W = (\text{sigma}_2(2) - \text{sigma}_{12}) / (\text{sigma}_1(2) + \text{sigma}_2(2) - 2 * \text{sigma}_{12})$

How to calculate the discount rate: Discount rate  $K = \text{RiskPremium} + r_f$

Arbitrage Pricing. No arbitrage price means that speculating future price is the same with speculating spot price. Law of One price: Put Call Parity:  $C = P + S - K/(1 + r)$  Price =  $C/(1 + k) + C/(1 + k)^2 + C/(1 + k)^3 + \dots = C/K$  The growth rate is  $g$ , Price =  $C * (1 + g)/(1 + k) + C * (1 + g)^2 / (1 + k)^2 + \dots$  Price =  $C/k - g$

Factor Investing:  $P/E = (1 - \text{plowback ratio}) / (k - g)$

Efficient market versus behavioral finance:

1. Weak form— You can't beat the market using only past prices— So prices are a random walk along a long-term trend 2. Semi-strong form— Can't beat the market using all publicly available information 3. Strong form — Can't beat the market using all information, including private information

Behavioral Finance 1 prices are sometimes wrong. 2 there are limits of arbitrage

Systematic errors of judgement: 1 Over-confidence 2 loss aversion 3 extrapolation from the past 4 limited attention

Limit of arbitrage: 1 arbitrage risk 2 short-sale constraints (the limit of short the market) 3 money manager incentives

Sharp ratio:  $U_p - r_f / \text{standard deviation of the portfolio}$  Treynor ratio:  $U_p - r_f / \beta_p$  (market beta)

Jensen's (CAPM) alpha = regression alpha Information Ratio =  $\alpha / \text{non-systematic risk} = \alpha / S.D(\epsilon_p)$

How to use several ratios to interpret the income

If the portfolio represents the entire investment for an individual: Use the Sharpe ratio, because investors dislike total risk

If this portfolio is part of an individual's larger holdings, use factor alphas or the Treynor ratio because we only care about the systematic risk The idiosyncratic part can be diversified away.

If we're compare fund managers who are trying to beat the same benchmark, use the information ratio: Which manager can beat the benchmark the most consistently?

Factor investment: CAPM:  $E(R_i - r_f) = \beta_i * E(R_M - r_f)$  Discount rate = Risk Premium +  $r_f$  (real rate plus expected inflation)

$B = \text{covariance}(r_i, r_M) / \text{var}(r_M)$ . This is the correlation of stock I with market.

Factors of investment: 1 value R(HML): Book/Market R(SMB) use Market Cap, Momentum: Low volatility Quality

Why Factor 1 pick portfolios that beat the market 2 arbitrage 3 Evaluate fund returns

How Calculate these factors, just pick out top 30% and the low 30% and then compare the difference of these factors

Arbitrage pricing theory: Morningstar diagrams: large Medium Small Value blend growth

Factor Investing:

This is a in-between view between passive investing and speculative investing. For CAPM, the common factor is market return.

The factor loading is just the asset's beta.

Fama-French three-factor model .  $R_{smb}$  and  $R_{hml}$ , these are two factors that need is factors for the market

$R_{hml}$  bookToMarket factor. Size Market Cap Momentum: stocks return based on previous year's returns.

Who beats the market: Mutual Fund: slightly beat the market before the fee, but underperform after the fee. However, there are some managers who add the value, but there is little persistence. Endowments: very picky about hedge fund manager.

Pension funds: underperformed because Political interference, Corruption and Low incentives of pension manager Hedge funds: some manager can produce the consistent alpha Private equity: Leveraged buyout, Venture capital High frequency traders.

## Question

$C(S,K)=P(S,K)+S \cdot K/(1+r)-D$ . put-call parity for a dividend paying stock

2 American put should be more expensive compared with its European counterparts. While the call option is the same  
Which of the following stock-picking strategies do you expect to yield the worst returns?

Momentum is stronger at shorter frequencies, long term reversals happen at 3-5 year frequencies So the winners for the last 3-5 years are more likely become loser

Stock price:  $DA = 100 - P$ . For group B, it is  $DB = 70 - P$ . For group C, it is  $DC = 50 - P$ . Market clearing requires  $1/3 [(100-P) + (70-P) + (50-P)] = 1$ , or  $P = 217/3 = 72$ . Suppose that investor C cannot short C will not short the stock Market clearing requires  $1/3 [(100-P) + (70-P)] = 1$ , or  $P = 167/2 = 84$ .

$E[r_A * r_M] = .01*(-.1)*(-.16) + .13*(-.01)*(-.01) + 0.86*.03*.15 = .004043$   $E[r_M] = .01*(-.1) + .13*(-.01) + .86*.03 = .0235$   
 $E[r_A] = .01*(-.16) + .13*(-.01) + .86*.15 = .1261$   $Cov(r_A, r_M) = E[r_A * r_M] - E[r_A] * E[r_M] = .004043 - .1261 * .0235 = .00107965$   
 $E[r_M^2] = .01*(.12) + .13*(.012) + .86*(.032) = .000887$   $Var(r_M) = E[r_M^2] - E[r_M]^2 = .000887 - (.0235)^2 = .00033475$

The beta of stock A is  $Cov(r_A, r_M) / Var(r_M) = 3.225$ .

A has beta=2, asset B has beta=4. Portfolio C is formed by combining with equal weights asset A and asset B The beta of asset C is a weighted average:  $\beta_C = 3$

Expected return on the market portfolio is 10% and the risk-free rate is 4%. Which asset is a better buy? For A, using CAPM, fair rate is  $4 + 2(10-4) = 16\%$ ,  $\alpha = 0.01$ . Buy with the alpha

With arbitrage opportunity: If you want to take advantage of an arbitrage opportunity. Use CAMP to judge whether the stocks have been priced fairly

A risky bond+ A credit default swap:  $1000/(C+200) = 1.05$

The variance of the portfolio is  $(0.5)^2 \square 10 + (0.5)^2 \square 20 + 2(5)(0.5)^2 = 10$ .  $r_P - r_f = 0.015 + 1.2(r_M - r_f) + e1 r_P - r_f = 0.009 + 1.1(r_M - r_f) - 0.2 SMB + 1.2 HML + e2$  Jensen's alpha is 0.015 or 1.5%. The Treynor ratio is calculated as follows:  $E(r_P - r_f) = 0.015 + 1.2 E(r_M - r_f) = 0.099$  Treynor =  $E(r_P - r_f) / \beta_{CAPM} = 0.099 / 1.2 = 0.0825$

Large cap, since the coefficient on SMB is  $< 0$ . Value, since the coefficient on HML is  $> 0$ .

minimum variance portfolio  $d(v)/dw = 0$ .

(a) Prospect Theory argues that investors have loss aversion: they suffer more disutility from losses than utility from gains. This implies that investors are more likely to sell "winners" rather than "losers".

The money in the stock if the client can just tolerate  $w^* \text{ stand } < \text{ the maximum deviation}$ . Then get the  $w$  and  $(1 - w)$  in money market