

NBA 5420 – Investments and Portfolio Management
Problem Set 2 – Portfolio Optimization

Problem 1:

Stock A's expected return and standard deviation are $E[R_A] = 5\%$ and $\sigma_A = 15\%$, while stock B's expected return and standard deviation are $E[R_B] = 7\%$ and $\sigma_B = 21\%$.

- a. Determine the expected return and standard deviation of the return on a portfolio with weights $\omega_A = .35$ and $\omega_B = .65$ for the following alternative values of correlation between A and B: $\rho_{AB} = 0.6$ and $\rho_{AB} = -0.4$.
- b. Assume now that $\rho_{AB} = -1.0$ and find the portfolio p of stocks A and B that has no risk (i.e. such that $\sigma_p = 0$). Can you do the same when $\rho_{AB} = 1.0$? If not, why? If so, find that portfolio.
- c. Finally, assume that $\rho_{AB} = 0$. Find the standard deviations of portfolios with the following expected returns: 4%, 5%, 6%, 7%, 8%, 9%, 10%. Plot the expected return—standard deviation pairs on a graph (with the standard deviations on the horizontal axis, and the expected returns on the vertical axis).
- d. Consider three stocks, A, B and C. Suppose that both $\rho_{AB} = 0$ and $\rho_{BC} = 0$. Does that imply $\rho_{AC} = 0$? Show that it is, or find a counter-example.

Problem 2:

The following table shows data on two risky assets, and a risk-free asset:

Asset	Expected Return	Standard Deviation
A	13%	40%
B	7%	10%
Risk-free	5%	0%

The returns of assets A and B are uncorrelated.

- a. Investor X does not mind taking a lot of risk, and reasons as follows: “The risk-free asset offers a very low return (5%). Asset B is better (7% expected return),

with some risk (10%), but Asset A is better still (13% expected return), with high risk (40%). Since risk doesn't bother me that much, the optimal strategy for me is to forget about Asset B and the risk-free asset, and invest only in Asset A." What do you think about this reasoning? Can you suggest a better strategy? Explain briefly how Investor X should choose her optimal portfolio. (A diagram may be helpful.)

- b.** Is the following statement TRUE or FALSE? Explain briefly, relating your answer to question C.1 above. (A correct answer with no explanation will receive little credit.)

"Investor Y is much more risk-averse than X. Therefore, his optimal portfolio will include less of asset A, and more of Asset B, than the optimal portfolio for Investor X."

Problem 3:

Suppose that you have n risky assets at your disposal. All n assets have the same expected return μ and the same standard deviation σ and are pairwise uncorrelated. Which of the following statement is true, and why?

- Any two portfolios made up with these assets will have the same expected returns and variances.
- Since the assets have the same expected return, variance and covariance, they also all have the same correlation with the market portfolio.
- Diversification is pointless in this case because since the assets have the same variance most of the risk is systematic.
- Diversification can reduce the risk really well in this case because most of the risk is idiosyncratic.

Problem 4:

Consider an economy with 2 stocks. Let stock 1 have a mean return $\mu_1=6$ and variance of $\sigma_1^2=10$. Stock 2 has a mean return of $\mu_2=8$ and a variance of $\sigma_2^2=20$. The covariance is $\sigma_{12}=5$.

- What is the variance of the minimum variance portfolio?
- If an investor has the utility function $U = \mu - \frac{1}{2} * \sigma^2$ and is only able to invest in those two stocks (and no riskless asset), what is the optimal portfolio?