NetID: hw544 Name: Hanwen Wang

Problem 1:

a. If the futures price on the S&P500 is above the current value of the index, investors expect the index to rise. True or false?

True, when futures prices is above the price.

For example now the S&P500 is 100, and future price is, say 110. In the future, if the price of S&P500 is above 110, the buyer of the contract will get money. While if the price is less than 100, the seller of the contract will earn money.

So from the general perspective, the expectation of S&P500 would be 110, so the price will be expected to increase.

b. Explain why a higher dividend rate makes a futures contract less valuable relative to the spot price of the underlying asset, other things equal.

Answer: compared with doing transaction now, the future contract will do the transaction in the future.

If the deal is done now, the dividend will belong to the buyer. For example if the dividend rate is 5%, and the price of stock is 100, say this is a 3-year contract. So if the buyer buy this stock now, he will get 100 * 5% * 3 = 15, the seller will get 100, which is the price of the stock. However if we do the future contract, then the dividend will belong to the sellers. so the seller will get 15 for holding the stock before the transaction period. To compensate the loss of the buyer, the future price may be set as 85 instead of 100 even though the exception of the price of the stock in the future can be 100. So with higher dividend rate, say r is larger, the future contract is less valuable.

Problem 2:

Using put-call parity:

Show how one can replicate a one-year pure discount Treasury bill with a face value of $100 using a share of stock, a put and a call.

we can

Suppose that S = $100, P = $15, and C = $35 (exercise prices = $100). What must be the one-year interest rate?

Show that if the one-year risk-free interest rate is lower than in your answer to part (b), there would be an arbitrage opportunity. (Hint: The price of the zero coupon bond would be too high.)

SOLUTION:We can buy a stock first, and then buy a put of the stock at prices of 100, and then sell a call at prices of 100. So in one year, when the price of the stock is above 100, then we just sell the stock from the call we sell and get 100. If the price is less than 100, we can sell the stock at price of 100 from the put well buy. So the price we pay the price of the stock we buy, and the money we get is 100;

b Suppose that S = $100, P = $15, and C = $35 (exercise prices = $100). What must be the one-year interest rate?

$$C - P = D(F - K)$$

$$C - P = S - D \cdot K.$$

20 = 100 - D*100

D = 0.8   1/(1 + r) = 0.8   r = 0.25

So the interest rate would be 0.25

c Show that if the one-year risk-free interest rate is lower than in your answer to part (b), there would be an arbitrage opportunity. (Hint: The price of the zero coupon bond would be too high.)

So if the interest if less than the answer in the question2, I can first borrow S from bank. dollor to buy the stock, then cause the interest rate is slower, so C - P + D*K > S.

The cash flow now can be larger then the price we paid now. So there is opportunity to arbitrage.

Problem 3:

There are two states of the world, up and down, to be realized in one year. A bond issued by firm XYZ pays out $1,000 in the up state; in the down state, it defaults and only pays out $200. A credit default swap (CDS) on XYZ is a derivative that makes you whole in the event of default: it pays out $0 in the up state, and $800 in the down state. The risk free rate is r_f= 5%. Suppose that the probability of up is 0.95, and the probability of down is 0.05. The CDS costs $20 to buy today. Using no arbitrage, what is the value of the XYZ bond today?

Assume that the price of the bound is x.

(1000/(1 + r)-20-x)*0.95 + (800/(1 + r) - 20 - x)*0.05 = (1000/(1 + r) - x)*0.95 + (0/(1 + r) - x)*0.05

If we buy the bond and CDS at the same time,

The pay off for the up and down state are all 1000. so if we can buy a bond and CDS we can get the result of 1000 at the end.

(x + 20)/(1 * r) = 1000

x = 932.4

The price for the bond should be 932.4, otherwise there could be arbitrage opportunity.