

The Cost of Capital for Alternative Investments

JAKUB W. JUREK and ERIK STAFFORD*

ABSTRACT

Traditional risk factor models indicate that hedge funds capture pre-fee alphas of 6% to 10% per annum over the period from 1996 to 2012. At the same time, the hedge fund return series is not reliably distinguishable from the returns of mechanical S&P 500 put-writing strategies. We show that the high excess returns to hedge funds and put-writing are consistent with an equilibrium in which a small subset of investors specialize in bearing downside market risks. Required rates of return in such an equilibrium can dramatically exceed those suggested by traditional models, affecting inference about the attractiveness of these investments.

LINEAR FACTOR REGRESSIONS (e.g., Capital Asset Pricing Model (CAPM), Fama–French three-factor model, Fung–Hsieh nine-factor model, and conditional variations thereof) indicate that hedge funds deliver statistically significant alphas (Sharpe (1964), Lintner (1965), Fama and French (1993), Agarwal and Naik (2004), Fung and Hsieh (2004), Hasanhodzic and Lo (2007)). Over the period from January 1996 to June 2012, pre-fee alpha estimates for diversified hedge fund *indices* range from 6% to 10% per annum, and thus, even after deducting fees, investors appear to earn large abnormal returns relative to commonly used risk models. These estimates indicate a degree of market inefficiency, that is dramatically different from other areas of investment management (Fama and French (2010)), and suggest that hedge fund returns cannot be replicated by portfolios combining traditional risk factors. Another interpretation of these results is that the proposed risk models fail to identify, or accurately measure, an important dimension of risk that hedge fund investors specialize in bearing. In this paper, we explore this explanation, focusing on downside market risks and their implications for cost of capital computations when the asset market equilibrium may involve investor specialization.

*Jurek is at Bendheim Center for Finance, Princeton University and NBER. Stafford is at Harvard Business School. We thank Joshua Coval, Ken French, Samuel Hanson, Campbell Harvey (Editor), Jonathan Lewellen, Andrew Lo (discussant), Burton Malkiel, Robert Merton, Gideon Ozik (discussant), André Perold, David Sraer, Jeremy Stein, Marti Subrahmanyam (discussant), Jules van Binsbergen (discussant), Russell Wermers (discussant), and seminar participants at Dartmouth College, Harvard Business School, Imperial College, Bocconi University, USI Lugano, Princeton University, Spring 2013 Q-Group Seminar, 2013 CEAR Workshop on Institutional Investors, the 2012 NBER Asset Pricing Summer Institute, the 2011 NYU Five-Star Conference, the 4th NYSE Liffe Hedge Fund Research Conference, and the BYU Red Rock Finance Conference for helpful comments and discussions.

DOI: 10.1111/jofi.12269

Merton (1987) explores a simple one-factor equilibrium model in which agents only trade subsets of assets, and demonstrates that linear factor pricing fails in this setting. In particular, assets that are borne by specialized investors appear to earn positive abnormal returns relative to the market portfolio. Equivalently, the equilibrium required rate of the return on these “specialized investments” exceeds the required rate of return implicit in linear factor regressions. We argue that a similar type of equilibrium may help rationalize the returns to alternative investments (and also index put-writing strategies). In practice, while alternatives comprise a modest 2% share of the global wealth portfolio, most investors hold none of these investments, leaving a few investors with large allocations relative to the aggregate supply.¹ For example, as of June 2010, 40% of the aggregated Ivy League endowment assets were allocated to nontraditional assets (Lerner, Schoar, and Wang (2008)). From this perspective, the high excess returns of alternatives may simply reflect fair compensation demanded by specialized investors, rather than unearned returns, or “alpha.”

We focus on the possibility that, in aggregate, hedge fund investors specialize in bearing downside market risks. These risks concentrate losses in highly adverse economic states, and are known to receive high equilibrium risk compensation. Importantly, the additional compensation demanded by investors that specialize in bearing these risks is likely to be large relative to that prevailing in the absence of segmentation, as these assets magnify the negative skewness of aggregate (market) shocks. While evidence of nonlinear systematic risk exposures resembling those of index put-writing is provided by Mitchell and Pulvino (2001) for risk arbitrage and Agarwal and Naik (2004) for a large number of equity-oriented strategies, the literature—aside from Lo (2001)—has been comparatively silent on *nonlinear* replicating strategies. We fill this gap by constructing the returns to a range of S&P 500 equity index option writing strategies designed to satisfy exchange margin requirements, as emphasized by Santa-Clara and Saretto (2009).² Specifically, we contrast the hedge fund index returns with two put-writing portfolios with different downside risk exposures, as measured by how far the put option is out-of-the-money and how much leverage is applied to the portfolio. Each of these strategies provides an unbiased proxy for *pre-fee* hedge fund returns, delivering a zero

¹ As of the end of 2010, the total assets under management held by hedge funds stood at roughly \$2 trillion (source: HFRI), in comparison to a combined global equity market capitalization of \$57 trillion (source: World Federation of Exchanges) and a combined global bond market capitalization of \$54 trillion, excluding the value of government bonds (source: TheCityUK, “Bond Markets 2011”).

² Our methodology for constructing put-writing strategy returns improves on several nonlinear risk factors proposed in the hedge fund literature. For example, we consider a wide range of option moneyness levels and leverage magnitudes, whereas Agarwal and Naik (2004) only use options that are 1% out-of-the-money. Fung and Hsieh (2004) construct factors based on the theoretical returns to lookback straddle portfolios, that are designed to capture *long* exposure to volatility. Instead, our focus is on strategies that are short volatility. The extreme volatility of the lookback straddle factors also suggests that—in order to ensure feasibility—short exposures would place severe margin requirements on the investor.

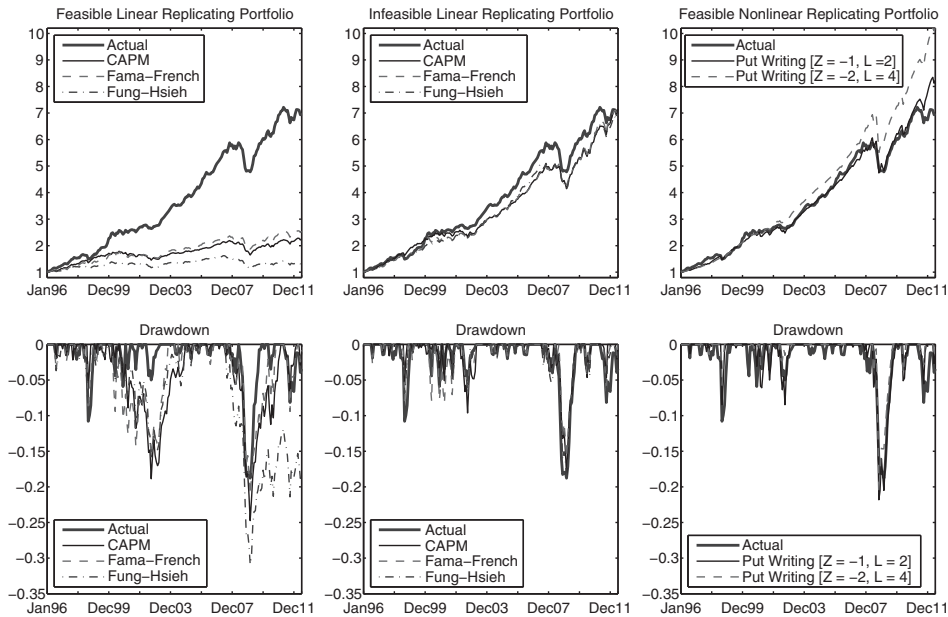


Figure 1. Replicating the risks and returns of the HFRI Fund Weighted Composite Index. The top panels plot the cumulative value of \$1 invested in the HFRI Fund Weighted Composite Index (pre-fees; “Actual”), along with various replicating strategies. The left-hand panel shows the cumulative return based on the fitted values from three common factor models (CAPM, Fama-French/Carhart, Fung-Hsieh) exclusive of the estimated intercept (feasible linear replication). The middle panel repeats the plot based on the fitted factor models, but returns are cumulated inclusive of the estimated intercept (infeasible linear replication). The right-hand panel plots the returns to the two put-writing strategies (feasible nonlinear replication). Relative to the $[Z = -1, L = 2]$ put-writing strategy, the $[Z = -2, L = 4]$ strategy applies a higher amount of leverage to options that are written further out of the money. The bottom panels plot the corresponding monthly drawdown series for the hedge fund index and the replicating strategies.

intercept and unit slope coefficient when the index excess returns are regressed onto the strategy excess returns.

Figure 1 plots the value of \$1 invested in the aggregate hedge fund index (pre-fees), along with various replicating strategies. The left-hand panel shows the cumulative return based on the fitted values from three common factor models exclusive of the estimated intercept (feasible linear replications), the middle panel repeats the plot but inclusive of the estimated intercept (infeasible linear replication), and the right-hand panel plots the returns to the two put-writing strategies (feasible nonlinear replication). The performance of the aggregate hedge fund index is impressive, accumulating wealth much more quickly than the risk-matched common factors predict would be fair. Popular common factor models explain most of the time series variation, but miss most of the mean, identifying this as alpha. The graph also shows that the common factors beyond the market factor explain little of the overall pattern, so much of our analysis emphasizes the market factor. On the other hand, simple

put-writing portfolios that explicitly bear downside market risks appear to track the economically important time series variation well, and also explain the mean return, suggesting that there is no alpha in the pre-fee returns relative to these passive benchmarks.

The empirical analysis of downside risk exposure in hedge fund returns is complicated by the presence of unconditional (Asness, Krail, and Liew (2001), Getmansky, Lo, and Makarov (2004)) and conditional (Bollen and Pool (2008)) return smoothing, reflecting asset illiquidity and discretion in marking portfolio net asset values (NAVs) (Cassar and Gerakos (2011), Cao et al. (2013)). These features of reported returns may effectively hide downside market risk exposure, making reported returns conform to linear risk models more than the true returns really do. For example, regressions that include both common factors and put-writing portfolios suggest that the common factors explain the time-series variation but that the put-writing factors are not reliably related to the reported returns. Yet, as we show, this conclusion hinges critically on the assumption that hedge fund portfolio returns were accurately marked in *two* key months (August 1998 and October 2008), out of 198 months in our sample. Even a modest degree of smoothing in these two months alone is sufficient to reestablish the significance of the put-writing factors and the conclusion of insignificant pre-fee alphas. These investments may thus have large downside market risk exposures, which would dramatically alter inferences about risks and required returns, especially when held in large allocations.

We evaluate the performance of the hedge fund indices and put-writing strategies from the perspective of an equilibrium with specialized investors, in the spirit of Merton (1987). To compute investor-required rates of return, we assemble a simple static portfolio selection framework that combines constant relative risk aversion (CRRA) utility preferences with a state-contingent asset payoff representation (Debreu (1959), Arrow (1964)), which specifies each asset's payoff as a function of the aggregate equity index (here, the S&P 500). In particular, we capture the downside risk exposure of a specialized investment with a portfolio of cash and equity index options, which provides a complete state-contingent description of an investable alternative to the aggregate hedge fund universe. We then determine the investor's required rate of return as a function of (a) portfolio concentration, (b) the payoff profile of the investments, and (c) the distribution of the common market factor. We show that, for a wide range of plausible parameter values, there exists a large wedge between the model-required rate of return and the one suggested by the product of an asset's market beta and the equilibrium equity risk premium. Put differently, fair pricing of downside market risk exposures in an equilibrium featuring specialized investors predicts large positive alphas in linear factor regressions onto the commonly traded risk factor (i.e., the equity index). In particular, we find that proper required rates of return that reflect the size of the allocation to these investments can vary widely, suggesting a promising path for rationalizing the high realized returns of the aggregate hedge fund index and of various index put-writing strategies within allocation sizes that are typical of many specialized end investors.

The remainder of the paper is organized as follows. Section I describes the risk profile of hedge funds. Section II presents a simple recipe for constructing feasible passive portfolios with known downside risk exposures with index put options, and empirically examines how these portfolios relate to the returns of the aggregate hedge fund indices. Section III develops a generalized asset allocation framework for computing the cost of capital for investors with large allocations to assets with downside risks. Finally, Section IV concludes the paper.

I. Properties of Hedge Fund Returns

We begin our investigation of hedge fund returns with an assessment of the risk properties of the aggregate asset class. We proxy for the performance of the hedge fund universe using two indices: the (value-weighted) Dow Jones/Credit Suisse (DJSC) Broad Hedge Fund Index, and the (equal-weighted) HFRI Fund Weighted Composite Index. These indices are not directly investable, and are thought to provide an upward biased assessment of hedge fund performance due to the presence of backfill and survivorship bias in the underlying databases (Malkiel and Saha (2005)). To the extent that these biases affect the measured risks, it is unlikely that the true risks are lower than those estimated from the realized returns over this period.

Table I reports summary statistics for the HFRI and DJCS aggregates computed using monthly returns, in excess of the one-month U.S. Treasury bill return, from January 1996 to June 2012 (198 months). Since our goal is to characterize compensation for bearing risk in capital markets rather than investor returns per se, we report summary statistics for *pre-fee* index returns. To estimate pre-fee returns, we treat the observed *net-of-fee* time series as if it represented the return of a representative fund that was at its high watermark throughout the sample, and charged a 2% flat fee and a 10% incentive fee, both payable monthly.³ The difference between the mean pre-fee and net-of-fee returns represents an approximation of the *all-in* investor fee. For comparison, using cross-sectional data from the TASS database for the period 1995 to 2009, Ibbotson, Chen, and Zhu (2010) find that the average fund collected an all-in annual fee of 3.43%. French (2008) reports an average total fee of 4.26% for U.S. equity-related hedge funds in the HFRI database using data from 1996 through 2007. We find that our simple estimates of all-in fees of 3.8% for the HFRI and DJCS coincide well with these values.

Table I also summarizes the excess returns to a variety of common factors used to describe the risks of risky investments. In particular, we focus on the factors suggested by the CAPM, the Fama–French three factor model (Fama

³ In practice most funds impose a “2-and-20” compensation scheme, comprised of a 2% flat fee and a 20% incentive allocation, subject to a high watermark provision. Our compensation scheme can therefore loosely be interpreted as describing the scenario where half of the funds in the universe are at their high watermark at each point in time. Our computation is also likely conservative in that the incentive component represents an option on the pre-fee return of a portfolio of funds, rather than a portfolio of options on the pre-fee returns of the underlying funds.

Table I
Summary Statistics

This table reports the excess returns of hedge fund indices, common risk factors, as well as S&P 500 put-writing strategies between January 1996 and June 2012 ($N = 198$ months). The HFRI Fund Weighted Composite Index (HFRI) and the Dow Jones/Credit Suisse Broad Hedge Fund Index (DJCS) series are *pre-fee* hedge fund index return series based on data from Hedge Fund Research Inc. and Dow Jones/Credit Suisse, respectively. To compute pre-fee returns, we treat the observed after-fee time series as if it represented the return of a representative fund that was at its high watermark throughout the sample, and charged a 2% flat fee and a 10% incentive fee, both payable monthly. The set of common risk factors includes the Fama-French (1993) factors, the momentum factor of Carhart (1997), and the Fung-Hsieh (2002) factors. The two S&P 500 put-writing strategies, $[Z, L]$, differ in the distance of the option strike price relative to the spot price (Z), and the amount of leverage applied at initiation (L). Means, volatilities, CAPM alphas ($\hat{\alpha}$), and Sharpe ratios (SR) are reported in annualized terms. Skewness and kurtosis estimates are based on monthly excess returns. JB and p_{JB} report the value of the Jarque-Bera test statistic for normality, and its associated p -value based on a finite-sample distribution obtained by Monte Carlo. CAPM $\hat{\alpha}$ and $\hat{\beta}$ report the intercept (annualized) and slope coefficient from a regression of the monthly excess return of each asset onto the monthly excess return of the market (S&P 500). AR(1) reports the value of the first-order return autocorrelation coefficient, and its associated t -statistic. Drawdown measures the largest peak-to-trough return loss for each strategy; the table reports the minimum drawdown over the full sample (Min) and in 1998 and 2008.

Asset	Mean	Vol.	Skew	Kurt.	JB	p_{JB}	SR	CAPM		AR(1)		Drawdown		
								$\hat{\alpha}$	$\hat{\beta}$	Coeff	t -stat	Min	1998	2008
HFRI	9.3%	7.9%	-0.46	4.86	35.6	0.00	1.18	7.3%	0.37	0.25	3.65	-18.8%	-10.8%	-18.3%
DJCS	9.6%	8.0%	-0.08	5.47	50.7	0.00	1.20	8.0%	0.29	0.16	2.26	-18.8%	-13.4%	-18.8%
HFRI (after-fee)	5.5%	7.4%	-0.64	5.27	56.2	0.00	0.74	3.6%	0.35	0.26	3.73	-21.4%	-11.4%	-20.5%
DJCS (after-fee)	5.8%	7.5%	-0.30	5.68	62.1	0.00	0.78	4.4%	0.27	0.17	2.44	-19.7%	-13.8%	-19.7%
HFRI (un-smoothed)	9.3%	9.5%	-1.99	15.43	1,406.8	0.00	0.98	7.0%	0.42	0.06	0.89	-22.2%	-19.1%	-22.2%
DJCS (un-smoothed)	9.6%	9.0%	-1.55	13.01	905.4	0.00	1.06	7.7%	0.34	0.01	0.20	-20.9%	-14.8%	-20.9%

(Continued)

Table I—Continued

Asset	Mean	Vol.	Skew	Kurt.	JB	p_{JB}	SR	CAPM		AR(1)		Drawdown		
								$\hat{\alpha}$	$\hat{\beta}$	Coeff	t -stat	Min	1998	2008
HFRI (un-smoothed; logs)	9.4%	9.4%	-1.75	14.12	1,122.3	0.00	1.00	7.2%	0.42	0.05	0.75	-21.8%	-18.4%	-21.8%
DJCS (un-smoothed; logs)	9.6%	8.9%	-1.41	12.15	756.7	0.00	1.08	7.9%	0.33	0.01	0.09	-20.6%	-14.2%	-20.6%
MKT-RF	5.6%	17.0%	-0.68	3.88	21.8	0.00	0.33	0.1%	1.03	0.13	1.77	-51.6%	-17.7%	-42.8%
SMB	3.3%	12.9%	0.81	10.44	478.3	0.00	0.25	2.8%	0.09	-0.08	-1.10	-27.6%	-22.7%	-8.7%
HML	2.9%	12.2%	0.04	5.46	49.9	0.00	0.23	3.5%	-0.13	0.12	1.68	-40.7%	-12.6%	-8.9%
MOM	5.5%	20.0%	-1.49	11.41	657.1	0.00	0.27	7.7%	-0.41	0.08	1.08	-57.6%	-5.2%	-8.5%
FH 1 (SP500)	5.4%	16.2%	-0.57	3.63	14.0	0.01	0.33	0.0%	1.00	0.09	1.30	-50.2%	-15.2%	-40.2%
FH 2 (SIZE)	1.0%	12.3%	0.24	7.44	164.3	0.00	0.08	0.7%	0.06	-0.13	-1.91	-35.9%	-30.1%	-8.3%
FH 3 (Treasury)	3.7%	7.3%	0.08	4.08	9.8	0.02	0.50	4.2%	-0.10	0.04	0.58	-10.0%	-1.7%	-3.5%
FH 4 (Credit)	-0.1%	5.3%	-0.15	6.52	102.8	0.00	-0.01	-1.0%	0.17	0.17	2.44	-19.8%	-4.5%	-19.8%
FH 5 (TF-BD)	-21.6%	51.8%	1.50	6.04	150.2	0.00	-0.42	-17.5%	-0.77	0.10	1.42	-99.8%	-56.3%	-98.9%
FH 6 (TF-FX)	-5.6%	63.4%	1.12	4.40	57.5	0.00	-0.09	-0.7%	-0.91	0.03	0.42	-97.1%	-59.0%	-78.9%
FH 7 (TF-COM)	-0.7%	49.1%	1.13	5.00	75.5	0.00	-0.02	2.0%	-0.52	-0.04	-0.58	-95.6%	-22.2%	-87.5%
FH 8 (TF-IR)	16.6%	99.2%	4.16	26.49	5,126.0	0.00	0.17	25.8%	-1.73	0.25	3.53	-98.0%	-49.4%	-87.3%
FH 9 (TF-STK)	-64.7%	46.6%	1.14	5.20	83.0	0.00	-1.39	-61.1%	-0.67	0.18	2.57	-100.0%	-64.3%	-100.0%
Put-Writing - Z = -1, L = 2]	10.3%	7.7%	-2.60	13.68	1,165.4	0.00	1.34	8.2%	0.39	0.13	1.88	-21.8%	-8.5%	-21.8%
Put-Writing - Z = -2, L = 4]	11.5%	6.1%	-3.67	24.66	4,312.6	0.00	1.88	10.1%	0.26	0.21	3.05	-20.6%	-5.8%	-20.6%

and French (1993)) augmented with the momentum factor (Carhart (1997)), and the Fung–Hsieh nine-factor model (Fung and Hsieh (2001, 2002, 2004)) originally developed to describe hedge fund returns. Finally, we report summary statistics for two mechanical put-writing strategies designed to function as downside risk factors. Each strategy holds one-month U.S. Treasury bills and writes short-dated, out-of-the-money S&P 500 index put options. The two strategies differ in their downside risk exposures, as measured by how far the put option is out-of-the-money and how much leverage is applied to the portfolio, and are described in detail in Section II.

The attraction of hedge funds over this time period is clear: mean returns on alternatives exceeded that of the stock market index, while incurring lower volatility. The realized pre-fee Sharpe ratios on alternatives are almost four times higher than that of the S&P 500 index. Hedge funds also perform well when evaluated on the dimension of drawdowns, which measure the magnitude of the strategy loss relative to its highest historical value (or high watermark). Both hedge fund indices have a minimum drawdown of approximately -20% , which is less than half of the -50% drawdown sustained by investors in public equity markets beginning in September 2008. Overall, the performance of hedge funds as an asset class is *not* market-neutral, and has been shown to be reminiscent of writing out-of-the-money put options on the aggregate index (Mitchell and Pulvino (2001), Agarwal and Naik (2004)).⁴ For example, hedge funds experience severe declines during extreme market events, such as the credit crisis during the fall of 2008 and the Long Term Capital Management (LTCM) crisis in August 1998. The Internet Appendix contrasts the cumulative investor returns for the HFRI and DJCS aggregates with the returns to investing in the S&P 500 and rolling over one-month T-bills (Figure IA.1).⁵

A critical issue in our analysis is the accuracy of the reported hedge fund returns. In general, the returns to hedge fund indices exhibit significant unconditional autocorrelation at the monthly horizon (Table I), reflecting the effects of stale prices and return smoothing (Asness, Krail, and Liew (2001), Getmansky, Lo, and Makarov (2004)). Cao et al. (2013) estimate that managerial discretion in marking the fund's NAV produces one-third of the autocorrelation in fund returns. There is also evidence of conditional return smoothing consistent with a higher propensity to underreport losses than gains (Bollen and Pool (2008)). The smoothing of downside returns coincident with adverse market events presents a meaningful challenge to the statistical identification of nonlinear market risk exposures similar to out-of-the-money put-writing. To investigate the effect of conditional return smoothing on the measured risks of hedge funds, we construct a hypothetical "unsmoothed" return series. This series adjusts the

⁴ Patton (2009) studies the neutrality of hedge funds with respect to market risks using correlation, tail exposures, and value-at-risk metrics. He finds that a quarter of the funds in the "market-neutral" category are significantly nonneutral at conventional significance levels, and an even greater proportion among funds in the equity hedge, equity nonhedge, event driven, and fund-of-funds categories.

⁵ The Internet Appendix is available in the online version of the article, on the *Journal of Finance* website.

reported returns in August 1998 and October 2008, when arbitrage markets were severely disrupted. In each of these episodes, we adjust returns under the assumption that funds reported a fraction, $\phi = 0.50$, of their true return in the event month, with the remainder being included in the subsequent monthly reported return, thus altering four observations in a sample covering 198 months.⁶ The unsmoothed return series preserves the arithmetic mean return of the hedge fund index, but induces more volatility, such that the Sharpe ratio declines. The measured skewness becomes more negative and kurtosis rises, as indicated by the Jarque–Bera (1980) statistic. The worst drawdown is slightly more negative and the CAPM β increases slightly, but not enough to meaningfully change the CAPM α estimates. Finally, the autocorrelation of the hedge fund returns becomes statistically indistinguishable from zero. Overall, these adjustments make the unsmoothed hedge fund returns look more similar to the downside risk factors. While there is no direct evidence that these adjustments produce a more accurate description of the true returns to broad hedge fund portfolios, they highlight the sensitivity of inferences regarding the underlying risks to a handful of influential observations.

The empirical literature on the characteristics of hedge fund returns focuses on regressions of index (and individual fund) returns onto various factors (Fung and Hsieh (2001, 2002, 2004), Agarwal and Naik (2004), Hasanhodzic and Lo (2007)). We consider several popular factor models, including the CAPM one-factor model, the Fama–French (1993)/Carhart (2004) four-factor model, and the Fung–Hsieh nine-factor model (2004), which was specifically developed to describe the risks of well-diversified hedge fund portfolios (Fung and Hsieh (2001, 2002, 2004)). Five of the Fung–Hsieh factors are based on lookback straddle returns, to mimic trend-following strategies, whose return characteristics are similar to being *long* options, or volatility (Merton (1981)). To facilitate comparison with the other factor models, we represent each of the factors in the form of equivalent zero-investment factor-mimicking portfolios. Specifically, we make the following adjustments: (a) returns on the S&P 500 and five trend-following factors are computed in excess of the return on the one-month T-bill (from Ken French’s website); (b) the bond market factor is computed as the difference between the monthly return on the 10-year Treasury bond and the return on the one-month T-bill; and (c) the credit factor is computed as the difference between the total return on the Barclays (Lehman) U.S. Credit Bond Index and the return on the 10-year Treasury bond.

Table II reports results from regressions of hedge fund excess returns onto factor-mimicking portfolio returns using quarterly data spanning January 1996 to June 2012 ($N = 66$ quarters). We focus on quarterly regressions to parsimoniously adjust for the effect of return autocorrelation observed at the monthly frequency. The common factor models tend to describe the

⁶ Table I also reports the results from applying this adjustment to the monthly *log* returns, which preserves the total reported compound return. The analysis in subsequent sections explores the sensitivity of inference to alternative values of the smoothing parameter, ϕ .

Table II
Comparison of Derivative-Based and Linear Factor Hedge Fund Replicating Models

This table reports coefficients from quarterly excess return regressions under several risk models over the period January 1996 through June 2012 ($N = 66$ quarters). The dependent variable is the quarterly excess return on the hedge fund index, computed as the difference between the quarterly pre-fee index return and the quarterly return from rolling investments in one-month T-bills, r_f . All independent variables represent zero-investment portfolios, and are obtained by compounding the corresponding monthly return series. Specifications (1) to (5) examine the excess returns of the HFRI Fund Weighted Composite Index; specifications (6) to (10) examine the excess returns of the DJCS Broad Hedge Fund Index. Specifications (1) and (6) correspond to the CAPM model with a single factor calculated as the total return on the S&P 500 minus r_f . Specifications (2) and (7) correspond to the Fama–French (1993) model (RMRF, SMB, HML) with the addition of the momentum factor (MOM) of Carhart (1997). Specifications (3) and (8) correspond to the nine-factor model proposed by Fung–Hsieh (2002). Specifications (4), (5), (9), and (10) correspond to derivative-based models with a single factor calculated as the quarterly return of the $[Z, L]$ put-writing strategy less the compounded return from rolling investments in one-month T-bills. OLS t -statistics are reported in brackets; *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Adj. R^2 is the adjusted R^2 of the linear regression. Adj. R^2 [feasible] is the goodness-of-fit computed net of the contribution of the intercept. Finally, we report the p -value of the joint test that the intercept and slope of a regression of the hedge fund index returns onto the returns of the feasible replicating portfolio are zero and one, respectively.

	HFRI (pre-fee)					DJCS (pre-fee)				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Intercept ($\times 100$)	1.76*** [5.18]	1.58*** [5.64]	2.50*** [6.25]	0.01 [0.02]	−0.80 [−1.31]	1.94*** [4.85]	1.57*** [4.13]	2.89*** [5.78]	0.66 [1.22]	0.04 [0.06]
RMRF	0.44*** [11.63]	0.40*** [12.53]	0.29*** [6.60]			0.35*** [7.82]	0.38*** [8.68]	0.21*** [3.79]		
SMB		0.22*** [3.78]					0.12 [1.49]			
HML		−0.05 [−1.13]					0.06 [1.19]			
MOM		0.06* [1.91]					0.12*** [3.05]			

(Continued)

Table II—Continued

	HFRI (pre-fee)			DJCS (pre-fee)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
SIZE			0.23*** [3.56]					0.16** [2.02]		
TSY			−0.15 [−1.32]					−0.02 [−0.15]		
CREDIT			0.24 [1.50]					0.26 [1.30]		
TF-BD			0.00 [−0.11]					−0.03** [−1.99]		
TF-FX			0.01 [1.03]					0.01 [0.79]		
TF-COM			0.00 [−0.03]					0.00 [0.20]		
TF-IR			−0.02*** [−2.96]					−0.02*** [−3.14]		
TF-STK			0.03* [1.83]					0.05*** [2.81]		
Put-Writing - $ Z =$ −1, $L = 2$				0.91*** [8.76]					0.68*** [5.94]	0.82*** [5.38]
Put-Writing - $ Z =$ −2, $L = 4$					1.09*** [7.56]					
Alpha fraction	73.7%	66.0%	104.6%	0.6%	−33.6%	79.8%	64.6%	118.7%	27.3%	1.6%
Adj. R^2	67.4%	80.3%	78.3%	53.8%	46.3%	48.0%	58.9%	60.8%	34.6%	30.1%
Adj. R^2 [feasible]	53.8%	68.8%	46.9%	53.8%	43.5%	28.6%	45.5%	11.6%	32.3%	30.1%
p -value ($H_0 : \alpha =$ 0, $\beta = 1$)	0.00	0.00	0.00	1.00	0.16	0.00	0.00	0.00	0.32	1.00

time-series variation in the monthly excess returns of the HFRI index better than those of the DJCS index, consistently achieving higher adjusted R^2 s for this index. The adjusted R^2 for the HFRI index ranges from 67% (CAPM) to 80% (Fama–French/Carhart), suggesting a good overall fit, while ranging from 48% (CAPM) to 61% (Fama–French/Carhart) for the DJCS index.

Table II shows that, relative to common factor models, hedge funds deliver *pre-fee* alphas between 6.3% and 10% per year over the sample period. The *pre-fee* alphas for both indices are highly statistically significant across all three common factor models, with t -statistics ranging from 5.2 to 6.2. Moreover, these alpha estimates are economically large, exceeding the realized mean risk premia for all of the considered common factors, with the exception of one of the Fung–Hsieh factors that has an annualized volatility of 99%. Remarkably, standard asset pricing factors never account for more than one-third of the risk premium earned by hedge funds over the sample period, and contribute no premium whatsoever in the Fung–Hsieh model. The net contribution of the nonmarket common factors (i.e., SMB, HML, MOM, etc.) in explaining the mean excess returns of the two hedge fund indices turns out to be minimal. In the Fama–French/Carhart model, only SMB has a factor exposure that differs reliably from zero, and, while two of the Fung–Hsieh factors are statistically different from zero, not all of these factors have positive risk premia and thus the net effect to the model implied risk premium beyond the market exposure is actually negative.⁷

The resulting alpha estimates imply an extreme form of capital market inefficiency relative to other areas of active investment management. The empirical evidence on risk-adjusted returns to actively managed mutual funds suggests that the average mutual fund produces *pre-fee* alphas that are statistically indistinguishable from zero (see Fama and French (2010) for a recent discussion of the mutual fund evidence). The estimated CAPM alpha for the HRFI *aggregate* hedge fund index corresponds to the 97th percentile of active mutual fund CAPM alphas.⁸ This suggests that hedge fund and mutual fund alphas are drawn from very different distributions. One possibility is that hedge fund managers are simply better than mutual fund managers and use the hedge fund structure to effectively create a separate labor market. Another possibility is that the common factor models are missing an important dimension of the systematic risks of hedge funds.

⁷ These regression results are robust to the choice of the data frequency as illustrated in Table IA.I of the Internet Appendix, which repeats the factor analysis using monthly excess returns.

⁸ The distribution of CAPM alphas is estimated from all mutual funds in the CRSP Mutual Fund database with at least 60 months of data over the sample period. Active funds are identified based on regression R^2 , with those below the 85th percentile of R^2 (0.85) being considered active funds. Lowering the R^2 threshold to 0.9 (around the 60th percentile) places the average hedge fund alpha at the 96th percentile of the distribution of active mutual fund alphas.

II. Proxying for Downside Risks with Index Put-Writing Portfolios

There are structural reasons to view the aggregate hedge fund exposure as similar to short index put option exposure. Many strategies explicitly bear risks that tend to realize when economic conditions are poor and when the stock market is performing poorly. For example, Mitchell and Pulvino (2001) document that the aggregate merger arbitrage strategy is like writing short-dated out-of-the money index put options because the underlying probability of deal failure increases as the stock market drops. Hedge fund strategies that are net long credit risk are effectively short long-dated put options on firm assets—in the spirit of Merton's (1974) structural credit risk model—such that their aggregate exposure is similar to writing long-dated index put options. Other strategies (e.g., distressed investing, leveraged buyouts) can be viewed as bets on business turnarounds at firms that have serious operating or financial problems. In the aggregate these assets are likely to perform well when purchased cheaply so long as market conditions do not get too bad. However, in a rapidly deteriorating economy these firms are likely to be the first ones to fail.

The potential for downside exposure of hedge funds is induced not only by the nature of the economic risks they are bearing, but also by the features of the institutional environment in which they operate. In particular, almost all of the strategies above make use of outside investor capital and financial leverage. Following negative price shocks, outside investors make additional capital more expensive, reducing the arbitrageur's financial slack and increasing the fund's exposure to further adverse shocks (Shleifer and Vishny (1997)). Brunnermeier and Pedersen (2008) provide a complementary perspective highlighting the fact that, in extreme circumstances, the withdrawal of funding liquidity (i.e., leverage) to arbitrageurs can interact with declines in market liquidity to produce severe asset price declines.

To explore the potential for downside risk to explain the attractive returns to hedge funds, we contrast the index returns of two put-writing portfolios with different downside risk exposures, as measured by how far the put option is out-of-the-money and how much leverage is applied to the portfolio. Each strategy writes a single short-dated put option, and is rebalanced monthly. We take seriously the problem of capital requirements (Santa-Clara and Saretto (2009)) and transaction costs to produce returns of feasible put-writing strategies, thus extending the linear hedge fund replication analysis of Hasanhodzic and Lo (2007).

A. Measuring Put-Writing Portfolio Returns

To calculate returns and characterize risks associated with put-writing portfolios, we begin by specifying feasible investment strategies. Implementing each strategy requires defining the (1) rebalancing frequency, (2) security selection rule, and (3) amount of financial leverage.

Each month from January 1996 through June 2012, we form a simple portfolio consisting of a short position in a single S&P 500 index put option,

$\mathcal{P}(K(Z), T)$, and equity capital, $\kappa_E(L)$, where $K(Z)$ is the option strike price, T is the option expiration date, and L is the leverage of the portfolio. The portfolio buys (sells) put options at the ask (bid) prevailing at the market close of the month-end trade date.⁹ If no market quotes are available for the option contract held by the agent at month-end, portfolio rebalancing is delayed until such quotes become available. The proceeds from shorting the option along with the portfolio's equity capital are invested at the risk-free rate for one month, earning $r_{f,t+1}$. This produces a terminal *accrued interest* payment of

$$AI_{t+1} = (\kappa_E(L) + \mathcal{P}_t^{bid}(K(Z), T)) \cdot (e^{r_{f,t+1}} - 1). \quad (1)$$

The monthly portfolio return, $r_{p,t+1}$, is the change in the value of the put option plus the accrued interest divided by the portfolio's equity capital:

$$r_{p,t+1} = \frac{\mathcal{P}_t^{bid}(K(Z), T) - \mathcal{P}_{t+1}^{ask}(K(Z), T) + AI_{t+1}}{\kappa_E(L)}. \quad (2)$$

We construct strategies that write options at fixed strike Z-scores. Selecting strikes on the basis of their corresponding Z-scores ensures that the systematic risk exposure of the options at the rebalancing dates is roughly constant, when measured using their Black–Scholes deltas. This contrasts with previous studies, such as Glosten and Jagannathan (1994), Coval and Shumway (2001), Bakshi and Kapadia (2003), and Agarwal and Naik (2004), that focus on strategies with fixed option moneyness (measured as the strike-to-spot ratio, K/S , or strike-to-forward ratio). Options selected by fixing moneyness have higher systematic risk, as measured by delta or market beta, when implied volatility is high, and lower risk when implied volatility is low.

In particular, we define the option strike corresponding to a Z-score, Z , as

$$K(Z) = S_t \cdot \exp(\sigma_{t+1} \cdot Z), \quad (3)$$

where S_t is the prevailing level of the S&P 500 index and σ_{t+1} is the one-month stock index implied volatility, observed at time t . We select the option whose strike is closest to but below the proposal value (3), and whose expiration date is closest to but after the end of the month. At trade initiation, the time to option expiration is roughly equal to seven weeks, since options expire on the third Friday of the following month. To measure volatility at the one-month horizon, σ_{t+1} , we use the CBOE VIX implied volatility index.

Option writing strategies require the posting of capital, or margin. The capital bears the risk of losses due to changes in the mark-to-market value of the liability. The inclusion of margin requirements plays an important role in determining the profitability of option writing strategies (Santa-Clara and Saretto (2009)), and further distinguishes our approach from papers in which

⁹ We aim to provide a conservative assessment of put-writing returns by assuming the strategy demands immediacy by executing at the opposing side of the bid-ask spread. Returns measured on the basis of the option midprice are considerably higher given the wide bid-ask spread, especially in the early part of the sample.

the option writer's capital contribution is assumed to be limited to the option price, as would be the case for a long position. In the case of put-writing strategies, the maximum loss per option contract is given by the option's strike value, K . Consequently, a put-writing strategy is fully funded or unlevered (i.e., can guarantee the terminal payoff) if and only if the portfolio's equity capital is equal to (or exceeds) the maximum loss at expiration. For European options, this requires an initial investment of unlevered asset capital, κ_A , equal to the discounted value of the exercise price less the proceeds of the option sale:

$$\kappa_A = e^{-r_{f,t+\tau}} \cdot K(Z) - \mathcal{P}_t^{bid}(K(Z), T), \quad (4)$$

where $r_{f,t+\tau}$ is the risk-free rate of interest corresponding to the time to option expiration, and is set on the basis of the nearest available maturity in the OptionMetrics zero curves. The ratio of the unlevered asset capital to the portfolio's equity capital represents the portfolio leverage, $L = \frac{\kappa_A}{\kappa_E}$. Allowable leverage magnitudes are controlled by broker and exchange limits, with values up to approximately 10 consistent with existing CBOE regulations.¹⁰ We consider two put-writing strategies, $[Z, L]$. In particular, we consider options at two strike levels, $Z \in \{-1.0, -2.0\}$, which at inception are on average between 7% ($Z = -1$) and 13% ($Z = -2.0$) below the prevailing index price. By contrast, Agarwal and Naik (2004) base their "out-of-the-money" put factor on options whose strike is 1% below the prevailing spot price. Consequently, their approach is essentially equivalent to a linear regression methodology that separately estimates the downside and upside betas, in the spirit of Glosten and Jagannathan (1994). For the $Z = -1$ strategy we apply two times leverage, $[Z = -1, L = 2]$, and for the $Z = -2$ strategy we apply four times leverage, $[Z = -2, L = 4]$.

Table I reports summary statistics for the excess returns to the two put-writing strategies over the period January 1996 to June 2012. The mean excess returns exceed 10% per annum, roughly matching those of the pre-fee hedge fund indices, with volatilities no greater than those of hedge funds. Consistent with being proxies for downside risks, these strategies exhibit significant negative skewness and excess kurtosis. Finally, the minimum drawdowns sustained by the put-writing strategies are roughly -20%, again matching the sample properties of the hedge fund returns. The cumulative performance of these feasible investment strategies contrasts with pre-fee returns to the noninvestable HFRI Fund-Weighted Composite in the right-hand panel of Figure 1.

¹⁰ The CBOE requires that writers of uncovered (i.e., unhedged) puts deposit/maintain 100% of the option proceeds plus 15% of the aggregate contract value (current index level) minus the amount by which the option is out-of-the-money, if any, subject to a minimum of [...] option proceeds plus 10% of the aggregate exercise amount:

$$\min \kappa_E^{CBOE} = \mathcal{P}_t^{bid}(K, S, T; t) + \max(0.10 \cdot K, 0.15 \cdot S - \max(0, S - K)).$$

A.1. An Example

To illustrate the portfolio construction mechanics, consider the first portfolio rebalancing trade of the $[Z = -1, L = 2]$ strategy. The initial positions are established at the closing prices on January 31, 1996, and are held until the last business day of the following month (February 29, 1996), when the portfolio is rebalanced. At the inception of the trade the closing level of the S&P 500 index was 636.02, and the implied volatility index (VIX) was at 12.53%. Together these values pin down a proposal strike price, $K(Z) = 613.95$, for the option to be written via (3). We then select an option maturing *after* the next rebalance date whose strike is closest from below to the proposal value, $K(Z)$. In this case, the selected option is the index put with a strike of 610 maturing on March 16, 1996. The $[Z = -1, L = 2]$ strategy writes the put, bringing in a premium of \$2.3750, which corresponds to the option's *bid* price at the market close. The required asset capital, κ_A , for that option is \$603.56, and, since the investor deploys leverage equal to $L = 2$, he posts capital of $\kappa_E = \$301.78$. The investor's capital is invested at the risk-free rate, with the positions held until February 29, 1996. The risk-free rates corresponding to the trade roll date (29 days) and maturity (45 days) are $r_{f,t+1} = 5.50\%$ and $r_{f,t+\tau} = 5.43\%$, respectively, and are obtained from the OptionMetrics zero-coupon yield curves. On the trade roll date, the option position is closed by repurchasing the index put at the close-of-business *ask* price of \$1.8750. This generates a profit of \$0.50 on the option and \$1.3150 of accrued interest, representing a 60 basis point return on investor capital. Finally, a new strike proposal value that reflects the prevailing market parameters is computed and the entire procedure repeats.

A.2. Comparison to Capital Decimation Partners

Lo (2001) and Hasanhodzic and Lo (2007) examine the returns to bearing “tail risk” using a related, naked put-writing strategy employed by a fictitious fund called Capital Decimation Partners (CDP). The strategy involves “shorting out-of-the-money S&P 500 put options on each monthly expiration date for maturities less than or equal to three months, and with strikes approximately 7% out of the money” (Lo (2001), p. 21). This strike selection is comparable to that of a $Z = -1.0$ strategy, which between 1996 to 2012 wrote options that were *on average* about 7% out-of-the-money. By contrast, given the margin rule applied in the CDP return computations, the leverage, L , at inception is roughly three and a half times greater than our preferred put-writing strategy. The CDP strategy assumes that 66 percent of the margin is required to be posted as collateral,” where the margin is set equal to $0.15 \cdot S - \max(0, S - K) - \mathcal{P}$. In what follows, we interpret this conservatively to mean that the strategy posts collateral that is 66% *in excess* of the minimum exchange requirement. Abstracting from the value of the put premium, which is significantly smaller than the other numbers in the computation, and setting the risk-free interest rate to zero, the strategy leverage given our definition is

$$L^{CDP} = \frac{\kappa_A}{\kappa_E} \approx \frac{0.93 \cdot S}{(1 + \frac{2}{3}) \cdot (0.15 \cdot S - \max(0, S - 0.93 \cdot S))} = 6.975. \quad (5)$$

This has led some to conclude that put-writing strategies do not represent a viable alternative to hedge fund replication, due to difficulties with surviving exchange margin requirements. As we demonstrate, this is not the case. The strategies we consider to match the risk exposure of the aggregate hedge fund universe are comfortably within exchange margin requirements at inception, and do not violate those requirements intra-month (unreported results).

B. Regression Analysis

Table II reports results from regressions of hedge fund excess returns onto the excess returns of the put-writing portfolios. As before, the regressions use quarterly data from January 1996 to June 2012. The estimated hedge fund risk exposures to each of the put-writing factors are highly statistically significant for both indices, with t -statistics ranging from 5.4 to 8.8. The adjusted- R^2 values from these regressions are somewhat lower than those from the common factor regressions reported earlier. The biggest difference is that the estimated pre-fee alphas are statistically indistinguishable from zero for all of the specifications, with economically small point estimates ranging from -3.2% to 0.0% for the HFRI index, and from 0.2% to 2.6% for the DJCS index. In all four regressions, we cannot reject the null hypothesis of a zero intercept and a unit slope coefficient. Given evidence of hedge fund return smoothing, the risk exposures may in fact be underestimated, inducing an upward bias in the alpha estimates. Consequently, benchmarking hedge fund risks with the put-writing factors suggests that hedge fund investors are barely covering their cost of capital *before* fees.

Aside from return smoothing, another conceptual issue arising in the regression analysis of hedge fund returns is the choice of which factors to include. For example, although the two put-writing strategies are static transformations of the underlying equity market index return and are closely related to one another, they are neither spanned by the equity market index nor spanned by one another. First, excess return regressions of the put-writing strategies onto the equity market index indicate significant CAPM alphas (see the Internet Appendix; Table IA.II). Second, a regression of the $[Z = -2, L = 4]$ portfolio excess returns onto the $[Z = -1, L = 2]$ portfolio excess returns produces a statistically significant annualized alpha of 3.7% (t -statistic = 7.3). Each of these put-writing strategies appears to be spanning a different dimension of downside risk. Since hedge funds represent an institutional structure that can flexibly engage in dynamic trading strategies, their periodic payoff profile can take on arbitrary shapes in relation to traditional risk factors (e.g., the equity market index). As a result, a single downside risk factor is unlikely to accurately describe their downside exposure accurately. Moreover, from a practical standpoint, conditional return smoothing may render the identification and differentiation of these exposures essentially impossible. Since these factors earn

large and statistically distinguishable risk premia, this further complicates inference regarding true hedge fund alphas.

B.1. Sensitivity of Inferences to Conditional Return Smoothing

Table II highlights that economic inference about capital market efficiency is quite different depending on whether one views the traditional factors (CAPM, Fama–French/Carhart, or Fung–Hsieh) or the put-writing factors as providing a better description of risks underlying hedge fund strategies. The standard approach to addressing this discrepancy is to combine the relevant factors in a single regression. However, our results highlight that statistical inference from this analysis is extremely sensitive to conditional return smoothing affecting a handful of influential economic observations.

The regressions in Table II indicate that, among the traditional risk factors, only the equity market factor (CAPM) plays a meaningful role in explaining the realized excess returns. We therefore focus our analysis on regressions combining the market factor with one of the two nonredundant put-writing strategies. To maximize the statistical power of the regressions, and explore the effects of return smoothing on inference, we focus on the monthly excess return regressions. Given the strong evidence of autocorrelation in hedge fund returns (Table I) and the literature on unconditional and conditional return smoothing, we include two monthly lags of the market factor (Scholes and Williams (1977)). The basic idea is that the factor portfolio returns are generally well marked each period, while the test asset may be marked less accurately. The lagged regressors help correct for the measurement errors created by any nonsynchronicity in returns.

Given the empirical evidence of conditional return smoothing and managerial discretion in marking hedge fund NAVs, simply adding lagged regressors may not properly capture the full risk exposure. In particular, this procedure will be less effective if the reporting errors in hedge fund returns are concentrated in certain episodes. Rather than some of the market “beta” simply being spread over time in a consistent manner because of constant reporting errors, consider the situation in which the “beta” of hedge fund returns is magnified in poor market environments, but more likely to be reported with error at these times. This would have the effect of altering a return series exposed to downside risks to look more like one with constant market exposure but state-dependent reporting errors.

To investigate the sensitivity of inferences to this possibility, we repeat the regression analysis using the reported return series and hypothetical “unsmoothed” hedge fund return series. We focus on two months for which hedge fund reporting errors are likely, August 1998 and October 2008. We adjust returns under the assumption that some fraction, ϕ , of the full return is reported in the event month, with the remaining return included in the subsequent monthly reported return, $r_t = \phi \cdot R_t$, and $r_{t+1} = R_{t+1} + (1 - \phi) \cdot R_t$, where r_t is the reported return and R_t is the full return in event month t . With two events, this procedure affects four monthly returns in the 198-month sample.

Tables III and IV report the results from these regressions. We report regression results as a function of the fraction, ϕ , of the true return that was marked contemporaneously during the two event periods. We consider four values of ϕ , namely, $\phi = 1$ (no smoothing) and three progressively increasing levels of smoothing, $\phi = \{0.67, 0.50, 0.33\}$. In the unadjusted (as reported) return series for both the HFRI and DJCS indices, the market factor and both of its lags are statistically significant, while neither of the put-writing factors is statistically reliable. These specifications suggest that the market factor provides a better description of the reported return series than either of the put-writing factors, pointing to statistically significant and economically large alphas. As the severity of the assumed return smoothing increases, the coefficients on the put-writing factors consistently increase and become statistically significant. At the same time, the sum of the coefficients on the market factor and its lags decrease as reporting errors increase, with the coefficients on the lagged market factor becoming statistically indistinguishable from zero. The adjusted hedge fund returns suggest that the put-writing factors are statistically relevant for explaining the time series variation, and imply statistically unreliable and economically small pre-fee alphas for the aggregate hedge fund indices. In the Internet Appendix, we demonstrate that these results are robust to applying the smoothing procedure to log returns, which preserves the geometric—rather than average—return of the reported hedge fund return series (Tables IA.III and IA.IV). Finally, it is important to emphasize that we do not obtain direct evidence that the adjusted returns more accurately describe the true hedge fund index returns; rather, the results simply demonstrate the extreme sensitivity of regression-based inference to the accuracy of reported returns in two economically important episodes.¹¹

C. Evaluating Hedge Fund Replication Strategies

Statistical analysis of hedge fund returns vis-a-vis traditional risk factors (CAPM, Fama–French/Carhart, Fung–Hsieh) and put-writing strategies paints a disparate picture regarding the feasibility of hedge fund replication. Regressions involving the traditional risk factors find alphas of 6% to 10% per annum, which suggests a high level of capital market inefficiency and infeasibility of replicating hedge fund returns by passively combining traditional risk factors. By contrast, regressions involving the put-writing factors suggest that hedge funds specialize in bearing downside market risks with the pricing of these risks well integrated across markets and pre-fee hedge fund returns are statistically indistinguishable from put-writing strategies such as

¹¹ Correcting the reporting errors requires getting inside the opaque entities that produce the return series. Cao et al. (2013) use a unique data set of separate accounts available on a platform offered by Societe Generale. The platform contracts with hedge funds to create accounts that trade pari passu with the assets in the main fund. The platform calculates and publishes the NAV and return weekly, while the main funds often provide self-reported performance relatively infrequently. They estimate that managerial discretion in marking the fund's NAV produces one-third of the autocorrelation in fund returns.

Table III
The Effect of Reporting Errors on Downside Risk Exposure Estimates (HFRI)

This table examines the effect of reporting errors on the results of regressions of monthly hedge fund index returns (HFRI Fund Weighted Composite Index) onto the contemporaneous market excess return ($RMRF_t$), two monthly lags of the market excess return ($RMRF_{t-1}$ and $RMRF_{t-2}$), and a contemporaneous excess return to a strategy that writes short-dated S&P 500 equity index put options ($PW[Z, L]_t$). The analysis assumes funds marked a fraction ϕ of their true return in August 1998 and October 2008, and accrued the remaining fraction, $1 - \phi$, in the subsequent month; the data reported in all other months are assumed to correctly reflect the true, realized fund returns. The regressions are carried out using the raw data ($\phi = 1$), and after unsmoothing the returns based on three different reporting schemes ($\phi = \{0.67, 0.50, 0.33\}$), which assume progressively more extreme errors in the data. Panel A reports results for the $[Z = -1, L = 2]$ put-writing strategy; Panel B reports results for the $[Z = -2, L = 4]$ put-writing strategy. All regressions are carried out using data from January 1996 through June 2012 ($N = 198$ months). The panels report coefficient estimates with OLS t -statistics in brackets; point estimates of intercepts have been multiplied by 100. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Put-Writing [$Z = -1, L = 2$]												
Pre-fee returns					After-fee returns							
ϕ	Intercept	$RMRF_t$	$RMRF_{t-1}$	$RMRF_{t-2}$	$PW[-1, 2]_t$	Adj. R^2	Intercept	$RMRF_t$	$RMRF_{t-1}$	$RMRF_{t-2}$	$PW[-1, 2]_t$	Adj. R^2
1.00	0.59*** [5.84]	0.41*** [12.62]	0.06*** [3.34]	0.04*** [2.11]	-0.06 [-0.90]	72.1%	0.26*** [2.77]	0.37*** [12.34]	0.06*** [3.48]	0.04*** [2.13]	-0.03 [-0.40]	72.5%
0.67	0.47*** [4.16]	0.37*** [10.29]	0.04*** [2.02]	0.03 [1.58]	0.10 [1.23]	69.3%	0.14 [1.32]	0.34*** [9.73]	0.04*** [2.12]	0.03 [1.54]	0.14* [1.83]	69.2%
0.50	0.36*** [2.60]	0.34*** [7.76]	0.03 [1.07]	0.03 [1.06]	0.26*** [2.68]	63.7%	0.03 [0.19]	0.30*** [7.08]	0.02 [0.99]	0.02 [0.99]	0.30*** [3.23]	62.8%
0.33	0.13 [0.65]	0.28*** [4.29]	-0.01 [-0.21]	0.01 [0.40]	0.58*** [4.10]	51.9%	-0.21 [-1.03]	0.24*** [3.68]	-0.01 [-0.30]	0.01 [0.32]	0.63*** [4.47]	50.4%

Panel B: Put-Writing [$Z = -2, L = 4$]												
Pre-fee returns					After-fee returns							
ϕ	Intercept	$RMRF_t$	$RMRF_{t-1}$	$RMRF_{t-2}$	$PW[-2, 4]_t$	Adj. R^2	Intercept	$RMRF_t$	$RMRF_{t-1}$	$RMRF_{t-2}$	$PW[-2, 4]_t$	Adj. R^2
1.00	0.50*** [4.72]	0.37*** [14.63]	0.06*** [3.49]	0.04*** [2.23]	0.05 [0.68]	72.0%	0.18* [1.78]	0.34*** [14.51]	0.06*** [3.63]	0.04*** [2.27]	0.08 [1.17]	72.7%
0.67	0.34*** [2.88]	0.35*** [12.53]	0.05*** [2.32]	0.04* [1.79]	0.23*** [3.05]	70.5%	0.01 [0.08]	0.32*** [12.10]	0.04*** [2.32]	0.03* [1.77]	0.27*** [3.67]	70.7%
0.50	0.18 [1.25]	0.33*** [9.91]	0.03 [1.24]	0.03 [1.31]	0.42*** [4.58]	66.0%	-0.16 [-1.16]	0.30*** [9.27]	0.03 [1.16]	0.03 [1.25]	0.46*** [5.14]	65.5%
0.33	-0.15 [-0.73]	0.29*** [6.03]	0.00 [-0.10]	0.02 [0.64]	0.80*** [5.95]	55.8%	-0.49*** [-2.41]	0.26*** [5.39]	-0.01 [-0.20]	0.02 [0.57]	0.84*** [6.31]	54.6%

Table IV
The Effect of Reporting Errors on Downside Risk Exposure Estimates (DJCS)

This table examines the effect of reporting errors on the results of regressions of monthly hedge fund index returns (Dow Jones/Credit Suisse Broad Hedge Fund Index) onto the contemporaneous market excess return ($RMRF_t$), two monthly lags of the market excess return ($RMRF_{t-1}$ and $RMRF_{t-2}$), and a contemporaneous excess return to a strategy that writes short-dated S&P 500 equity index put options ($PW[Z, L]_t$). The analysis assumes funds marked a fraction ϕ of their true return in August 1998 and October 2008, and accrued the remaining fraction, $1 - \phi$, in the subsequent month; the data reported in all other months are assumed to correctly reflect the true, realized fund returns. The regressions are carried out using the raw data ($\phi = 1$), and after unsmoothing the returns based on three different reporting schemes ($\phi = \{0.67, 0.50, 0.33\}$), which assume progressively more extreme errors in the data. Panel A reports results for the $|Z| = -1, L = 2$ put-writing strategy; Panel B reports results for the $|Z| = -2, L = 4$ put-writing strategy. All regressions are carried out using data from January 1996 through June 2012 ($N = 198$ months). The panels report coefficient estimates with OLS t -statistics in brackets; point estimates of intercepts have been multiplied by 100. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Put-Writing - $ Z = -1, L = 2$												
Pre-fee returns						After-fee returns						
ϕ	Intercept	$RMRF_t$	$RMRF_{t-1}$	$RMRF_{t-2}$	$PW[-1, 2]_t$	Adj. R^2	Intercept	$RMRF_t$	$RMRF_{t-1}$	$RMRF_{t-2}$	$PW[-1, 2]_t$	Adj. R^2
1.00	0.65*** [4.95]	0.34*** [8.10]	0.05** [2.09]	0.09*** [3.97]	-0.10 [-1.12]	50.1%	0.33*** [2.70]	0.31*** [7.85]	0.05** [2.18]	0.09*** [3.95]	-0.07 [-0.81]	50.1%
0.67	0.55*** [3.83]	0.33*** [7.20]	0.03 [1.36]	0.09*** [3.59]	0.02 [0.24]	50.2%	0.23* [1.74]	0.28*** [6.69]	0.03 [1.42]	0.08*** [3.51]	0.08 [0.87]	50.2%
0.50	0.45*** [2.93]	0.28*** [5.78]	0.02 [0.72]	0.08*** [3.04]	0.19 [1.76]	48.4%	0.12 [0.83]	0.25*** [5.31]	0.02 [0.68]	0.08*** [2.91]	0.23*** [2.23]	48.0%
0.33	0.24 [1.21]	0.22*** [3.50]	-0.01 [-0.28]	0.07** [2.04]	0.48*** [3.45]	42.5%	-0.09 [-0.46]	0.19*** [2.99]	-0.01 [-0.36]	0.07* [1.87]	0.53*** [3.85]	41.6%
Panel B: Put-Writing - $ Z = -2, L = 4$												
Pre-fee returns						After-fee returns						
ϕ	Intercept	$RMRF_t$	$RMRF_{t-1}$	$RMRF_{t-2}$	$PW[-2, 4]_t$	Adj. R^2	Intercept	$RMRF_t$	$RMRF_{t-1}$	$RMRF_{t-2}$	$PW[-2, 4]_t$	Adj. R^2
1.00	0.55*** [3.97]	0.29*** [8.86]	0.05** [2.22]	0.10*** [4.06]	0.03 [0.33]	49.8%	0.23* [1.80]	0.27*** [8.66]	0.05** [2.32]	0.09*** [4.06]	0.06 [0.65]	50.1%
0.67	0.41*** [2.82]	0.28*** [8.05]	0.04 [1.56]	0.09*** [3.77]	0.20** [2.12]	51.4%	0.08 [0.62]	0.25*** [7.75]	0.04 [1.58]	0.09*** [3.72]	0.23*** [2.59]	51.7%
0.50	0.26* [1.65]	0.26*** [6.90]	0.02 [0.88]	0.09*** [3.30]	0.37*** [3.59]	50.8%	-0.07 [-0.45]	0.23*** [6.48]	0.02 [0.84]	0.08*** [3.20]	0.40*** [4.11]	51.0%
0.33	-0.04 [-0.17]	0.22*** [4.63]	-0.01 [-0.16]	0.08** [2.34]	0.71*** [5.39]	46.9%	-0.37* [-1.87]	0.19*** [4.13]	-0.01 [-0.25]	0.07** [2.18]	0.75*** [5.81]	46.5%

the $[Z = -1, L = 2]$ and $[Z = -2, L = 4]$ strategies, which yields a simple recipe for replicating hedge fund returns. These two perspectives suggest a starkly different view of hedge funds and capital markets, but are difficult to disentangle via statistical analysis given concerns about conditional return smoothing in the reported data (Bollen and Pool (2008), Bollen and Pool (2009), Cao et al. (2013)).

Figure 1 summarizes the fit of various replicating strategies. The top panels display the value of an initial \$1 investment in the HFRI Fund Weighted Composite and the various replicating portfolios. The fitted factor model-replicating portfolios, including the intercept, are shown in the left-hand panel, the feasible version of the common factor model-replicating portfolios (i.e., excluding the intercept) are in the middle panel, and the two put-writing portfolios are displayed in the right-hand panel. The bottom panels plot the time series of drawdowns for each of the replicating strategies, measured as the cumulative loss relative to the previous peak. The Internet Appendix displays results of the corresponding analysis for the DJCS Broad Hedge Fund Index (Figure IA.2).

The figure highlights the central role of the drift in explaining the overall fit of the various models, and therefore how one's prior belief about the competitiveness of capital markets essentially determines which perspective is most compelling. Including the 6% to 10% per annum fitted intercept (alpha) in the total return of the factor-replicating portfolio produces a great match (left-hand panel), but requires the belief that capital market competition is highly imperfect as well as the belief that conditional return smoothing is a negligible concern. A strong belief that capital markets are highly competitive would lead one to place more weight on feasible strategies, with large unexplained means allocated to omitted or mismeasured risks owing to the opaqueness of the hedge fund return generating process. From this perspective, the feasible replicating portfolios based on traditional factors (middle panel) produce return series that are highly dissimilar to the HFRI return series. On the other hand, the feasible put-writing strategies produce time series that look virtually identical to the aggregate hedge fund index, matching the losses during the fall of 2008 and the LTCM crisis, the flat performance during the bursting of the Internet bubble, and the strong returns during boom periods. While the put-writing strategies fail to explain some of the return variation in economically benign times like the bull market between 2002 and 2007, they capture well the variation in economically important times. The similarities between hedge fund returns and put-writing suggest that hedge funds may be bearing downside risks, and that the premia for bearing these risks are equalized across markets.

III. Required Rates of Return for Downside Market Risks

The evidence presented so far suggests that the high realized returns of the aggregate hedge fund indices may reflect compensation for bearing downside market risks. However, given evidence that equity index options may themselves be expensive relative to standard asset pricing models, we are left with

the possibility that both hedge fund returns and put-writing strategies have exceeded their cost of capital.¹² To explore this conjecture in greater detail and elucidate the conditions necessary to rationalize the high observed excess returns for these two strategies, we employ a simple static asset pricing framework inspired by Merton's (1987) model of investor specialization. In practice, there is evidence that end users of hedge fund investments (Lerner, Schoar, and Wang (2008)) and the marginal price setters in equity index options, typically viewed to be the market makers (Garleanu, Pedersen, and Poteshman (2009)), are specialized and therefore hold concentrated portfolios. Merton (1987) emphasizes that, in the presence of market segmentation, inference based on linear factor regressions may be misleading. In particular, assets requiring specialization will appear to have alpha relative to common market factors, as is the case for hedge fund indices (Table II) and the put-writing strategies (see the Internet Appendix; Table IA.II).

To investigate the conditions under which the observed returns to hedge fund investments and put-writing strategies can be viewed as having exceeded their proper cost of capital, we take equilibrium investor allocations as given and solve for the required rates of return. In the spirit of Merton (1987), we assume that markets are segmented and only a small subset of investors is allowed access to the market for alternatives, which we proxy with the put-writing strategies introduced earlier. The equilibrium we consider explicitly involves segmentation, with a small group of specialized investors forced to hold large allocations of the nontraditional risks, even though they comprise a small share of the aggregate wealth portfolio. The availability of a clear state-contingent payoff representation enables us to characterize the required rates of return on various assets as a function of (a) portfolio concentration, (b) the payoff profile of the investments, and (c) the distribution of the common market factor.

A. The Investor's Cost of Capital

To study investor required rates of return for downside market risk exposures in the context of concentrated portfolios, we employ a static framework that combines power utility (CRRA) preferences with a state-contingent asset payoff representation originating in Arrow (1964) and Debreu (1959). We specify the joint structure of asset payoffs by describing each security's payoff as a function of the log return, \tilde{r}_m , on the aggregate equity index (here, the S&P 500). For every \$1 invested, the state-contingent payoffs of the three assets are as follows: the risk-free asset pays $\exp(r_f \cdot \tau)$ in all states; the equity index payoff is, by definition, $\exp(r_m)$; and the payoff to the downside risk investment (alternative investment) is $f(r_m, \mathcal{P})$, where \mathcal{P} is the price of the alternative investment. Given a realization of the market return, \tilde{r}_m , the agent's utility is given by

¹² Coval and Shumway (2001), Bakshi and Kapadia (2003), Bondarenko (2003), Frazzini and Pedersen (2012), and Constantinides Jackwerth, and Savov (2013) document evidence of large excess returns to delta-hedged options.

$$U(\tilde{r}_m) = \frac{1}{1-\gamma} \cdot ((1-\omega_m - \omega_a) \cdot \exp(r_f \cdot \tau) + \omega_m \cdot \exp(\tilde{r}_m) + \omega_a \cdot f(\tilde{r}_m, \mathcal{P}))^{1-\gamma}, \quad (6)$$

where ω_m and ω_a are the agent's allocations to the equity market and alternatives, respectively. Under power utility the investor prefers more positive values for the odd moments of the terminal portfolio return distribution (mean, skewness), and penalizes for large values of even moments (variance, kurtosis).¹³ Finally, to operationalize the cost of capital computations we need to specify the investor risk aversion, γ , and the distribution of the log market index return, $\phi(r_m)$.

We are interested in studying the asset pricing implications of a segmented market equilibrium in which the investor's allocation to alternatives, ω_a , is pre-specified exogenously to satisfy market clearing (i.e., it is determined by the equilibrium supply of this type of risk relative to the aggregate wealth of specialized investors). Taking the alternative allocation, ω_a , as given, we solve for the equilibrium equity market allocation and a valuation for the put option, $(\omega_m^*, \mathcal{P}^*)$, which jointly satisfy the investor's two first-order conditions with respect to the portfolio weights. At the constrained equilibrium, the specialized investor's subjective valuations of the equity index and the alternative payoff, $f(\tilde{r}_m, \mathcal{P})$, match their market prices, which are both normalized to one:

$$E_t[\Lambda(\omega_m^*, \omega_a, \mathcal{P}^*) \cdot \exp(\tilde{r}_m)] = 1, \quad (7a)$$

$$E_t[\Lambda(\omega_m^*, \omega_a, \mathcal{P}^*) \cdot f(\tilde{r}_m, \mathcal{P}^*)] = 1, \quad (7b)$$

where $\Lambda = \exp(-r_f \cdot \tau) \cdot \frac{U'(\cdot)}{E_t[U'(\cdot)]}$ is the investor's subjective pricing kernel. The first equation ensures the investor is at his optimal allocation to equities, and therefore that subsequent required rate of return computations are based on (constrained) optimal portfolios. The second equation pins down his subjective valuation for the put option embedded in the alternative investment, \mathcal{P}^* , and is used to determine the required rate of return on alternatives via (8). Both of these equations are computed taking the distribution of equity index returns as exogenous, reflecting the assumption that the specialized investor is assumed to be a price taker in this market. After we solve for the equilibrium value of \mathcal{P}^* , the specialized investor's required excess rate of return on the alternative investment is given by

$$\begin{aligned} r_a^*(\omega_a) &= \frac{1}{\tau} \cdot \ln E_t \left[\frac{f(\tilde{r}_m, \mathcal{P}^*)}{E_t[\Lambda(\omega_m^*, \omega_a, \mathcal{P}^*) \cdot f(\tilde{r}_m, \mathcal{P}^*)]} \right] - r_f \\ &= \frac{1}{\tau} \cdot \ln E_t[f(\tilde{r}_m, \mathcal{P}^*)] - r_f. \end{aligned} \quad (8)$$

¹³ By specifying the joint distribution of returns using state-contingent payoff functions, we can allow security-level exposures to depend on the market state nonlinearly, generalizing the linear correlation structure implicit in mean-variance analysis. Patton (2004), Harvey, et al. (2010), and Martellini and Ziemann (2010) emphasize the importance of higher-order moments and the asset return dependence structure for portfolio selection.

Due to our focus on a single-factor payoff representation, we contrast the proper required rate of return, (8), with the corresponding rate of return based on the linear (Gaussian) CAPM rule, $\beta \cdot \lambda^{CAPM}$, where $\beta = \frac{\text{Cov}[r_a, r_m]}{\text{Var}[r_m]}$ is the CAPM β of the alternative on the equity index. The market risk premium in the Gaussian CAPM is given by $\lambda^{CAPM} = \tilde{\gamma} \cdot \sigma_m^2$, where $\tilde{\gamma}$ is the risk aversion of an agent who is fully invested in the equity index at his optimum, and σ_m is the volatility of the equity index return.

B. Alternative Investment, $f(r_m, \mathcal{P})$

The payoff of the alternative investment is represented using a levered, naked put-writing portfolio, as in the empirical analysis in Section II. Specifically, we assume that the investor places his capital, ω_a , in a limited liability company (LLC) to eliminate the possibility of losing more than his initial contribution. Limited liability structures are standard in essentially all alternative investments, private equity, and hedge funds alike, effectively converting their payoffs into put spreads. In practice, the cost of establishing this structure is minimal relative to the assets under management, and thus we approximate its cost as zero. Given a leverage of L , the quantity of puts that can be supported per \$1 of investor capital is given by

$$q = \frac{L}{\exp(-r_f \cdot \tau) \cdot K(Z) - \mathcal{P}(K(Z), \tau)}, \quad (9)$$

where $K(Z)$ is the strike corresponding to a Z -score, Z . The put premium and the agent's capital grow at the risk-free rate over the life of the trade, and are offset at maturity by any losses on the index puts to produce a terminal state-contingent payoff:

$$f(\tilde{r}_m, \mathcal{P}) = \max(0, \exp(r_f \cdot \tau) \cdot (1 + q \cdot \mathcal{P}(K(Z), \tau)) - q \cdot \max(K(Z) - \exp(\tilde{r}_m), 0)). \quad (10)$$

The terminal payoff of the alternative depends on the initial put premium, \mathcal{P} , and the terminal realization of the equity index. We substitute this payoff function into the endowment investor's first-order conditions to determine his shadow valuation of the put option, and therefore his required rate of return on the alternative investment.

C. Equity Index Distribution, $\phi(r_m)$

Given our focus on pricing payoffs with nonlinear downside risk exposures, we choose a parameterization for the equity index distribution, $\phi(r_m)$, that can accommodate the empirical evidence of skewness and kurtosis in index returns. Specifically, we rely on the normal inverse Gaussian (NIG) distribution, which

allows us to flexibly specify the first four moments (see Appendix). This pins down the (conditional) distribution from which we simulate τ -period log index returns:

$$r_m = (r_f + \lambda_t - k_Z(1)) \cdot \tau + \tilde{Z}_\tau, \quad \tilde{Z}_\tau \sim \text{NIG}(0, \mathcal{V}_t, S, \mathcal{K}), \quad (11)$$

where $\mathcal{V}_t = \sigma_t^2 \cdot \tau$ is the τ -period variance, and S and \mathcal{K} are the skewness and kurtosis of the τ -period returns. The market risk premium, $\lambda_t = k_Z(-\tilde{\gamma}) + k_Z(1) - k_Z(1 - \tilde{\gamma})$, and the convexity adjustment (Jensen) term $k_Z(1)$ depend on the cumulant generating function, $k_Z(u)$, of the shock, \tilde{Z}_τ , which is given in the Appendix. We fix the equilibrium equity risk premium by imposing the condition that a hypothetical investor, with $\tilde{\gamma} = 2$, would be fully invested in the equity market in the absence of alternatives. In a Gaussian setting, this is equivalent to an optimal equity allocation of 60% for an investor with a risk aversion of 3.3 ($= \frac{\tilde{\gamma}}{0.6}$), since the risky asset allocation is an inverse function of the coefficient of relative risk aversion. Since a portfolio of 40% cash and 60% equities corresponds to an allocation commonly used as a benchmark by endowments and pension plans, we set the risk aversion of traditional and specialized investors equal to $\gamma = 3.3$.

We calibrate the distribution to match the properties of historical returns. We estimate volatility as 0.8 times the sample average of the VIX. The remaining moments are chosen to roughly match historical features of monthly S&P 500 Z-scores, obtained by demeaning the time series of monthly log returns and scaling them by 0.8 of the VIX as of the preceding month-end. Specifically, we target a monthly Z-score skewness, S , of -1 , and kurtosis, \mathcal{K} , of 7. These parameters combine to produce a left-tail “event” once every five years that results in a mean monthly Z-score of -3.6 . For comparison, the mean value of the Z-score under the standard normal (Gaussian) distribution, conditional on being in the left 1/60 percent of the distribution, is -2.5 .¹⁴

D. Comparative Statics

The novel components of the framework are the explicitly nonlinear downside exposure of the alternative investment and the potentially large allocation to this investment among the few specialized investors who bear this risk in equilibrium. Figure 2 explores the consequences of this friction for the specialized investor as a comparative static in ω_a . In the absence of market frictions these risks will be diffusely held and ω_a will be small, whereas if market frictions force high degrees of specialization, then ω_a can be quite large.

The upper left panel plots the payoff profiles of the three risky assets, plotted as a function of the equity market return. By construction, the payoff to the equity index is a 45-degree line. The two downside risk portfolios differ in their

¹⁴ Based on a preceding month-end VIX value of 22.4%, and our parameterization of the NIG distribution, the -21.6% return of the S&P 500 index in October 1987 corresponds to a Z-score of -4.7 . The probability of observing a monthly return at least as bad as this is 0.2% under the NIG distribution, and 0.0001% under the Gaussian distribution.

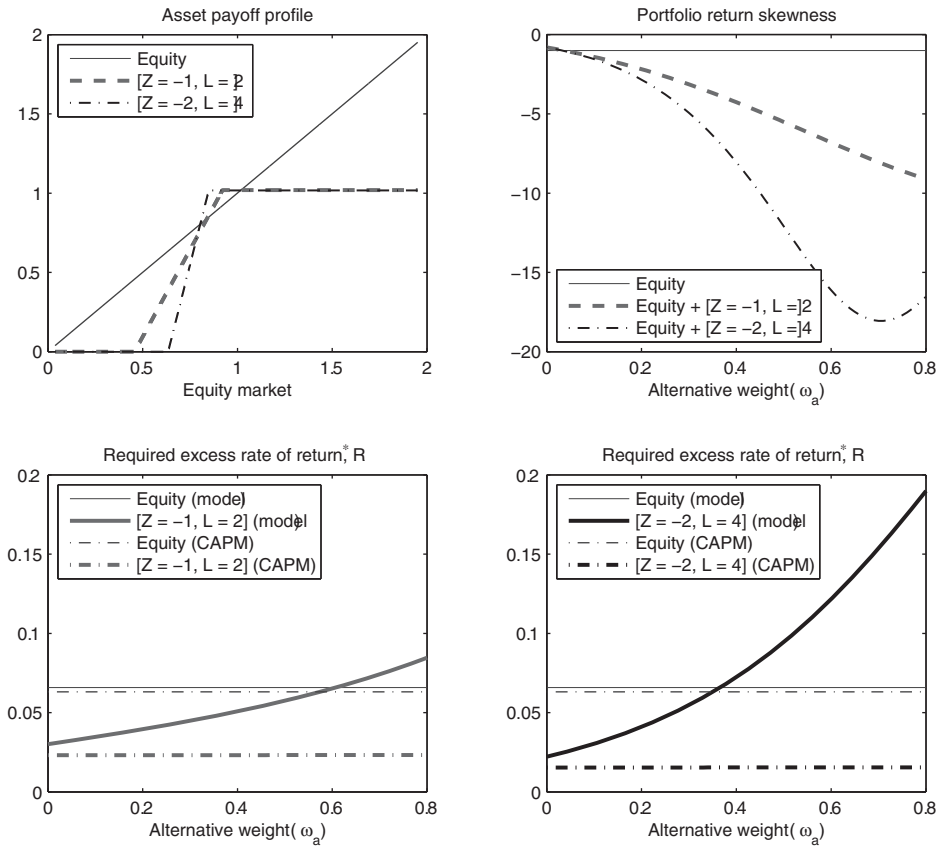


Figure 2. Required rates of return for large allocations to nonlinear risk exposures.

This figure illustrates the comparative statics of the investor's required rate of return for various assets as a function of the asset's payoff profile and the investor's portfolio allocation. The top left panel plots the payoff profile of the equity index and two levered, put-writing portfolios ($[Z = -1, L = 2]$ and $[Z = -2, L = 4]$) as a function of the equity index realization. Relative to the $[Z = -1, L = 2]$ put-writing strategy, the $[Z = -2, L = 4]$ strategy applies a higher amount of leverage to options that are written further out of the money. The top right panel plots the skewness of the investor's optimally invested portfolio as a function of the allocation to the put-writing strategy (i.e., the alternative investment). The bottom panels plot the required excess rates of return for the equity index and the alternative investment as a function of the portfolio allocation to the alternative. The bottom panels display the model required rate of return (solid lines) and the risk premium under the linear CAPM benchmark ($\beta \cdot \tilde{\gamma} \sigma^2$; dash-dot lines) for the equity index and the two put-writing strategies. The bottom left (bottom right) panel plots the required excess rates of returns for the $[Z = -1, L = 2]$ ($[Z = -2, L = 4]$) strategy. The underlying equity index distribution is assumed to follow a normal inverse Gaussian (NIG) distribution with volatility equal to 17.8% per annum (0.8 times the sample average of the VIX index), skewness equal to -1 , and kurtosis equal to 7 .

state-contingent profiles in that the $[Z = -1, L = 2]$ portfolio is more exposed to smaller market shocks than the $[Z = -2, L = 4]$ portfolio, but less exposed to larger market shocks. At the baseline parameter values, the CAPM betas of the $[Z = -1, L = 2]$ and $[Z = -2, L = 4]$ portfolios are 0.37 and 0.24, respectively, which accords well with the values realized in the data (Table I).¹⁵

The upper right panel plots the skewness of the wealth portfolio as a function of the allocation to the specialized asset, ω_a , along with the skewness of the underlying equity index for comparison. When market frictions are minor, allowing for small allocations to the specialized investment, the portfolio skewness is lower than that of the underlying equity index. However, as market frictions increase, larger allocations are required of the specialized investor and skewness becomes more negative, quickly exceeding in magnitude that of the underlying index. This is especially true for the further out-of-the-money investment with the lower CAPM beta, illustrating that the relative riskiness of the two downside assets flips as the allocation increases. This highlights the challenge of identifying required returns from the realized returns of assets with downside market risk exposures, when neither the risk profile nor the equilibrium weight is known.

The bottom panels plot the required excess returns for the two specialized investments as allocations change, along with their CAPM required excess returns, with the $[Z = -1, L = 2]$ portfolio on the left and the $[Z = -2, L = 4]$ portfolio on the right. The proper required return for the specialized investor is increasing and convex in ω_a . Even at a small allocation to alternatives, the proper required return exceeds the CAPM calculation due to the nonlinear risk profile. These plots further illustrate the challenges in identifying the proper required return in this setting. There are large regions in which the risky asset with the lowest CAPM beta has the highest required return—a higher required return than both the higher beta downside risk portfolio and the market portfolio that has a beta four times higher.

Figure 3 explores the comparative statics of the investor required excess rates of return as a function of the moments of the underlying equity index distribution, holding the investor's allocation to alternatives fixed at 35%. The top (bottom) two panels examine the $[Z = -1, L = 2]$ ($[Z = -2, L = 4]$) portfolio, plotting its required excess rate of return against those of the underlying equity index. The left-hand panels explore the comparative static in variance, holding skewness and kurtosis fixed at their baseline values. The right-hand panels explore the comparative static in skewness, setting variance at its baseline value and kurtosis at the minimum value allowable under the NIG parameterization ($\mathcal{K} = 3 + \frac{5}{3} \cdot S^2$).

The proper required rate of return for the equity index is essentially linear in variance, as in the standard Gaussian CAPM model ($\lambda = \tilde{\gamma} \cdot \sigma^2$), but is somewhat higher, reflecting the departures from Gaussianity (see Appendix). Since

¹⁵ We compute the CAPM beta for the nonlinear payoff profile directly from its periodic returns and those of the equity index, $\beta = \frac{\text{Cov}[f(r_m) - 1, \exp(r_m) - 1]}{\text{Var}[\exp(r_m) - 1]}$, evaluating the relevant moments using the postulated NIG probability distribution.

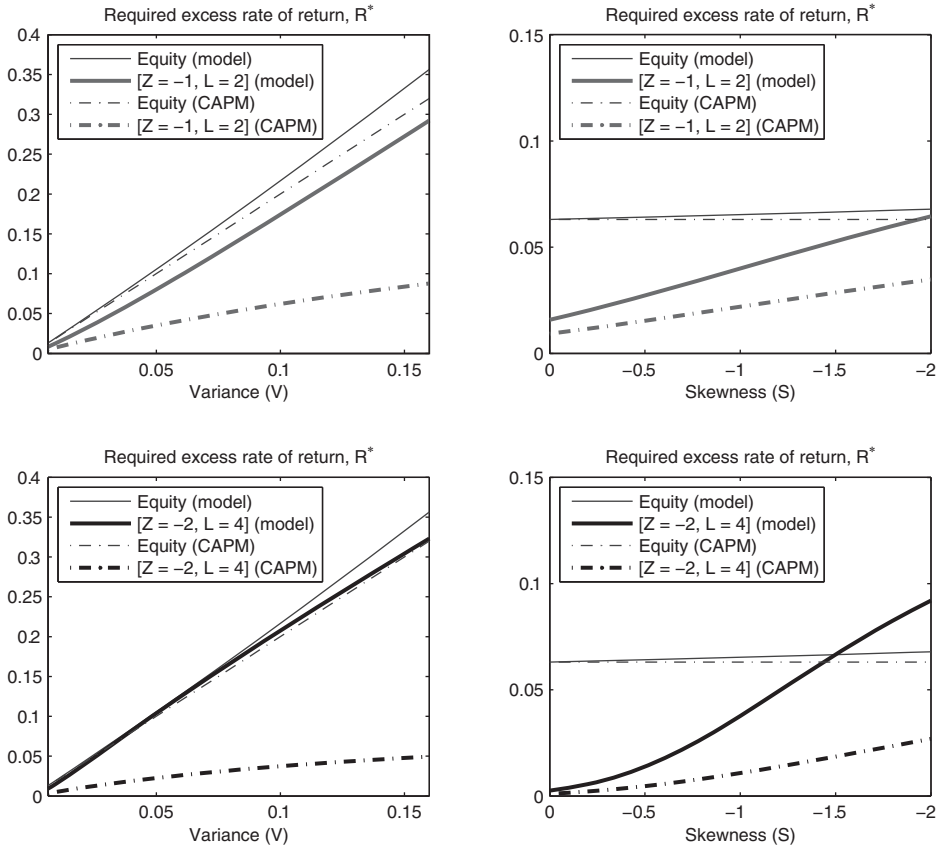


Figure 3. Sensitivity of required rates of return to the variance and skewness of the equity index distribution. This figure illustrates the comparative statics of the required rates of return for the model and the linear CAPM benchmark ($\beta \cdot \tilde{\gamma} \sigma^2$) as a function of the moments of the underlying equity index return distribution. The top (bottom) panels plot the required rates of return for the equity index and the $[Z = -1, L = 2]$ ($[Z = -2, L = 4]$) put-writing portfolios. Relative to the $[Z = -1, L = 2]$ put-writing strategy, the $[Z = -2, L = 4]$ strategy applies a higher amount of leverage to options that are written further out of the money. The left (right) panels examine the sensitivity with respect to the variance (skewness) of the underlying equity index return distribution, which is parameterized using a NIG density. Each panel plots the model (CAPM) required rate of return for the equity index using a solid (dash-dot) line. The model (CAPM) required rate of return for the alternative investments are displayed using bold solid (dash-dot) lines. In each case, the investor is assumed to allocate 35% of his wealth to the put-writing strategy (i.e., the alternative investment). For the variance comparative static, the skewness (S) is set to -1 and kurtosis (K) is set to 7 , as in the baseline distribution. For the skewness comparative static, the volatility is fixed at 0.8 times the sample average of the VIX index ($\sigma = 17.8\%$) and kurtosis is set equal to the minimum value for which the NIG density is well defined ($K = 3 + \frac{5}{3} \cdot S^2$).

there is no role for higher moments in the Gaussian CAPM, the required rate of return for the equity index does not depend on skewness (right-hand panels). By contrast, when the common factor follows an NIG distribution, the required excess rate of return on the equity increases very modestly, rising from 6.3% (Gaussian; $S = 0$) to just under 7.0% per annum when skewness reaches a value of -2 . The modest increase in the model equity risk premium as a function of skewness contrasts with the intuition from consumption-based disaster risk models (Barro (2006), Barro and Ursua (2008)). These models typically rely on much higher levels of risk aversion and/or more extreme assumptions regarding the distribution of consumption growth disasters than are consistent with observed equity index option prices. For example, using the parameter values from Backus, Chernov, and Martin (2011), a typical Poisson consumption growth disaster model attributes more than 50% of the equity risk premium, defined as the expectation of the log equity return over the log risk-free rate, to high-order moments (skewness, kurtosis, etc.) of the risk factor. By contrast, the disaster distribution implied from equity index options suggests that less than 5% of the total equity risk premium is due to higher-order moments.

The required excess rates of return for the alternative investments exhibit considerably more interesting patterns. First, holding skewness fixed (left-hand panels), the proper required excess rates of return are much more similar to those of the equity index than is suggested by the comparatively low CAPM betas of these investments. Second, there is a significant wedge between the CAPM and model required excess rates of return, reflecting the joint effect of the inadequacy of using the CAPM beta to characterize the nonlinear state-contingent payoff profile of the put-writing portfolio, and the magnification of the underlying equity index return skewness by the put-writing portfolio. Increasing the magnitude of skewness in the underlying equity index return distribution (right-hand panels) has a similarly stark effect on the required excess rates of return for the put-writing portfolios, causing them to eventually exceed those of the equity index itself. Consistent with intuition, the required rates of return for payoff profiles that reallocate losses to the tail more aggressively—by applying higher leverage to further out-of-the-money short positions in options—are much more sensitive to the skewness of the underlying distribution.

E. Evaluating Downside Risks against a Proper Cost of Capital

To evaluate the realized performance of various portfolios with explicitly nonlinear downside market risks (alternatives), we use the state-contingent payoff model to produce a time series of required rates of return for the endowment investor introduced in the previous section. The investor without access to alternatives would allocate 60% to stocks and 40% to risk-free securities, but we assume that his constrained equilibrium allocation requires either 35% or 50% to alternatives to match the holdings of various Ivy League endowments.

We produce a time series of proper required rates of return for each considered downside risk profile according to the following procedure. On each

rebalancing date we supply the specific composition of the put-writing portfolio along with parameters characterizing the terminal distribution of the (log) equity index return. At each point in time, the composition of the put-writing replicating portfolio is pinned down by the option strike, $K(Z)$, and the option price, $\mathcal{P}(Z)$, which jointly with L determine the quantity of options sold and the investor's capital. For parsimony, we hold the skewness and kurtosis of the market return distribution fixed at their baseline values, and only let the market return volatility, σ_t , vary over time, by setting it equal to 0.8 times the prevailing value of the VIX on each rebalancing date. The time series of market volatility also pins down the time series of the equilibrium market risk premium, λ_t (see Appendix). For comparison, we also produce a time series of required rates of return for hedge funds based on the Gaussian CAPM model by multiplying the β_t of the option-replicating portfolio by the CAPM market risk premium, $\tilde{\gamma} \cdot \sigma_t^2$, where $\tilde{\gamma}$ is the risk aversion of the all-equity investor. We compute the β_t of the option portfolio directly from the joint distribution of its returns and those of the equity index, as in the comparative statics analysis.

Before comparing the model required rates of return to the realized returns of the put-writing strategies, we convert the continuously compounded required rates of return, $r_a^*(\omega_a)$, given by (8) into discretely compounded net returns, and compute the required rate of return given the investor's allocation to alternatives, ω_a . To obtain the discretely compounded monthly return, we scale the annualized continuously compounded rate by $\frac{1}{12}$, exponentiate it, and subtract one. We repeat this procedure at each rebalancing date to produce a monthly time series of average required rates of return for use in performance evaluation.

E.1. Estimates of Required Returns

Panel A of Table V illustrates how we calculate required excess returns. The table reports the annual time series from 1996 through 2012 of various measures of volatility, as well as the excess realized and required returns for the S&P 500 index and the (pre-fee) [$Z = -1$, $L = 2$] put-writing portfolio. The table shows that the simple estimate of volatility ($\sigma_t = 0.8 \cdot VIX_t$) corresponds closely to realized volatility year-by-year and on average. *Mean* reports the full-sample average with *t*-statistics reported in square brackets. Over the sample period, the stock market index realizes, on average, an annualized excess return of 5.4%, while the traditional investor with no allocation to alternatives requires 7.6% per year, given the realized path of volatility over the sample. For comparison, we also report the Gaussian CAPM required return for the equity index, $r_{m,t}^* = \tilde{\gamma} \cdot \sigma_t^2$, which averages 7.2%. The small 40 basis point wedge in required returns for the equity index is created by accounting for skewness and kurtosis of the assumed equity index distribution. These estimates reflect the severe consequences of 2008 and 2009, when realized returns were low and realized volatility was high. Over the more economically benign period of 1996 through 2007, the stock market index average annual excess return is 6.5%, and the

Table V
Comparison of Required Rates of Return (1996:1–2012:6)

Panel A of this table compares the realized excess rates of return for the S&P 500 index and the $|Z| = -1, L = 2|$ put-writing strategy, with ex ante required risk premia. Investor required rates of return are computed at the beginning of each month in the sample (January 1996 to June 2012) using investor portfolios and an estimate of equity market volatility ($0.8 \cdot VIX_t$) based on the CBOE VIX index. Realized volatility is computed using the standard deviation of daily returns within each month, annualized, and reported as a year-by-year average. The required risk premia are computed based on the Gaussian CAPM benchmark ($\beta_t \cdot \hat{\gamma} \sigma_t^2$) and the nonlinear model introduced in Section III. The CAPM benchmark is computed using the risk aversion of an all-equity investor ($\hat{\gamma} = 2$), and the market beta of the put-writing portfolio at inception (β_t). The model required rate of return is computed assuming the distribution of the monthly equity index return follows a normal inverse Gaussian (NIG) distribution, with an annualized volatility equal to $0.8 \cdot VIX_t$, and skewness and kurtosis fixed at -1 and 7 , respectively. The model required rate of return is computed for two investor types: a traditional investor with no allocation to alternatives ($\omega_a = 0$), and a specialized investor with a large allocation to alternatives ($\omega_a = 0.35$ or $\omega_a = 0.50$). Each investor is assumed to have a CRRA risk aversion, γ , equal to 3.3 , such that—in the absence of alternatives—their optimal portfolio consists roughly of 60% equities and 40% cash. The table reports the sum of monthly excess returns within each year, as well as the mean annualized excess return for the full sample (*Mean*). The t -statistic for the mean excess return is reported in square brackets. Panel B reports the annualized values of the arithmetic mean monthly (excess) returns for the equity index (S&P 500) and two put-writing strategies ($|Z| = -1, L = 2|$ and $|Z| = -2, L = 4|$), and computes investor alphas as the difference in the realized and required excess returns with respect to the linear CAPM benchmark and the model implied excess return for the traditional and specialized investors (t -statistics in brackets). Investor alphas are reported for two skewness levels (-1 and -1.5) of the underlying normal inverse Gaussian (NIG) distribution describing monthly equity index returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Excess Returns											
Volatility			S&P 500 Index				Put-Writing [$Z = -1, L = 2$]				
			Realized		Required		Realized		Required		
					CAPM ($\hat{\gamma}\sigma_t^2$)	Traditional ($\omega_a = 0$)	Put-Writing [$Z = -1, L = 2$]		CAPM ($\beta_t \cdot \hat{\gamma}\sigma_t^2$)	Traditional ($\omega_a = 0$)	Specialized ($\omega_a = 0.35$)
Year	0.8 · VIX	Realized	S&P 500								Specialized ($\omega_a = 0.50$)
1996	13.1%	11.4%	16.5%		3.5%	3.6%	10.0%		1.5%	1.9%	2.6%
1997	18.4%	17.2%	25.4%		7.0%	7.4%	13.8%		2.8%	3.9%	5.9%
1998	20.8%	18.6%	23.4%		9.4%	10.0%	14.7%		3.6%	5.4%	8.2%
1999	19.6%	18.0%	15.7%		7.8%	8.2%	19.5%		3.1%	4.3%	6.5%
2000	18.5%	21.9%	-13.1%		7.1%	7.4%	7.5%		2.8%	3.8%	5.8%
2001	20.6%	21.0%	-14.6%		8.8%	9.2%	2.6%		3.5%	5.0%	7.6%
2002	20.9%	24.6%	-24.0%		9.3%	9.9%	1.7%		3.8%	5.5%	8.3%
2003	18.1%	16.5%	25.1%		7.0%	7.3%	20.6%		2.9%	3.9%	5.9%
2004	12.5%	11.0%	9.5%		3.1%	3.2%	13.4%		1.4%	1.7%	2.5%

Table V—Continued

Panel A: Excess Returns									
Volatility		S&P 500 Index			Put-Writing $[Z = -1, L = 2]$				
		Realized		Required	Realized		Required		
				CAPM ($\gamma\sigma^2$)	Traditional ($\omega_a = 0$)	Put-Writing $[Z = -1, L = 2]$	CAPM ($\beta_t \cdot \gamma\sigma^2$)	Traditional ($\omega_a = 0$)	Specialized ($\omega_a = 0.35$)
Year	0.8 · VIX	Realized	S&P 500						Specialized ($\omega_a = 0.50$)
2005	10.4%	10.1%	2.4%	2.2%	2.3%	8.7%	1.0%	1.2%	1.7%
2006	10.1%	9.8%	10.1%	2.1%	2.1%	9.8%	0.9%	1.1%	1.6%
2007	13.5%	15.1%	1.5%	3.9%	4.0%	9.8%	1.7%	2.2%	3.2%
2008	24.1%	35.2%	−44.0%	14.1%	15.4%	−10.9%	5.3%	9.2%	13.9%
2009	26.7%	25.0%	25.9%	15.2%	16.4%	20.5%	6.2%	10.0%	15.0%
2010	19.3%	16.8%	15.8%	7.8%	8.2%	11.9%	3.6%	4.9%	7.2%
2011	18.5%	20.4%	2.9%	7.8%	8.2%	11.0%	3.5%	5.0%	7.3%
2012	15.7%	12.4%	19.4%	5.1%	5.3%	11.3%	2.3%	2.9%	4.3%
Mean	17.7%	17.9%	5.4%	7.2%***	7.6%***	10.3%***	2.9%***	4.3%***	6.4%***
			[1.3]	[16.7]	[15.8]	[5.4]	[19.4]	[14.5]	[14.3]
									[14.0]
Panel B: Investor Alphas									
NIG Skewness = −1									
		S&P 500		Put-Writing $[Z = -1, L = 2]$	Put-Writing $[Z = -2, L = 4]$	NIG Skewness = −1.5			
						S&P 500	Put-Writing $[Z = -1, L = 2]$	Put-Writing $[Z = -2, L = 4]$	
Realized excess return, R* CAPM R* alpha		5.4%	10.3%	11.5%	11.5%	5.4%	10.3%	11.5%	
		7.2%	2.9%	2.3%	2.3%	7.2%	3.7%	2.9%	
		−1.8%	7.4%***	9.2%***	9.2%***	−1.8%	6.7%***	8.6%***	
		[−0.5]	[3.9]	[6.1]	[6.1]	[−0.5]	[3.5]	[5.8]	
Model R* (traditional, $\omega_a = 0$) alpha		7.6%	4.3%	3.8%	3.8%	7.8%	5.0%	4.5%	
		−2.3%	6.1%***	7.8%***	7.8%***	−2.4%	5.3%***	7.1%***	
		[−0.6]	[3.2]	[5.2]	[5.2]	[−0.6]	[2.8]	[4.8]	
Model R* (specialized, $\omega_a = 0.35$) alpha		7.6%	6.4%	9.1%	9.1%	7.8%	7.1%	10.0%	
		−2.3%	3.9%***	2.4%	2.4%	−2.4%	3.2%*	1.5%	
		[−0.6]	[2.1]	[1.6]	[1.6]	[−0.6]	[1.7]	[1.0]	
Model R* (specialized, $\omega_a = 0.50$) alpha		7.6%	7.7%	12.8%	12.8%	7.8%	8.2%	13.6%	
		−2.3%	2.7%	−1.3%	−1.3%	−2.4%	2.1%	−2.1%	
		[−0.6]	[1.4]	[−0.8]	[−0.8]	[−0.6]	[1.1]	[−1.3]	

required excess return for the traditional investor is 6.2%. These computations suggest that our model calibration produces sensible required rates of return for traditional investments and that the sample period is not particularly unusual.

We now turn to an evaluation of the investment with an explicit downside risk exposure. The $[Z = -1, L = 2]$ put-writing portfolio has a mean realized excess return of 10.3% per annum. A specialized investor who is forced to allocate 35% to this investment requires an excess return of 6.4% per annum, or 7.7% if forced to allocate 50%. Both of these requirements are considerably higher than the CAPM required excess rate of return of 2.9% or the 4.3% excess return that would be required if the investor was able to hold a small allocation. The model risk premium is very volatile, averaging 15% to 17% for specialized investors with large allocations in 2008 and 2009, when both the VIX and realized volatility were high. The Internet Appendix presents the corresponding analysis for the $[Z = -2, L = 4]$ put-writing portfolio.

Panel B of Table V reports estimated alphas based on the Gaussian CAPM and the generalized model required rate of return for the S&P 500 index and the two put-writing portfolios under various assumptions. The CAPM model consistently indicates the highest annualized alphas for put-writing strategies, with point estimates of 7.4% (t -statistic = 3.9) for the $[Z = -1, L = 2]$ put-writing portfolio and 9.2% (t -statistic = 6.1) for the $[Z = -2, L = 4]$ put-writing portfolio. These high values reflect the implicit assumption of an infinitesimal allocation to the put-writing strategy, along with the failure to account for the non-Gaussian distribution of the market portfolio. Maintaining infinitesimal allocations but accounting for the non-Gaussian distribution increases the required rates of return (traditional, $\omega_a = 0$) by roughly 1.5% per annum relative to the CAPM, but the alphas remain statistically significant. Finally, when the allocation is fixed to be large at either 35% or 50% of the specialized investor's wealth, the required rates of return become sufficiently high to render the abnormal returns indistinguishable from zero. In particular, note that the required rate of return on the $[Z = -2, L = 4]$ put-writing portfolio begins to exceed that of the $[Z = -1, L = 2]$ portfolio, even though the latter has a higher measured CAPM beta in the sample (Table I). At an allocation of 35%, the annualized alpha is 3.9% (t -statistic = 2.1) for the $[Z = -1, L = 2]$ put-writing portfolio and a statistically unreliable 2.4% (t -statistic = 1.6) for the $[Z = -2, L = 4]$ put-writing portfolio. At a 50% allocation, the alphas to both put-writing portfolios are insignificant. Panel B illustrates the effect of a more negative skewed equity index distribution (skewness = -1.5) on the required rates of return on various investments. While the required rate of return for the equity index increases by only 20 basis points per annum, the corresponding required rates of return for the put-writing portfolios held by specialized investors increase nearly 1%.

E.2. A New Perspective on the Expensiveness of Index Put Options

A specialized investor with a 35% allocation to the put-writing portfolios realizes a statistically significant alpha from the $[Z = -1, L = 2]$ put-writing portfolio, and a statistically insignificant alpha for the $[Z = -2, L = 4]$ put-writing

portfolio, despite the fact that this portfolio looks better based on traditional mean-variance metrics. Neither of the portfolios produces statistically significant alphas once the specialized investor is forced to have a 50% allocation. These findings contrast with much of the existing literature, which documents high negative (positive) risk-adjusted returns to buying (selling) index options (e.g., Coval and Shumway (2001), Bakshi and Kapadia (2003), Bondarenko (2003), Frazzini and Pedersen (2012), Constantinides, Jackwerth, and Savov (2013)).¹⁶ The conclusion of index put options being highly expensive implicitly assumes that an investor who is short these portfolios would earn the negative of the long portfolio returns. This is far from the reality, as an investor with a short position would be required to post sufficient margin to initiate the position and maintain sufficient margin to survive the sample paths realized ex post in the data (Santa-Clara and Saretto (2009)).

The annualized alphas we report are an order of magnitude lower than reported in previous papers. This difference is due to (1) incorporating margin requirements, as emphasized by Santa-Clara and Saretto (2009), (2) a cost of capital computation that explicitly accounts for the nonlinearity of the payoff profiles, and, importantly, (3) investor portfolio concentration. The large margin requirements for short positions in index put options effectively make these positive net supply assets; the supplier of these payoffs has to allocate considerably more capital to this activity than that implied in the frictionless models of Black and Scholes (1973) and Merton (1973). Moreover, this is a risk that is not well distributed throughout the economy, as the suppliers of these securities are typically highly specialized in bearing this risk. The same channel highlighted in our paper—concentrated portfolios require additional risk premium above the frictionless model, especially when a nonlinear downside exposure is present—manifests here. From the perspective of the frictionless model, both alternative investments and index put options seem expensive, but much less so from the perspective of specialized investors (see also Garleanu, Pedersen, and Poteshman (2009)). These two frictionless model anomalies are fairly consistent with one another after accounting for these notable features.

Finally, it is important to recall that these calculations rely upon a specific distributional assumption about the underlying stock market index, which is roughly consistent with historical experience. A slightly worse left tail will have a meaningful effect on the required returns for these portfolios, given their nonlinear risk profiles and the large allocation sizes, as illustrated in the right-hand panel of Panel B.

¹⁶ Coval and Shumway (2001) report that zero-beta, at-the-money straddle positions produce average losses of approximately 3% per week. Bakshi and Kapadia (2003) conclude in favor of a negative volatility risk premium by examining delta-hedged options returns. Frazzini and Pedersen (2012) report mean monthly delta-hedged excess returns between -9.5% (at-the-money) and -30% (deep out-of-the-money) for one-month index put options.

IV. Conclusion

Standard linear factor models (CAPM, Fama–French, Fung–Hsieh) indicate that the required excess rate of return for hedge fund indices is equal to roughly 40% of the excess rate of return on the equity index, which implies that hedge funds have earned pre-fee alphas between 6% to 10% per annum (January 1996 to June 2012). We demonstrate, however, that the pre-fee returns of broad hedge fund indices are well matched by mechanical S&P 500 put-writing strategies, suggesting that hedge fund managers do not earn alpha, but rather are compensated for bearing downside market risks. Distinguishing between these two views of capital market efficiency hinges critically on a strong prior belief in the accuracy of the hedge fund return reporting process, which has been challenged by a growing literature documenting conditional and unconditional return smoothing (Asness, Krail, and Liew (2001), Getmansky, Lo, and Makarov (2004), Bollen and Pool (2008)), as well as manager discretion in marking portfolio NAVs (Cassar and Gerakos (2011), Cao et al. (2013)). In particular, we show that smoothing two reported monthly returns (September 1998 and October 2008) is sufficient to statistically obscure the exposure to downside market risks, which can lead one to conclude in favor of high pre-fee alphas. An investor who is skeptical about the quality of the data is not able to reliably reject the presence of downside risk.

We show that the high realized excess returns to put-writing strategies and pre-fee hedge fund index returns are consistent with an equilibrium in which a small subset of investors bears the aggregate supply of downside risks. In practice, there is evidence that end users of hedge fund investments (Lerner, Schoar, and Wang (2008)) and the marginal price setters in equity index options, typically viewed to be the market makers (Garleanu, Pedersen, and Poteshman (2009)), are specialized and therefore hold concentrated portfolios. Merton (1987) emphasizes that, in an equilibrium with segmentation, assets requiring specialization will produce positive intercepts in regressions onto common market factors even though there is no true alpha, with the intercept capturing a concentration premium. We demonstrate that the required concentration premium is particularly large when the specialized asset has a payoff profile with downside risk relative to the market factor.

The transparency of the state-contingent payoffs of various put-writing portfolios allows us to develop cost of capital estimates for potentially large allocations to investments explicitly exposed to downside market risks. The model required rates of return vary as a function of investor preferences and allocations, the nonlinearity of the portfolio (option strike price and leverage), and the properties of the underlying equity market return distribution (volatility and tail risks). We find that the proper required excess rates of return—reflecting the nonlinearity of the payoff profile and large allocation—can be significantly higher than those indicated by models focusing on marginal deviations from allocations in a frictionless equilibrium. For example, using two put-writing strategies that are statistically indistinguishable from the pre-fee returns of the hedge fund index and portfolio allocations comparable to allocations to

alternatives at endowments, the proper required excess rates of return range from 6% to 14% per annum. This offers a dramatically different perspective on the cost of capital for alternative investments, which cannot be reliably rejected by investors concerned about return reporting errors.

Initial submission: April 25, 2012; Final version received: March 4, 2015
Editor: Campbell Harvey

Appendix: Asset Pricing with NIG Distributions

The normal inverse Gaussian (NIG) distribution is characterized by four parameters, (a, b, c, d) . The first two parameters control the tail heaviness and asymmetry, and the second two—the location and scale of the distribution. The density of the NIG distribution is given by

$$f(x; a, b, c, d) = \frac{a \cdot d \cdot K_1 \left(a \cdot \sqrt{d^2 + (x - c)^2} \right)}{\pi \cdot \sqrt{d^2 + (x - c)^2}} \cdot \exp(d \cdot \eta + b \cdot (x - c)), \quad (\text{A1})$$

where K_1 is the modified Bessel function of the third kind with index 1 (Abramowitz and Stegun (1965)) and $\eta = \sqrt{a^2 - b^2}$ with $0 \leq |b| < a$. Given the desired set of moments for the NIG distribution—mean (\mathcal{M}), variance (\mathcal{V}), skewness (\mathcal{S}), and kurtosis (\mathcal{K})—the parameters of the distribution can be obtained from

$$a = \sqrt{\frac{3 \cdot \mathcal{K} - 4 \cdot \mathcal{S}^2 - 9}{\mathcal{V} \cdot (\mathcal{K} - \frac{5}{3} \cdot \mathcal{S}^2 - 3)^2}}, \quad (\text{A2})$$

$$b = \frac{\mathcal{S}}{\sqrt{\mathcal{V}} \cdot (\mathcal{K} - \frac{5}{3} \cdot \mathcal{S}^2 - 3)}, \quad (\text{A3})$$

$$c = \mathcal{M} - \frac{3 \cdot \mathcal{S} \cdot \sqrt{\mathcal{V}}}{3 \cdot \mathcal{K} - 4 \cdot \mathcal{S}^2 - 9}, \quad (\text{A4})$$

$$d = \frac{3^{\frac{3}{2}} \cdot \sqrt{\mathcal{V} \cdot (\mathcal{K} - \frac{5}{3} \cdot \mathcal{S}^2 - 3)}}{3 \cdot \mathcal{K} - 4 \cdot \mathcal{S}^2 - 9}. \quad (\text{A5})$$

In order for the distribution to be well-defined we need, $\mathcal{K} > 3 + \frac{5}{3} \cdot \mathcal{S}^2$. The NIG-distribution has closed-form expressions for its moment-generating and characteristic functions, which are convenient for deriving equilibrium risk premia and option prices. Specifically, the moment generating function is

$$E[\exp(u \cdot x)] = \exp(c \cdot u + d \cdot (\eta - \sqrt{a^2 - (b + u)^2})). \quad (\text{A6})$$

A.1. Pricing Kernel and Risk Premia

Suppose the value of the aggregate wealth portfolio evolves according to

$$W_{t+\tau} = W_t \cdot \exp((\mu - k_Z(1)) \cdot \tau + Z_{t+\tau}), \quad (\text{A7})$$

where $k_Z(u)$ is the cumulant generating function of random variable $Z_{t+\tau}$:

$$k_Z(u) = \frac{1}{\tau} \cdot \ln E_t[\exp(u \cdot Z_{t+\tau})] = c \cdot u + d \cdot \left(\eta - \sqrt{a^2 - (b + u)^2} \right). \quad (\text{8})$$

If markets are complete, there will exist a unique pricing kernel, $\Lambda_{t+\tau}$, which prices the wealth portfolio as well as the risk-free asset. Assuming the representative agent has CRRA utility with coefficient of relative risk aversion, γ , the pricing kernel in the economy is an exponential martingale given by

$$\frac{\Lambda_{t+\tau}}{\Lambda_t} = \exp(-r_f \cdot \tau - \gamma \cdot Z_{t+\tau} - k_Z(-\gamma) \cdot \tau). \quad (\text{A9})$$

Now consider assets whose terminal payoff has a linear loading, β , on the aggregate shock, $Z_{t+\tau}$, and an independent idiosyncratic shock, $Z_{i,t+\tau}$:

$$P_{t+\tau} = P_t \cdot \exp((\mu(\beta) - k_Z(\beta) - k_{Z_i}(1)) \cdot \tau + \beta \cdot Z_{t+\tau} + Z_{i,t+\tau}), \quad (\text{A10})$$

where $\mu(\beta)$ is the equilibrium rate of return on the asset, and the two $k(\cdot)$ terms compensate for the convexity of the systematic and idiosyncratic innovations. For example, when $\beta = 1$ and the variance of the idiosyncratic shocks goes to zero, the asset converges to a claim on the aggregate wealth portfolio. Assets with $\beta < 1$ ($\beta > 1$) are concave (convex) with respect to the aggregate wealth portfolio.

To derive the equilibrium risk premium for such assets, we make use of the equilibrium pricing condition:

$$\Lambda_t \cdot P_t = E_t[\Lambda_{t+\tau} \cdot P_{t+\tau}] \quad \Leftrightarrow \quad 0 = \frac{1}{\tau} \cdot \ln E_t \left[\frac{\Lambda_{t+\tau}}{\Lambda_t} \cdot \frac{P_{t+\tau}}{P_t} \right]. \quad (\text{A11})$$

Substituting the payoff function into the above condition and taking advantage of the independence of the aggregate and idiosyncratic shocks yields the following expression for the equilibrium risk premium on an asset with loading β on the aggregate wealth shock:

$$\mu(\beta) - r_f = k_Z(-\gamma) + k_Z(\beta) - k_Z(\beta - \gamma). \quad (\text{A12})$$

This expression generalizes the standard CAPM risk-premium expression from mean-variance analysis to allow for the existence of higher moments in the shocks to the aggregate market portfolio. For a Gaussian-distributed shock, $Z_{t+\tau}$, the cumulant generating function is given by $k_Z(u) = \frac{1}{\tau} \cdot \frac{(\sigma \cdot \sqrt{\tau} \cdot u)^2}{2}$, such that, (A12), specializes to

$$\begin{aligned}\mu(\beta) - r_f &= \frac{\sigma^2}{2} \cdot ((-\gamma)^2 + \beta^2 - (\beta - \gamma)^2) \\ &= \beta \cdot \gamma \cdot \sigma^2 = \beta \cdot (\mu(1) - r_f).\end{aligned}\quad (\text{A13})$$

In our generalized setting, the risk premium on an asset with loading β on the innovations to the market portfolio does not equal β times the market risk premium, unlike in the standard CAPM. The discrepancy is specifically related to the existence of higher moments in the shocks to the aggregate market portfolio.

Equilibrium risk premia can also be linked to the moments of the underlying distribution of the shocks to the aggregate portfolio by taking advantage of an infinite series expansion of the cumulant generating function and the underlying cumulants of the distribution of $Z_{t+\tau}$:

$$\mu(\beta) - r_f = \frac{1}{\tau} \cdot \sum_{n=2}^{\infty} \frac{\kappa_n \cdot ((-\gamma)^n + \beta^n - (\beta - \gamma)^n)}{n!}.\quad (\text{A14})$$

The consecutive cumulants, κ_n , are obtained by evaluating the n^{th} derivative of the cumulant generating function at $u = 0$. The cumulants can then be mapped to central moments: $\kappa_2 = \mathcal{V}$, $\kappa_3 = \mathcal{S} \cdot \mathcal{V}^{\frac{3}{2}}$, and $\kappa_4 = \mathcal{K} \cdot \mathcal{V}^2$. Using the value for the first four terms, the equilibrium risk premium is approximately equal to

$$\begin{aligned}\mu(\beta) - r_f \approx \frac{1}{\tau} \cdot \left\{ \beta \cdot \gamma \cdot \mathcal{V} + \frac{\beta^2 \cdot \gamma - \beta \cdot \gamma^2}{2} \cdot \mathcal{S} \cdot \mathcal{V}^{\frac{3}{2}} \right. \\ \left. + \frac{2 \cdot \beta^3 \cdot \gamma - 3 \cdot \beta^2 \cdot \gamma^2 + 2 \cdot \beta \cdot \gamma^3}{12} \cdot \mathcal{K} \cdot \mathcal{V}^2 \right\}\end{aligned}\quad (\text{A15})$$

This expression demonstrates the degree to which the agent demands compensation for exposure to higher moments, and illustrates the degree to which the standard Gaussian CAPM over- or understates the required rate of return for an asset with a given market beta, β .

A.2. The Risk-Neutral Distribution

Suppose the historical (\mathbb{P} -measure) distribution of the shocks, $Z_{t+\tau}$, is NIG(a, b, c, d). The risk-neutral distribution, $\pi^{\mathbb{Q}} = \pi^{\mathbb{P}} \cdot \frac{\Lambda_{t+\tau}}{\Lambda_t}$, can also be shown to be the NIG class, but with perturbed parameters NIG($a, b - \gamma, c, d$). To see this, substitute the expression for the \mathbb{P} -density into the definition of the \mathbb{Q} -density to obtain:

$$\begin{aligned}\pi^{\mathbb{Q}} &= \frac{a \cdot d \cdot K_1 \left(a \cdot \sqrt{d^2 + (Z_{t+\tau} - c)^2} \right)}{\pi \cdot \sqrt{d^2 + (Z_{t+\tau} - c)^2}} \\ &\quad \cdot \exp \left(d \cdot \eta + (b - \gamma) \cdot (Z_{t+\tau} - c) - \gamma \cdot c - k_Z(-\gamma) \cdot \tau \right),\end{aligned}\quad (\text{A16})$$

where $\eta = \sqrt{a^2 - b^2}$. Making use of the expression for the cumulant generating function of the NIG distribution, the above formula can be rearranged to yield

$$\pi^{\mathbb{Q}} = \frac{a \cdot d \cdot K_1 \left(a \cdot \sqrt{d^2 + (Z_{t+\tau} - c)^2} \right)}{\pi \cdot \sqrt{d^2 + (Z_{t+\tau} - c)^2}} \cdot \exp(d \cdot \tilde{\eta} + \tilde{b} \cdot (Z_{t+\tau} - c)), \quad (\text{A17})$$

where we have introduced the perturbed parameters, $\tilde{b} = b - \gamma$, and $\tilde{\eta} = \sqrt{a^2 - \tilde{b}^2}$. This verifies that the risk-neutral (\mathbb{Q} -measure) distribution is also an NIG distribution, but with shifted parameters, (a, \tilde{b}, c, d) .

REFERENCES

- Agarwal, Vikas, and Narayan Y. Naik, 2004, Risks and portfolio decisions involving hedge funds, *Review of Financial Studies* 17, 63–98.
- Arrow, Kenneth J., 1964, The role of securities in the optimal allocation of risk bearing, *Review of Economic Studies* 31, 91–96.
- Asness, Clifford, Robert Krail, and John Liew, 2001, Do hedge funds hedge? *Journal of Portfolio Management* 28, 1–19.
- Backus, David K., Mikhail Chernov, and Ian Martin, 2011, Disasters implied by equity index options, *Journal of Finance* 66, 1969–2012.
- Bakshi, Gurdip, and Nikunj Kapadia, 2003, Delta-hedged gains and the negative market volatility risk premium, *Review of Financial Studies* 16, 527–566.
- Barro, Robert J., 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.
- Barro, Robert J., Emi Nakamura, Jón Steinsson, and José F. Ursúa, 2013, Crises and recoveries in an empirical model of consumption disasters, *American Economic Journal: Macroeconomics* 5, 35–74.
- Barro, Robert J., and José F. Ursua, 2008, Macroeconomic crises since 1870, *Brookings Papers on Economic Activity* 39, 255–350.
- Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 617–654.
- Bollen, Nicolas P. B., and Veronika K. Pool, 2008, Conditional return smoothing in the hedge fund industry, *Journal of Financial and Quantitative Analysis* 43, 267–298.
- Bollen, Nicolas P. B., and Veronika K. Pool, 2009, Do hedge fund managers misreport returns? Evidence from the pooled distribution, *Journal of Finance* 64, 2257–2288.
- Bondarenko, Oleg, 2003, Why are put options so expensive? *Quarterly Journal of Finance* 4, 64–113.
- Brunnermeier, Markus, and Lasse H. Pedersen, 2009, Market liquidity and funding liquidity, *Review of Financial Studies* 22, 2201–2238.
- Cao, Charles, Grant V. Farnsworth, Bing Liang, and Andrew W. Lo, 2013, Liquidity costs, return smoothing, and investor flows: Evidence from a separate account platform, Working paper, MIT.
- Carhart, Mark, 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.
- Cassar, Gavin, and Joseph Gerakos, 2011, Hedge funds: Pricing controls and the smoothing of self-reported returns, *Review of Financial Studies* 14, 1689–1734.
- Constantinides, George, Jens C. Jackwerth, and Alexei Z. Savov, 2013, The puzzle of index option returns, *Review of Asset Pricing Studies* 3, 229–257.
- Coval, Joshua D., and Tyler Shumway, 2001, Expected option returns, *Journal of Finance* 56, 983–1009.
- Debreu, Gérard 1959, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium* (Yale University Press, New Haven and London).
- Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

- Fama, Eugene F., and Kenneth R. French, 2010, Luck versus skill in the cross-section of mutual fund returns, *Journal of Finance* 65, 1915–1947.
- Frazzini, Andrea, and Lasse H. Pedersen, 2012, Embedded leverage, Working paper, NYU Stern School of Business.
- French, Kenneth, 2008, Presidential address: The cost of active investing, *Journal of Finance* 58, 1537–1573.
- Fung, William, and David Hsieh, 2001, The risk in hedge fund strategies: Theory and evidence from trend followers, *Review of Financial Studies* 14, 313–341.
- Fung, William, and David Hsieh, 2002, Asset-based hedge-fund styles factors for hedge funds, *Financial Analysts Journal* 58, 16–27.
- Fung, William, and David Hsieh, 2004, Hedge fund benchmarks: A risk based approach, *Financial Analysts Journal* 60, 65–80.
- Garleanu, Nicolae, Lasse H. Pedersen, and A. Potesman, 2009, Demand-based option pricing, *Review of Financial Studies* 22, 4259–4299.
- Getmansky, Mila, Andrew W. Lo, and Igor Makarov, 2004, An econometric model of serial correlation and illiquidity in hedge fund returns, *Journal of Financial Economics* 74, 529–609.
- Glosten, Lawrence R., and Ravi Jagannathan, 1994, A contingent claim approach to performance evaluation, *Journal of Empirical Finance* 1, 133–160.
- Harvey, Campbell R., John C. Liechty, Merrill W. Liechty, and Peter Muller, 2010, Portfolio selection with higher moments, *Quantitative Finance* 10, 469–485.
- Hasanhodzic, Jasmina, and Andrew W. Lo, 2007, Can hedge-fund returns be replicated?: The linear case, *Journal of Investment Management* 5, 5–45.
- Ibbotson, Roger G., Peng Chen, and Kevin X. Zhu, 2010, The ABCs of hedge funds: Alphas, betas, and costs, Working paper, Yale School of Management.
- Jarque, Carlos M., and Anil K. Bera, 1980, Efficient tests for normality, homoscedasticity, and serial independence of regression residuals, *Economic Letters* 6, 255–259.
- Lerner, Josh, Antoinette Shoar, and Jialan Wang, 2008, Secrets of the academy: The drivers of university endowment success, *Journal of Economic Perspectives* 22, 207–222.
- Linter, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics of Statistics* 47, 13–37.
- Lo, Andrew W., 2001, Risk management for hedge funds: Introduction and overview, *Financial Analysts Journal* 6, 16–33.
- Malkiel, Burton G., and Atanu Saha, 2005, Hedge funds: Risk and return, *Financial Analysts Journal* 61, 80–88.
- Martellini, Lionel, and Volker Ziemann, 2010, Improved estimates of higher-order comoments and implications for portfolio selection, *Review of Financial Studies* 23, 1467–1502.
- Merton, Robert C., 1973, Theory of rational option pricing, *Bell Journal of Economics and Management Science* 4, 141–183.
- Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.
- Merton, Robert C., 1981, On market timing and investment performance: An equilibrium theory of value for market forecasts, *The Journal of Business* 54, 363–406.
- Merton, Robert C., 1987, A simple model of capital market equilibrium with incomplete information, *Journal of Finance* 42, 483–510.
- Mitchell, Mark, and Todd Pulvino, 2001, Characteristics of risk in risk arbitrage, *Journal of Finance*, 56, 2135–2175.
- Patton, Andrew J., 2004, On the out-of-sample importance of skewness and asymmetric dependence for asset allocation, *Journal of Financial Econometrics* 2, 130–168.
- Patton, Andrew J., 2009, Are “market neutral” hedge funds really market neutral?, *Review of Financial Studies* 22, 2495–2530.
- Santa-Clara, Pedro, and Alessio Saretto, 2009, Option strategies: Good deals and margin calls, *Journal of Financial Markets* 12, 391–417.
- Scholes, Myron, and Joseph Williams, 1977, Estimating beta from non-synchronous data, *Journal of Financial Economics* 5, 309–327.

Sharpe, William F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Shleifer, Andrei, and Robert W. Vishny, 1997, The limits of arbitrage, *Journal of Finance* 52, 35–55.

Todorov, Viktor, 2010, Variance risk premium dynamics: The role of jumps, *Review of Financial Studies* 23, 345–383.

Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.