JOHNSON
Cornell University

NBA 5420: Investment and Portfolio Management

Class 7: Fixed Income I

Professor Matt Baron

March 16, 2016

Midterm

- We need to split the class into two classrooms
  - so you have enough space to take the exam
- Morning class (10:10 – 11:25 AM)
  - last name starts with A-H: go to Sage B10
  - last name starts with I-Z: normal classroom (Sage B09)
- Afternoon class (1:25 – 2:40 PM)
  - last name starts with A-L: go to Sage B08
  - last name starts with M-Z: normal classroom (Sage B05)

Midterm Review

- You can choose which of these (if any) you want to attend:

- **Thursday**, March 17, Sage B05
  - **6:30 – 7:15 PM:** Kate will go over selected problems on the board from the practice midterm questions
  - **7:15 – 8:00 PM:** Kate will informally answer individual questions office-hours-style

- **Friday**, March 18, Sage B05
  - **2:00 – 2:45 PM:** Sam will go over selected problems on the board from the practice midterm questions
  - **2:45 – 3:30 PM:** Sam will informally answer individual questions office hours-style

- My office hours
  - I will also have office hours on Friday from 3:30 – 4:30 PM

For midterm day

- Bring a pen and calculator
- Bring double-sided 8.5’ x 11’ “cheat sheet”
- No books, computers, tablets, cell phones allowed

Fixed Income

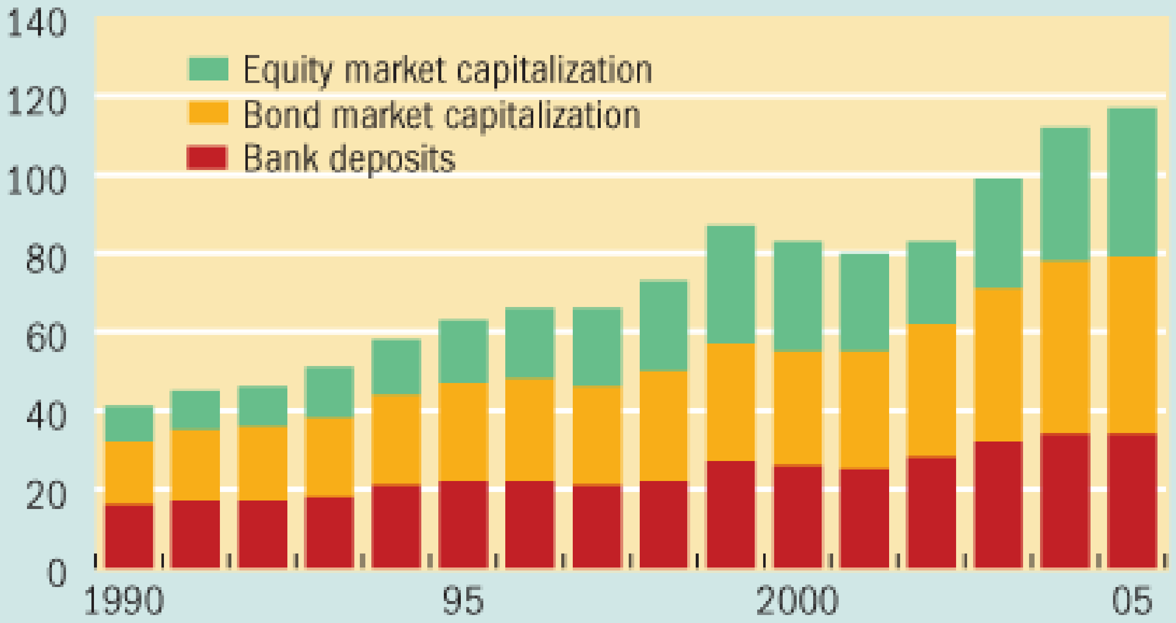
1. Overview of bond markets
2. Bond pricing
3. Duration and convexity
4. The term structure & the expectations hypothesis

Chart 3

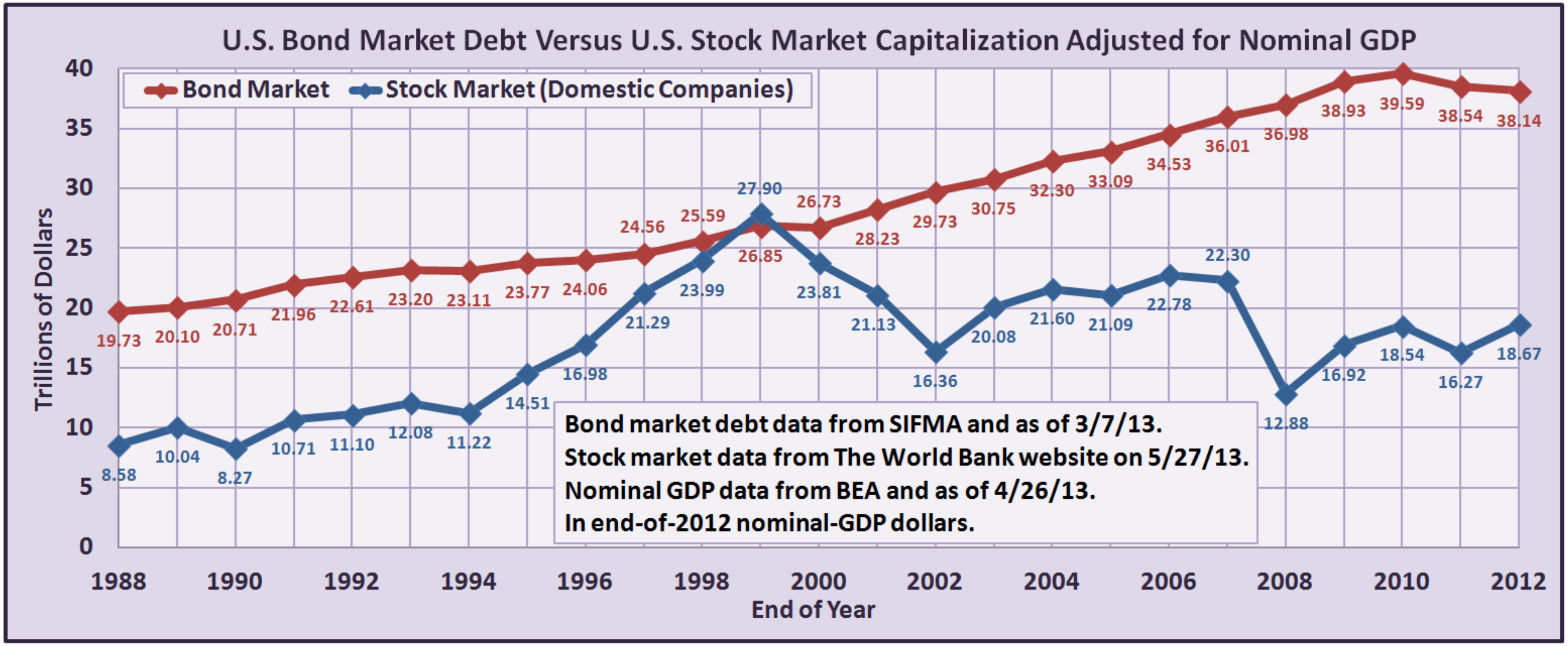
Expanding markets

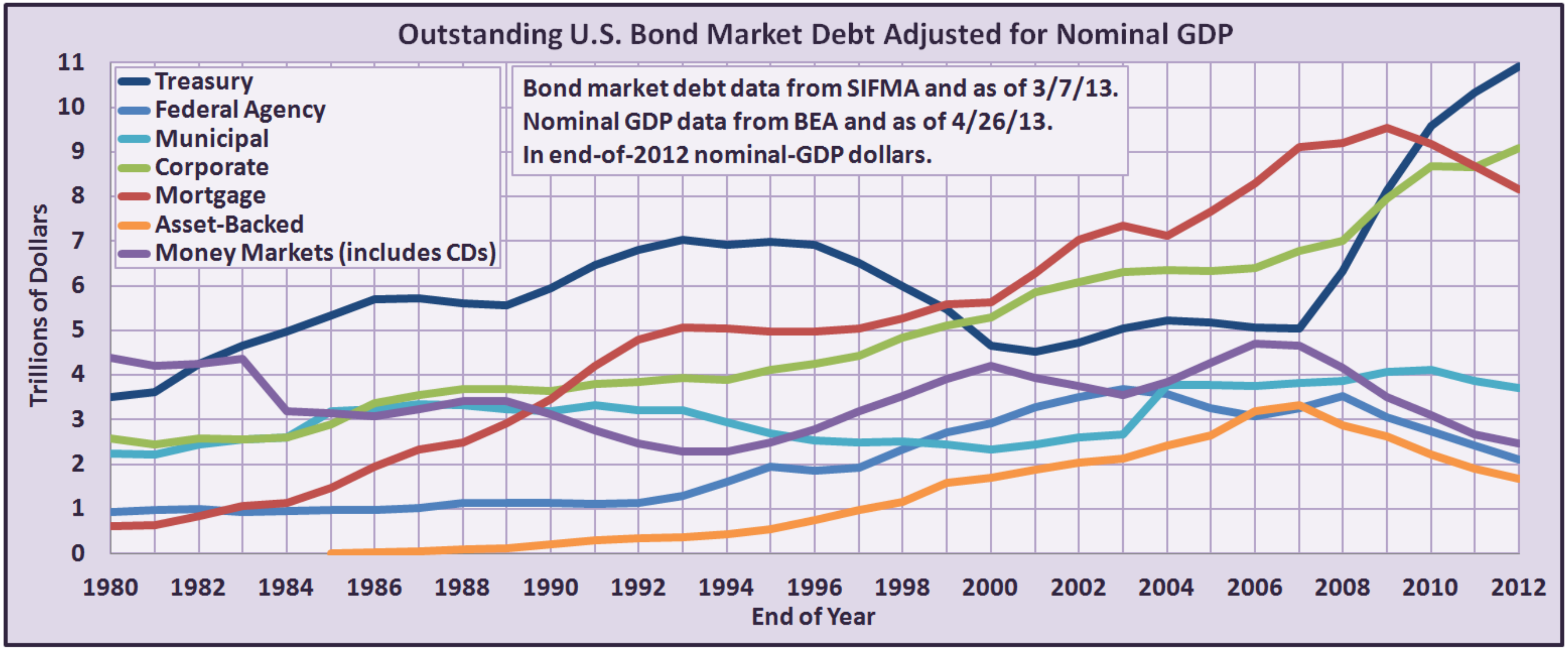
Global bond and equity markets have witnessed unprecedented growth in the past 15 years.

(trillion dollars)



Sources: IMF staff estimates based on S&P/IFC Emerging Market database; World Federation of Exchanges; Datastream; Bank for International Settlements; and International Finance Corporation.


U.S. Bond Market Debt Versus U.S. Stock Market Capitalization Adjusted for Nominal GDP
Bond Market
Stock Market (Domestic Companies)
Trillions of Dollars
End of Year
0
5
10
15
20
25
30
35
40
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
19.73
20.10
20.71
21.96
22.61
23.20
23.11
23.77
24.06
24.56
25.59
26.85
26.73
28.23
29.73
30.75
32.30
33.09
34.53
36.01
36.98
38.93
39.59
38.54
38.14
8.58
10.04
8.27
10.71
11.10
12.08
11.22
14.51
16.98
21.29
23.99
27.90
23.81
21.13
16.36
20.08
21.60
21.09
22.78
22.30
12.88
16.92
18.54
16.27
18.67
Bond market debt data from SIFMA and as of 3/7/13.
Stock market data from The World Bank website on 5/27/13.
Nominal GDP data from BEA and as of 4/26/13.
In end-of-2012 nominal-GDP dollars.


Outstanding U.S. Bond Market Debt Adjusted for Nominal GDP
Treasury
Federal Agency
Municipal
Corporate
Mortgage
Asset-Backed
Money Markets (includes CDs)
Bond market debt data from SIFMA and as of 3/7/13.
Nominal GDP data from BEA and as of 4/26/13.
In end-of-2012 nominal-GDP dollars.
Trillions of Dollars
0
1
2
3
4
5
6
7
8
9
10
11
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
End of Year

Bond characteristics

- Issuers
  - Governments or sovereigns
    - US Treasuries:
      - “Bills” < 1 yr maturity
      - “Notes” between 1 and 10 year maturities
      - “Bonds” > 10 maturity
    - UK bonds are called “gilts”
  - Corporations (investment grade vs. junk)
  - Municipalities (usually tax-advantaged)
  - Structured Finance
    - Mortgage backed securities (MBS)
    - Collateralized default obligations (CDOs)

Trading

- Over the Counter (OTC) search markets
  - Intermediated via broker-dealers
  - Buyers and sellers are usually large institutional investors who hold long-term
    - e.g. insurance companies and pension funds

- Illiquid, high transaction costs
  - SEC mandated TRACE (post-trade transparency) hopefully is leading to more liquidity and competitive pricing

Bond characteristics

- Credit Ratings
  – Moody's, Standard and Poor's, Fitch

Grade	Risk	Moody's	S&P/Fitch
Investment	Quality Highest	Aaa	AAA
Investment	High Quality	AA	Aa
Investment	Strong	A	A
Investment	Medium Grade	Baa	BBB
Junk	Speculative	Ba.B	BB,B
Junk	Highly Speculative	Caa/Ca/C	CCC/CC/C
Junk	In Default	C	D

$$\boxed{P_B = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{\text{Par}}{(1+y)^T}}$$

1. Par = Par Value (or Face Value)
   – the cash you get paid at maturity T (e.g., $1000)

2. C = Coupon Rate
   – e.g., 5% semi-annually (meaning $25 paid twice a year)
   – in this class, we'll assume everything is annual for simplicity
   – In practice, coupon could also be floating or inflation-indexed

3. y = yield to maturity (i.e. the discount rate)
   – The effective interest rate of the bond
   – Which is determined by market forces and varies over time

An example

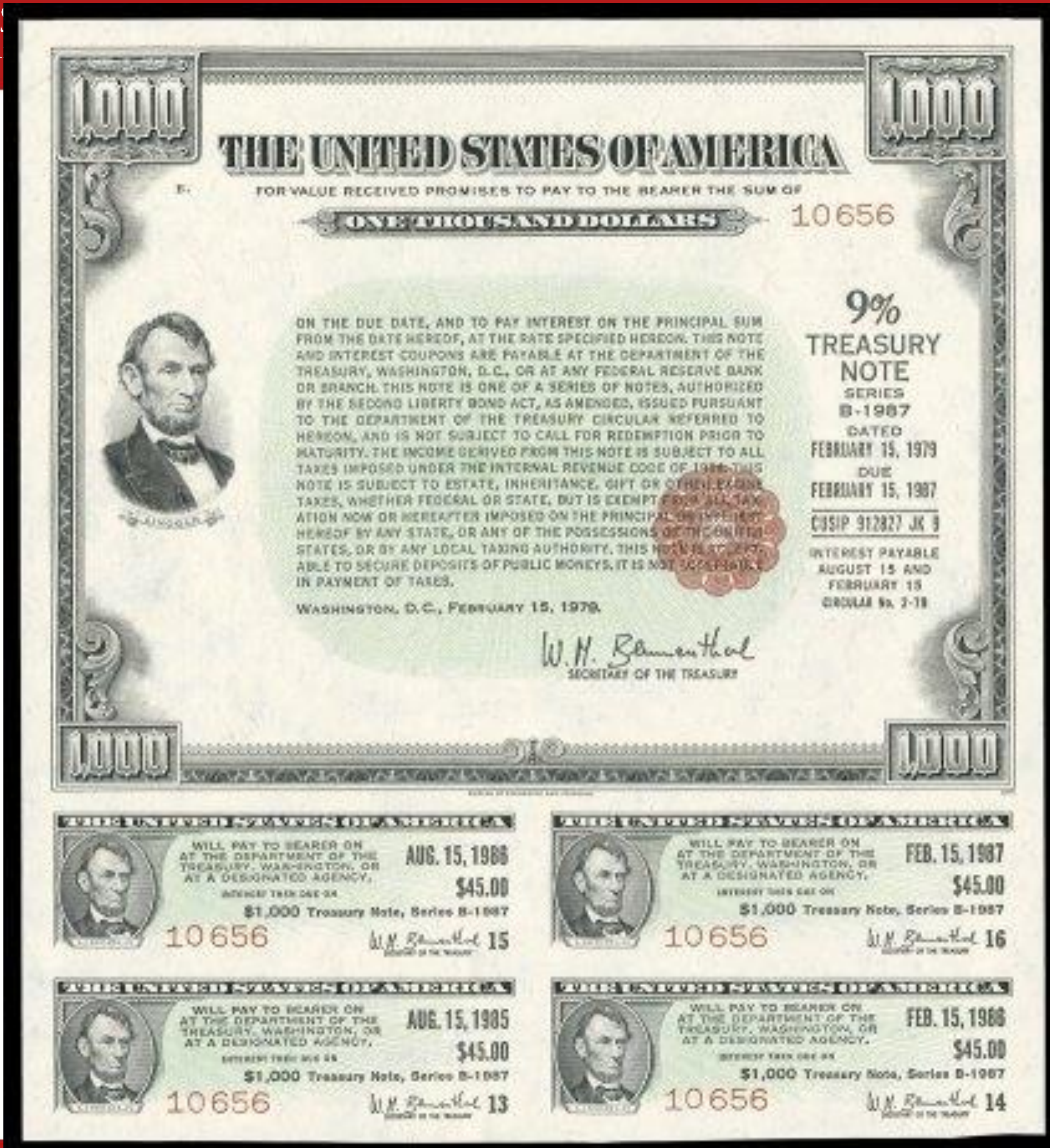
- What is the price of a 5% coupon bond (Par = $1000) making annual coupon payments if it has 5 years until maturity and YTM of 6%?

Years	Cash Flow	Discounted Cash Flow
1	50	47.17
2	50	44.50
3	50	41.98
4	50	39.60
5	1050	784.62

	SUM:	957.88

Bond pricing

4. Extra Features/Provisions will affect pricing
   - Secured v. unsecured
   - Priority in bankruptcy (junior vs. senior)
   - Options:
     - Call provisions
     - Pre-payment option
   - Convertibility (into equity)


1,000
THE UNITED STATES OF AMERICA
FOR VALUE RECEIVED PROMISES TO PAY TO THE BEARER THE SUM OF
ONE THOUSAND DOLLARS
10656
ON THE DUE DATE, AND TO PAY INTEREST ON THE PRINCIPAL SUM FROM THE DATE HEREOF, AT THE RATE SPECIFIED HEREON. THIS NOTE AND INTEREST COUPONS ARE PAYABLE AT THE DEPARTMENT OF THE TREASURY, WASHINGTON, D.C., OR AT ANY FEDERAL RESERVE BANK OR BRANCH. THIS NOTE IS ONE OF A SERIES OF NOTES, AUTHORIZED BY THE SECOND LIBERTY BOND ACT, AS AMENDED, ISSUED PURSUANT TO THE DEPARTMENT OF THE TREASURY CIRCULAR REFERRED TO HEREON, AND IS NOT SUBJECT TO CALL FOR REDEMPTION PRIOR TO MATURITY. THE INCOME DERIVED FROM THIS NOTE IS SUBJECT TO ALL TAXES IMPOSED UNDER THE INTERNAL REVENUE CODE OF 1954. THIS NOTE IS SUBJECT TO ESTATE, INHERITANCE, GIFT OR OTHER EXCISE TAXES, WHETHER FEDERAL OR STATE, BUT IS EXEMPT FROM ALL TAXATION NOW OR HEREAFTER IMPOSED ON THE PRINCIPAL OR INTEREST HEREOF BY ANY STATE, OR ANY OF THE POSSESSIONS OF THE UNITED STATES, OR BY ANY LOCAL TAXING AUTHORITY. THIS NOTE IS ACCEPTABLE TO SECURE DEPOSITS OF PUBLIC MONEYS. IT IS NOT ACCEPTABLE IN PAYMENT OF TAXES.
WASHINGTON, D.C., FEBRUARY 15, 1979.
W. M. Blumenthal
SECRETARY OF THE TREASURY
9%
TREASURY NOTE
SERIES B-1987
DATED FEBRUARY 15, 1979
DUE FEBRUARY 15, 1987
CUSIP 912827 JK 9
INTEREST PAYABLE AUGUST 15 AND FEBRUARY 15
CIRCULAR No. 2-79
THE UNITED STATES OF AMERICA
WILL PAY TO BEARER ON AT THE DEPARTMENT OF THE TREASURY, WASHINGTON, OR AT A DESIGNATED AGENCY,
AUG. 15, 1986
$45.00
$1,000 Treasury Note, Series B-1987
10656
15
FEB. 15, 1987
$45.00
$1,000 Treasury Note, Series B-1987
10656
16
AUG. 15, 1985
$45.00
$1,000 Treasury Note, Series B-1987
10656
13
FEB. 15, 1986
$45.00
$1,000 Treasury Note, Series B-1987
10656
14

Coupons


$ 15. THE $ 15.
Highland Street Railway Company
Will pay to Bearer Dollars on
Jan 1st 1896 at the Rockland Bank
Boston for Six Months Interest on
No 4
Treasurer.


THE UNITED STATES OF AMERICA
36 88 CTS.
JUNE, 1935
ON $50
LIBERTY LOAN OF 1917
1926903
THE UNITED STATES OF AMERICA
32 88 CTS.
JUNE, 1933
ON $50
LIBERTY LOAN OF 1917
1926903
THE UNITED STATES OF AMERICA
28 88 CTS.
JUNE, 1931
ON $50
LIBERTY LOAN OF 1917
1926903
THE UNITED STATES OF AMERICA
35 87 CTS.
DEC., 1934
ON $50
LIBERTY LOAN OF 1917
1926903
THE UNITED STATES OF AMERICA
31 87 CTS.
DEC., 1932
ON $50
LIBERTY LOAN OF 1917
1926903
THE UNITED STATES OF AMERICA
27 87 CTS.
DEC., 1930
ON $50
LIBERTY LOAN OF 1917
1926903
THE UNITED STATES OF AMERICA
34 88 CTS.
JUNE, 1934
ON $50
LIBERTY LOAN OF 1917
1926903
THE UNITED STATES OF AMERICA
30 88 CTS.
JUNE, 1932
ON $50
LIBERTY LOAN OF 1917
1926903
THE UNITED STATES OF AMERICA
26 88 CTS.
JUNE, 1930
ON $50
LIBERTY LOAN OF 1917
1926903

Yield to maturity (YTM)

- y* = YTM
  - Interest rate that makes the present value of the bond's payments equal to its price

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+y^*)^t} + \frac{\text{Par}}{(1+y^*)^T}$$

  - Where P_B is the observed bond price

Yield to maturity (YTM)

- Example:
  - What's the YTM of a bond with 10yr maturity, 7% coupon, face value of 1000, semi-annual payments, and a price of 950?

$$950 = \sum_{t=1}^{20} \frac{35}{(1+y)^{t/2}} + \frac{1000}{(1+y)^{10}}$$

- y = 7.12%
  - Have to solve by guess-and-check

Yield to maturity (YTM)

- Consider a 1-yr bond:
  - P = 900, Par = 1000, C = 50

- The return (YTM) comes from two sources:
  1. The discount
     - Since you buy at P=900, get paid Par=1000 in a year:
     - Return = (1000-900)/1000 = 11.1%

  2. The coupon
     - You get a coupon C=50 in a year on a price of 900
     - Coupon yield = 50/900 = 5.6%

  - Total return = YTM = 11.1% + 5.6% = 16.7%

Another example

- 10-year bond, face value = $1000
- C = semi-annual 8% coupon
- YTM = 6% annualized

$$P = \sum_{t=1}^{20} \frac{40}{1.06^{t/2}} + \frac{1000}{1.06^{10}} = 1155.9$$

- Decomposing the YTM
  - 10-year coupon return = $\frac{\sum_{t=1}^{20} 40*1.06^{t/2}}{1155.9}$ = 92.6%
  - 10-year capital gain = $(\frac{1000-1155.9}{1155.9})$ = -13.6%
  - 10-year total return = 92.6% + (-13.6%) = 79.0%
  - YTM = $(1 + \text{10-year total return})^{1/10} - 1$ = 6%

Facts about YTM

1. When p = par
   - Then: YTM = the coupon rate
   - Because only source of returns is the coupon

2. For a zero-coupon bond
   - Then: YTM = $\sqrt[T]{(p / par)}$ - 1
     - because the only source of returns is the discount
   - For example, if p=90, par = 100 on a zero-coupon bond:
     - Then YTM would be $\frac{100}{90} - 1 \approx 11\%$

3. If p < par, then coupon rate < YTM (& vice versa)

Main risks with bonds

1. Interest rate risk
   – When interest rates rise, the YTM of all bonds must rise (by no arbitrage), so the bond price will fall
2. Inflation risk
   – Fixed, nominal coupon payments are less valuable if there's an increase in inflation
3. Credit risk
   – Borrower goes bankrupt, doesn't repay loan
4. Pre-payment risk
   – If interest rates fall, borrower will prepay their loans by re-financing at a lower interest rate
   – You no longer get the high interest payments you were receiving

Interest rate risk

- A simple example with a "perpetuity" (an infinite maturity bond)

Before:
- Market interest rates = 5%
- C = 5, 5, 5, 5, 5, ….
- P must be 100
- To make YTM = 5 / 100 = 5% equal to the going market rate

After:
- Market interest rates increase to 10%
- C is unchanged = 5, 5, 5, 5, 5, ….
- So P must fall: P = 50
- To make YTM = 5 / 50 = 10% equal to the going market rate

Interest rate risk

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{\text{Par}}{(1+y)^T}$$

- Easy to see that, in general, y and P are inversely related
- So when market rates (and thus y) go up, P must go down

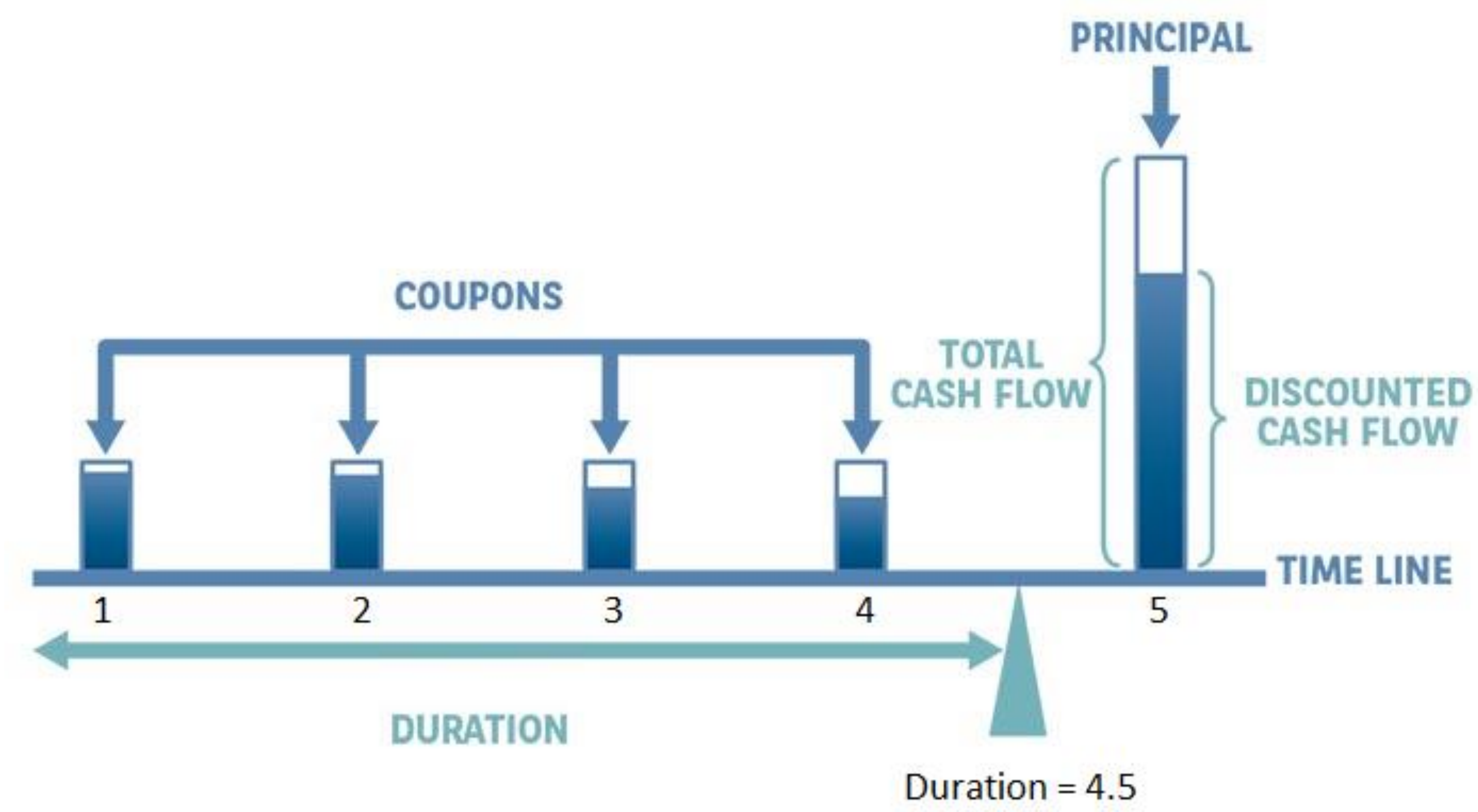
Duration

- Modified duration:

$$D = \frac{1}{(1+y)} \sum_{t=1}^{T} t \underbrace{\frac{Cashflows_t/(1+y)^t}{P}}_{\text{Discounted value weights}}$$

  - Intuitively, it's the average time (weighted by the discounted-value of when the cash gets paid out)
    - And divided by (1+y) – we'll see in a bit why
  - *Cashflows*$_t$ here represents both coupon payments and principal repayment (par value).

Duration


PRINCIPAL
COUPONS
TOTAL CASH FLOW
DISCOUNTED CASH FLOW
TIME LINE
1
2
3
4
5
DURATION
Duration = 4.5

Duration

Some facts about duration:

1. For a zero-coupon bond:

   $$(\text{modified duration}) = \frac{1}{(1+y)} * (\text{maturity})$$

2. Holding everything else fixed, duration:
   – increases with maturity
   – decreases with higher coupon rate
   – decreases with higher YTM

Duration

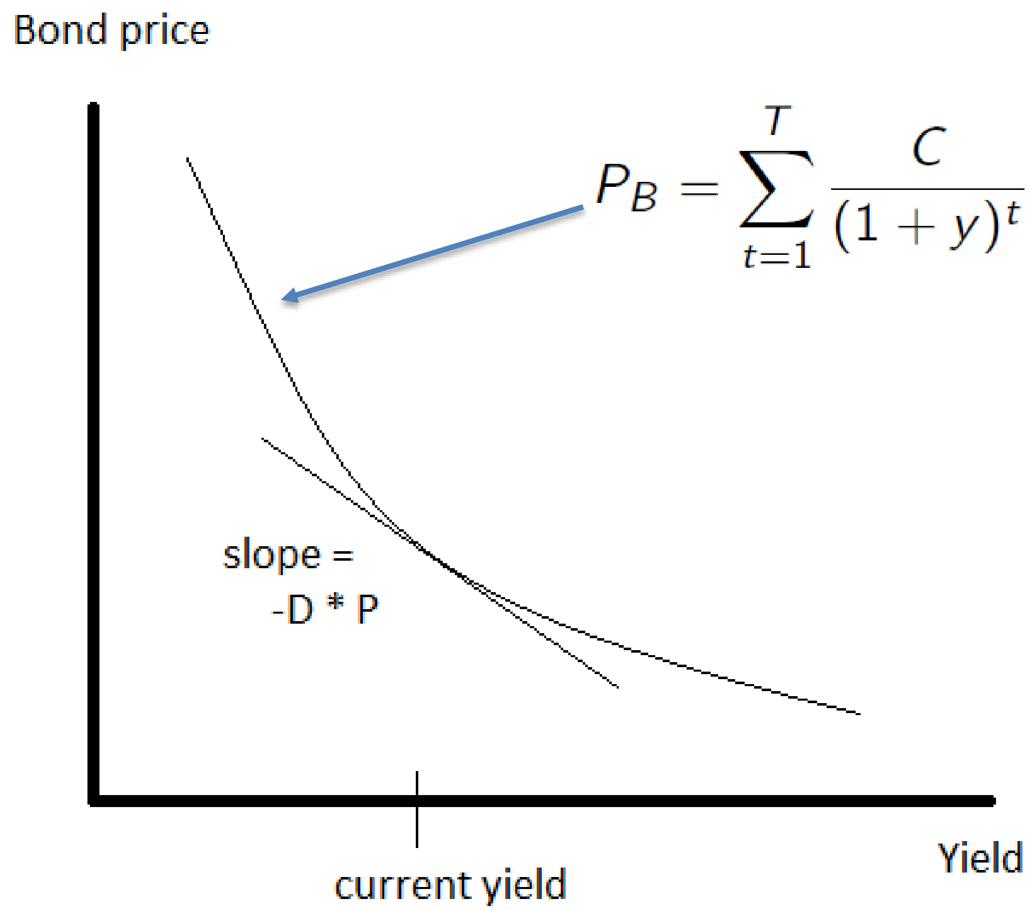
- But the most important thing about duration is that it's also the interest rate risk:

$$\underbrace{\frac{1}{P}\frac{dP}{dy}}_{\text{Bond return}} = -D$$

  - D = modified duration
  - Higher duration bonds will be hit harder if interest rates rise

Duration

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{\mathrm{Par}}{(1+y)^T}$$



A proof

1. Start with the standard bond pricing formula:

$$\mathrm{P} = \sum_{t=1}^{T} \frac{Cashflows_t}{(1+y)^t}$$

2. Differentiate with respect to y:

$$\frac{dP}{dy} = -\frac{1}{(1+y)} \sum_{t=1}^{T} t \frac{Cashflows_t}{(1+y)^t}$$

3. Divide both sides by P and recognize that the RHS is -D:

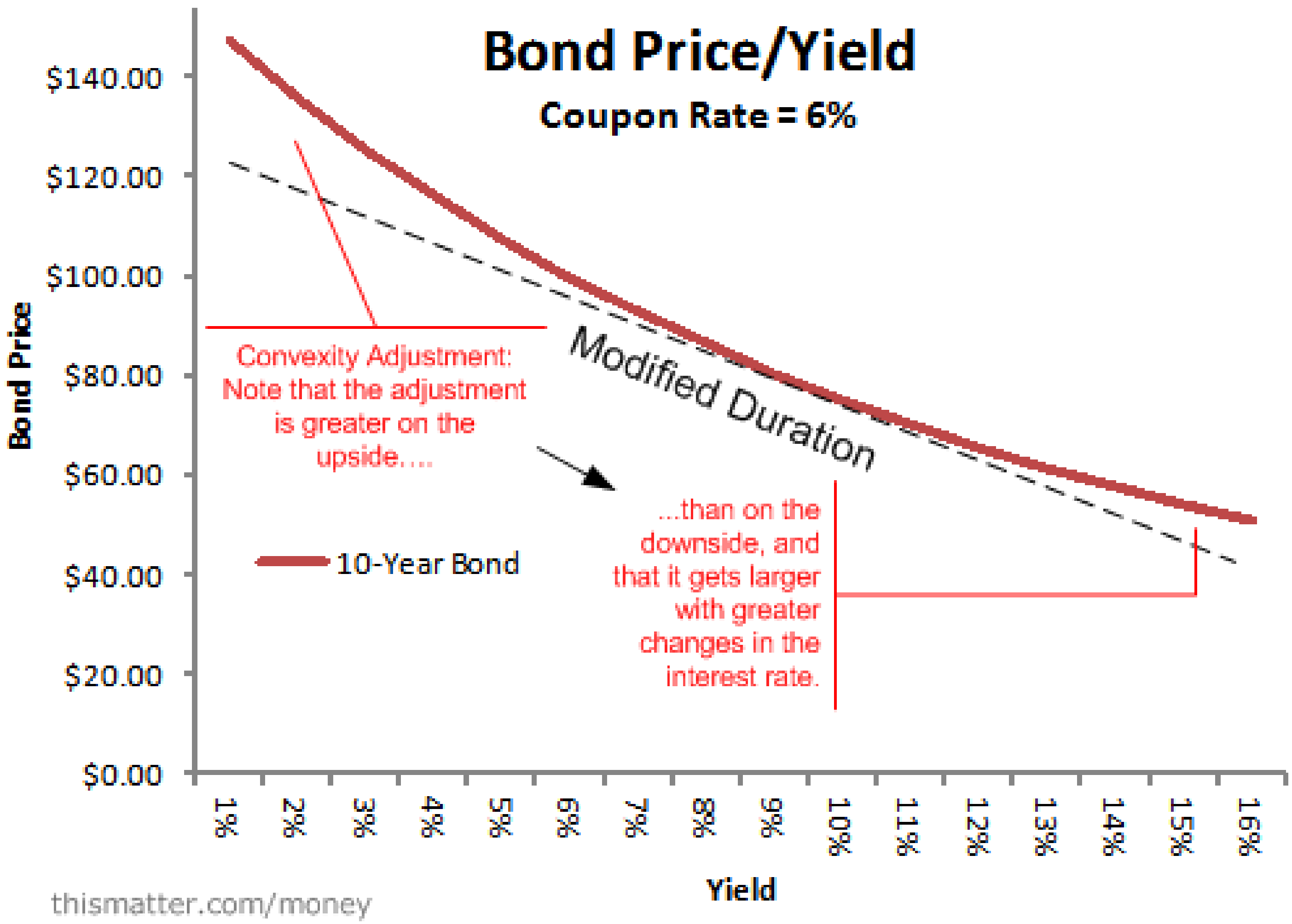
$$\frac{1}{P}\frac{dP}{dy} = -\frac{1}{(1+y)} \sum_{t=1}^{T} t \frac{\frac{Cashflows_t}{(1+y)^t}}{P} = -D$$

Approximation of the bond pricing curve

- Linear approximation:

$$\boxed{bond\ return = \frac{\Delta P}{P} = -D \cdot \Delta y}$$

Duration & Convexity



Approximation of the bond pricing curve

- Linear approximation:

$$\boxed{bond\ return = \frac{\Delta P}{P} = -D \cdot \Delta y}$$

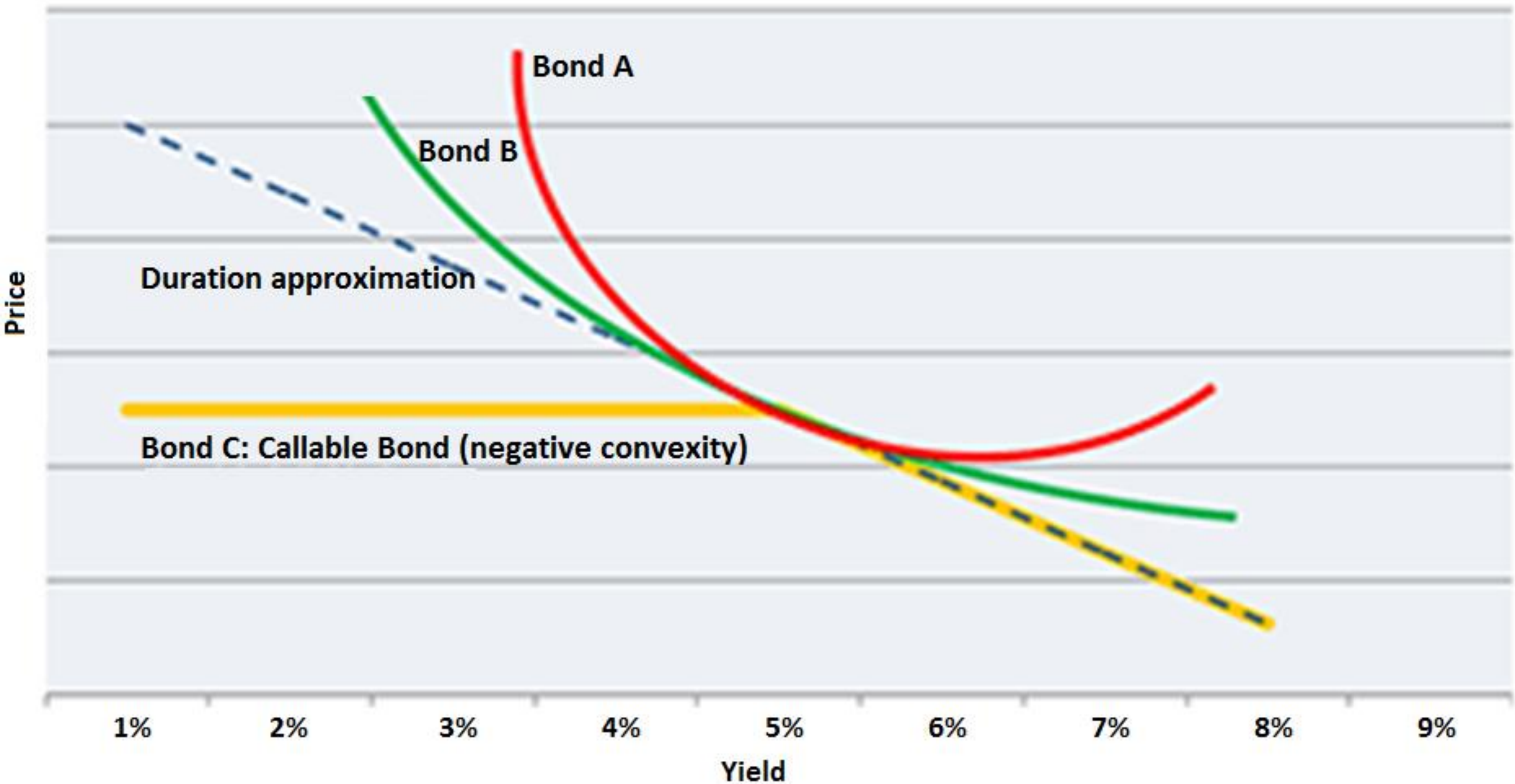
- Quadratic approximation

$$\boxed{bond\ return = \frac{\Delta P}{P} = -\,D \cdot \Delta y\ +\ ½\,\mathrm{Convexity} \cdot (\Delta \mathrm{y})^2}$$

where $\mathrm{Convexity} = \frac{1}{P}\frac{d^2P}{dy^2}$

$$= \frac{1}{(1+y)^2}\sum_{t=1}^{T} t(t+1)\ \frac{Cashflows_t/(1+y)^t}{P}$$

Various convexity



What is the actual safe asset?

"If one uses conventional mean-variance analysis, it is hard to explain why any investors hold large positions in bonds. Mean-variance analysis treats cash as the riskless asset and bonds as merely another risky asset like stocks. Bonds are valued only for their potential contribution to the short-run excess return, relative to risk, of a diversified risky portfolio. ...

“A long-horizon analysis treats bonds very differently, and assigns them a much more important role in the optimal portfolio. For long-term investors, [long-term bonds are the riskless asset] and money market investments are not riskless because they must be rolled over at uncertain future interest rates.”

– John Campbell and Luis Viceira, *Strategic Asset Allocation*

Interest rate risk

Recall the simple example with a “perpetuity” (an infinite maturity bond)

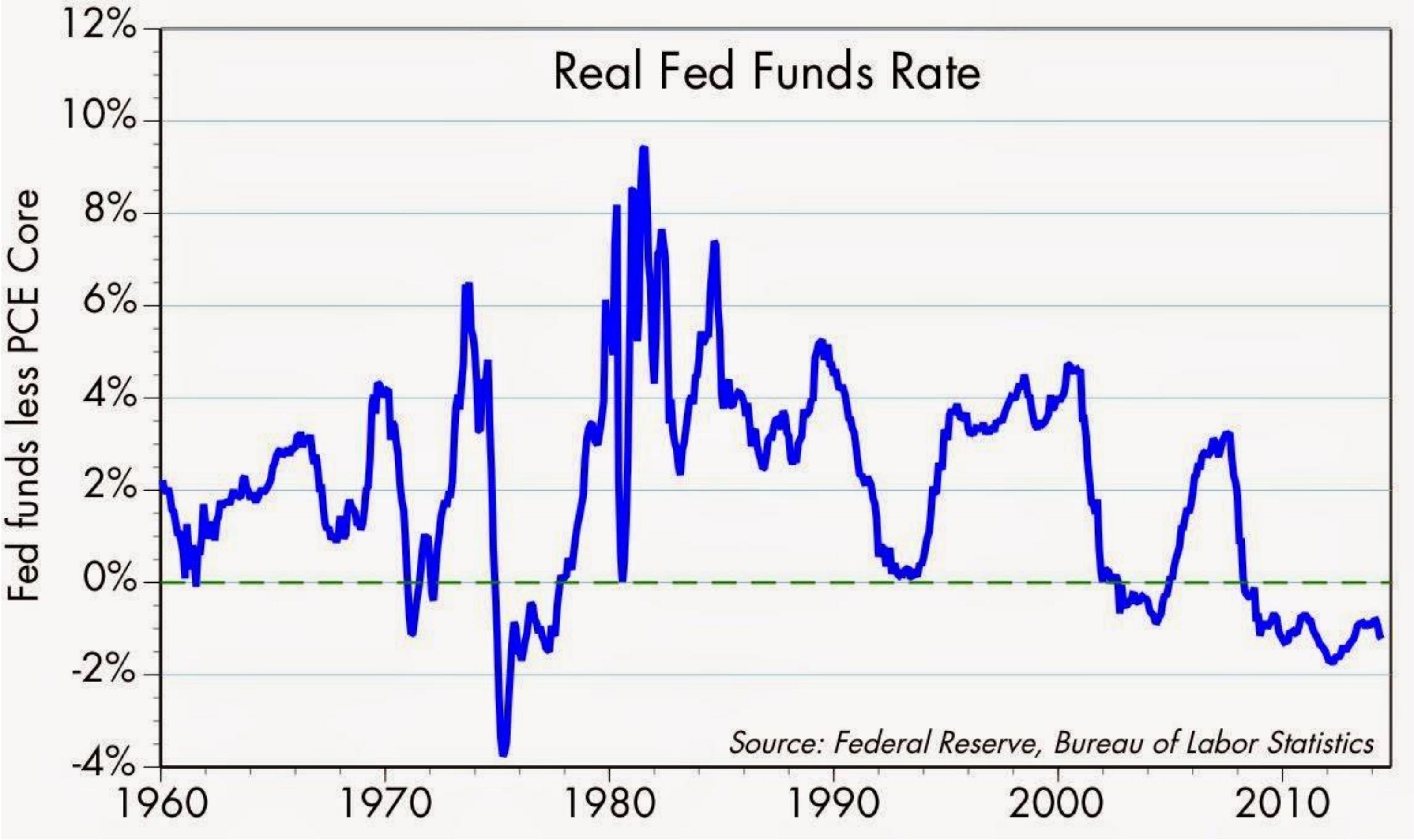
Before:

- Market interest rates = 5%
- C = 5, 5, 5, 5, 5, ….
- P must be 100
- To make YTM = 5 / 100 = 5% equal to the going market rate

After:

- Market interest rates increase to 10%
- C is unchanged = 5, 5, 5, 5, 5, ….
- So P must fall: P = 50
- To make YTM = 5 / 50 = 10% equal to the going market rate

The riskiness of the risk-free rate



THE TERM STRUCTURE OF INTEREST RATES

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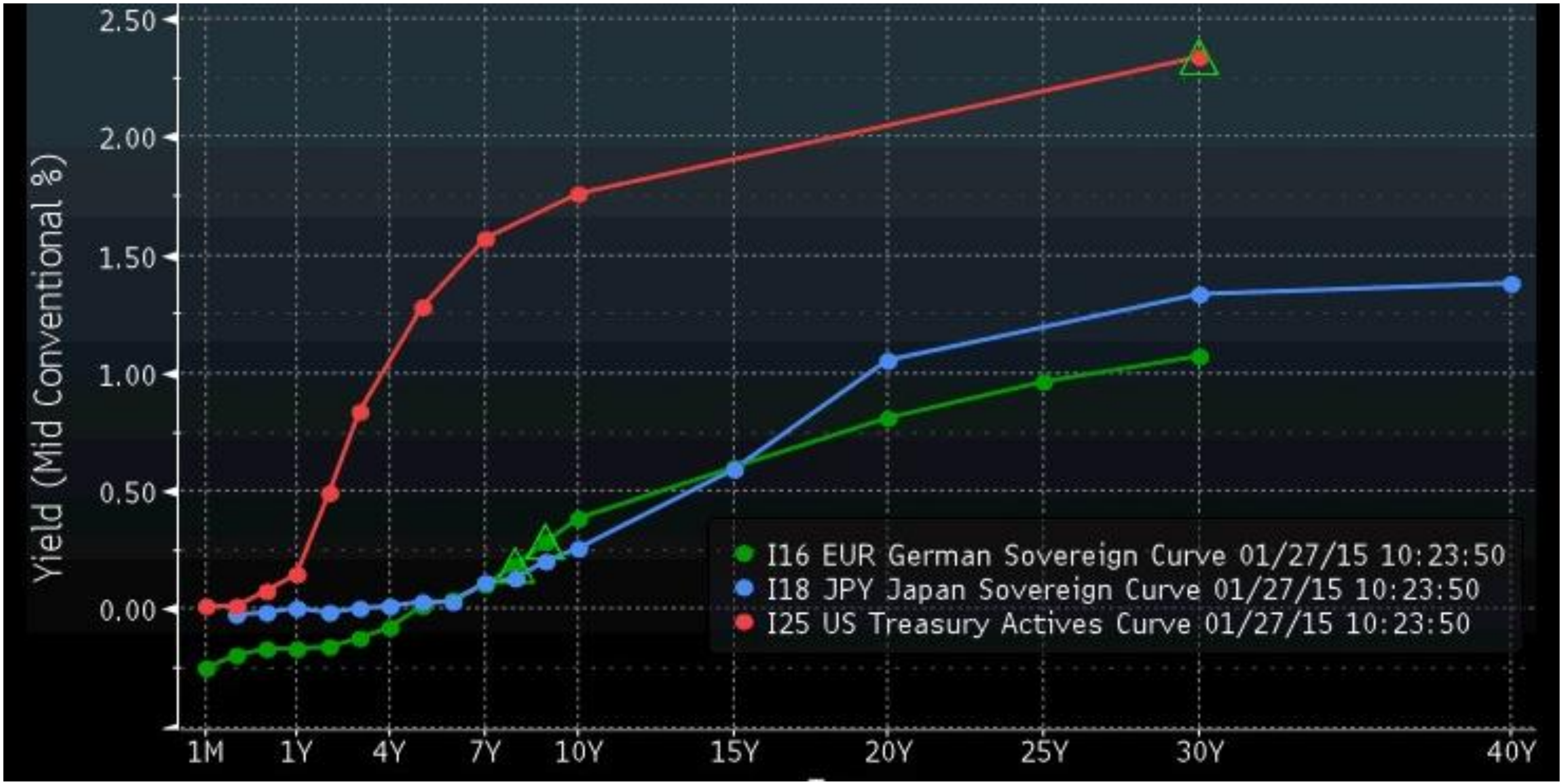
Term structure of interest rates



The Expectations Hypothesis

Consider two ways to invest for 2 years:

1. Buy and hold 2-year bond
2. Buy 1-year bond and reinvest proceeds in another 1-year zero one year from now (i.e. roll it over)

- By No-Arbitrage Pricing:

$$(1+r_{0,2})^2 = (1+r_{0,1})(1+Er_{1,2})$$

  - $r_{0,1}$ = one-year bond yield
  - $r_{0,2}$ = two-year bond yield
  - $Er_{1,2}$ = expected one-year bond yield a year from now

The Expectations Hypothesis

- In general,

$$(1+r_{0,n})^n = (1+r_{0,n-i})^{n-i}\ (1+Er_{n-i,n})^i$$

  – $r_{0,n}$ = n-year bond yield
  – $Er_{n-i,n}$ = expected i-year bond yield (n-i) years from now

- So long-term rates should predict the forward path of short-term rates

An example

- Inverted yield curve:
  - 1yr = 12%, 2yr = 11.75%, 3yr = 11.25%, 4yr = 10%, 5yr = 9.25%

- Then, expected forward rates are:
  - $Er_{1,2} = [(1.1175)^2 / 1.12] - 1 = 11.5\%$
  - $Er_{2,3} = [(1.1125)^3 / (1.1175)^2] = 10.3\%$
  - $Er_{3,4} = [(1.1)^4 / (1.1125)^3] = 6.3\%$
  - $Er_{4,5} = [(1.0925)^5 / (1.11)^4] = 6.3\%$

Term structure of interest rates

The Expectations Hypothesis (the benchmark):

- Downward (upward) sloping means expectation for future interest rates to be falling (rising)

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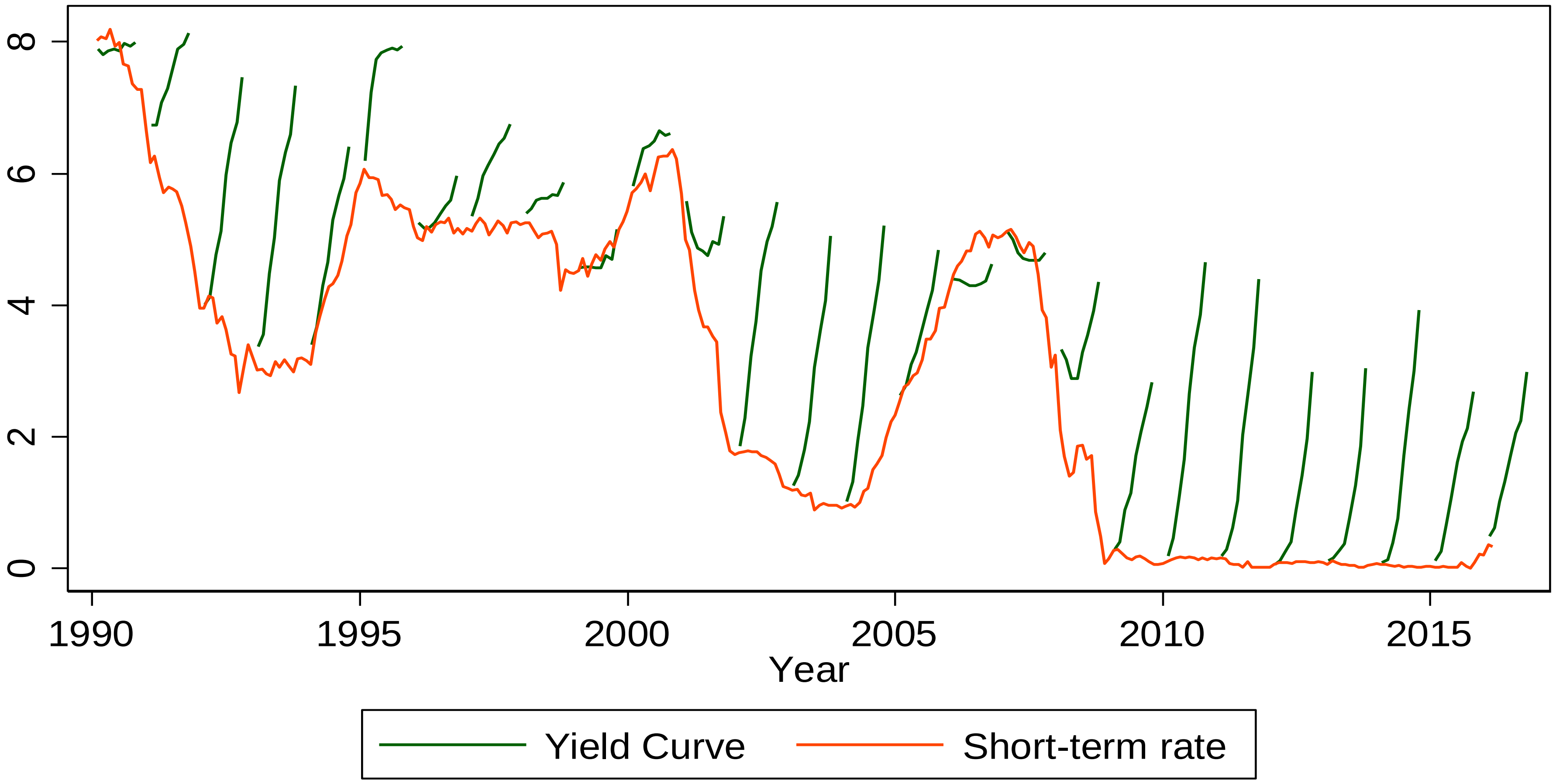
The Expectations Hypothesis



Term structure of interest rates

Deviations from the Expectations Hypothesis:

- Term premium:
  - "liquidity premium": Long-term bonds less liquid and pay a higher interest rates relative to the Expectations Hypothesis
  - "safety premium": Short-term bonds are like cash and pay a lower interest rates relative to the Expectations Hypoth.
- Habitat hypothesis:
  - Markets are somewhat segmented; different types of investors buy/trade long-term vs. short-term bonds
    - Long-term bonds: pension funds, insurance companies
    - Short-term bonds: individual investors, corporate cash holdings

Actual vs. predicted short-rates (2-years-ahead, 1984-2012)