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NBA 5420: Investment and Portfolio Management

Class 3: Arbitrage Pricing

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Topics

- Futures and Swaps
  - How forward and futures contracts work
  - Hedging vs. speculating
  - Relationship between futures and spot prices

- Options
  - Put-call parity
  - Binomial option pricing model
  - Black-Scholes

No Arbitrage / Law of One Price

- If two contracts yield identical cash flows in all future states of the world, then their price <u>today</u> must be equal.
  - Otherwise, an arbitrageur would…
    - Buy the one with the lower price
    - Short the one with the higher price
  - No risk involved
    - Since all future cash flows would perfectly cancel each other out in all future states of the world

No Arbitrage / Law of One Price

Example:

			Sunny		Rainy
Contract 1:	P = 10,	Cash flows:	-1	or	+2
Contract 2:	P = 11,	Cash flows:	-1	or	+2

FUTURES

Forward contracts

- Agreement today for purchase in the future. Traders agree on:
  - Asset to be delivered (called the underlying asset)
  - Date of delivery AND payment
  - Amount of payment (called the forward price)

- Both parties are protected from price fluctuations, which are often substantial. For example:
  - Oil producers (Exxon-Mobil) worried the price will go down.
  - Oil consumers (airlines) worried the price will go up.
  - Therefore, **they agree to lock in a price today** for delivery and payment a year from now.

Futures contracts

1. **Formalize and standardize** *forward* contracts, providing a matching mechanism for buyers and sellers.
   – Standard terms
   – Contract size
   – Acceptable grade of commodity
   – Delivery date
   – Place of delivery, etc.

2. Most importantly, **minimize counterparty risk**
   – Requiring initial margin accounts on both sides
   – Transferring money on a day to day basis
   – Requiring the loser to meet margins on a daily basis

Underlying asset

- The asset that can be bought/sold with the derivative is called the underlying asset.

- Futures / Swaps / Options are written on a variety of assets:
  - stocks, indexes, bonds,
  - interest rates, futures,
  - foreign currencies, commodities, etc.

Long and short positions

- The *long* position commits to **buy** the commodity at the delivery date
  - Benefits if the price of underlying goes **up**
- The *short* position commits to **sell** the commodity at the delivery date
  - Benefits if the price of underlying goes **down**
- Although it is common to talk of purchases and sales of futures contracts, a futures contract is not really “bought” or “sold” like a bond or stock
  - The contract is entered into by mutual agreement
  - No money changes hand when the contract is signed.

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Term structure

- Various expirations (or maturities):
- If you want to hold a position in a commodity long-term:
  - You just 'roll-over' your position from one front-month contract (usually the most liquid) to the next

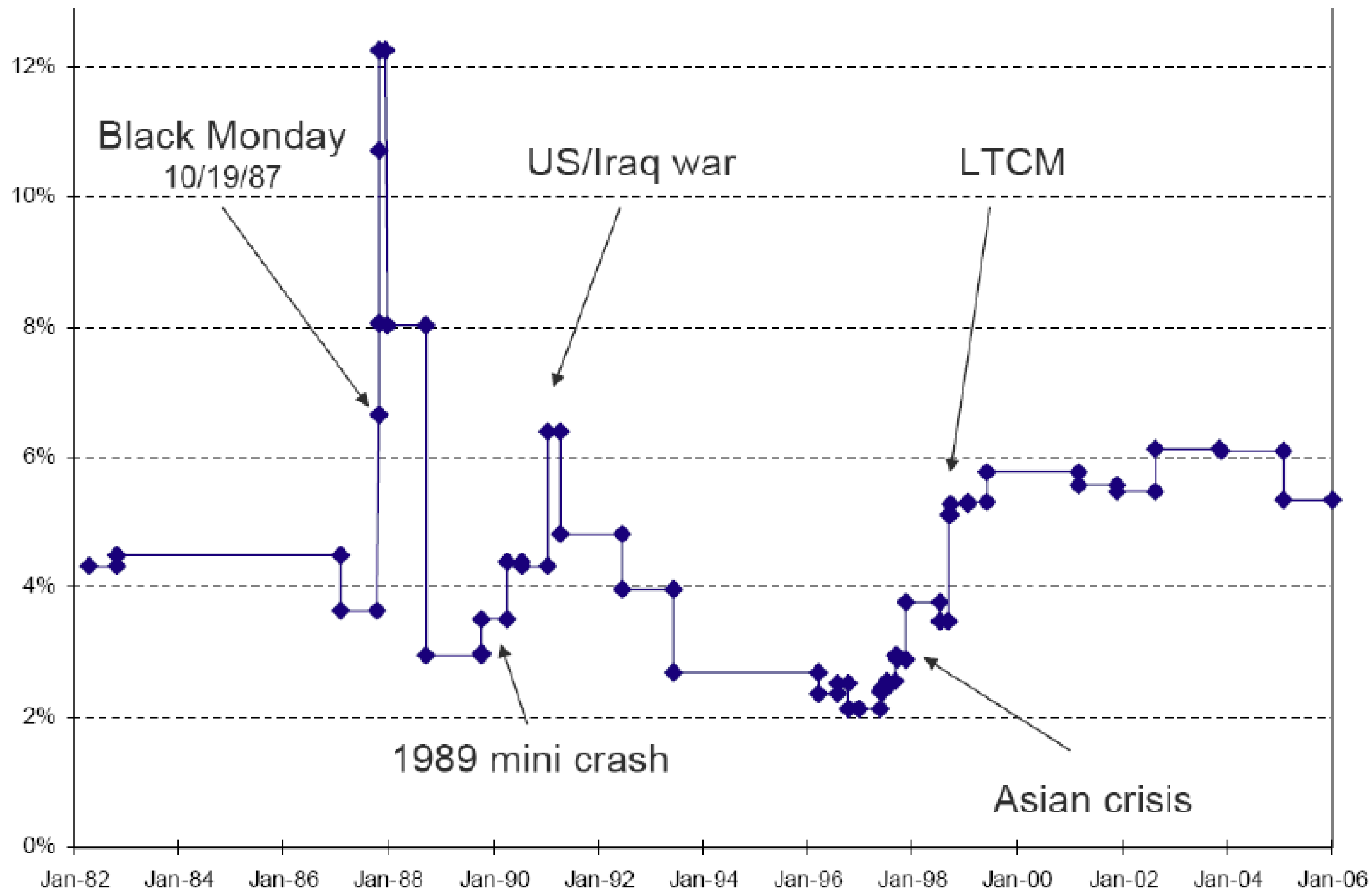
Month	Options	Charts	Last	Change
MAR 2016	OPT		33.74	+0.52
APR 2016	OPT		35.41	+0.70
MAY 2016	OPT		36.86	+0.83
JUN 2016	OPT		38.06	+0.97
JUL 2016	OPT		38.99	+1.04
AUG 2016	OPT		39.62	+0.99
SEP 2016	OPT		40.34	+1.16
OCT 2016	OPT		40.72	+1.09
NOV 2016	OPT		41.08	+1.02
DEC 2016	OPT		41.67	+1.16

Trading futures

- In practice, trading futures feels like buying (or shorting) the underlying
  - To buy a futures that trades at $100, you put $100 cash into your margin account.
  - Then, if the futures price goes up to $110, you sell.
  - Your margin account now has $110 in it. Profit = $10.

- Actually, **a leveraged bet**: In practice, you don't have to put up 100% margins, more like 10%
  - You need $10 down to buy that $100 contract of oil.
  - If the futures price goes up to $110, you've doubled your money (made $10 on the original $10)
  - 10-1 leveraged bet

Margins on S&P 500 futures



Trading futures

- Usually, traders don't hold to expiration (and take delivery)
  - They close out their position by selling

- Suppose you are an oil consumer (airlines) trying to hedge oil risk.
  - Oil (both spot and futures price) is currently at $100.
  - Buy a futures: if oil goes up to $110, then you make $10 in the futures
  - Then, when you buy a barrel of oil for $110, effective price of oil is $100

    (= $110 - $10)
  - Equivalent to locking in a price of $100

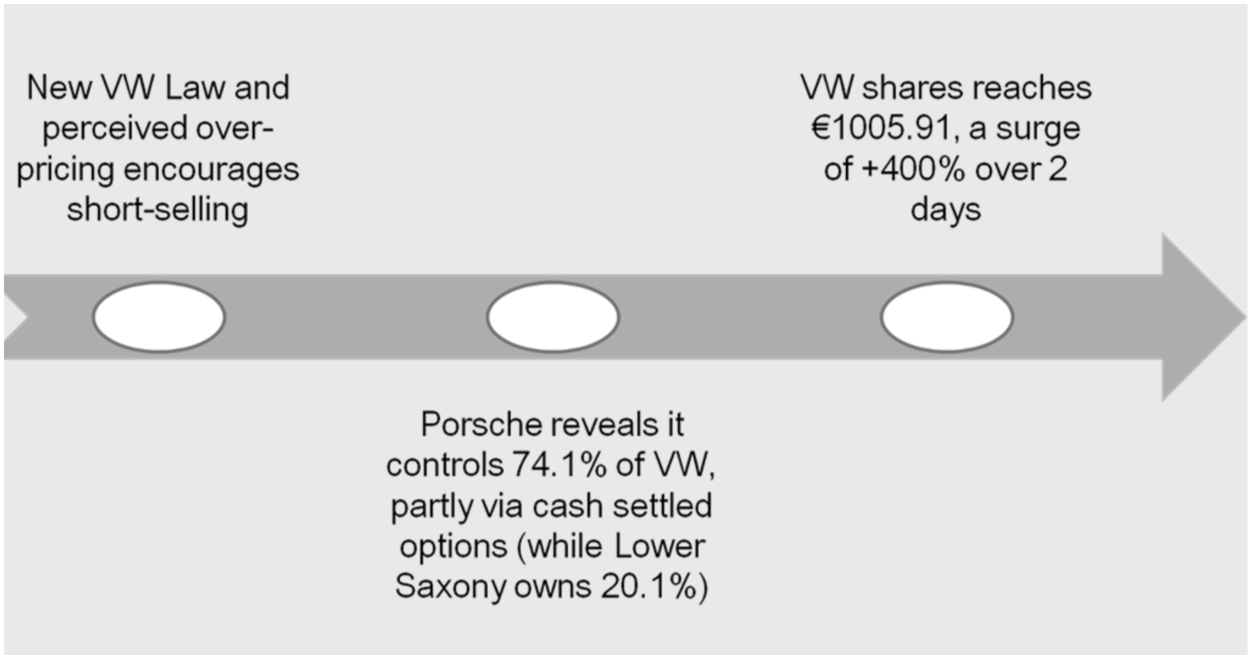
Trading futures

- Works because the offsetting positions (long and short) over the trading day (to various anonymous counterparties) are **netted out** by the exchange
  - The exchange automatically transfers cash between margin accounts (on a daily basis) as the future price fluctuates.
  - Also, contracts now often “cash settled” (rather than by “physical delivery”).

Market manipulation (not recommended!)

- Here’s how to “corner the market”:
  1. Buy a long position in copper futures
  2. *Secretly* buy up much of the world’s copper in the spot market, pushing the price up.
     - The world will mistake this for fundamental demand
  3. Convert your long futures position to a short position
     - Making a big profit from your long futures position
  4. Dump your physical copper and send the price of copper plummeting
  5. Exit your short positions (making a big profit)
  6. **Get prosecuted and go to jail**

A modern “corner”



What Happened Next?

1. Porsche settled 5% of VW options to ease the short squeeze. VW shares fell 44.2%
2. Porsche reported a profit of €6.8 billion from the VW options trade, compared to €1 billion from car sales

Market manipulation (not recommended!)

- More recently, a popular way of manipulating the market is “hammering the close”
  - Buying up a lot of the underlying (usually equity) to manipulate the price of the underlying just as the futures / derivative contract is about to expire. **(This is also illegal.)**

Notation

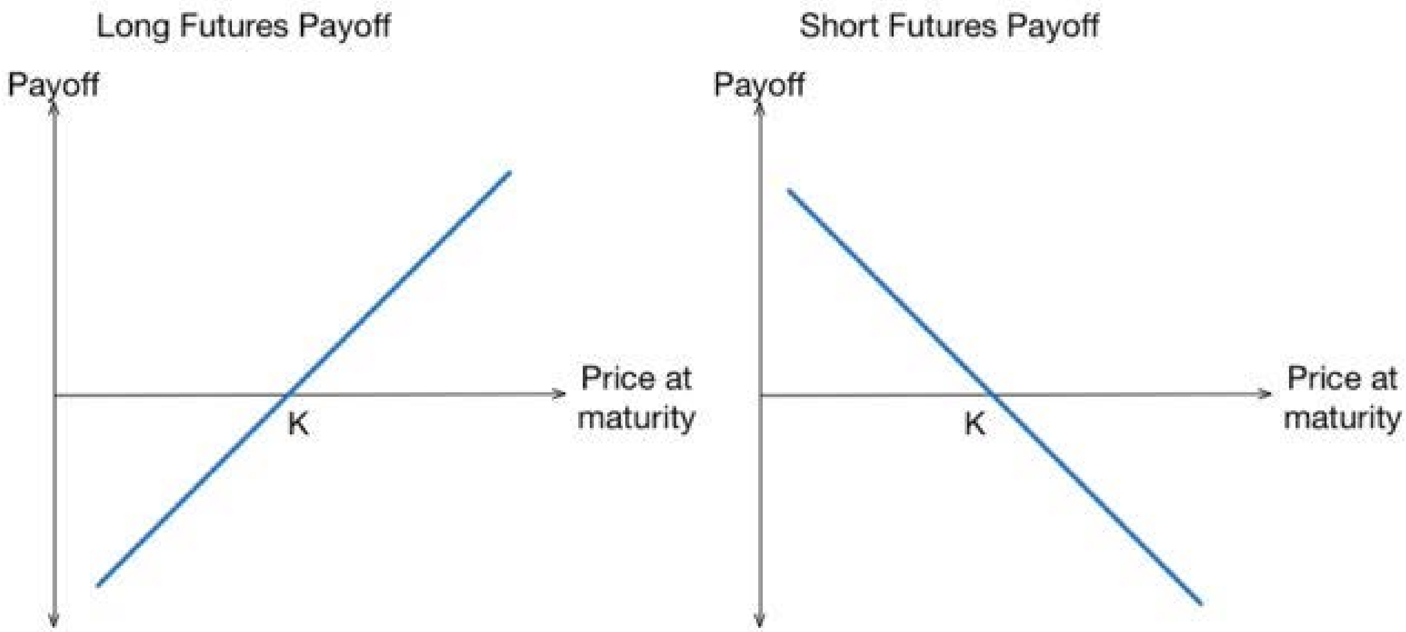
t = current date

T = delivery date specified by contract (maturity or expiration)

S_T = spot price of the underlying at time T

$F_{t,T}$ = market price of the contract at t for delivery at T

Payoff Diagram for Futures



- Payoff to long = (S_T – K), Payoff to short = (K – S_T)
- K = the price at which you buy/sell the futures

- So this is a zero-sum game
  – Payoff to long + Payoff to short = 0.

No arbitrage pricing

Two equivalent ways of locking in the price of oil for a year as an oil consumer:

1. Buy a futures contract
   - Oil (both spot and futures price) is currently at $100.
   - Buy a futures: if futures goes up to $110, then you make $10 in the futures
   - Then, when you buy a barrel of oil for $110, effective price of oil is $100 (= $110 - $10)
     - Equivalent to locking in a price of $100

2. Buy a barrel of oil now for $100 and store it for a year
   - Assuming (for simplicity) no storage costs or interests costs

No arbitrage pricing

- This suggests a no-arbitrage pricing formula (adding back in potential storage costs and interest costs):

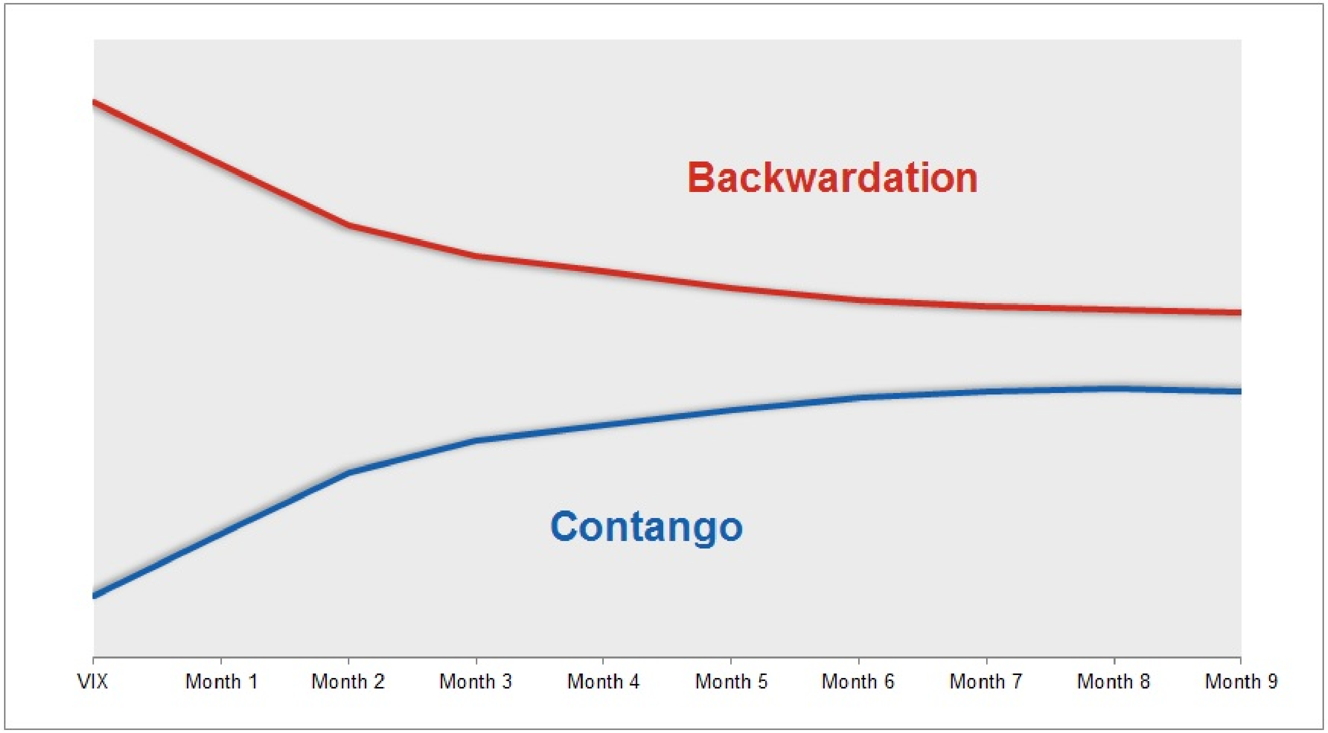
$$\boxed{Futures_{t,T} = Spot_t\ (1+r)^{T-t} - D}$$

  - Where D represents: storage costs, dividends, convenience yield, etc. (paid at time T)

- Formula suggests that the futures price is essentially equivalent to the spot price (adjusting for interest & storage costs).
  - So speculating on the futures is essentially the same as speculating on the spot – but without actually having to deal with barrels of oil

Contango vs. Backwardation

- **Contango**: **Futures$_{t,T}$ > Spot$_t$**
  (upward sloping term structure)
- **Backwardation**: **Futures$_{t,T}$ < Spot$_t$**
  (downward sloping term structure)



Contango vs. Backwardation

- Keynes thought that contango vs. backwardation was driven by whether the long or the short was more risk-averse
  - i.e. willing to pay a higher risk-premium to lock in the forward price

- Most people still believe this, but it's actually in direct conflict with no-arbitrage pricing:

$$\text{Futures}_{t,T} = \text{Spot}_t\,(1+r)^{T-t} - D$$

  - The no-arbitrage formula says the futures-spot spread just depends on interest and storage costs
  - The arbitrage strategy from the previous slide doesn't involve taking any risk, so there shouldn't be any risk premium built into the futures-spot spread (unless the commodity is not storable)

Contango vs. Backwardation

- Technical aside:
  - Now, there could be a risk premium in the spot price depending on who (the consumer or producer) is more risk-averse
    - The spot price would appreciate (or depreciate) over time, and the futures would appreciate (or depreciate) in parallel.
    - But the futures-spot spread would NOT depend on the risk-premium because it is pinned down by no-arbitrage

Futures contracts

- Commodities
  - Energy: Crude Oil (WTI or Brent) & Natural Gas
  - Grains: Corn & Soybeans
  - Metals: Gold and copper
  - “Softs”: Cotton, Cocoa, Sugar, Coffee
  - Electricity
- Non-commodities
  - Eurodollar
  - E-mini S&P 500
  - EUR/USD & JPY/USD
  - Swaps (as a result of Dodd-Frank)

Example: WTI Crude (traded at the CME)

- Oil Benchmark: WTI crude
  - Based on the spot price of Light Sweet Crude traded at Cushing, OK
  - Still useful for oil consumers/producers of other grades
    - Even though the price for different grades can vary somewhat relative to the benchmark.

	Volume										Open Interest	
	Venue Detail				Trade Type Detail							
Month	Globex	Open Outcry	PNT / ClearPort	Total Volume	Block Trades	EFP	EFR	EFS	TAS	Deliveries	At Close	Change
MAR 16	611,564	28	6,518	618,110	3,080	1,169	0	0	25,671	0	605,746	-3,085
APR 16	175,868	0	2,589	178,457	2,192	2	0	0	9,915	0	209,740	4,291
MAY 16	85,443	0	739	86,182	343	1	0	0	820	0	128,020	9,585

Futures Exchanges

- Two major futures exchanges
  - CME (merger of CME, CBOT, NYMEX, COMEX, etc.)
  - ICE (merger of IPE [Brent], NYBOT [softs], etc.)

- Other exchanges: London Metal Exchange, Shanghai metal exchange

- Recent trend is exchange consolidation:
  - Multiple venues historically → CME and ICE today
  - Opposite trend from equities:
    - 3 major venues historically (NYSE, NASDAQ, AMEX)
      → 30+ venues today (NYSE Arca, ISE, BATS, Turquoise, IEX, etc.)

SWAPS

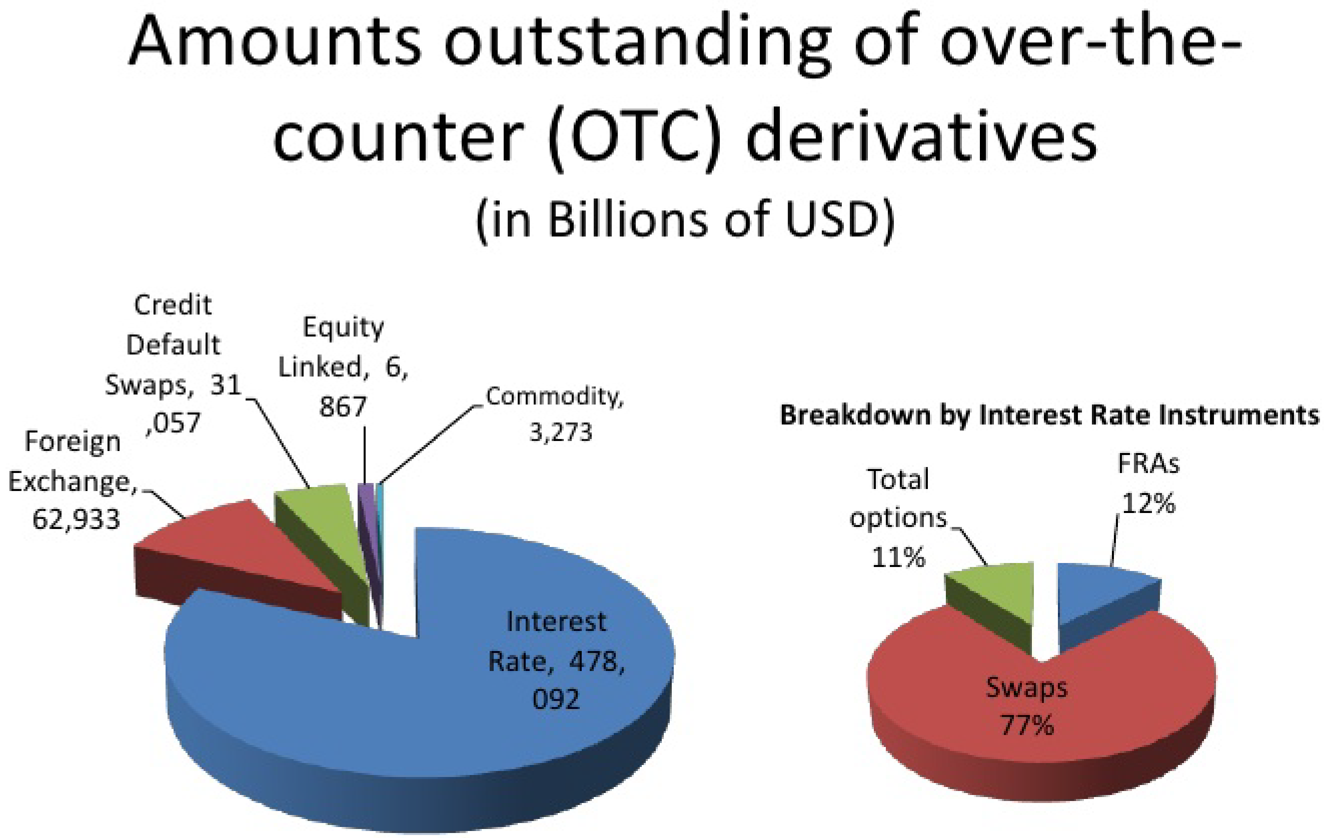
Class announcements

- As I said in an email last week,
  Problem Set 3 not due this Friday

1. **Due date pushed back** a week to:
   Friday, February 26

2. **I removed a problem**, so please re-download Problem Set 3 from Blackboard.

Various kinds of swaps

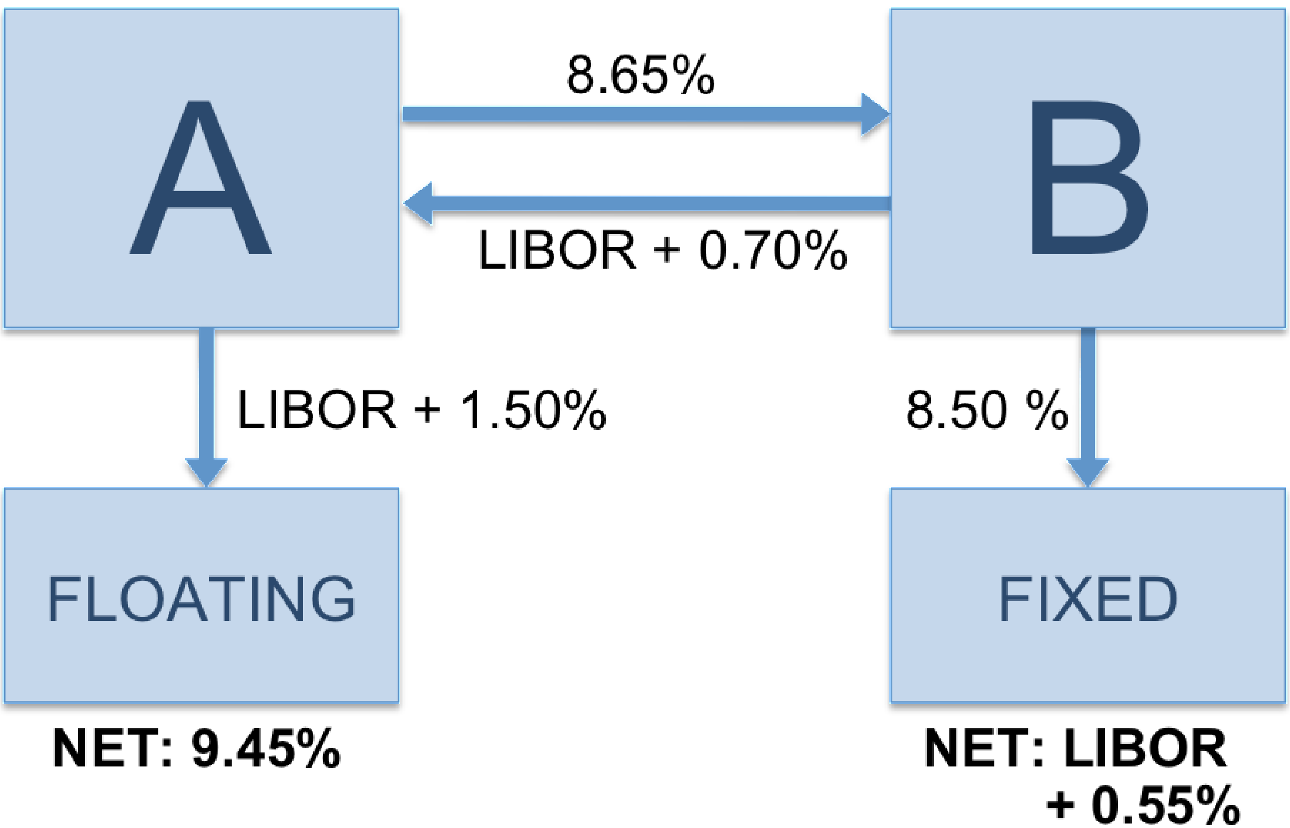


Interest rate swaps

- Derivatives are also commonly used by both financial and nonfinancial firms when they raise capital.
  - A Japanese firm might want to borrow yen at a floating rate. However, there might be more demand for its debt from dollar-based investors who want to be paid a fixed rate.

- Banks use interest-rate derivatives to manage potential mismatches between their assets (loans) and their liabilities (checking accounts, for instance).
  - Banks often have assets with a fixed rate of interest but pay a floating rate on their liabilities.
  - Or they could purchase options that, for example, "cap" what they might be forced to pay out, or put a "floor" on the rate they would receive.

Interest rate swaps

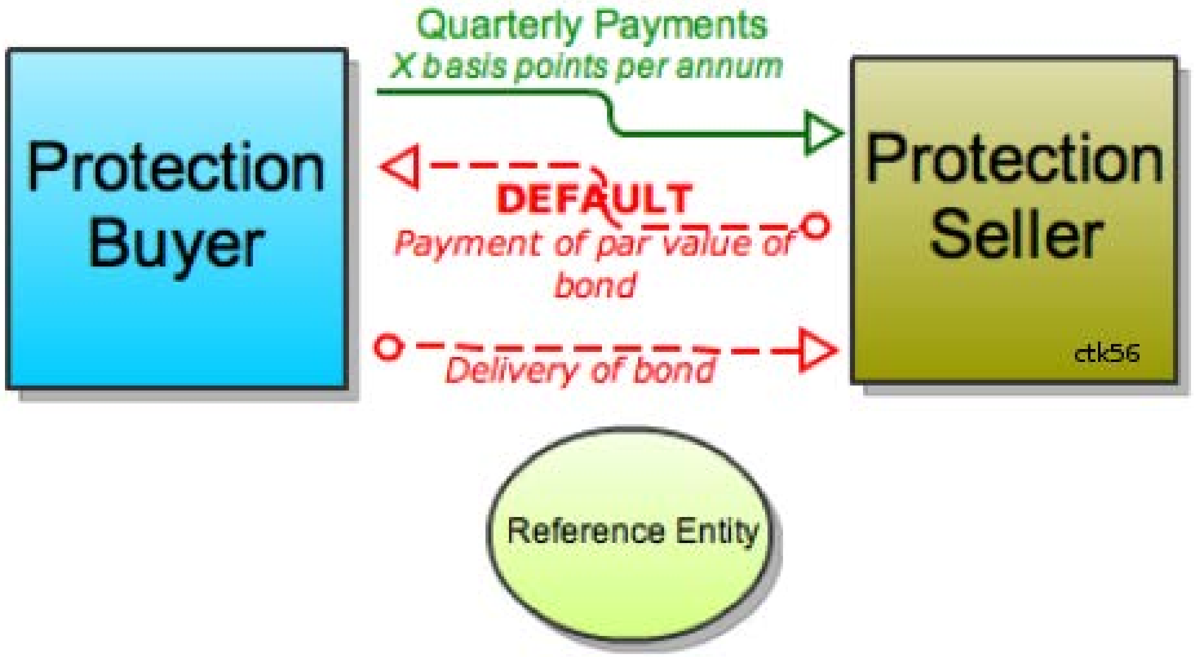


Credit Default Swaps (CDS)

- The CDS seller **insures** the buyer against some bond defaulting
  - The buyer of the CDS makes a series of payments (the CDS "fee" or "spread") to the seller

- In exchange, the buyer receives a payoff if the loan defaults.
  - Traditionally, in the event of default, seller of the CDS pays the full par value of the bond and takes possession of it

Credit Default Swaps (CDS)



Credit Default Swaps (CDS)

- However, in many cases, anyone can purchase a CDS, even buyers who do not hold the bond:

- No direct insurable interest

  - Some critics assert that this should be banned
  - Buying fire insurance on your neighbor's house? Pure speculation, not a hedge.

- In this case, a protocol exists to hold a credit event auction to determine the recovery payment

Hedging vs. Speculating

- Hedging:
  - If used properly, futures, options, swaps, and other ‘synthetics’ can reduce risk in the world (or smooth it out over more investors)
    - Oil producers (Exxon-Mobil) worried the price will go down.
    - Oil consumers (airlines) worried the price will go up.
    - Therefore, **they agree to lock in a price today**
- Speculating:
  - People with different beliefs place bets.
  - Now, someone is going to win and someone is going to lose.
  - So aggregate risk has increased
    - Relative to before, when no money was changing hands = zero risk
- Dick Thaler and Selena Gomez explain:
  - https://www.youtube.com/watch?v=sD3ZSqCKOCg

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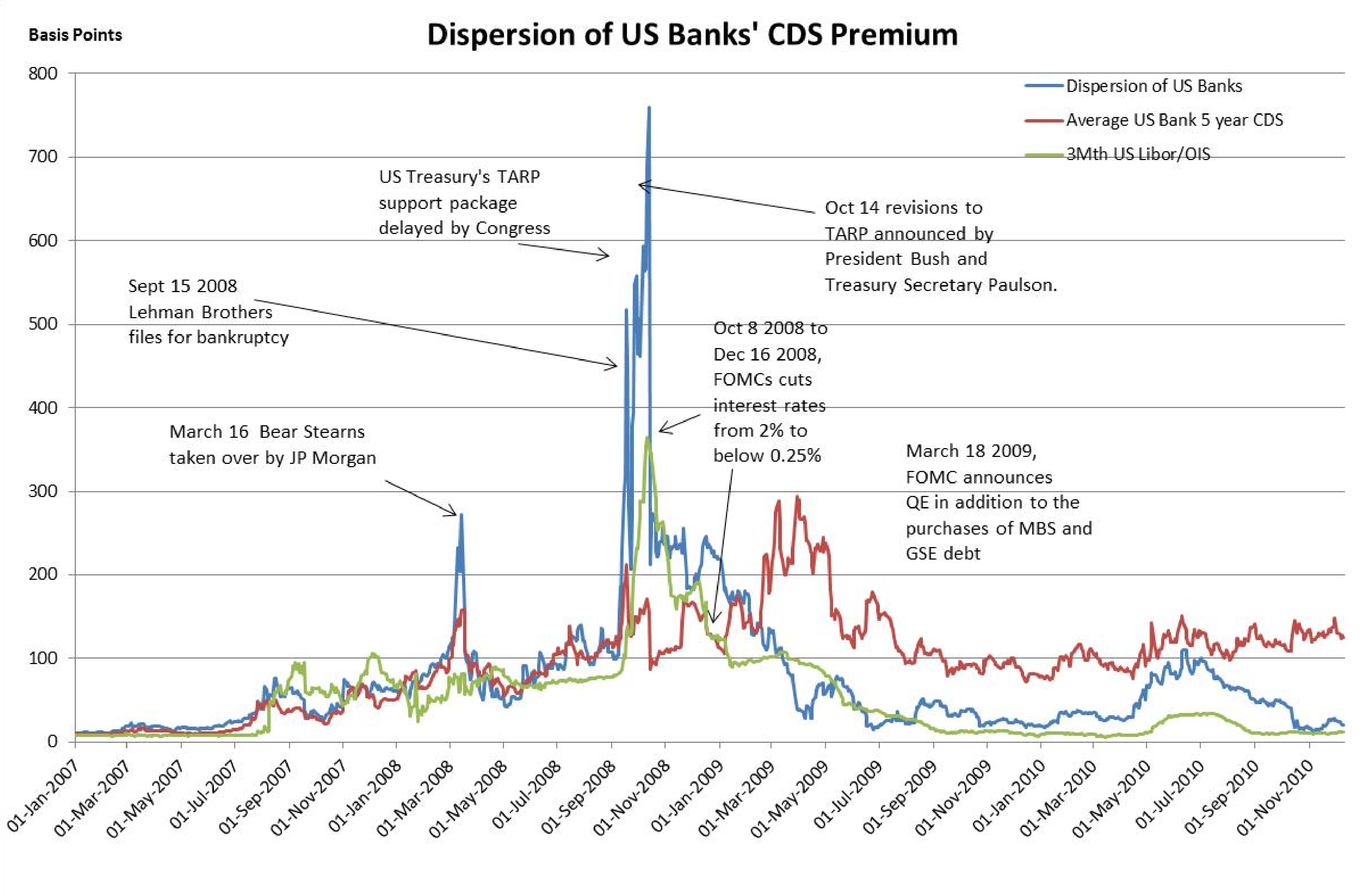
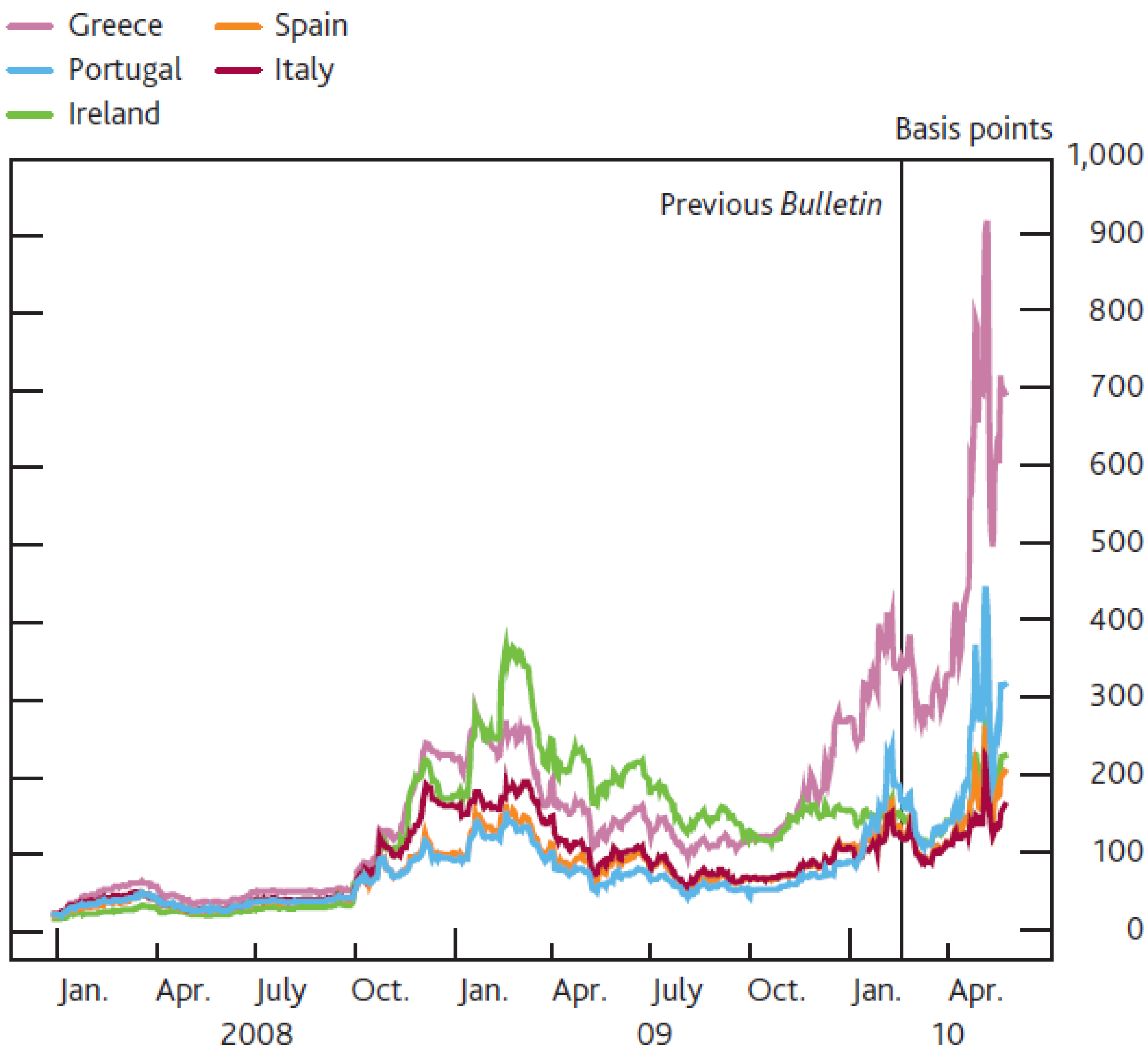


Chart 9 Selected European sovereigns' CDS premia[(a)]



Counterparty Risk

- Counterparty risk is a huge issue with swaps
  – AIG nearly took down the financial system

- AIG got paid ~10 cents as an insurance premium for every $100 in mortgage-backed securities (MBS) that it ensured against default.
  – They basically believed the probability of default was negligible.
    - But after Lehman's collapse, expected defaults spiked

  – And suddenly AIG was on the hook for $100+ billion
    - Of course, it didn't have that money set aside, as is required for traditional insurance

Counterparty Risk

- Financial regulation (Dodd-Frank)
  - Put swaps on exchanges, limit issuance to situations where one party has a legitimate business need (hedging, not speculating)
    - Advantages: collateral, netting
    - Disadvantages: Will reduce size of swaps market (but this might be a good thing, since you probably shouldn’t be selling insurance unless you have the collateral to pay up later)
  - We will talk a lot more about counterparty risk and swaps near the end of the course when we return to financial regulation

OPTIONS

Calls and puts

- Call option
  – Gives its owner the right (but not the obligation) to buy the asset at a fixed price, called the strike price
- A put option
  – Gives its owner the right to sell the asset at a fixed price

American vs. European options

- European option:
  - Can exercise only at a fixed date, called the maturity or expiration date of the option.
- The American option
  - Gives the right to exercise at any time prior to, and including the expiration date. (“early exercise”)
- Virtually all options traded in the US are American
  - Except for foreign currency options and S&P500 index options traded at the CBOE
  - But we will focus mainly on European options in subsequent slides because they are easier to analyze.

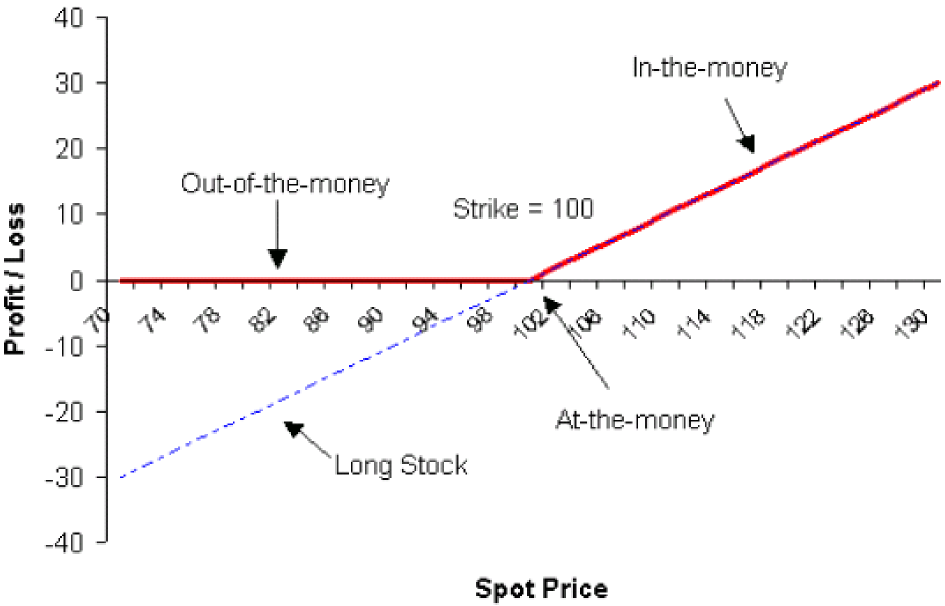
Notation

t = today's date
T = expiration
S_t = price of the underlying asset today
S_T = price of the underlying asset at expiration (a random variable)
K = strike price
r = the risk-free interest rate

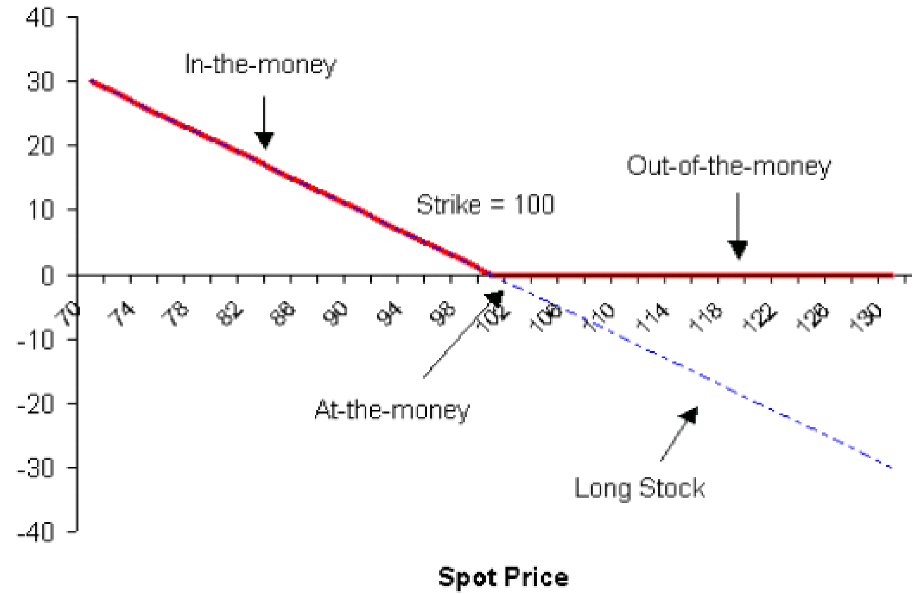
$C_{t,K}$ = price of a call option of strike K
$P_{t,K}$ = price of a put option of strike K

Payoff Diagrams for Options

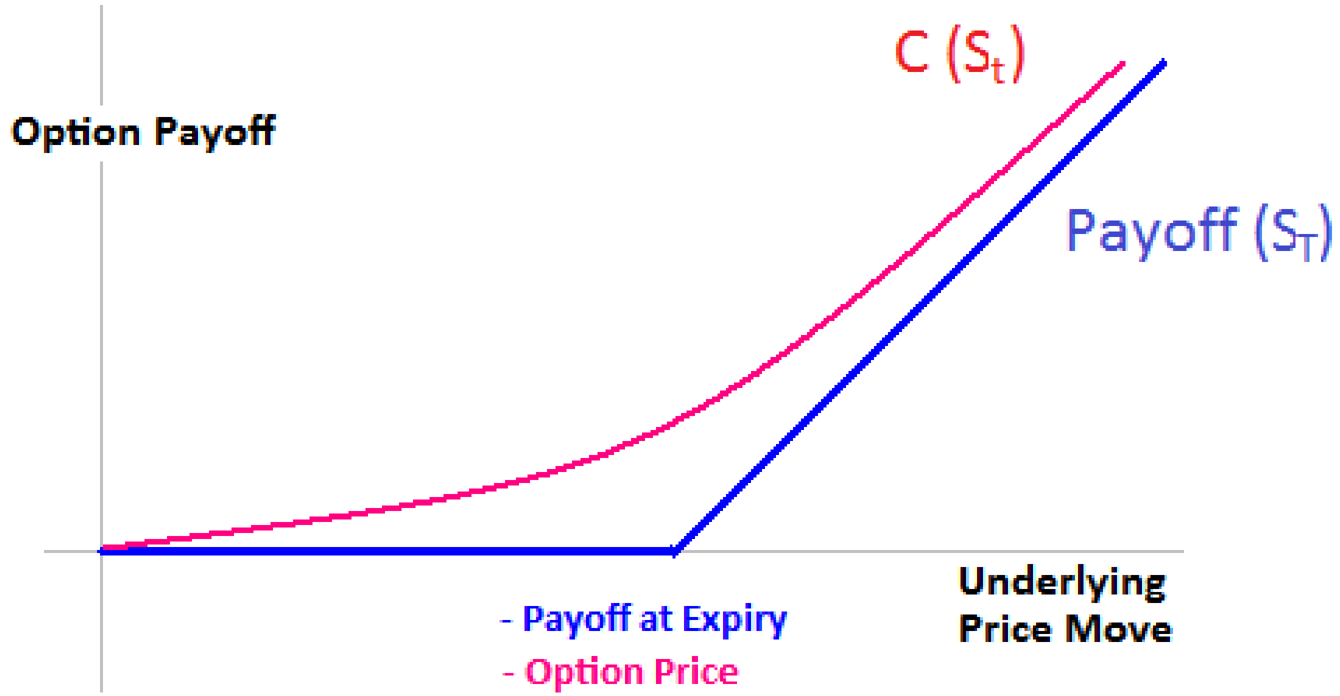
Call



Put



Price vs. Payoff



General notes about the price

1. Price always positive
   – Option always has upside potential, but no downside

2. Price always greater than final payoff for a given stock price
   – Because there's always 'optionality'

3. Price decreases as you approach expiration
   – Optionality decreases over time

4. American options are at least as valuable as their European counterparts
   – Having the extra option of early exercise is always a good thing, because you can always choose not to use it

Option Trading

- Both parties deal only with the clearinghouse, which guarantees contract performance and nets out buying / selling
- Option writers ("sellers") post margins to guarantee that they will fulfill their obligations.
  - Margin requirements apply *only* to the option writer.
  - Since the option buyer cannot harm the writer once the option price has been paid, which is always done in full at initiation.
- Like futures contracts, terms for options are standardized
  - This increases the depth of the trading in any particular option
  - Most trading is around at-the-money strikes

Put Call Parity

$$C = P + S - \frac{K}{(1+r)}$$

where C and P have the same K strike price

- Proof:
  - Use the payoff diagrams to show the payoffs on both sides of the equation are equal in every future state (S_T) of the world.
  - Therefore, by no arbitrage, the prices today must be equal
- Put-Call Parity does not apply to American options because of early exercise

Early exercise (American options)

1. It is never optimal to early exercise an American **call** on a stock **paying no dividend**

2. It is sometimes optimal to early exercise an American call on a dividend paying stock just before the payment of a large dividend
   - Stock price will drop on the ex-dividend date by the dividend amount.

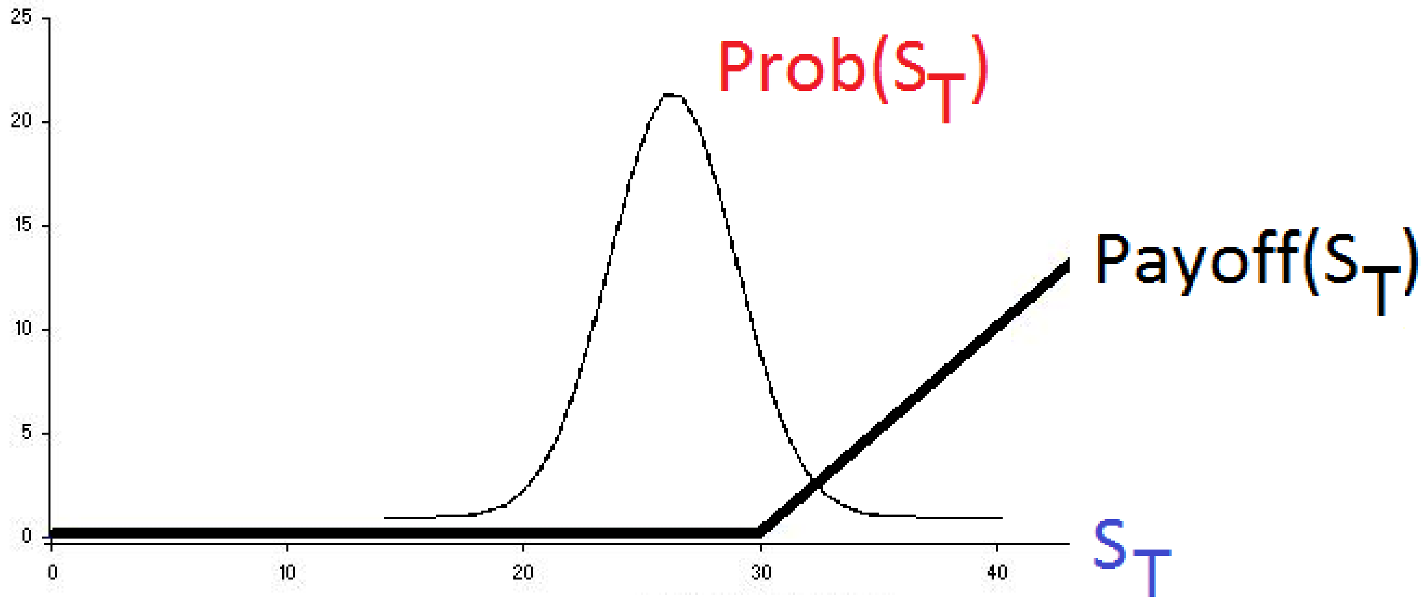
3. Early exercising an American **put** can be optimal whether the stock pays a dividend or not
   - Buying a put is like selling the stock but not receiving the proceeds (=K) until maturity.
     - Exercising early accelerates the repayment of the loan, and can be optimal.

   - Example: Suppose the firm goes bankrupt, so $S_t = 0$. You then want to exercise immediately because the stock price cannot go any lower.
     - There is no point in waiting: get K now (instead of at T).

How NOT to price options

- Expected value pricing:

$$Price \;=\; E[S_T] = \int Prob \cdot S_T \, dS_T$$



- Why? Because the market risk premium is not built into the option price
  – We need to infer market probabilities and risk premia from stock valuations and transfer that into option pricing

A no-arbitrage pricing idea

- An option can be ‘dynamically replicated’ using stocks and bonds.
  – If the payoffs from the option and the ‘dynamic portfolio’ of stocks and bonds are equal in all future states of the world:
  – Then, by no-arbitrage, the price of the option must equal the price of the stock and bond portfolio

Black-Scholes (1973)

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$
$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

N( ) is the cumulative distribution function of the standard normal distribution
T-t is time to maturity
S is the spot price
K is the strike price
r is the risk-free interest rate
σ is the (future) volatility of returns of the underlying asset
which is both **unknown** and **assumed to be constant across time and strikes**

Black-Scholes

- Notice that the probabilities of the stock moving up or down are not used
  - This is a general fact about no arbitrage pricing
- Black-Scholes formula is a function of **volatility**
  - If volatility is efficiently priced, then derivatives are a way of transferring risk
    - From those that don't want → to those that do (in exchange for compensation)
  - If you believe that the underlying stock is efficiently priced, buying an option is equivalent to taking a bet on volatility

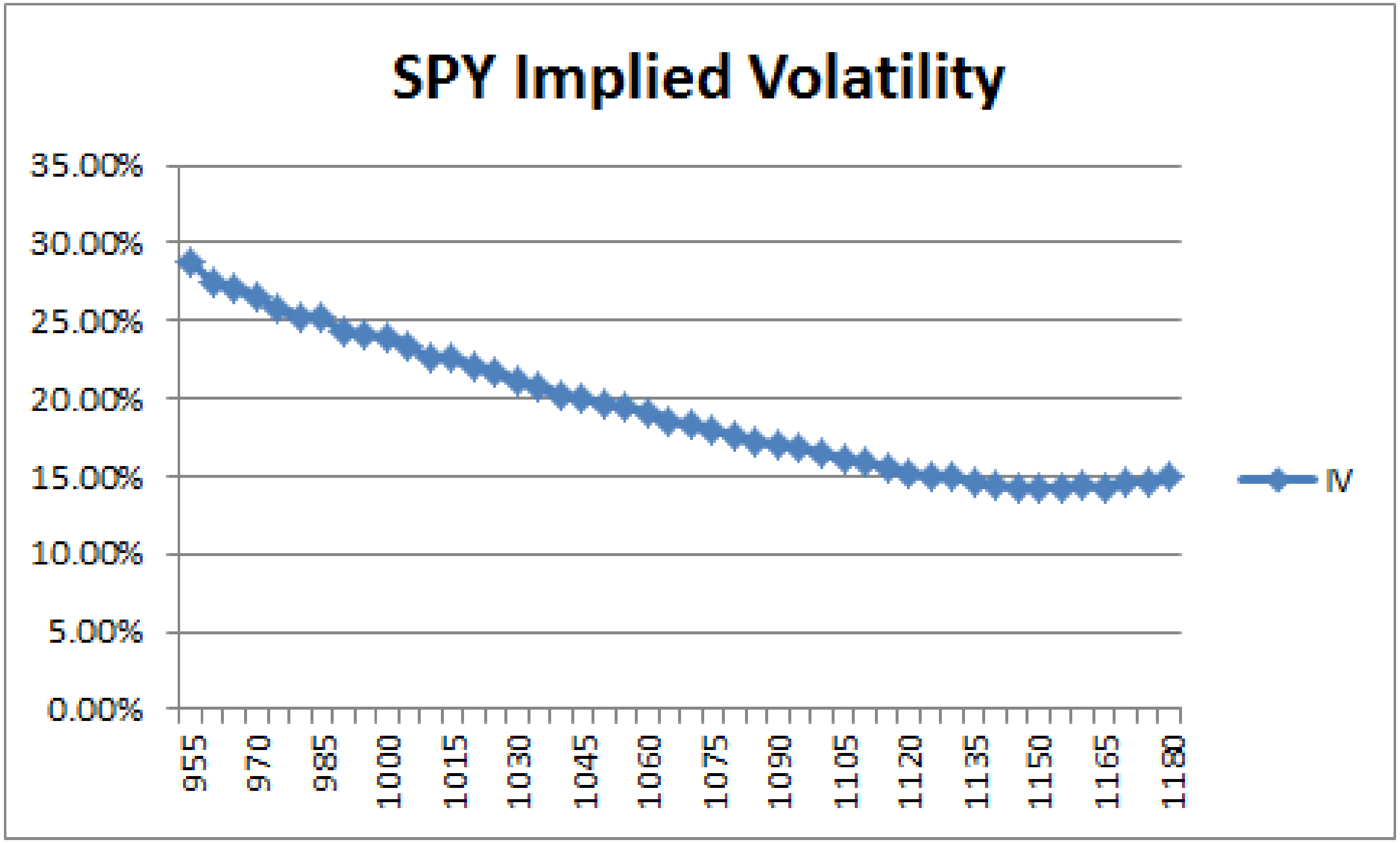
Implied volatility

- A measure of expected future market volatility
  – Use the B-S formula in reverse: take options prices as given & back out implied volatility

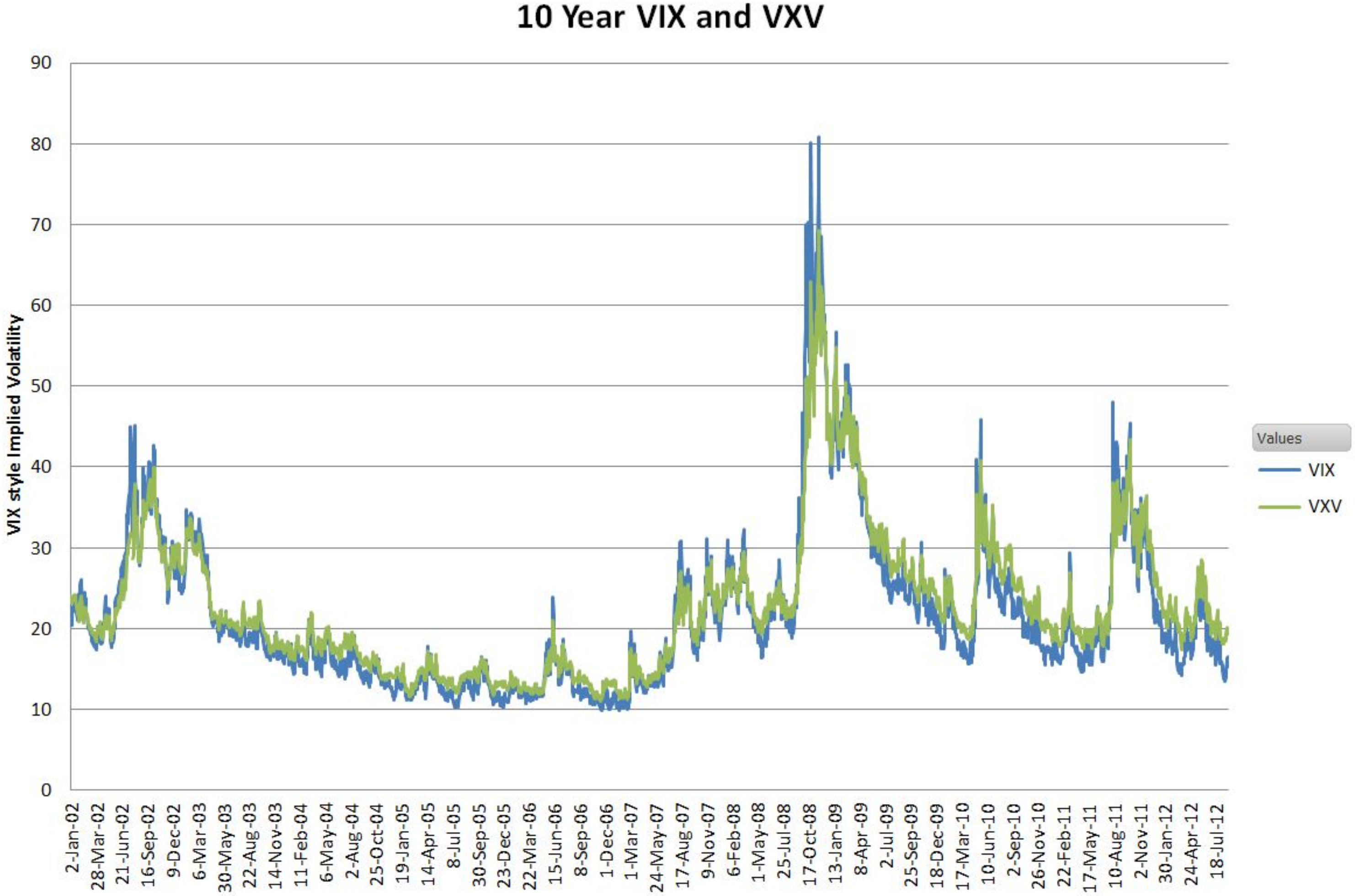
- Implied volatility is not constant across strikes (K)
  – Higher at more extreme strikes (the “volatility smile”),
  – Suggesting that tail risk is priced differently from normal volatility
  – Or that assumption of Normally Distributed stock returns used in B-S formula is not accurate

- Implied volatility is generally higher than future realized volatility, suggesting that investors get a premium for bearing volatility risk

Volatility smile

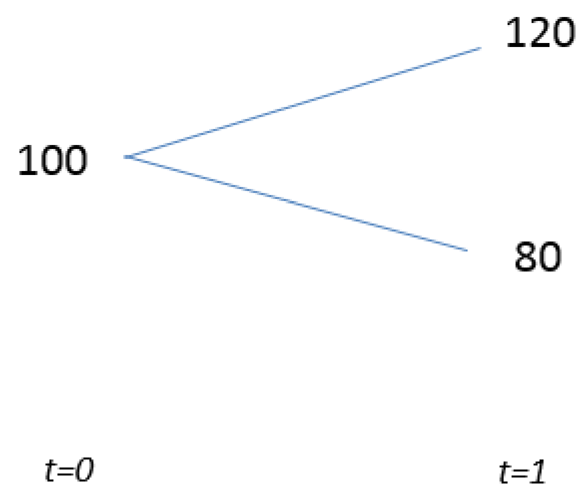


VIX


10 Year VIX and VXV
VIX style Implied Volatility
0
10
20
30
40
50
60
70
80
90
2-Jan-02
28-Mar-02
21-Jun-02
16-Sep-02
9-Dec-02
6-Mar-03
30-May-03
22-Aug-03
14-Nov-03
11-Feb-04
6-May-04
2-Aug-04
25-Oct-04
19-Jan-05
14-Apr-05
8-Jul-05
30-Sep-05
23-Dec-05
22-Mar-06
15-Jun-06
8-Sep-06
1-Dec-06
1-Mar-07
24-May-07
17-Aug-07
9-Nov-07
6-Feb-08
1-May-08
25-Jul-08
17-Oct-08
13-Jan-09
8-Apr-09
2-Jul-09
25-Sep-09
18-Dec-09
17-Mar-10
10-Jun-10
2-Sep-10
26-Nov-10
22-Feb-11
17-May-11
10-Aug-11
2-Nov-11
30-Jan-12
24-Apr-12
18-Jul-12
Values
VIX
VXV

OPTIONAL MATERIAL ON OPTIONS (WILL NOT BE TESTED ON EXAMS)

A binomial options example



- Assume r = 0.10.
- What is the price of a **European put** option on this stock (exercising at t=2) with strike price K=104?

A binomial options example

For no-arbitrage price, we want to build a 'replicating' stock/bond portfolio at t=0 that has the same payoff as the option in t=1 in both the 'up' and 'down' state

- Then the price of the option at t=0 would have to equal the price of the stock/bond portfolio at t=0
- Let x = number of stock shares, y = bonds
  - Note that bonds cost $1 at t=0, payoff $(1+r) at t=1

<u>Step 0: Calculate the option payoff in each state at t=1</u>

- Payoff of put = max(0, K – S_T), which is max(0,104-120) = 0 in the 'up' state and max(0,104-80)=24 in the 'down' state

<u>Step 1: Replicate the payoffs at t=1</u>

- Setting the payoffs at t=1 of the stock/bond portfolio equal to the payoff of the option
  - 'Up' State: 120x + 1.1y = 0
  - 'Down' State: 80x + 1.1y = 24
- Solve for x and y: replicating portfolio needs x = -3/5 shares of stock and y = 720/11 dollars in bonds

A binomial options example

Step 2: Calculate the price at t=0

- From Step 1, we found the replicating portfolio needs x = -3/5 shares of stock and y = 720/11 dollars in bonds
- The price of the option at t=0 must equal the price of the replicating portfolio of stocks/bonds at t=0

$$
\begin{aligned}
\text{Price of put } P_t(S_t) &= \text{price of stock/bond portfolio} \\
&= 100\,x + y \\
&= 100\,(-3/5) + 720/11 \\
&= 5.45
\end{aligned}
$$

A binomial pricing formula

$$C = \frac{qC_u + (1-q)C_d}{1+r}$$

- Where $q = \frac{(1+r)-d}{u-d}$ is called the 'risk-neutral probability'
- u, d are the stock prices in the up and down states
- C_u and C_d are the payoffs of the option in the up/down states

- The **probabilities** of the stock moving up or down are NOT used in the final formula
  – This is a general fact about no arbitrage pricing