THE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH2241 Introduction to Mathematical Analysis

Tutorial 2

1. Use the definition of convergence of a sequence to prove that

$$\lim_{n\to\infty} \frac{n^2+n+1}{3n^2+2} = \frac{1}{3}.$$

2. (a) Prove that if $\{a_n\}$ is a sequence converging to L, then the sequence $\{|a_n|\}$ of absolute values converges to $|L|$.

   (b) Suppose now that the sequence $\{|a_n|\}$ of absolute values converges to $|L|$. Is it necessarily that $\{a_n\}$ converges to L?
   (*Hint*: Consider separately the cases $L = 0$ and $L \neq 0$.)

3. (a) Let $\{a_n\}$ and $\{b_n\}$ be sequences. Prove that if $\lim_{n\to\infty} a_n = 0$ and if $\{b_n\}$ is bounded, then $\lim_{n\to\infty} a_n b_n = 0$.

   (b) Suppose now that $\lim_{n\to\infty} a_n = L \neq 0$ and that $\{b_n\}$ is bounded. What can we conclude about the convergence of the sequence $\{a_n b_n\}$?

4. Give an example of each of the following, or prove that such a situation is impossible:

   (a) sequences $\{a_n\}$ and $\{b_n\}$, where $\{a_n\}$ converges, $\{b_n\}$ unbounded and $\{a_n b_n\}$ converges.

   (b) sequences $\{a_n\}$ and $\{b_n\}$, where $\{a_n\}$ unbounded, $\{b_n\}$ converges and $\{a_n - b_n\}$ bounded.

   (c) sequences $\{a_n\}$ and $\{b_n\}$, where $\{a_n\}$ converges, $\{b_n\}$ diverges and $\{a_n + b_n\}$ converges.

   (d) sequences $\{a_n\}$ and $\{b_n\}$, which both diverge, but $\{a_n + b_n\}$, $\{a_n b_n\}$ and $\{a_n/b_n\}$ all converge.

5. Use the Sandwich Theorem to prove that

$$\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n}) = 0.$$