Homework 1

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Question 1. Explain for each procedure why it works or does not work. If the procedure does not work, provide an example input on which it fails; otherwise, give a correctness proof.

- Algorithm 0
  Claim: This algorithm is correct.
  Loop invariant: For all $n > 2$ we have $F_0(n-m) = i; F_0(n-m+1) = j; F_0(n-m+2) = k$

  Base case: For inputs of 0, 1, 2, the outputs are 0, 1 and 1, respectively. So this is correct.
  Induction Hypothesis: $F_0(l)$ is correct for all values of $l <= n$, where $n, l \in \mathbb{N}$
  Inductive Step:
  let $F_0(3l)$ is correct for all values until n, where l is the number of iterations.
  So $i_l = F_0(3l), j_l = F_0(3l+1), k_l = F_0(3l+2)$
  then, for $F_0(3(l+1))$, we have:
  $i_{l+1} = j_l + k_l = F(3(l+1))$,
  $j_{l+1} = i_{l+1} + k_l = F(3(l+1)+1)$ and
  $k_{l+1} = i_{l+1} + j_{l+1} = F(3(l+1)+2)$.
  It holds true for the Fibonacci formula.
  Also, if the actual input n is $n \, mod \, 3 = 0$, the m will be 0 eventually and will return i_l, then n+1 will return $i_{l+1} = j_l$, which satisifies the Fibonacci formula. Likewise we could have correct answer on the situations that equals 1 and 2.
  Soundness: for each valid input, the program will return correct value.
  Termination: the program will terminate after m¡3, and with every valid input n¿0, it will definitely terminate because the m is deducted by 3 every loop. It will not cause an infinte loop.

- Algorithm 1
  Claim: This algorithm is incorrect.
  Counterexample: When n = 3, the output is 3.
  However, the result should be 2.
  Hence, the algorithm is not correct.

- Algorithm 2
  Claim: This algorithm is incorrect.
  Counterexample: When n = 3, the output is 1.
  However, the result should be 2.
  Hence, the algorithm is not correct.

- Algorithm 3
  Claim: This algorithm is incorrect.
  Counterexample: When n = 3, the output is 1.
  However, the result should be 2.
  Hence, the algorithm is not correct.