# Formalising Mathematics

Project 1

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## Introduction

For the first project in *Formalising Mathematics*, I decided to formalise a result from measure theory known as Egorov's theorem. Egorov's theorem is a useful result establishing a relation between convergence almost everywhere and uniform convergence. In particular, Egorov's theorem states that a sequence of almost everywhere convergent functions converge uniformly everywhere except on an arbitrarily small set.
Egorov's theorem is used to prove the Vitali convergence theorem (a generalisation of the monotone convergence theorem for uniformly integrable functions) and is also useful to simplify more elementary results such as convergence almost everywhere implies convergence in measure.

## Formalisation of Egorov's Theorem

We will in this section outline the proof of Egorov's theorem and comment on the formalisation effort. For the remainder of this document, let us assume $\alpha$ is a measure space with measure $\mu$ and $\beta$ is a second-countable metric space.

**Theorem 1** (Egorov's Theorem)**.** If $(f_n : \alpha \to \beta)_{n=0}^{\infty}$ is a sequence of measurable functions which converge almost everywhere on a set $s \subseteq \alpha$ of finite measure to $g : \alpha \to \beta$, then, for any $\epsilon > 0$, there exists some $t \subseteq s$ with measure $\mu(t) \leq \epsilon$ and $f_n$ converges uniformly to $g$ on $s \setminus t$.

In Lean, this statement is represented by

```
lemma tendsto_uniformly_on_of_ae_tendsto' [is_finite_measure μ]
(hf :   n, measurable (f n)) (hg : measurable g)
(hfg :   x μ, tendsto ( n, f n x) at_top ( (g x))) { : } (h : 0 <  ) :
 t, measurable_set t μ  t  ennreal.of_real   tendsto_uniformly_on f g at_top  t :=
```