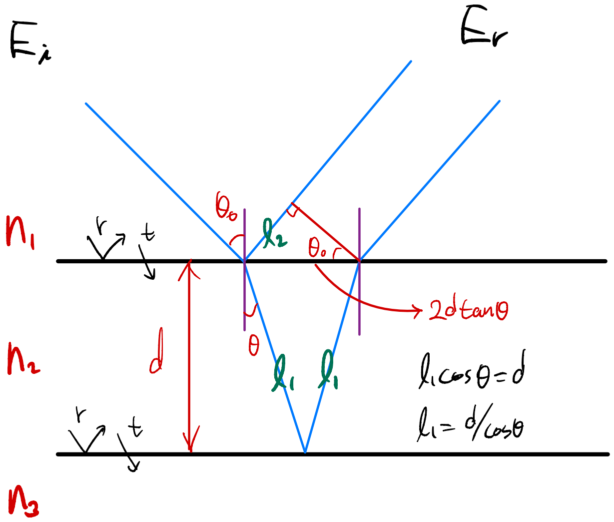
Thin film reflection

Ref. Fujiwara p.43



$l_1\cos\theta = d$

$l_1 = d/\cos\theta$

$l_2 = 2d\tan\theta\cdot\sin\theta_0$

$= 2d\frac{\sin\theta}{\cos\theta}\cdot\sin\theta_0$

$(n_1\sin\theta_0 = n_2\sin\theta)$

$= 2d\frac{\sin\theta}{\cos\theta}\cdot\frac{n_2}{n_1}\sin\theta$

$= 2d\frac{\sin^2\theta}{\cos\theta}\cdot\frac{n_2}{n_1}$

$\langle E = E_0 e^{i(\omega t - kx + \delta)} \rangle$

- phase diff in $n_2 \rightarrow \frac{2\pi}{\lambda}\cdot n_2\cdot 2l_1 = \frac{2\pi}{\lambda}\cdot n_2\cdot\frac{2d}{\cos\theta}$
  $(k\cdot x = k\cdot n\cdot x)$
- phase diff in $n_1 \rightarrow \frac{2\pi}{\lambda}\cdot n_1\, l_2 = \frac{2\pi}{\lambda}\cdot\cancel{n_1}\cdot 2d\,\frac{\sin^2\theta}{\cos\theta}\,\frac{n_2}{\cancel{n_1}}$
  - sum $= \frac{4\pi d n_2}{\lambda}\left(\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}\right) = \frac{4\pi d n_2}{\lambda}\cdot\cos\theta \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$$r_{123} = E_r/E_i = r_{12} + t_{12}\cdot r_{23}\cdot t_{21}\cdot e^{i\alpha} + t_{12}r_{23}r_{21}r_{23}t_{21}e^{i2\alpha} + \cdots$$

$$= r_{12} + t_{12}t_{21}\cdot r_{23}e^{i\alpha} + t_{12}t_{21}r_{23}\,r_{21}r_{23}e^{i2\alpha} + \cdots$$

$a_0 = t_{12}t_{21}r_{23}e^{i\alpha} \quad r = r_{21}r_{23}e^{i\alpha}$

$$\sum_{n=1}^{\infty} ar^n = \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}\ (0<r<1)$$

$$r_{123} = r_{12} + \frac{t_{12}t_{21}r_{23}e^{i\alpha}}{1 - r_{21}r_{23}e^{i\alpha}} \quad (\text{def. } \beta = \alpha/2)$$

$$\boxed{r_{123} = r_{12} + \frac{t_{12}t_{21}r_{23}e^{-i2\beta}}{1 - r_{21}r_{23}e^{-i2\beta}}}$$

$$\beta = \frac{2\pi d n_2}{\lambda}\cos\theta$$

Stokes relations $r_{12} = -r_{21}$

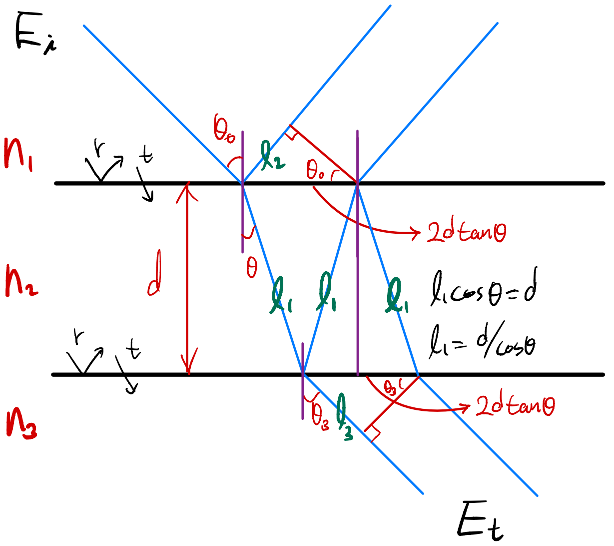
$t_{12}\cdot t_{21} = 1 - r_{12}^2$

$$\frac{r_{12} - r_{12}r_{21}r_{23}e^{-i2\beta} + t_{12}t_{21}r_{23}e^{-i2\beta}}{1 - r_{21}r_{23}e^{-i2\beta}}$$

$$\frac{r_{12} - \underline{r_{12}r_{21}}^{\,r_{12}^2}r_{23}e^{-i2\beta} + (1-r_{12}^2)\,r_{23}e^{-i2\beta}}{1 - r_{21}r_{23}e^{-i2\beta}}$$

$$\boxed{r_{123} = \frac{r_{12} + r_{23}e^{-i2\beta}}{1 + r_{12}r_{23}e^{-i2\beta}}}$$

Thin film transmission



$\sin\theta_3 = \frac{l_3}{\bigcirc}$

$l_3 = 2d\tan\theta \cdot \sin\theta_3$ $\quad(n_2\sin\theta = n_3\sin\theta_3)$

$= 2d\,\frac{\sin\theta}{\cos\theta}\cdot\sin\theta\,\frac{n_2}{n_3}$

$= 2d\,\frac{\sin^2\theta}{\cos\theta}\,\frac{n_2}{n_3}$

$\langle E = E_0 e^{i(\omega t - kx + \delta)} \rangle$

- phase diff in $n_2 \rightarrow \frac{2\pi}{\lambda}\cdot n_2 \cdot 2l_1 = \frac{2\pi}{\lambda}\cdot n_2\,\frac{2d}{\cos\theta}$
  $(k\cdot x = k\cdot n\cdot x)$
- phase diff in $n_3 \rightarrow \frac{2\pi}{\lambda}\cdot n_3\, l_3 = \frac{2\pi}{\lambda}\cdot \cancel{n_3}\cdot 2d\,\frac{\sin^2\theta}{\cos\theta}\,\frac{n_2}{\cancel{n_3}}$
  - sum $= \frac{4\pi d n_2}{\lambda}\left(\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}\right) = \underbrace{\frac{4\pi d n_2}{\lambda}\cdot\cos\theta}_{\text{same}}$ $\quad(\because \sin^2\theta + \cos^2\theta = 1)$

$e^{i\beta}$ ~ No need. But, to match.

$$t_{123} = E_t/E_i = t_{12}t_{23} + \underline{t_{12}r_{23}r_{21}t_{23}e^{i\alpha}}^{+\beta} + \underline{t_{12}r_{23}r_{21}}\,r_{23}r_{21}\,\underline{t_{23}e^{i2\alpha}}^{+\beta} + \cdots$$

$a_0 = t_{12}r_{23}r_{21}t_{23}e^{i\alpha} \quad r = r_{23}r_{21}e^{\alpha}$

$\sum_{n=1}^{\infty} ar^n = \frac{a(1-r^n)}{1-r}$

$= \frac{a}{1-r} \quad (0<r<1)$

$$t_{123} = t_{12}t_{23}\overset{e^{i\beta}}{} + \frac{t_{12}r_{23}r_{21}t_{23}e^{i\alpha}}{1-r_{23}r_{21}e^{i\alpha}} = \frac{t_{12}t_{23} - \cancel{t_{12}t_{23}r_{23}r_{21}e^{i\alpha}} + \cancel{t_{12}t_{23}r_{23}r_{21}e^{i\alpha}}}{1-r_{23}r_{21}e^{i\alpha}}$$

$$= \frac{t_{12}t_{23}e^{-i\beta}}{1-r_{23}r_{21}e^{-i2\beta}} \qquad (\text{def. } \beta = \alpha/2)$$

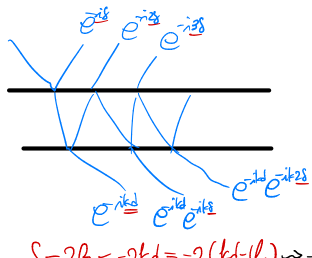
Etalon

$n_1 = n_3$

$\delta = 2\beta = \frac{4\pi n d}{\lambda}\cos\theta$

Eq 5 $\quad r_{123} = \frac{r_{12} + r_{23}e^{-i2\beta}}{1 + r_{12}r_{23}e^{-i2\beta}} = \frac{r_{12} + r_{23}e^{-i\delta}}{1 + r_{12}r_{23}e^{-i\delta}} \qquad \mathbb{r} = \frac{r_{12} + r_{21}e^{-i\delta}}{1 + r_{12}r_{21}e^{-i\delta}} = \frac{\sqrt{R}(e^{-i\delta}-1)}{1-Re^{-i\delta}}$

Eq 7 $\quad t_{123} = \frac{t_{12}t_{23}e^{-i\beta}}{1 - r_{12}r_{23}e^{-i2\beta}} = \frac{t_{12}t_{23}e^{-ikd}}{1 - r_{12}r_{23}e^{-i\delta}} \qquad \tau = \frac{t_{12}t_{21}e^{-ikd}}{1 - r_{12}r_{21}e^{-i\delta}} = \frac{(1-R)e^{-ikd}}{1-Re^{-i\delta}}$



$\delta = 2\beta = -2kd = -2(kd - \varphi_r) \rightsquigarrow -2kd$

Eq 6 $\rightarrow \delta = 2\varphi_r - 2kd = 2(\varphi_r - kd)$

$E = E_0 e^{-i(kx - \omega t + \varphi_r)}$

Eq 9.10 $\quad \delta = -2\cdot n\cdot\frac{2\pi\nu}{c}\cdot d = -\frac{4\pi n d}{c}\cdot\nu = -\frac{2nd}{c}\omega$

c: velocity; ν: freq. $= f$ (Hz); ω: Ω in book

$\omega = 2\pi f$ (rad/s) — angular freq.

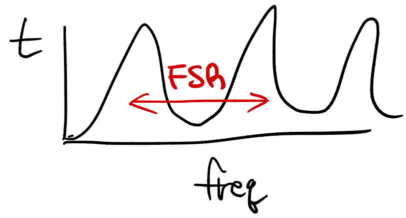
$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{2\pi\nu}{c} = \frac{\omega}{c}$

$(c = f\cdot\lambda)$

when $\theta = 0°$, $\delta = 2kd = \frac{4\pi n d}{\lambda} = 4\pi n d\cdot\nu$

when $\theta \neq 0°$, $\delta = \frac{4\pi d n}{\lambda}\cos\theta$

* $n = n(\Omega)$, also $\delta(\Omega)$ or $\delta(\nu) \to t(\Omega)$ or $t(\nu) \to$ freq dependent.



Free spectral range (FSR)

: spacing in frequency between two consecutive transmission maxima.

happens when phase diff (δ) is 2π.

Eq 9.10 $$\delta = -\frac{4\pi n d}{c}\cdot \underset{\text{Hz}}{\nu} = -\frac{2nd}{c}\cdot \underset{\text{rad/s}}{\omega}^{=\Omega}$$

$$2\pi = \frac{2nd\,\Delta\Omega_{fsr}}{c}$$

✓ $$\Delta\Omega_{fsr} = \frac{\pi c}{nd}$$

✓ $$\Delta\nu_{fsr} = \frac{c}{2nd}\,(\text{Hz}) = \frac{1}{2nd}\,(\text{cm}^{-1}) = \frac{\lambda^2}{2nd}\,(\text{nm})$$

$$T = |t|^2 = \left|\frac{(1-R)e^{-ikd}}{1-Re^{-i\delta}}\right|^2 = \frac{(1-R)^2}{|1-Re^{-i\delta}|^2} \quad (\because |e^{-ikd}|^2 = 1)$$

$z = |z| e^{i\theta}$ — magnitude; phase factor does not affect magnitude

$$= \frac{(1-R)^2}{(1-Re^{-i\delta})(1-Re^{i\delta})} \quad (\because \; |z|^2 = z\cdot z^* \quad z = 1-Re^{-i\delta},\; z^* = 1-Re^{i\delta})$$

$$= \frac{(1-R)^2}{1-Re^{-i\delta}-Re^{i\delta}+R^2} = \frac{(1-R)^2}{1-2R\cos\delta+R^2}$$

$$\left(\because \; e^{i\theta} = \cos\theta + i\sin\theta,\quad e^{-i\theta} = \cos\theta - i\sin\theta\right)$$

Near the peaks (maxima), δ (phase diff) ~ 0.

$\cos\delta \approx 1 - \frac{\delta^2}{2}$ (Taylor series, when δ is small)

$$T \approx \frac{(1-R)^2}{1-2R\cdot\left(1-\frac{\delta^2}{2}\right)+R^2} = \frac{(1-R)^2}{1-2R+R\delta^2+R^2} = \frac{(1-R)^2}{(1-R)^2+R\delta^2} = \frac{1}{1+\frac{R}{(1-R)^2}\delta^2}$$

반각공식. $\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2} \to \cos x = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$$\boxed{T = \frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\left(\frac{\delta}{2}\right)}}$$

$$\delta = 2\cdot\frac{2\pi}{\lambda}\cdot nd\cos\theta$$

→ angle inside (?)

$$\boxed{FSR = \frac{c}{2nd}\,(\text{Hz}) = \frac{\lambda^2}{2nd}\,(\text{nm})}$$

FWHM is when $T=\frac{1}{2}$.

$$\frac{1}{2}=\frac{1}{1+\frac{\cancel{4}R}{(1-R)^2}\cdot\frac{\delta^2}{\cancel{4}}}$$

$$1=\frac{R\cdot\delta^2}{(1-R)^2}$$

$$\delta^2=\frac{(1-R)^2}{R}$$

$$\Delta\delta=\delta_{FWHM}=\frac{2(1-R)}{\sqrt{R}}$$

$$\Delta\nu\cdot\frac{\cancel{4}\pi nd}{c}=\frac{\cancel{2}(1-R)}{\sqrt{R}}$$

$$\Delta\nu=\nu_{FWHM}=\frac{c(1-R)}{2\pi nd\sqrt{R}}\ (Hz)$$

$$\boxed{FWHM=\frac{c(1-R)}{2\pi nd\sqrt{R}}}$$

$$Finesse=\frac{FSR}{FWHM}=\frac{\frac{\cancel{c}}{\cancel{2nd}}}{\frac{\cancel{c}(1-R)}{\cancel{2}\pi \cancel{nd}\sqrt{R}}}=\frac{\pi\sqrt{R}}{1-R}$$

$$\boxed{Finesse=\frac{\pi\sqrt{R}}{1-R}}$$

$d\cos\theta$

$$\delta=2\cdot kd=2\cdot\frac{2\pi}{\lambda}\cdot n\cdot d$$

$c=\nu\lambda$

$2\pi f=\omega$

$$=\frac{4\pi nd}{\lambda}=\frac{4\pi nd}{c}\cdot\nu=\frac{2nd}{c}\omega$$

$$\Delta\delta=\Delta\nu\cdot\frac{4\pi nd}{c}$$