CS 510 **Homework 2a** **Nov. 26, 2017**

Due in class (PROBABLY) on Dec. 4, 2017. The "2a" refers to the current version. I may change or correct or add to this HW and will proceed through the alphabet with each such modification.

You can "discuss" questions with others but you should write up your own answers completely independent of others.

- Show ALL your work and explain what you are doing, so it is clear.
- Cite sources if you have used books or the internet. Also name the people with whom you have discussed solutions.
- A (*) denotes an optional, harder question that you are not required to solve, but could receive a little extra credit if you do.
- When you do submit this homework, please write "These solutions are completely my own work" and then sign your name. If we DO encounter clear evidence of copying, little credit will be given, in view of the pledge.

1. (Careful operation counting) Let A denote a real matrix with n rows and columns. We will do the QR factorization algorithm on A to reduce it to the product of matrix Q (with n mutually orthogonal columns whose pairwise inner products are zero) and matrix R which is *UNIT* upper triangular. A handout was posted for this algorithm (temporarily removed, but will return).
   (a) Carefully compute the number of multiplication and division operations you might be forced to perform during this procedure. (So when you compute the squared length of the vector $\underline{x} = (x_1, \dots, x_n)$ as $x_1^2 + ... + x_n^2$ you should account for n (*) product computations, even though it is possible that some x_i's *might* be zero.)

2. Consider the matrix
$$A = \begin{pmatrix} 3 & 4 \\ 4 & 3 \end{pmatrix}.$$
   (a) Find the eigenvalues and unit eigervectors of A.
   (b) Now do two steps of the power method starting with $\underline{x}^T = (1, 0)$. Carefully describe what you think would happen as you continued to iterate, and why.
   (c) With A as above, take $\underline{b}^T = (-1, 1)$ and do two steps of Jacobi's FPI ($\underline{x}_{k+1} \leftarrow M^{JAC}\underline{x}_k + \underline{c}^{JAC}$) starting from $\underline{x}_0^T = (-1, 1)$. Show how you got the matrix M^{JAC} and vector $\underline{c}^{JAC}$ you used in this iteration.
   (d) Repeat for the Gauss-Seidel FPI, but still using M^{JAC} and $\underline{c}^{JAC}$ from above.
   (e) Now find M^{GS} and $\underline{c}^{GS}$ and use them *directly* to repeat the above Gauss-Seidel iterations.

3. For each of the following statements, state whether it is true or false. If you said TRUE, justify your response. If you said FALSE, give a counterexample. Explain your answer in any case (no justification, no credit).
   (a) Let A be an n by n real matrix with distinct, real eigenvalues $\lambda_1 > \cdots > \lambda_n > 0$. Then $B = A^2$ has eigenvalues λ_i^2, $i = 1, \dots, n$, and the *same* eigenvectors as A.

(b) (*) Jacobi iteration may converge to *different* limits depending on $\underline{x}_0$, the initial point of the iteration $\underline{x}_{k+1} \leftarrow M^{JAC}\underline{x}_k + \underline{c}^{JAC}$.

4. (*) Let f denote a differentiable function. Derive a formula for $P_2(t)$, the quadratic polynomial that interpolates f at two given collocation points x_0 and x_1, and which has the same slope at x_0 as f does [Hint: One way would be to write P_2 in standard form and impose the three conditions. Another would be to use Newtons form of the interpolating polynomial]. (If you do BOTH, I will give extra).

5. (*) As above, but we now want $P_3(t)$, the degree $= 3$ polynomial that agrees with f at x_0 and x_1, and also its derivatives agree with those of f at these points.

6. $f(x) = x^3$ on $[1, 2]$. Find $M_1(x)$ the minimax straight line approximation to f. As always, explain what you are doing.

7. (*) Using only equioscillation, find the quadratic minimax approximation to $g(x) = x^3$, $-1 \leq x \leq 1$, and explain your reasoning. Then verify that it is the degree two Tchebycheff interpolation for g (you could leave this out if we have not covered Tchebycheff interpolation yet).