Jason Lai
204995126
Dis 1A

CS 118
April 9, 2020

Homework 1

1 Problem 1

Suppose users share a 100 Mbps link. Also suppose each user requires 10 Mbps when transmitting, but each user transmits only 25% of the time.

(a) When circuit switching is used, how many users can be supported?
(b) For the remainder of the problem, suppose packet switching is used. Find the probability that a given user is transmitting.
(c) Suppose there are 100 users. Find the probability that at any given time, exactly n users are transmitting simultaneously. (Hint: Use the binomial distribution)
(d) Find the probability that there are 21 or more users transmitting simultaneously (Hint: only the formula is required).

a) When using circuit switching, 10 users can be supported on this communication link, since the 100 Mbps link must reserve/dedicate 10 Mbps per user.

b) The probability that a given user is transmitting is 0.25 or 25%, since each user transmits only 25% of the time.

c) The probability that at any given time, exactly n users are transmitting simultaneously is given by the formula:

Let X be the random variable that is the number of users who are transmitting simultaneously at a given time.

$$\mathrm{P}(\mathrm{X} = n) = \tbinom{100}{n}(0.25)^n(0.75)^{100-n}$$

d) The probability that there are 21 or more users transmitting simultaneously is given by the formula:

Let X be the random variable that is the number of users who are transmitting simultaneously at a given time.

$$\mathrm{P}(\mathrm{X} \geq 21) = \sum_{x=21}^{100} \binom{100}{x}(0.25)^x(0.75)^{100-x}$$

Work:

c)

This is the binomial distribution that gives the probability that out of 100 "trials" (users), there are exactly n successes (n users are transmitting).

$$\text{P}(\text{X} = x) = \binom{N}{x}(p)^x(1-p)^{N-x}$$

Since we have 100 users, $N = 100$; we have n successes, so $x = n$; and a single success is the probability that a user transmits, so $p = 0.25$

$$\text{P}(\text{X} = n) = \binom{100}{n}(0.25)^n(0.75)^{100-n}$$

d)

Using the binomial distribution again, we now want to know the probability that 21 or more users are simultaneously transmitting. This is the same as finding the probability that exactly n users are transmitting, but find finding the sum of the probabilities for n from 21 to 100. So, we have:

$$\text{P}(\text{X} \geq n) = \sum_{x=n}^{N} \binom{N}{x}(p)^x(1-p)^{N-x}$$

We have $N = 100$, $n = 21$, and $p = 0.25$.

$$\text{P}(\text{X} \geq 21) = \sum_{x=21}^{100} \binom{100}{x}(0.25)^x(0.75)^{100-x}$$

Jason Lai
204995126
Dis 1A

2 Problem 2

Queuing delay.

(a) Suppose N packets arrive simultaneously to a link at which no packets are currently being transmitted or queued. Each packet is of length L and the link has transmission rate R. What is the average queuing delay for N packets?
(b) Now suppose that a batch of packets arrives to the link every $\frac{LN}{R}$ seconds and each batch has N packets. What is the average queuing delay of a packet?

a) The average queuing delay for N packets is $\frac{L(N-1)}{2R}$.

b) The average queuing delay for a packet is still $\frac{L(N-1)}{2R}$.

This is because it takes $\frac{LN}{R}$ seconds for a batch to finish transmitting i.e. for the last packet in a batch to finish transmitting. That means each batch will finish transmitting by the time the next batch arrives, so each batch will arrive at the link when the queue is empty, so the average queuing delay we found for an initially empty queue and N packets in part **a)** still applies.

Homework 1

Jason Lai
204995126
Dis 1A

Work:

a)

Since there are no packets at the link initially, the first packet in the queue does not have to wait. However, the second packet must wait for the first packet to be transmitted onto the next link, so it has a queuing delay of $\frac{L}{R}$, since the length of every packet is L and the transmission rate is R. Every subsequent packet i in the queue must wait for the i-1 packets before it to finish transmitting. So, we have to sum up the queuing delay time of all packets and divide this sum by N to get the average queuing delay, therefore we have:

$$\frac{\sum_{i=1}^{N}(i-1)\frac{L}{R}}{N} => \frac{L}{NR}\sum_{i=1}^{N}(i-1)$$

We use the sum of an arithmetic series: $S = n(\frac{a_1+a_n}{2})$ to get:

$$\frac{L}{NR}(N(\frac{0+(N-1)}{2})) => \frac{L(N-1)}{2R}$$

b)

Now that batches of packets are coming into the link constantly at a rate of $\frac{LN}{R}$, we have to consider if there are any packets left in the queue at the link when a batch j arrives; that is, are there any packets from any previous batches that arrived at the link prior to batch j's arrival. To do this, we have to consider the length of time it takes for an entire batch of N packets to be fully transmitted.

In this case, since each packet has a transmission time of $\frac{L}{R}$, then for N packets in a batch, we have that it takes $N\frac{L}{R} = \frac{LN}{R}$ seconds for a batch to be completely transmitted. Since it takes $\frac{LN}{R}$ for a batch to completely transmit and its takes $\frac{LN}{R}$ for the next batch of packets to arrive from the time that the current batch arrived, we see that all batches of N packets will finish transmitting just as the next batch arrives, so the first packet of next the batch will not have to wait for any leftover packets to finish transmitting from the previous batch. This means that every batch of N packets will arrive at an empty queue at the link, so the average queuing delay for a packet across all batches will be the same as before: $\frac{L(N-1)}{2R}$.

3 Problem 3

Review the car-caravan analogy in lecture #1 slides (for Chapter 1). Assume a propagation speed of 100 km/h.

(a) Suppose the caravan (5 cars) travels 100 km, beginning in front of one tollbooth, passing through a second tollbooth, and finishing just after a third tollbooth. The distance between two tollbooths is 50 km. Each car takes 12 sec to serve. The caravan can only dispatch a tollbooth after all cars in the caravan are served. What is the end-to-end delay (from when the caravan is lined up before 1st tollbooth till the caravan is served by the 3rd tollbooth)?

(b) Repeat (a), now assuming that there are 8 cars in the caravan instead of 5.

a) The end-to-end delay for a 5-car caravan is 63 minutes.

b) The end-to-end delay for an 8-car caravan is 64.8 minutes.

Homework 1

Jason Lai
204995126
Dis 1A

Work:

a)

Propagation speed: 100 km/h
Distance between booth 1 2: 50 km
Distance between booth 2 3: 50 km
Bit transmission time (service time): 12 sec/car

Time to push entire caravan through toll booth $= 12 \times 5 = 60$ seconds $= 1$ minute
Time for last car to propagate from toll booth to toll booth $= \frac{50\text{km}}{100\text{km/h}} = 0.5$ hours $= 30$ minutes

End-to-end delay $= 1 + 30 + 1 + 30 + 1 = 63$ minutes

b)

Time to push entire caravan through toll booth $= 12 \times 8 = 96$ seconds
Time for last car to propagate from toll booth to toll booth $= \frac{50\text{km}}{100\text{km/h}} = 0.5$ hours $= 30$ minutes $= 1800$ seconds

End-to-end delay $= 96 + 1800 + 96 + 1800 + 96 = 3888$ seconds $= 64.8$ minutes

4 Problem 4

In this problem, we consider sending real-time voice from Host A to Host B over a packet-switched network (VoIP). Host A converts analog voice to a digital 64 Kbps bit stream on the fly, which means it take 1 second to create 64K bits from the analog signal. Host A then groups the bits into 56-byte packets. There is one link between Hosts A and B; its transmission rate is 2 Mbps and its propagation delay is 10 msec. As soon as Host A gathers a 56-byte packet, it sends it to Host B. As soon as Host B receives an entire packet, it converts the packet's bits to an analog signal. How much time elapses from the time the first bit of one packet is created (from the original analog signal at Host A) until the packet is received at Host B)?

The amount of time that elapses from the time the first bit of one packet is created until the packet is received at Host B is 0.017224 seconds.

Work:

64 Kbps = 64 × 1000 = 64000 bps
56-bytes = 448 bits
2 Mbps = 2 × 1000000 = 2000000 bps
10 msec = 0.01 seconds

End-to-end delay = processing delay + transmission delay + propagation delay
*No queuing delay since we're only concerned about one packet (the first one).
Processing delay for one packet = $\frac{448\text{bits}}{64000\text{bps}}$ = 0.007 seconds
Transmission delay for one packet = $\frac{448\text{bits}}{2000000\text{bps}}$ = 0.000224 seconds
Propagation delay = 0.01 seconds

End-to-end delay = 0.007 + 0.000224 + 0.01 = 0.017224 seconds

5 Problem 5

Suppose you would like to urgently deliver 50 terabytes data from Boston to Los Angeles. You have available a 2 Gbps dedicated link for date transfer. Would you prefer to transmit the data via this link or to use FedEx overnight delivery instead? Explain your choice.

I would prefer to use FedEx overnight delivery to deliver the 50 terabytes of data, since using the 2 Gbps dedicated link for data transfer would take approximately 2.31 days to transfer all 50 terabytes of data.

Homework 1

Jason Lai
204995126
Dis 1A

Work:

50 terabytes $= 50 \times 10^{12} \times 8 = 4 \times 10^{14}$ bits
4×10^{14} bits $= \frac{4 \times 10^{14}}{10^9} = 400000$ Gigabits

Time to finish transmission $= \frac{400000\text{Gigabits}}{2\text{Gbps}} = 200000$ seconds $= 2.31$ days